TOPICS IN HALO EFT

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OUTLINE

- Subtractive renormalization
- Seduced by the $\Lambda \rightarrow \infty$ limit
- Some wins for perturbative power counting
- Coulomb energies and isospin symmetry in Halo EFT

SUBTRACTIVE RENORMALIZATION

Afnan & Phillips, PRC, 2004   Yang, Elster, Phillips, PRC, 2008

- Consider the zero-energy amplitude resulting from a long-range potential v, and a contact term, C

$$T(p,0;0) = [v(p,0) + C] + \int_0^\Lambda dp'\, p'^2 [v(p,p') + C] G_0(p';0) T(p',0;0)$$

$$G_0(q;E) = \frac{1}{E - q^2/(2m_R)}$$

- Take difference of T(p,0;0) and T(0,0;0)

$$T(p,0;0) = T(0,0;0) + [v(p,0) - v(0,0)] + \int_0^\Lambda dp'\, p'^2 [v(p,p') - v(0,p')] G_0(p';0) T(p',0;0)$$

Hammer & Mehen, NPA, 2001

- Same trick suffices to compute T(p,p';0):

$$T(p,p';0) = T(0,p';0) + [v(p,p') - v(0,p')] + \int_0^\Lambda dq\, q^2 [v(p,q) - v(0,q)] G_0(q;0) T(q,p';0)$$

WHAT DO WE LEARN?

- Nothing here that can't be done in original formulation
- Avoiding computation of $C(\Lambda)$ allows $\Lambda \rightarrow \infty$ limit to be straightforwardly taken
- Off-shell behavior of zero-energy amplitude entirely determined by scattering length and differences of v
- Kernel is negative definite, therefore equation can be used, together with RG for T with Λ, to show that effects of cutoff in T are of relative order p^3

FINITE ENERGIES

- Off-shell behavior at one energy suffices to get T for all energies

$$T(E) = [v + C] + [v + C]G_0(E)T(E)$$

$$T(E) = T(0) + T(0)[G_0(E) - G_0(0)]T(E)$$

"First-resolvent method"

