

EMMI Rapid Reaction Task Force
**The systematic treatment of the Coulomb interaction
in few-body systems,
*Second meeting.***
May 31-June 3, 2016

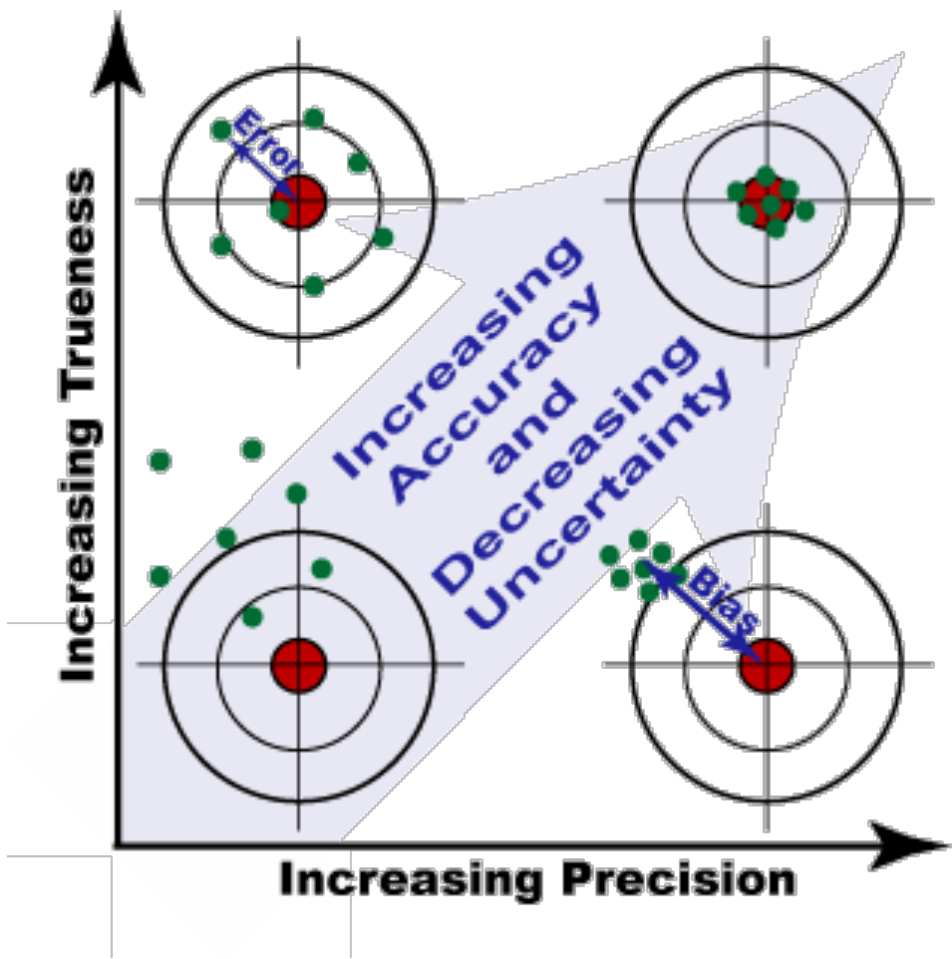
Updates on solar proton-proton fusion from π EFT



Doron Gazit, Hilla De-Leon
Racah Institute of Physics
Hebrew University of Jerusalem



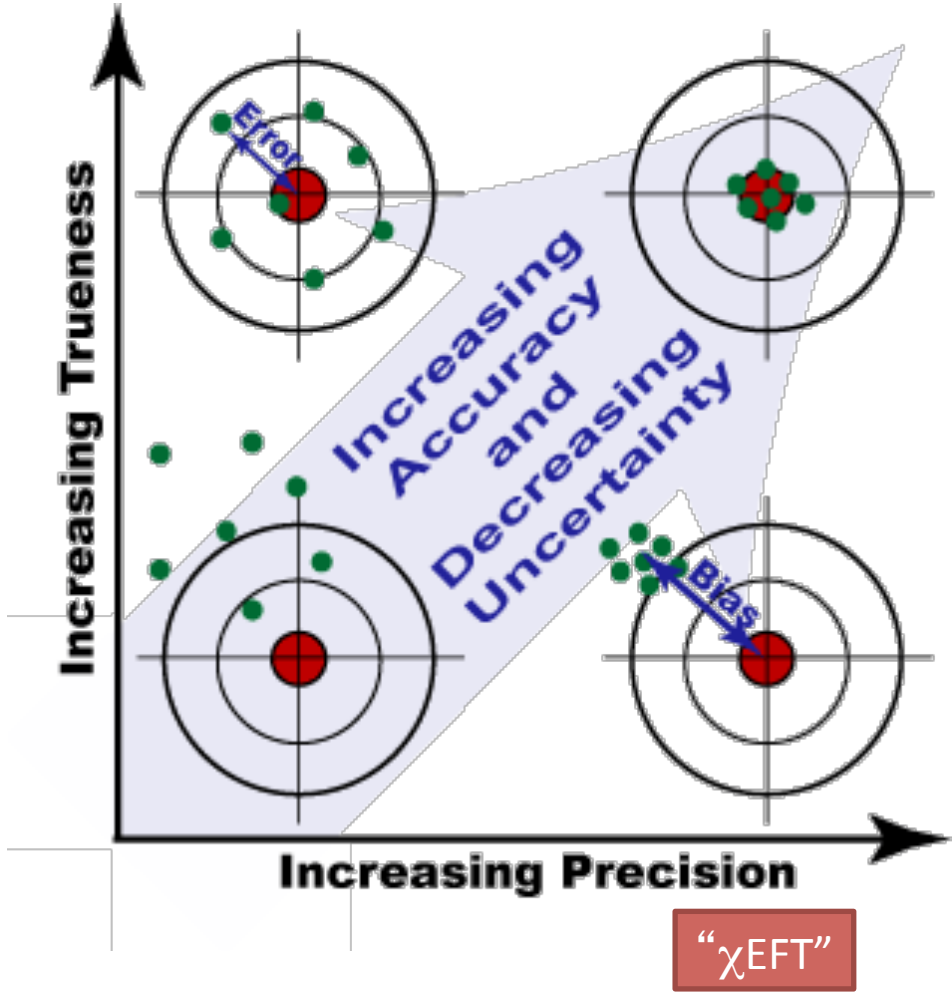
Precision, Uncertainty, and predictions





Precision, Uncertainty, and predictions

Widely believed:





Weak proton-proton fusion in the Sun – theory standards

SFII – Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)

- $4.01(1 \pm 0.009) \times 10^{-25}$ MeV b potential models,
- $4.01(1 \pm 0.009) \times 10^{-25}$ MeV b EFT*,
- $3.99(1 \pm 0.030) \times 10^{-25}$ MeV b pionless EFT.



SFII recommended value (2011): $S_{11}(0) = 4.01(1 \pm 0.009) \times 10^{-25}$ MeV b.

“ χ EFT” calculation by Marcucci et al., Phys. Rev. Lett. (2013):
Use consistent ^3H decay-rate to constrain consistently axial MEC (DG, Quaglioni, Navratil, PRL 2009), and predict pp-fusion rate.

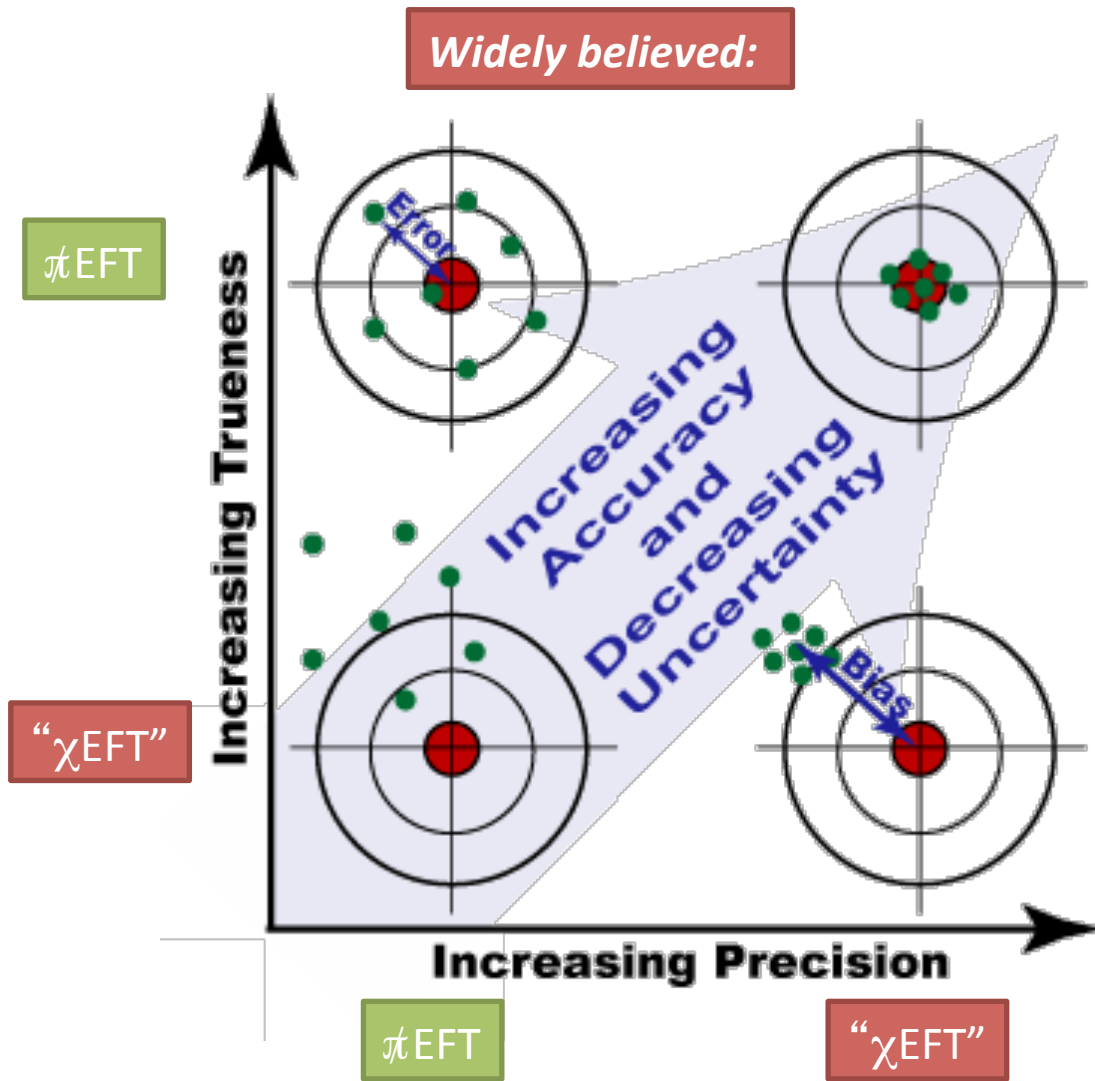
$$S(0) = (4.030 \pm 0.006) \times 10^{-23} \text{ MeV fm}^2$$

Including: p-wave contribution (+0.005%), full EM (-0.0025-(-0.0075)%), difference between 500 and 600 MeV cutoff and potential models.

