Bound states and scattering in 3N and 4N systems

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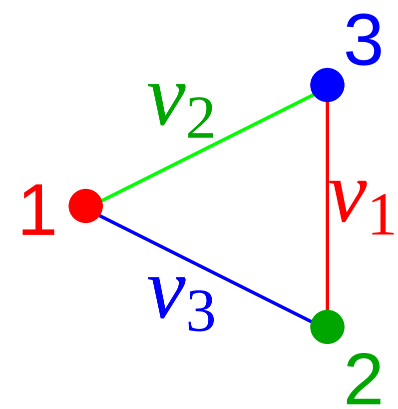
Outline

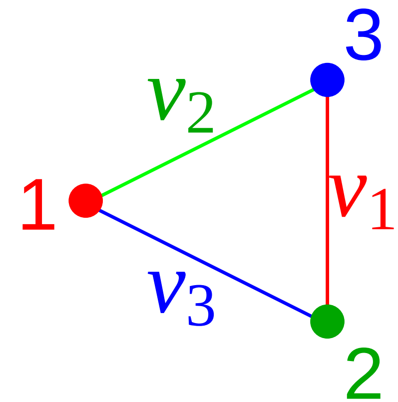
- 3N
- 4N
- preliminary results with EFT-like interactions

Three-particle system



Hamiltonian $H_0 + \sum_{\alpha} v_{\alpha}$

Three-particle system



Hamiltonian $H_0 + \sum_{\alpha} v_{\alpha}$

- Faddeev equations

$$(E - H_0 - v_{\alpha})|\psi_{\alpha}\rangle = v_{\alpha} \sum_{\gamma} \bar{\delta}_{\alpha\gamma} |\psi_{\sigma}\rangle$$

$$|\Psi\rangle = \sum_{\alpha} |\psi_{\alpha}\rangle$$

Alt, Grassberger, and Sandhas equations

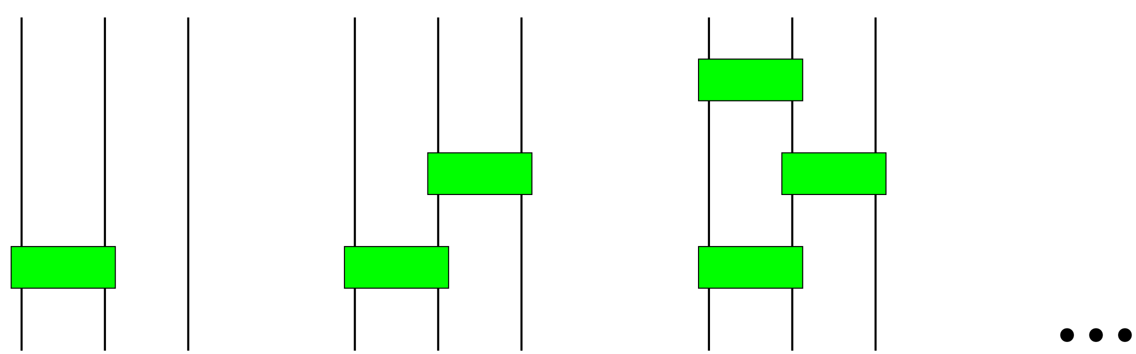
$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\gamma} \bar{\delta}_{\beta\gamma} T_\gamma G_0 U_{\gamma\alpha}$$

$$U_{0\alpha} = G_0^{-1} + \sum_{\gamma} T_\gamma G_0 U_{\gamma\alpha}$$

$$T_\gamma = v_\gamma + v_\gamma G_0 T_\gamma$$

$$G_0 = (E + i0 - H_0)^{-1}$$

channel states $\quad (E - H_0 - v_\alpha)|\phi_\alpha\rangle = 0$



AGS equations with 3BF

$$V_{3BF} = \sum_{\alpha=1}^{3} w_\alpha$$

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_\gamma \bar{\delta}_{\beta\gamma} T_\gamma G_0 U_{\gamma\alpha}$$

$$+ w_\alpha + \sum_\gamma w_\gamma G_0 (1 + T_\gamma G_0) U_{\gamma\alpha}$$

AGS equations: numerical solution

$$U = PG_0^{-1} + PTG_0U + (1+P)w + (1+P)wG_0(1+TG_0)U$$

- symmetrized for 3N: $P = P_{12}P_{23} + P_{13}P_{23}$
- momentum-space partial wave basis
- set of coupled 2-variable integral equations
- integrable singularities in kernel
- Coulomb interaction: screening and renormalization

[PRC 71, 054005; PRC 72, 054004; PRC 80, 064002]

