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Radiative Corrections on $\bar{p}p \to e^+e^-$ A Numerical Implementation

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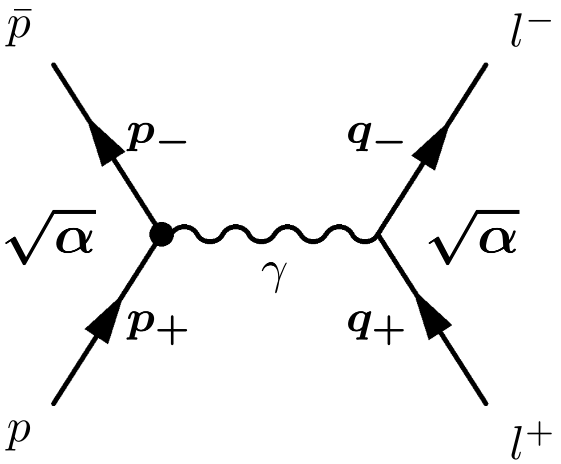
PRISMA

The $\bar{p}p \to e^+e^-$ cross section : leading order

- **at leading order (BORN approximation), only one diagram contributes to the amplitude:**



$$s = (p_+ + p_-)^2 \qquad t = (p_- - q_-)^2 \qquad u = (p_+ - q_-)^2$$

$$\Rightarrow \mathcal{M}_B \sim \alpha \quad \Rightarrow \quad d\sigma_B \sim |\mathcal{M}_B|^2 \text{ is } \mathcal{O}(\alpha^2)$$

A.Zichichi et al., Nuovo Cimento XXIV, 170 (1962):

$$\frac{d\sigma_B}{d\cos\theta^*} = \frac{\pi\alpha^2}{2s\beta}\left\{|G_M|^2(1+\cos^2\theta^*) + |G_E|^2(1-\beta^2)(1-\cos^2\theta^*)\right\}$$

θ^* **= angle (positron, antiproton) in $\bar{p}p$ CM frame,** $\qquad 1-\beta^2 = 4M_p^2/s$

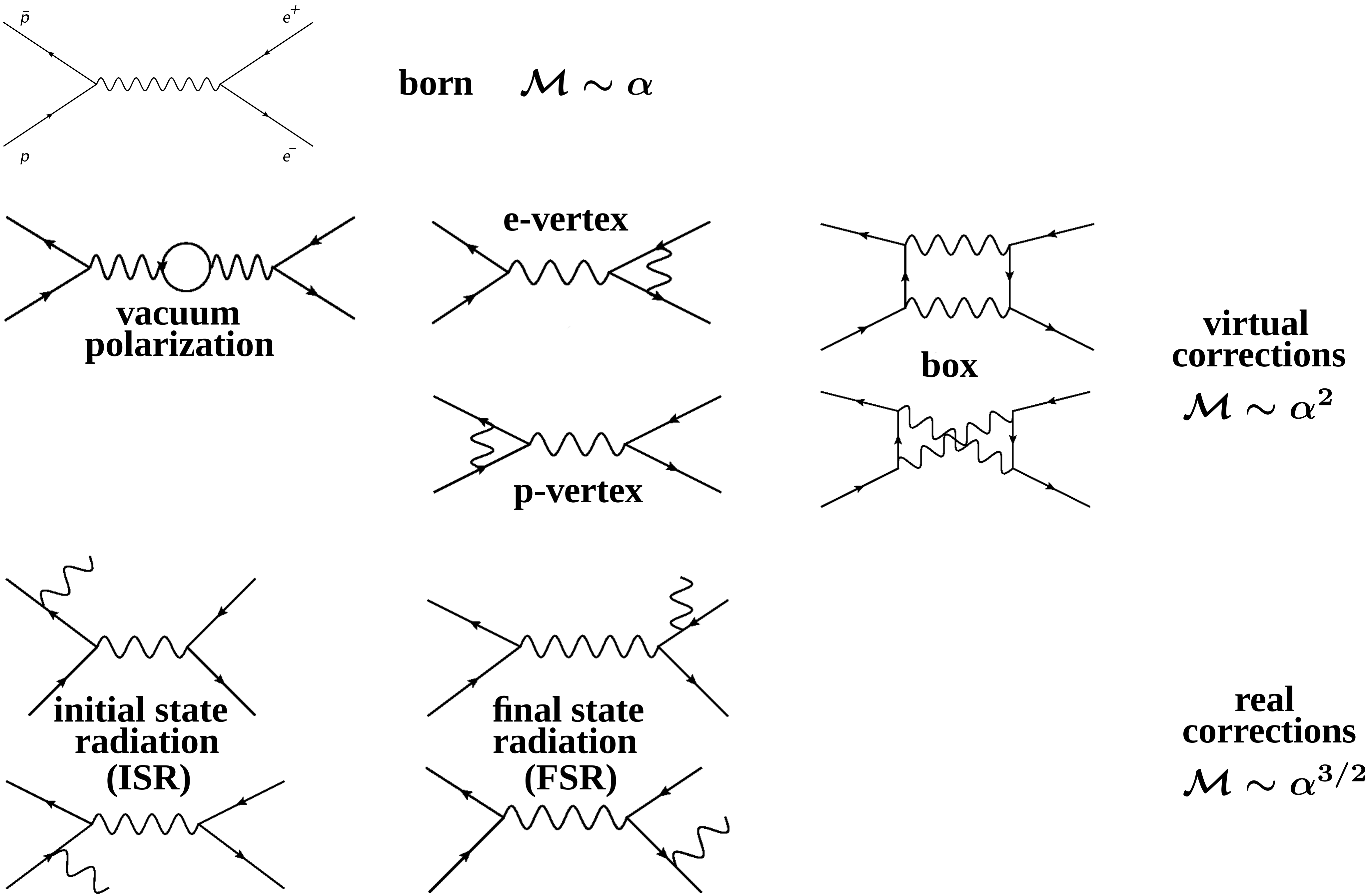
$\Rightarrow$ **fit function to extract $|G_E|, |G_M|$ at leading order**