Recently Archaya et al (1603.01593) χ EFT: $S(0) = (4.081^{+0.024}_{-0.032}) \times 10^{-23} \text{ MeV fm}^2$



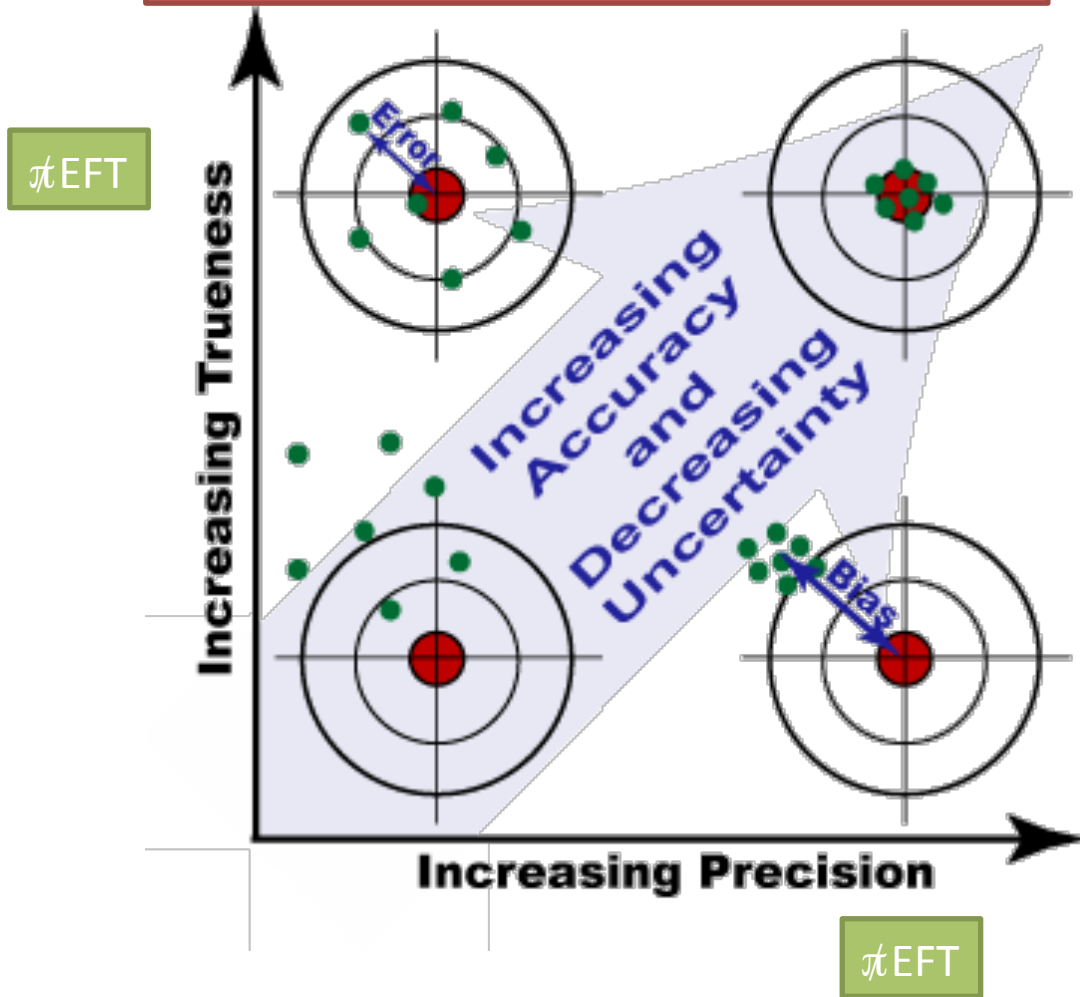
Precision, Uncertainty, and predictions





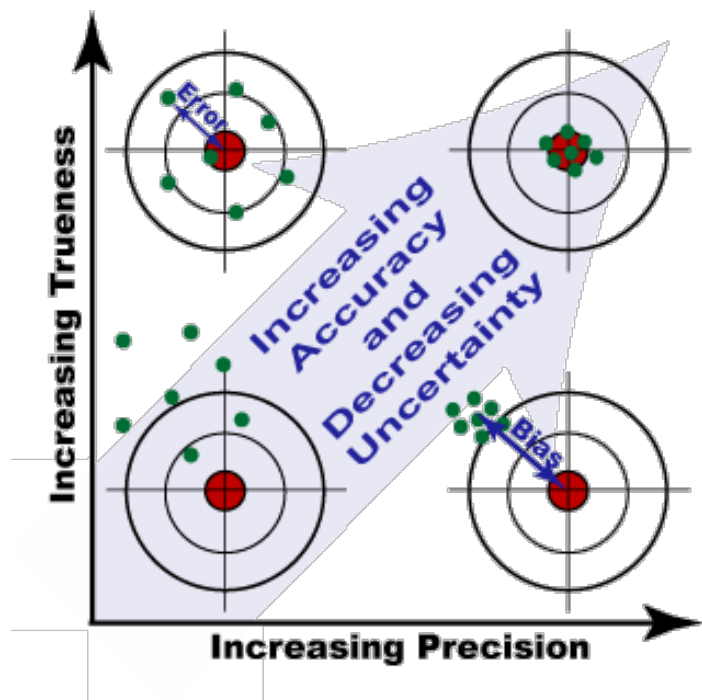
Precision, Uncertainty, and predictions

Can we reach precision physics with π EFT?





Precision, Uncertainty, and predictions

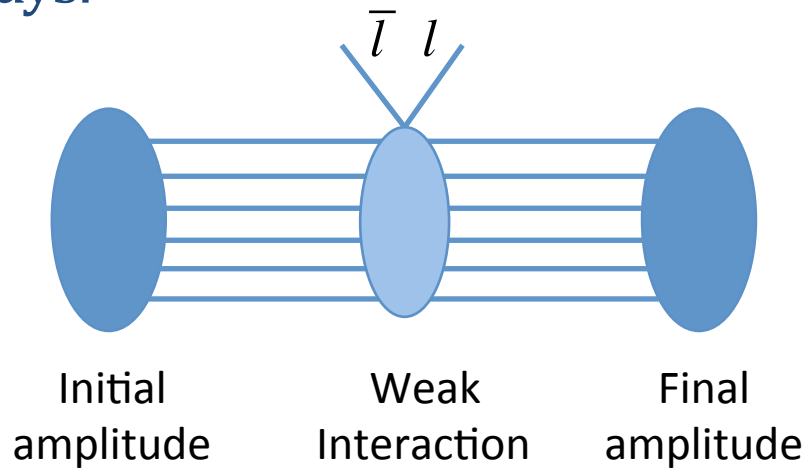


Role of π EFT:
Coherent and systematic (theoretical) uncertainty quantification.
Big question: is precision physics a possible frontier of π EFT?

We revisit the pp-fusion problem within pionless EFT, fixing the unknown LEC using triton decay.

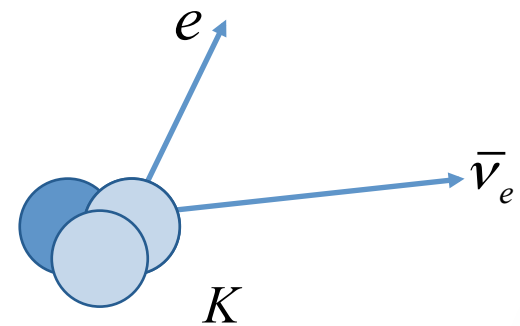
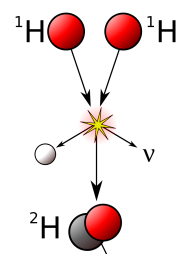


Weak decays:



$$\langle \psi_i | \mathcal{J}_\mu | \psi_f \rangle$$

$$\mathcal{J}_\mu^\pm = \frac{\tau_\pm}{2} (\mathcal{V}_\mu^\pm - \mathcal{A}_\mu^\pm)$$



$$\langle pp | \mathcal{A}_\mu^- | {}^2\text{H} \rangle$$

$$ft = \frac{K}{G_F^2 V_{ud}^2 \left[\left| \langle {}^3\text{H} | \mathcal{V}_\mu^+ | {}^3\text{He} \rangle \right|^2 + \frac{f_A}{f_V} \left| \langle {}^3\text{H} | \mathcal{A}_\mu^+ | {}^3\text{He} \rangle \right|^2 \right]}$$



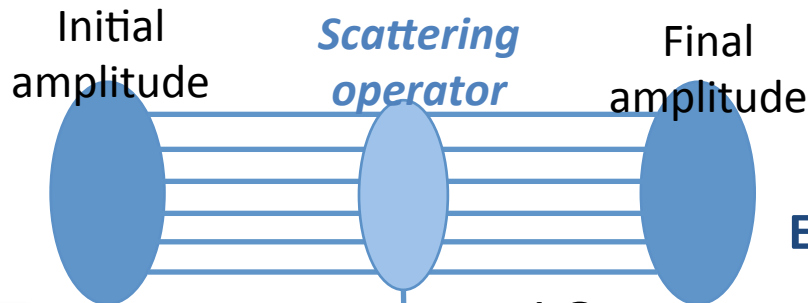
Advantages of π EFT for proton-proton fusion:

