Prospects with Extended RPA Theories

P. Papakonstantinou

Institut für Kernphysik, T.U.Darmstadt

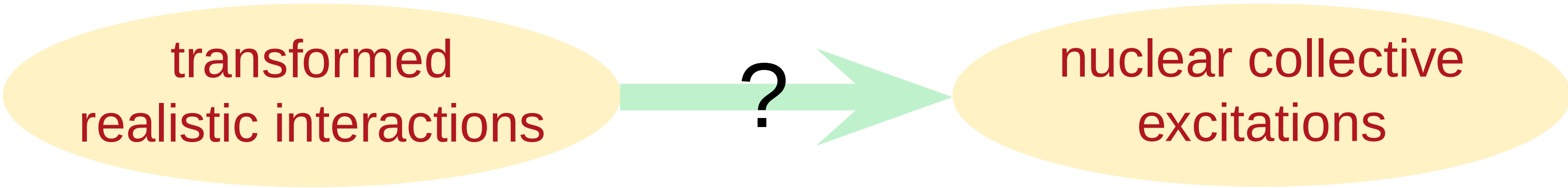
TECHNISCHE UNIVERSITÄT DARMSTADT

SFB 634

Overview

- Introduction
- From RPA to Second RPA
  - Formalism and technicalities
  - Results on Giant Resonances
  - Issues to be considered
- Conclusion and Outlook

Introduction



- Description based on RPA theories
- Why extended RPA theories?
  - More physics
  - Convergence issues with respect to model space
- What kind of extension is appropriate?

  ... remains to be seen

  - Second RPA to begin with

From the textbook

- **RPA**
  - Microscopic theory of small-amplitude density fluctuations
  - Single-particle excitation operators $f_L(r)Y_{LM}(\hat{r})$ (+isospin)
  - **GRs**: coherent superpositions of ph excitations
  - Change in single-particle Hamiltonian treated self-consistently
- **Why beyond RPA**
  - **Damping** of GRs due to

    coupling of ph state to 2p2h states and higher

    coupling to surface vibrations

    increases the width of GRs Γ_ν
  - **But also**: energetically shifts them by Δ_ν

    Dispersion relation: $\Delta_\nu(E) = \frac{\mathcal{P}}{2\pi} \int d\epsilon \frac{\Gamma_\nu(\epsilon)}{E-\epsilon}$

Present Work

- Two-body UCOM Hamiltonian

  ☞ Only state-independent, short-range correlations are treated

- A Second-order RPA Method

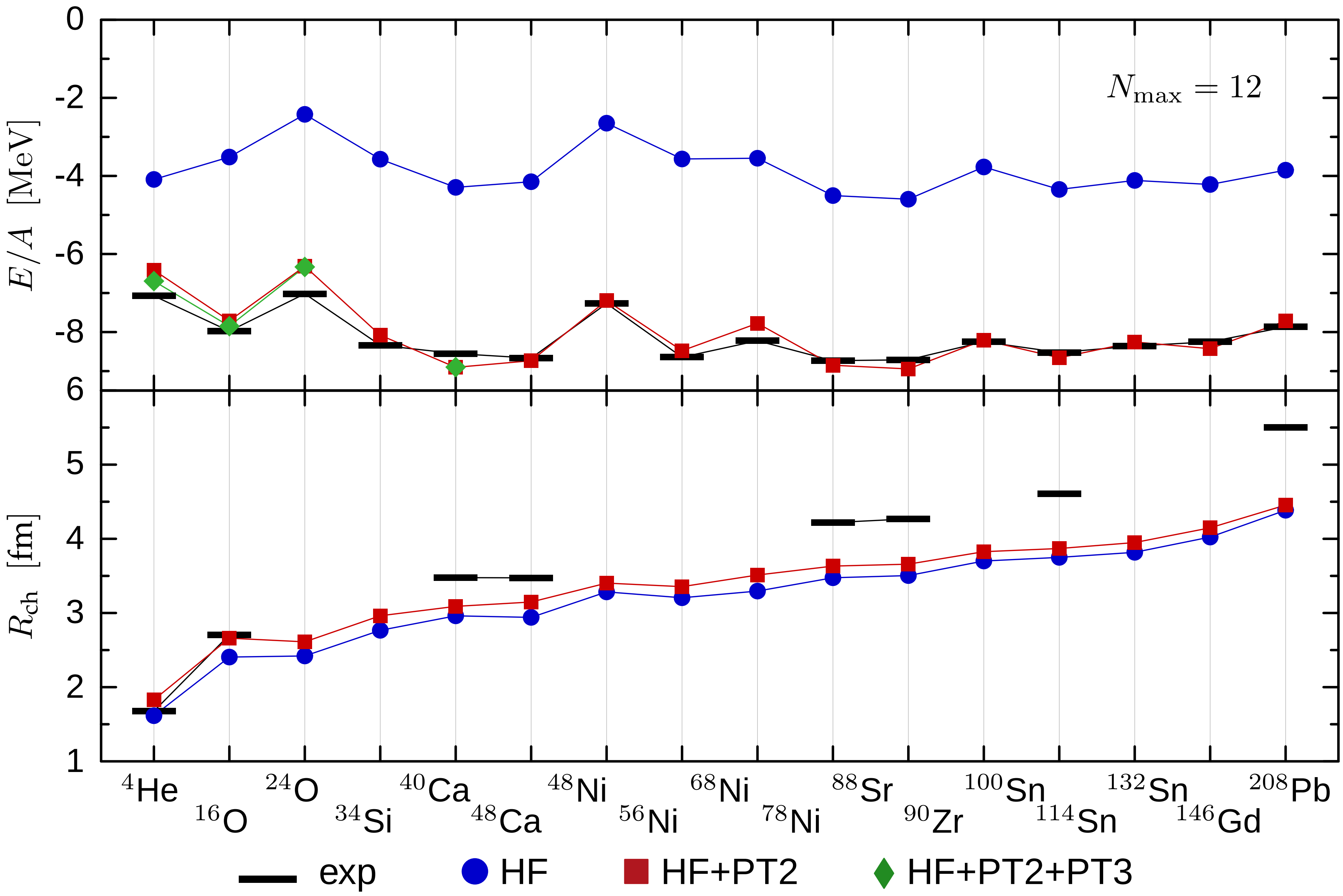
  ☞ Large-scale calculations in closed-shell nuclei

- Interesting results on Giant Resonances
- Learning about the inter-action and the method!

- Technical issues to be dealt with
- Formalism and consistency issues of the present SRPA method

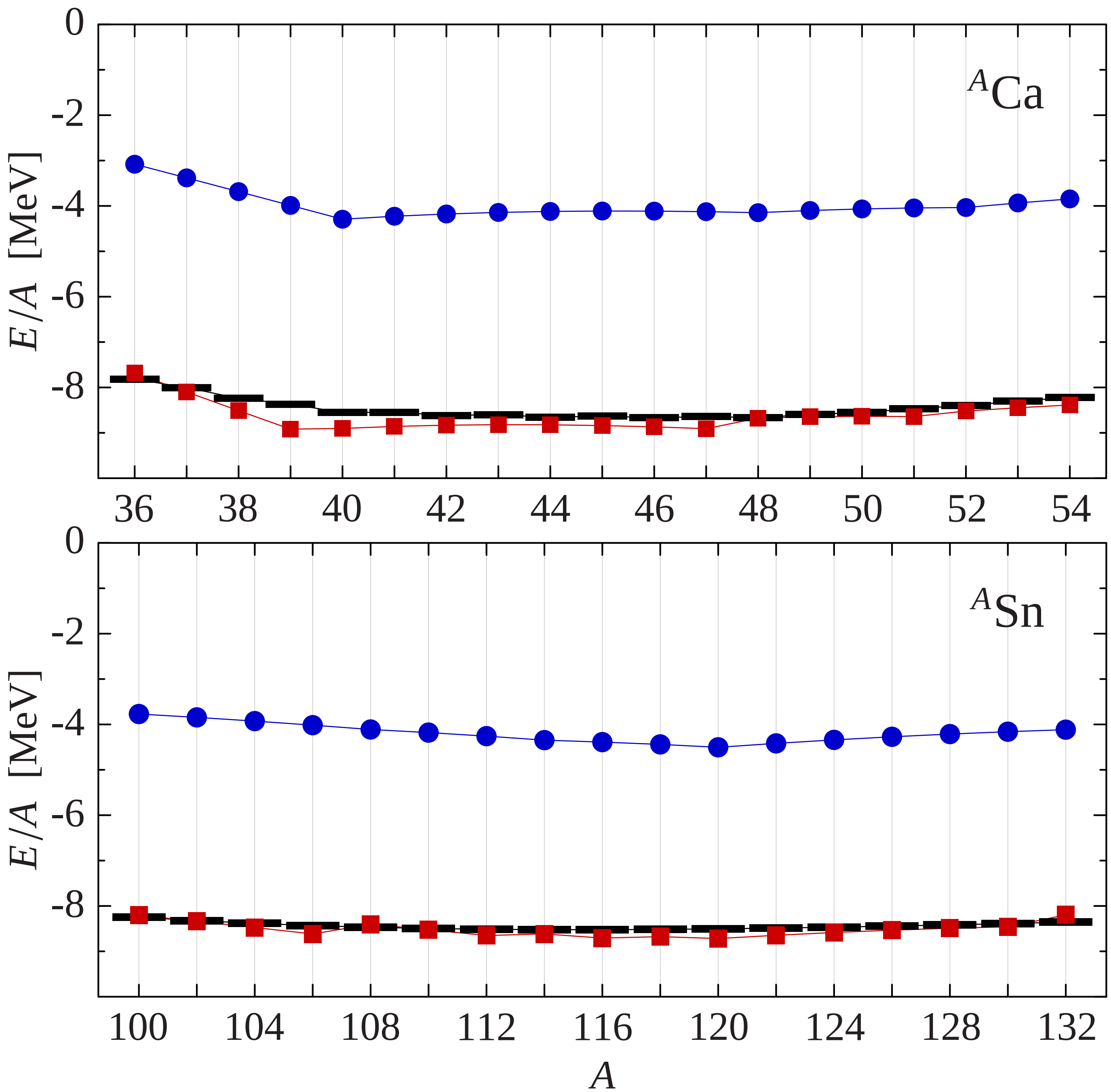
☞ In most of what follows a UCOM-transformed Argonne V18 potential is used