- **having sufficiently precise data, we would like to attemp a form factor extraction at next to leading order:**

$\Rightarrow$ **we need an $\mathcal{O}(\alpha^3)$ differential cross section**

The $\bar{p}p \rightarrow e^+e^-$ cross section : next to leading order

at next to leading order (NLO), many more diagrams contribute to the amplitude:



The $\bar{\mathrm{p}}\mathrm{p} \to \mathrm{e}^+\mathrm{e}^-$ cross section : next to leading order

at NLO, we have two different (i.e. non-interfering) final states:

- $*$ $\underline{e^+e^-}$: **born, virtual corrections (vacuum polarization, ...)**
- $*$ $\underline{e^+e^-\gamma}$: **real corrections (initial state radiation, final state radiation)**

$$\begin{aligned}
&\qquad\quad \mathcal{O}(\alpha) \qquad \mathcal{O}(\alpha^2) \qquad\qquad\qquad \mathcal{O}(\alpha^{3/2}) \\
\Rightarrow d\sigma &\sim |\mathcal{M}_B + \textstyle\sum_i \mathcal{M}_i|^2 + |\mathcal{M}_{ISR} + \mathcal{M}_{FSR}|^2 \qquad\qquad \text{truncated at } \mathcal{O}(\alpha^3) \\
&\sim |\mathcal{M}_B|^2 + \textstyle\sum_i 2\mathrm{Re}(\mathcal{M}_\mathrm{B}\mathcal{M}_\mathrm{i}^*) + |\mathcal{M}_\mathrm{ISR}|^2 + |\mathcal{M}_\mathrm{FSR}|^2 + 2\mathrm{Re}(\mathcal{M}_\mathrm{ISR}\mathcal{M}_\mathrm{FSR}^*) \\
&\sim |\mathcal{M}_B|^2 \left\{ 1 + \sum_i \frac{2\mathrm{Re}(\mathcal{M}_\mathrm{B}\mathcal{M}_\mathrm{i}^*)}{|\mathcal{M}_B|^2} + \frac{|\mathcal{M}_{ISR}|^2}{|\mathcal{M}_B|^2} + \frac{|\mathcal{M}_{FSR}|^2}{|\mathcal{M}_B|^2} + \frac{2\mathrm{Re}(\mathcal{M}_\mathrm{ISR}\mathcal{M}_\mathrm{FSR}^*)}{|\mathcal{M}_B|^2} \right\} \\
&\sim |\mathcal{M}_B|^2 \left\{ 1 + \delta_\text{vacuum} + \delta_\text{e-vertex} + \delta_\text{p-vertex} + \delta_\text{box} + \underbrace{\delta_\mathrm{ISR} + \delta_\mathrm{FSR} + \delta_\text{interference}}_{\equiv\, \delta_\gamma} \right\}
\end{aligned}$$

therefore, at $\mathcal{O}(\alpha^3)$, we write

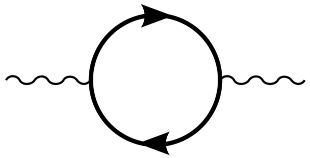
$$\boxed{d\sigma = d\sigma_B \left\{ 1 + \delta_\text{vacuum} + \delta_\text{e-vertex} + \delta_\text{p-vertex} + \delta_\text{box} + \delta_\gamma \right\}}$$

Calculation at NLO : virtual corrections

- **virtual corrections relying heavily on**

  A.I. Ahmadov et al. Phys. Rev. D **82**, 094016 (2010)
  G. Bonneu et al. Nucl. Phys. B **27**, 381 (1971)

∗ **vacuum polarization:** e, μ, τ, **and point-like pion in loop**



∗ **electron-vertex:** **ultra-relativistic limit** $s \gg m^2$ [G. Bonneu et al. (1971)]

∗ **proton-vertex:** **point-like proton**

∗ **box diagram:** **point-like proton**

[J. Van de Wiele and S. Ong, Eur. Phys. J. A **49**, 18 (2013) contains a first order study of radiative corrections to $\bar{p}p \to e^+e^-$, including full virtual and real corrections (soft and hard photon emission) in partial overlapping with A. I. Ahmadov et al. (2010). Our calculation does not make use of this source.]

Calculation at NLO : real corrections, cancellation of divergences

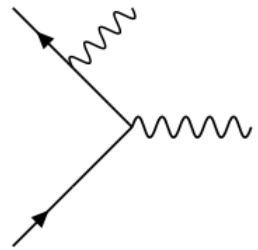
- **new calculation of real corrections (ISR, FSR, interference)**
  [Ahmadov et al. does not separate explicitly ISR/FSR terms;
  [kinematic regime $s, t, u \gg M^2, m^2$ assumed in interference term not valid for PANDA]

  following F. A. Berends et al. Nucl. Phys. B **57** 381 (1973)
  F. A. Berends et al. Nucl. Phys. B **63** 381 (1973) on $e^+e^- \to \mu^+\mu^-$

  done in the soft photon limit:

  * $E_\gamma < \omega = b\sqrt{s}/2$ $\quad b < 10^{-2}$
  * **integration over photon degrees of freedom (no experimental detection)**

- **infrared divergences regularized with a cutoff**
  * **real diagrams:** $E_\gamma > \lambda = a\sqrt{s}$ $\quad 10^{-6} < a < 10^{-3}$
  * **virtual diagrams:** $\int_0 dk^2 \to \int_{\lambda^2} dk^2$

