Transformation of nuclear potentials
from partial wave representation into operator representation

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Motivation

$$\langle k_1 (L_1 S) J; T | V_{NN} | k_2 (L_2 S) J; T \rangle$$

partial wave matrix elements

realistic interaction (Argonne, chiral)

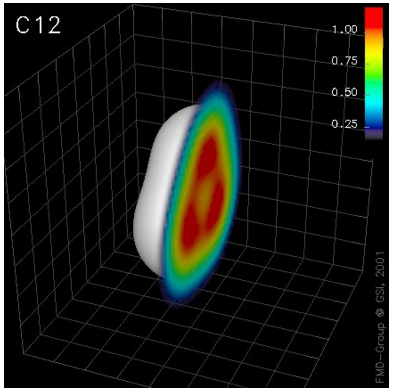
effective interaction (UCOM, SRG)

operator representation

$$V^{(\gamma,\kappa)}(r) \simeq \sum_i \gamma_i e^{-\frac{r^2}{2\kappa_i}}$$

model (FMD)

$$\begin{aligned} V_{NN} &= \sum_{S,T} V_{ST}^{Z}(r) \boldsymbol{\Pi}_{ST} \\ &+ \sum_{T} V_{1T}^{LS}(r) \boldsymbol{\Pi}_{1T} \vec{\boldsymbol{L}} \cdot \vec{\boldsymbol{S}} \\ &+ \sum_{T} V_{1T}^{T}(r) \boldsymbol{\Pi}_{1T} \boldsymbol{S}_{12} \ldots \end{aligned}$$



Motivation

realistic interaction (Argonne, chiral)

effective interaction (UCOM, SRG)

$$\left\langle k_1(L_1 S) J ; T \middle| \boldsymbol{V}_{NN} \middle| k_2(L_2 S) J ; T \right\rangle$$

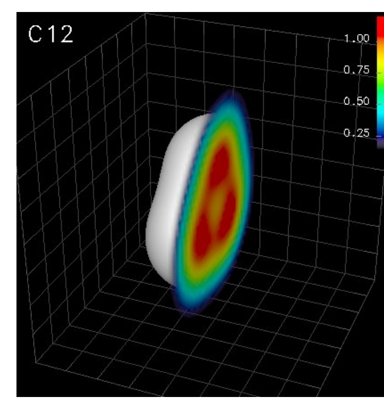
partial wave matrix elements

operator representation

$$V^{(\gamma,\kappa)}(r) \simeq \sum_i \gamma_i e^{-\frac{r^2}{2\kappa_i}}$$

model (FMD)

$$\begin{aligned} \boldsymbol{V}_{NN} \quad = \quad & \sum_{S,T} V_{ST}^{Z}(\boldsymbol{r}) \boldsymbol{\Pi}_{\boldsymbol{ST}} \\ + \quad & \sum_{T} V_{1T}^{LS}(\boldsymbol{r}) \boldsymbol{\Pi}_{\boldsymbol{1T}} \vec{\boldsymbol{L}} \cdot \vec{\boldsymbol{S}} \\ + \quad & \sum_{T} V_{1T}^{T}(\boldsymbol{r}) \boldsymbol{\Pi}_{\boldsymbol{1T}} \boldsymbol{S}_{12} \ldots \end{aligned}$$



Motivation

realistic interaction (Argonne, chiral)

effective interaction (UCOM, SRG)

$$\langle k_1 (L_1 S) J ; T | \boldsymbol{V}_{NN} | k_2 (L_2 S) J ; T \rangle$$

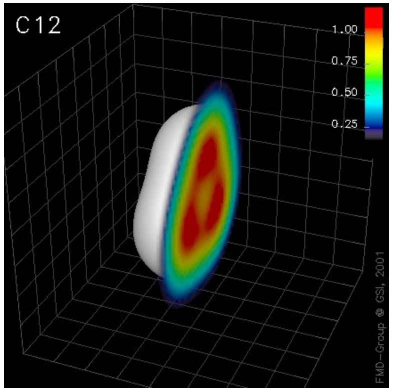
partial wave matrix elements

~~operator representation~~

$$V^{(\gamma,\kappa)}(r) \simeq \sum_i \gamma_i e^{-\frac{r^2}{2\kappa_i}}$$

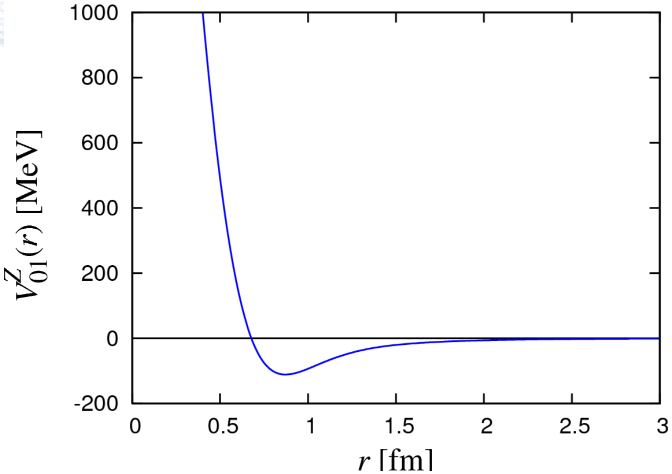
model (FMD)

$$\begin{aligned} \boldsymbol{V}_{NN} &= \sum_{S,T} V_{ST}^{Z}(\boldsymbol{r}) \boldsymbol{\Pi}_{ST} \\ &+ \sum_{T} V_{1T}^{LS}(\boldsymbol{r}) \boldsymbol{\Pi}_{1T} \vec{\boldsymbol{L}} \cdot \vec{\boldsymbol{S}} \\ &+ \sum_{T} V_{1T}^{T}(\boldsymbol{r}) \boldsymbol{\Pi}_{1T} \boldsymbol{S}_{12} \ldots \end{aligned}$$



Argonne potential

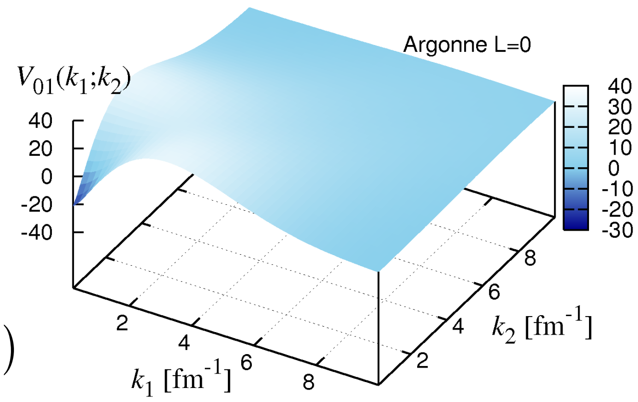
- realistic potential: nucleon-nucleon phase shifts, deuteron data
- operator representation:



$$
\begin{aligned}
\boldsymbol{V}_{Argonne} &= \sum_{S,T} V_{ST}^{Z}(\boldsymbol{r})\boldsymbol{\Pi}_{ST} \\
&+ \sum_{S,T} V_{ST}^{L2}(\boldsymbol{r})\boldsymbol{\Pi}_{ST}\vec{\boldsymbol{L}}^2 \\
&+ \sum_{T} V_{1T}^{LS}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}} \\
&+ \sum_{T} V_{1T}^{T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\boldsymbol{S}_{12} \\
&+ \sum_{T} V_{1T}^{TLL}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\boldsymbol{s}_{12}(\vec{L},\vec{L})
\end{aligned}
$$

$$\boldsymbol{S}_{12}=\frac{3}{r^2}(\vec{\boldsymbol{r}}\cdot\vec{\boldsymbol{\sigma}}_1)(\vec{\boldsymbol{r}}\cdot\vec{\boldsymbol{\sigma}}_2)-\vec{\boldsymbol{\sigma}}_1\cdot\vec{\boldsymbol{\sigma}}_2$$

$$\boldsymbol{s}_{12}(\vec{a},\vec{b})=\frac{3}{2}[(\vec{\boldsymbol{\sigma}}_1\cdot\vec{\boldsymbol{a}})(\vec{\boldsymbol{\sigma}}_2\cdot\vec{\boldsymbol{b}})+(\vec{\boldsymbol{\sigma}}_1\cdot\vec{\boldsymbol{b}})(\vec{\boldsymbol{\sigma}}_2\cdot\vec{\boldsymbol{a}})]-\frac{1}{2}(\vec{\boldsymbol{\sigma}}_1\cdot\vec{\boldsymbol{\sigma}}_2)(\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{b}}+\vec{\boldsymbol{b}}\cdot\vec{\boldsymbol{a}})$$