3N bound state

$$|\Psi\rangle = (1+P)|\psi\rangle$$

decomposed into Faddeev components

$$|\psi\rangle = G_0 T P|\psi\rangle + (1+G_0 T)G_0 w(1+P)|\psi\rangle$$

Nuclear interaction (S-waves only)

$$v_{2N} = |g\rangle \lambda_2 \langle g|$$

$$\langle p|g\rangle = [1 + c_2 (p/\Lambda)^2] e^{-p^2/\Lambda^2}$$

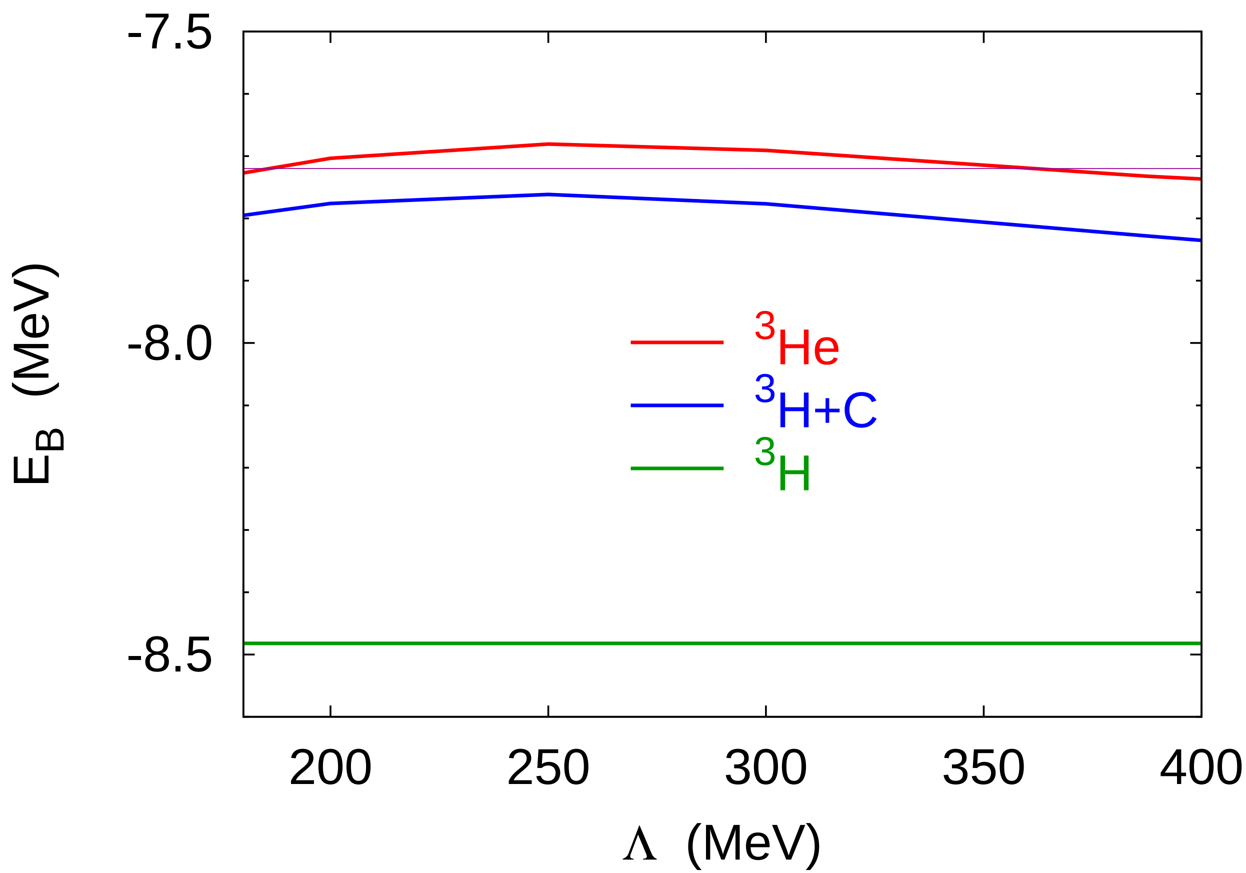
(λ_2, c_2): fit to (a_{NN}, r_{NN})

