Calculations at NLO A=3 bound state transitions

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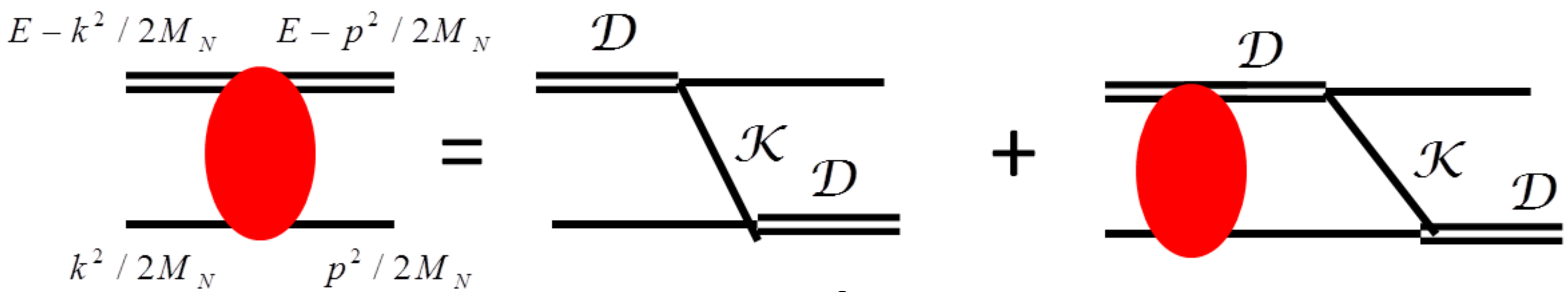
EMMI RRTF Workshop
Darmstadt

Outline

- Faddeev equation
- LO normalization.
- LO matrix element
- NLO matrix element
- Summary

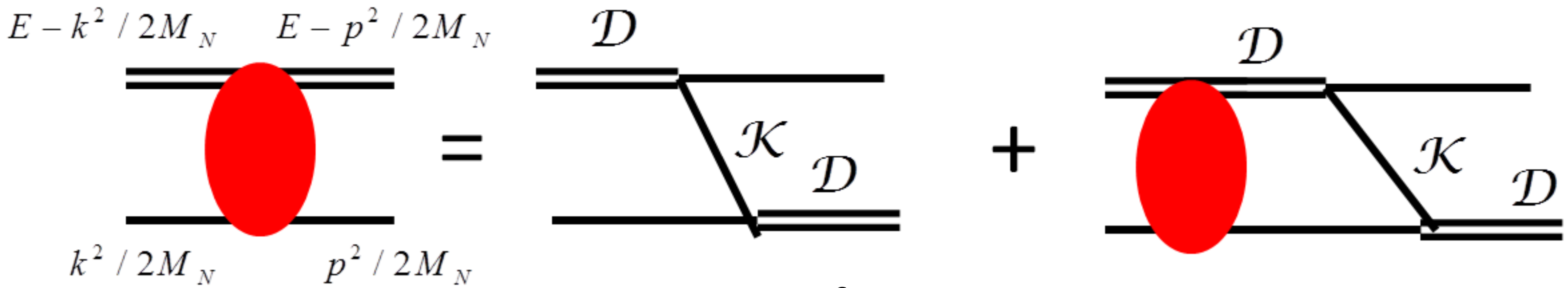
Faddeev equation- scattering amplitude for bound state



$$T(k,p) = \mathcal{K}(k,p) + \int dq T(k,q)\mathcal{D}(q)\mathcal{K}(q,p)$$

König thesis, 2013

Faddeev equation- scattering amplitude for bound state



$$T(k,p) = \mathcal{K}(k,p) + \int dq T(k,q)\mathcal{D}(q)\mathcal{K}(q,p)$$

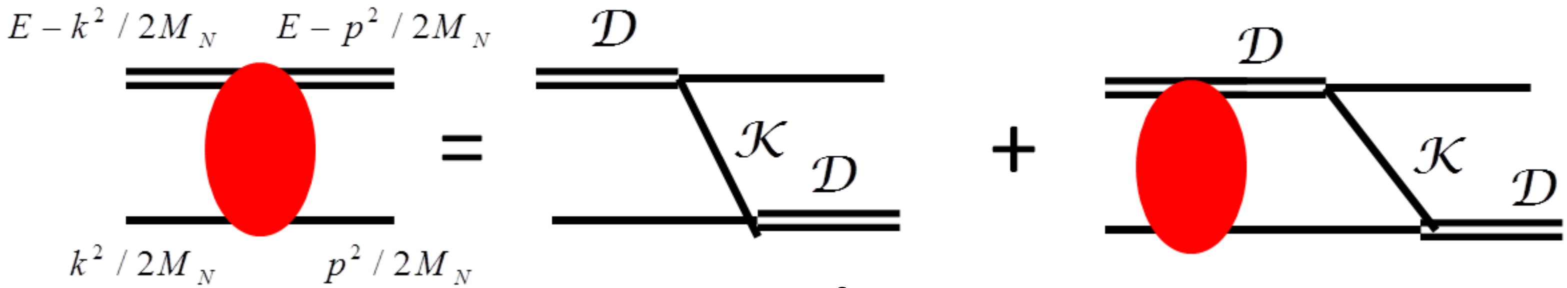
- For a bound state:

$$T(k,p) = \frac{\mathcal{B}(k)\mathcal{B}(p)}{E - E_B} + \not{R}$$

$\mathcal{B}(k)$ - Bethe-Slapeter wave function

Kӧnig thesis, 2013

Faddeev equation- scattering amplitude for bound state



$$T(k,p) = \mathcal{K}(k,p) + \int dq\, T(k,q)\mathcal{D}(q)\mathcal{K}(q,p)$$

- For a bound state:

$$T(k,p) = \frac{\mathcal{B}(k)\mathcal{B}(p)}{E - E_B} + \not{\mathcal{R}}$$

$\mathcal{B}(k)$ - Bethe-Slapeter wave function

$$T(k,q) \otimes \left\{\mathcal{D}(q)\left[\left(\hat{I} - \mathcal{K}\right)_{E=-E_B}\right]\mathcal{D}(q')\right\} \otimes T(q',p) = T(k,p)$$

$$1 = (\mathcal{B}(q))^T \otimes \left\{\mathcal{D}(q)\left[\frac{d}{dE}\left(\hat{I} - \mathcal{K}\right)_{E=-E_B}\right]\mathcal{D}(q')\right\} \otimes \mathcal{B}(q')$$

$$A(\dots,\mathrm{q}) \otimes B(q,\dots) = \int \frac{d^3q}{(2\pi)^3} A(\dots,\mathrm{q})B(q,\dots)$$

König thesis, 2013

Diagrammatic form of Bethe–Salpeter normalization

$$1 = (\mathcal{B}(q))^T \otimes \left\{\mathcal{D}(q)\left[\frac{d}{dE}\left(\hat{I} - \mathcal{K}\right)_{E=-E_B}\right]\mathcal{D}(q')\right\} \otimes \mathcal{B}(q')$$

$$\hat{I}(q,q') = \frac{2\pi^2}{q^2}\delta(q-q')\mathcal{D}(q,E)^{-1}$$

Diagrammatic form of Bethe–Salpeter normalization

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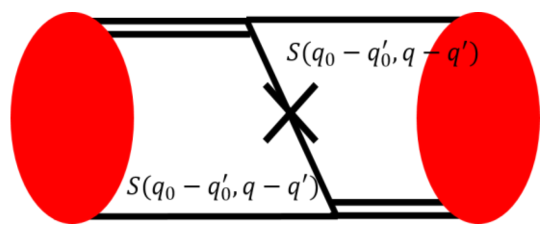
$$\hat{I}(q, q') = \frac{2\pi^2}{q^2} \delta(q - q') \mathcal{D}(q, E)^{-1}$$

- The Bethe–Salpeter normalization is equivalent to all the diagrams that connect the two bubbles:

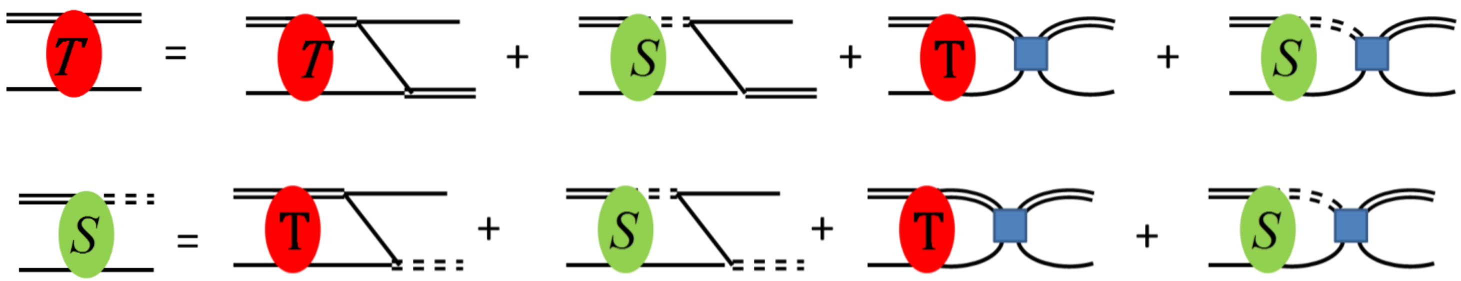
$\mathcal{D}(q) \left[\frac{d}{dE} \mathcal{K} \right] \mathcal{D}(q')$:



$\mathcal{D}(q) \left[\frac{d}{dE} \hat{I} \right] \mathcal{D}(q')$:

^{3}H bound state in $\not{\pi}$EFT

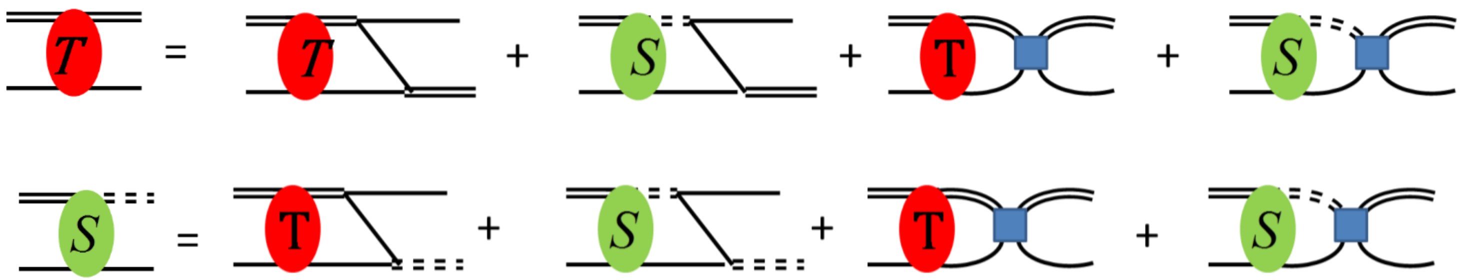
- ^{3}H can be written as a *n-d* amplitude:



- The red bubble represents the deuteron channel, while the green bubble represents the triplet channel.
- These equations are coupled integral equations and can be solved for $E_B(^3\mathrm{H}) = 8.48$ MeV with the 3-body three body force $\left(H(\Lambda)\right)$

P.F. Bedaque , H.-W. Hammer and U. van Kolck 1999

Triton normalization:



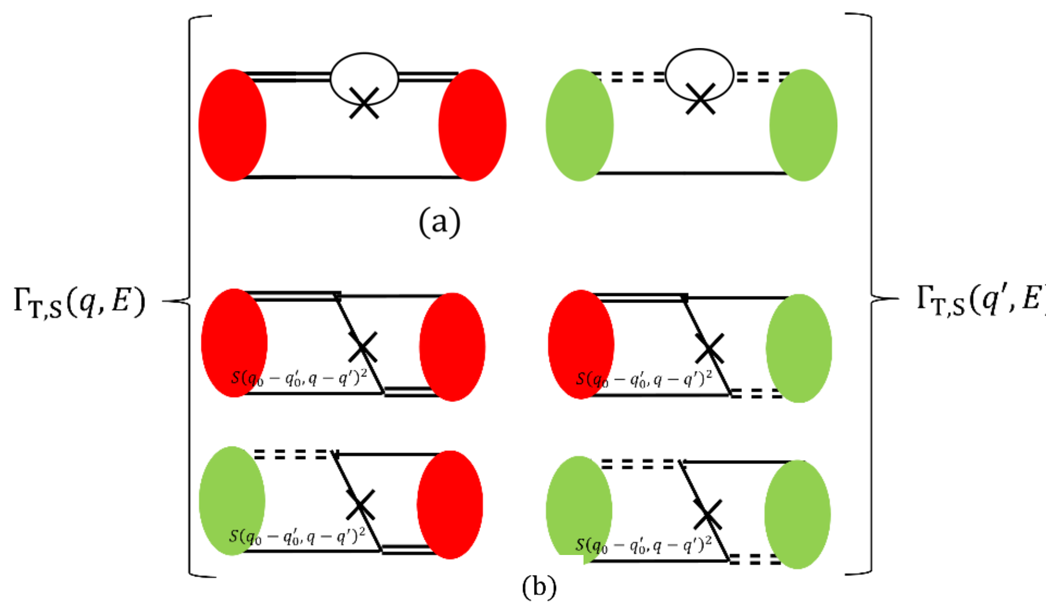
$$I_{t,s} = \frac{2\pi^2}{q^2}\delta(q-q')\mathcal{D}_{t,s}(q,E)^{-1}$$

$$\vec{\mathcal{B}}(q,E) = \begin{pmatrix}\Gamma_T(q,E)\\ \Gamma_S(q,E)\end{pmatrix}$$

$$\hat{\mathcal{D}}(p,E) = \begin{pmatrix}\mathcal{D}_T(q,E)\\ \mathcal{D}_S(q,E)\end{pmatrix}$$

(a)= $\frac{d}{dE}\hat{I}(q,q',E)$

(b)= $\frac{d}{dE}\mathcal{K}(q.q',E)$

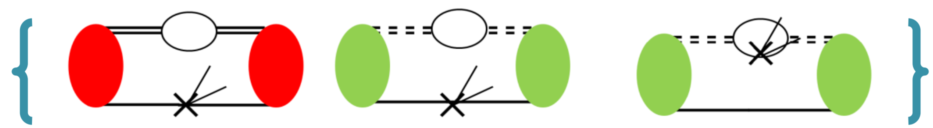


Fermi transition with $\alpha = 0$:

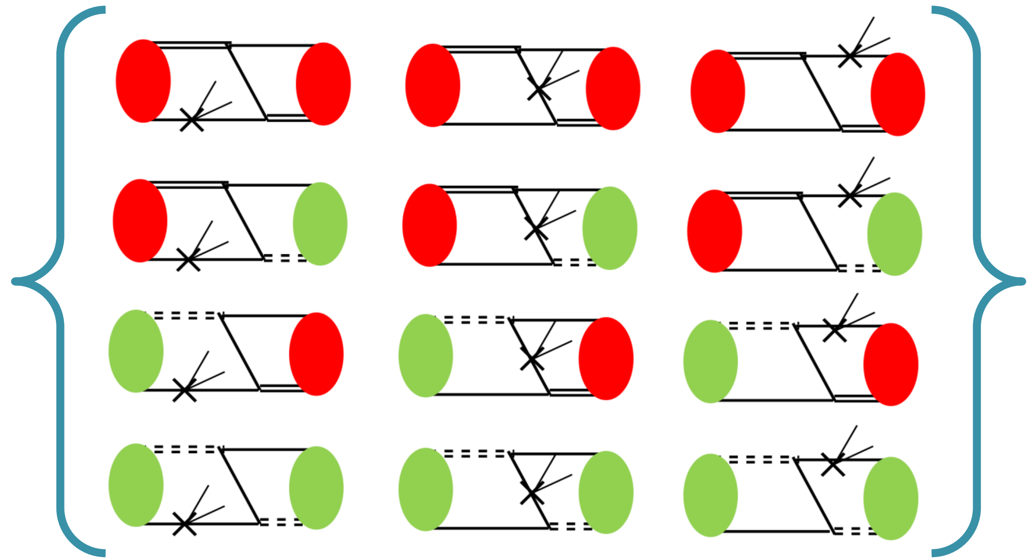
$$\langle F \rangle = \frac{\langle {}^3\mathrm{He} || \tau^- || {}^3\mathrm{H} \rangle}{\sqrt{2J+1}}$$

If $\alpha = 0$: $a_{np} = a_{pp}$, $E_{3H} = E_{3He}$

$$\hat{\mathcal{D}}(q)\left[\frac{d}{dE}\hat{I}(q,q',E)\right]\hat{\mathcal{D}}(q'):$$



$$\hat{\mathcal{D}}(q)\left[\frac{d}{dE}\mathcal{K}(q,q',E)\right]\hat{\mathcal{D}}(q'):$$



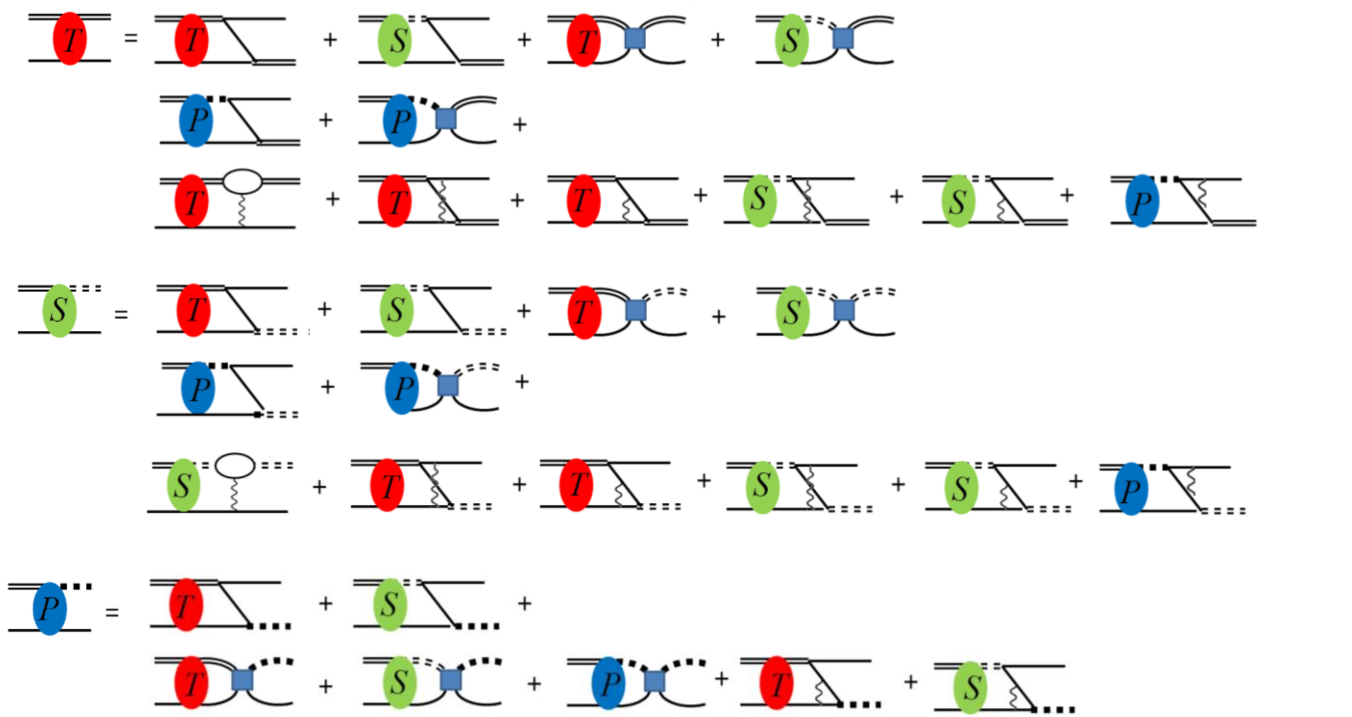
In LO with $\alpha = 0$:

$$\langle F \rangle = 1$$

H. D.L, and D. Gazit 2016…

^{3}He bound state in π̸EFT

- ^{3}He can be written as a result of *p-d* scattering (in LO):