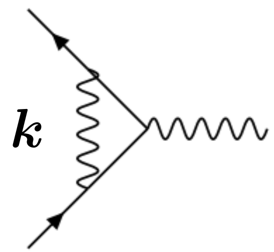


- **cancellation of divergences:**

  $\delta_{\text{e-vertex}} + \delta_{\text{FSR}} = \text{finite}$

  $\delta_{\text{p-vertex}} + \delta_{\text{ISR}} = \text{finite}$

  $\delta_{\text{box}} + \delta_{\text{interference}} = \text{finite}$



- **full cross section is λ-independent (more accurately, λ-stable)**

The code

- **the program is a radiative-corrected $\bar{p}p \to e^+e^-$ cross section calculator/integrator, including all virtual corrections and soft photon emission**

- **C++ code in a four-directory structure:** `/inc /src /output /plots`

- `/src`

  $\to$ **user parameters passed to the program through the infile** `mzgen.in`

  $\to$ **compilation (no need!) by** `Makefile` **defined rules**

  ```
  make clean
  make
  ```

  $\to$ **run:** `./run.sh`

- `/output`

  `output.log` **(ASCII file with tables, cross sections values, etc.)**

  `output.root` **(ROOT file with histos)**

- `/plots`

  `mzplots.C` **(ROOT macro to create .eps files out of histos in** `output.root`**)**

The infile : providing parameters

```
1                  ! RANLUX seed
100                ! #trials for MC integration
1.5                ! pbar momentum LAB-frame (GeV)
-0.8               ! cos(theta*) min
0.8                ! cos(theta*) max
2                  ! FFs
3.0                ! |G_E|
1.0                ! |G_M|
2                  ! FFs model
0.00001            ! infrared cutoff
0.01               ! soft photon cutoff
1                  ! vacuum polarization
0                  ! box
1                  ! electron vertex
1                  ! proton vertex
3                  ! soft photon
2.0                ! luminosity (fb^-1)
../output          ! pathout
output             ! outfile
```

The infile : information for the user

```
PARAMETER RANGE
===============

RANLUX seed:                [1,2^31-1]
#trials MC integration:     > 0 (integer!)
pbar momentum:              > 0.0 GeV
                            (recommended in PANDA range 1.5 < P < 15.0 GeV)
angular range:              -1 < cos(theta*) min < cos(theta*) max < 1
                            (boundaries excluded!)
FFs:                        1 (input from user) or 2 (taken from model)
|G_E|:                      >= 0.0
|G_M|:                      >= 0.0
FFs model:                  1 = pQCD, 2 = vector meson dominance (Iachello), 3 = dipole
infrared cutoff:            [10^{-6},10^{-3}]
                            [def: E_gamma_min = IRcutoff * sqrt(s)]
                            (recommended value 10^{-5})
soft photon cutoff:         (2.0*IRcutoff, 0.01]
                            [def: E_gamma_max = SOFTcutoff * sqrt(s) / 2]
vacuum polarization:        0 (OFF), 1 (leptons), 2(leptons + pions)
box:                        0 (OFF), 1 (ON)
electron vertex:            0 (OFF), 1 (ON)
proton vertex:              0 (OFF), 1 (ON)
soft photon:                0 (OFF), 1 (FSR), 2 (ISR), 3 (ISR + FSR),
                            4 (ISR + FSR + interference), 5 (interference)
luminosity:                 > 0.0 fb^{-1}


Divergences cancellation forbids the following flags combinations:

(electron vertex, soft photon) = {(0,1), (0,3), (0,4), (1,2), (1,5)}
(proton vertex, soft photon)   = {(0,2), (0,3), (0,4), (1,1), (1,5)}
(box, soft photon)             = {(0,4), (0,5), (1,1), (1,2), (1,3)}
```

The output : reading the infile

- **at the early stage, the program reads in the infile and prints it out to `output.log`**

```
------------------------------------
   Reading infile
------------------------------------

RANLUX seed:                     1
#trials MC integration:          100
pbar momentum:                   1.50 GeV
cos(theta*) min:                 -0.80
cos(theta*) max:                 0.80
FFs:                             2
|G_E|:                           3.00
|G_M|:                           1.00
FFs model:                       2
IR cutoff:                       0.000010
SOFT photon cutoff:              0.010000
vacuum polarization:             1
box:                             0
electron vertex:                 1
proton vertex:                   1
soft photon:                     3
luminosity:                      2.00 fb^-1
pathout:                         ../output
outfile:                         output
```

The output : checking parameters

- **then, the program checks that all parameters are fine, otherwise printing an error message and killing the job; examples:**