$$V_{3N} = 3w = |\xi\rangle \lambda_3 \langle \xi|$$

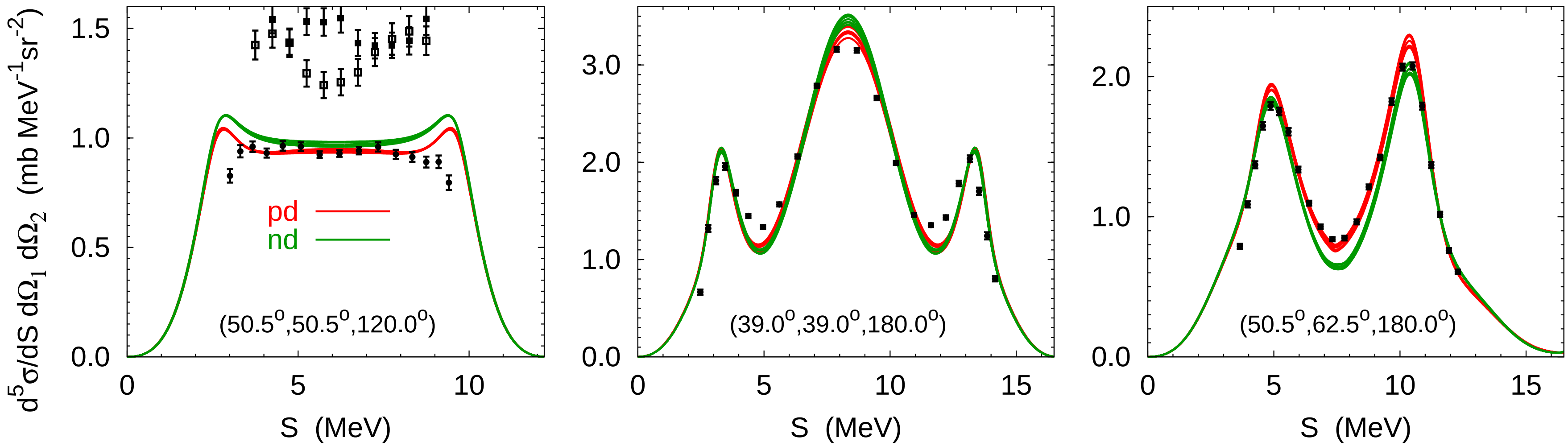
$$\langle pq|\xi\rangle \sim e^{-(p^2 + \frac{3}{4}q^2)/\Lambda^2}$$

λ_3: fit to $E_{^3H}$

3N bound states



Nucleon-deuteron breakup at $E_N = 13$ MeV



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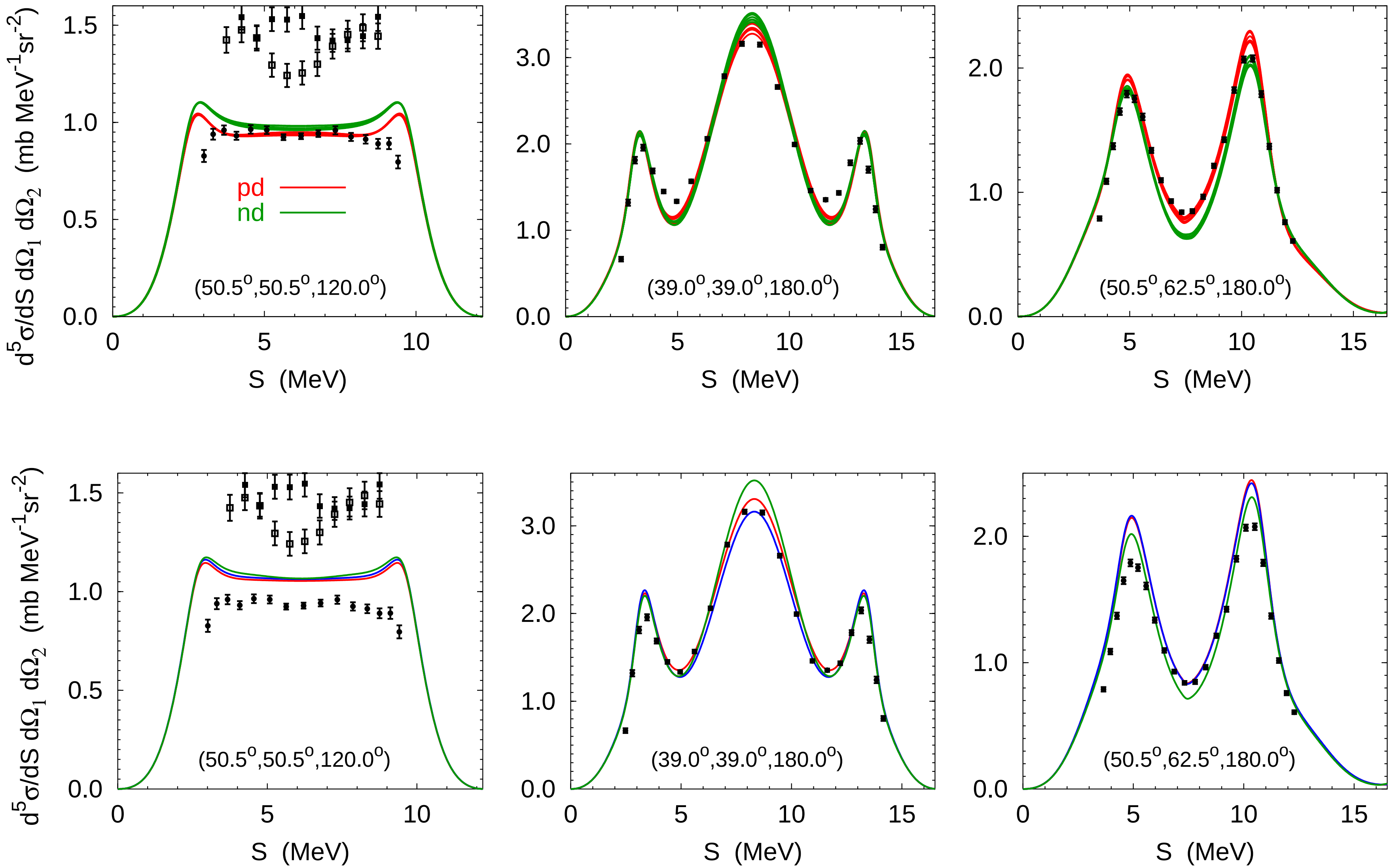
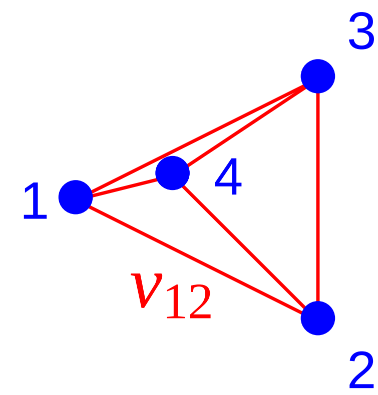