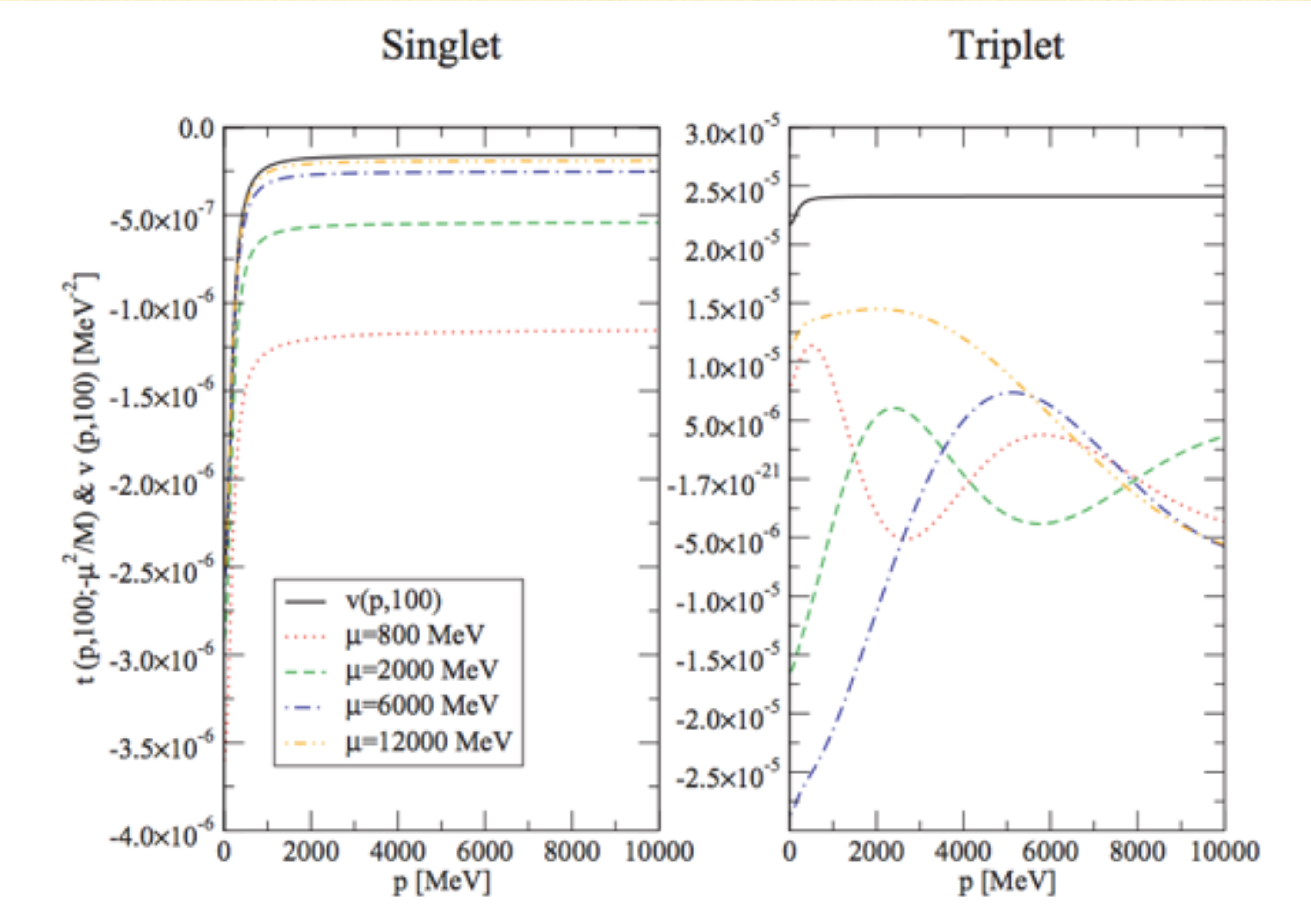
- For small E, high-momentum behavior will be as for T(0)

- Extensions:
  - Higher partial waves — Yang, Elster, Phillips, PRC, 2009
  - Energy-dependent potential — Afnan, Phillips, PRC, 2004
  - Contact terms ~ p^2 — Yang Elster, Phillips, PRC, 2009

NOT **THAT** SUBTRACTION METHOD

- Cf. Frederico et al., who perform difference with $T(-\mu^2/2m_R)$ and then assume Born approximation valid for latter