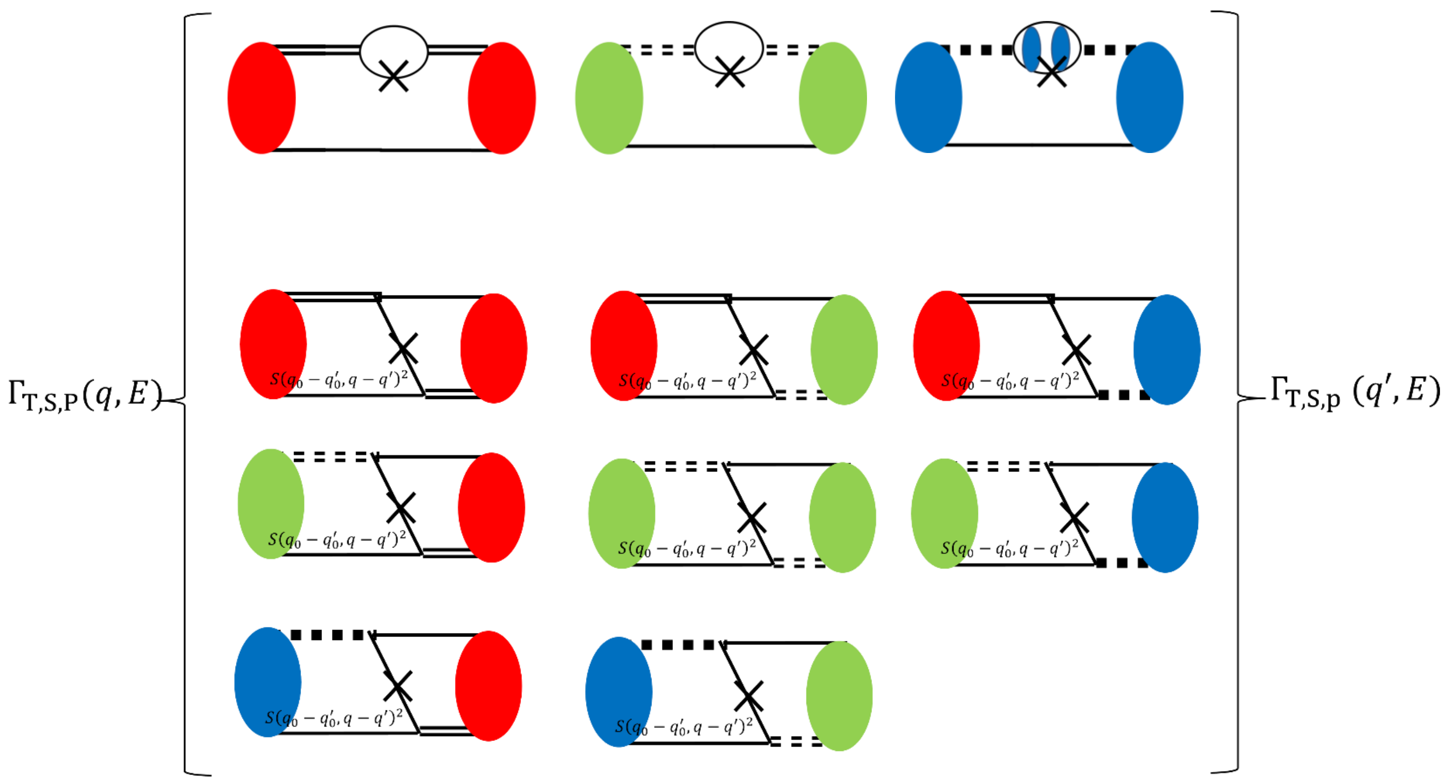


- The red bubble represents the deuteron channel, the green bubble represents the np and the blue bubble represents the pp channel .

M. C. Birse and S.-I. Ando. 2010. S König and H.-W. Hammer, 2011.

^{3}He normalization:

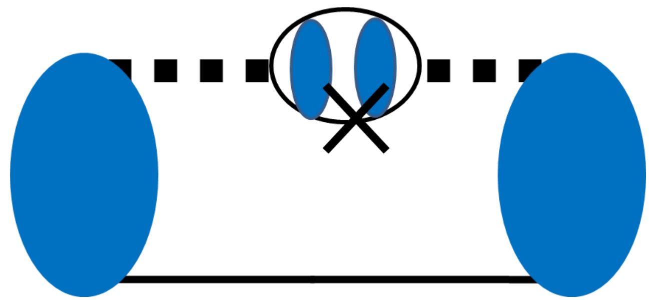
$$I_{t,s,pp} = \frac{2\pi^2}{q^2}\delta(q-q')\mathcal{D}_{t,s,pp}(q,E)^{-1}, \qquad \vec{\mathcal{B}}(q,E) = \begin{pmatrix}\Gamma_T(q,E)\\ \Gamma_S(q,E)\\ \Gamma_P(q,E)\end{pmatrix}, \hat{\mathcal{D}}(p,E) = \begin{pmatrix}\mathcal{D}_T(q,E)\\ \mathcal{D}_S(q,E)\\ \mathcal{D}_{pp}(q,E)\end{pmatrix}$$



3He normalization:

$$I_{t,s,pp} = \frac{2\pi^2}{q^2}\delta(q-q')\mathcal{D}_{t,s,pp}(q,E)^{-1}, \qquad \vec{\mathcal{B}}(q,E) = \begin{pmatrix}\Gamma_T(q,E)\\ \Gamma_S(q,E)\\ \Gamma_P(q,E)\end{pmatrix}, \hat{\mathcal{D}}(p,E) = \begin{pmatrix}\mathcal{D}_T(q,E)\\ \mathcal{D}_S(q,E)\\ \mathcal{D}_{pp}(q,E)\end{pmatrix}$$

$$\mathcal{D}_{pp}(q)\left[\frac{d}{dE}I_{pp}\right]\mathcal{D}_{pp}:$$



$$\frac{\partial}{\partial E}\frac{1}{\mathcal{D}_{pp}(q,E)} = \frac{M_N y_s^2}{4\pi\sqrt{3q^2-4M_N E}} + 2\alpha^2 M_N^4 y_s^2 \frac{\psi^{(1)}\left(\frac{2M_N\alpha}{\sqrt{\frac{3q^2}{4}-4M_N E}}\right)}{\pi(3q^2-4M_N E)} - \frac{\alpha^2 M_N^3 y_s^2}{\pi\alpha^2 M_N^4 y_s^2}$$

$\psi^{(1)}$ is the first derivation of PolyGamma function

^{3}He normalization – adding a photon:

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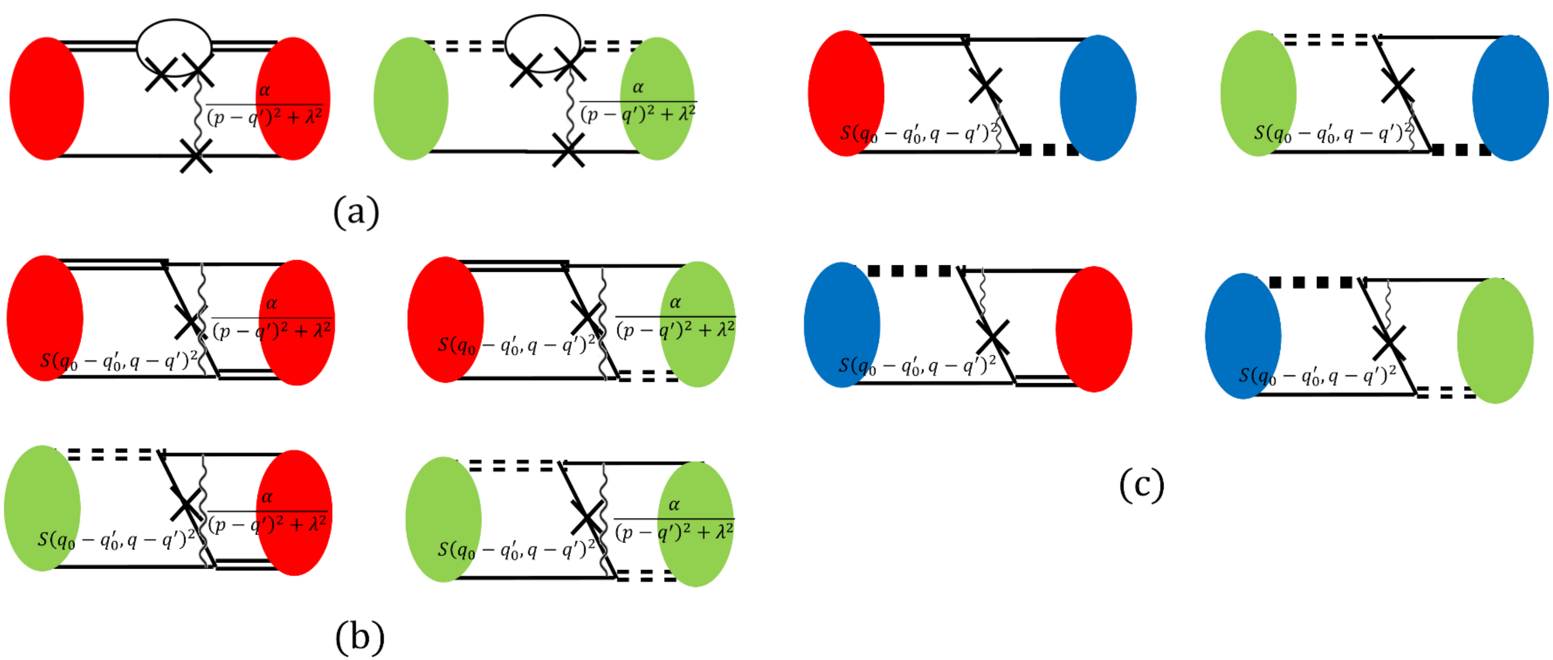