4N scattering



Hamiltonian $$H_0 + \sum_{i>j} v_{ij}$$

- Wave function:
  Schrödinger equation (HH + Kohn VP, r-space)
  [M. Viviani, A. Kievsky, L. E. Marcucci, S. Rosati, L. Girlanda]
- Wave function components:
  Faddeev-Yakubovsky equations (r-space)
  [R. Lazauskas, J. Carbonell]
- Transition operators:
  Alt-Grassberger-Sandhas equations (p-space)
  [AD, A. C. Fonseca]

4-body scattering: AGS equations

4-body transition operators

$$t_i = v_i + v_i G_0 t_i$$

$$G_0 = (E + i0 - H_0)^{-1}$$

$$U_\gamma^{jk} = G_0^{-1} \bar{\delta}_{jk} + \sum_i \bar{\delta}_{ji} t_i G_0 U_\gamma^{ik}$$

$$\mathcal{U}_{\beta\alpha}^{ji} = (G_0 t_i G_0)^{-1} \bar{\delta}_{\beta\alpha} \delta_{ji} + \sum_{\gamma k} \bar{\delta}_{\beta\gamma} U_\gamma^{jk} G_0 t_k G_0 \, \mathcal{U}_{\gamma\alpha}^{ki}$$

i, j, k: pairs ($\equiv$ three-cluster (2+1+1) partitions)
α, β, γ: two-cluster (1+3 or 2+2) partitions

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i, j, k: pairs ($\equiv$ three-cluster (2+1+1) partitions)
α, β, γ: two-cluster (1+3 or 2+2) partitions

wave function

$$|\Psi_\alpha\rangle = |\Phi_\alpha\rangle + \sum_{\gamma jki} G_0 t_j G_0 U_\gamma^{jk} G_0 t_k G_0 \mathcal{U}_{\gamma\alpha}^{ki} |\phi_\alpha^i\rangle$$

$$|\Phi_\alpha\rangle = \sum_i |\phi_\alpha^i\rangle, \qquad |\phi_\alpha^i\rangle = G_0 \sum_j \bar{\delta}_{ij} t_j |\phi_\alpha^j\rangle$$

