

Multi cluster correlations studied with UCOM-like treatment

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To study intruder states or exotic structure (second 0^+ state of ^{12}C)

Model approach



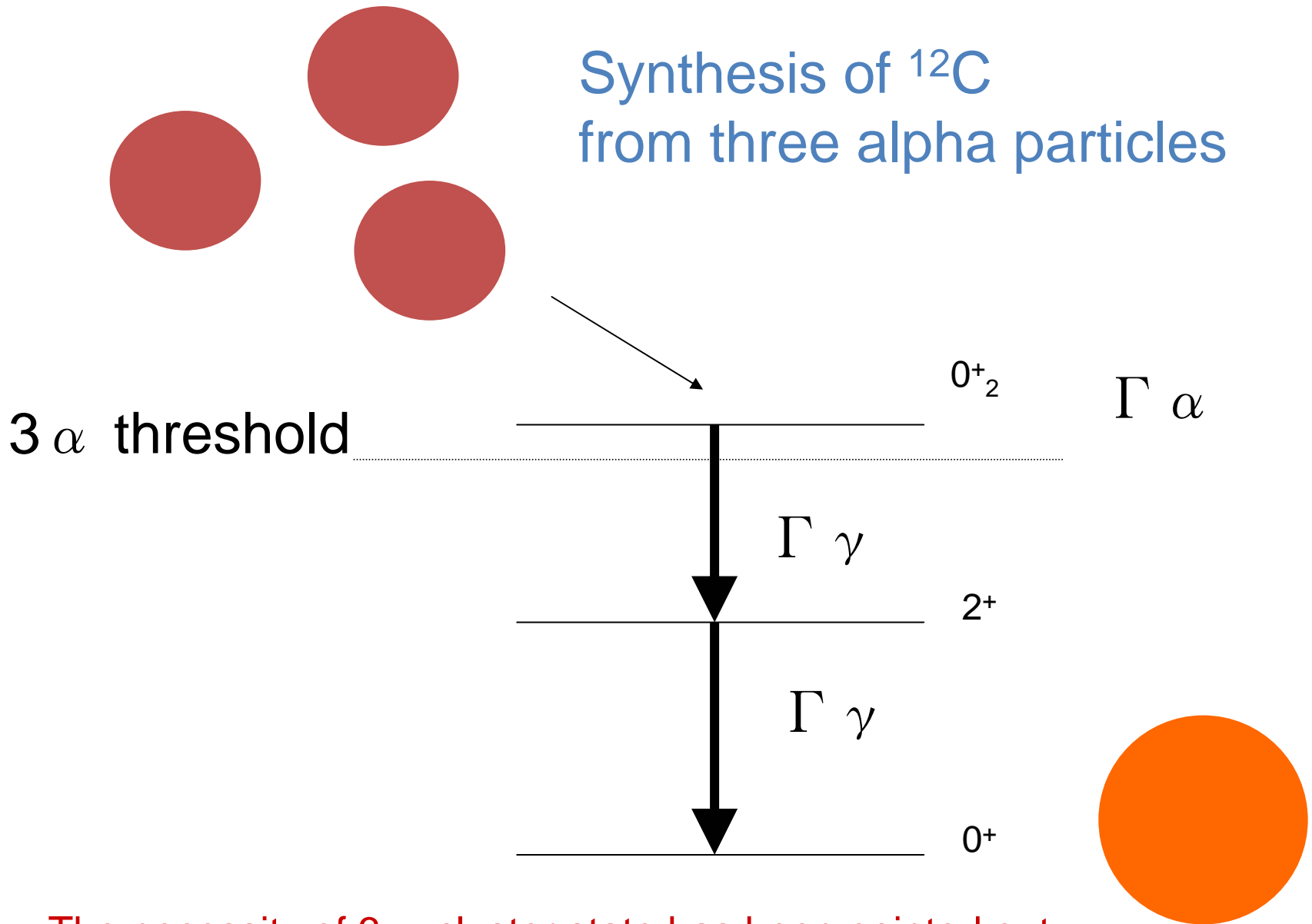
Effective interactions

Nuclear Structure Physics



From the analyses on nucleon-nucleon interactions
(GFMC, NCSM, $V_{\text{low-k}}$, , , , ,)

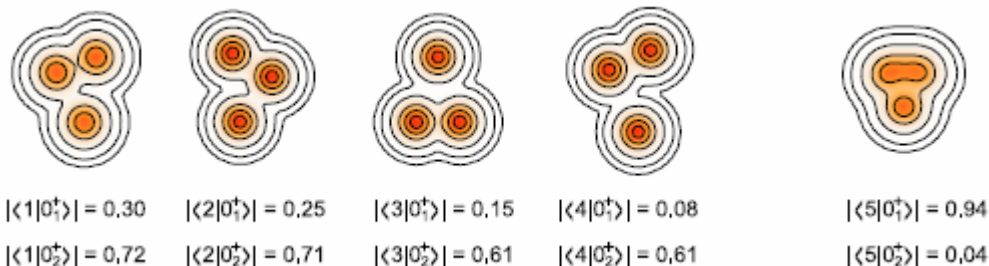
Synthesis of ^{12}C from three alpha particles



The necessity of 3α -cluster state has been pointed out from astrophysical point of view, and experimentally confirmed afterwards

TABLE I. Energies, radii, and transition strengths. Units of energies are MeV, of radii fm, $M(E0)$ e fm², and $B(E2)$ e² fm⁴. Data are from [28], BEC results from [13,15].

	Experimental	FMD	α cluster	BEC
$E(0_1^+)$	-92.16	-92.64	-89.56	-89.52
$E^*(0_2^+)$	7.65	9.50	7.89	7.73
$E(0_2^+) - E(3\alpha)$	0.38	0.44	0.38	0.26
$E^*(0_3^+)$	(10.3)	11.90	10.33	
$E^*(2_1^+)$	4.44	5.31	2.56	2.81
$E^*(2_2^+)$	(11.16)	11.83	9.21	9.03
$E(3\alpha)$	-84.89	-83.59	-82.05	-82.05
$r_{\text{charge}}(0_1^+)$	2.47(2)	2.53	2.54	
$r(0_1^+)$		2.39	2.40	2.40
$r(0_2^+)$		3.38	3.71	3.83
$r(0_3^+)$		4.62	4.75	
$r(2_1^+)$		2.50	2.37	2.38
$r(2_2^+)$		4.43	4.02	
$M(E0, 0_1^+ \rightarrow 0_2^+)$	5.4(2)	6.53	6.52	6.45
$B(E2, 2_1^+ \rightarrow 0_1^+)$	7.6(4)	8.69	9.16	
$B(E2, 2_1^+ \rightarrow 0_2^+)$	2.6(4)	3.83	0.84	



Structure of the Hoyle State in ¹²C

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(Received 15 September 2006; published 17 January 2007)

Alpha Cluster Condensation in ^{12}C and ^{16}O

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(Received 29 June 2001; published 17 October 2001)

A new α -cluster wave function is proposed which is of the α -particle condensate type. Applications to ^{12}C and ^{16}O show that states of low density close to the 3 and 4 α -particle thresholds in both nuclei are possibly of this kind. It is conjectured that all self-conjugate $4n$ nuclei may show similar features.

DOI: 10.1103/PhysRevLett.87.192501

PACS numbers: 21.60.Gx, 03.75.Fi, 21.10.Gv, 27.20.+n

$$\begin{aligned}
 \Phi &= \int d\vec{R}_1 d\vec{R}_2 \cdots d\vec{R}_n \\
 & \mathcal{A} G_1(\vec{R}_1) G_2(\vec{R}_2) G_3(\vec{R}_3) \cdots G_n(\vec{R}_n) \\
 & \times \exp[-(\vec{R}_1^2 + \vec{R}_2^2 + \vec{R}_3^2 \cdots \vec{R}_n^2)/\sigma^2] \\
 &= \mathcal{A} \prod_{i=1}^n \int d\vec{R}_i G_i(\vec{R}_i) \exp[-\vec{R}_i^2/\sigma^2],
 \end{aligned}$$

One of the merit of THSR wave function we can introduce “size of the condensate”

We simplify the THSR wave function

- We need to study multi-alpha cases
- We need to study the case with core nuclei
- The coupling with normal states should be included
- Di-neutron condensate around the core can be studied

$$\Psi = \sum_{k=1}^m P^\pi P_{MK}^J \Psi_k,$$

$$\Psi_k = [\mathcal{A} G_1(\vec{R}_1) G_2(\vec{R}_2) G_3(\vec{R}_3) \cdots G_n(\vec{R}_n)]_k.$$

For the α clusters in the condensed state

$$W(\vec{R}_i) \propto \exp[-\vec{R}_i^2/\sigma^2].$$

Virtual THSR wave function

N. Itagaki, M. Kimura, M. Ito, C. Kurokawa, and W. von Oertzen,
Phys. Rev. C **75**, 037303 (2007)

Content

- Di-neutron correlation in ${}^8\text{He}$
- Connection to realistic interaction.
Application of softcore potential
with UCOM-like treatment
- Study alpha-cluster states by applying THSR
wave function to heavier nuclei (with core,
coupling with the low-lying states)

Result 1:

di-neutron-like correlation

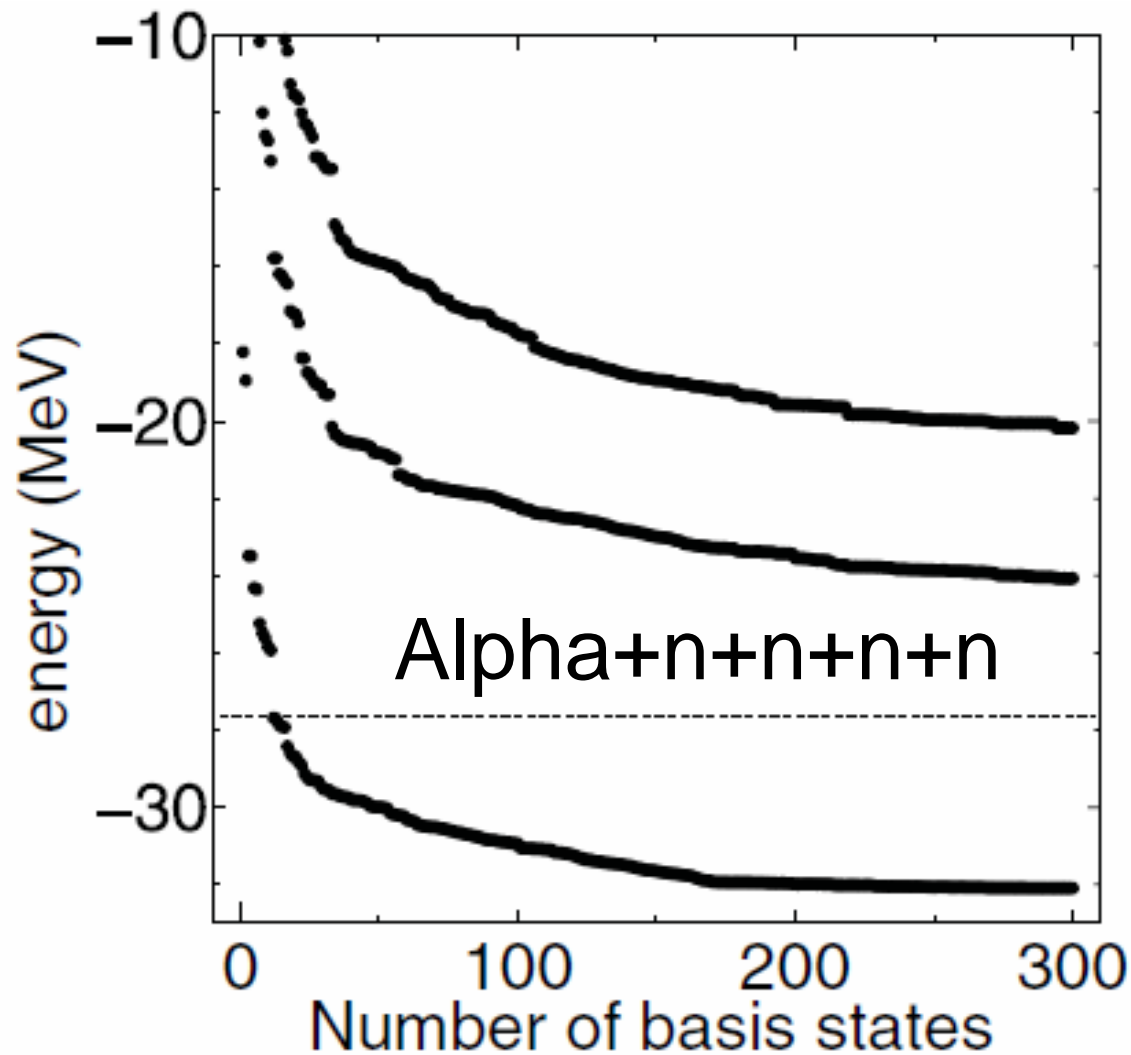
Di-neutron correlation in ${}^8\text{He}$?

- ${}^8\text{He}$ is well-bound compared with ${}^6\text{He}$ and ${}^{11}\text{Li}$
- Sub-closed configuration of spin-orbit-favored shell

Di-neutron \rightarrow no spin-orbit

Condensation of $S=0$ di-neutron clusters
components really contributes?

${}^4\text{He}+4n$ model space (AMD)



${}^8\text{He } 0^+$

As for the N - N interaction, for the central part, we use the Volkov No.2 effective potential [16]:

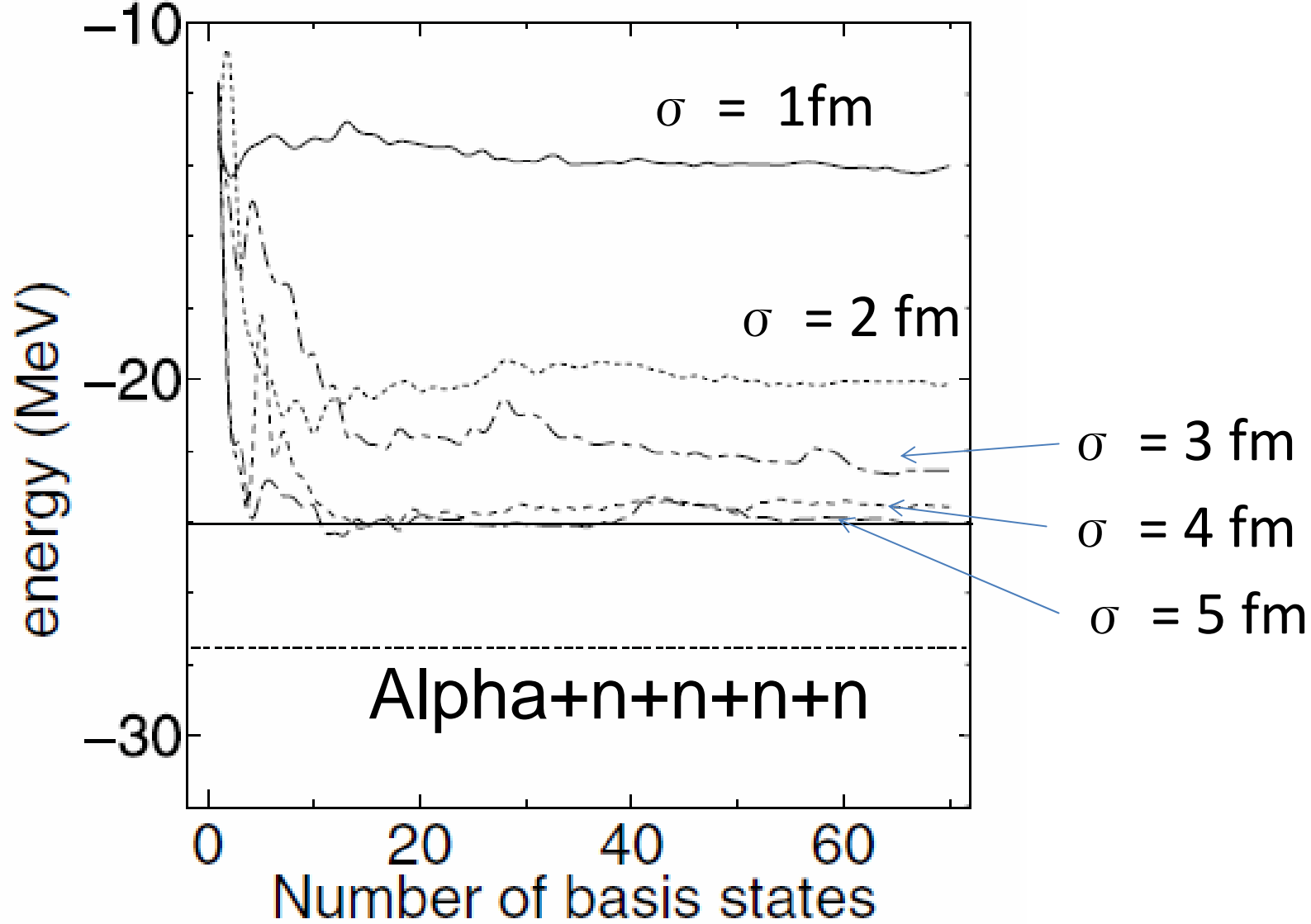
$$V(r) = (W - MP^\sigma P^\tau + BP^\sigma - HP^\tau) \\ (V_1 \exp(-r^2/c_1^2) + V_2 \exp(-r^2/c_2^2)), \quad (9)$$

where $c_1 = 1.01$ fm, $c_2 = 1.8$ fm, $V_1 = 61.14$ MeV, $V_2 = -60.65$ MeV, $W = 1 - M$ and $M = 0.60$. The

$$V_{ls} = V_0(e^{-d_1 r^2} - e^{-d_2 r^2})P(^3O)\vec{L} \cdot \vec{S}, \quad (10)$$

where $d_1 = 5.0$ fm⁻², $d_2 = 2.778$ fm⁻², $V_0 = 2000$ MeV,

**Determined to reproduce
the alpha-alpha and alpha+N
Phase shifts**



${}^8\text{He } 0^+$

${}^4\text{He} + 2$ di-neutron clusters

B = H = 0 case

σ (fm)	$0_1^+(\Psi_{\text{AMD}})$	$0_2^+(\Psi_{\text{AMD}})$	$0_3^+(\Psi_{\text{AMD}})$
1	0.43	0.01	0.02
2	0.53	0.01	0.02
3	0.61	0.08	0.03
4	0.45	0.36	0.00
5	0.24	0.48	0.07

σ (fm)	$0_1^+(\Psi_{\text{AMD}} + \Psi_d)$	$0_2^+(\Psi_{\text{AMD}} + \Psi_d)$	$0_3^+(\Psi_{\text{AMD}} + \Psi_d)$
1	0.43	0.01	0.02
2	0.53	0.01	0.03
3	0.61	0.07	0.04
4	0.46	0.36	0.00
5	0.25	0.53	0.05

In ${}^7\text{H}$ case

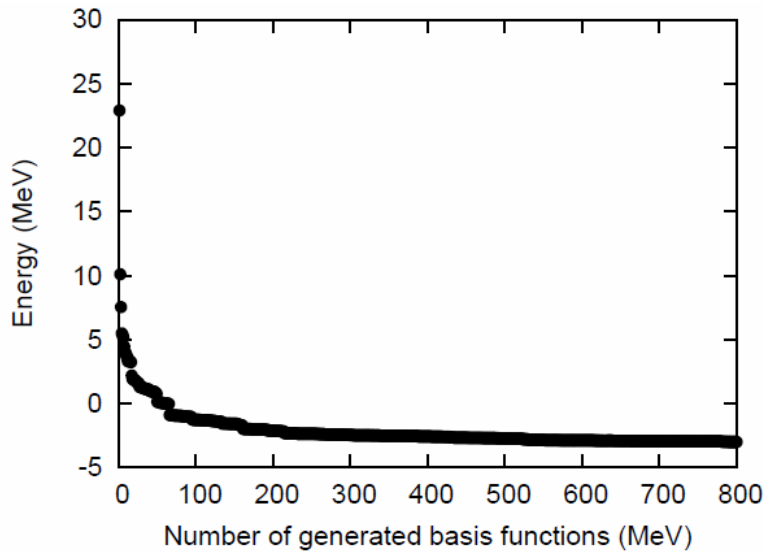
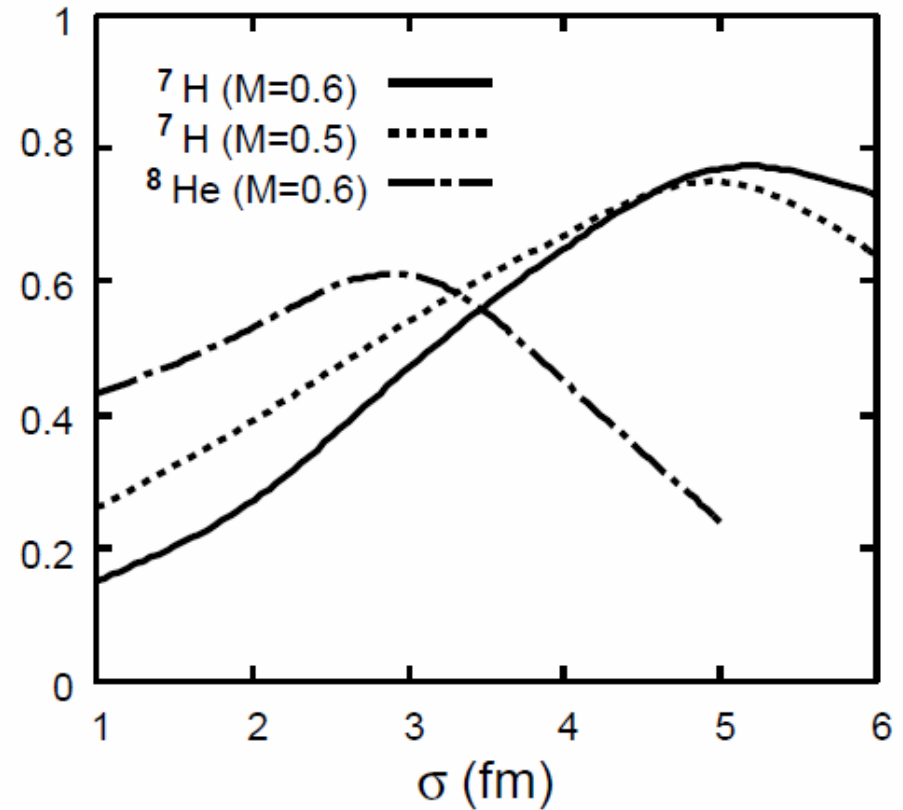


FIG. 1: The energy convergence of the ground state ($1/2^+$) of ${}^7\text{H}$ based on the GCM. The energy is obtained by diagonalizing the Hamiltonian and plotted as a function of the number of the generated basis functions (Slater determinants).