^{3}He Energy Shift matrix element- LO

S. König et al 2015

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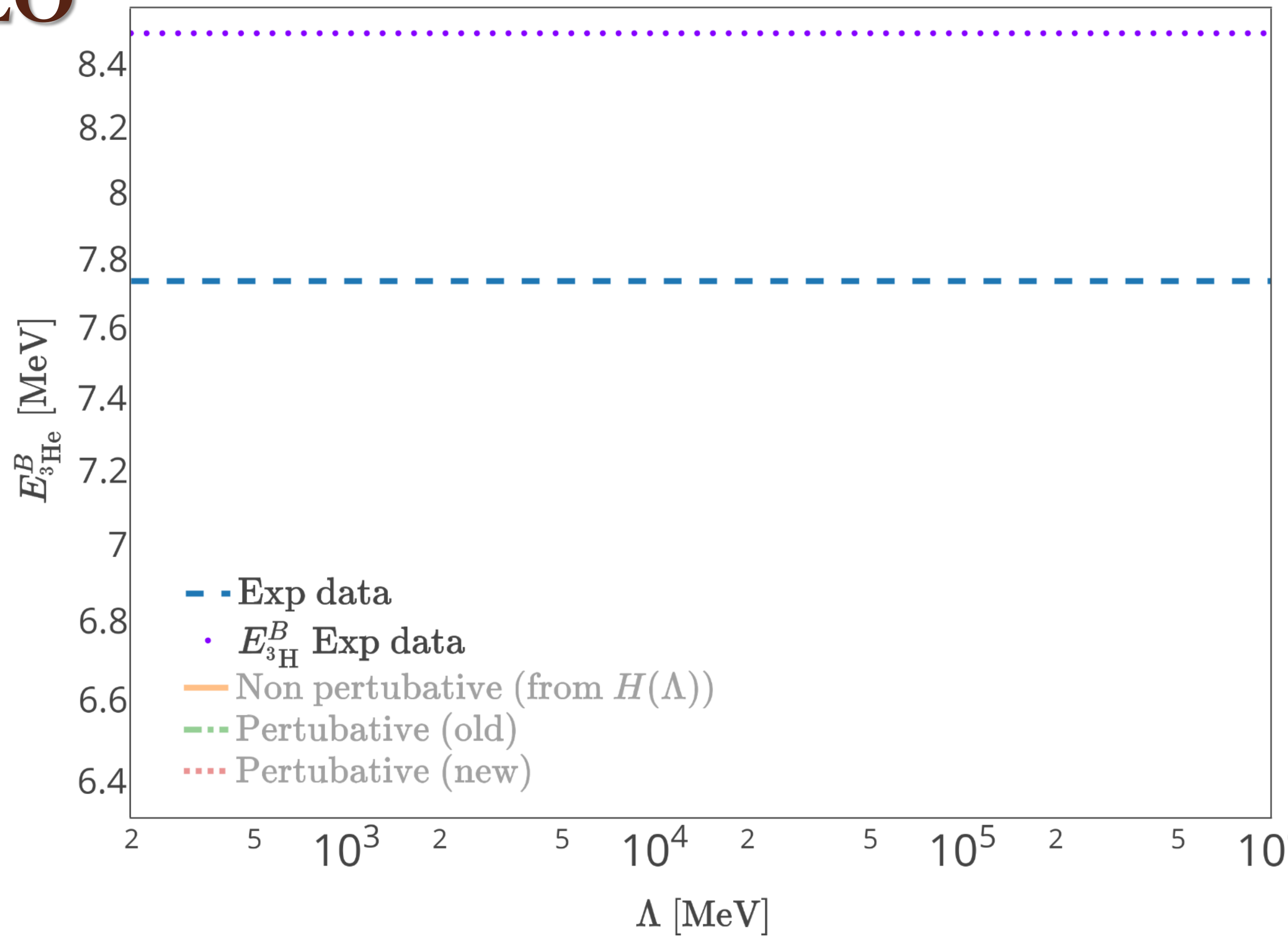

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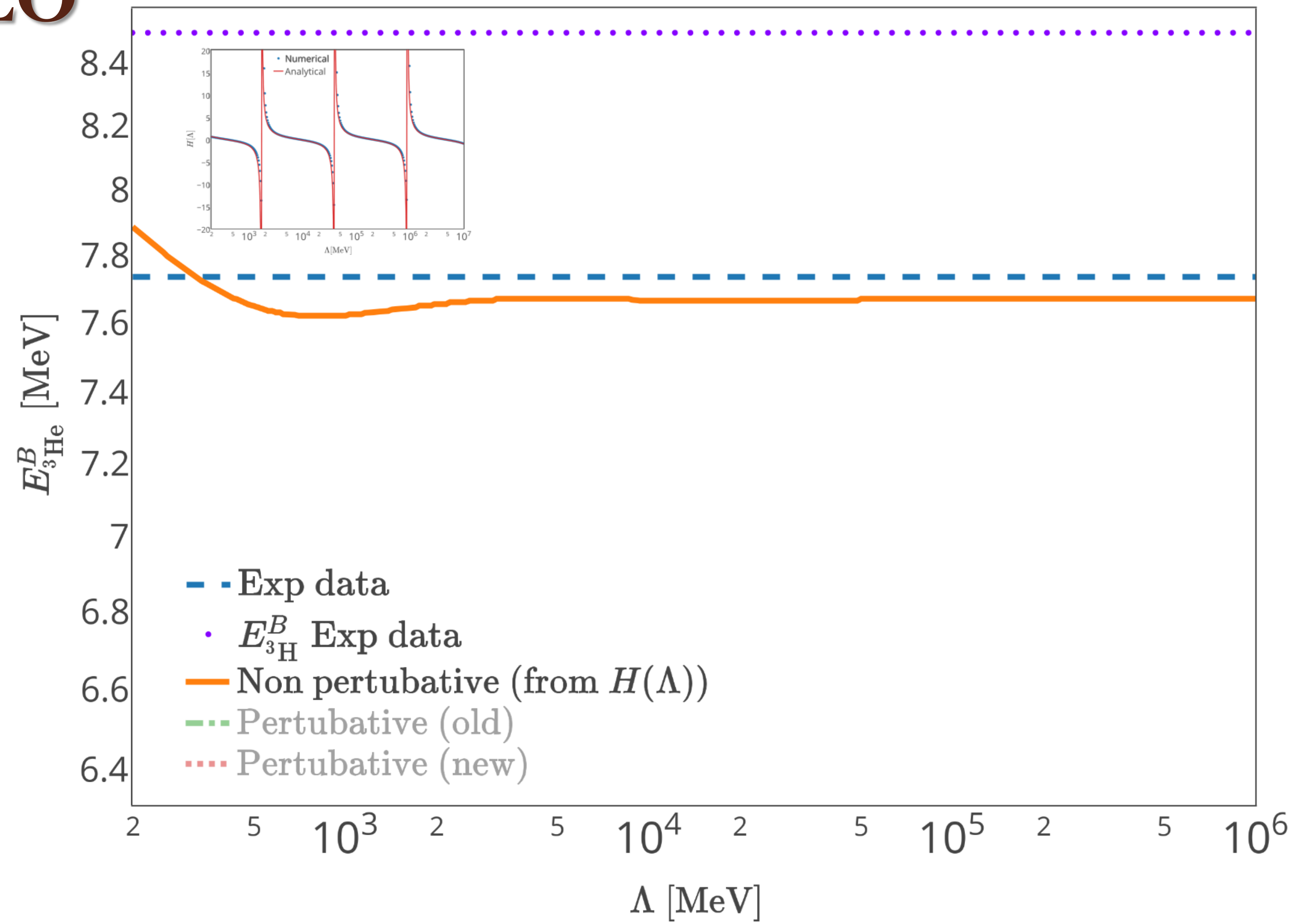

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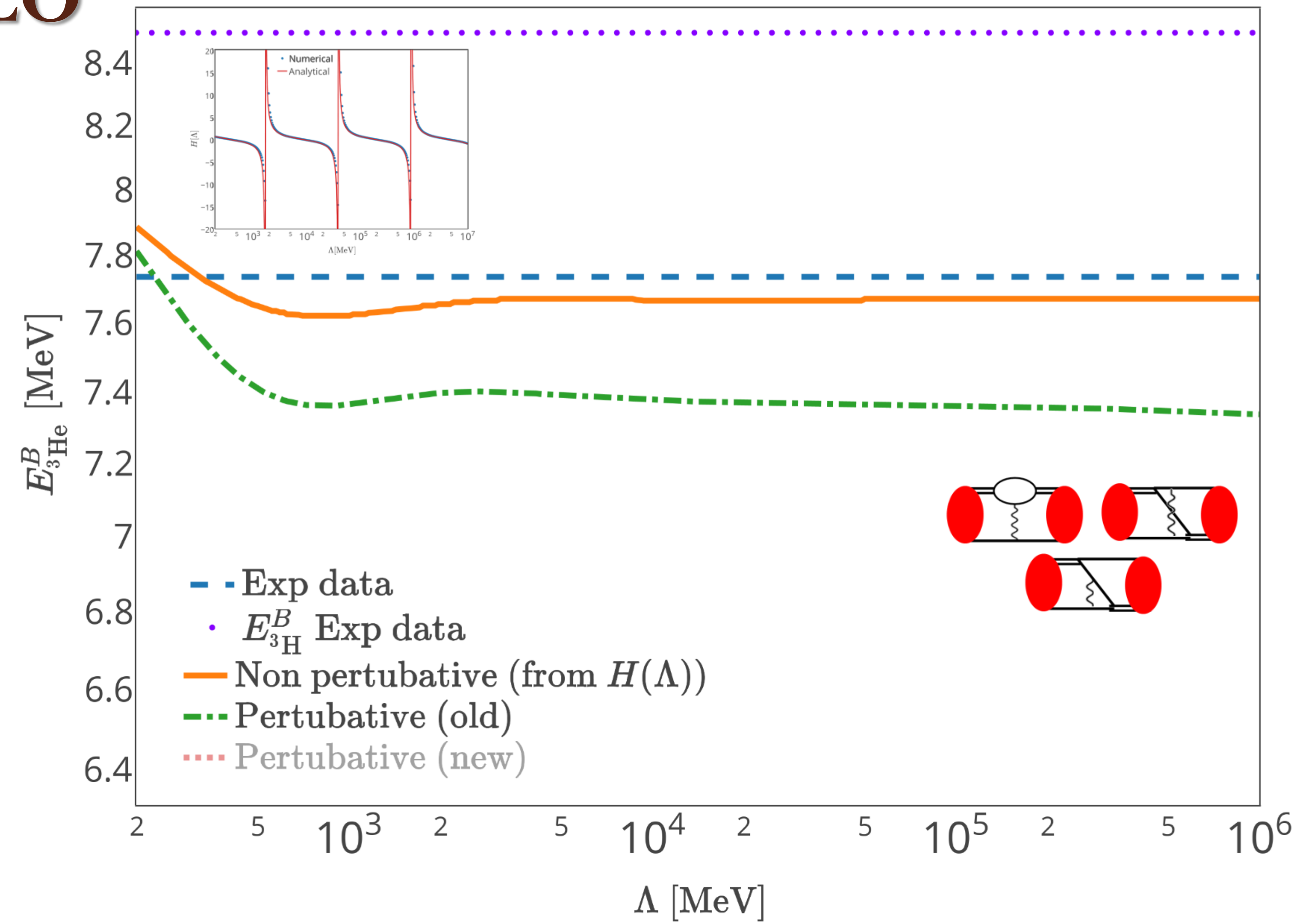