1. Small number of parameters.
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A fully perturbative pionless EFT $A=2, 3$ calculation @NLO

- LO Parameters:
 - nn and 2-np Scattering lengths: ${}^3S_1, {}^1S_0$.
 - pp scattering length.
 - Fine structure constant.
 - Three body force strength to prevent Thomas collapse.
- NLO parameters:
 - 2 effective ranges.
 - Renormalizations of pp and 3NF.
 - (isospin dependent 3NF to prevent logarithmic divergence in the binding energy of ${}^3\text{He}$).
- **Weak Interaction: LO ($g_A - 1$ body), NLO ($L_{1A} - 2$ body)**
- **EM Interaction: LO ($\kappa_S, \kappa_V - 1$ body), NLO ($L_1, L_2 - 2$ body)**



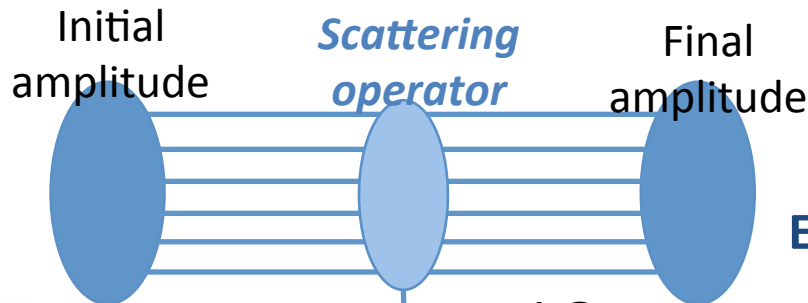
• LO: $N^\dagger \frac{\tau^-}{2} N + \frac{g_A}{2} N^\dagger \sigma \tau^- N$

Fermi

Gamow-Teller

• LO: $\frac{e}{2M_N} N^\dagger (\kappa_0 + \kappa_1 \tau_3) \sigma \cdot B N$

Single nucleon interaction



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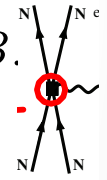
- NLO (correction to GT):

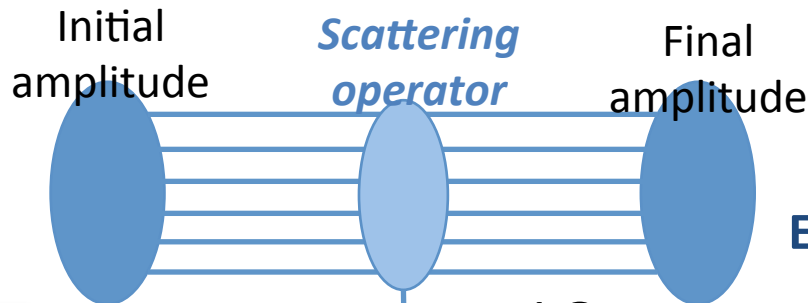
$$-L'_{1A}(t^\dagger s + s^\dagger t)$$

- NLO:

$$-L'_1(t^\dagger s + s^\dagger t) \cdot B + L'_2(t^\dagger t) \cdot B$$

Simultaneous interaction with two nucleons coupled to singlet (s) / triplet (t)





- LO: $N^\dagger \frac{\tau^-}{2} N + \frac{g_A}{2} N^\dagger \sigma \tau^- N$

Fermi Gamow-Teller

- LO: $\frac{e}{2M_N} N^\dagger (\kappa_0 + \kappa_1 \tau_3) \sigma \cdot B N$

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$$-L'_{1A} (t^\dagger s + s^\dagger t)$$

- NLO:

$$-L'_1 (t^\dagger s + s^\dagger t) \cdot B + L'_2 (t^\dagger t) \cdot B$$

Simultaneous interaction with two nucleons coupled to singlet (s) / triplet (t)

$$L'_{1A} = g_A \left[\frac{1}{2} \frac{\rho_t + \rho_s}{\sqrt{\rho_t \rho_s}} - L_{1A} \frac{1}{2\pi \sqrt{\rho_t \rho_s} g_A} \left(\mu - \frac{1}{a_t} \right) \left(\mu - \frac{1}{a_s} \right) \right]$$

$$L'_1 = \frac{e}{2M_N} \left[-\frac{1}{2} \frac{\rho_t + \rho_s}{\sqrt{\rho_t \rho_s}} (\kappa_p - \kappa_n) + L_1 \frac{M_N}{\pi \sqrt{\rho_t \rho_s}} \left(\mu - \frac{1}{a_t} \right) \left(\mu - \frac{1}{a_s} \right) \right]$$

$$L'_2 = \frac{e}{2M_N} \left[L_2 \frac{2M_N}{\pi \rho_t} \left(\mu - \frac{1}{a_t} \right)^2 - (\kappa_p + \kappa_n) \right]$$

Should be cutoff invariant, nu + photon bremsstrahlung



Advantages of π EFT for proton-proton fusion:

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The role of the deuteron tail

- Many low energy reactions depend on deuteron normalization.

$$Z_d^{-1} = i \frac{\partial}{\partial p_0} \frac{1}{i\mathcal{D}_t(p_0, p)} \Big|_{p_0 = \frac{\gamma_t^2}{M_N}, p=0}$$

- One has a choice of rearranging the expansion:

- rho-parameterization: $Z_d = \frac{1}{1 - \gamma\rho} \approx 1 + \gamma\rho + (\gamma\rho)^2 + \dots$
- Z(ed)-parameterization: $Z_d = \frac{1}{1 - \gamma\rho} \approx 1 - (Z_d - 1) + 0 + \dots$

Both are valid rearrangements!
Z-parameterization has quicker convergence, especially for observables sensitive to the deuteron tail.

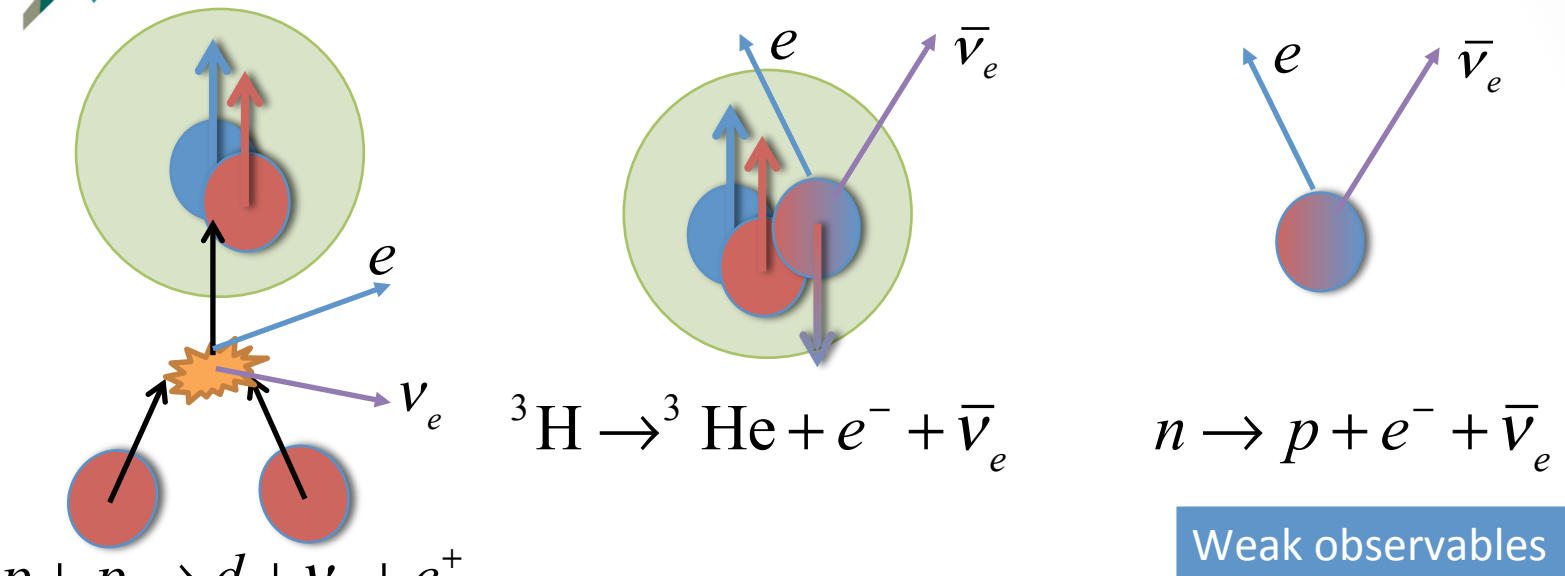
Phillips, Rupak, Savage, Phys. Lett. **B473**, 209 (2000)
Grißhammer, Nucl. Phys. **A744**, 192 (2004)