Symmetrized AGS equations

$$t = v + vG_0 t$$

$$G_0 = (E + i\varepsilon - H_0)^{-1}$$

$$U_j = P_j G_0^{-1} + P_j t G_0 U_j$$

$$3+1: \quad P_1 = P_{12} P_{23} + P_{13} P_{23}$$

$$2+2: \quad P_2 = P_{13} P_{24}$$

$$\mathcal{U}_{11} = (G_0 t G_0)^{-1} \zeta P_{34} + \zeta P_{34} U_1 G_0 t G_0 \, \mathcal{U}_{11} + U_2 G_0 t G_0 \, \mathcal{U}_{21}$$

$$\mathcal{U}_{21} = (G_0 t G_0)^{-1} (1 + \zeta P_{34}) + (1 + \zeta P_{34}) U_1 G_0 t G_0 \, \mathcal{U}_{11}$$

$$\mathcal{U}_{12} = (G_0 t G_0)^{-1} + \zeta P_{34} U_1 G_0 t G_0 \, \mathcal{U}_{12} + U_2 G_0 t G_0 \, \mathcal{U}_{22}$$

$$\mathcal{U}_{22} = (1 + \zeta P_{34}) U_1 G_0 t G_0 \, \mathcal{U}_{12}$$

$\zeta = -1 \; (+1)$ for fermions (bosons)

basis states partially symmetrized

Scattering amplitudes: $E + i\varepsilon \to E + i0$

2-cluster reactions:

$$T_{fi} = s_{fi} \langle \phi_f | \mathcal{U}_{fi} | \phi_i \rangle$$

$$|\phi_j\rangle = G_0 t P_j |\phi_j\rangle$$

$$|\Phi_j\rangle = (1 + P_j) |\phi_j\rangle$$

3-cluster breakup/recombination:

$$T_{3i} = s_{3i} \langle \phi_3 | [(1 + \zeta P_{34}) U_1 G_0 t G_0 \mathcal{U}_{1i} + U_2 G_0 t G_0 \mathcal{U}_{2i}] | \phi_i \rangle$$

4-cluster breakup/recombination:

$$T_{4i} = s_{4i} \{ \langle \phi_4 | [1 + (1 + P_1) \zeta P_{34}] (1 + P_1) t G_0 U_1 G_0 t G_0 \mathcal{U}_{1i} | \phi_i \rangle + \langle \phi_4 | (1 + P_1)(1 + P_2) t G_0 U_2 G_0 t G_0 \mathcal{U}_{2i} | \phi_i \rangle \}$$

Bound state

Wave function

$$|\Psi\rangle = [1 + (1 + P_1)\zeta P_{34}](1 + P_1)|\psi_1\rangle + (1 + P_1)(1 + P_2)|\psi_2\rangle$$

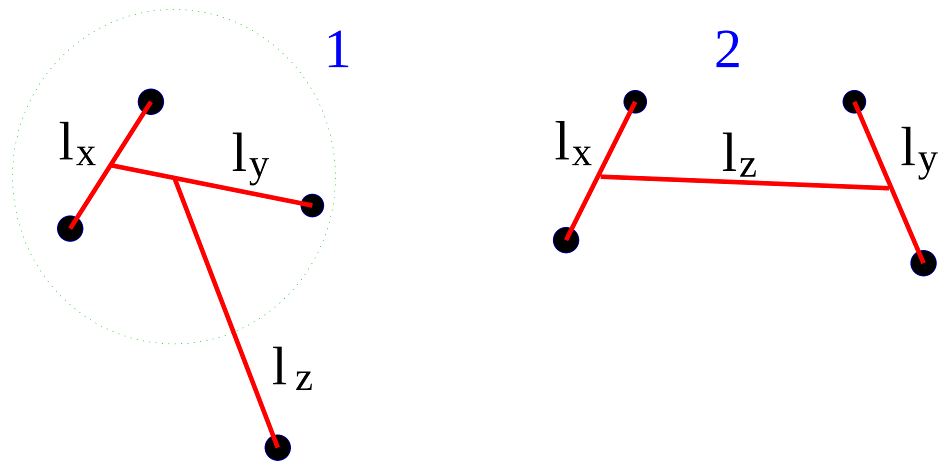
FY components:

$$|\psi_1\rangle = G_0 t G_0 U_1 (\zeta P_{34}|\psi_1\rangle + |\psi_2\rangle)$$

$$|\psi_2\rangle = G_0 t G_0 U_2 (1 + \zeta P_{34})|\psi_1\rangle$$

Solution of 4N AGS equations

$$\mathcal{U}_{11}|\phi_1\rangle = -G_0^{-1}P_{34}P_1|\phi_1\rangle - P_{34}U_1G_0\,t\,G_0\,\mathcal{U}_{11}|\phi_1\rangle + U_2G_0\,t\,G_0\,\mathcal{U}_{21}|\phi_1\rangle$$