Tensor correlations in the unitary correlation operator method

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Received 4 July 2002; received in revised form 5 September 2002; accepted 2 October 2002

$$|\Psi\rangle = \mathcal{A}\{|\psi_1\rangle \otimes \cdots \otimes |\psi_A\rangle\}, \quad \hat{H} = C^\dagger H C$$

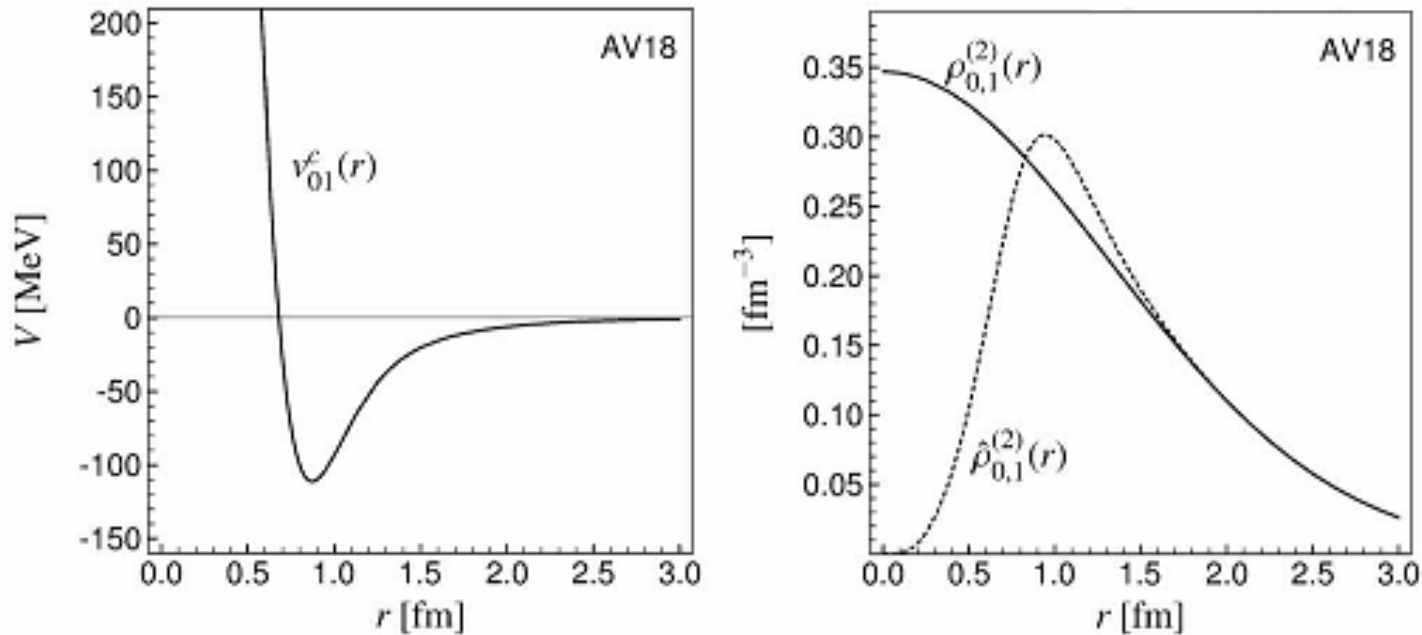


Fig. 1. The Argonne V18 potential in the $S, T = 0, 1$ channel is plotted on the left-hand side. Its strong repulsion at short distances causes a pronounced depletion in the correlated two-body density $\hat{\rho}_{0,1}^{(2)}$ when compared with the uncorrelated two-body density $\rho_{0,1}^{(2)}$ of the ${}^4\text{He}$ shell-model state.

UCOM transformation

$$V(r) \rightarrow V(R_+(r))$$

$$R_+(r) = r + \alpha (r / \beta)^\eta \exp(-\exp(r / \beta))$$

$$\alpha = 0.94 \text{ fm} , \quad \beta = 1.00 \text{ fm}$$

$$\eta = 0.37$$

Our strategy

- UCOM-like transformation is performed only for the treatment of the short-range correlation of the central interaction. Hamiltonian becomes extremely simple.
- Non-central interactions (spin-orbit and tensor) are just introduced with the original form.
- We superpose many Slater determinants

Gaussian 3-Range Softcore (G3RS)

R. Tamagaki, Prog. Theor. Phys. 39 (1968)

$$V(r) = 2000 \exp[-5.005r^2] \\ -250 \exp[-1.127r^2] \\ -5 \exp[-0.16r^2]$$

Each range has

Wigner, Majorana, Heisenberg, Bartlet
dependence

Gaussian 3-Range Softcore (G3RS)

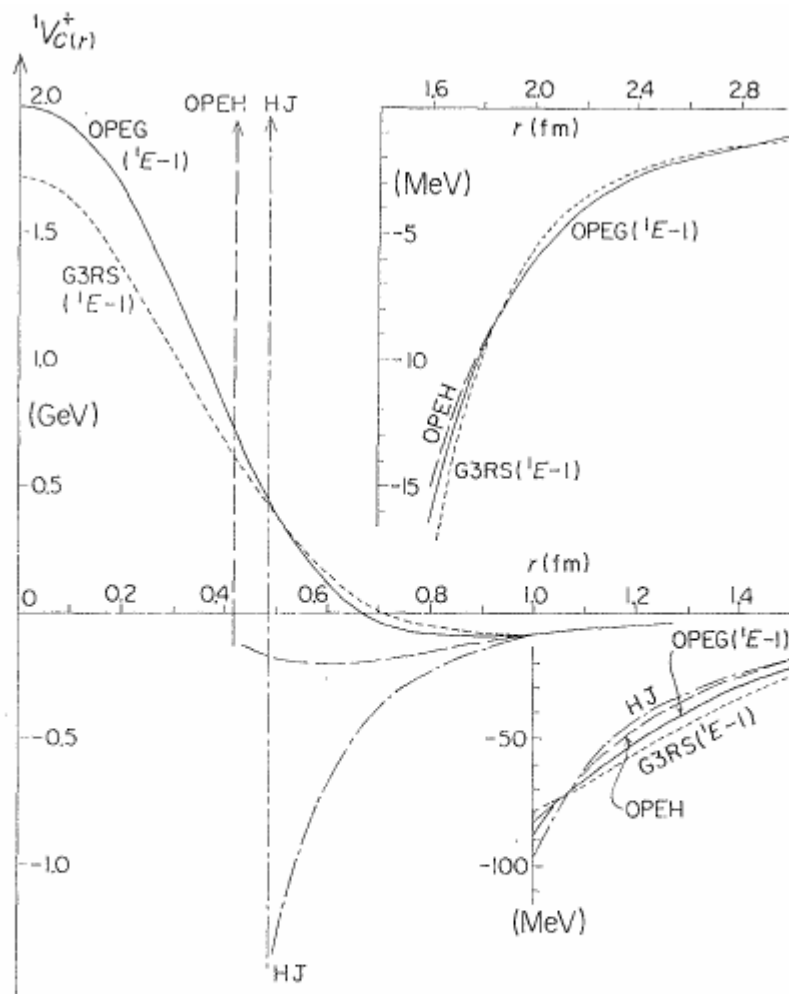


Fig. 1. Singlet even central potentials ${}^1V_{C^+}(r)$ which reproduce the 1S_0 -phase shifts up to 660 MeV. The potential parameters are given in Tables I, II and IV. The curves for $r < 1.0$ fm illustrates that the essential feature of the 1S_0 repulsive core lies in its steepness. The curves for $r > 1.6$ fm show the goodness of the Gaussian 3-range approximation for the tail part.

What we do is the following:

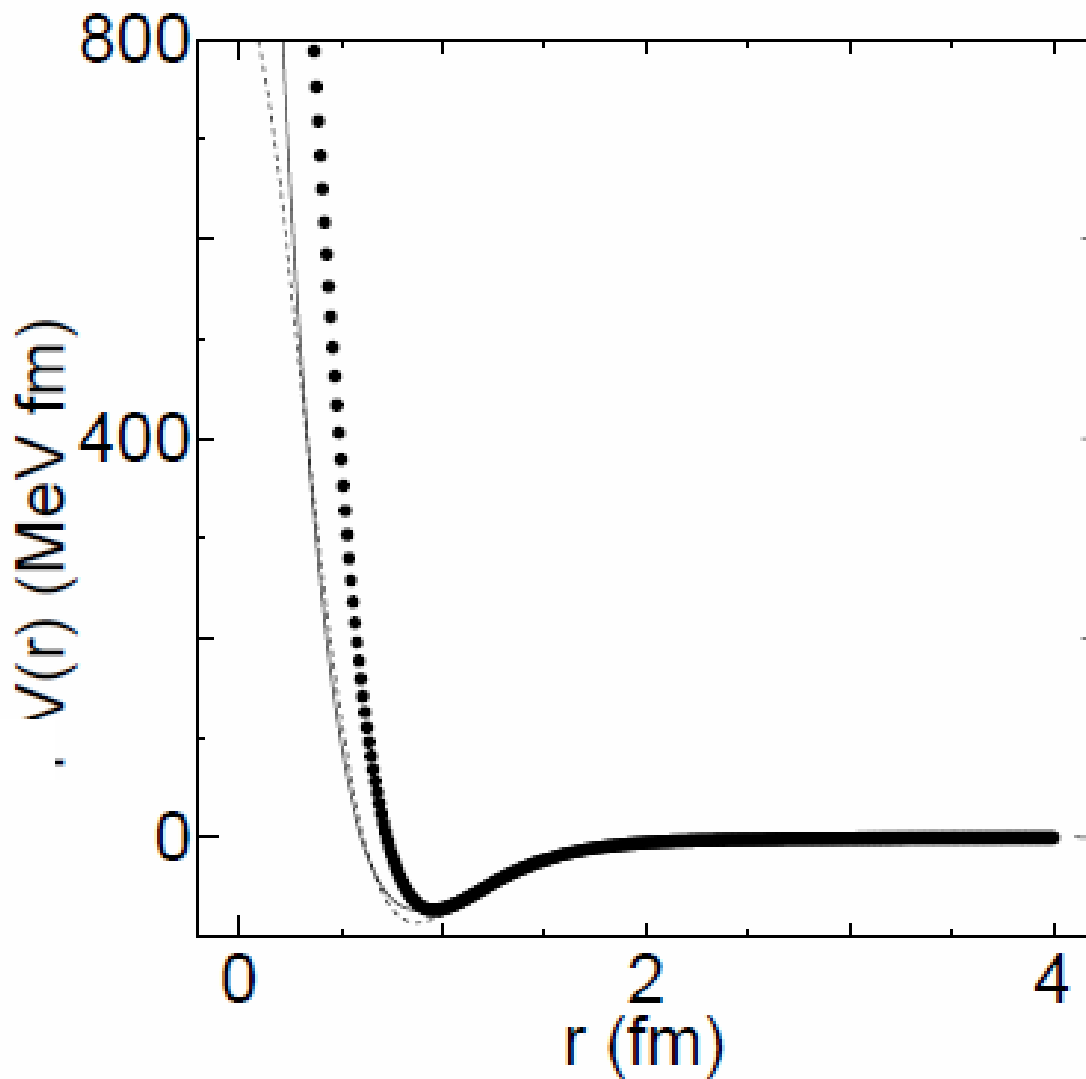
- We start with the G3RS interaction with a softcore
- We perform the UCOM-like transformation for the central interaction
- We refit the transformed central interaction by adjusting the amplitude of 3 Gaussians
- The best fit is to reduce the amplitude of the shortest range 2000 MeV \rightarrow 1100 MeV