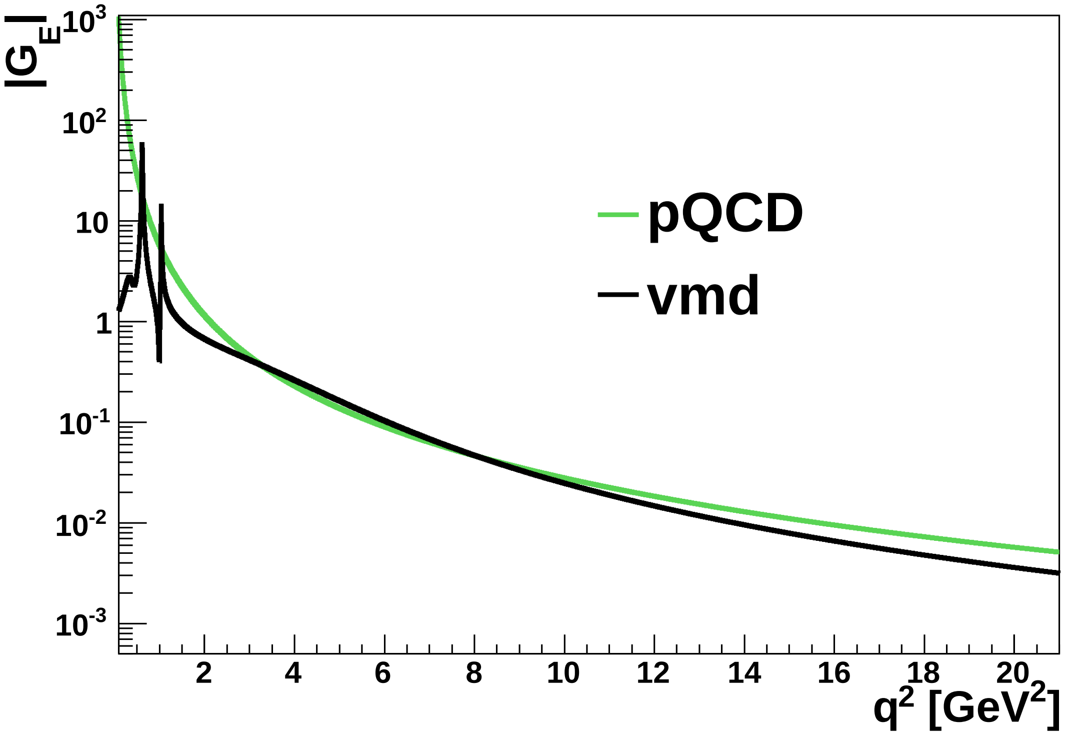
```
WARNING: anti-proton momentum larger than PANDA upper limit 15.00 GeV
         [P = 20.00 GeV]


ERROR: cos(theta*) range NOT allowed! [-0.80 < cos(theta*) < -0.80]
    => cos(theta*) range [cos(theta*) min, cos(theta*) max]
       should be contained within the interval [-1.0,1.0]
    => job killed!!!


ERROR: soft photon cutoff out of range! [SOFT photon cutoff = 0.020000]
    => soft photon cutoff should be contained within the interval
       (2.0 * infrared cutoff, 0.010000]
       (with user's choice infrared cutoff = 0.000010, this interval is
       (0.000020,0.010000] )
    => job killed!!!


ERROR: vacuum polarization flag out of range! [flag = 3]
    => vacuum polarization flag should take the value 0, 1 or 2
    => job killed!!!
```

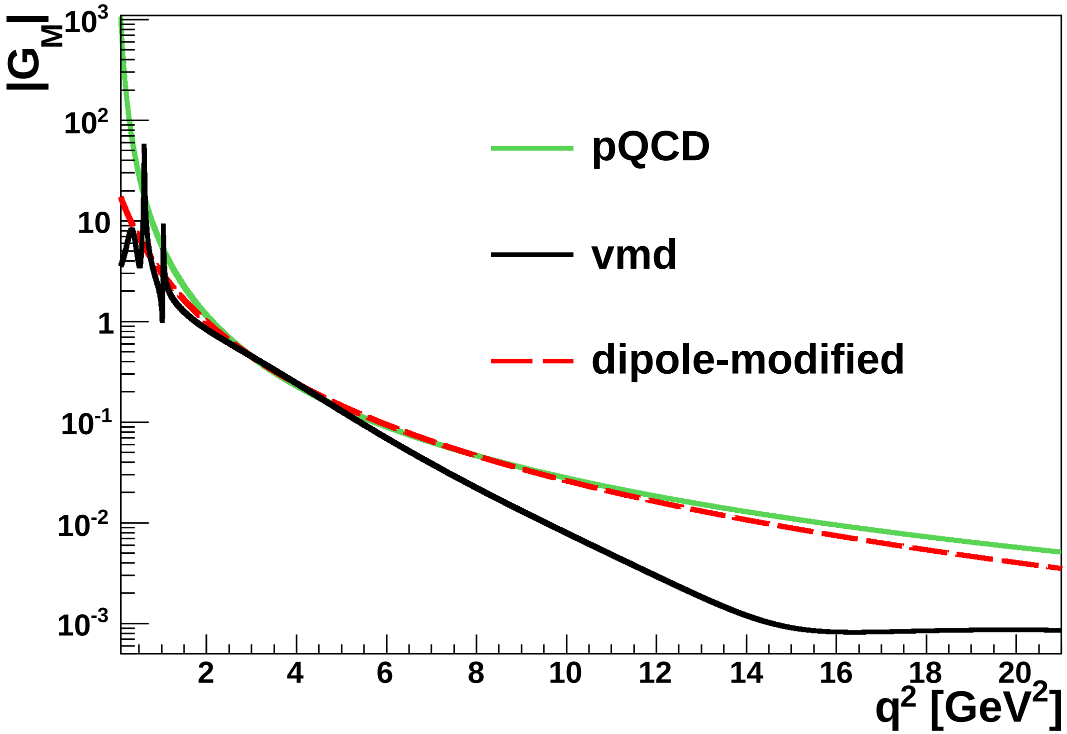
⇒ **no way to run a job with wrong parameters!**

The output : kinematics, form factors

```
Kinematics
----------

anti-proton momentum (lab frame):  P = 1.50 GeV
                    __
=> CM energy:     \/s = 2.25 GeV
   CM energy^2:     s = 5.08 GeV^2

   soft photon minimum energy (CM frame):  0.000023 GeV
   soft photon maximum energy (CM frame):  0.011270 GeV



Form Factors (for BORN cross section)
-------------------------------------

Form Factors: values from vector meson dominance model (Iachello)

q2 = s = 5.08 GeV^2 => |G_E| =  0.157
                       |G_M| =  0.123
                        R = |G_E|/|G_M| =  1.279
```

Models for the form factors





- **“perturbative QCD - inspired”: [1], and references therein**
- **“vector meson dominance”: [2]**
- **“dipole-modified”: [1], and references therein**

[1] M. Sudol et al., Eur. Phys. J. A **44**, 373 (2010)

[2] F. Iachello, and Q. Wan, Phys. Rev. C **69**, 055204 (2004)