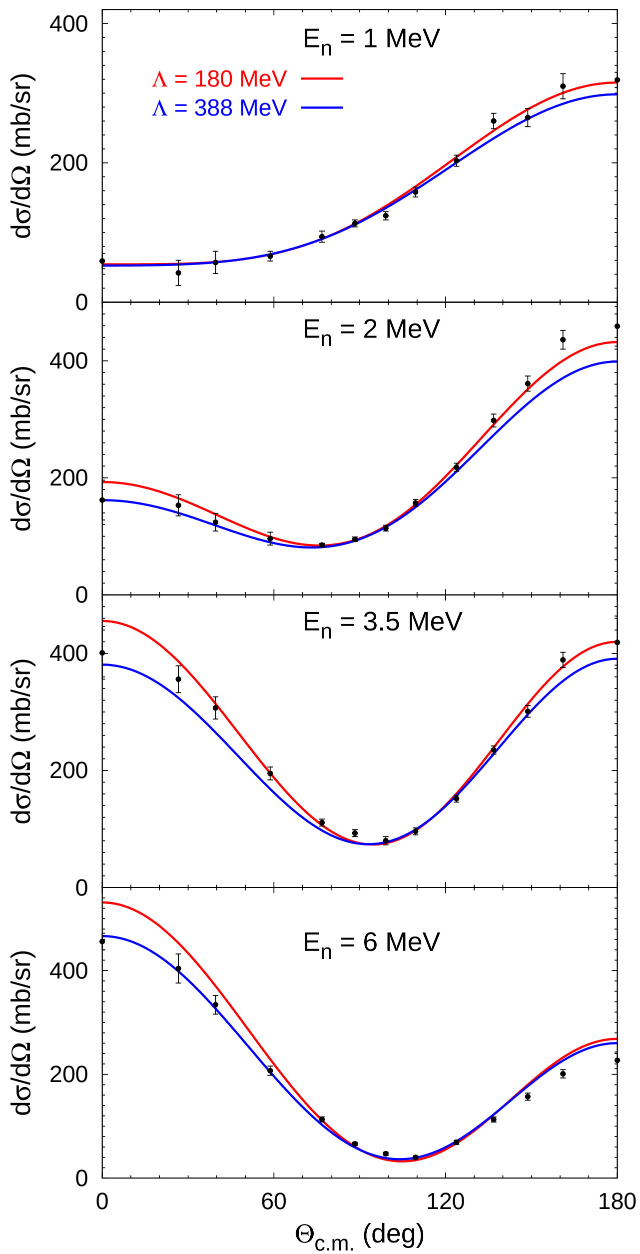


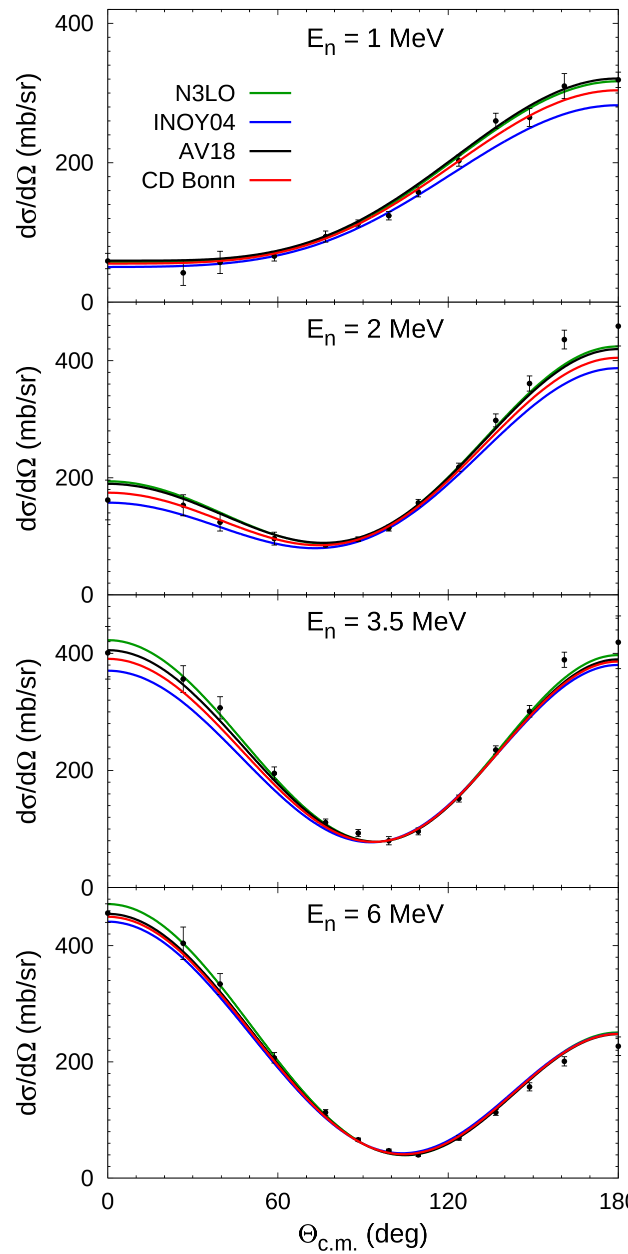
- momentum-space partial-wave basis
  $|k_xk_yk_z[l_z(\{l_y[(l_xS_x)j_xs_y]S_y\}J_ys_z)S_z]JM,[(T_xt_y)T_yt_z]TM_T\rangle_1$
  $|k_xk_yk_z[l_z\{(l_xS_x)j_x[l_y(s_ys_z)S_y]j_y\}S_z]JM,[T_x(t_yt_z)T_z]TM_T\rangle_2$
- large system (up to 30000) of coupled 3-variable integral equations with integrable singularities
- Coulomb interaction: screening and renormalization [PRC 75, 014005, PRL 98, 162502]