$$\langle k_1(LS)JT|V_{ST}(\boldsymbol{r})|k_2(LS)JT\rangle$$

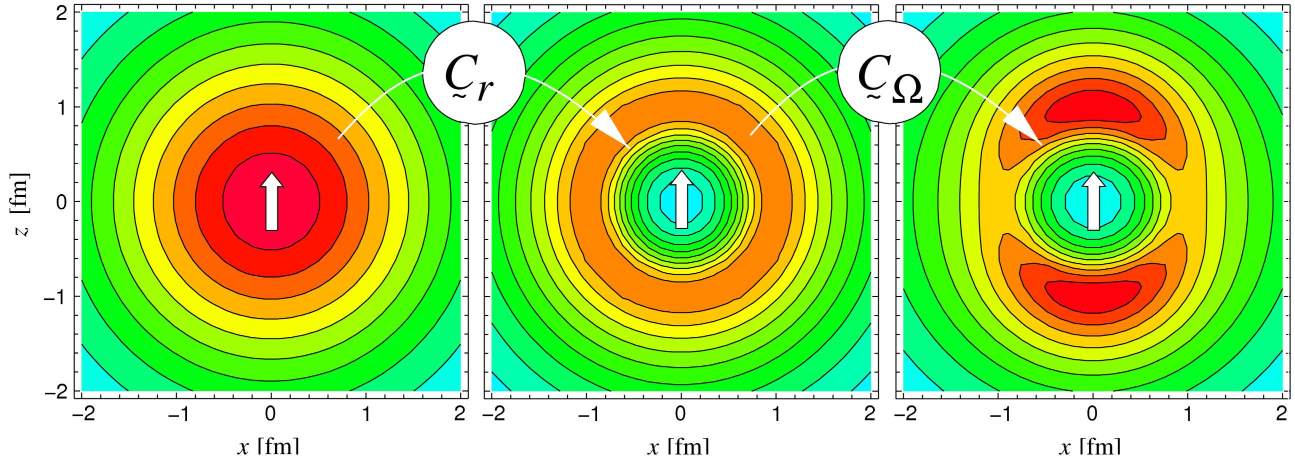
Unitary Correlation Operator Method

- realistic interaction cannot be used in model spaces consisting of one or a few Slater determinants
- Short range correlations ('hard core') and correlations induced by the tensor force cannot be described in model space
- Correlations can be inserted by the 'Unitary Correlation Operator Method' UCOM:
  - correlated state

    $$|\hat{\psi}\rangle = \boldsymbol{C}\,|\psi\rangle = \boldsymbol{C}_{\Omega}\boldsymbol{C}_{r}\,|\psi\rangle$$

  - correlated operator

    $$\langle\hat{\phi}|\boldsymbol{B}|\hat{\psi}\rangle = \langle\phi|\boldsymbol{C}^{+}\ \boldsymbol{B}\,\boldsymbol{C}|\psi\rangle \equiv \langle\phi|\widehat{\boldsymbol{B}}|\psi\rangle$$



T. Neff, H. Feldmeier, Nucl. Phys. A713

Unitary Correlation Operator Method

- UCOM interaction $V_{UCOM} = \widehat{\boldsymbol{H}}^{[2]} = \widehat{\boldsymbol{T}}^{[2]} + \widehat{\boldsymbol{V}}^{[2]}$
- operator representation:

$$
\begin{aligned}
\boldsymbol{V}_{UCOM} = & \sum_{S,T} V_{ST}^{Z}(\boldsymbol{r})\,\boldsymbol{\Pi}_{ST} \\
+ & \sum_{S,T} V_{ST}^{L2}(\boldsymbol{r})\,\boldsymbol{\Pi}_{ST}\,\vec{\boldsymbol{L}}^{2} \\
+ & \sum_{S,T} 1/2\left(\vec{\boldsymbol{p}}^{2}\,V_{ST}^{p}(\boldsymbol{r}) + V_{ST}^{p}(\boldsymbol{r})\,\vec{\boldsymbol{p}}^{2}\right)\boldsymbol{\Pi}_{ST} \\
+ & \sum_{T} V_{1T}^{LS}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}} \\
+ & \sum_{T} V_{1T}^{L2LS}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\vec{\boldsymbol{L}}^{2}\,\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}} \\
+ & \sum_{T} V_{1T}^{T}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\boldsymbol{S}_{12} \\
+ & \sum_{T} V_{1T}^{TLL}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\boldsymbol{s}_{12}(\vec{L},\vec{L}) \\
+ & \sum_{T} V_{1T}^{Tpp}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\bar{\boldsymbol{s}}_{12}(\vec{p}_{\Omega},\vec{p}_{\Omega}) \\
+ & \sum_{T} \left(\boldsymbol{p}_{r}\,V_{1T}^{Trp}(\boldsymbol{r}) + V_{1T}^{Trp}(\boldsymbol{r})\,\boldsymbol{p}_{r}\right)\boldsymbol{\Pi}_{1T}\,\boldsymbol{s}_{12}(\vec{r},\vec{p}_{\Omega})
\end{aligned}
$$

Similarity Renormalization Group

- flow equation $\frac{d\,\boldsymbol{H}_s}{ds} = \left[\boldsymbol{\eta}_s, \boldsymbol{H}_s\right]$
- partial wave basis: generator $\boldsymbol{\eta}_s = \left[\boldsymbol{T}_{rel}, \boldsymbol{H}_s\right]$
- fixpoint $\frac{d\,\boldsymbol{H}_s}{ds} = 0$ when $\boldsymbol{H}_s$ is diagonal in partial wave basis

- evolution to a band-diagonal structure

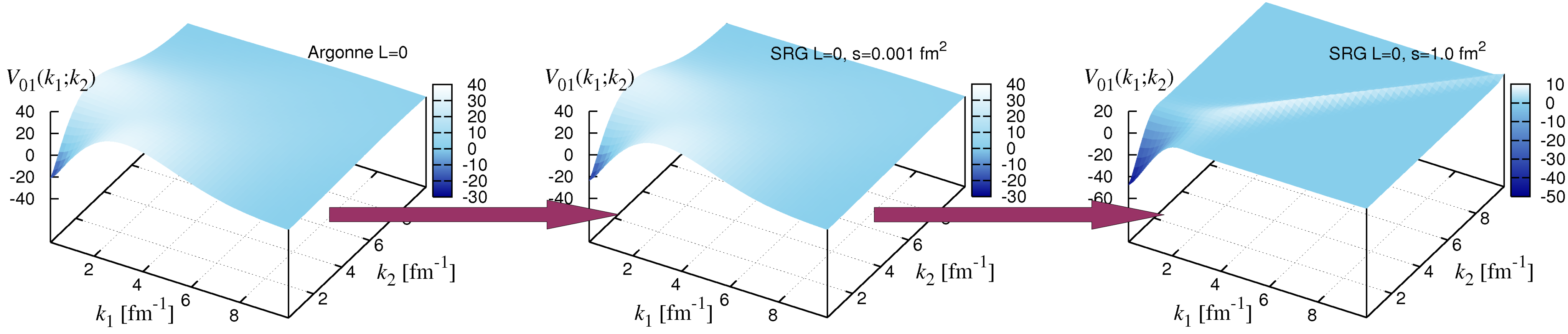
- calculation of matrix elements $\langle k_1 (L_1 S) J ; T | \boldsymbol{V} | k_2 (L_2 S) J ; T \rangle$

  for given quantum numbers L, S, T and J

$$\frac{d\,\boldsymbol{V}_s(k_1,k_2)}{ds} = -\frac{1}{(2\mu)^2}\left(k_1^2 - k_2^2\right)^2 V_s(k_1,k_2) + \frac{1}{2\mu}\int_0^\infty dp\, p^2 (k_1^2 + k_2^2 - 2\mathrm{p}^2)\, V_s(k_1,p)\, V_s(p,k_2)$$

Similarity Renormalization Group

- matrix elements, but no operator representation



matrix elements in *MeV fm*^{3}

From operator representation to partial wave matrix elements

- operator representation

$$\boldsymbol{V} = \sum_{S',T'} V^{Z}_{S'T'}(\boldsymbol{r})\,\boldsymbol{\Pi}_{S'T'} + \sum_{T'} V^{LS}_{1T'}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T'}\,\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}} + \ldots$$

- partial wave matrix elements

$$\begin{aligned}\left\langle k_1(L_1S)J;T\middle|\boldsymbol{V}\middle|k_2(L_2S)J;T\right\rangle \quad = \quad & \sum_{S',T'}\left\langle k_1(L_1S)J;T\middle|V^{Z}_{S'T'}(\boldsymbol{r})\,\boldsymbol{\Pi}_{S'T'}\middle|k_2(L_2S)J;T\right\rangle \\ & +\sum_{T'}\left\langle k_1(L_1S)J;T\middle|V^{LS}_{1T'}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T'}\,\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}}\middle|k_2(L_2S)J;T\right\rangle + \ldots\end{aligned}$$

From operator representation to partial wave matrix elements

- operator representation

$$\boldsymbol{V} = \sum_{S',T'} V^{Z}_{S'T'}(\boldsymbol{r})\,\boldsymbol{\Pi}_{S'T'} + \sum_{T'} V^{LS}_{1T'}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T'}\,\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}} + \ldots$$

- partial wave matrix elements

$$\begin{aligned}
\left\langle k_1(L_1S)J;T\right|\boldsymbol{V}\left|k_2(L_2S)J;T\right\rangle \quad &= \quad \sum_{S',T'}\left\langle k_1(L_1S)J;T\right|V^{Z}_{S'T'}(\boldsymbol{r})\boldsymbol{\Pi}_{S'T'}\left|k_2(L_2S)J;T\right\rangle \\
&\qquad + \sum_{T'}\left\langle k_1(L_1S)J;T\right|V^{LS}_{1T'}(\boldsymbol{r})\boldsymbol{\Pi}_{1T'}\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}}\left|k_2(L_2S)J;T\right\rangle + \ldots \\
&= \quad \left\langle k_1L_1\right|V^{Z}_{ST}(\boldsymbol{r})\left|k_2L_2\right\rangle\cdot\left\langle (L_1S)J\right|\boldsymbol{1}\left|(L_2S)J\right\rangle \\
&\qquad + \left\langle k_1L_1\right|V^{LS}_{ST}(\boldsymbol{r})\left|k_2L_2\right\rangle\cdot\left\langle (L_1S)J\right|\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}}\left|(L_2S)J\right\rangle + \ldots
\end{aligned}$$