The output : radiative corrections (FULL)

$$\frac{d\sigma}{d\cos\theta^*} = \frac{d\sigma_B}{d\cos\theta^*}(1 + \delta_{\text{vacuum}} + \delta_{\text{box}} + \cdots)$$

Radiative corrections (FULL)

theta* [deg]	VACUUM	BOX	e-VERTEX	p-VERTEX	soft-GAMMA	TOTAL
10.0	0.010145	-0.124423	-0.486899	-0.005677	0.290710	-0.316144
20.0	0.010145	-0.117403	-0.486899	-0.005702	0.287012	-0.312847
30.0	0.010145	-0.106432	-0.486899	-0.005741	0.281209	-0.307717
40.0	0.010145	-0.092352	-0.486899	-0.005791	0.273724	-0.301174
50.0	0.010145	-0.076005	-0.486899	-0.005848	0.264986	-0.293622
60.0	0.010145	-0.058107	-0.486899	-0.005905	0.255372	-0.285395
70.0	0.010145	-0.039214	-0.486899	-0.005953	0.245182	-0.276740
80.0	0.010145	-0.019740	-0.486899	-0.005986	0.234649	-0.267831
90.0	0.010145	0.000000	-0.486899	-0.005997	0.223958	-0.258794
100.0	0.010145	0.019740	-0.486899	-0.005986	0.213267	-0.249734
110.0	0.010145	0.039214	-0.486899	-0.005953	0.202734	-0.240759
120.0	0.010145	0.058107	-0.486899	-0.005905	0.192544	-0.232008
130.0	0.010145	0.076005	-0.486899	-0.005848	0.182929	-0.223668
140.0	0.010145	0.092352	-0.486899	-0.005791	0.174191	-0.216002
150.0	0.010145	0.106432	-0.486899	-0.005741	0.166706	-0.209358
160.0	0.010145	0.117403	-0.486899	-0.005702	0.160903	-0.204150
170.0	0.010145	0.124423	-0.486899	-0.005677	0.157206	-0.200803

box diagram, proton vertex and ISR/FSR interference are t, u-dependent

The output : radiative corrections (USER)

$$\frac{d\sigma}{d\cos\theta^*} = \frac{d\sigma_B}{d\cos\theta^*}(1 + \delta_{\text{vacuum}} + \delta_{\text{box}} + \cdots)$$

Radiative corrections (USER)

theta* [deg]	VACUUM	BOX	e-VERTEX	p-VERTEX	soft-GAMMA	TOTAL
10.0	0.030963	0.000000	-0.486899	-0.005677	0.223958	-0.237656
20.0	0.030963	0.000000	-0.486899	-0.005702	0.223958	-0.237681
30.0	0.030963	0.000000	-0.486899	-0.005741	0.223958	-0.237720
40.0	0.030963	0.000000	-0.486899	-0.005791	0.223958	-0.237770
50.0	0.030963	0.000000	-0.486899	-0.005848	0.223958	-0.237827
60.0	0.030963	0.000000	-0.486899	-0.005905	0.223958	-0.237883
70.0	0.030963	0.000000	-0.486899	-0.005953	0.223958	-0.237932
80.0	0.030963	0.000000	-0.486899	-0.005986	0.223958	-0.237965
90.0	0.030963	0.000000	-0.486899	-0.005997	0.223958	-0.237976
100.0	0.030963	0.000000	-0.486899	-0.005986	0.223958	-0.237965
110.0	0.030963	0.000000	-0.486899	-0.005953	0.223958	-0.237932
120.0	0.030963	0.000000	-0.486899	-0.005905	0.223958	-0.237883
130.0	0.030963	0.000000	-0.486899	-0.005848	0.223958	-0.237827
140.0	0.030963	0.000000	-0.486899	-0.005791	0.223958	-0.237770
150.0	0.030963	0.000000	-0.486899	-0.005741	0.223958	-0.237720
160.0	0.030963	0.000000	-0.486899	-0.005702	0.223958	-0.237681
170.0	0.030963	0.000000	-0.486899	-0.005677	0.223958	-0.237656

- **no $\pi^+\pi^-$ contribution to vacuum polarization**
- **no box diagram**
- **no interference term ISR/FSR**

The output : differential cross sections

$$\frac{d\sigma}{d\cos\theta^*} = \frac{d\sigma_B}{d\cos\theta^*}(1 + \delta_{\text{vacuum}} + \delta_{\text{box}} + \cdots)$$

```
Differential cross section
---------------------------

theta*      BORN              BORN + RAD. CORR. (FULL)          BORN + RAD. CORR. (USER)
[deg]       [fb]              [fb]                              [fb]
-----       ----              ------------------------          ------------------------

 10.0       348.425921        238.272996                        265.620476
 20.0       350.457105        240.817695                        267.160273
 30.0       353.569060        244.769675                        269.518715
 40.0       357.386438        249.750975                        272.410605
 50.0       361.448807        255.319599                        275.486565
 60.0       365.266185        261.021217                        278.375466
 70.0       368.378139        266.433355                        280.729310
 80.0       370.409324        271.202336                        282.265046
 90.0       371.114747        275.072485                        282.798281
100.0       370.409324        277.905536                        282.265046
110.0       368.378139        279.687615                        280.729310
120.0       365.266185        280.521632                        278.375466
130.0       361.448807        280.604389                        275.486565
140.0       357.386438        280.190231                        272.410605
150.0       353.569060        279.546690                        269.518715
160.0       350.457105        278.911354                        267.160273
170.0       348.425921        278.461028                        265.620476
```