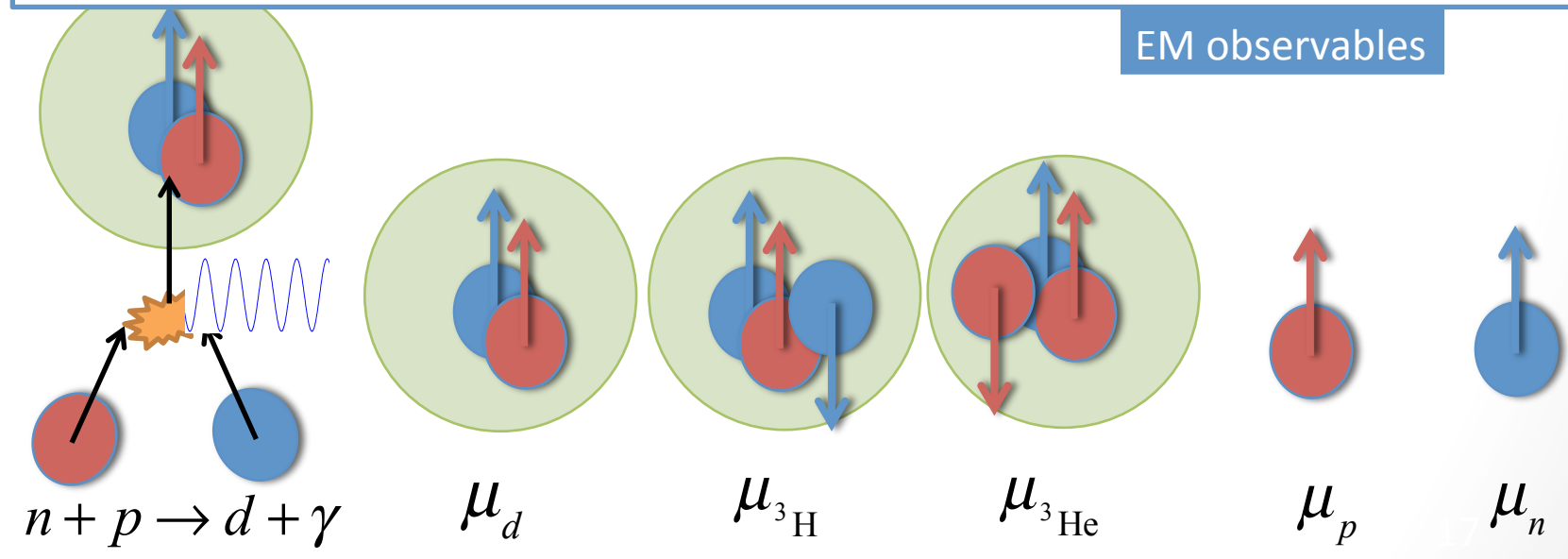
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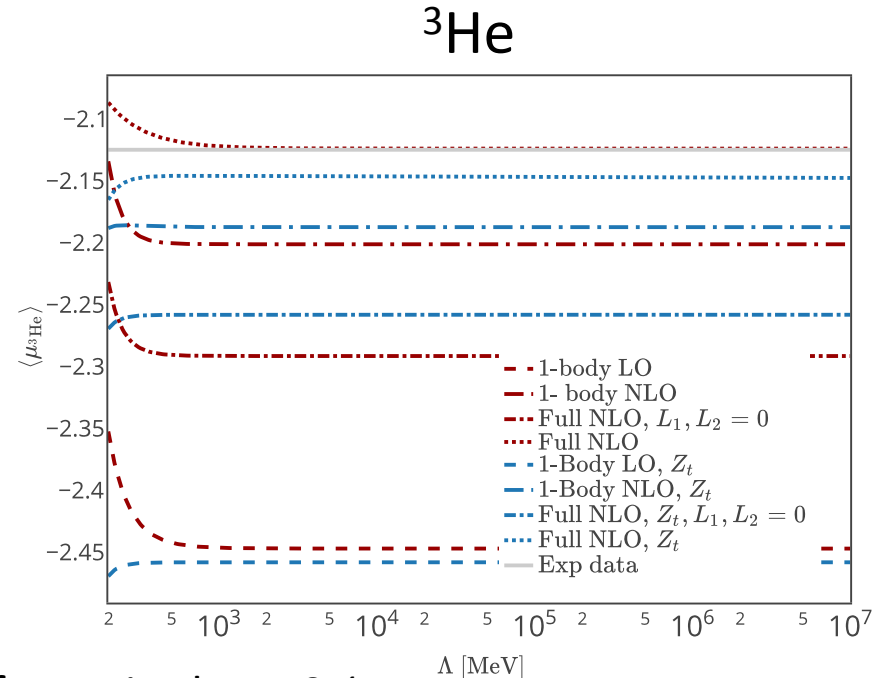
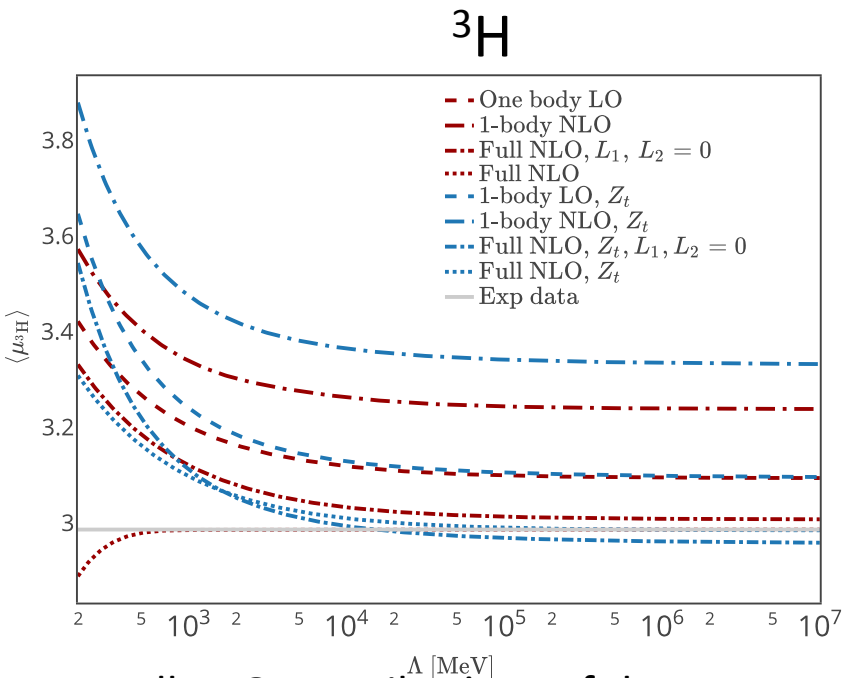
Analogy between weak and EM:



Use the same strategy in both cases: fix probe LECs at A=3 and predict A=2.



A=3 magnetic moments calculations:



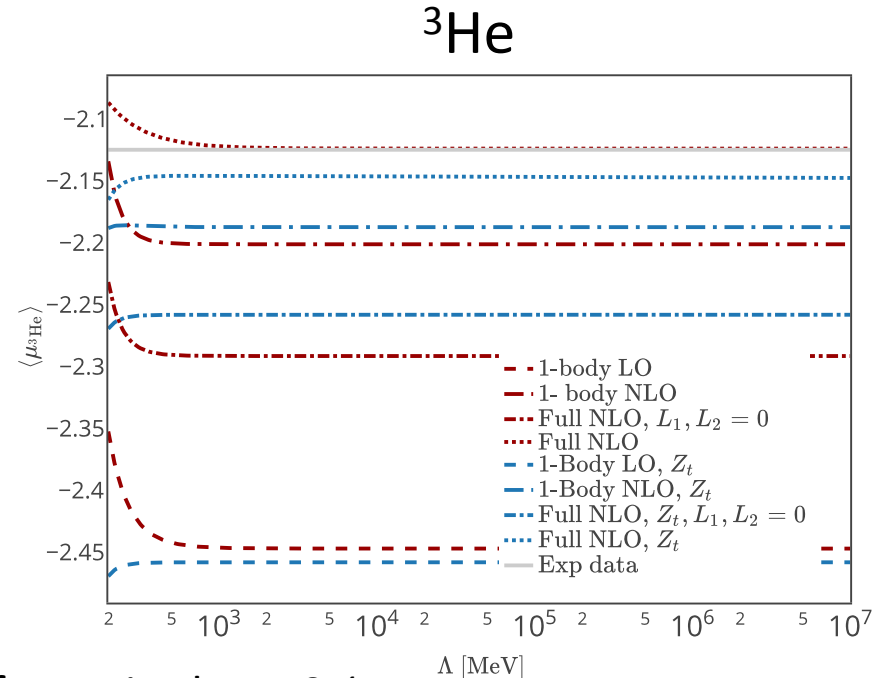
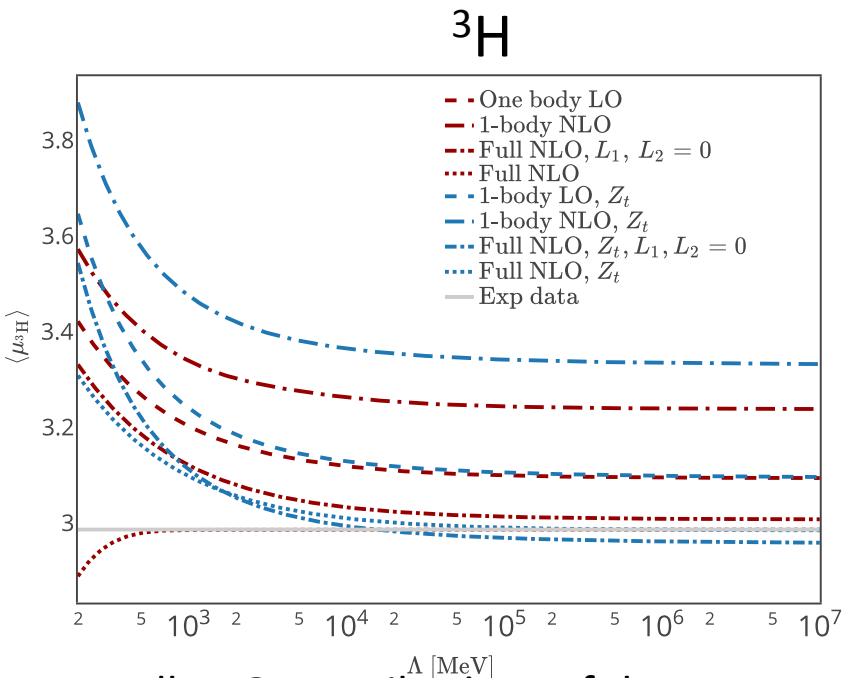
- All NLO contributions of the same order of magnitude 5-10% – Natural NLO contributions – useful for theoretical error estimates!
- No effect due to Zed-Rho parameterizations.
- Cutoff independence.
- When L_1 and L_2 are fixed from **A=2 observables**:

LO: $\mu_{^3\text{H}}^{LO} = 3.09 \pm_{Z_d} 0.01$ $\mu_{^3\text{He}}^{LO} = -2.455 \pm_{Z_d} 0.005$

NLO: $\mu_{^3\text{H}}^{NLO} = 3.005 \pm_{Z_d} 0.01$ $\mu_{^3\text{He}}^{NLO} = -2.13 \pm_{Z_d} 0.01$

exp: $\mu_{^3\text{H}}^{\text{exp}} = 2.9789\dots$ $\mu_{^3\text{He}}^{\text{exp}} = -2.1276\dots$

A=3 magnetic moments calculations:



- All NLO contributions of the same order of magnitude 5-10% – Natural NLO contributions – useful for theoretical error estimates!
- No effect due to Zed-Rho parameterizations.
- Cutoff independence.
- When L_1 and L_2 are fixed from **A=3 magnetic moments**:

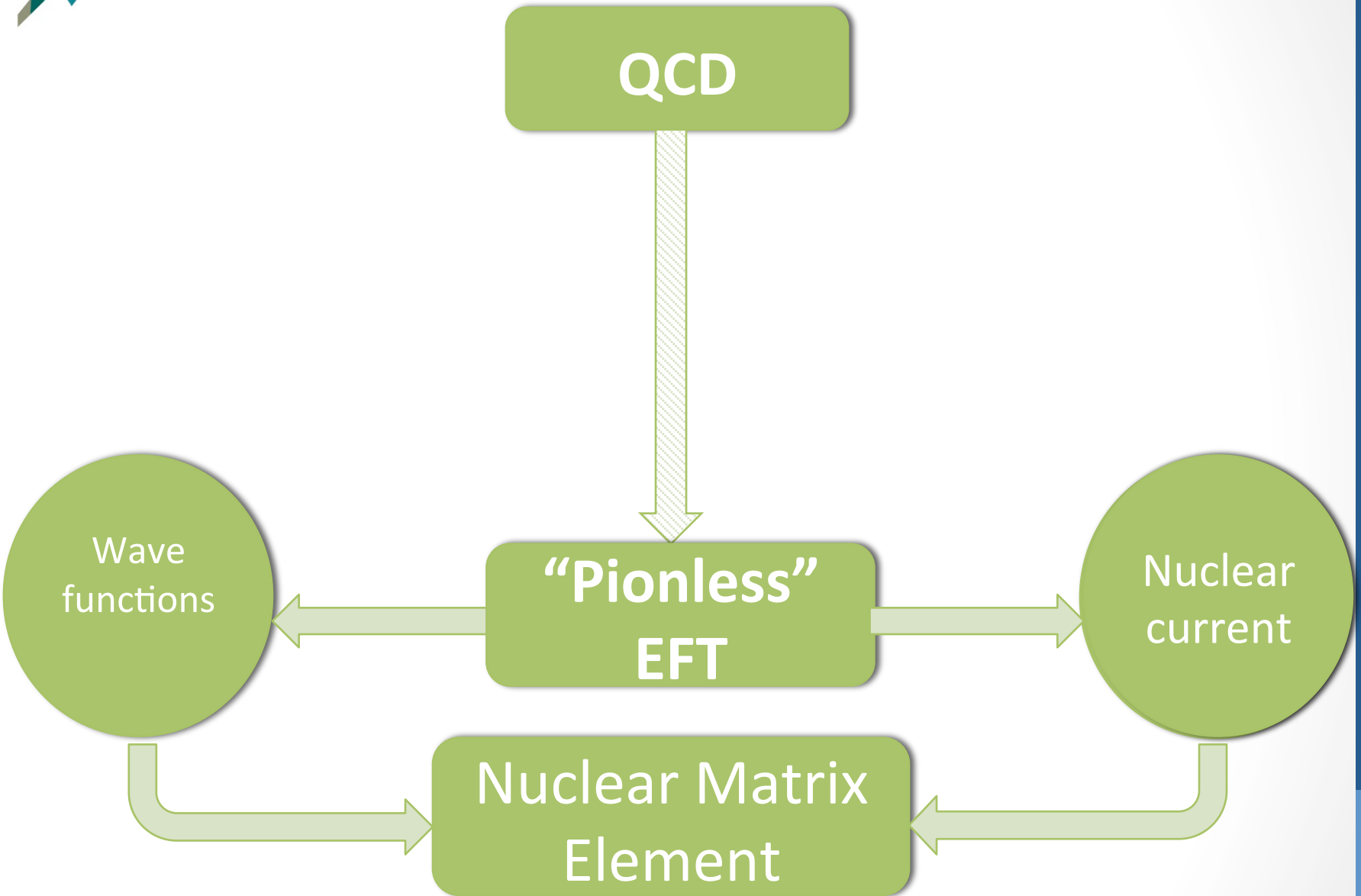
LO: $\mu_d^{LO} = 0.8798$ $\sigma_{np}^{LO} = 298.2 \text{ mb}$

NLO: $\mu_d^{NLO} = 0.8617 \pm_{Z_d} 0.0002$ $\sigma_{np}^{NLO} = 335(Z_d) - 320(\rho)$

exp: $\mu_d^{\text{exp}} = 0.8574\dots$ $\sigma_{np}^{\text{exp}} = 334.2 \pm 0.5 \text{ mb}\dots$

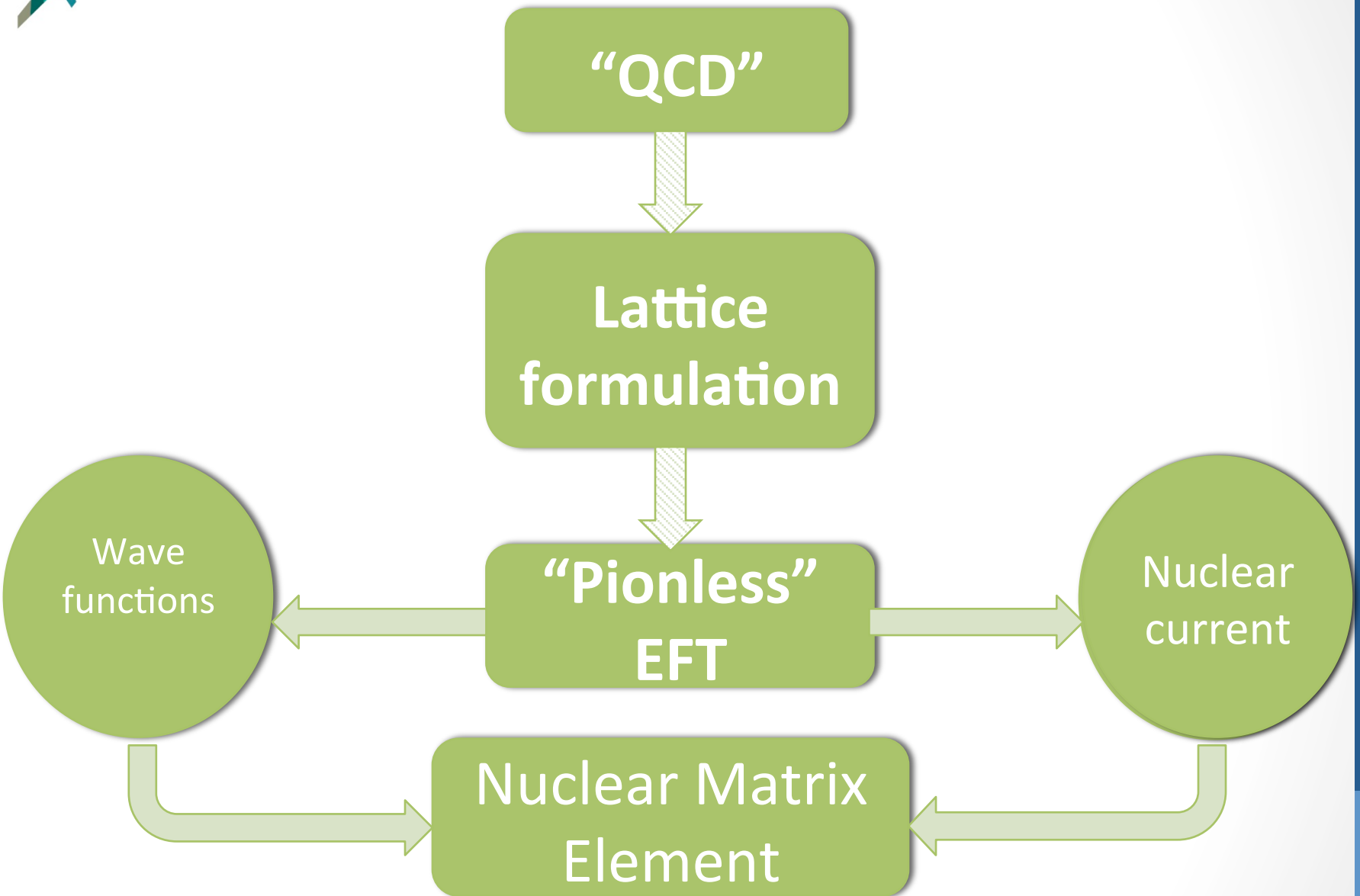


The interaction of a nucleus with an external (weak) probe





The interaction of a nucleus with an external (weak) probe



Lattice QCD calculation of l_1

PRL **115**, 132001 (2015)

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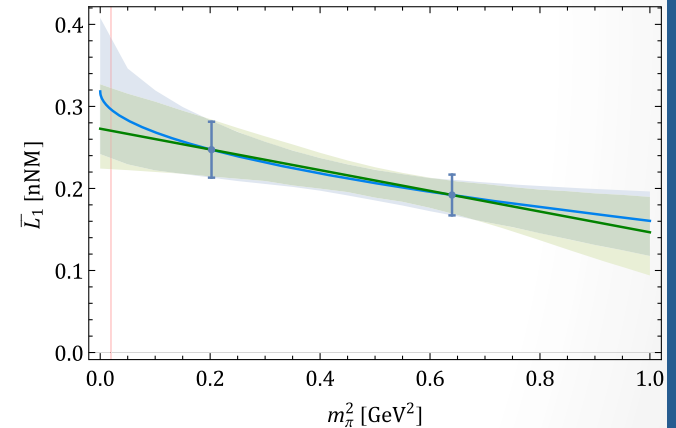
week ending
25 SEPTEMBER 2015