From operator representation to partial wave matrix elements

- operator representation

$$\boldsymbol{V} = \sum_{S',T'} V^{Z}_{S'T'}(\boldsymbol{r})\,\boldsymbol{\Pi}_{S'T'} + \sum_{T'} V^{LS}_{1T'}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T'}\,\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}} + \dots$$

- partial wave matrix elements

$$\begin{aligned}
\left\langle k_1(L_1S)J;T\right|\boldsymbol{V}\left|k_2(L_2S)J;T\right\rangle &= \sum_{S',T'}\left\langle k_1(L_1S)J;T\right|V^{Z}_{S'T'}(\boldsymbol{r})\boldsymbol{\Pi}_{S'T'}\left|k_2(L_2S)J;T\right\rangle \\
&\quad + \sum_{T'}\left\langle k_1(L_1S)J;T\right|V^{LS}_{1T'}(\boldsymbol{r})\boldsymbol{\Pi}_{1T'}\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}}\left|k_2(L_2S)J;T\right\rangle + \dots \\
&= \left\langle k_1L_1\right|V^{Z}_{ST}(\boldsymbol{r})\left|k_2L_2\right\rangle \cdot \left\langle (L_1S)J\right|\boldsymbol{1}\left|(L_2S)J\right\rangle \\
&\quad + \left\langle k_1L_1\right|V^{LS}_{ST}(\boldsymbol{r})\left|k_2L_2\right\rangle \cdot \left\langle (L_1S)J\right|\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}}\left|(L_2S)J\right\rangle + \dots
\end{aligned}$$

- matrix elements of the operators

$$\left\langle (L_1S)J\right|\boldsymbol{1}\left|(L_2S)J\right\rangle,\quad \left\langle (L_1S)J\right|\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}}\left|(L_2S)J\right\rangle,\quad \dots$$

- transformed radial dependencies: local $\left\langle \vec{r}\right|V^{o}_{ST}(\boldsymbol{r})\left|\vec{r}\,'\right\rangle = \delta^3(\vec{r}-\vec{r}\,')\,V^{o}_{ST}(r)$

$$\left\langle k_1L_1\right|V^{o}_{ST}(\boldsymbol{r})\left|k_2L_2\right\rangle = \frac{2}{\pi}\int_0^{\infty} dr\, r^2\, j_{L_1}(k_1r)\,V^{o}_{ST}(r)\,j_{L_2}(k_2r)$$

From partial wave matrix elements to operator representation

- representation of radial dependencies by a sum of gaussians:

$$V^o(r) = \sum_j \gamma_j^o \, e^{-\frac{r^2}{2\kappa_j}} \qquad o \in \{\mathbf{1},\ \vec{\boldsymbol{L}} \cdot \vec{\boldsymbol{S}},\ \ldots\}$$

- choose appropriate set of operators

$$\boldsymbol{V} = \sum_{S,T} V_{ST}^{Z}(\boldsymbol{r}) \boldsymbol{\Pi}_{ST} + \sum_{T} V_{1T}^{LS}(\boldsymbol{r}) \boldsymbol{\Pi}_{1T} \vec{\boldsymbol{L}} \cdot \vec{\boldsymbol{S}} + \ldots$$

- calculate partial wave matrix elements analytically

$$\langle k_1 L_1 | V_{ST}^o(\boldsymbol{r}) | k_2 L_2 \rangle = \frac{2}{\pi} \sum_j \gamma_{ST,j}^o \int_0^\infty dr\, r^2\, j_{L_1}(k_1 r)\, e^{-\frac{r^2}{2\kappa_j}}\, j_{L_2}(k_2 r)$$

- choose parameters $\kappa_n = \kappa_1 \cdot \alpha^{n-1}$

$$\kappa = \{0.05,\ 0.05\cdot\sqrt{2},\ 0.1,\ 0.1\cdot\sqrt{2},\ \ldots,\ 12.8\}\ fm^2$$

  for each L_1, L_2, S, J and T: $\langle k_1 (L_1 S) J; T | \boldsymbol{V} | k_2 (L_2 S) J; T \rangle \rightarrow V_{ST}^{L_1 L_2 J}(k_1; k_2)$

- fit to partial wave basis matrix elements

$$\rightarrow\ \gamma_{ST,j}^o\ \rightarrow\ V^o(r) \simeq \sum_j \gamma_{ST,j}^o\, e^{-\frac{r^2}{2\kappa_j}}$$

From partial wave matrix elements to operator representation

- Ansatz for $\boldsymbol{V}_{UCOM}$

$$
\begin{aligned}
\boldsymbol{V}_{UCOM} &= \sum_{S,T} V^{Z}_{ST}(\boldsymbol{r})\boldsymbol{\Pi}_{ST} \\
&+ \sum_{S,T} V^{L2}_{ST}(\boldsymbol{r})\boldsymbol{\Pi}_{ST}\vec{\boldsymbol{L}}^2 \\
&+ \sum_{S,T} 1/2(\vec{\boldsymbol{p}}^2 V^{p2}_{ST}(\boldsymbol{r})+V^{p2}_{ST}(\boldsymbol{r})\vec{\boldsymbol{p}}^2)\boldsymbol{\Pi}_{ST} \\
&+ \sum_{T} V^{LS}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}} \\
&+ \sum_{T} V^{L2LS}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\vec{\boldsymbol{L}}^2\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}} \\
&+ \sum_{T} V^{T}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\boldsymbol{S}_{12} \\
&+ \sum_{T} V^{TLL}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\boldsymbol{s}_{12}(\vec{L},\vec{L}) \\
&+ \sum_{T} V^{Tpp}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\bar{\boldsymbol{s}}_{12}(\vec{p}_\Omega,\vec{p}_\Omega) \\
&+ \sum_{T} (\boldsymbol{p}_r V^{Trp}_{1T}(\boldsymbol{r})+V^{Trp}_{1T}(\boldsymbol{r})\boldsymbol{p}_r)\boldsymbol{\Pi}_{1T}\boldsymbol{s}_{12}(\vec{r},\vec{p}_\Omega)
\end{aligned}
$$

From partial wave matrix elements to operator representation

- Ansatz for $\boldsymbol{V}_{UCOM}$

$S=0$

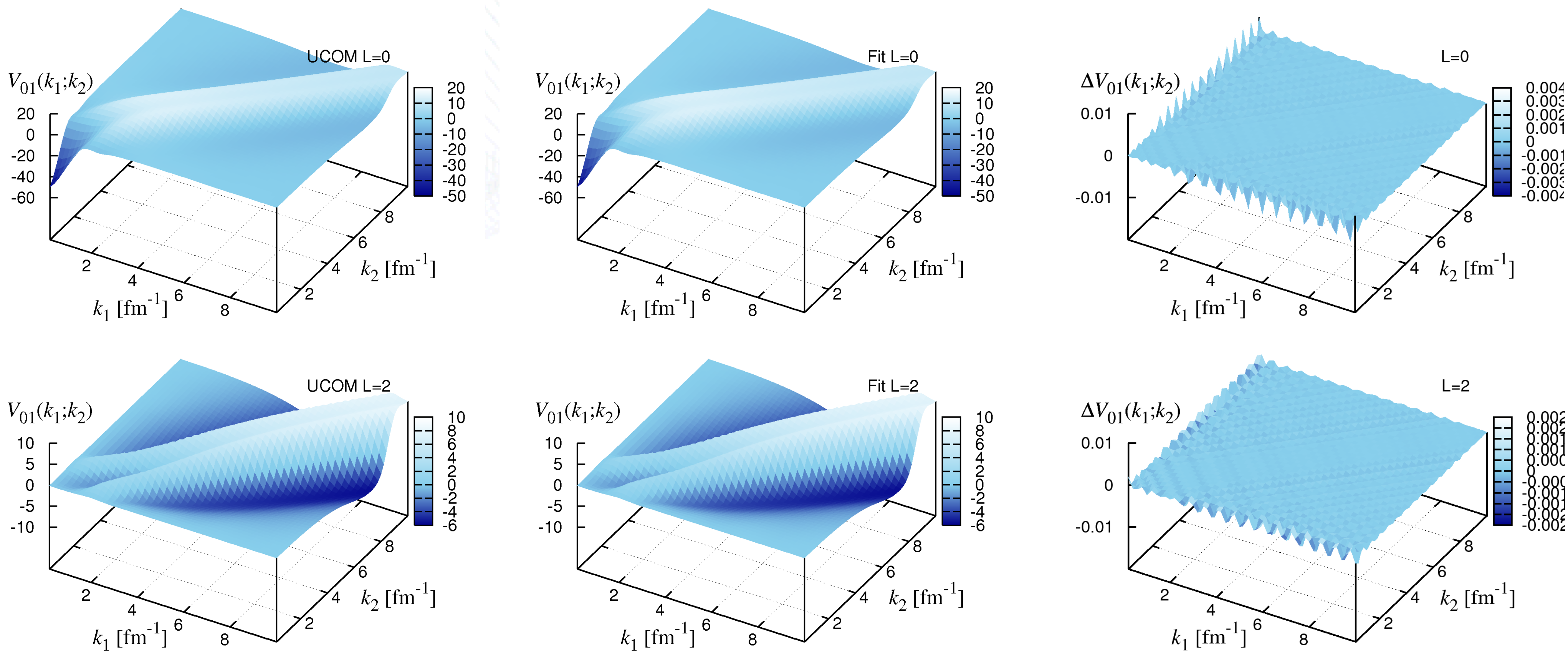
$$
\begin{aligned}
\boldsymbol{V}_{UCOM} &= \sum_{S,T} V^{Z}_{ST}(\boldsymbol{r})\boldsymbol{\Pi}_{ST} \\
&+ \sum_{S,T} V^{L2}_{ST}(\boldsymbol{r})\boldsymbol{\Pi}_{ST}\vec{\boldsymbol{L}}^{2} \\
&+ \sum_{S,T} 1/2\left(\vec{\boldsymbol{p}}^{2}V^{p2}_{ST}(\boldsymbol{r})+V^{p2}_{ST}(\boldsymbol{r})\vec{\boldsymbol{p}}^{2}\right)\boldsymbol{\Pi}_{ST} \\
&+ \cancel{\sum_{T} V^{LS}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}}} \\
&+ \cancel{\sum_{T} V^{L2LS}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\vec{\boldsymbol{L}}^{2}\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}}} \\
&+ \cancel{\sum_{T} V^{T}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\boldsymbol{S}_{12}} \\
&+ \cancel{\sum_{T} V^{TLL}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\boldsymbol{s}_{12}(\vec{L},\vec{L})} \\
&+ \cancel{\sum_{T} V^{Tpp}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\bar{\boldsymbol{s}}_{12}(\vec{p}_{\Omega},\vec{p}_{\Omega})} \\
&+ \cancel{\sum_{T} \left(\boldsymbol{p}_{r}V^{Trp}_{1T}(\boldsymbol{r})+V^{Trp}_{1T}(\boldsymbol{r})\boldsymbol{p}_{r}\right)\boldsymbol{\Pi}_{1T}\boldsymbol{s}_{12}(\vec{r},\vec{p}_{\Omega})}
\end{aligned}
$$