UCOM-HF + PT



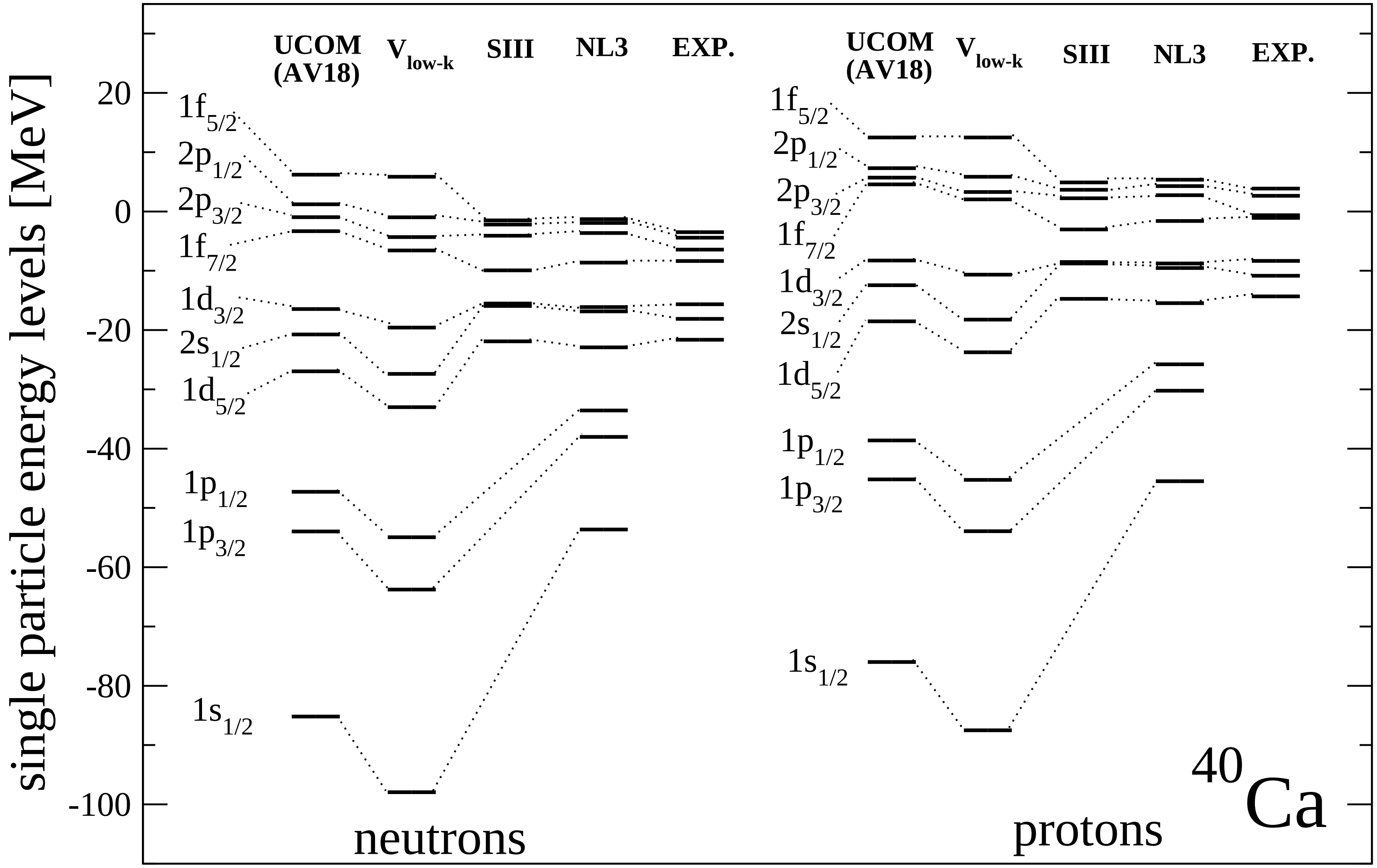
$N_{max} = 12$
E/A [MeV]
0
-2
-4
-6
-8
6
5
4
3
2
1
R_{ch} [fm]
^{4}He
^{16}O
^{24}O
^{34}Si
^{40}Ca
^{48}Ca
^{48}Ni
^{56}Ni
^{68}Ni
^{78}Ni
^{88}Sr
^{90}Zr
^{100}Sn
^{114}Sn
^{132}Sn
^{146}Gd
^{208}Pb
exp
HF
HF+PT2
HF+PT2+PT3

UCOM-HF + PT



0
-2
-4
-6
-8
E/A [MeV]
^{A}Ca
36
38
40
42
44
46
48
50
52
54
0
-2
-4
-6
-8
E/A [MeV]
^{A}Sn
100
104
108
112
116
120
124
128
132
A

UCOM-HF


single particle energy levels [MeV]
20
0
-20
-40
-60
-80
-100
UCOM (AV18)
V_{low-k}
SIII
NL3
EXP.
$1f_{5/2}$
$2p_{1/2}$
$2p_{3/2}$
$1f_{7/2}$
$1d_{3/2}$
$2s_{1/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$
neutrons
protons
^{40}Ca

Standard RPA

- Vibration creation operator:

$$Q_\nu^\dagger = \sum_{ph} X_{ph}^\nu O_{ph}^\dagger - \sum_{ph} Y_{ph}^\nu O_{ph} \quad ; \quad Q_\nu|\mathrm{RPA}\rangle = 0 \quad ; \quad Q_\nu^\dagger|\mathrm{RPA}\rangle = |\nu\rangle$$

- Standard RPA - the RPA vacuum is approximated by the HF ground state:

$$\langle\mathrm{RPA}|\ldots|\mathrm{RPA}\rangle \rightarrow \langle\mathrm{HF}|\ldots|\mathrm{HF}\rangle \quad ; \quad O_{ph}^\dagger \rightarrow a_p^\dagger a_h$$

- RPA equations in $ph-$space:

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \hbar\omega_\nu \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix}$$

$$A_{ph,p'h'} = \delta_{pp'}\delta_{hh'}(e_p - e_h) + H_{hp',ph'} \;\; ; \;\; B_{ph,p'h'} = H_{hh',pp'} \;\; ; \;\; H = H_{\mathrm{int}} = T_{\mathrm{rel}} + V_{\mathrm{UCOM}}$$

☞ Self-consistent HF+RPA: spurious state and sum rules

Second RPA

- Vibration creation operator: Includes $2p2h$ configurations

$$Q_\nu^\dagger = \sum_{ph} X_{ph}^\nu O_{ph}^\dagger - \sum_{ph} Y_{ph}^\nu O_{ph} + \sum_{p_1h_1p_2h_2} \mathcal{X}_{p_1h_1p_2h_2}^\nu O_{p_1h_1p_2h_2}^\dagger$$
$$- \sum_{p_1h_1p_2h_2} \mathcal{Y}_{p_1h_1p_2h_2}^\nu O_{p_1h_1p_2h_2}$$

- The SRPA vacuum is approximated by the HF ground state:

$$\langle \mathrm{SRPA}|\ldots|\mathrm{SRPA}\rangle \rightarrow \langle \mathrm{HF}|\ldots|\mathrm{HF}\rangle$$

- SRPA equations in $ph \oplus 2p2h-$space:

$$\left(\begin{array}{cc|cc} A & \mathcal{A}_{12} & B & 0 \\ \mathcal{A}_{21} & \mathcal{A}_{22} & 0 & 0 \\ \hline -B^* & 0 & -A^* & -\mathcal{A}_{12}^* \\ 0 & 0 & -\mathcal{A}_{21}^* & -\mathcal{A}_{22}^* \end{array}\right) \left(\begin{array}{c} X^\nu \\ \mathcal{X}^\nu \\ \hline Y^\nu \\ \mathcal{Y}^\nu \end{array}\right) = \hbar\omega_\nu \left(\begin{array}{c} X^\nu \\ \mathcal{X}^\nu \\ \hline Y^\nu \\ \mathcal{Y}^\nu \end{array}\right)$$

$$A_{ph,p'h'} = \delta_{pp'}\delta_{hh'}(e_p - e_h) + H_{hp',ph'} \;\; ; \;\; B_{ph,p'h'} = H_{hh',pp'} \;\; ; \;\; H = H_{\mathrm{int}} = T_{\mathrm{rel}} + V_{\mathrm{UCOM}}$$

$\mathcal{A}_{12}$: interactions between ph and $2p2h$ states

$\mathcal{A}_{22}$: $\delta_{p_1p_1'}\delta_{h_1h_1'}\delta_{p_1p_1'}\delta_{h_1h_1'}(e_{p_1} + e_{p_2} - e_{h_1} - e_{h_2})$ + interactions among $2p2h$ states

Second RPA

- Large model spaces:
  - Number of states up to $\approx 10^6$ for the present cases – can get larger
  - But SRPA matrix is sparse and reduction to half the size is always possible

Second RPA

- Large model spaces:
  - Number of states up to $\approx 10^6$ for the present cases – can get larger
  - But SRPA matrix is sparse and reduction to half the size is always possible
- Use Lanczos
  - Find only the lowest eigenvalues $|\epsilon_\nu|$
  - ... or the ones closest to a set value E_0, e.g.

$$\mathrm{H}\mathrm{X}_\nu = \epsilon_\nu \mathrm{X}_\nu \iff \mathrm{H}'\mathrm{X}_\nu = \epsilon'_\nu \mathrm{X}_\nu \ , \quad \left\{ \begin{array}{l} \mathrm{H}' \equiv \mathrm{H} - E_0 \mathrm{I} \\ \epsilon'_\nu \equiv \epsilon_\nu - E_0 \end{array} \right\}$$

Second RPA

- Large model spaces:
  - Number of states up to $\approx 10^6$ for the present cases – can get larger
  - But SRPA matrix is sparse and reduction to half the size is always possible
- Use Lanczos
  - Find only the lowest eigenvalues $|\epsilon_\nu|$
  - ... or the ones closest to a set value E_0, e.g.

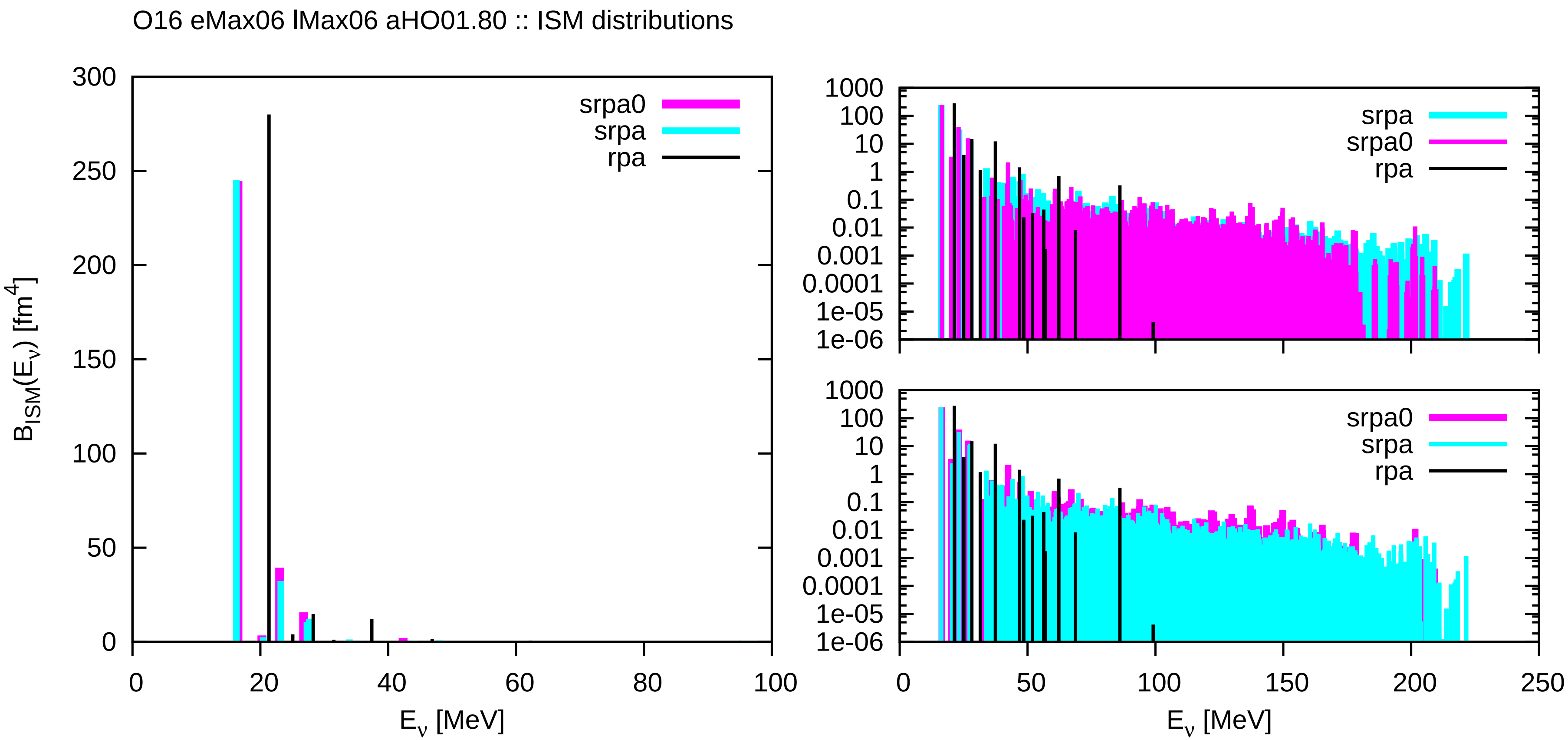
$$\mathrm{H}\mathrm{X}_\nu = \epsilon_\nu \mathrm{X}_\nu \iff \mathrm{H}'\mathrm{X}_\nu = \epsilon'_\nu \mathrm{X}_\nu \ , \quad \left\{ \begin{array}{l} \mathrm{H}' \equiv \mathrm{H} - E_0 \mathrm{I} \\ \epsilon'_\nu \equiv \epsilon_\nu - E_0 \end{array} \right\}$$