Binding energies and $n+{}^3$H scattering lengths

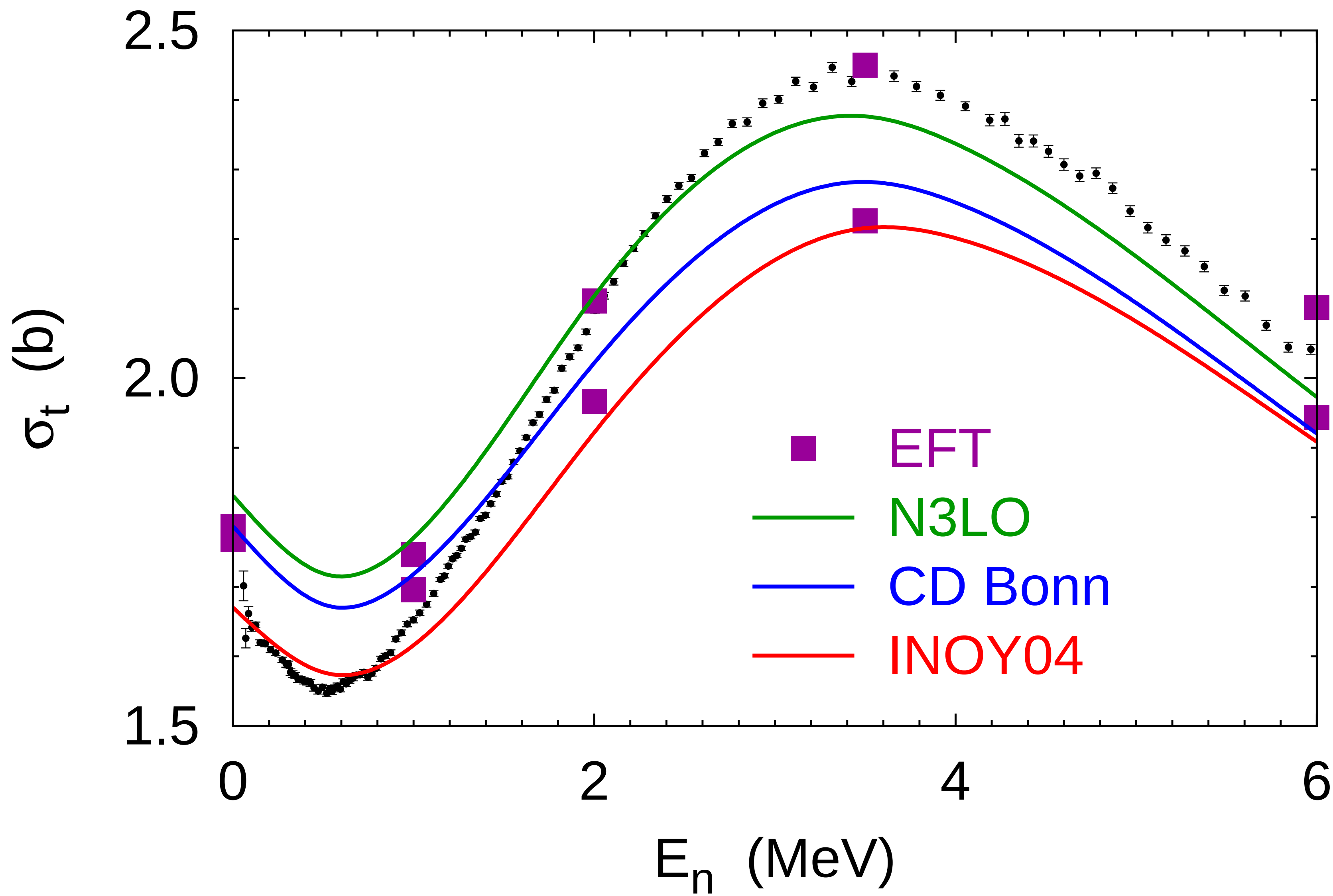
	ε_t	ε_α	a_0	a_1	σ_t (0)	σ_t (3.5)
AV18	7.621	24.24	4.28	3.71	1.88	2.33
Nijmegen II	7.653	24.50	4.27	3.71	1.87	2.31
Nijmegen I	7.734	24.94	4.25	3.69	1.85	2.30
N3LO	7.854	25.38	4.23	3.67	1.83	2.38
CD Bonn	7.998	26.11	4.17	3.63	1.79	2.28
INOY04	8.493	29.11	4.02	3.51	1.67	2.22
$\Lambda = 180$ MeV	8.482	29.41	4.14	3.64	1.79	2.45
$\Lambda = 388$ MeV	8.482	29.41	4.12	3.62	1.77	2.23