Even-parity channel

Dots → original interaction

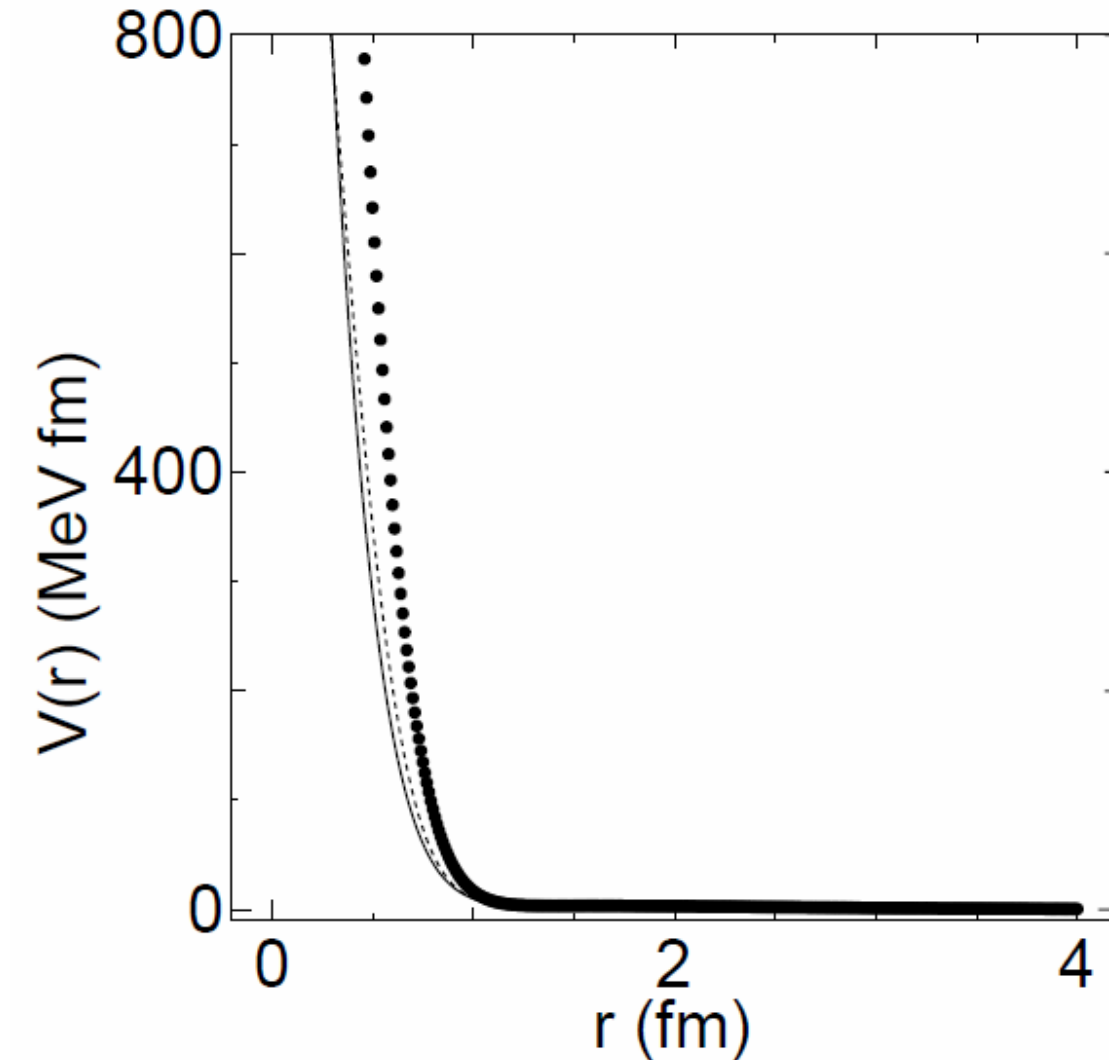
Solid line → UCOM-like transformation

Dotted line → refit

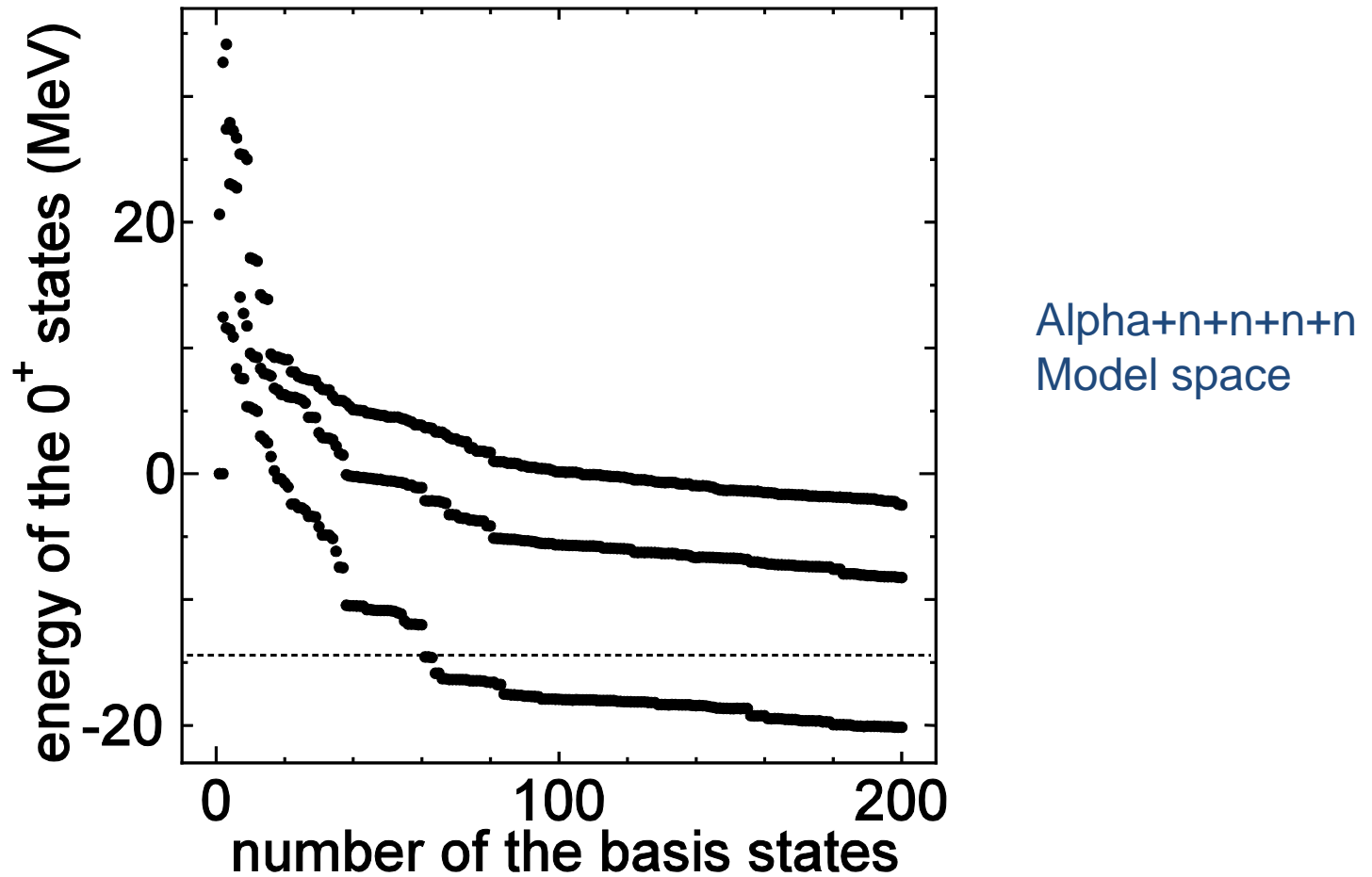


Odd channel

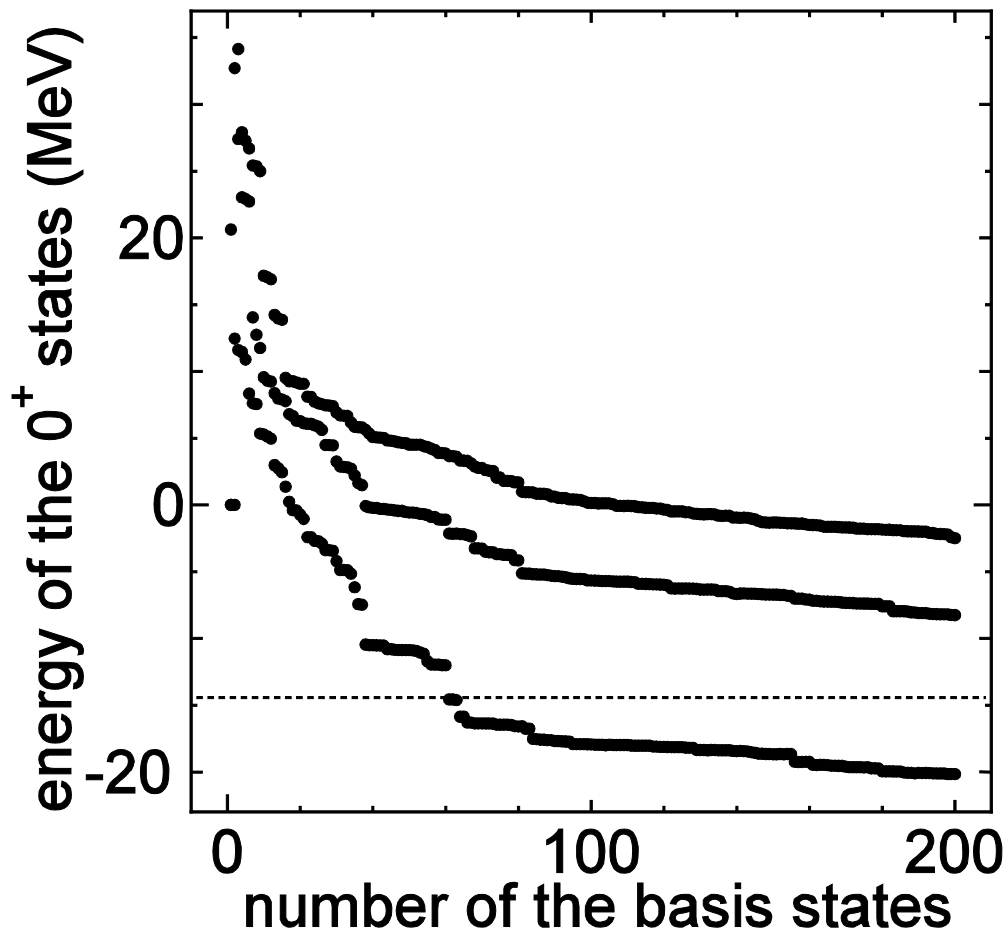
Dots → original interaction
Solid line → UCOM-like transformation
Dotted line → refit



Energy convergence of ${}^8\text{He}$



^8He ground state, squared overlap with alpha+2 dineutrons THSR wave function



G3RS+UCOM-like

sigma = 2 fm 0.41
sigma = 3 fm 0.33
sigma = 4 fm 0.22

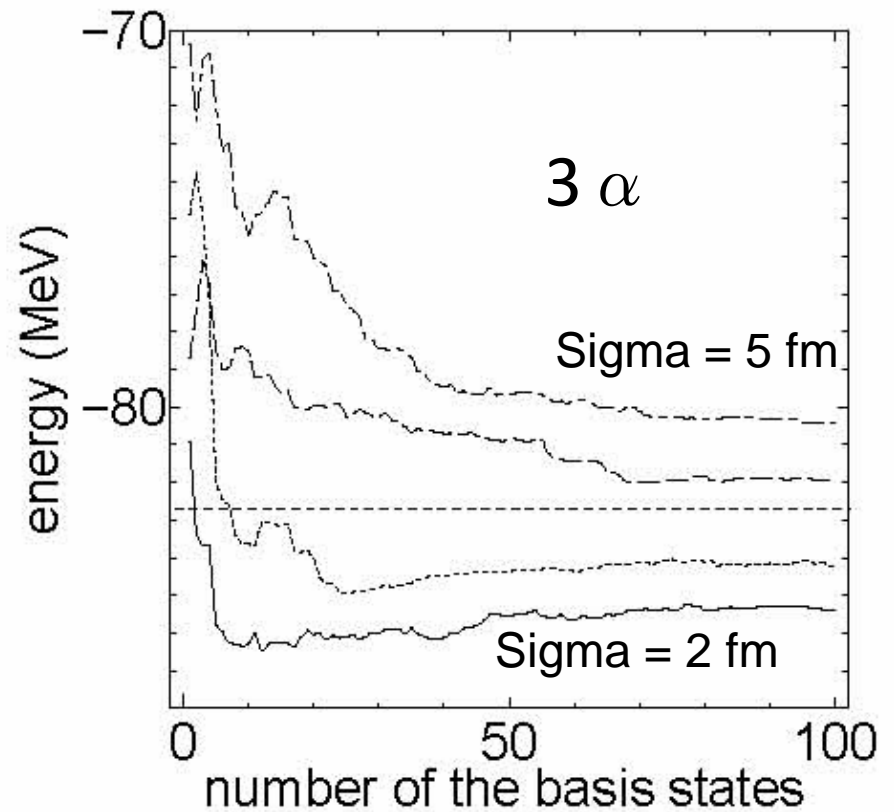