From partial wave matrix elements to operator representation - UCOM

- $S=0$ and $T=1$: $V_{01}^{Z}(r)$, $V_{01}^{L2}(r)$, $V_{01}^{p2}(r)$
- fit to lowest $L \rightarrow L=0$, $L=2$

From partial wave matrix elements to operator representation - UCOM

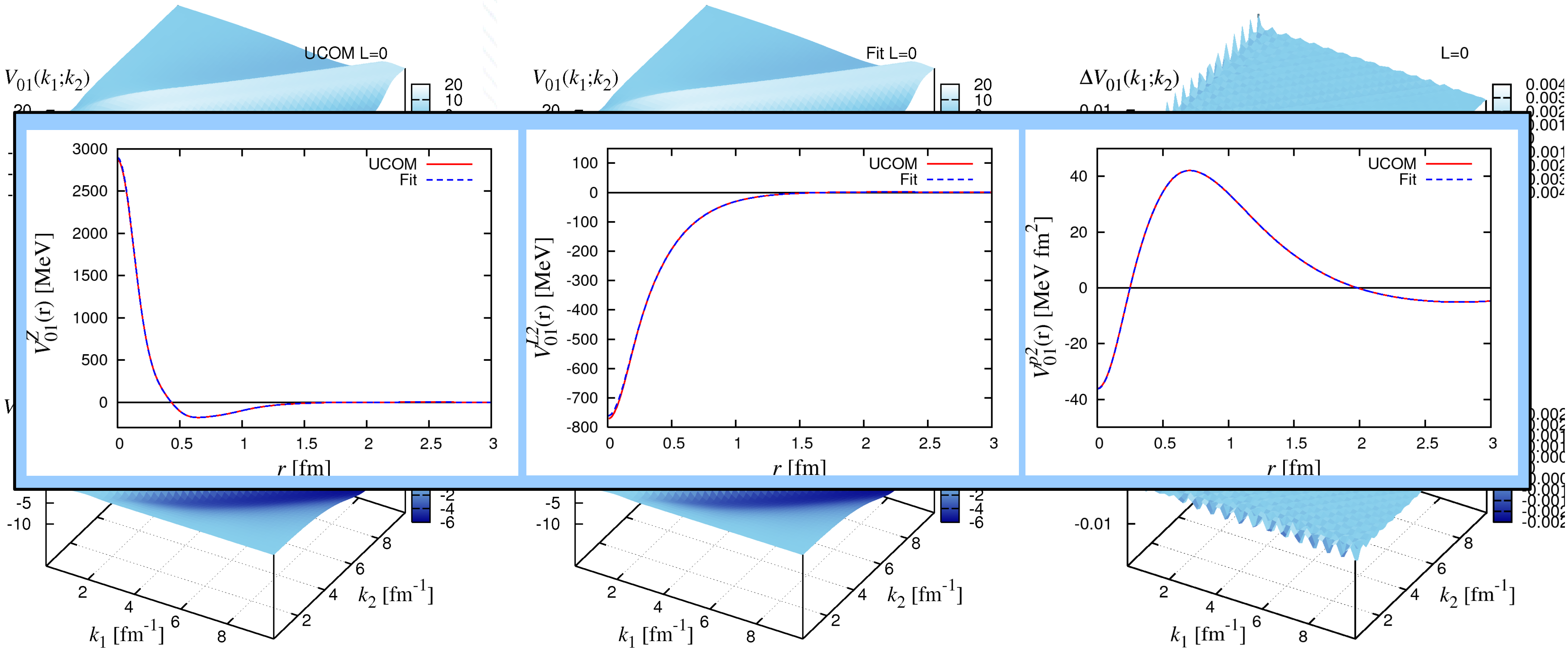
- $S=0$ and $T=1$: $V_{01}^{Z}(r)$, $V_{01}^{L2}(r)$, $V_{01}^{p2}(r)$
- fit to lowest $L \rightarrow L=0$, $L=2$



matrix elements in *MeV fm*^{3}

From partial wave matrix elements to operator representation - UCOM

- $S=0$ and $T=1$: $V^{Z}_{01}(r)$, $V^{L2}_{01}(r)$, $V^{p2}_{01}(r)$
- fit to lowest $L \rightarrow L=0$, $L=2$



matrix elements in *MeV fm*^{3}

From partial wave matrix elements to operator representation - UCOM

- $S=1$: isolation of tensor, spin-orbit and central components

- linear combination for given S, T and L

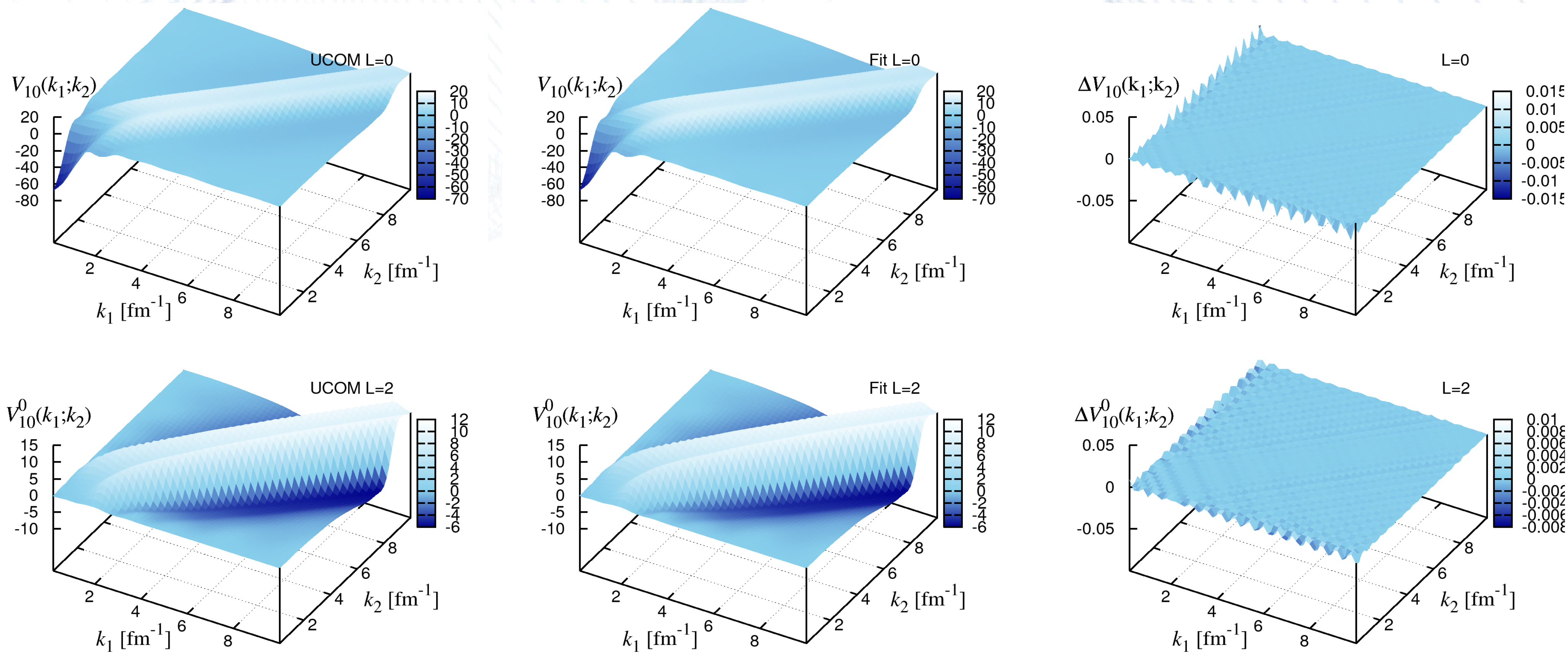
$$\sum_{J=L-1}^{L+1} \alpha_{ST}^{LJ} \langle k_1 (LS) J; T | \boldsymbol{V} | k_2 (LS) J; T \rangle \begin{array}{l} \rightarrow V_{ST}^{Z}(\boldsymbol{r}),\ V_{ST}^{L2}(\boldsymbol{r}),\ V_{ST}^{p2}(\boldsymbol{r}) \\ \rightarrow V_{ST}^{LS}(\boldsymbol{r}),\ V_{ST}^{L2LS}(\boldsymbol{r}) \\ \rightarrow V_{ST}^{T}(\boldsymbol{r}),\ V_{ST}^{TLL}(\boldsymbol{r}) \end{array}$$