Frederico, Timoteo, Tomio, PRC, 2007



Yang, Elster, Phillips, PRC, 2007

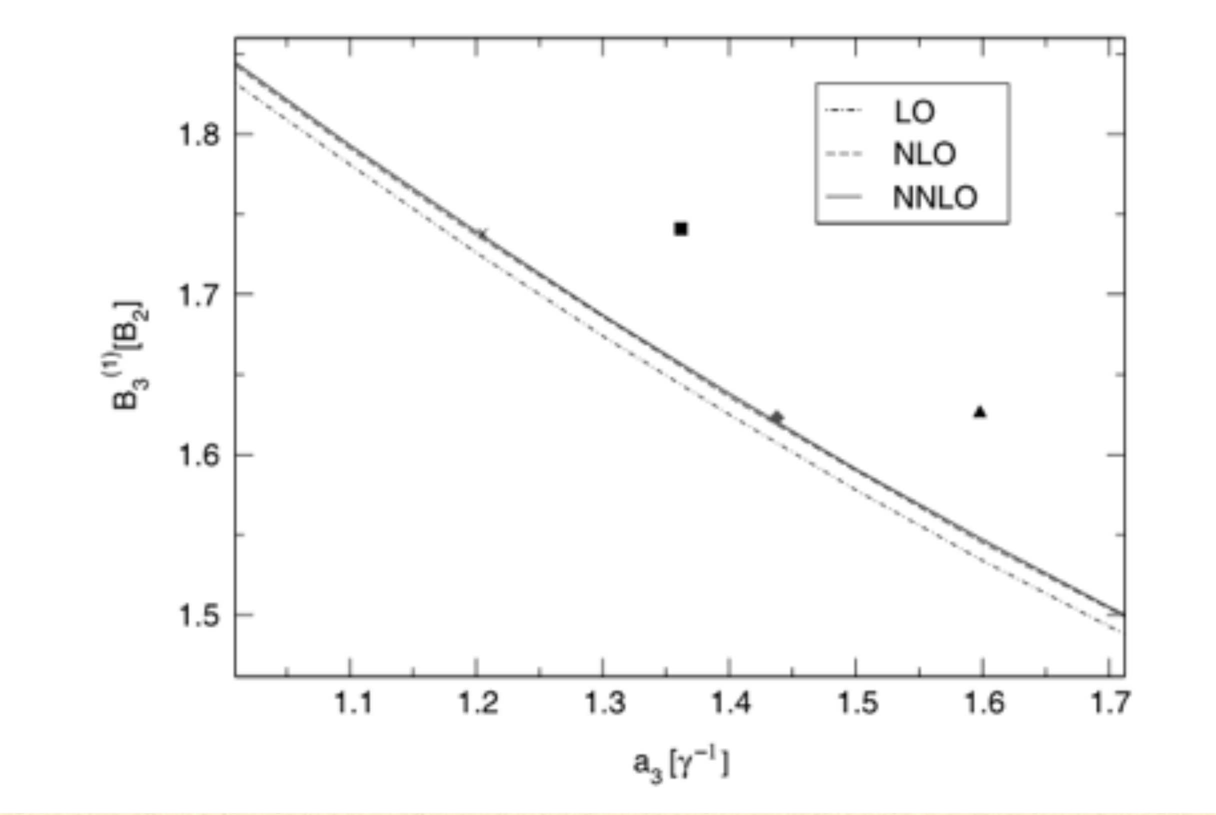
Born approximation never holds for singular potential

APPLICATION: ATOM-DIMER SCATTERING AT N^2LO

Platter, Phillips, FBS, 2006

- Employ "partial resummation": take nth-order kernel in three-body integral equation, and solve for amplitude
  - "Weinberg counting" Bedquae, Griesshammer, Hammer, Rupak, NPA, 2003

$$K^{(n)}(p,0;-B_2) = \frac{2m}{p^2+\gamma^2} + \frac{4m^2}{3}\int_0^{\Lambda} dq \frac{1}{pq} \log\left(\frac{p^2+q^2+pq+\gamma^2}{p^2+q^2-pq+\gamma^2}\right) \times S^{(n)}(-B_2;q) K^{(n)}(q,0;-B_2),$$

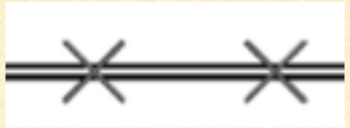


$S^{(n)} \sim q^{n+1}$ for large q

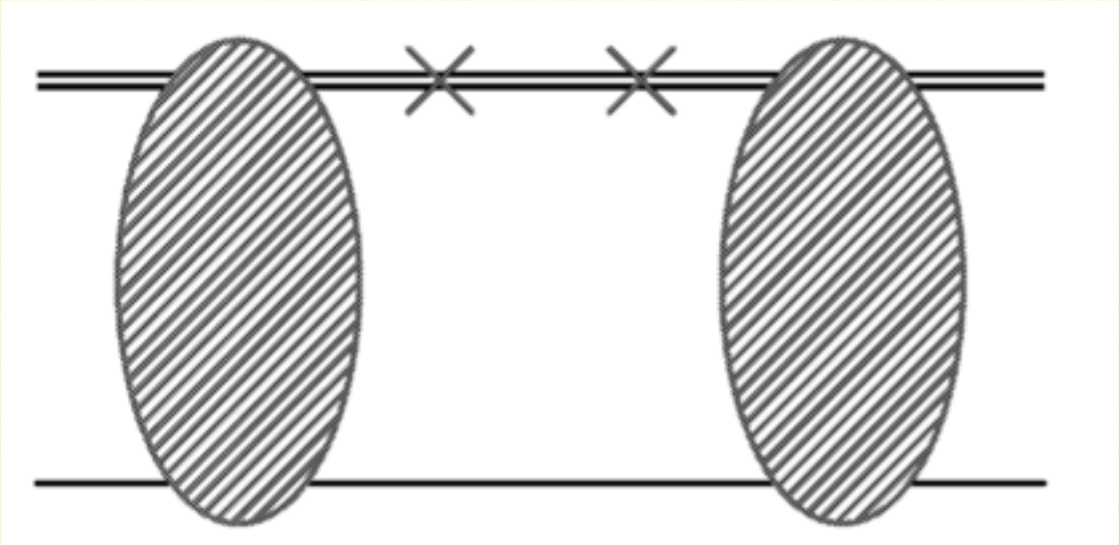
- After subtraction, can be solved numerically for $\Lambda \to \infty$
- Cutoff independent results at N^2LO