n+^{3}H differential cross section

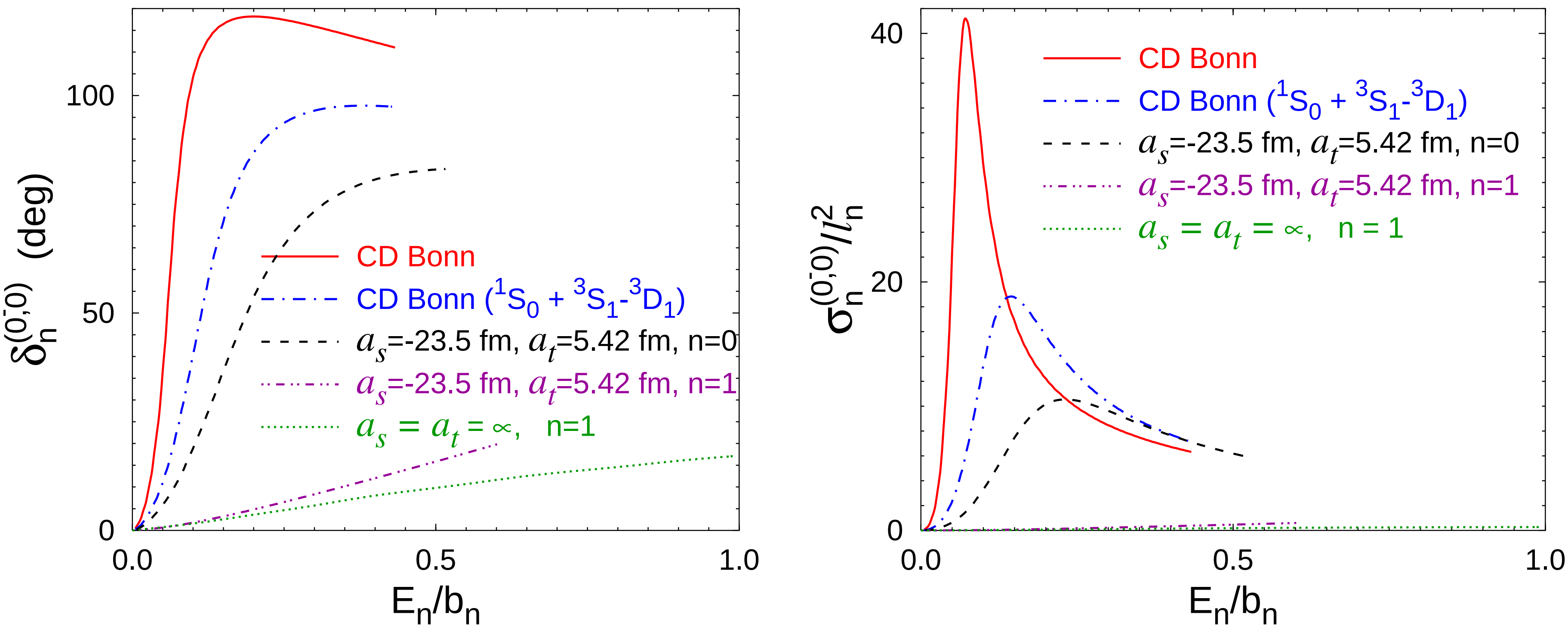




n+^{3}H total cross section



Four-nucleon $(J^{\pi}, T) = (0^{-}, 0)$ resonance



Few-nucleon systems at low energies

- Faddeev-Yakubovsky and/or
  Alt-Grassberger-Sandhas theory
- Local and nonlocal potentials
- Difficulties with Coulomb at very low energies
- Treatment of 3NF depends on its form