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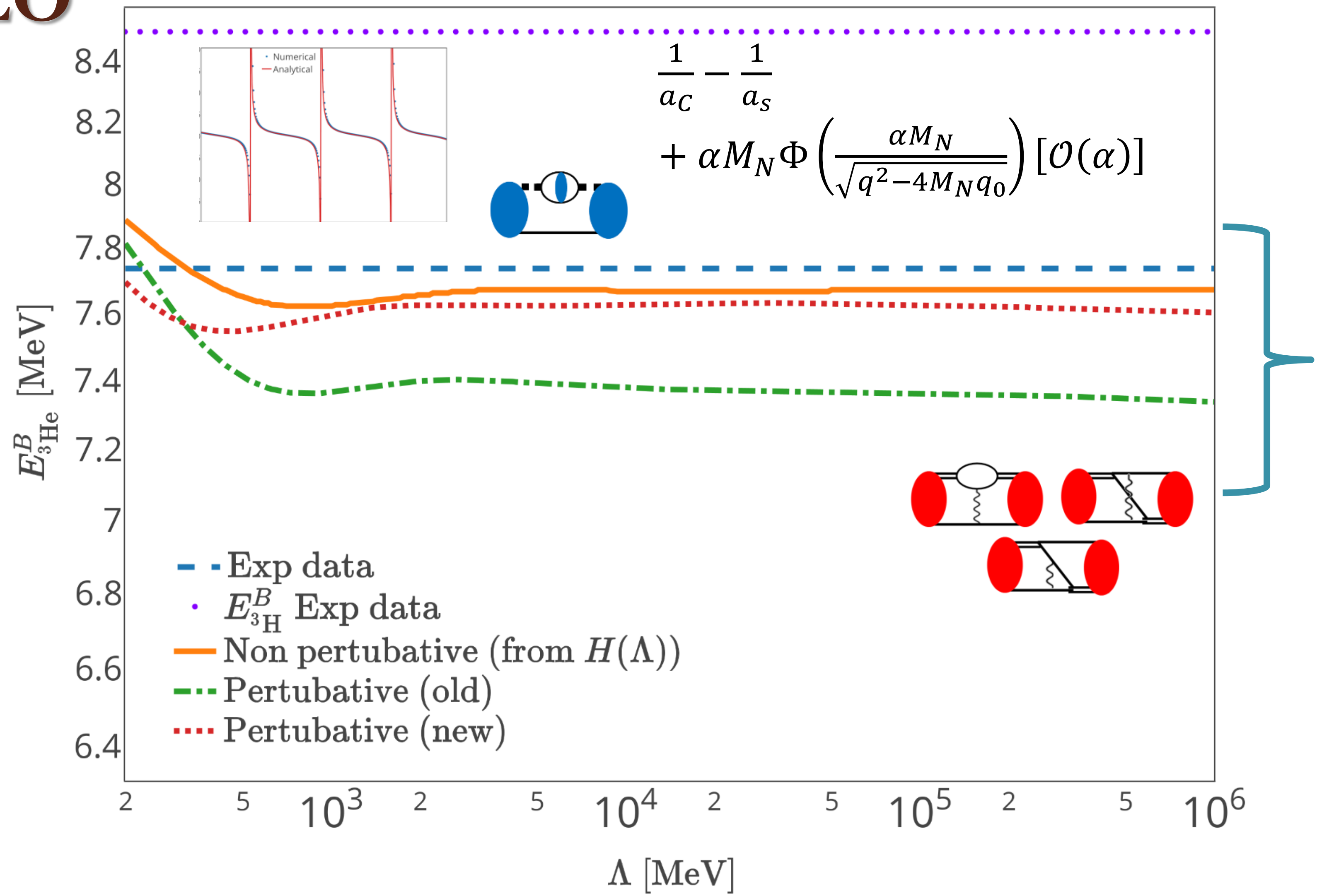
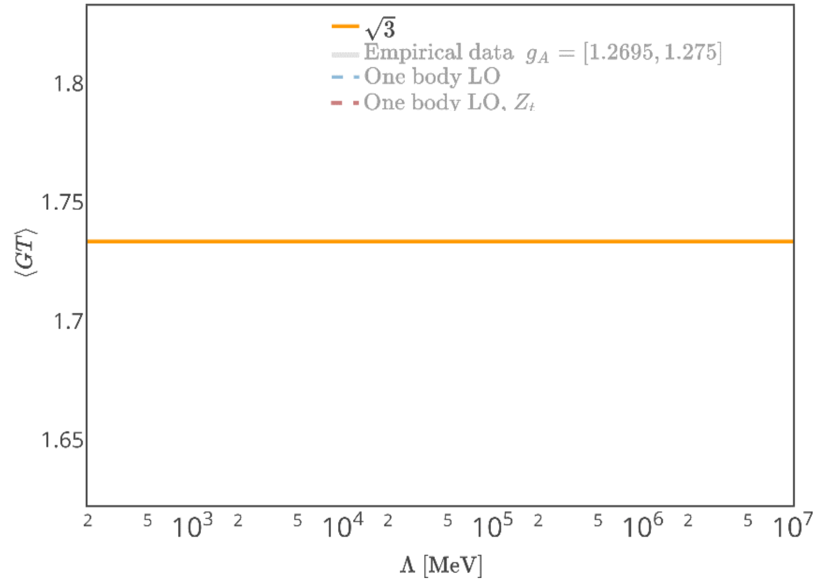


S. König et al 2015

Gamow -Teller transition - LO

$$\langle GT \rangle = \frac{\langle {}^3\mathrm{He} || \boldsymbol{\sigma}\tau^- || {}^3\mathrm{H} \rangle}{\sqrt{2J+1}}$$

- Yellow - $\sqrt{3}$ (LO with $\alpha = 0$)



H. D.L, and D. Gazit 2016…

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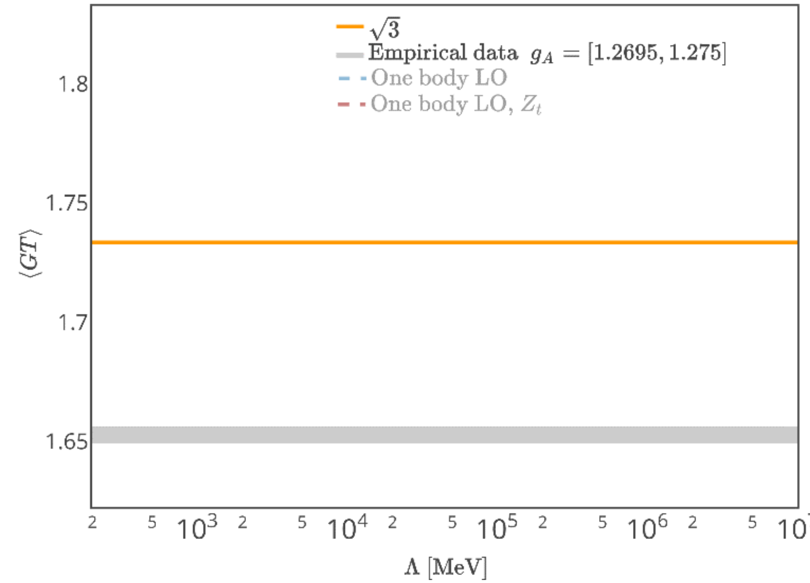
$$\langle GT\rangle = \sqrt{3}\,\frac{1.213}{g_A}$$



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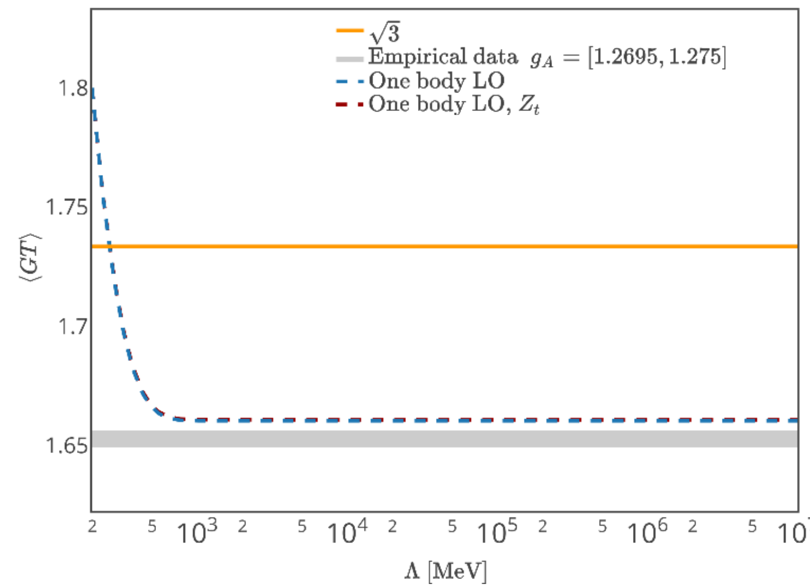
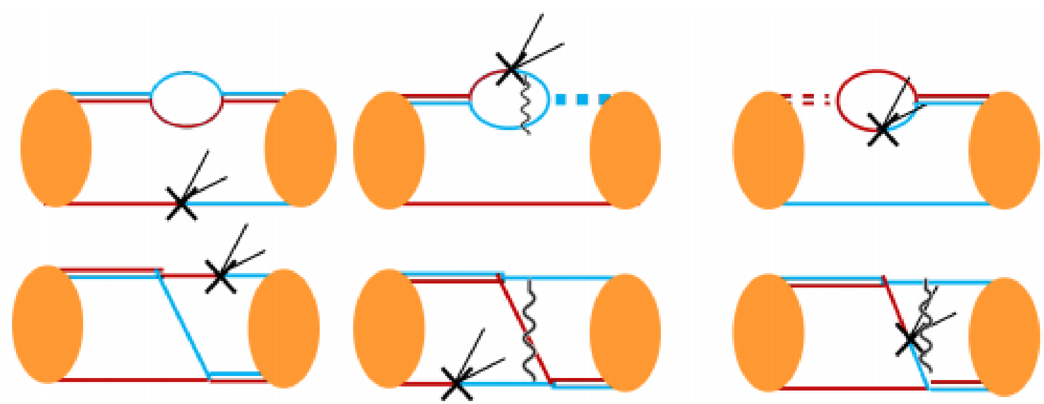
- Dashed – one body LO