- Alternatively, reduce to an $\omega-$dependent problem of RPA size
  - ... especially if you ignore interactions within 2p2h space:

$$A_{php'h'} \longrightarrow A_{php'h'}(\epsilon) = A_{php'h'} + \sum_{PHP'H'} \frac{A^*_{ph\,PHP'H'} A_{p'h'\,PHP'H'}}{\hbar\epsilon - (\epsilon_P + \epsilon_{P'} - \epsilon_H - \epsilon_{H'}) + \mathrm{i}\eta}$$

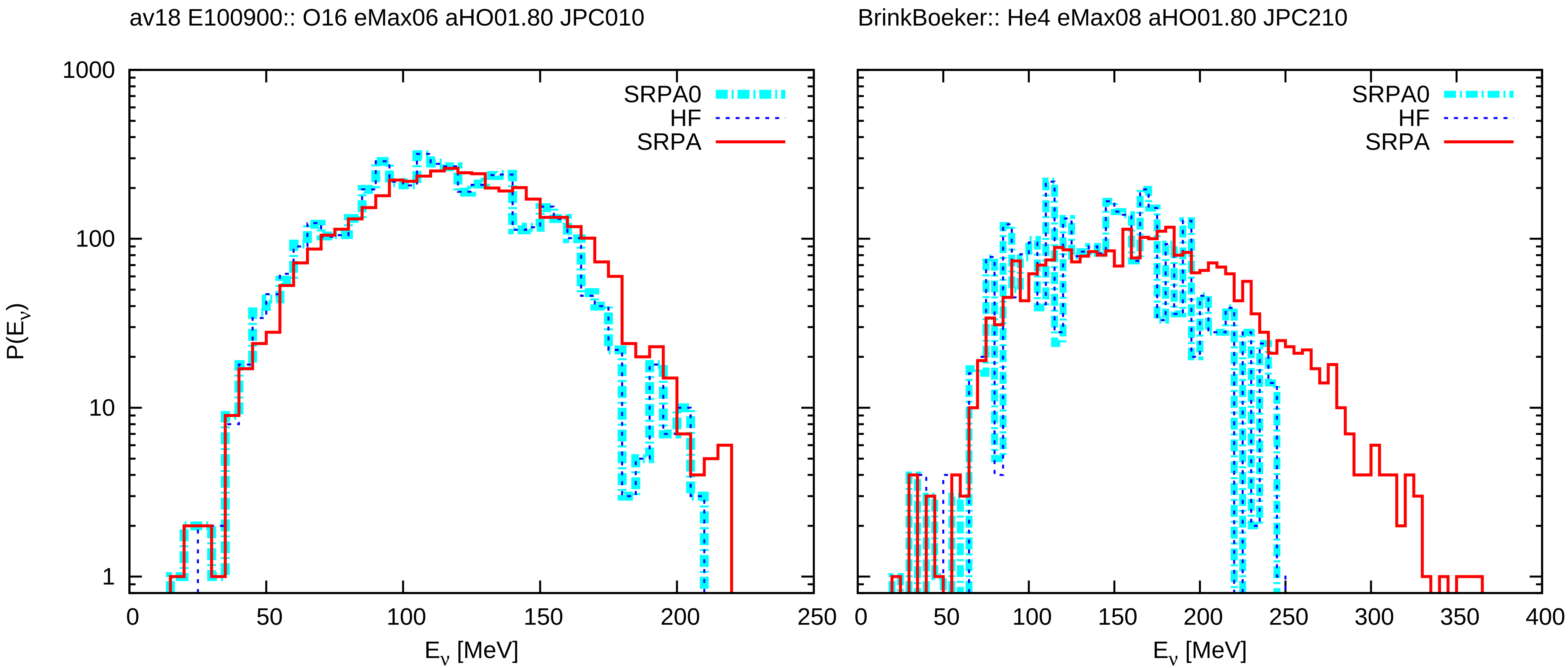
SRPA Eigenstates