THE H_2 CONFLICT

- This suggests no additional three-body force is needed at N^2LO
- In contradistinction to the findings of BGHR



$$\sim \frac{2\pi}{m_R}\frac{1}{\gamma+ip}\left(\frac{\rho}{2}\frac{\gamma^2+p^2}{\gamma+ip}\right)^2 \sim p \text{ as } p\to\infty$$



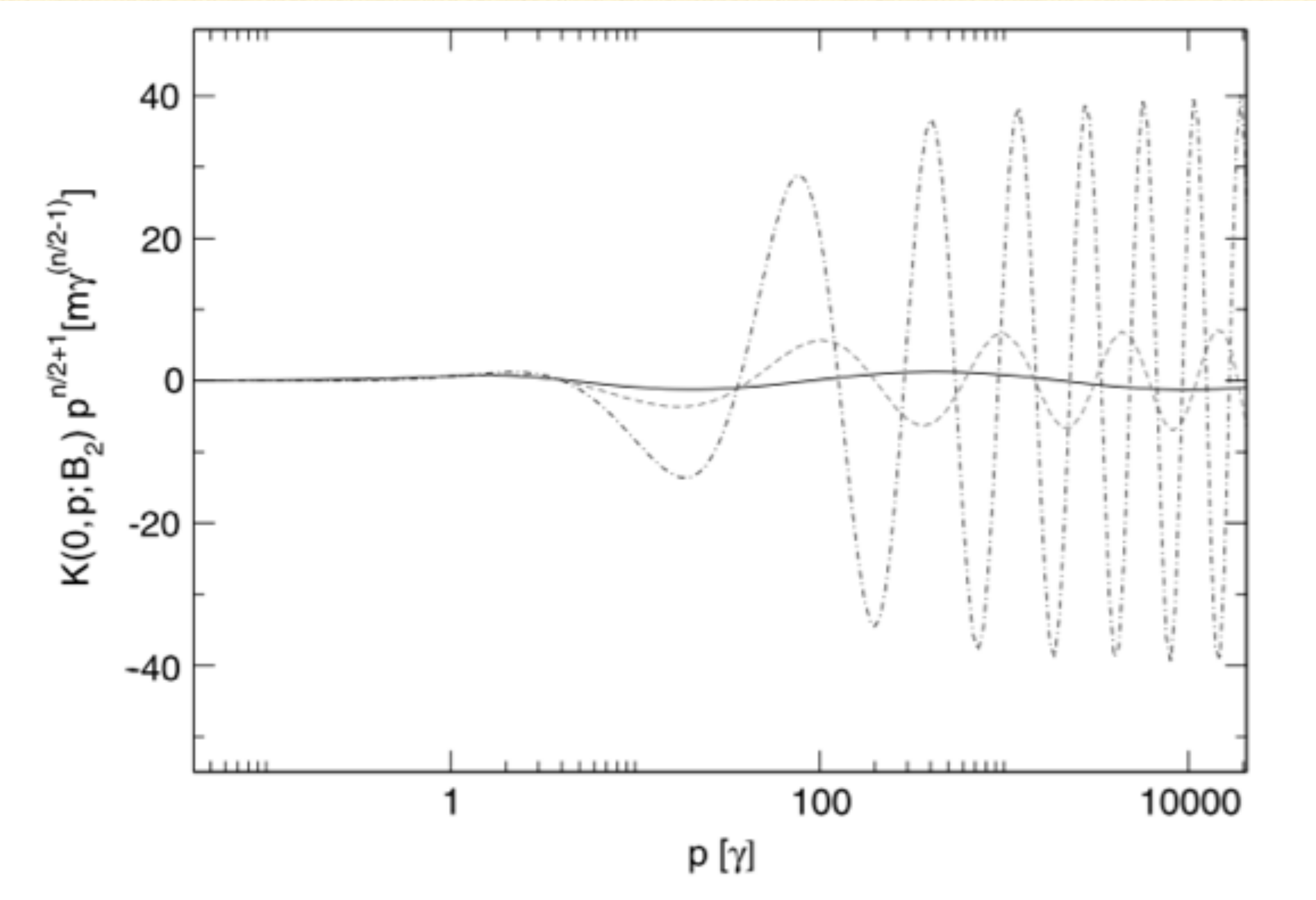
$$\sim \frac{1}{p^{1+is_0}}p\frac{1}{p^{1+is_0}}p^3$$