H. D.L, and D. Gazit 2016…

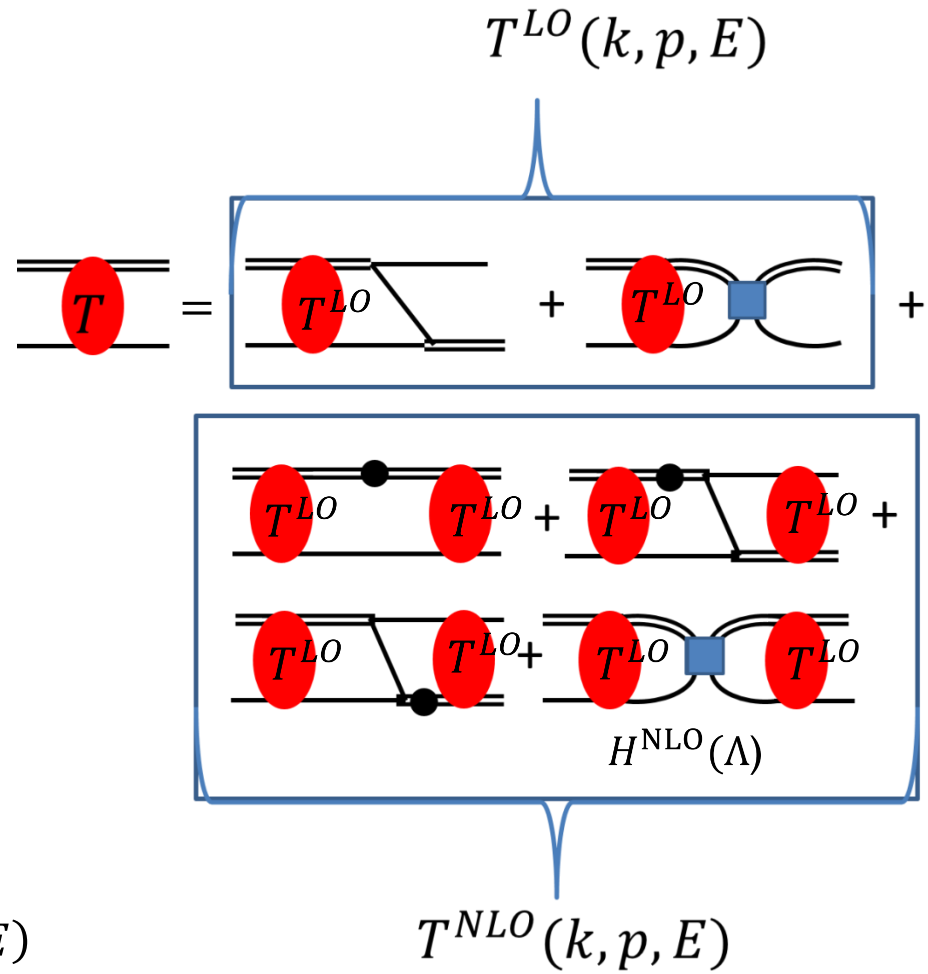
LO – summary

- Diagrammatic form of the B.S normalization
- LO matrix element can be calculated as all possible connections between two amplitude.
- ^{3}He LO energy shift: Summing over all possible diagrams is almost equivalent to the non-perturbative solution.
- Done with LO, moving to NLO…

NLO A=3, T-matrix

UP to NLO:

- The NLO insertion are:
  $\rho, H^{NLO}(\Lambda)$
- $T^{LO}(k,p,E)$ is describing a bound state:
  $$\frac{\mathcal{B}^{LO}(k)\mathcal{B}(p)^{LO}}{E-E_B}$$
- $\mathcal{B}^{LO}(k)$ is normalized.
- Black circle – effective range insertion



$$T(k,p,E)^{NLO} = \frac{a(\Lambda)T^{LO}(k,p,E)}{E-E_B}$$

Vanasse et al 2014. H.D.L and D. Gazit 2016…

NLO A=3, bound state

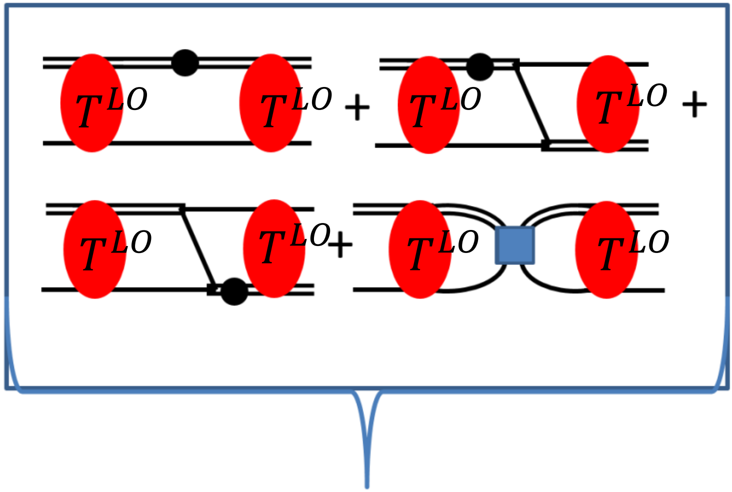
$$Z^{LO}(k,p) = \lim_{E\to E_B} (E - E_B) T^{LO}(k,p,E)$$

$$\frac{Z^{LO}(k,p) + Z^{NLO}(k,p)}{E - E_B^{LO} - \Delta E_B} = T^{LO}(k,p,E) + T^{NLO}(k,p,E) \to$$

$$\Delta E_B = \lim_{E\to E_B} \frac{(E - E_B)^2 T^{NLO}(k,p,E)}{Z^{LO}(k,p)}$$

$$T(k,p,E)^{NLO} = \frac{a(\Lambda) T^{LO}(k,p,E)}{E - E_B} \to$$

$$\Delta E_B = \lim_{E\to E_B} (E - E_B)^2 \frac{a(\Lambda) T^{LO}(k,p,E)}{(E - E_B)^2 T^{LO}(k,p,E)} = a(\Lambda)$$



$T^{NLO}(k,p,E)$

- For ^{3}H : $a(\Lambda) = B_1 = 0$
- For ^{3}He : $\frac{B_1}{E_B^{LO}} \ll 1$

C. Ji and D. R. Phillips 2013 , Vanasse et al 2014, H.D.L and D. Gazit 2016…

NLO A=3, scattering amplitude

- Up to NLO :

$$T(k,p) = T^{LO}(k,p) + T^{NLO}(k,p) = \frac{Z^{LO}(k,p)}{E - E_B^{LO} - \Delta E_B} + \frac{Z^{NLO}(k,p)}{E - E_B - \Delta E_B}$$

For the case that $\Delta E_B = 0$,

$$T^{NLO}(k,p)(E - E_B) = Z^{NLO}(k,p) = 0$$

So there is no change of the normalizing for: $\Delta E_B = 0$.

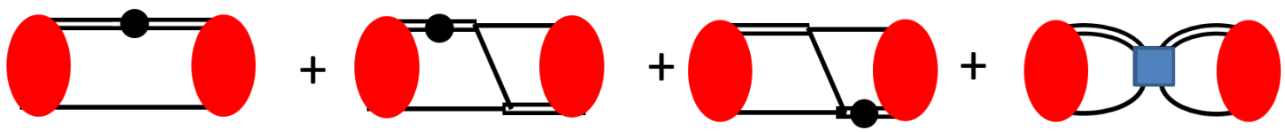
The B.S wave–function is not necessarily zero.

Similarly to LO, the NLO matrix element is set using $\mathcal{B}(q)$

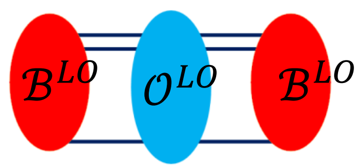
H.D.L and D. Gazit 2016…

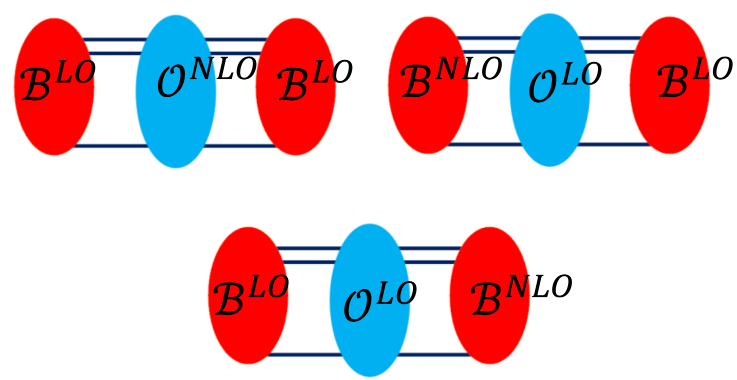
NLO A=3, matrix element

$$a(\Lambda) = \Delta E_B = \langle \mathcal{B}^{LO}(q) | M^1(q,q') | \mathcal{B}^{LO}(q') \rangle = \langle \mathcal{B}^{LO} | \mathcal{B}^{NLO} \rangle$$



$$\mathcal{O}^{LO} + \mathcal{O}^{NLO} = \underbrace{\langle B^{LO} | \mathcal{O}^{LO} | B^{LO} \rangle}_{\mathcal{O}^{LO}} + \underbrace{\langle B^{LO} | \mathcal{O}^{NLO} | B^{LO} \rangle + \langle B^{NLO} | \mathcal{O}^{LO} | B^{LO} \rangle + \langle B^{LO} | \mathcal{O}^{LO} | B^{NLO} \rangle}_{\mathcal{O}^{NLO}}$$





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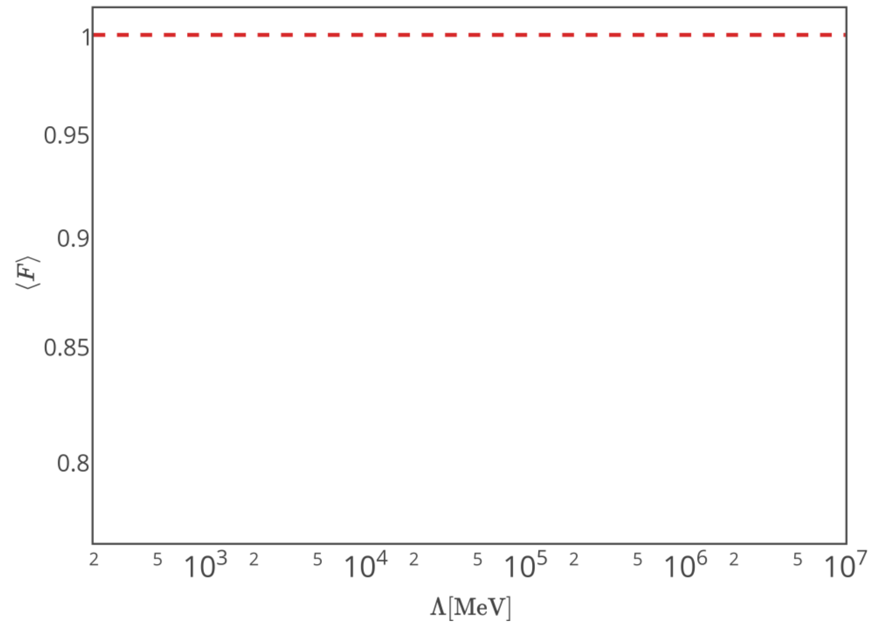
Fermi transition

$$\langle F \rangle = \frac{\langle ^3\mathrm{He}||\tau^-||^3\mathrm{H}\rangle}{\sqrt{2J+1}}$$