SRPA and its diagonal approximation (”srpa0”) vs RPA

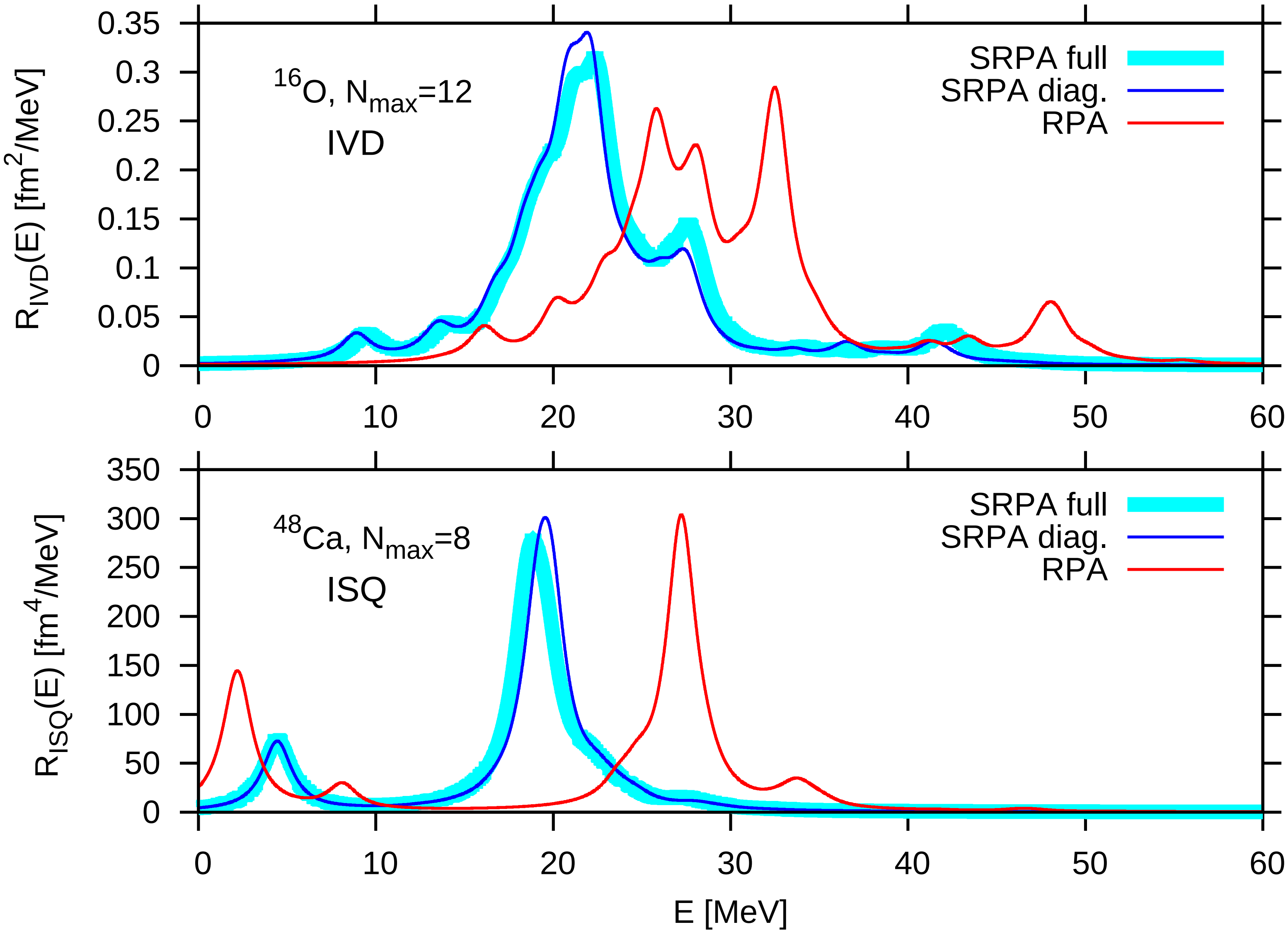


SRPA Eigenstate Density

SRPA vs its diagonal approximation and unperturbed states

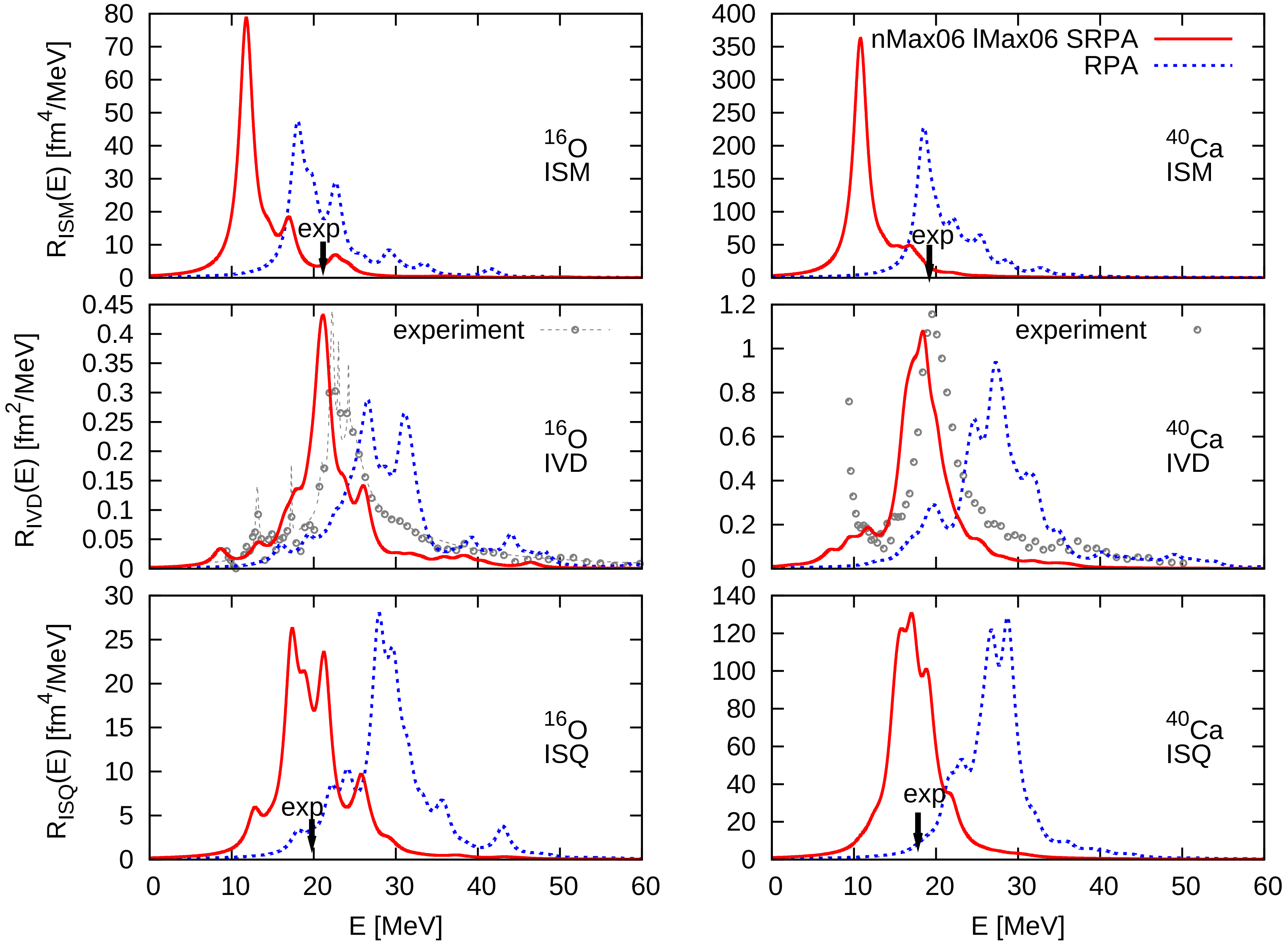


SRPA - Diagonal approximation

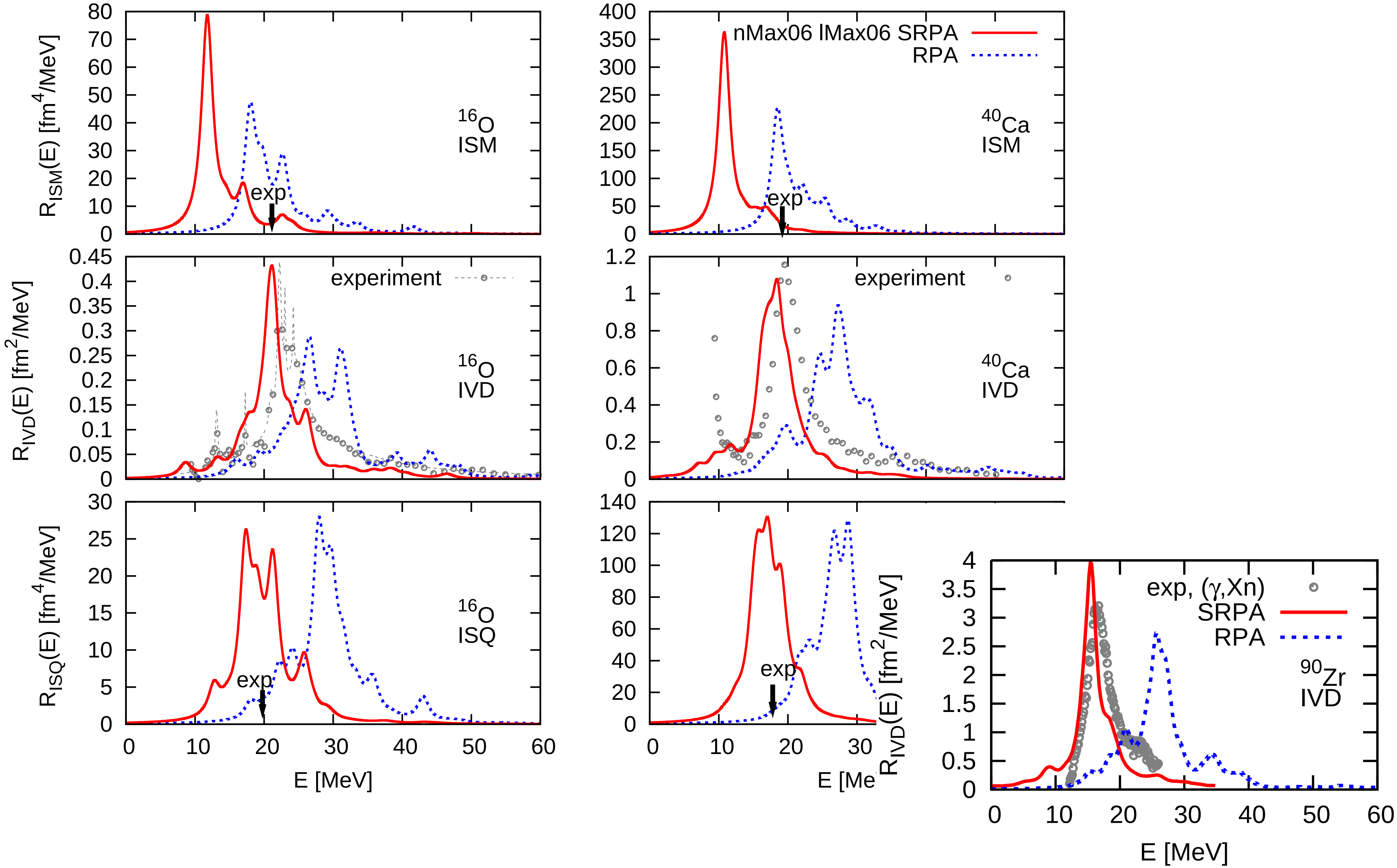


Results on GRs

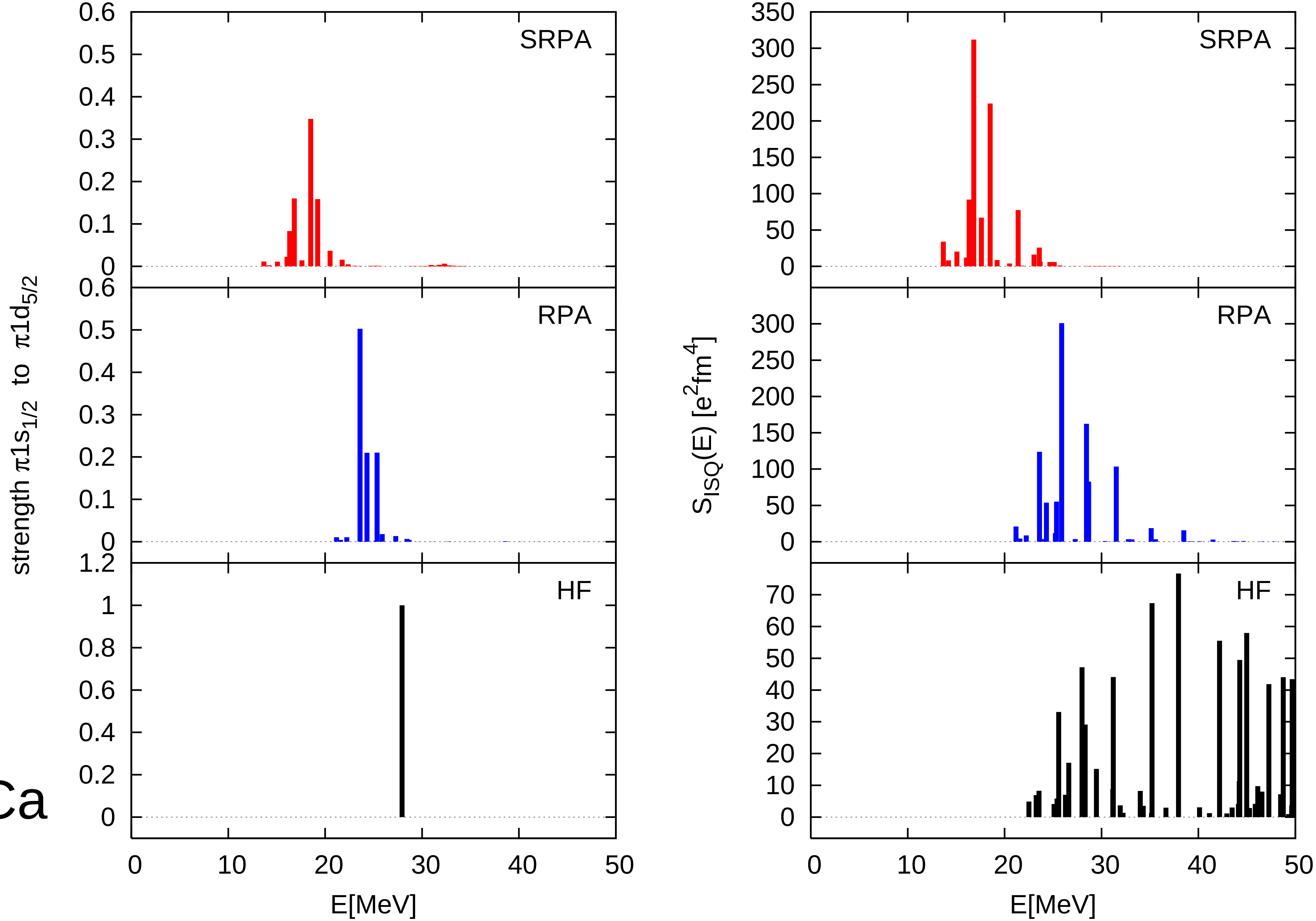
UCOM :: RPA and SRPA



UCOM :: RPA and SRPA