- nondiagonal matrix elements $\langle k_1 (L_1 S) J; T | \boldsymbol{V} | k_2 (L_2 S) J; T \rangle, \quad L_1 \neq L_2$

  only tensor contribution $\rightarrow V_{ST}^{T}(\boldsymbol{r}),\ V_{ST}^{Tpp}(\boldsymbol{r}),\ V_{ST}^{Trp}(\boldsymbol{r})$

From partial wave matrix elements to operator representation - UCOM

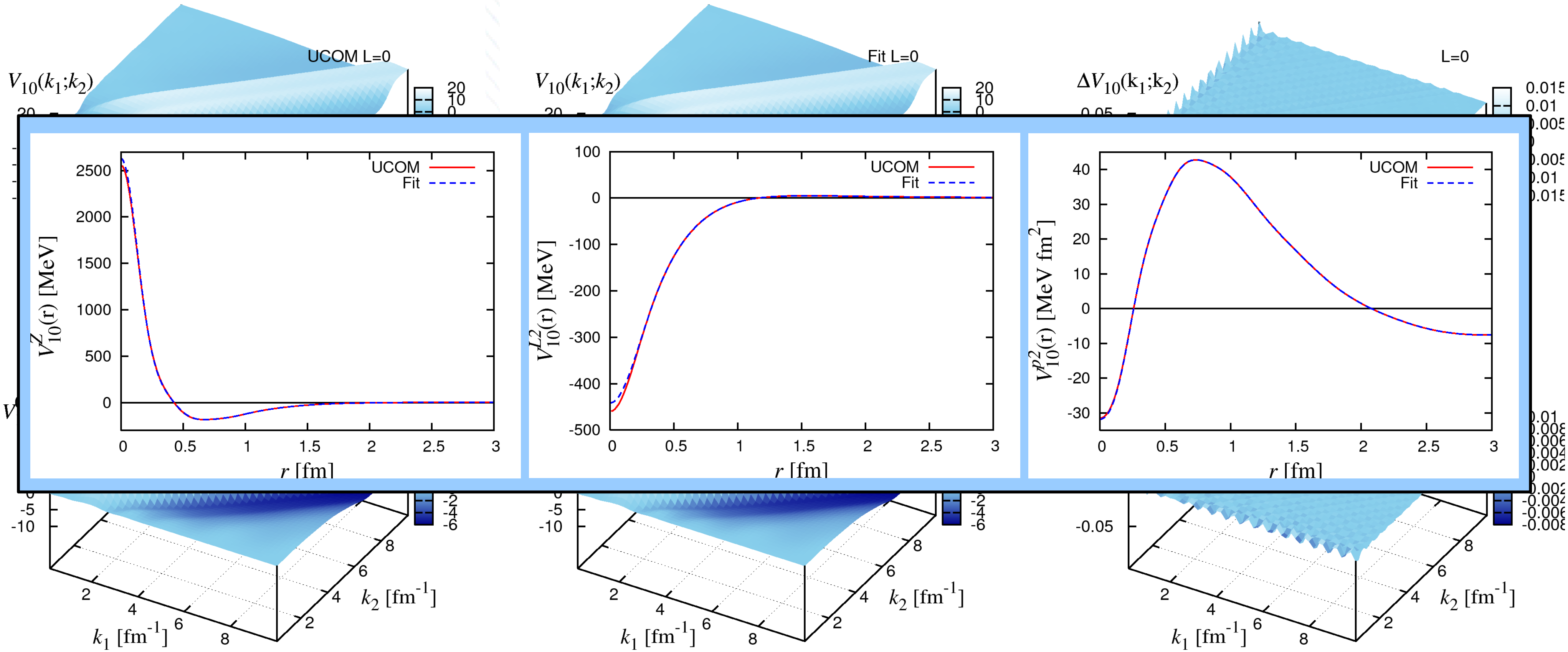
- $S=1$ and $T=0$: $V_{10}^{Z}(r)$, $V_{10}^{L2}(r)$, $V_{10}^{p2}(r)$
- fit to lowest $L \rightarrow L=0$, $L=2$



matrix elements in *MeV fm*^{3}

From partial wave matrix elements to operator representation - UCOM

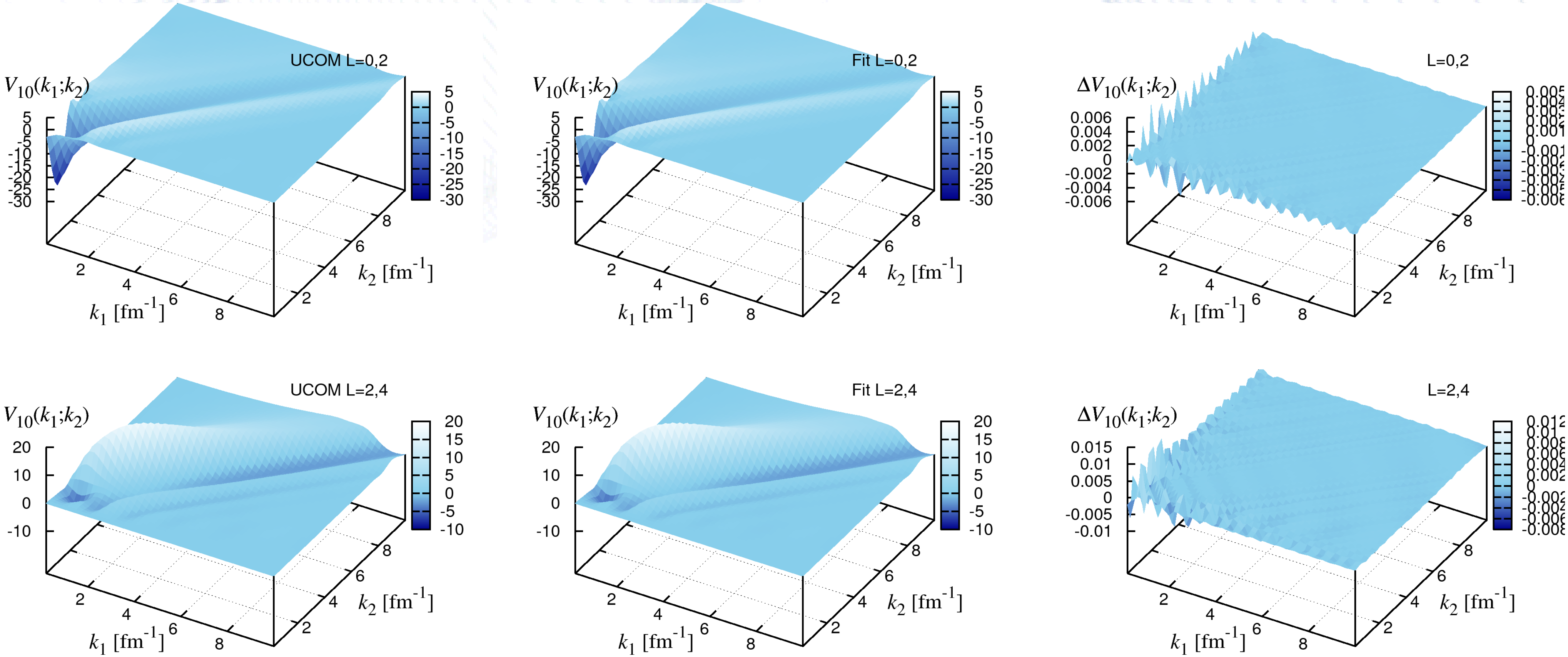
- $S=1$ and $T=0$: $V^{Z}_{10}(r)$, $V^{L2}_{10}(r)$, $V^{p2}_{10}(r)$
- fit to lowest $L \rightarrow L=0$, $L=2$