- **Red** –Former calculation of the Fermi reduced matrix elements for total isospin, $\langle F \rangle = 0.993$



T.-Y. Saito, et al 1990. H. D.L, and D. Gazit 2016…

Fermi transition

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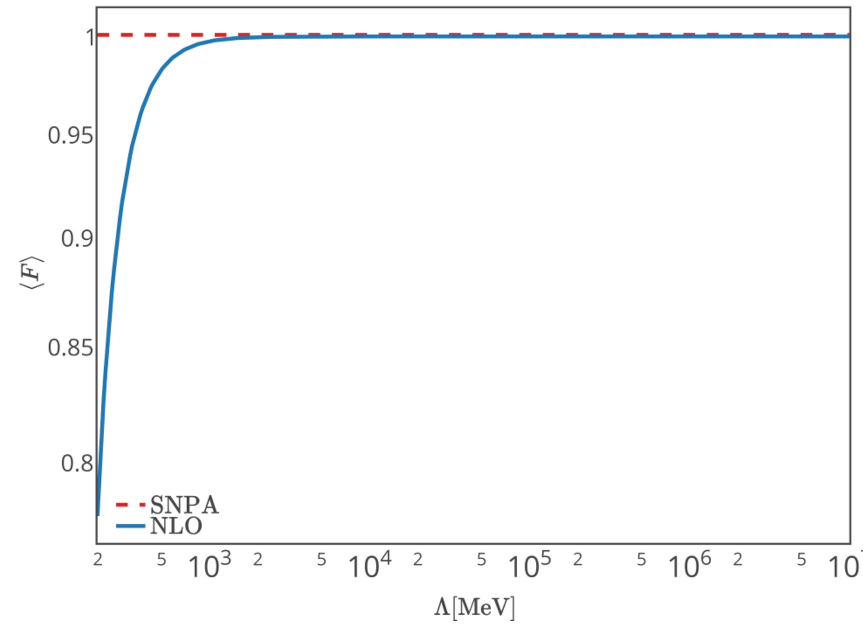
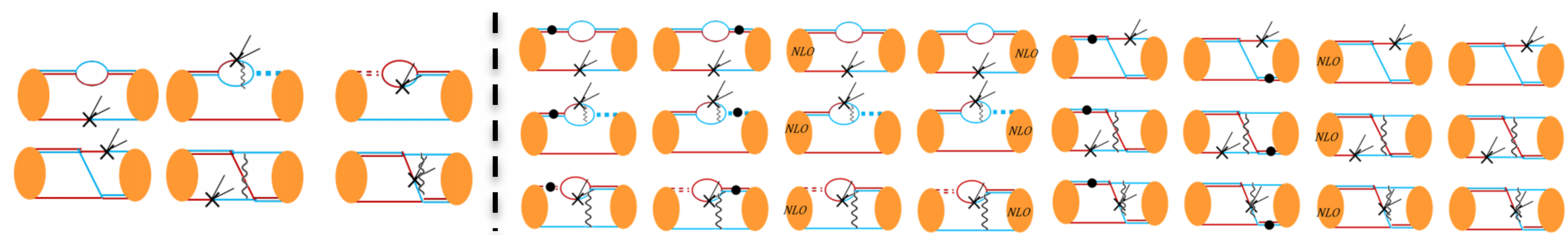
- **Red** –Former calculation of the Fermi reduced matrix elements for total isospin, $\langle F \rangle = 0.9993$
- **Blue** – calculated Fermi transition (contains only the one body diagrams). $\langle \text{F} \rangle = 0.9985 \pm 0.0005$





One body LO

One body NLO

T.-Y. Saito, et al 1990. H. D.L, and D. Gazit 2016…

Deuteron normalization

$$Z_d = \frac{1}{1-\gamma_t\rho_t} = \underbrace{1}_{\text{LO}} + \underbrace{\gamma_t\rho_t}_{\text{NLO}} + \underbrace{(\gamma_t\rho_t)^2}_{\text{N}^2\text{LO}} + \underbrace{(\gamma_t\rho_t)^3}_{\text{N}^3\text{LO}} + \ldots$$

which are equivalent to Q expansion around $k = 0$

$$C_2^t = 2\pi \frac{Z_d^{NLO} - 1}{M_N\gamma_t(\mu - \gamma_t)^2}$$

expansion around the deuteron pole: $Z_d = \frac{1}{1-\gamma_t\rho_t} = 1.69$

$$Z_d = \frac{1}{1-\gamma_t\rho_t} = \underbrace{1}_{\text{LO}} + \underbrace{Z_d - 1}_{\text{NLO}} + \underbrace{0}_{\text{N}^2\text{LO}} + \underbrace{0}_{\text{N}^3\text{LO}} + \ldots$$

And therefore:

$$C_2^t = 2\pi \frac{Z_d^{NLO} - 1}{M_N\gamma_t(\mu - \gamma_t)^2} = 2\pi \frac{0.69}{M_N\gamma_t(\mu - \gamma_t)^2}$$

$$y_t^2 = \frac{(C_0^t)^2}{C_2^t} = \frac{8\pi}{M_N 0.69}$$

X. Kong and F. Ravndal. 1999,2001, H.W. Grießhamme 2004

Gamow -Teller transition

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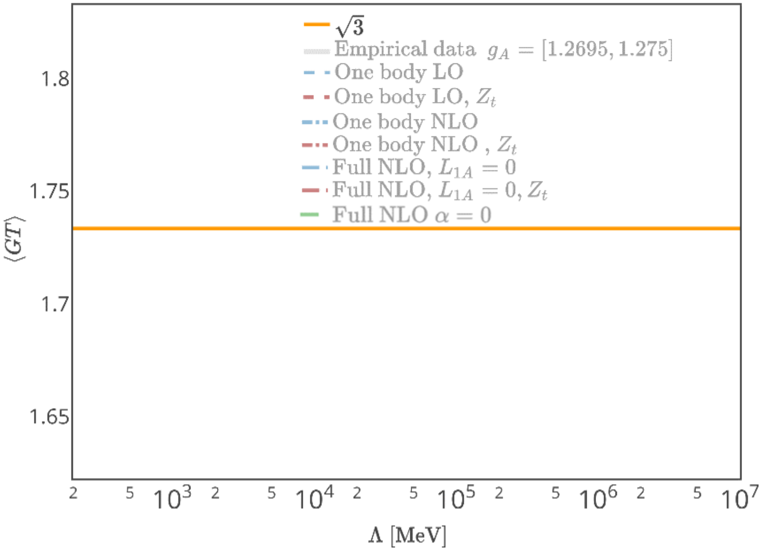

H. D.L, and D. Gazit 2016…

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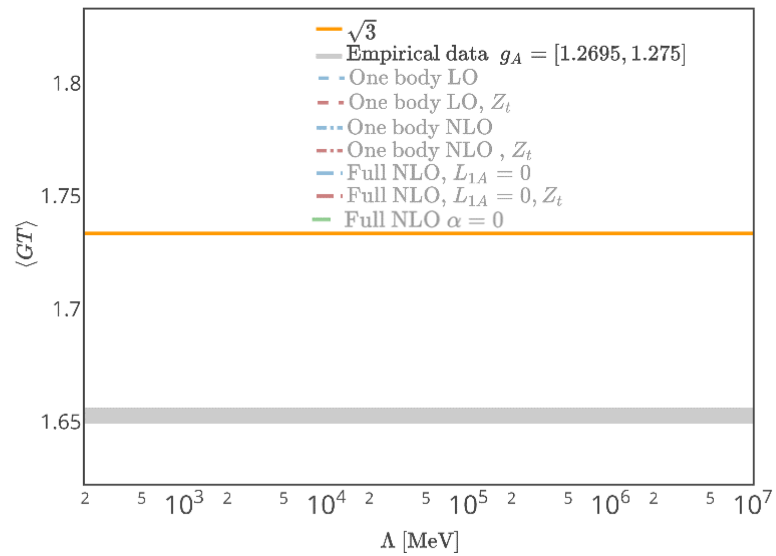
$$\langle GT \rangle = \sqrt{3}\frac{1.213}{g_A}$$



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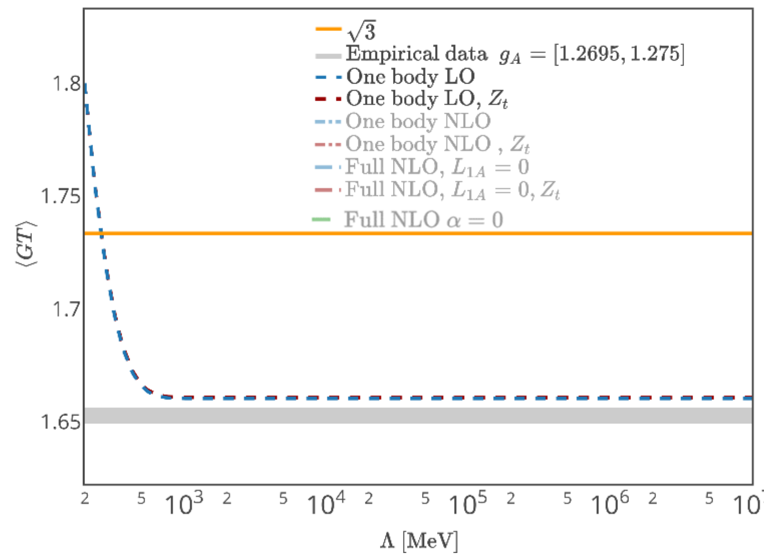
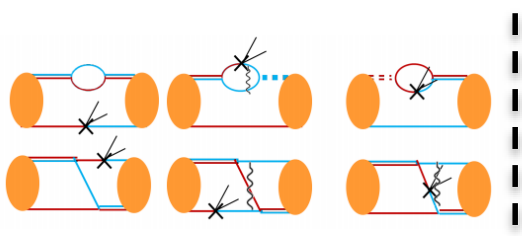