Anticipate $\Lambda^2 + m_R E\ln(\Lambda)$

- Buttressed by slope of Lepage plot at low cutoffs
- Confirmed in subsequent analysis

Chen, Phillips, FBS, 2013
Vnnasse, PRC, 2014

CONFLICT RESOLUTION



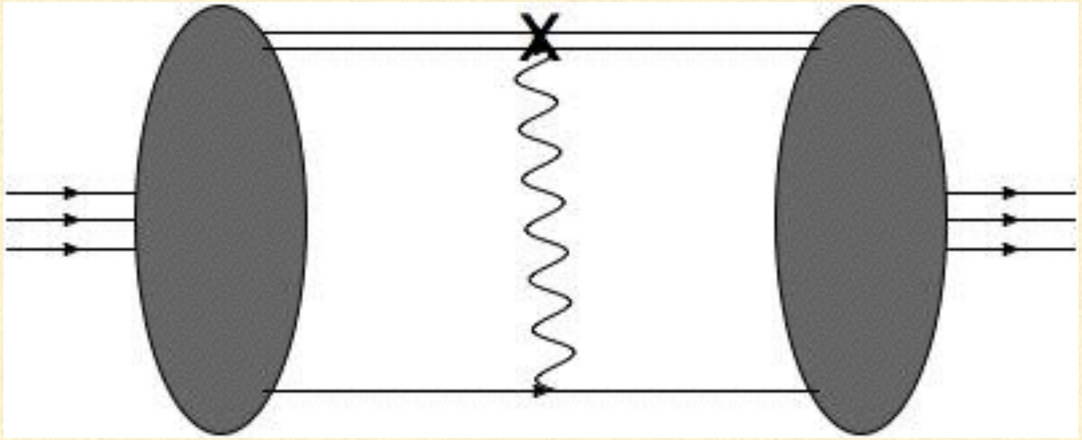
Non-perturbative treatment of kernel modifies high-p amplitude

$T^{(n)} \sim 1/p^{1+n/2}$ (Oscillatory) as $p \to \infty$

- Harder kernel⇒softer amplitude (cf. NN scattering)
- Platter-Phillips results emerge only if $|a| \gg |r| \gg \ell$

PERTURBATIVE COULOMB

- NLO graph:



$$\text{Graph} \sim \frac{1}{p^{1+is_0}} \frac{1}{p} \frac{1}{(p-p')^2} \frac{1}{p'} \frac{1}{p'^{1+is_0}} p^3 p'^3$$