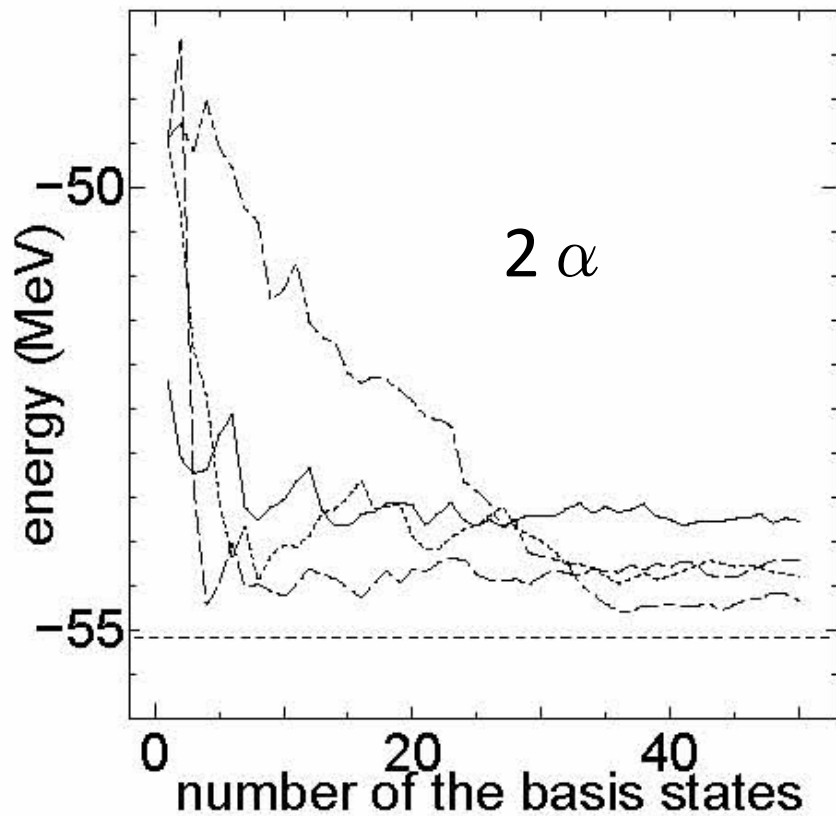
previous

Effect interaction case:

sigma = 2 fm 0.53
sigma = 3 fm 0.61
sigma = 4 fm 0.45

Result 2:

Alpha-like correlation

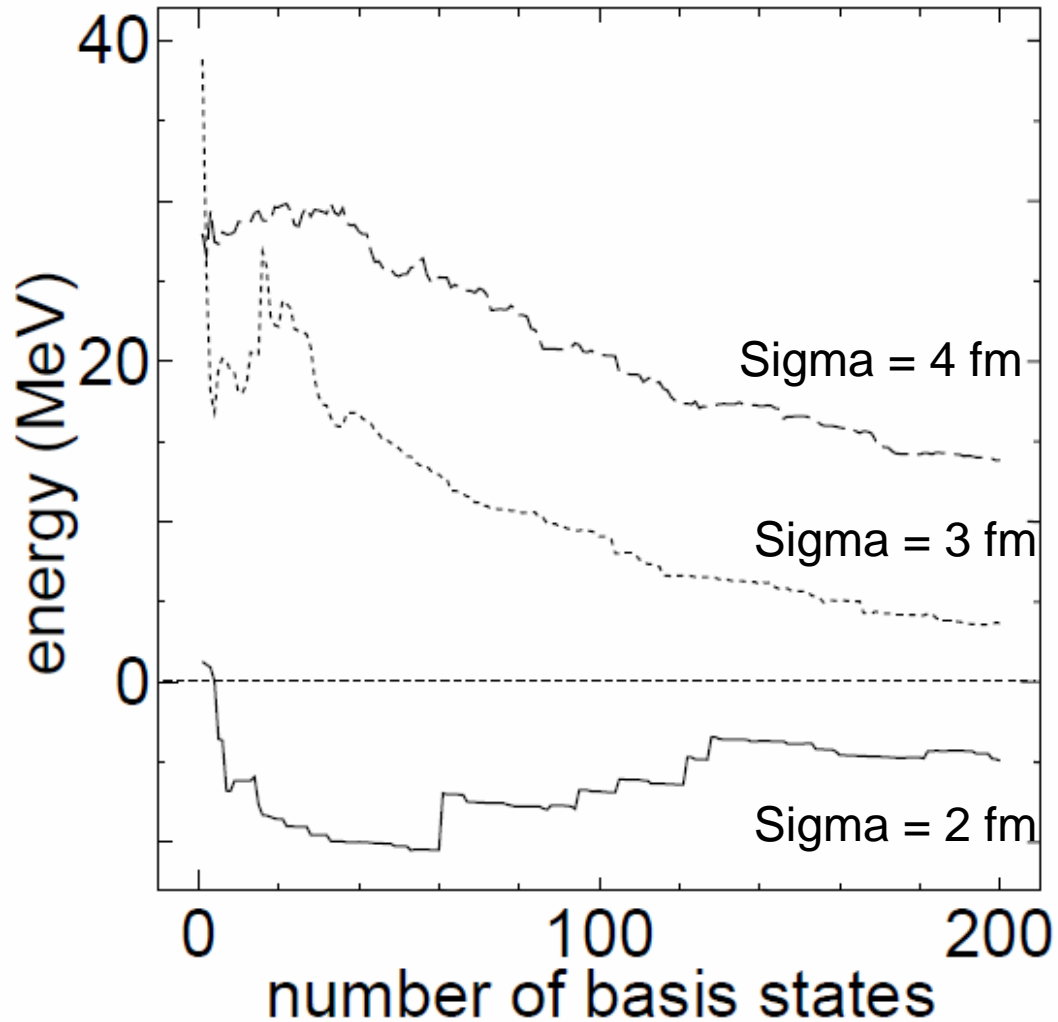


Solid, dotted, dashed, dash-dotted $\rightarrow \sigma = 2,3,4,5$ fm

r.m.s. radius
of ^{12}C

$\sigma = 3$	$\sigma = 4$	$\sigma = 5$	Micro	Cond
3.06	3.60	4.38	3.47	3.83

$^{20}\text{Ne} = 5$ alpha's



N. Itagaki, Tz. Kokalova, M. Ito, M. Kimura, and W. von Oertzen,
Phys. Rev. C 77 037301 (2008)

0^+ states of the 5 α system (^{20}Ne)