One body LO

H. D.L, and D. Gazit 2016…

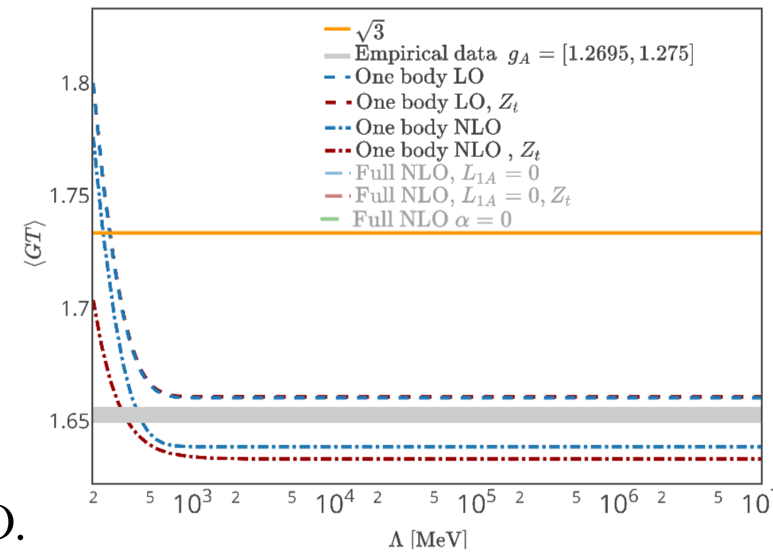
Gamow -Teller transition

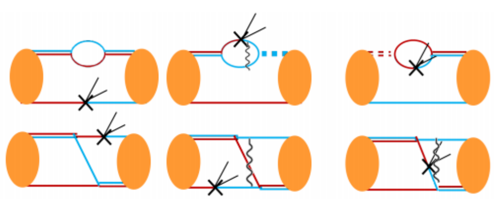
$$\langle GT \rangle = \frac{\langle {}^3\text{He} || \boldsymbol{\sigma}\tau^- || {}^3\text{H} \rangle}{\sqrt{2J+1}}$$

- Yellow - $\sqrt{3}$ (LO with $\alpha = 0$)
- Gray – empirical results:

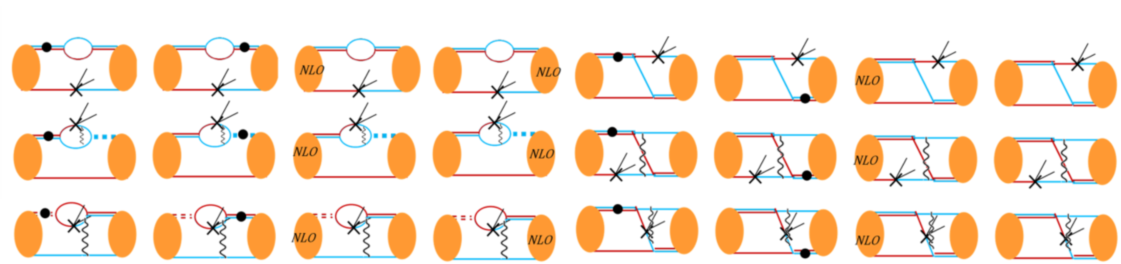
$$\langle GT \rangle = \sqrt{3}\frac{1.213}{g_A}$$

- Dashed – one body LO
- Dotted-dashed - one body NLO.





One body LO



One body NLO

H. D.L, and D. Gazit 2016…

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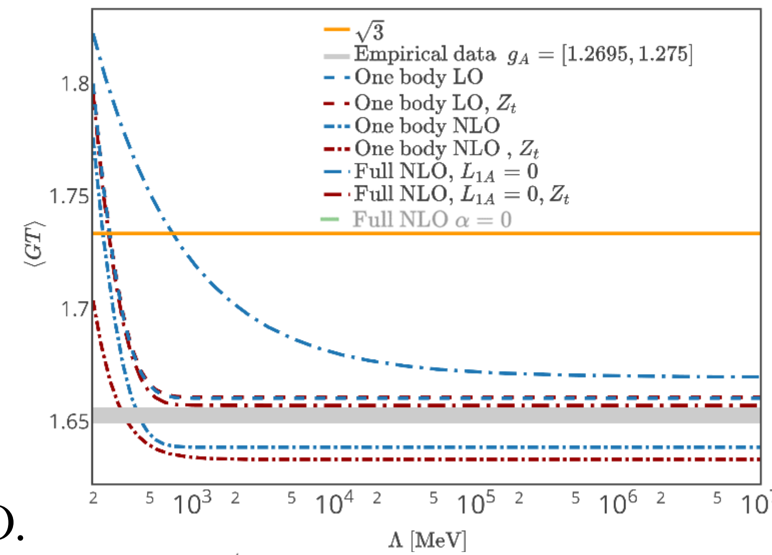
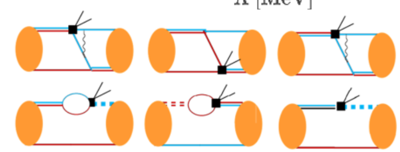
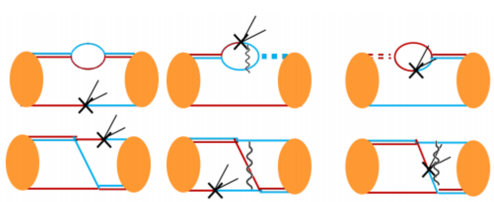
Long Dotted-dashed – full NLO with $L_{1A} = 0$

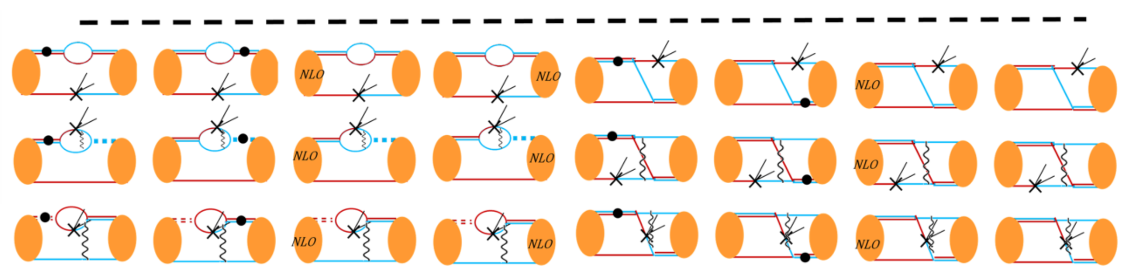




Two body NLO



One body LO

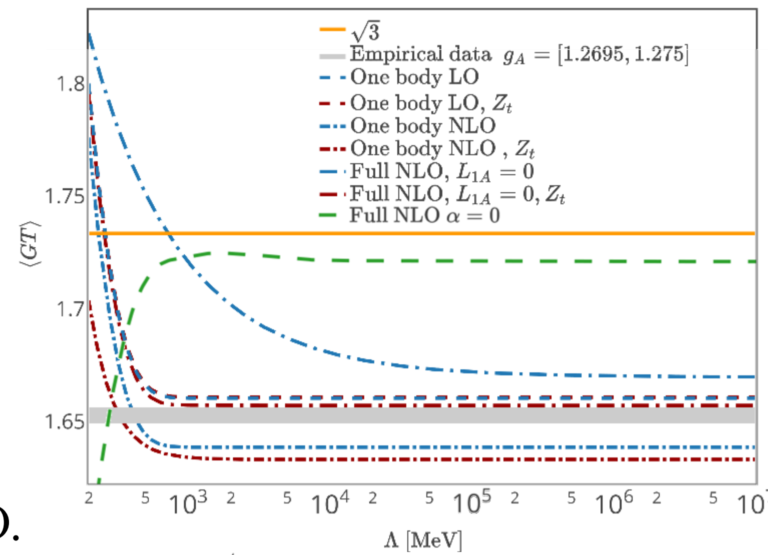


One body NLO

H. D.L, and D. Gazit 2016…

Gamow -Teller transition

$$\langle GT \rangle = \frac{\langle ^3\mathrm{He}||\boldsymbol{\sigma}\tau^-||^3\mathrm{H}\rangle}{\sqrt{2J+1}}$$



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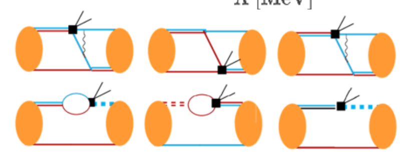
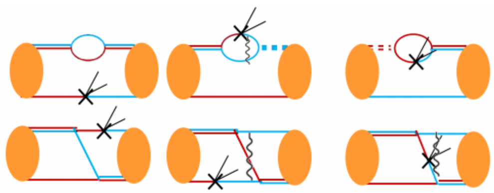
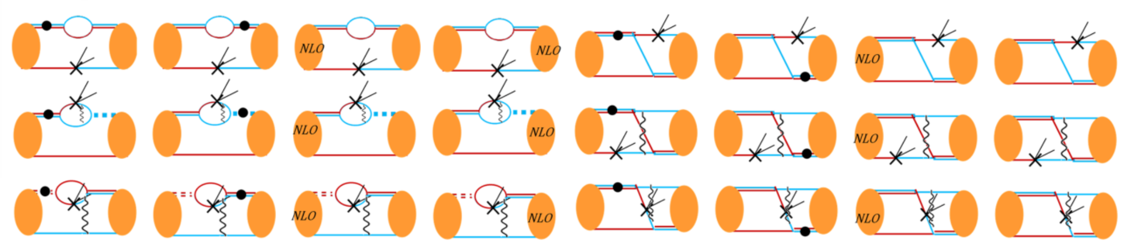
- Dashed – one body LO
- Dotted-dashed - one body NLO.
- Long Dotted-dashed – full NLO with $L_{1A} = 0$
- Green – Full NLO with $\alpha = 0$



Two body NLO



One body LO



One body NLO

H. D.L, and D. Gazit 2016…

Summary

- Diagrammatic representation of the B.S normalization.
- At LO:
  - B.S normalization is equivalent to all possible connections between two amplitudes.
  - Matrix element is equivalent to all possible connections between two amplitudes.
- NLO:
  - No normalization correction for $\Delta E_B = 0$
  - T-matrix ("wave-function") correction includes all possible NLO insertions between LO T-matrix amplitudes.
  - Matrix element calculation includes all possible single NLO insertion: "wave function", operator.