matrix elements in *MeV fm*^{3}

From partial wave matrix elements to operator representation - UCOM

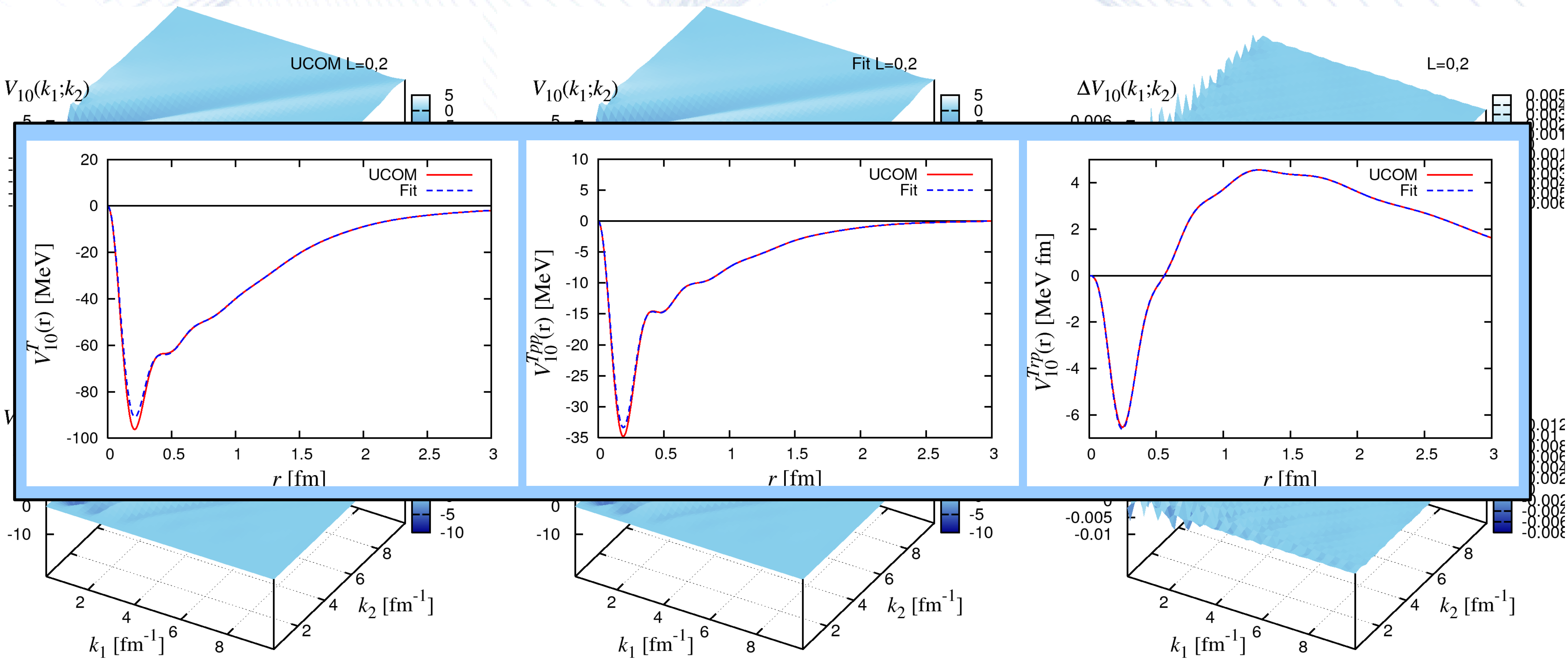
- $S=1$ and $T=0$: $V^{T}_{10}(r)$, $V^{Tpp}_{10}(r)$, $V^{Trp}_{10}(r)$
- fit to lowest $L \rightarrow \langle k_1(01)1;0|V|k_2(21)1;0\rangle, \langle k_1(21)1;0|V|k_2(41)1;0\rangle$



matrix elements in *MeV fm*^{3}

From partial wave matrix elements to operator representation - UCOM

- $S=1$ and $T=0$: $V^{T}_{10}(r)$, $V^{Tpp}_{10}(r)$, $V^{Trp}_{10}(r)$
- fit to lowest $L \rightarrow \langle k_1(01)1;0|V|k_2(21)1;0\rangle, \langle k_1(21)1;0|V|k_2(41)1;0\rangle$



matrix elements in *MeV fm*^{3}

From partial wave matrix elements to operator representation - UCOM

- which operators are important ?

$$
\begin{aligned}
V_{UCOM} &= \sum_{S,T} V^{Z}_{ST}(\boldsymbol{r})\boldsymbol{\Pi}_{ST} \\
&+ \sum_{S,T} V^{L2}_{ST}(\boldsymbol{r})\boldsymbol{\Pi}_{ST}\vec{\boldsymbol{L}}^2 \\
&+ \sum_{S,T} 1/2(\vec{\boldsymbol{p}}^2 V^{p2}_{ST}(\boldsymbol{r}) + V^{p2}_{ST}(\boldsymbol{r})\vec{\boldsymbol{p}}^2)\boldsymbol{\Pi}_{ST} \\
&+ \sum_{T} V^{LS}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}} \\
&+ \sum_{T} V^{L2LS}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\vec{\boldsymbol{L}}^2\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}} \\
&+ \sum_{T} V^{T}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\boldsymbol{S}_{12} \\
&+ \sum_{T} V^{TLL}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\boldsymbol{s}_{12}(\vec{L},\vec{L}) \\
&+ \sum_{T} V^{Tpp}_{1T}(\boldsymbol{r})\boldsymbol{\Pi}_{1T}\bar{\boldsymbol{s}}_{12}(\vec{p}_{\Omega},\vec{p}_{\Omega}) \\
&+ \sum_{T} (\boldsymbol{p}_r V^{Trp}_{1T}(\boldsymbol{r}) + V^{Trp}_{1T}(\boldsymbol{r})\boldsymbol{p}_r)\boldsymbol{\Pi}_{1T}\boldsymbol{s}_{12}(\vec{r},\vec{p}_{\Omega})
\end{aligned}
$$

NCSM calculation:

$E_B(^3He)$	$E_B(^4He)$
7.71 MeV	28.30 MeV
7.48 MeV	28.33 MeV

From partial wave matrix elements to operator representation - UCOM

- which operators are important ?

$$
\begin{aligned}
\boldsymbol{V}_{UCOM} = & \sum_{S,T} V^{Z}_{ST}(\boldsymbol{r})\,\boldsymbol{\Pi}_{ST} \\
+ & \sum_{S,T} V^{L2}_{ST}(\boldsymbol{r})\,\boldsymbol{\Pi}_{ST}\,\vec{\boldsymbol{L}}^{2} \\
+ & \sum_{S,T} 1/2\left(\vec{\boldsymbol{p}}^{2}\,V^{p2}_{ST}(\boldsymbol{r})+V^{p2}_{ST}(\boldsymbol{r})\,\vec{\boldsymbol{p}}^{2}\right)\boldsymbol{\Pi}_{ST} \\
+ & \sum_{T} V^{LS}_{1T}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}} \\
& \cancel{+ \sum_{T} V^{L2LS}_{1T}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\vec{\boldsymbol{L}}^{2}\,\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}}} \\
+ & \sum_{T} V^{T}_{1T}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\boldsymbol{S}_{12} \\
+ & \sum_{T} V^{TLL}_{1T}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\boldsymbol{s}_{12}(\vec{L},\vec{L}) \\
+ & \sum_{T} V^{Tpp}_{1T}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\bar{\boldsymbol{s}}_{12}(\vec{p}_{\Omega},\vec{p}_{\Omega}) \\
+ & \sum_{T} \left(\boldsymbol{p}_{r}\,V^{Trp}_{1T}(\boldsymbol{r})+V^{Trp}_{1T}(\boldsymbol{r})\,\boldsymbol{p}_{r}\right)\boldsymbol{\Pi}_{1T}\,\boldsymbol{s}_{12}(\vec{r},\vec{p}_{\Omega})
\end{aligned}
$$

NCSM calculation:

$E_B(^3He)$	$E_B(^4He)$
7.71 MeV	28.30 MeV
7.48 MeV	28.33 MeV
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From partial wave matrix elements to operator representation - UCOM

- which operators are important ?

$$
\begin{aligned}
V_{UCOM} &= \sum_{S,T} V^{Z}_{ST}(\boldsymbol{r})\,\boldsymbol{\Pi}_{ST} \\
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&+ \sum_{S,T} 1/2\,(\vec{\boldsymbol{p}}^{2}\,V^{p2}_{ST}(\boldsymbol{r}) + V^{p2}_{ST}(\boldsymbol{r})\,\vec{\boldsymbol{p}}^{2})\,\boldsymbol{\Pi}_{ST} \\
&+ \sum_{T} V^{LS}_{1T}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}} \\
&\cancel{+ \sum_{T} V^{L2LS}_{1T}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\vec{\boldsymbol{L}}^{2}\,\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}}} \\
&+ \sum_{T} V^{T}_{1T}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\boldsymbol{S}_{12} \\
&+ \sum_{T} V^{TLL}_{1T}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\boldsymbol{s}_{12}(\vec{L},\vec{L}) \\
&\cancel{+ \sum_{T} V^{Tpp}_{1T}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\bar{s}_{12}(\vec{p}_{\Omega},\vec{p}_{\Omega})} \\
&+ \sum_{T} (\boldsymbol{p}_r\,V^{Trp}_{1T}(\boldsymbol{r}) + V^{Trp}_{1T}(\boldsymbol{r})\,\boldsymbol{p}_r)\,\boldsymbol{\Pi}_{1T}\,\boldsymbol{s}_{12}(\vec{r},\vec{p}_{\Omega})
\end{aligned}
$$