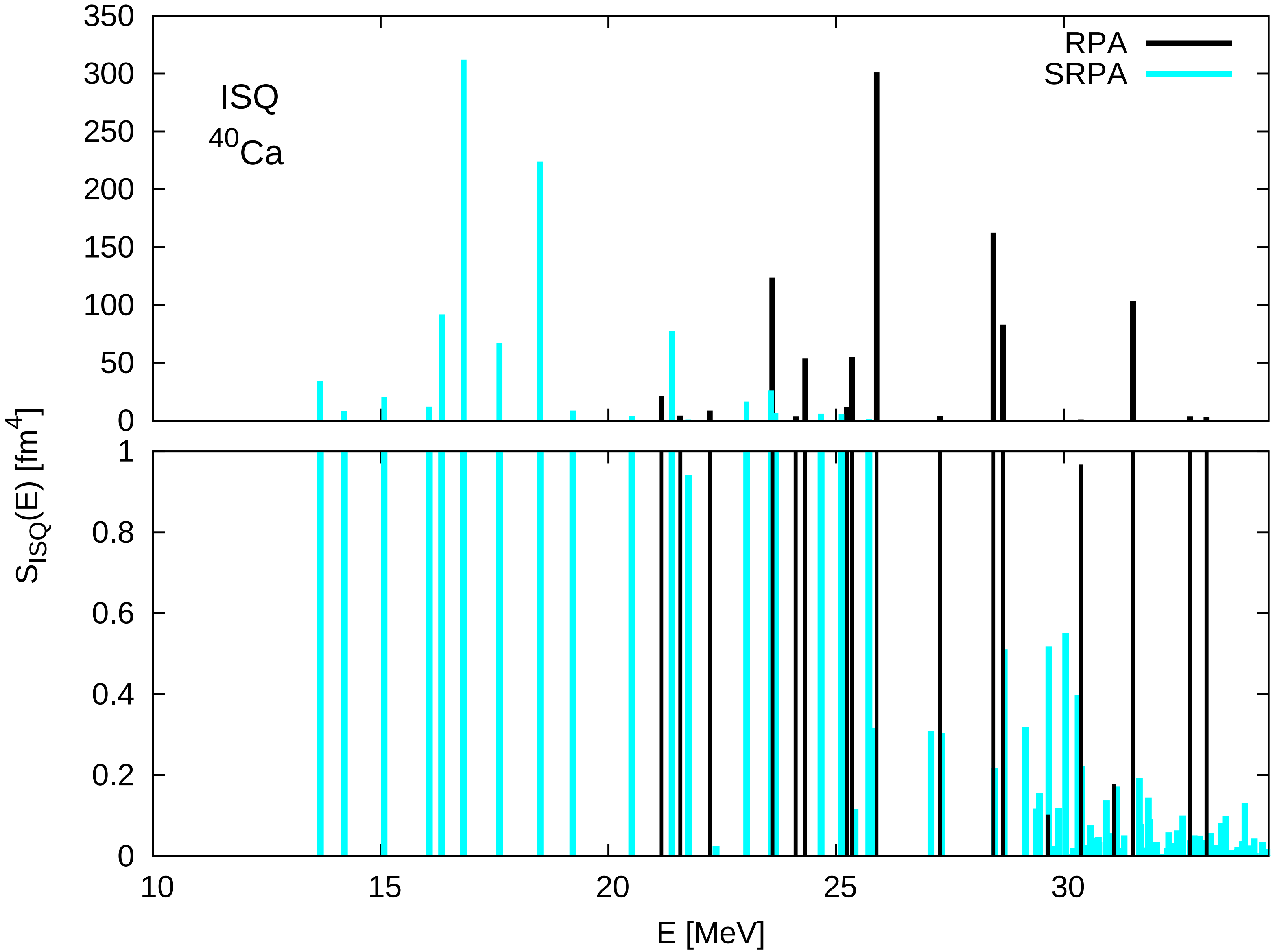


Fragmentation of ph states

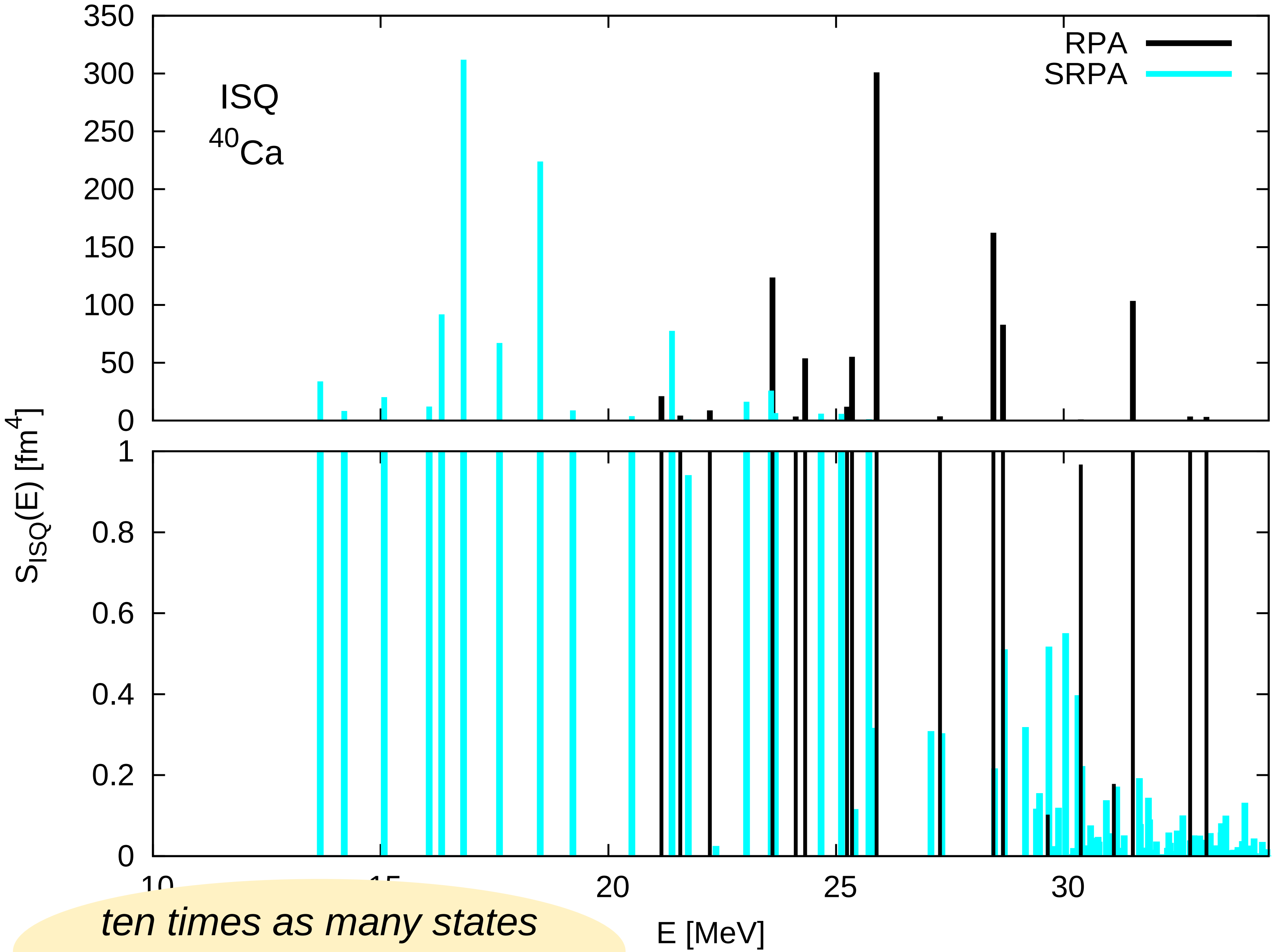


^{40}Ca

Fragmentation of resonances



Fragmentation of resonances



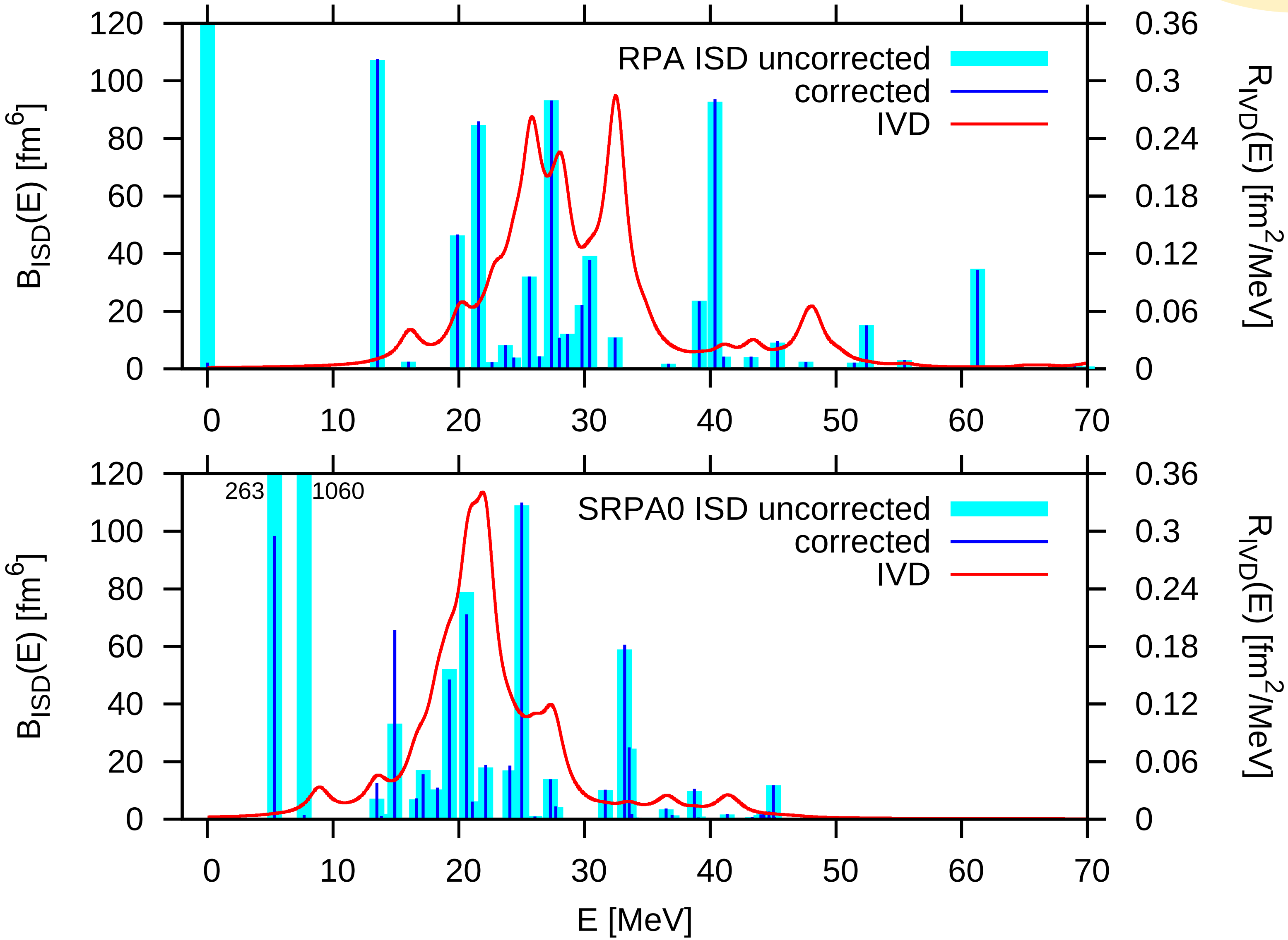
ten times as many states below 35 MeV

To consider

Spurious states

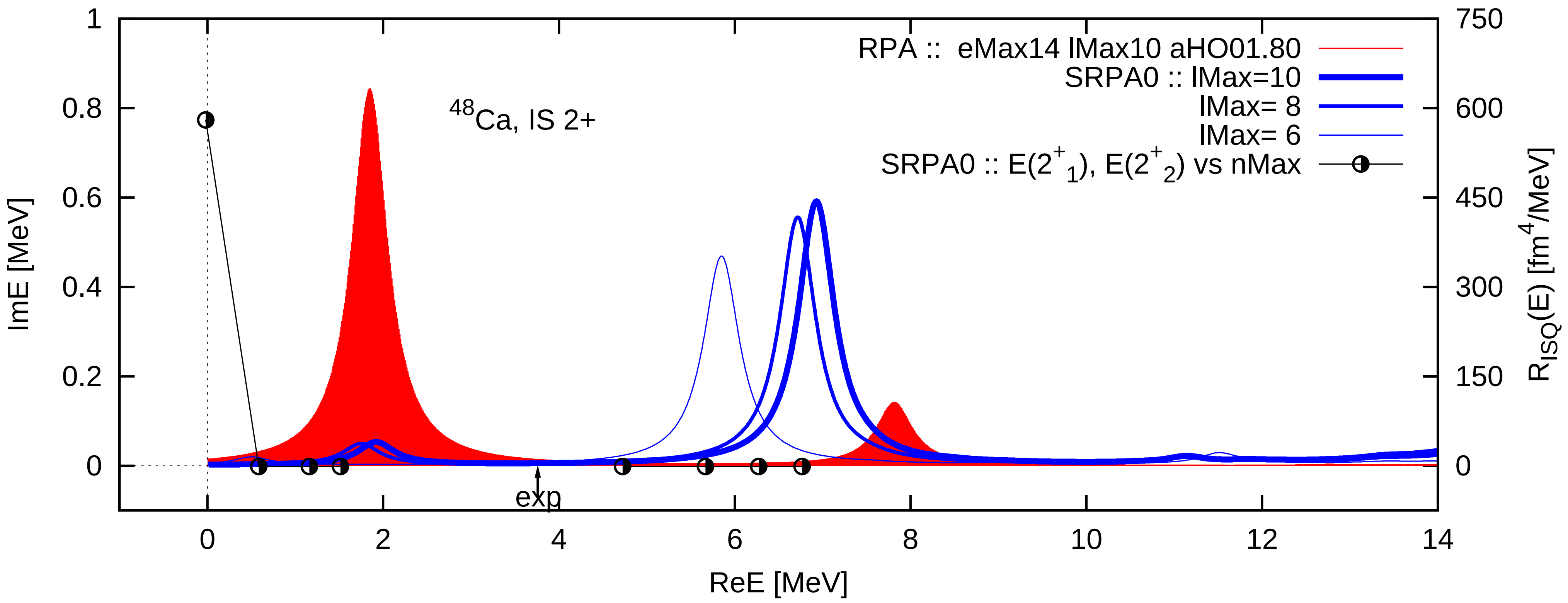
ISD corrected radial operator $r^3 - \frac{5}{3}\langle r^2 \rangle r$ vs r^3