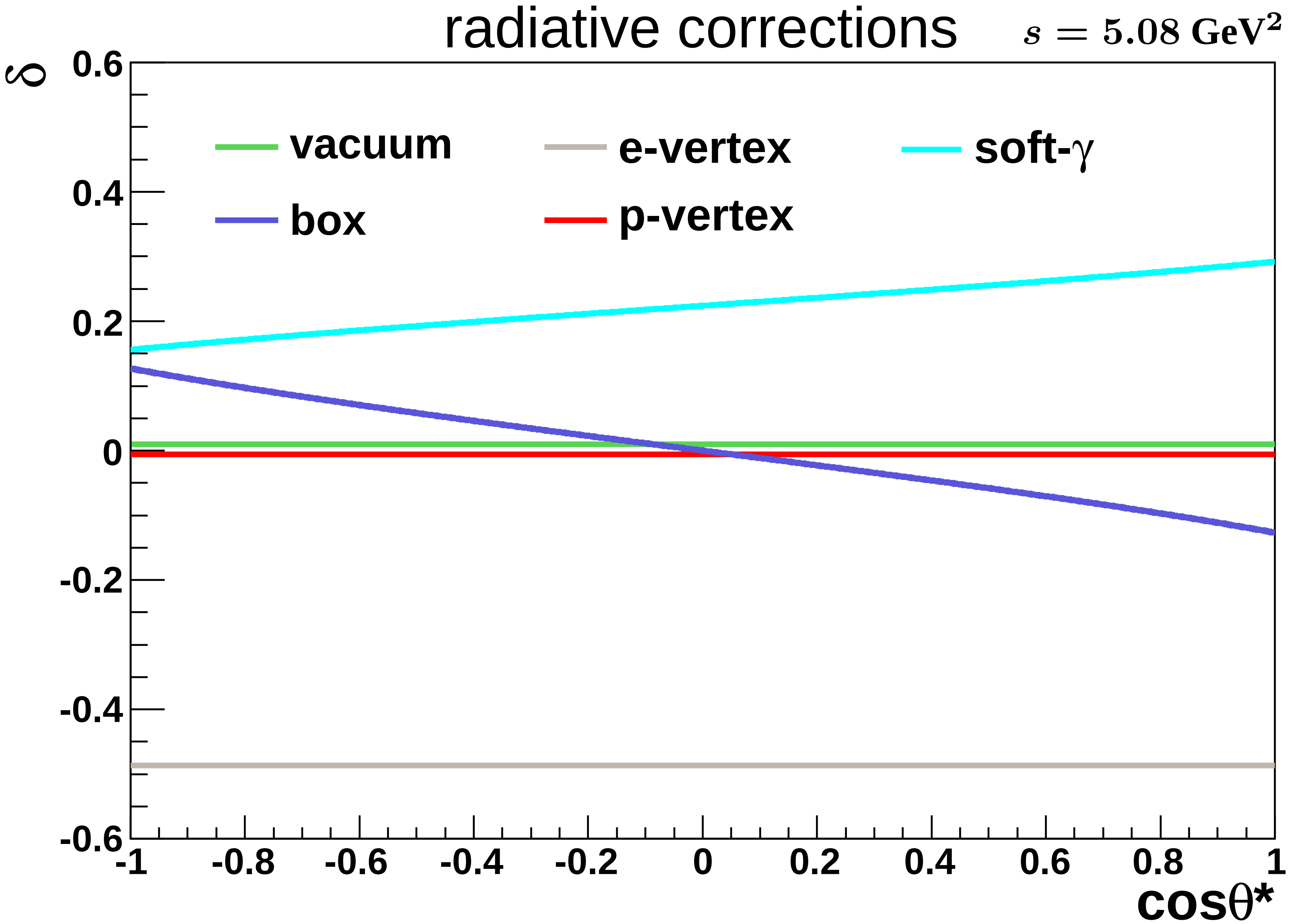
The output : integrated cross sections / statistics

```
Integrated cross section / statistics
-------------------------------------

-0.8 < cos(theta*) < 0.8,     luminosity = 2.0 fb^-1


BORN:                         sigma = 585.80 pb,             N = 1171596 events
BORN + RAD. CORR. (FULL)      sigma = 434.20 +/- 1.55 pb,    N = 868404 events
BORN + RAD. CORR. (USER)      sigma = 447.90 +/- 0.48 pb,    N = 895790 events
```