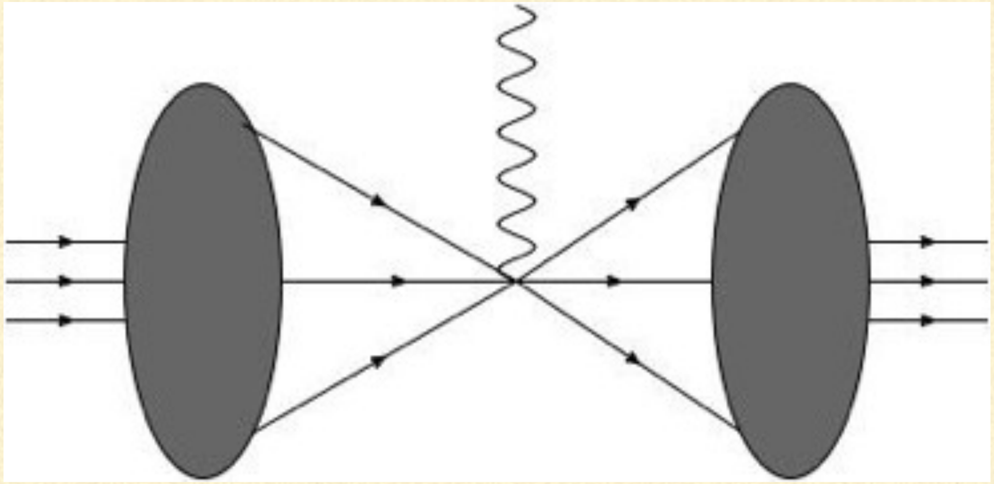
- In co-ordinate space

$$\text{Graph} \sim \int_R^\infty dr_{pd}\, r_{pd}^2 \frac{1}{r_{pd}} \frac{\alpha_{\rm em}}{r_{pd}} \frac{1}{r_{pd}} (\text{Oscillatory in } \mathrm{r_{pd}})$$

- Anticipate α_{em} ln(R) divergence at NLO

IMPLICATIONS FOR EM OPERATORS

- Short-distance contribution to tri-nucleon form factors



- Unitary limit, analog of reduced radial wave function

$$f_n(\rho) = 2\kappa_n\sqrt{\frac{\sinh(\pi s_0)}{\pi s_0}}\rho^{1/2}K_{is_0}(\sqrt{2}\kappa_n\rho)$$

$$\langle\rho^2\rangle = \kappa^2\int_R^\infty K_{is_0}(\sqrt{2}\kappa\rho)\rho^3 K_{is_0}(\sqrt{2}\kappa\rho)d\rho$$

Short-distance contribution ~ $\kappa^2 R^4$ cf. $1/\kappa^2$ at LO

Vanasse result for ^{3}H: $\langle r^2\rangle^{1/2} = 1.13 + 0.46 + 0.27 \pm 0.07$ fm

Vanasse, 2016

THE STRONG PC SCATTERING LENGTH

- 'Strong" proton-core scattering length is defined as the proton-core scattering length when the Coulomb potential is off

- In Halo EFT with PDS it is $\frac{1}{a_{pc}} = \frac{2\pi}{C_0(\mu) m_R} + \mu$

- Relationship to observable, a_C:

$$\frac{1}{a_{pc}^{MS}(\mu)} = \frac{1}{a_C} + 2k_C \left[\ln\left(\frac{\sqrt{\pi}\mu}{2k_C} \right) + 1 - \frac{3C_E}{2} \right]$$

Kong & Ravndal, 1998; Gegelia, 2001; Higa, Hammer, van Kolck, 2008; Ryberg et al., 2014

- Scheme and scale dependent

- Includes effects of photons "above μ": but dressed by strong interactions, "Coulomb-nuclear interference"

BINDING ENERGY SHIFTS DUE TO COULOMB

$$H_{\rm strong}|\psi_s\rangle = E_{\rm strong}|\psi_s\rangle;$$
$$(H_{\rm strong} + V_C)|\psi\rangle = E|\psi\rangle$$

- Coulomb energy then defined as $E-E_{strong}$
- Coulomb energy of a proton halo is scheme and scale dependent
- Thomas-Ehrman shift?
- Recent evaluation of ^{3}He-^{3}H binding-energy difference in expansion about unitary limit: B(^{3}He) - B(^{3}H)=-0.86 ± 0.17 MeV