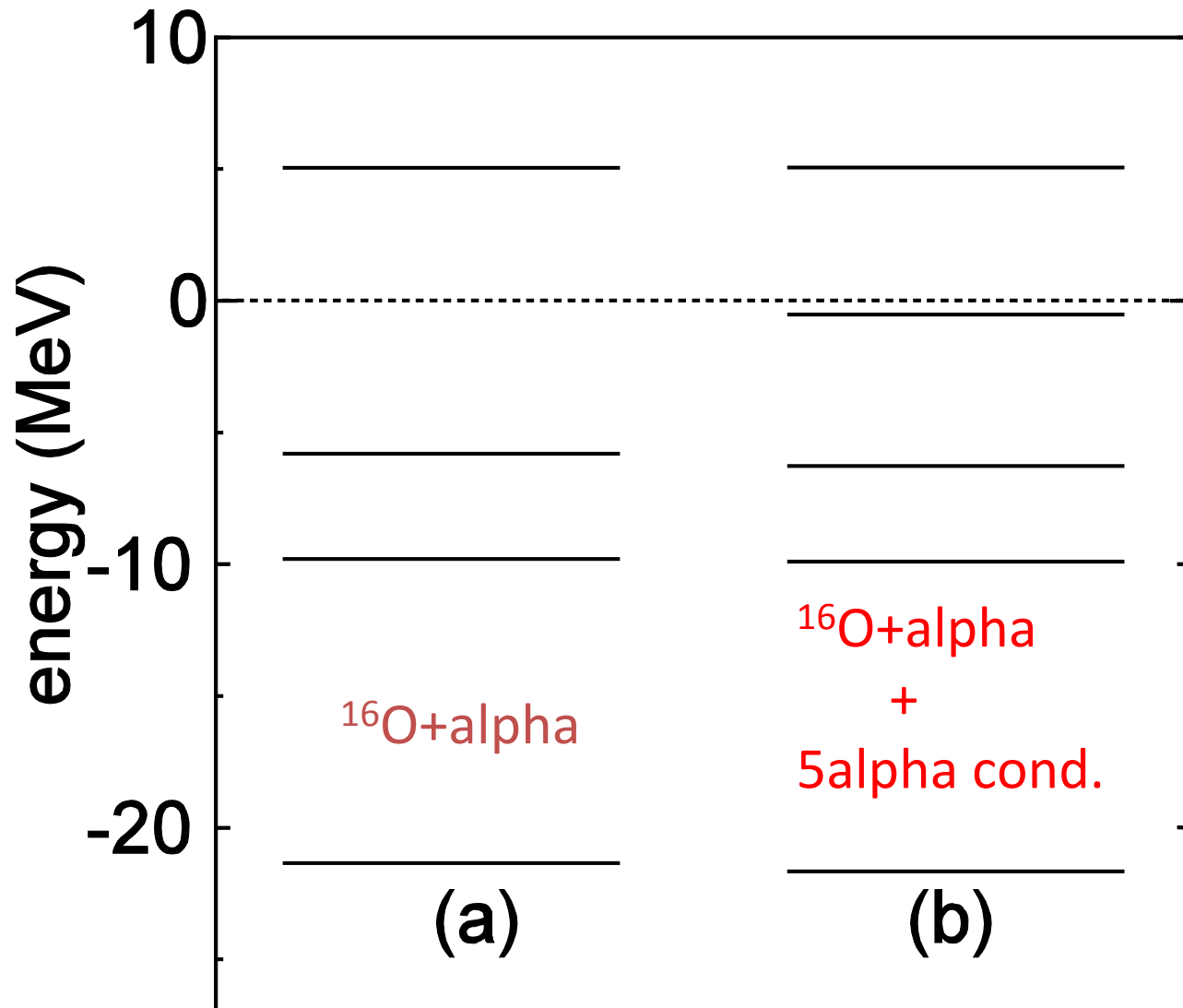
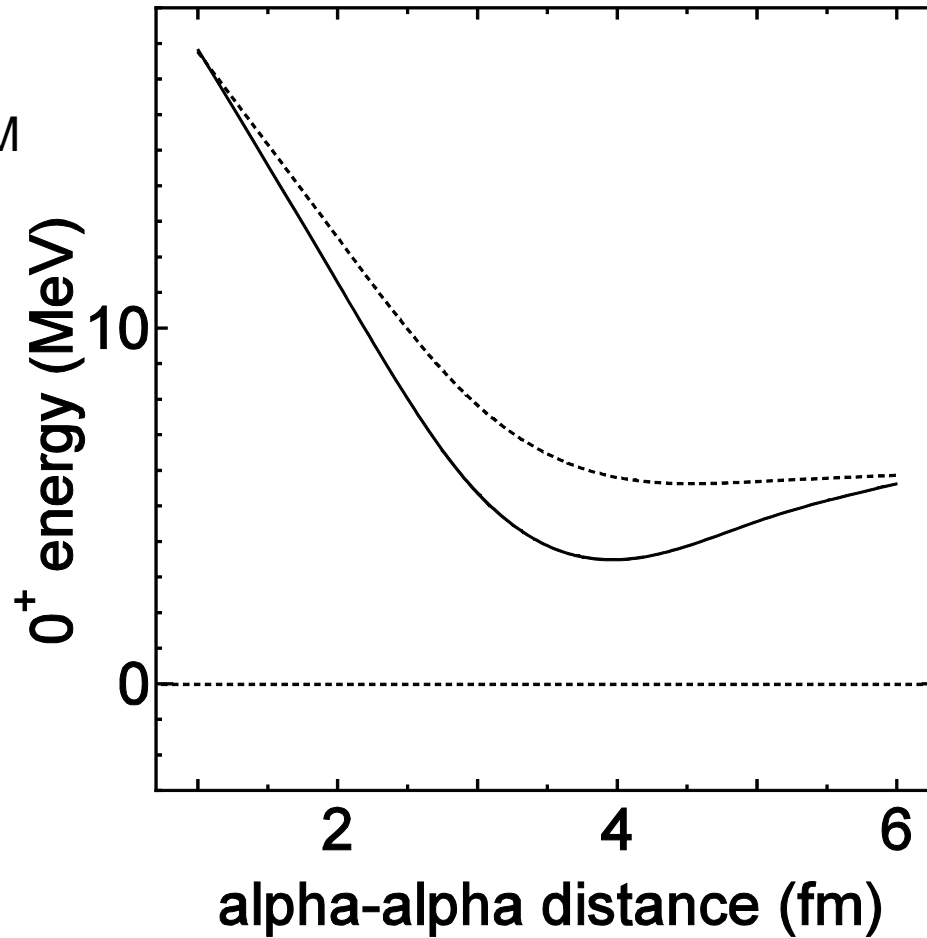


TABLE I: The squared overlap between the wave function of each 0^+ state of ^{20}Ne (Fig. 4 (b)) and virtual Schuck wave function with $\sigma = 2, 3, 4$ fm.

state	$\sigma = 2$ fm	$\sigma = 3$ fm	$\sigma = 4$ fm
0_1^+	0.19	0.03	0.00
0_2^+	0.02	0.01	0.00
0_3^+	0.09	0.04	0.01
0_4^+	0.68	0.51	0.13
0_5^+	0.00	0.04	0.03