The output : histograms

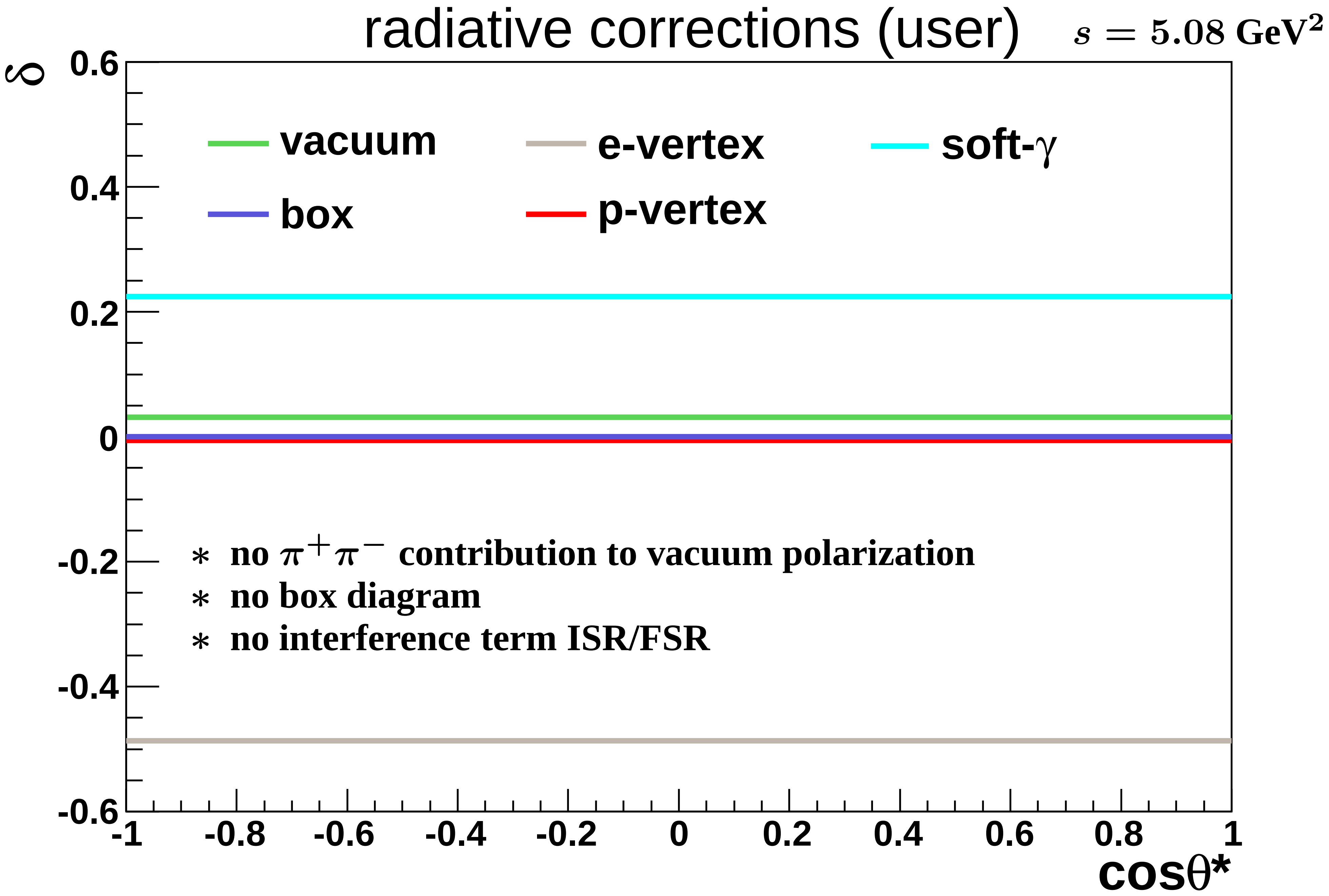
```
Writing histos to file:
     ../output/output.root   (root file)


HISTOGRAM DESCRIPTION
=====================

h1    -> delta vacuum       (FULL)
h2    -> delta box          (FULL)
h3    -> delta e-vertex     (FULL)
h4    -> delta p-vertex     (FULL)
h5    -> delta soft photon  (FULL)
h6    -> total delta        (FULL)

h10   -> delta vacuum       (USER)
h20   -> delta box          (USER)
h30   -> delta e-vertex     (USER)
h40   -> delta p-vertex     (USER)
h50   -> delta soft photon  (USER)
h60   -> total delta        (USER)

h100  -> dsigma(e+e-)/dcos(theta*), BORN
h101  -> dsigma(e+e-)/dcos(theta*), BORN + RAD. CORR. (FULL)
h102  -> dsigma(e+e-)/dcos(theta*), BORN + RAD. CORR. (USER)



Execution time... 0.43s
```

Example plots : full radiative corrections



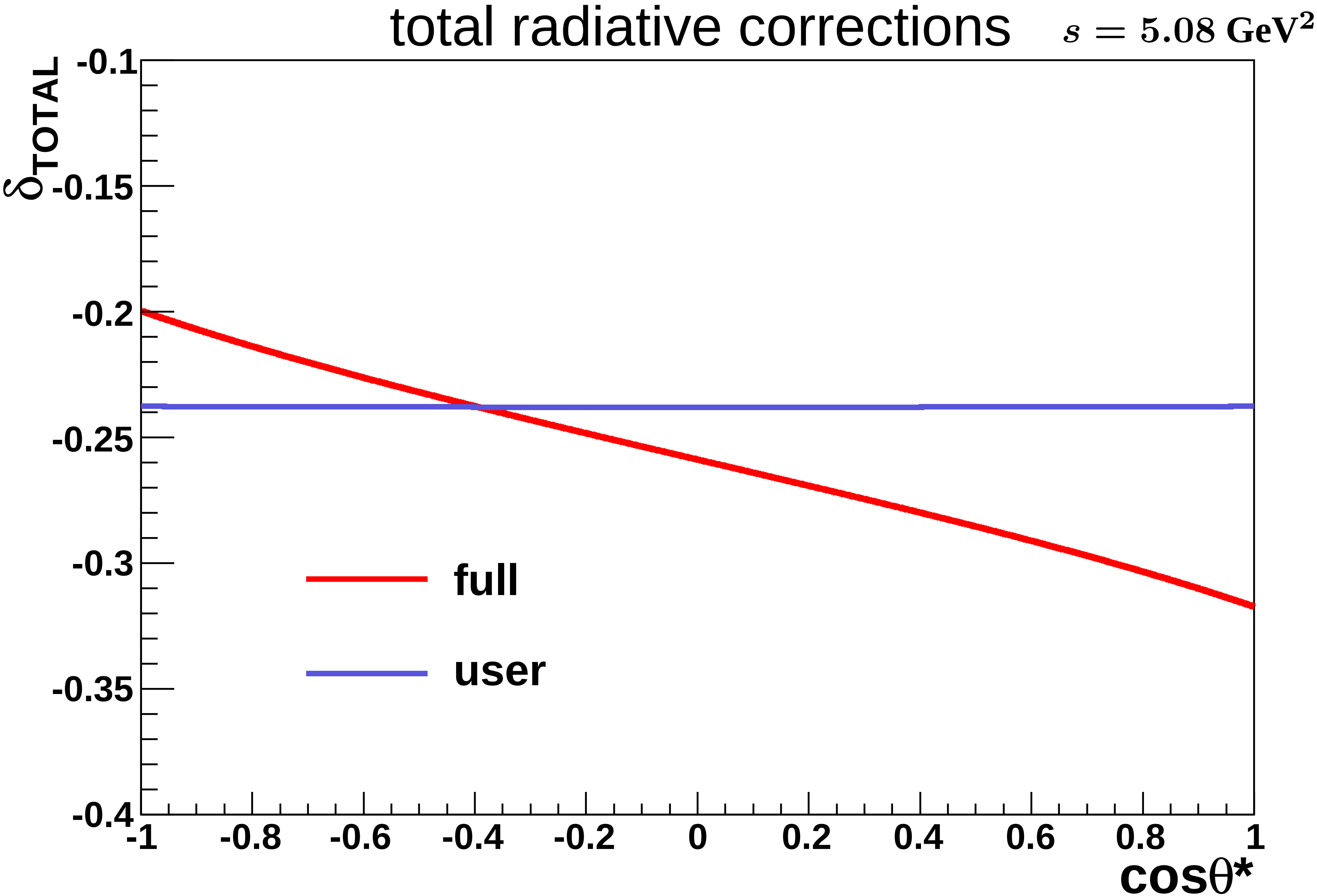
box, proton vertex and ISR/FSR interference are t, u-dependent