Koenig, Griesshammer, Hammer, van Kolck, JPG, 2016

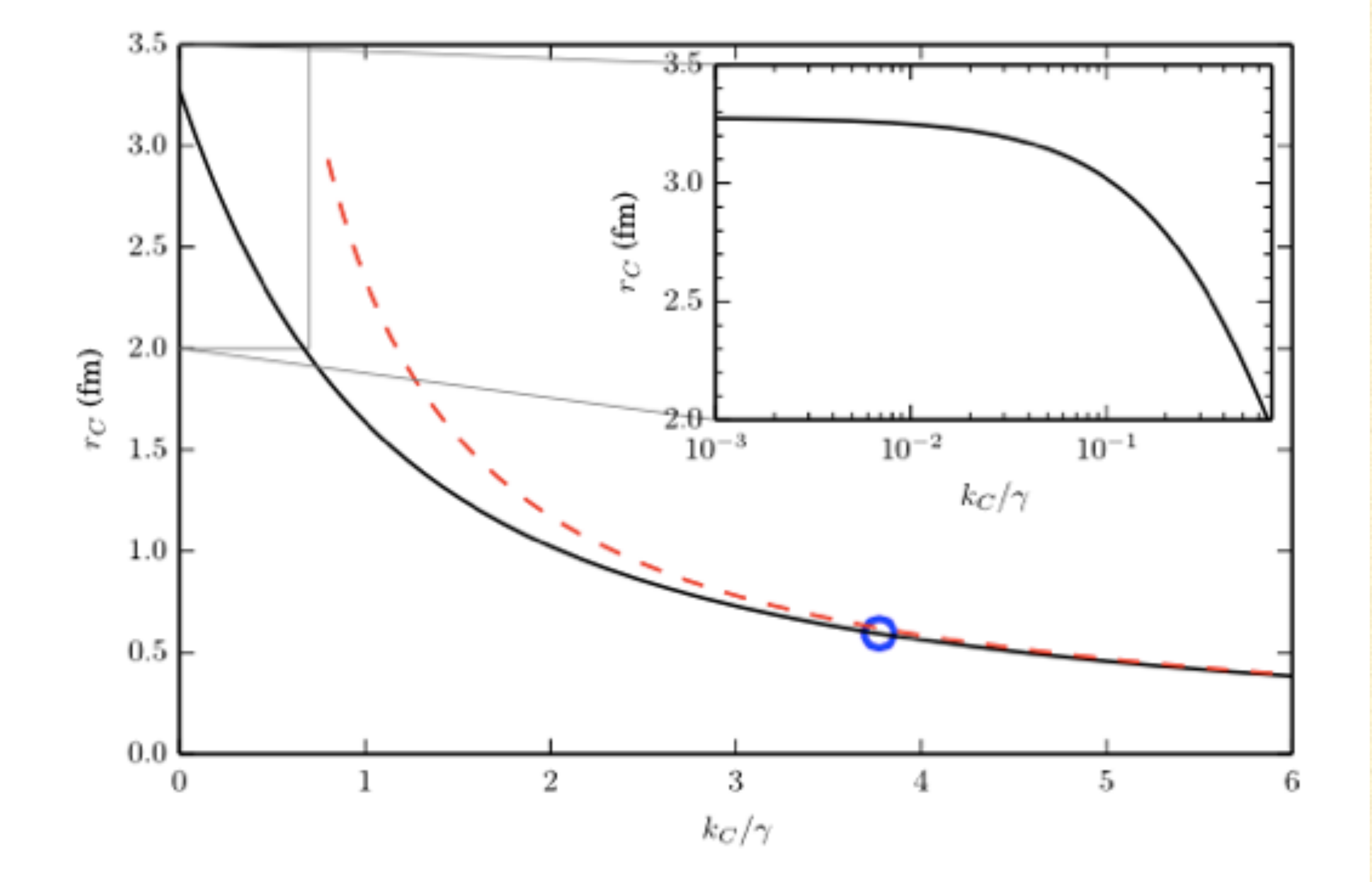
- Possible because Coulomb does not require additional renormalization at LO in α_{em} in pionless EFT
- Coulomb energy of 2p halos? "Three-body Thomas-Ehrman shift" (^{16}Ne/^{16}C)?