Ab initio Calculation of the $np \rightarrow d\gamma$ Radiative Capture Process

Silas R. Beane,¹ Emmanuel Chang,² William Detmold,³ Kostas Orginos,^{4,5} Assumpta Parreño,⁶
Martin J. Savage,² and Brian C. Tiburzi^{7,8,9}

(NPLQCD Collaboration)

$$\Delta E_{3S_1, 1S_0}(\mathbf{B}) = 2(\kappa_1 + \gamma_0 Z_d^2 \tilde{l}_1) \frac{e}{M} |\mathbf{B}| + \mathcal{O}(|\mathbf{B}|^2),$$



$$\tilde{X}_{M1} = \frac{Z_d}{-\frac{1}{a_1} + \frac{1}{2} r_1 |\mathbf{p}|^2 - i|\mathbf{p}|} \times \left[\frac{\kappa_1 \gamma_0^2}{\gamma_0^2 + |\mathbf{p}|^2} \left(\gamma_0 - \frac{1}{a_1} + \frac{1}{2} r_1 |\mathbf{p}|^2 \right) + \frac{\gamma_0^2}{2} l_1 \right]$$

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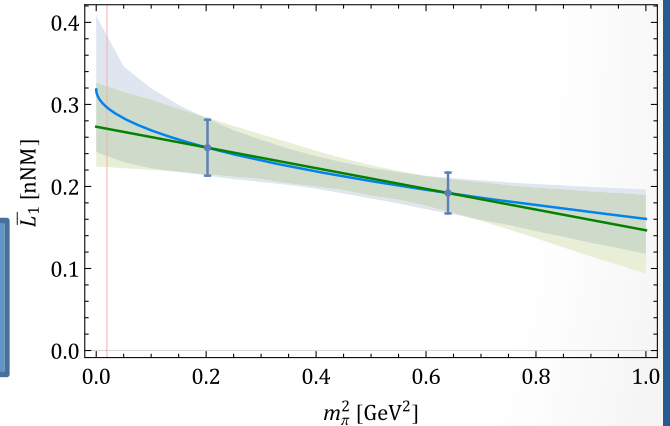
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$$l_1^{\text{qcd}} = -4.41 \left(\begin{array}{c} +15 \\ -16 \end{array} \right) \text{ fm.}$$

$$\sigma^{\text{qcd}} = 332.4 \left(\begin{array}{c} +5.4 \\ -4.7 \end{array} \right) \text{ mb}$$



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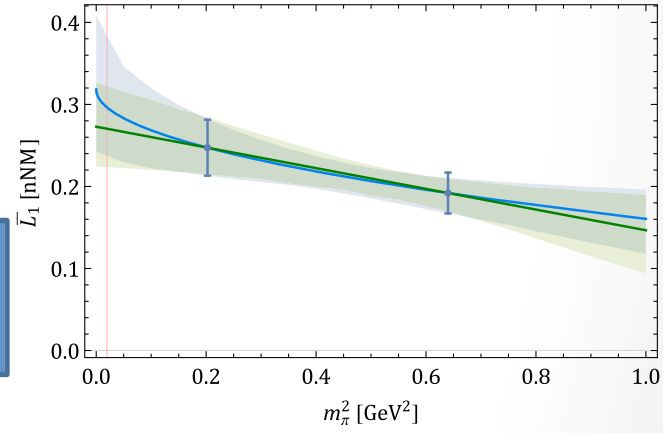
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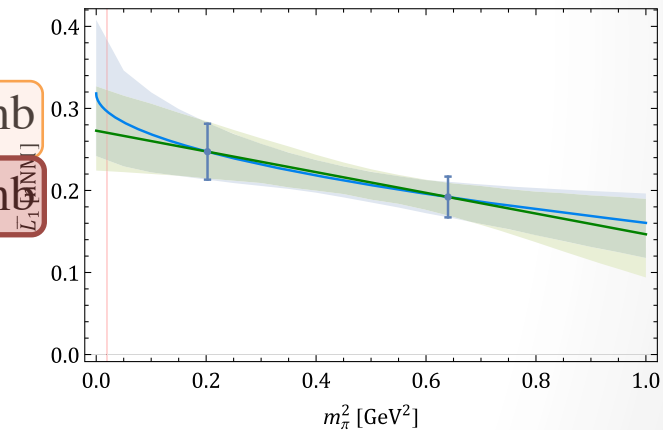
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$$\Delta E_{3S_1, 1S_0}(\mathbf{B}) = 2(\kappa_1 + \gamma_0 \tilde{l}_1) \frac{e}{M} |\mathbf{B}| + \mathcal{O}(|\mathbf{B}|^2),$$

Rho paramet. $l_1 = -3.934 \text{ fm} \rightarrow \sigma_{np} = 322.9 \text{ mb}$

Z-paramet. $l_1 = -5.48 \text{ fm} \rightarrow \sigma_{np} = 342.6 \text{ mb}$

This could be regarded as a measure of the NPLQCD uncertainty in predicting n+p fusion, due to the EFT Expansion.



$$\tilde{X}_{M1} = \frac{Z_d}{-\frac{1}{a_1} + \frac{1}{2} r_1 |\mathbf{p}|^2 - i|\mathbf{p}|} \times \left[\frac{\kappa_1 \gamma_0^2}{\gamma_0^2 + |\mathbf{p}|^2} \left(\gamma_0 - \frac{1}{a_1} + \frac{1}{2} r_1 |\mathbf{p}|^2 \right) + \frac{\gamma_0^2}{2} l_1 \right]$$



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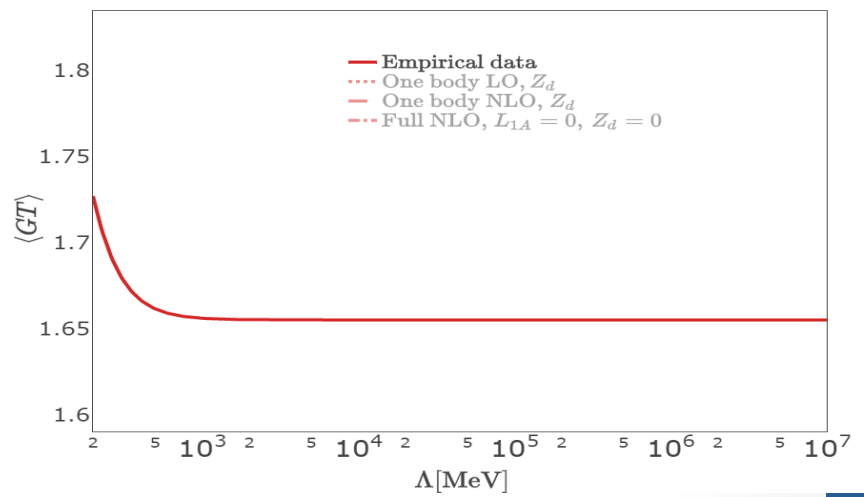
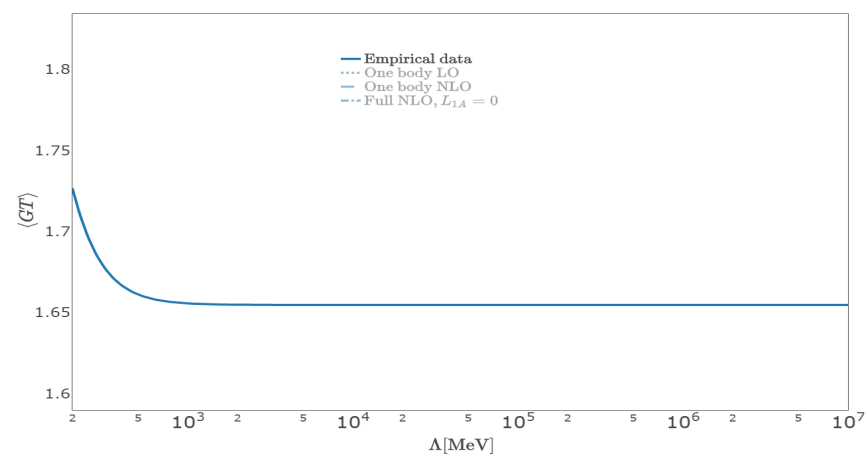


Triton decay – GT cutoff independence

Rho-parameterization

Z-parameterization

$$ft = \frac{K}{G_F^2 V_{ud}^2 \left[\left| \langle {}^3\text{H} \| \mathcal{V}_\mu^+ \| {}^3\text{He} \rangle \right|^2 + \frac{f_A}{f_V} \left| \langle {}^3\text{H} \| \mathcal{A}_\mu^+ \| {}^3\text{He} \rangle \right|^2 \right]}$$



“Empirical” extraction of GT (using calculated F strength)