Example plots : user radiative corrections



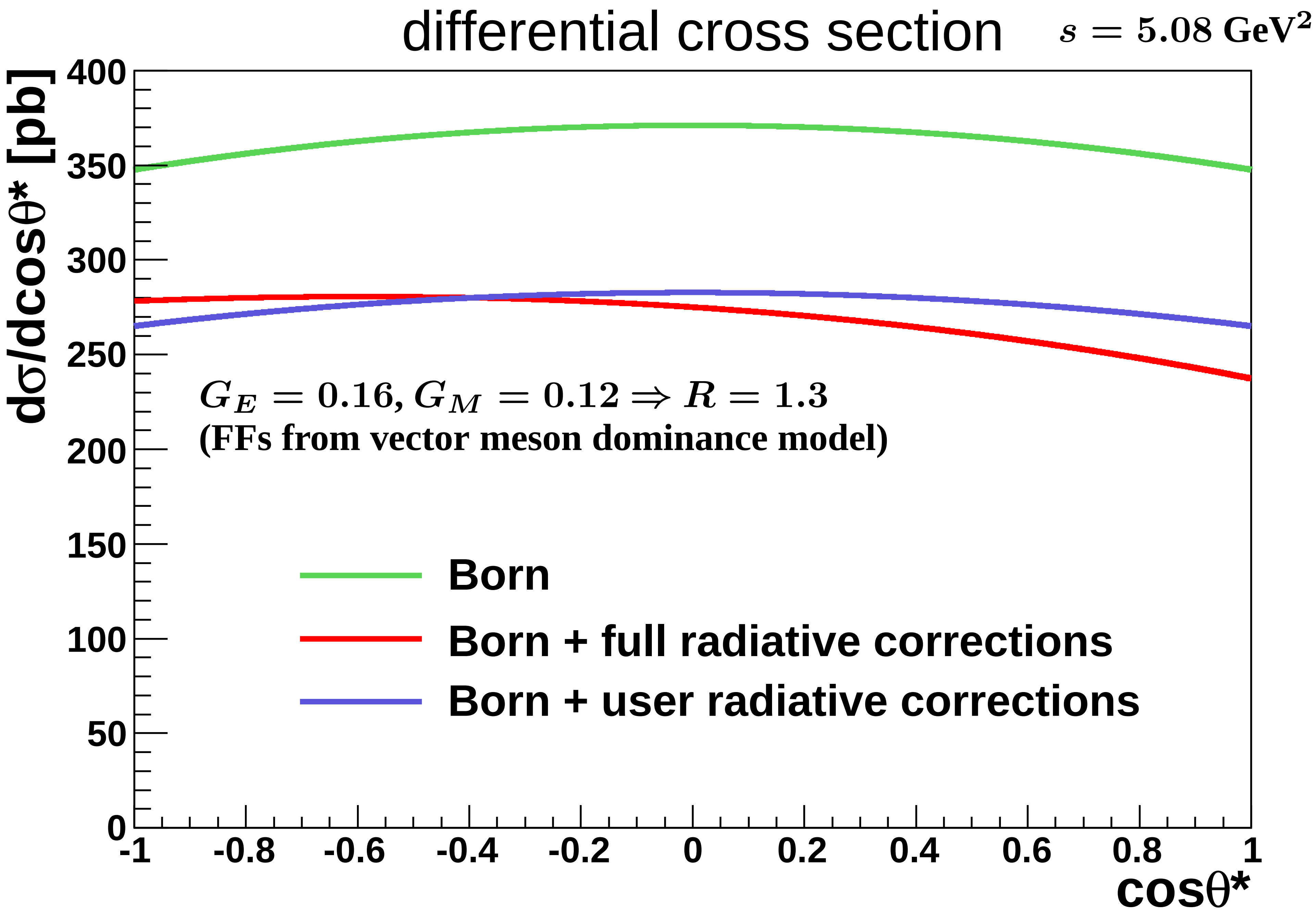
Example plots : full / user total radiative corrections comparison

$$d\sigma/d\cos\theta^* = (d\sigma_B/d\cos\theta^*)(1+\delta_T)$$



Example plots : differential cross sections comparison

$$d\sigma/d\cos\theta^* = (d\sigma_B/d\cos\theta^*)(1+\delta_T)$$



Summary, conclusions, and what is next

- **complete, user-oriented C++ code to calculate radiative corrections to $\bar{p}p \to e^+e^-$, including all virtual corrections and soft photon emission**

next steps:

$\to$ **event generator, including virtual corrections and soft photon emission (already under development)**

$\to$ **complete real corrections with hard photon emission**

$\to$ **cross section calculator and event generator including all real/virtual corrections (including interface to PandaRoot, full documentation and public availability within the PANDA Collaboration)**