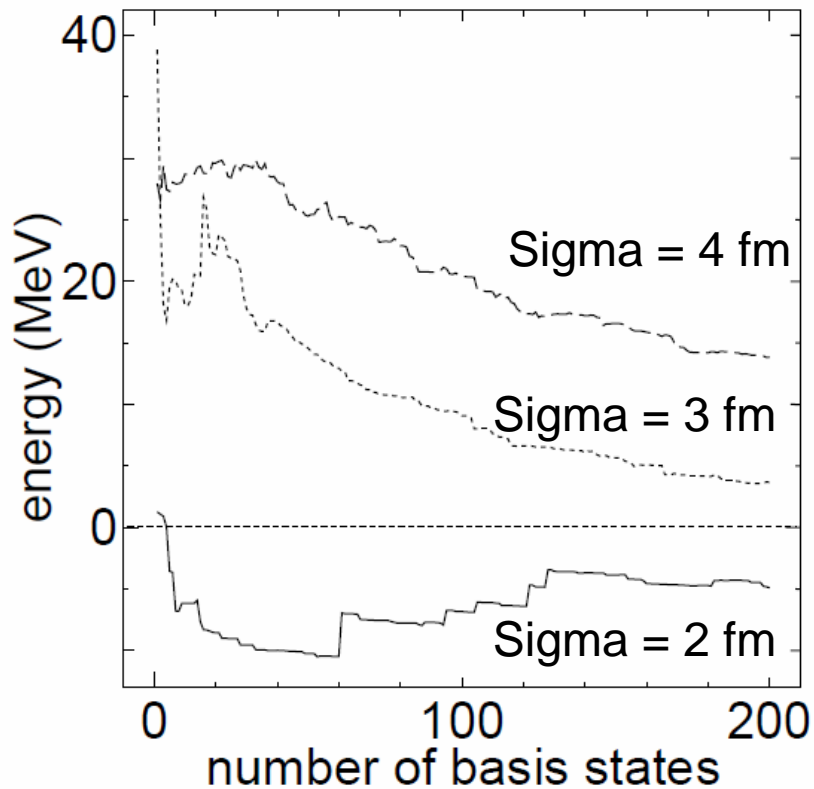
^8Be energy curve

Solid : effective
Dotted : G3RS+UCOM

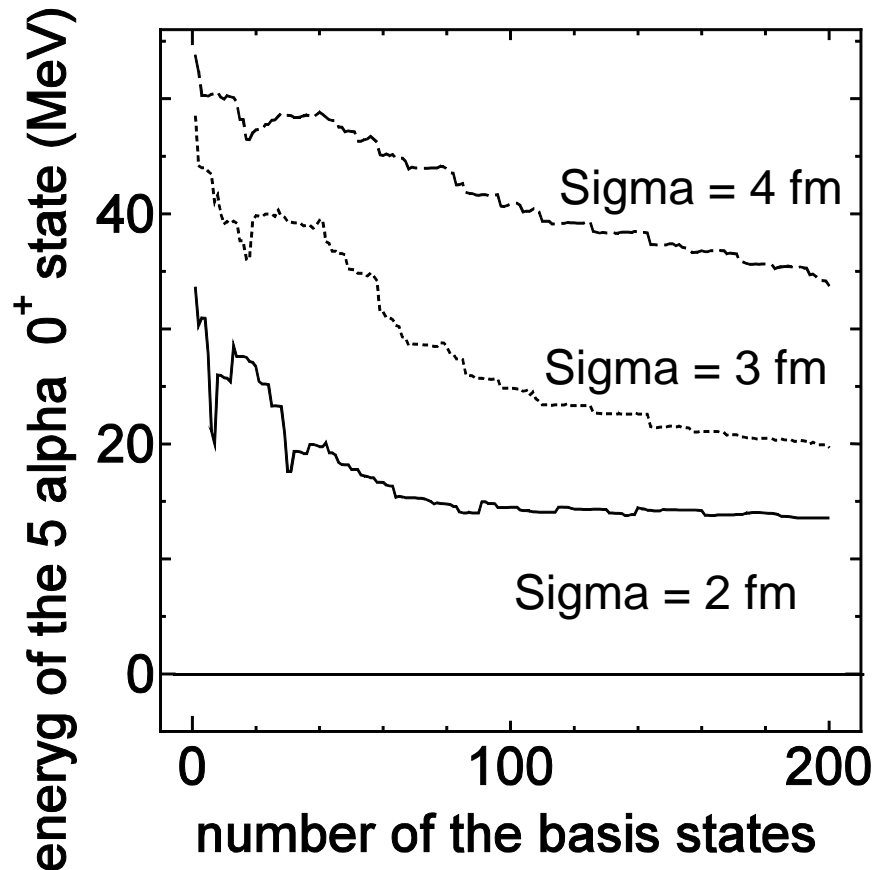


$^{20}\text{Ne} = 5$ alpha's

Effective interaction



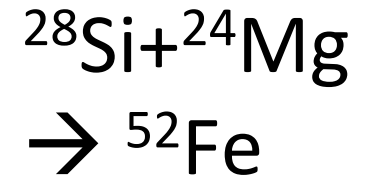
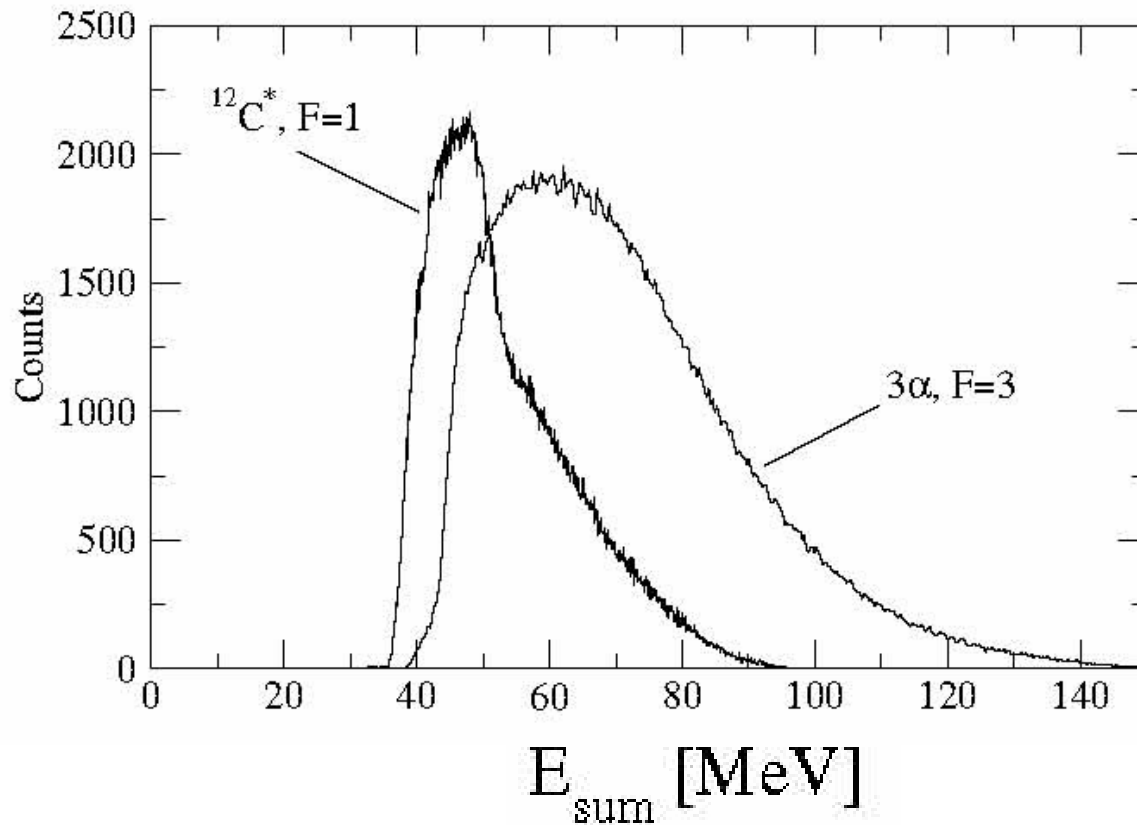
G3RS+UCOM-like



alpha condensation, the next step.....

condensation around core nucleus

Tz. Kokalova et al. Eur. Phys. J A23 (2005)



Enhancement of emissions of the condensed state from the compound state compared with the sequential α emissions

Is it due to the lowering of the Coulomb barrier for the condensed state?

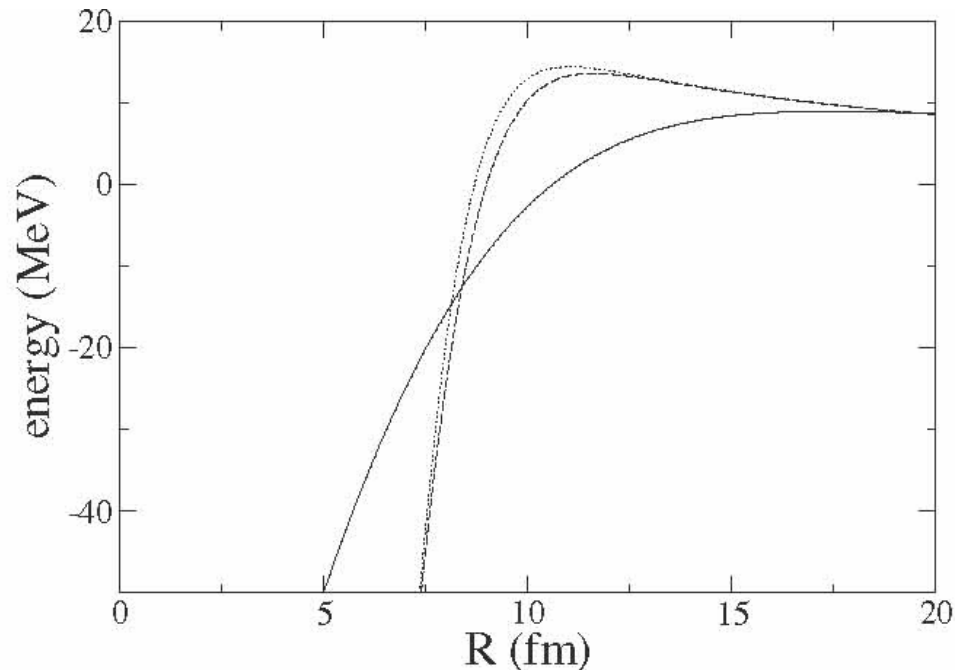
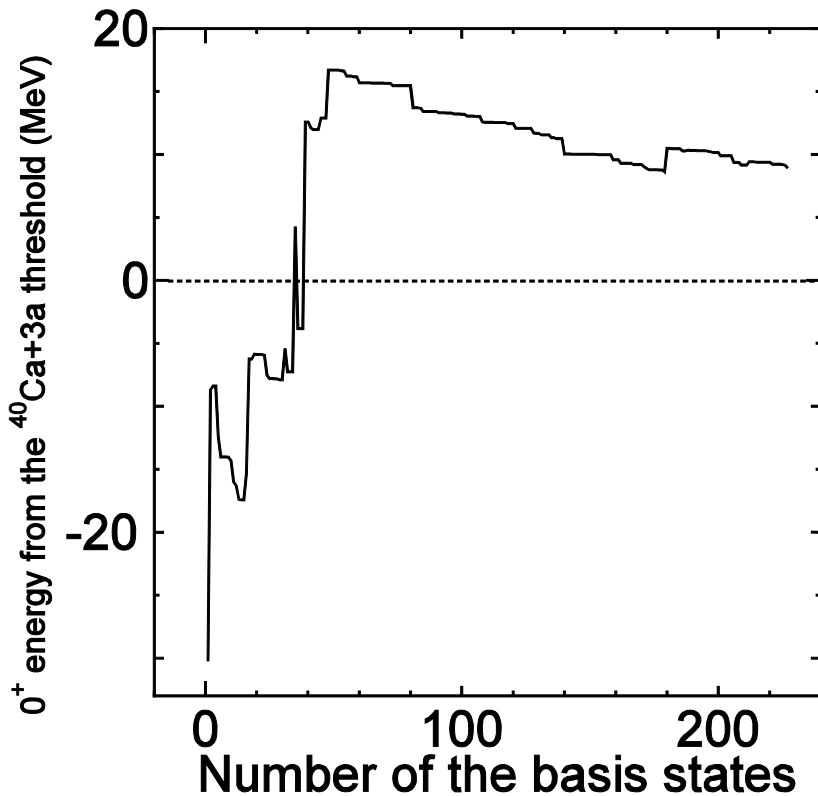


FIG. 1. The folded potential (V in the text) for ^{12}C emission as a function of the distance (R in the text) between the ^{12}C and the ^{40}Ca core. The solid, dashed, and dotted lines correspond to the condensed, cluster, and ground states of ^{12}C , respectively.

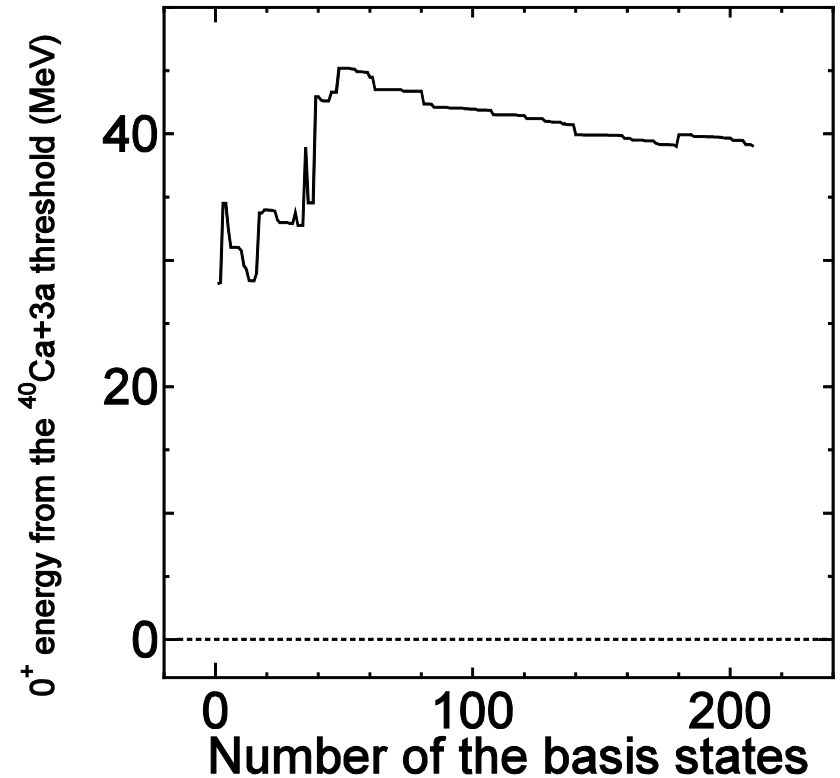
Tz. Kokalova, N.Itagaki., C. Wheldon, and W von Oertzen,
Physical Review Letters **96**, 192502 (2006).