^{16}O
$N_{\max} = 12$

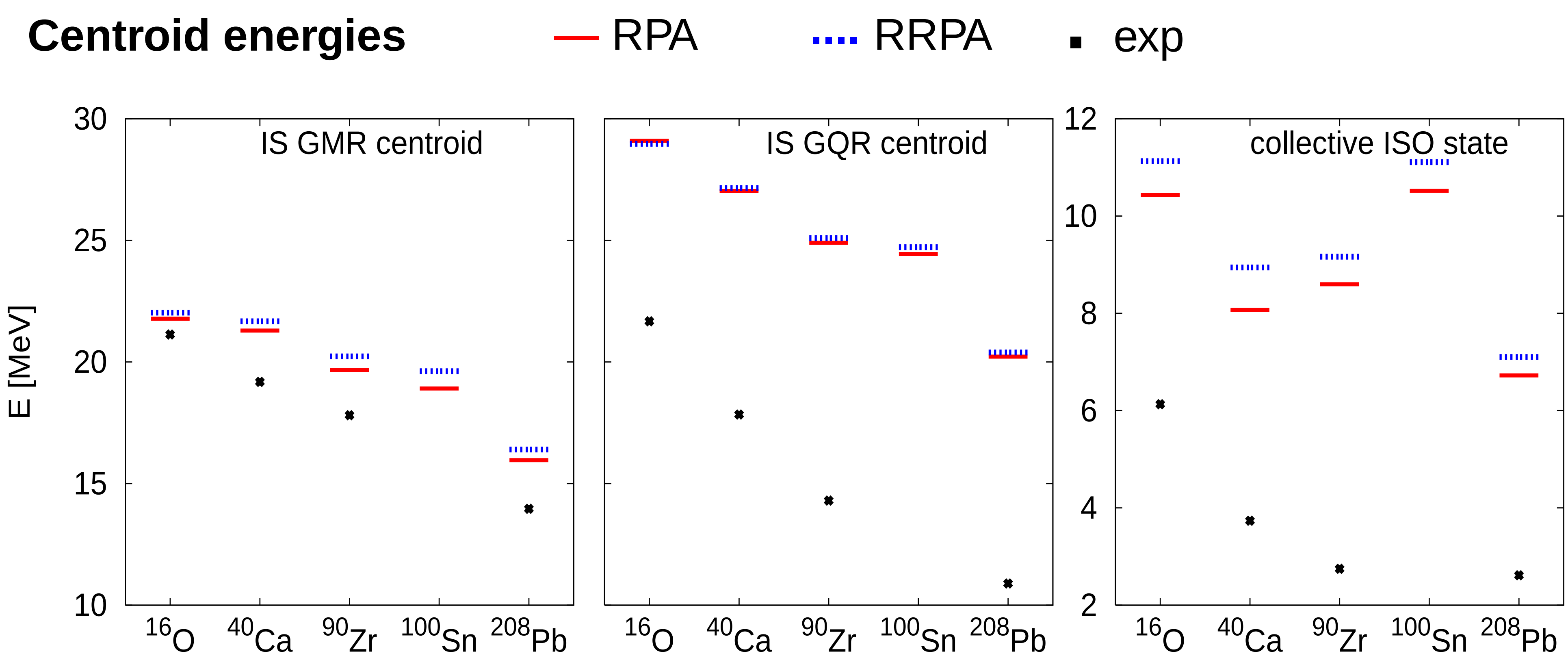


Low-lying states

SRPA0: convergence and stability of low-lying ISQ states



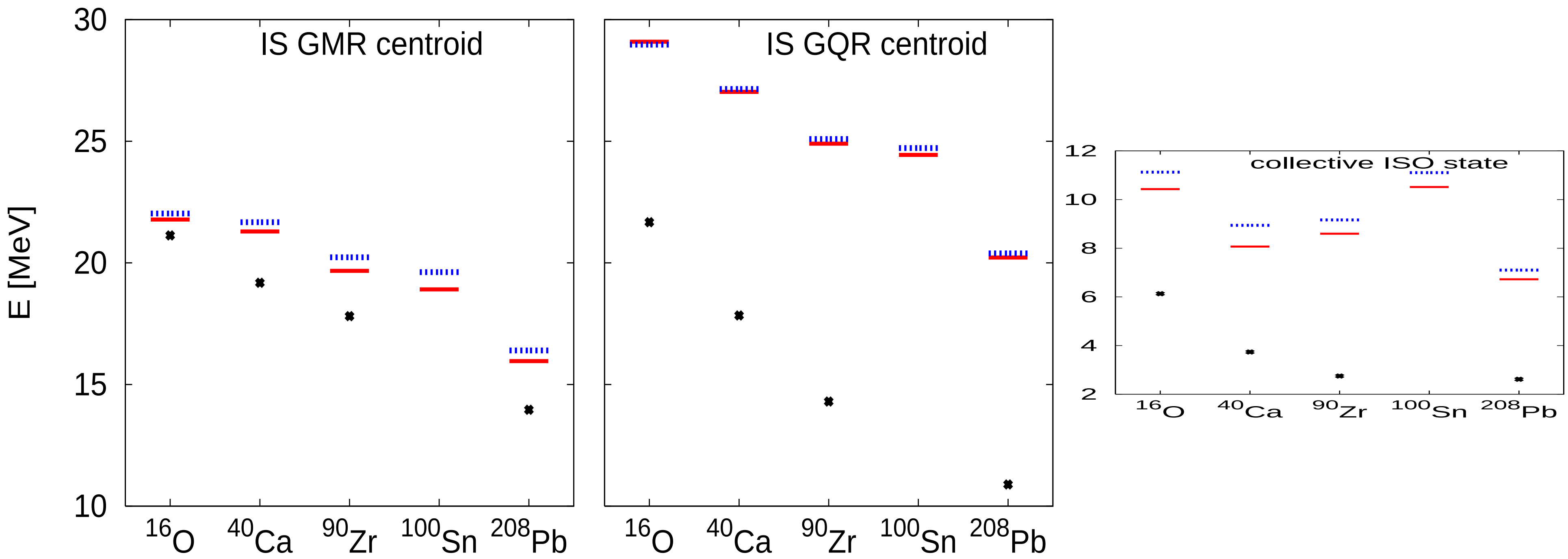
Renormalized RPA



Fermi-sea depletion: 2.6-5.0%

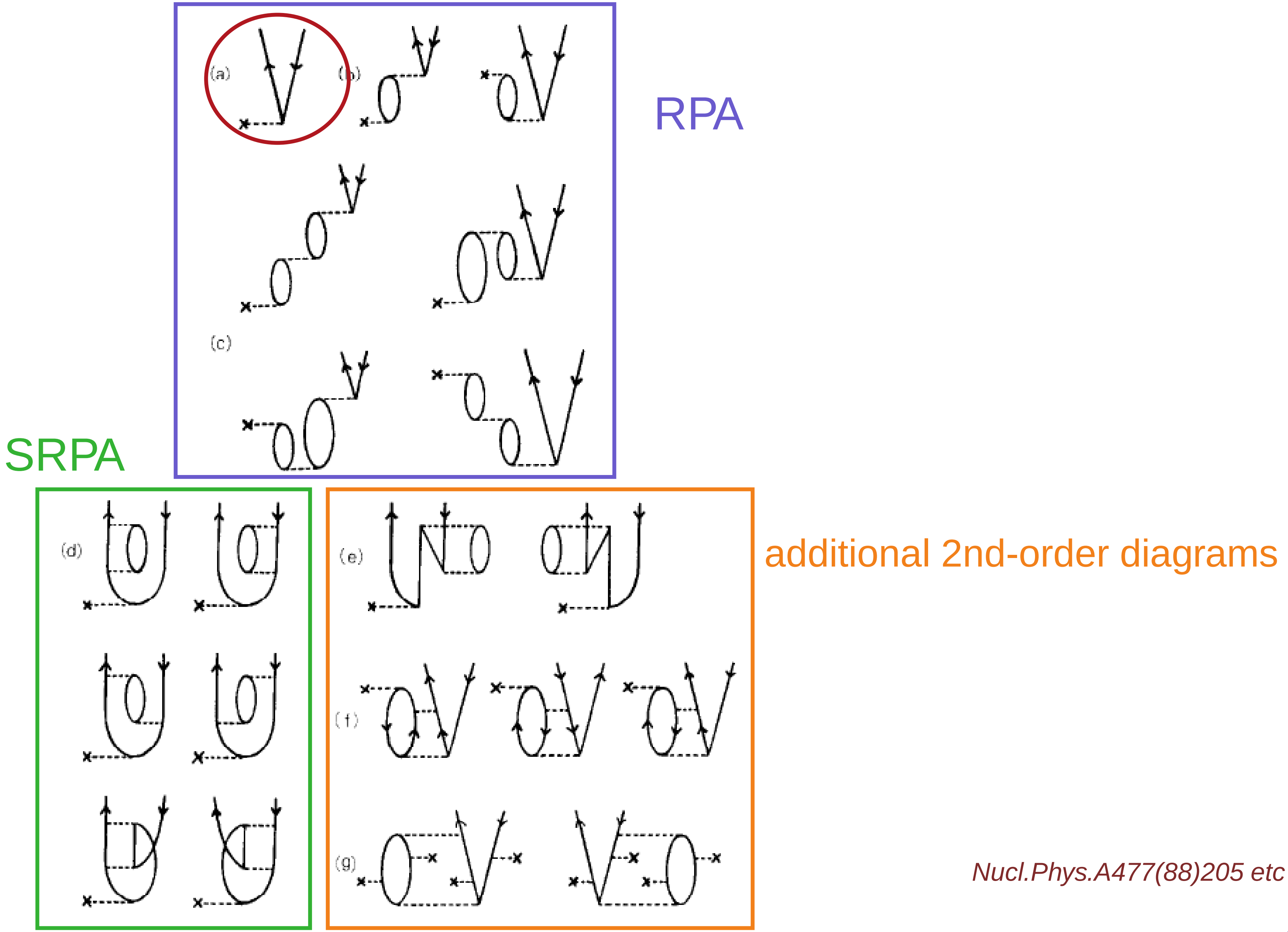
Renormalized RPA

Centroid energies — RPA ···· RRPA ▪ exp



Fermi-sea depletion: 2.6-5.0%

RPA, SRPA, and extensions



Nucl.Phys.A477(88)205 etc

Summary and Outlook

Extended-RPA calculations using
correlated realistic interactions

Effective interactions for extended RPA?

- ✔ Avoiding conceptual problems
- ✔ More fundamental treatment of nucleon self energy, m^* (ISQ, IVD)
- ✘ Two-body UCOM: Soft nuclei due to residual three body effects?
- Second RPA:
  - ✔ Great improvement over RPA results
  - ✔ model space should be flexible enough to describe residual LRC
  - ✘ Instabilities and inconsistencies
- Extensions of the present simple SRPA method

Summary and Outlook

Extended-RPA calculations using
correlated realistic interactions

Effective int… for extended RPA?

- ✔ Avoiding co…
- ✔ More …
- ✘ Tw…
- Se…
  - ✔ Imp…
  - ✔ Model spa…
  - ✘ Instabilities and inconsist…
- Extensions of the present simple SRPA method

Thank you!

Thank you!

Work in collaboration with:

- A.Günther, H.Hergert, S. Reinhardt, **R.Roth** J.Wambach, ...

  Institut für Kernphysik, TU Darmstadt, Germany

- H. Feldmeier, K. Langanke, G. Martinez-Pinedo, T. Neff, ...