NCSM calculation:

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&+ \sum_{S,T} 1/2\left(\vec{\boldsymbol{p}}^{2}\,V^{p2}_{ST}(\boldsymbol{r}) + V^{p2}_{ST}(\boldsymbol{r})\,\vec{\boldsymbol{p}}^{2}\right)\boldsymbol{\Pi}_{ST} \\
&+ \sum_{T} V^{LS}_{1T}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}} \\
&\cancel{+ \sum_{T} V^{L2LS}_{1T}(\boldsymbol{r})\,\boldsymbol{\Pi}_{1T}\,\vec{\boldsymbol{L}}^{2}\,\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}}} \\
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\end{aligned}
$$

NCSM calculation:

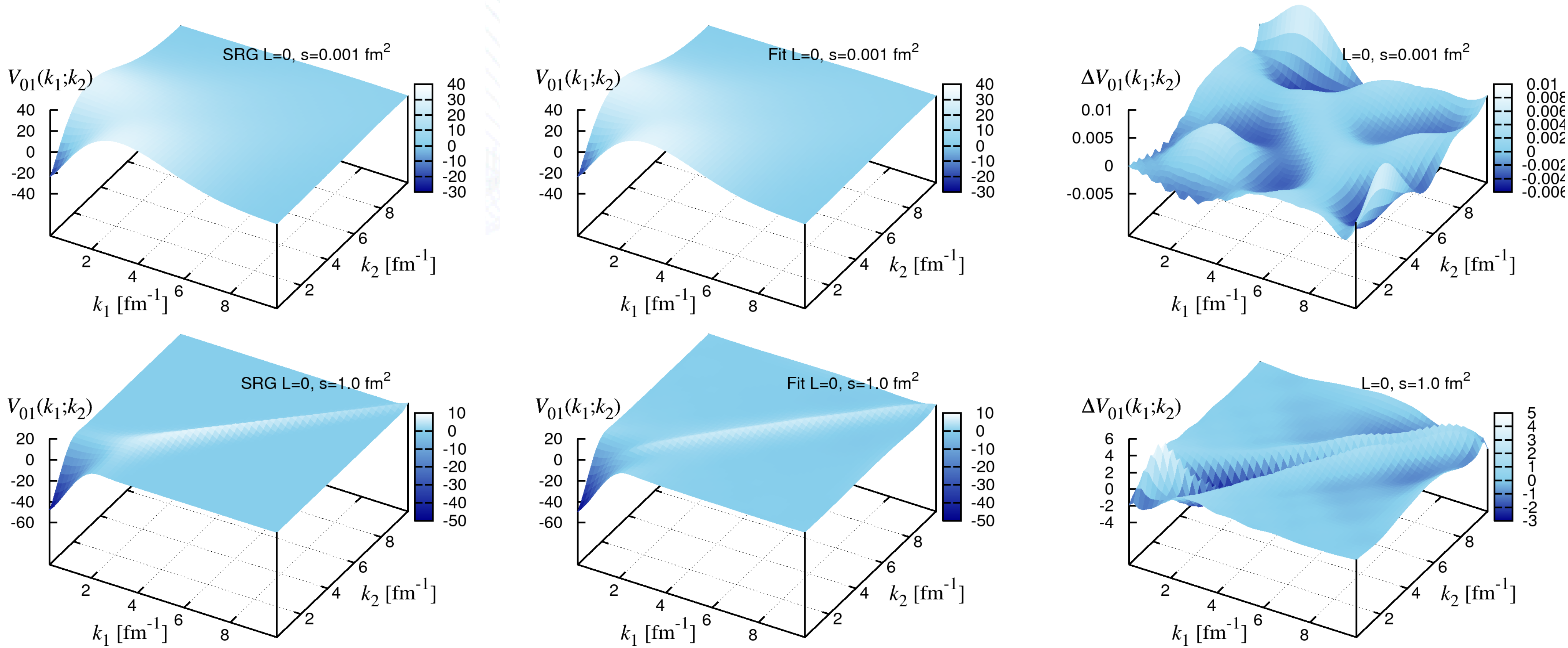
$E_B({}^3He)$	$E_B({}^4He)$
7.71 MeV	28.30 MeV
7.48 MeV	28.33 MeV
7.48 MeV	28.33 MeV
7.48 MeV	28.34 MeV
5.75 MeV	25.60 MeV

From partial wave matrix elements to operator representation - SRG

- $S=0,\ T=1$ and $L=0$
- Ansatz: $\boldsymbol{V}=V^{Z}(\boldsymbol{r})+1/2\,(\vec{\boldsymbol{p}}^{2}V^{p2}(\boldsymbol{r})+V^{p2}(\boldsymbol{r})\,\vec{\boldsymbol{p}}^{2})$ ~ UCOM

From partial wave matrix elements to operator representation - SRG

- $S=0,\ T=1$ and $L=0$
- Ansatz: $V = V^{Z}(\boldsymbol{r}) + 1/2\,(\vec{\boldsymbol{p}}^{\,2} V^{p2}(\boldsymbol{r}) + V^{p2}(\boldsymbol{r})\,\vec{\boldsymbol{p}}^{\,2})$ ~ UCOM



matrix elements in *MeV fm*^{3}

From partial wave matrix elements to operator representation - SRG

- SRG potential cannot be described by quadratic momentum dependence

$$\boldsymbol{V}=V^{Z}(\boldsymbol{r})+1/2(\vec{\boldsymbol{p}}^{2}V^{p2}(\boldsymbol{r})+V^{p2}(\boldsymbol{r})\vec{\boldsymbol{p}}^{2})$$

- higher powers of momentum operator: ~ better, but fit diverges for large k

- $\rightarrow$ nonlocal potential:

$$V(\boldsymbol{r}) \rightarrow V(\boldsymbol{r},\boldsymbol{p})$$

$$\langle\vec{r}_1|V(\boldsymbol{r})|\vec{r}_2\rangle=\delta^3(\vec{r}_1-\vec{r}_2)\sum_j \gamma_j e^{-\frac{\vec{r}_1^2}{2\kappa_j}} \rightarrow \langle\vec{r}_1|V(\boldsymbol{r},\boldsymbol{p})|\vec{r}_2\rangle=\sum_{k,l}\gamma_{kl}\sqrt{(2\lambda_l)^{-3}}\,e^{-\frac{\vec{s}^2}{2\lambda_l}}\cdot e^{-\frac{\vec{r}^2}{2\kappa_k}}$$

$$\vec{r}=\frac{1}{2}(\vec{r}_1+\vec{r}_2) \qquad \vec{s}=\vec{r}_1-\vec{r}_2$$

- λ shows how “local” the potential is:

$$\lambda=0 \rightarrow \langle\vec{r}_1|V(\boldsymbol{r},\boldsymbol{p})|\vec{r}_2\rangle\simeq\delta^3(\vec{r}_1-\vec{r}_2)\sum_k \gamma_k e^{-\frac{\vec{r}\,'^2}{2\kappa_k}}$$

$$\lambda k_F^2 \ll 1 \rightarrow \boldsymbol{V}(\boldsymbol{r},\boldsymbol{p})\approx V^{Z}(\boldsymbol{r})+1/2(\vec{\boldsymbol{p}}^{2}V^{p2}(\boldsymbol{r})+V^{p2}(\boldsymbol{r})\vec{\boldsymbol{p}}^{2})$$

From partial wave matrix elements to operator representation - SRG

- Ansatz:

$$\boldsymbol{V} = V^Z(\boldsymbol{r}) + 1/2\left(\vec{\boldsymbol{p}}^2 V^{p2}(\boldsymbol{r}) + V^{p2}(\boldsymbol{r})\vec{\boldsymbol{p}}^2\right) \;\rightarrow\; V(\boldsymbol{r},\boldsymbol{p})$$

- partial wave matrix elements

$$\langle k_1 L | V(\boldsymbol{r},\boldsymbol{p}) | k_2 L\rangle =$$

$$8i^L \sum_{k,l} \gamma_{kl} \sqrt{(2\lambda_l)^{-3}} \int_0^\infty dr_1 r_1^2 \int_0^\infty dr_2 r_2^2 e^{-\left(\frac{1}{4\kappa_k}+\frac{1}{\lambda_l}\right)(r'^2+r^2)} j_L\left(\frac{i}{2}\left(\kappa_k^{-1} - 4\lambda_l^{-1}\right) r_1 r_2\right) j_L(k_1 r_1)\, j_L(k_2 r_2)$$