Gas-like state of 3 alpha around ^{40}Ca



- $\sigma = 3$ fm

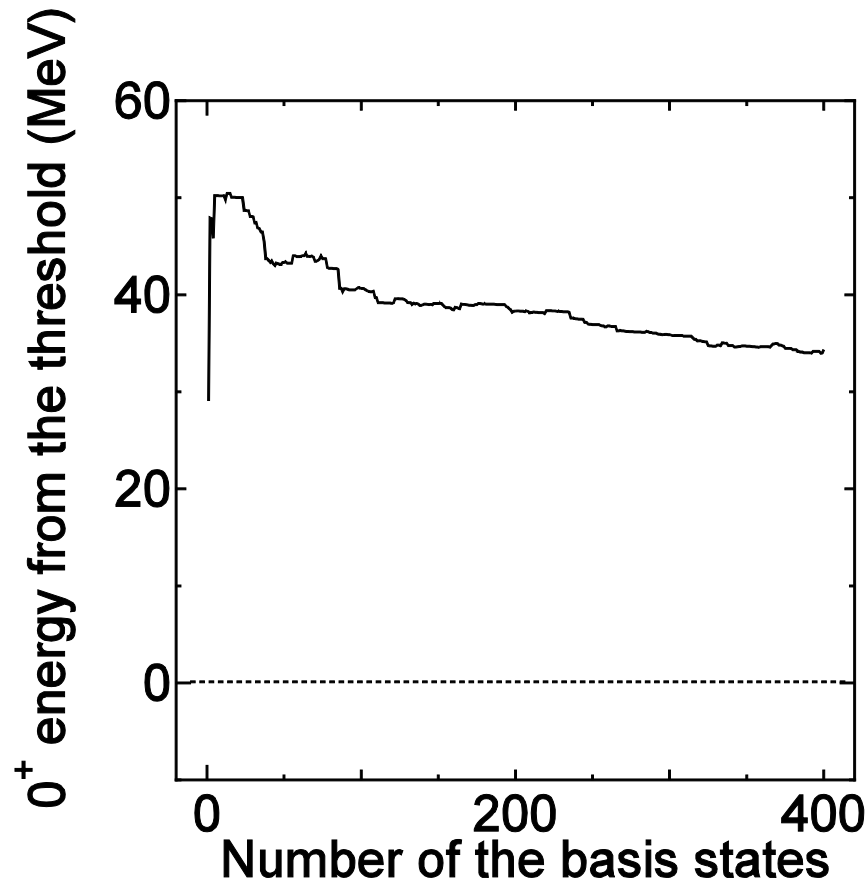
Effective interaction



- $\sigma = 3$ fm

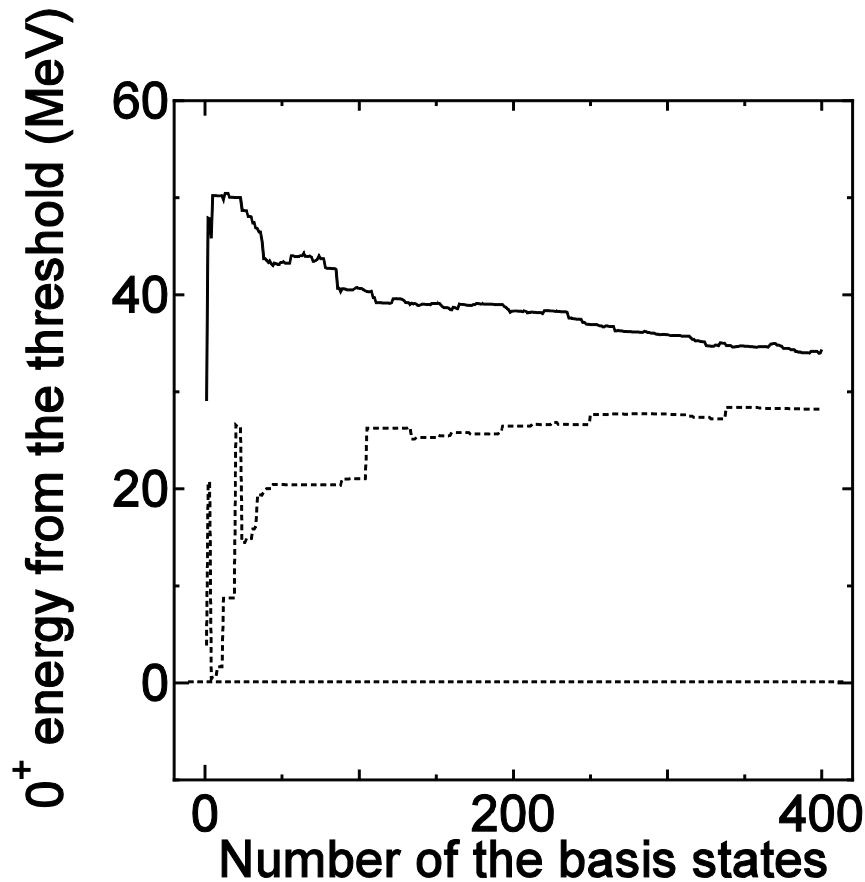
G3RS interaction

6-alpha gas-like state



- $\sigma = 3$ fm Effective interaction

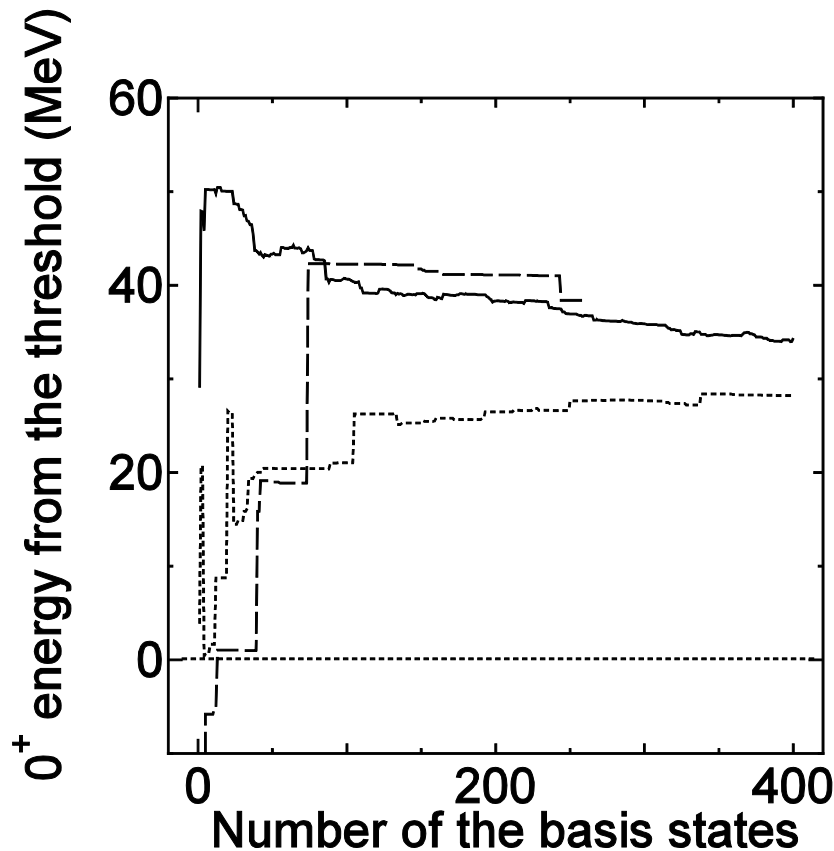
6-alpha gas-like state



Dotted line
Around ¹⁶O

- $\sigma = 3$ fm Effective interaction

6-alpha gas-like state

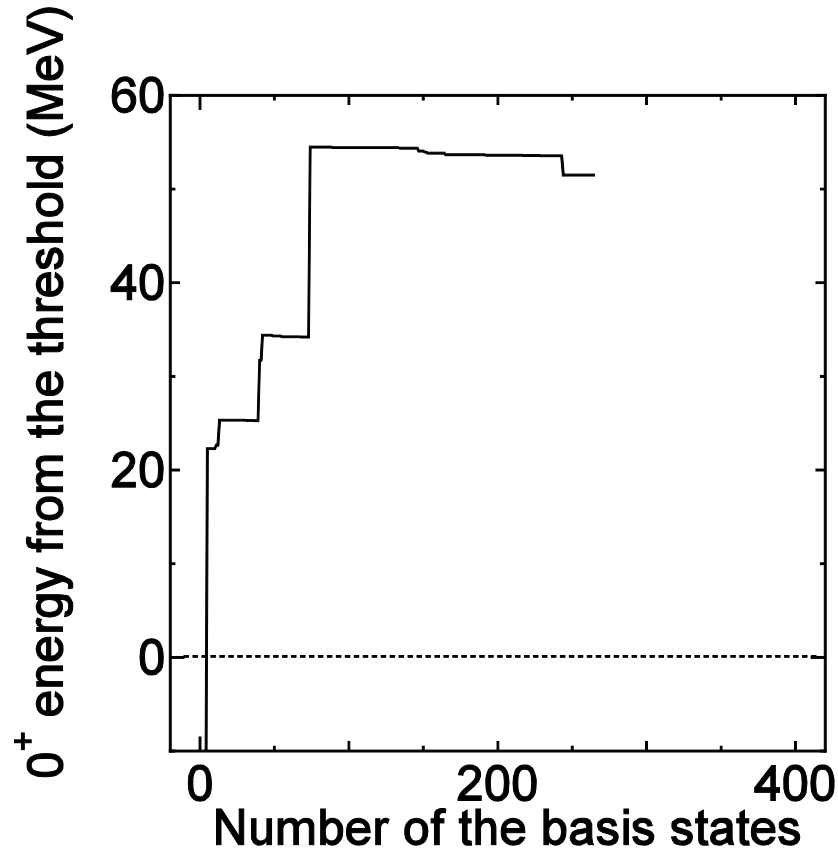


Dotted line
Around ^{16}O

Dashed line
Around ^{40}Ca

- $\sigma = 3 \text{ fm}$ Effective interaction

6alpha around ^{40}Ca



- $\sigma = 3$ fm G3RS+UCOM-like treatment

Summary:

We have shown the application of THSR wave functions using Monte Carlo technique

- * The ground state of ${}^8\text{He}$ contains large amount of di-neutron component
- * 5 alpha condensate survives after imposing the coupling with the ${}^{16}\text{O}+\text{alpha}$ configurations
- * Alpha condensate around core nucleus can be attacked
- * The analysis using softcore potential with UCOM treatment is under development

Then, let us use realistic interaction seriously:

- Which realistic interaction should be used?
- What kind of UCOM transformation is good?

Discussion with German side is essential