  GSI, Darmstadt, Germany

Some related references

- P. P., R. Roth, PLB**671**, 356 (2009)
- P. P., R. Roth, N.Paar, Phys. Rev. C**75**, 014310 (2007)
- N. Paar, P. P., H. Hergert, R. Roth, Phys. Rev. C**74**, 014318 (2006)
- and many more: **http://crunch.ikp.physik.tu-darmstadt.de/tnp/**

SFB 634

Reduction of the RPA problem

[P.P.,EPL78(07)12001]

- 2Nx2N RPA problem, with A and B NxN symmetric:

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X_\nu \\ Y_\nu \end{pmatrix} = \epsilon_\nu \begin{pmatrix} X_\nu \\ Y_\nu \end{pmatrix}$$

- Reduction to NxN is always possible in various ways

... even when $A \pm B$ are not positive definite

- Simplest way:

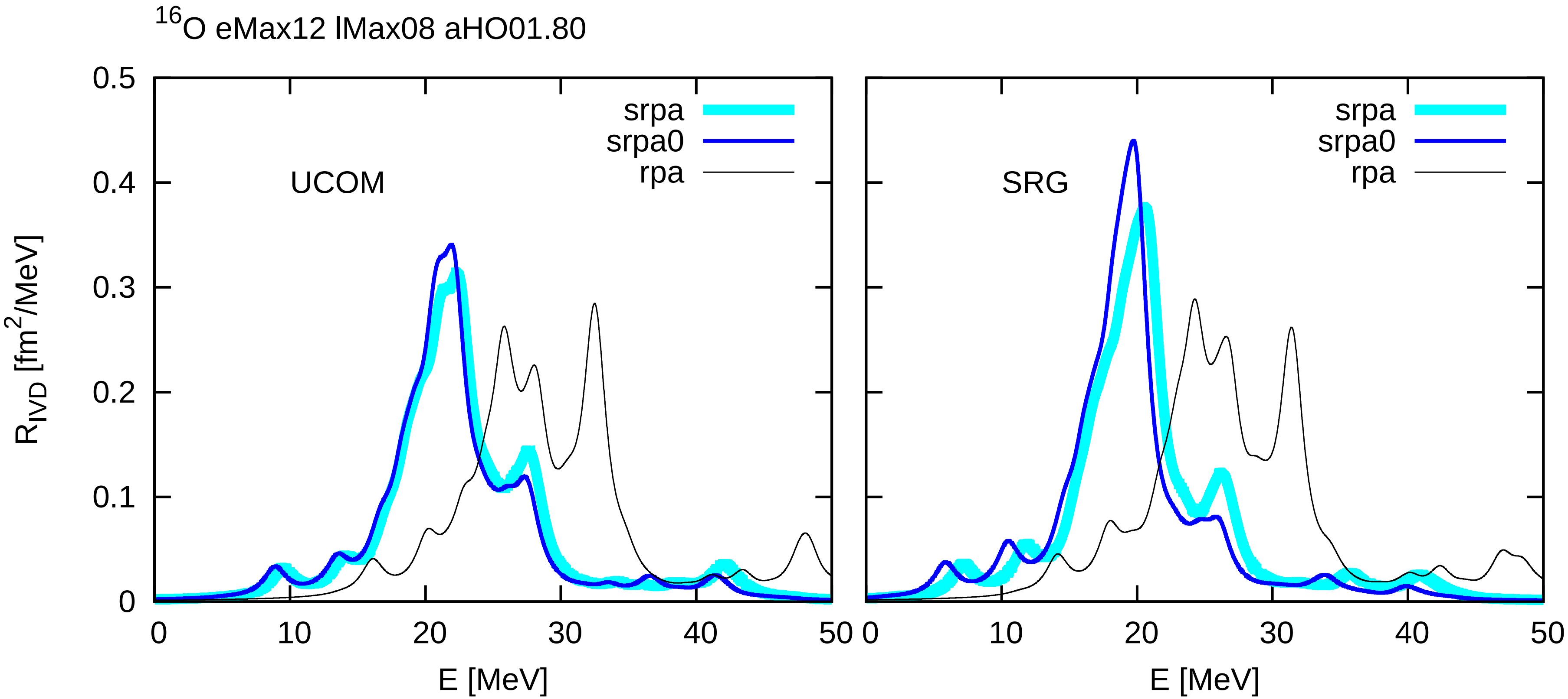
$[(A-B)(A+B)]R_\nu = \epsilon_\nu^2 R_\nu$, with $R_\nu = \epsilon_\nu^{-1/2}(X_\nu + Y_\nu)$

For real, positive solutions,

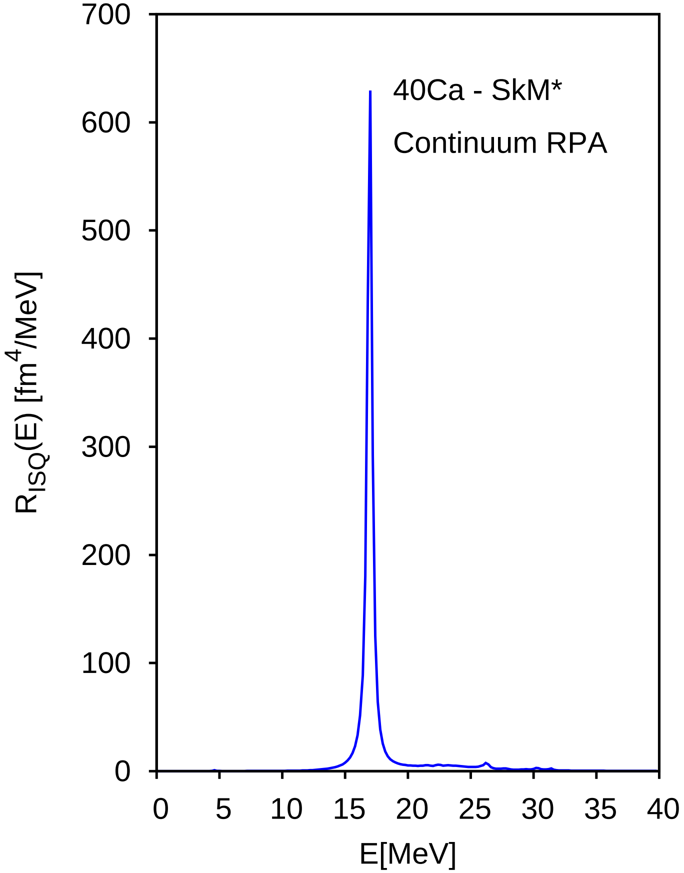
$$X_\nu = \tfrac{1}{2}[\epsilon_\nu^{1/2} I + \epsilon_\nu^{-1/2}(A+B)]R_\nu$$

$$Y_\nu = \tfrac{1}{2}[\epsilon_\nu^{1/2} I - \epsilon_\nu^{-1/2}(A+B)]R_\nu$$

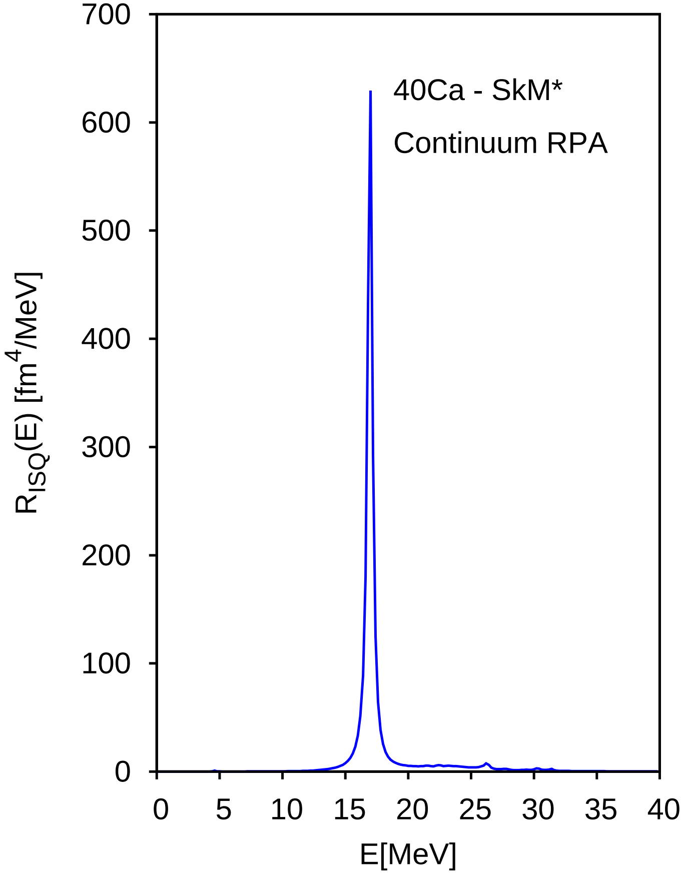
Second RPA with 2p2h coupling

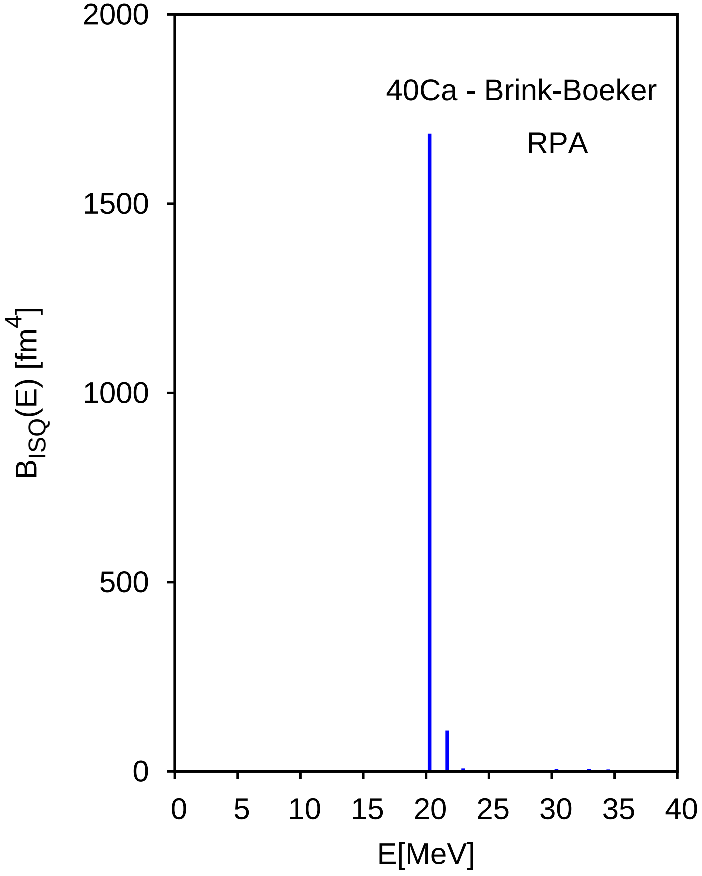


The ISQ resonance of ^{40}Ca - Width?

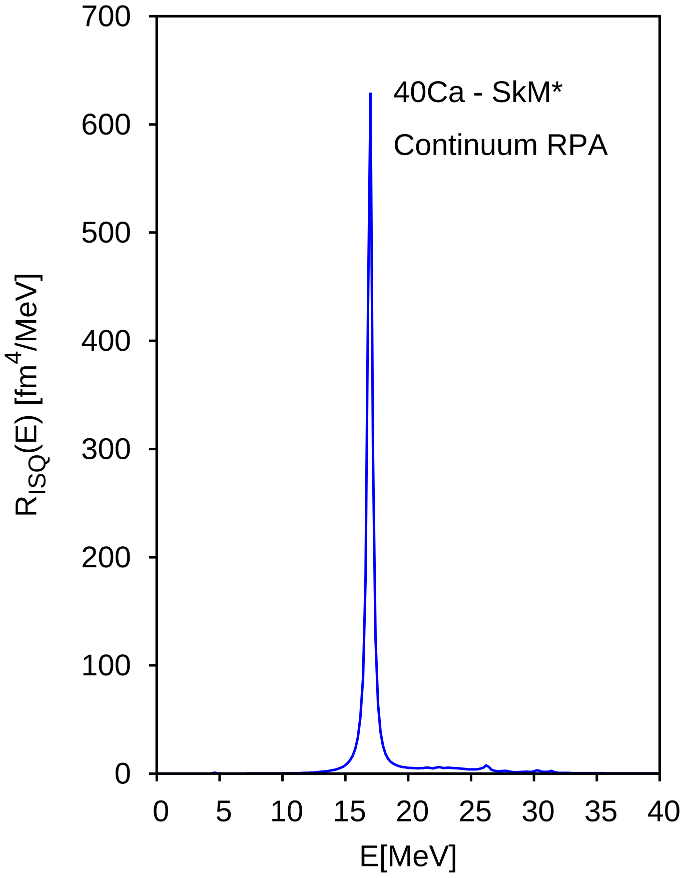


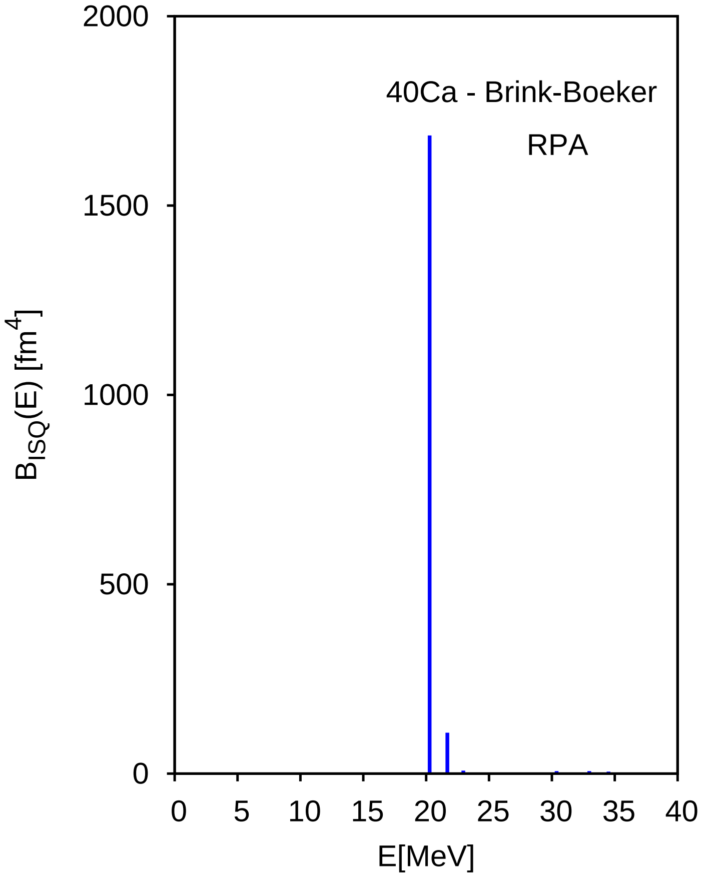
The ISQ resonance of ^{40}Ca - Width?