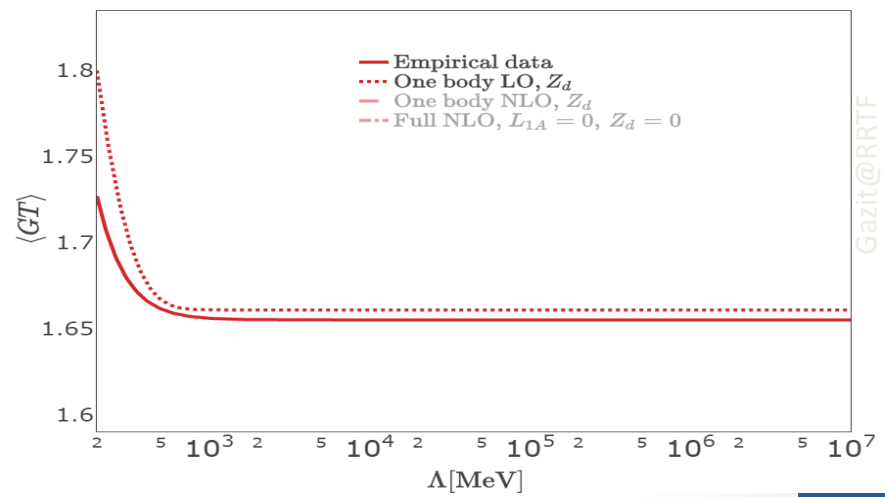
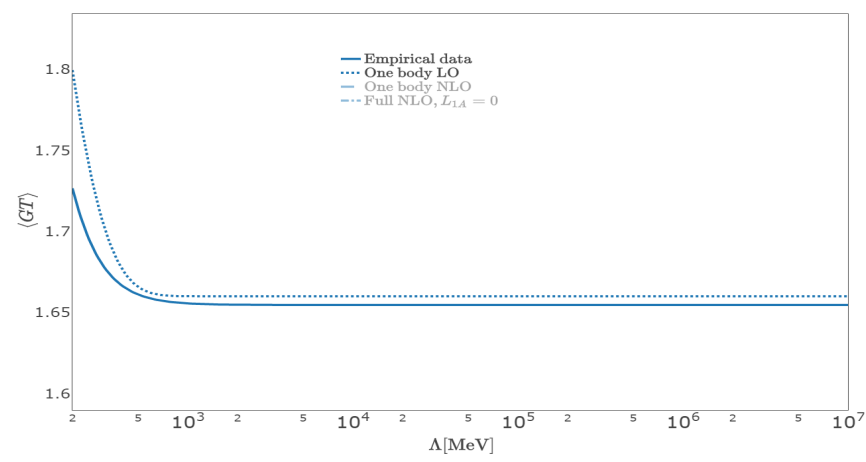


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Adding the LO 1-body contribution

Gazit@RRTF

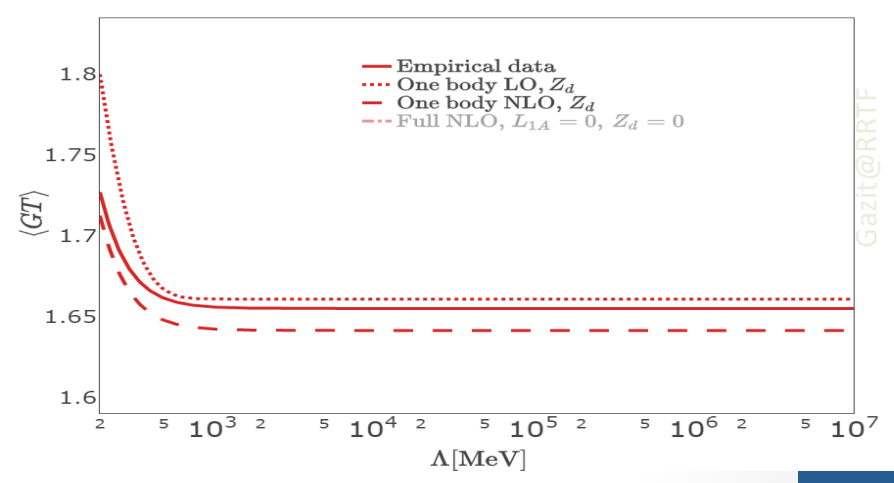
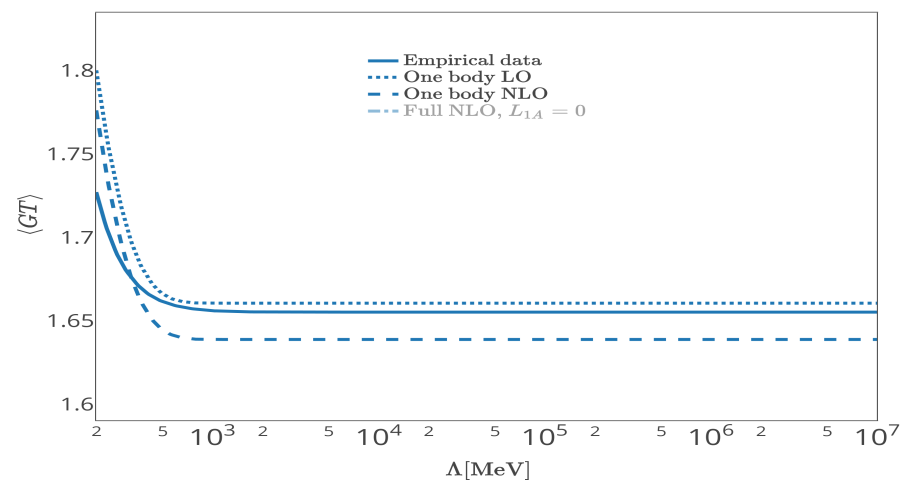


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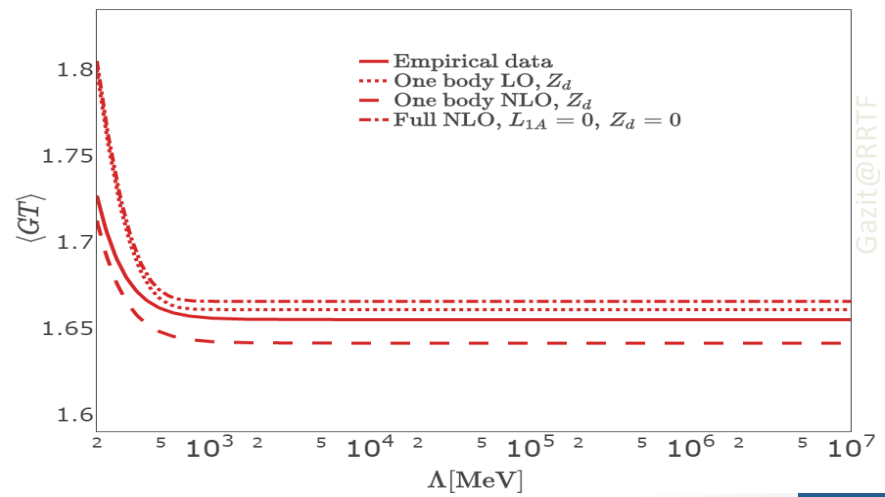
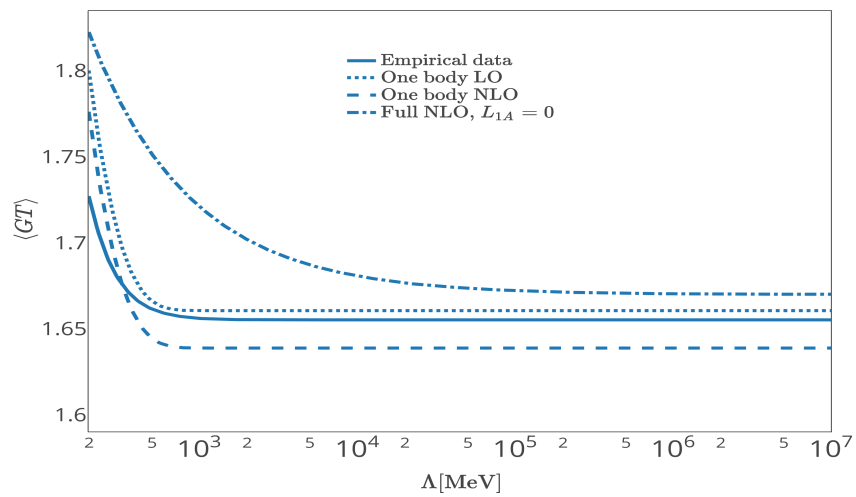


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Adding all contribution, but L_{1A}

1st estimate of theoretical uncertainty:
All NLO contributions are of the same order (1-2%),
one can estimate higher order effects as the NLO contribution.

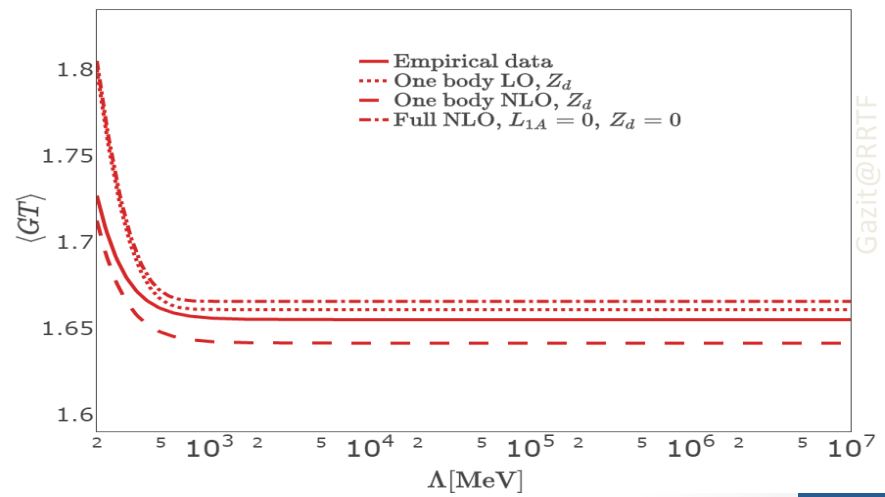
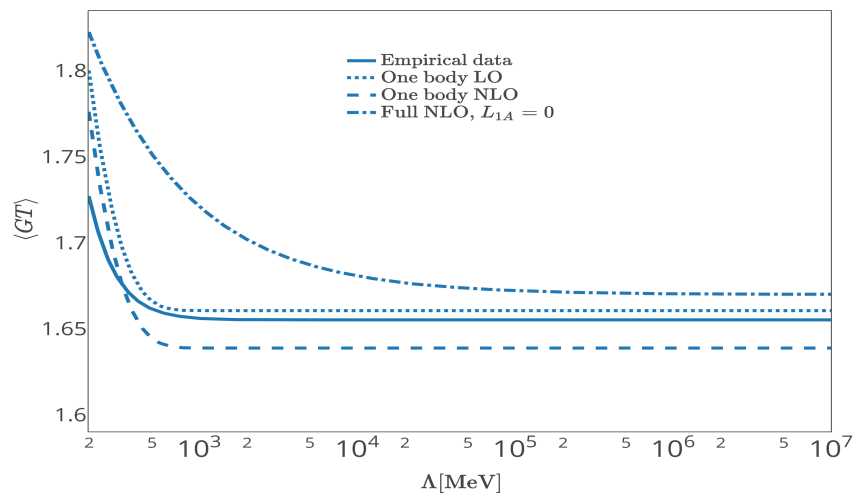