PROTON HALOS FROM NEUTRON HALOS?

- Can we predict the energy of a proton halo from its isospin mirror?
- First problem: neutron halo already fine tuned, require another fine tuning to also have proton halo bound or nearly bound
- Note that if neutron is bound by little enough to be in a halo then Coulomb must be treated non-perturbatively

Theory seems to work for $^{17}F^*$, but this state is in the deep Coulomb regime, with $k_c\, r \sim 1$

Ryberg et al., Ann. Phys., 2016



ISOSPIN SYMMETRY IN HALO EFT

- Second problem: Halo EFT is typically asserted to have isospin symmetry, but does it?
- Simplest case: pp vs. nn systems
- $C_{0,nn}$ and $C_{0,pp}$ differ in their μ-dependence, due to $C_{0,pp}$ having to account for Coulomb interactions
- So at what scale does isospin apply? $\mu = m_\pi$? Predicts:

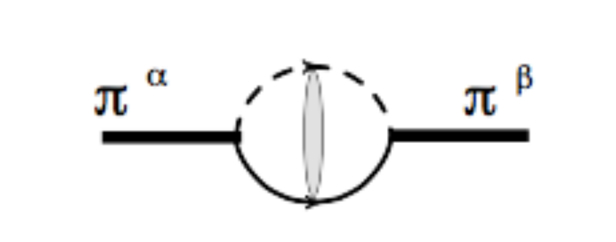
$$\frac{1}{a_{C,pp}} = \frac{1}{a_{nn}} - 2k_C\left[\ln\left(\frac{\sqrt{\pi}m_\pi}{2k_C}\right) + 1 - \frac{3C_E}{2}\right]$$

For a_{nn}=-18.6 fm, predicts $a_{C,pp}$=-6.45 fm cf. $a_{C,pp}$=-7.8063(26) fm

AND IT GETS WORSE…

	^{7}Li-n	^{7}Be-p
γ_1 (MeV)	57.8	15
r_1 (fm^{-1})	-1.43	-0.34

- Consider a p-wave proton-core system, need to calculate:



- Results for effective-range parameters in terms of Lagrangian

$$\frac{(-)}{a_1} \equiv (-)6\pi\frac{M_{\mathrm{R}}\Delta}{h_{Pt}^2} + 2k_C^3\left(\frac{1}{3-d} + \ln\frac{\mu\sqrt{\pi}}{k_C} - \frac{3}{2}C_E + \frac{4}{3}\right) - 3\mu k_C^2\left(1 + \frac{\pi^2}{3}\right) ,$$

$$\frac{r_1}{2} \equiv (-)\frac{3\pi}{h_{Pt}^2} + 2k_C\left(\frac{1}{3-d} + \ln\frac{\mu\sqrt{\pi}}{k_C} - \frac{3}{2}C_E + \frac{4}{3}\right) - 3\mu ,$$

Zhang, Nollett, Phillips, 2014 and in preparation

CONCLUSION

- Subtractive renormalization illuminates aspects of the three-body problem in pionless EFT/halo EFT, but is not a silver bullet
- Higher-order corrections should **not** be iterated at arbitrarily large cutoffs: they change the asymptotic behavior of the amplitude. This tends to produce erroneous conclusions about the order at which counterterms are needed.
- Perturbative analysis should permit extraction of order at which a particular effect becomes sensitive to short-distance pieces of the 3B wave function; unitary limit wf in hyperspherical co-ordinates can be useful for this
- UV piece of Coulomb-nuclear interference is associated with non-observability of Coulomb energies
- Complicates implementation of isospin symmetry in Halo EFT: how does isospin relate, e.g. ^{3}He(^{4}He,γ)^{7}Be and ^{3}H(^{4}He,γ)^{7}Li