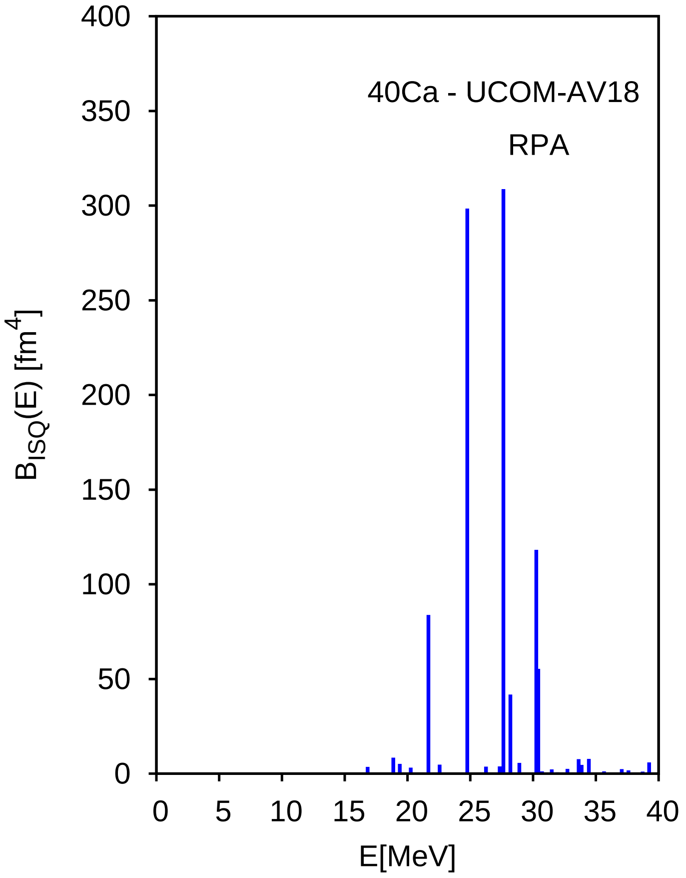




The ISQ resonance of ^{40}Ca - Width?







Wavelet transform – Morlet

- Consider spectrum E_i, $B(E_i)$; fold with, e.g., a Gaussian

$$S(E) = \frac{1}{\sqrt{2\pi}\gamma} \sum_i B(E_i) \exp^{-(E-E_i)^2/2\gamma^2}$$

- mother wavelet: Morlet

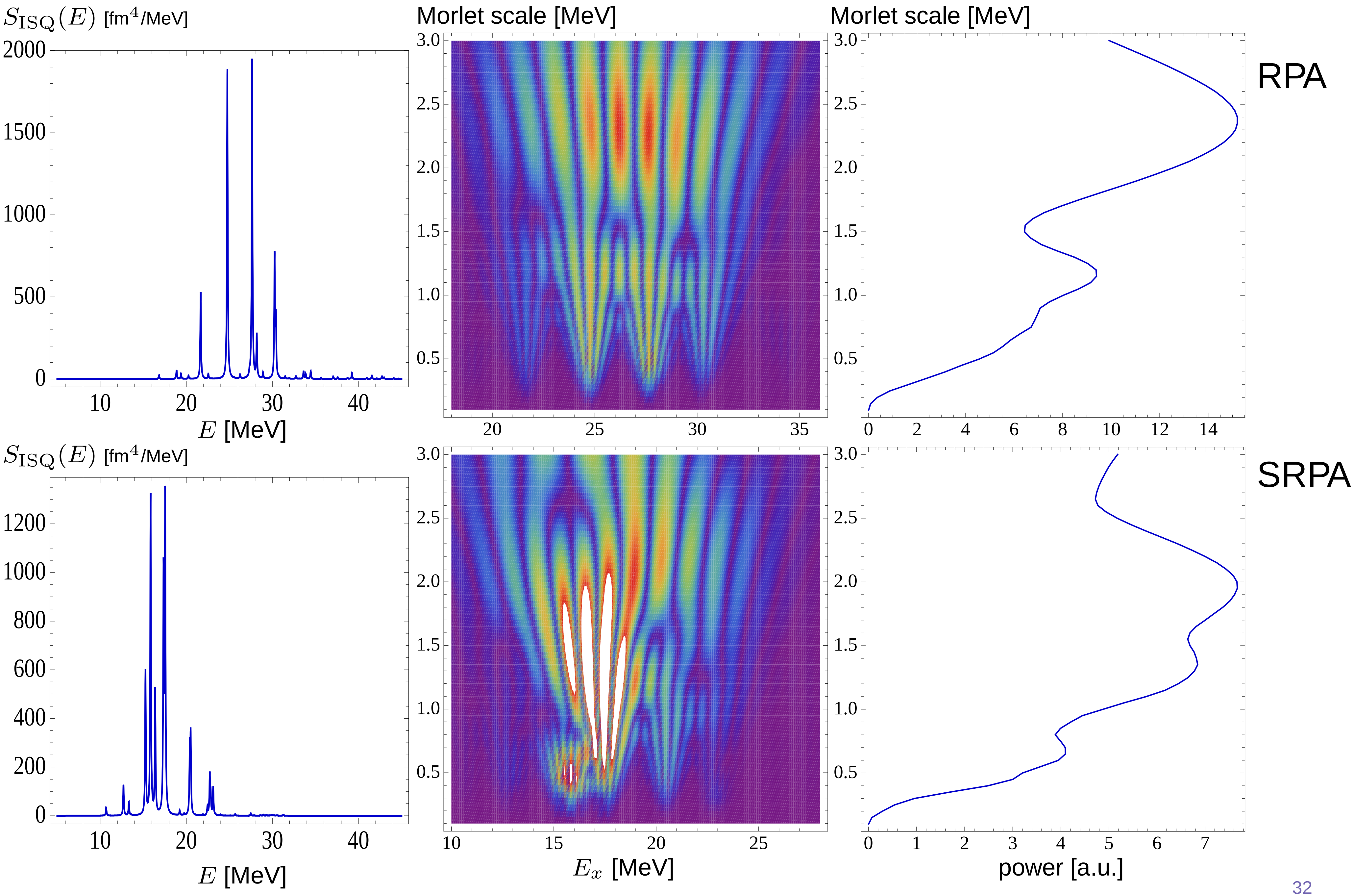
$$\psi(x) = \pi^{-1/4} \cos kx \exp^{-x^2/2} \; ; \; k = 5$$

- Wavelet coefficients: (δE the scale)

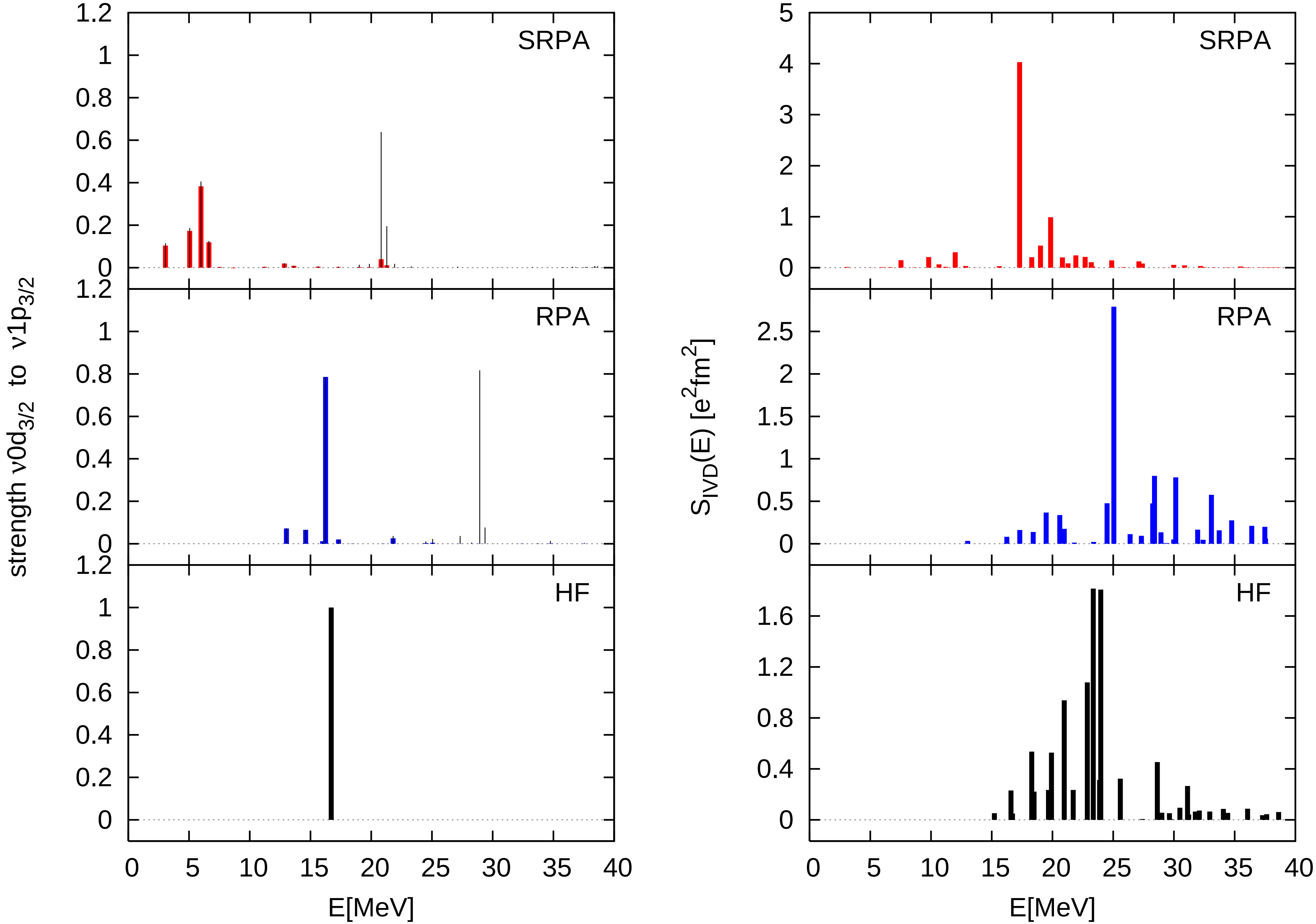
$$C(E_x, \delta E) = \frac{1}{\sqrt{\delta E}} \int S(E) \psi(\frac{E_x - E}{\delta E}) dE$$

- Next we plot $|C(E_x, \delta E)|$ $(\gamma = 50\mathrm{keV})$

The ISQ resonance of ^{40}Ca - Wavelet transform



Fragmentation of ph states



Fragmentation of resonances