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Adding all contributions

Translates to ±2% difference in pp fusion

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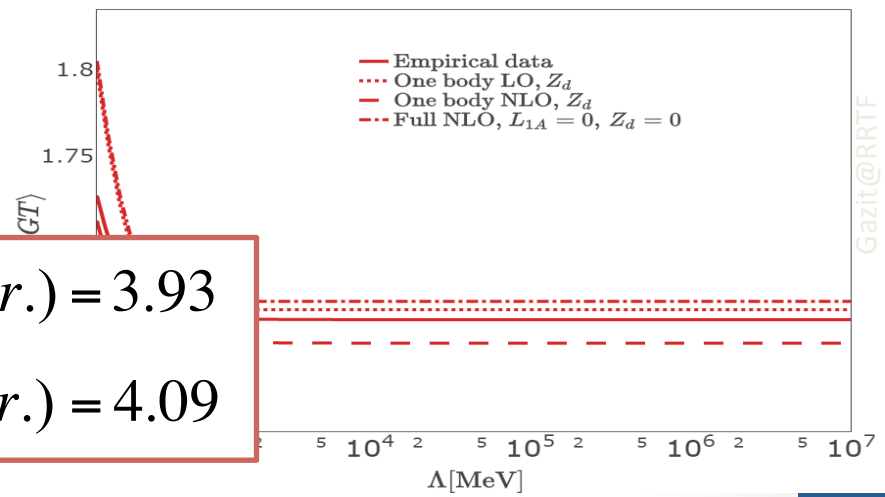
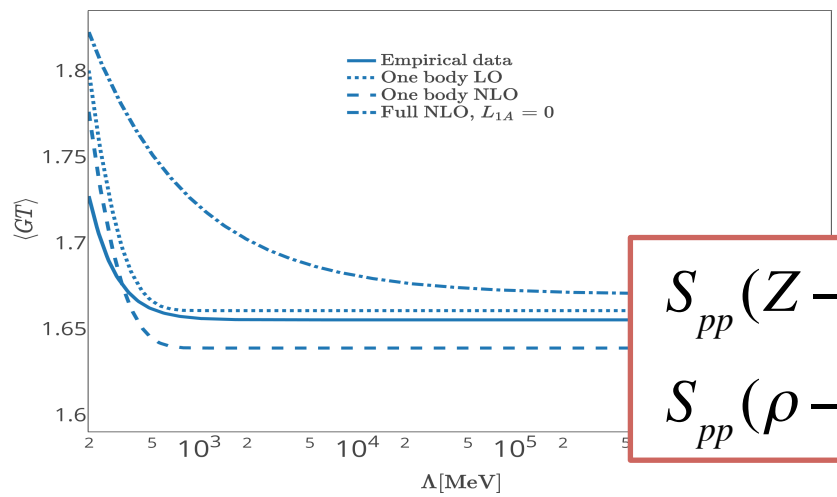


Triton decay – GT cutoff independence

Rho-parameterization

Z-parameterization

$$ft = \frac{K}{G_F^2 V_{ud}^2 \left[\left| \langle {}^3\text{H} \| \mathcal{V}_\mu^+ \| {}^3\text{He} \rangle \right|^2 + \frac{f_A}{f_V} \left| \langle {}^3\text{H} \| \mathcal{A}_\mu^+ \| {}^3\text{He} \rangle \right|^2 \right]}$$



$S_{pp} (Z - par.) = 3.93$
 $S_{pp} (\rho - par.) = 4.09$

2nd estimate of theoretical uncertainty:
 difference between Zed and Rho Parameterizations.

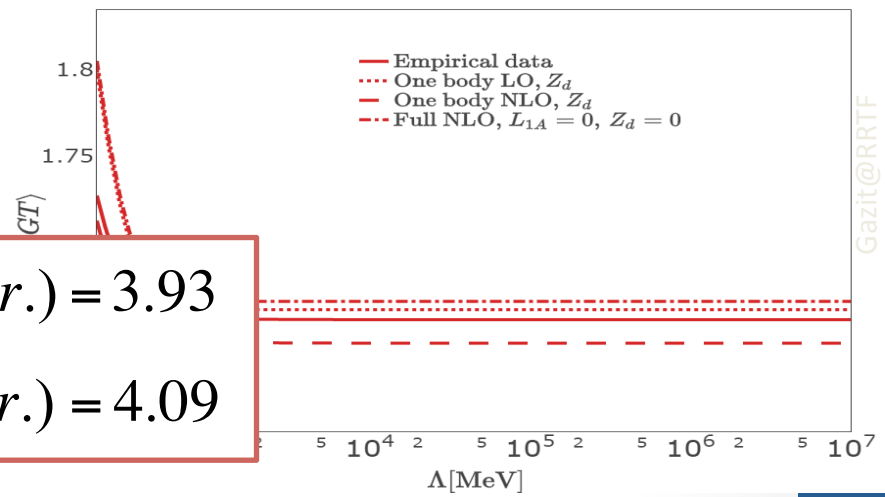
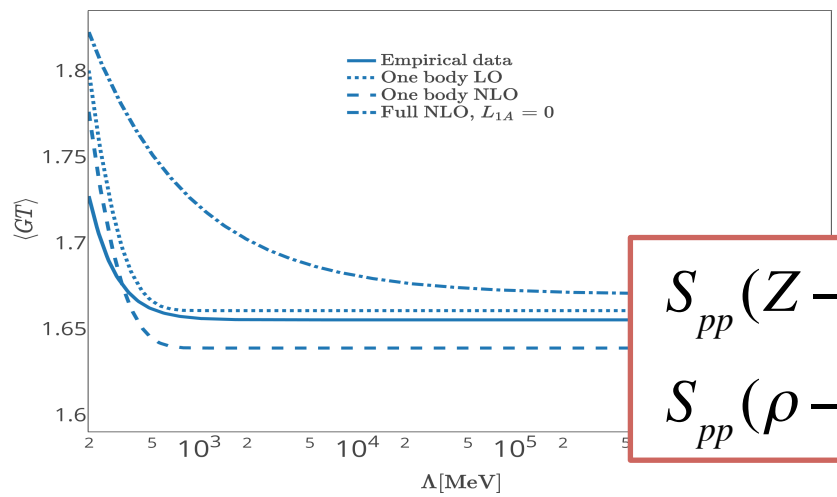


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$S_{pp} (Z - par.) = 3.93$
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Translates to $\pm 2\%$ difference in pp fusion

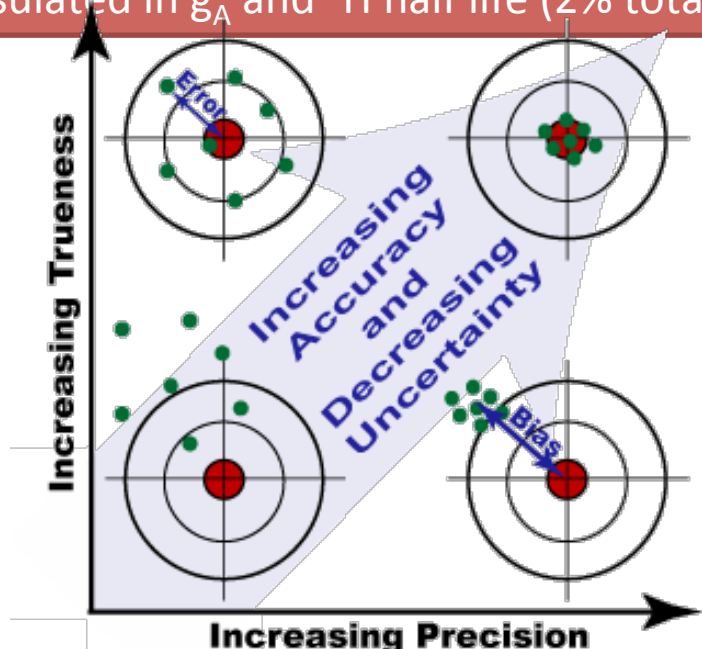
2nd estimate of theoretical uncertainty:
difference between Zed and Rho Parameterizations.



So... is 3% too big to be called precision physics?

$S_{pp}(g_A = 1.2701) =$	4.01	\pm	0.08 \pm	0.07 \pm	0.04
$S_{pp}(g_A = 1.275) =$	4.12	\pm	0.08 \pm	0.07 \pm	0.04
<div style="border: 1px solid blue; padding: 5px; display: inline-block;">g_A systematic uncertainty</div>			theoretical uncertainty	g _A stat. unc.	³ H half-life syst. unc.

i.e., theoretical uncertainty of the same order of systematic experimental error encapsulated in g_A and ³H half life (2% total).





Summary

- Pionless EFT reproduces low-energy electroweak observables to a very good precision ($\sim 1\%$), even at NLO, and allows reliable uncertainty estimates.
- Theoretical uncertainty estimated from:
 - (Natural) Size of NLO contribution (all NLO contributions are of the same order of magnitude).
 - Difference between Zed and Rho parameterizations.
 - Both error estimates lead to about 2% uncertainty.
- EM sector confirms calculation procedure.
- Lattice QCD for nuclei is a new front for π EFT
- Based on the EM sector, a theoretical prediction for pp fusion:

$$S_{pp}(g_A = 1.2701) = 4.01 \pm_{theory} 0.08 \pm_{g_A(1\sigma)} 0.07 \pm_{^3\text{H half life}} 0.04$$

$$S_{pp}(g_A = 1.275) = 4.12 \pm_{theory} 0.08 \pm_{g_A(1\sigma)} 0.07 \pm_{^3\text{H half life}} 0.04$$

- Better determination of g_A is necessary!
- (^3H half life is also an open exp. issue).