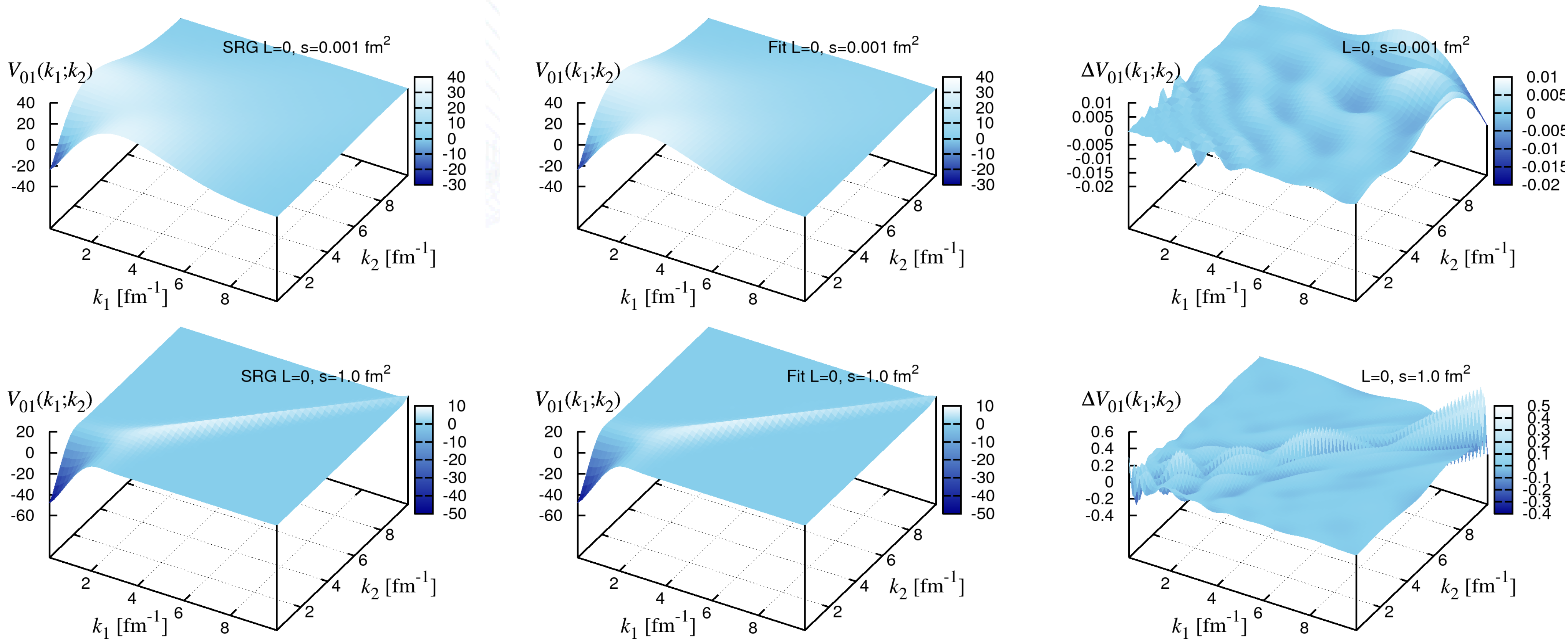
- parameters

$$\kappa = \{0.1\cdot\sqrt{2},\ 0.2,\ 0.2\cdot\sqrt{2},\ 0.4,\ \ldots,\ 25.6\}\, fm^2$$

$$\lambda = \{0,\ 0.25,\ 0.5,\ 1,\ 2\}\, fm^2$$

From partial wave matrix elements to operator representation - SRG

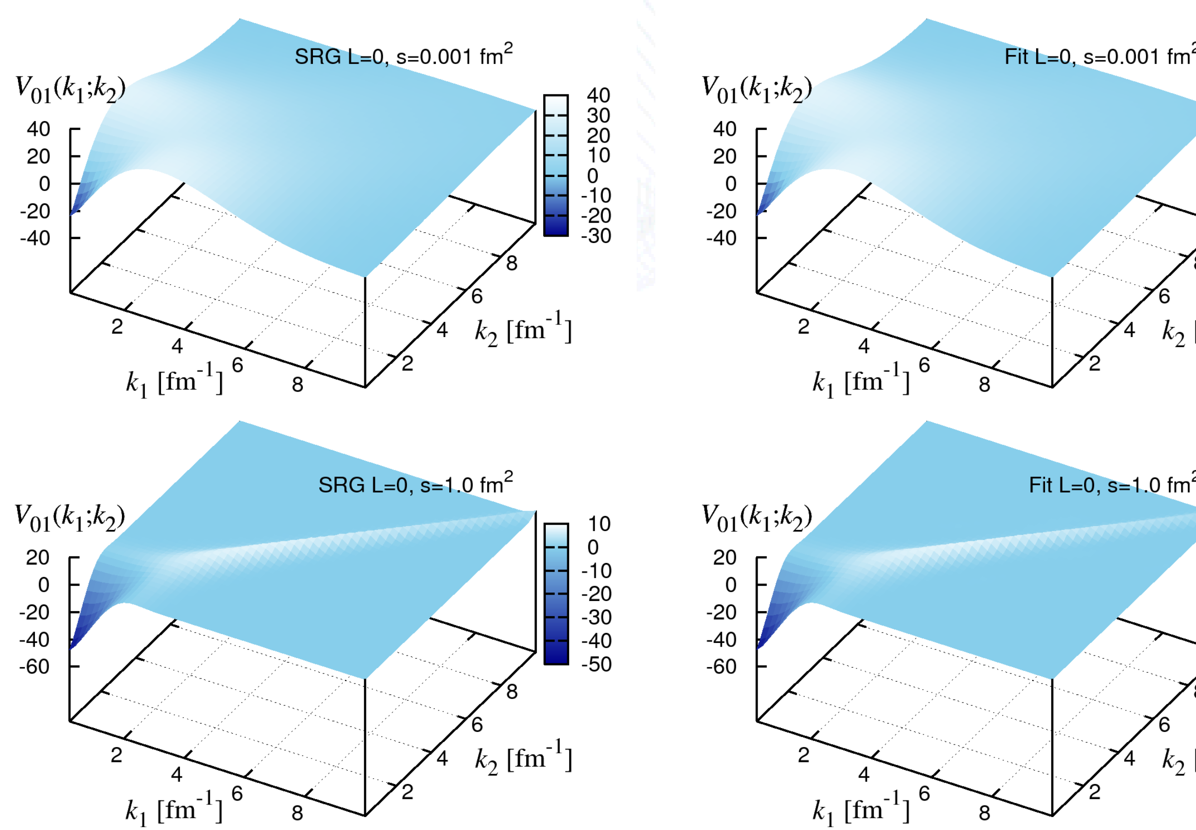
- $S=0,\ T=1$ and $L=0$
- Ansatz: $\boldsymbol{V}=V(\boldsymbol{r},\boldsymbol{p})$

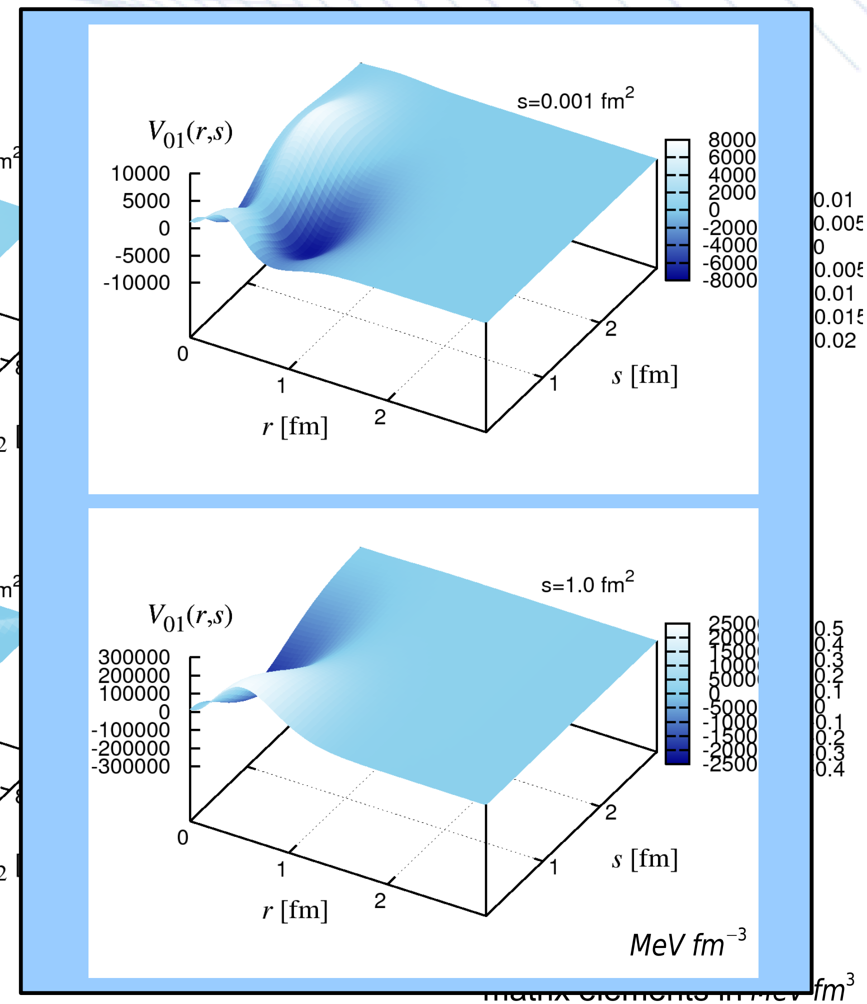


matrix elements in *MeV fm*^{3}

From partial wave matrix elements to operator representation - SRG

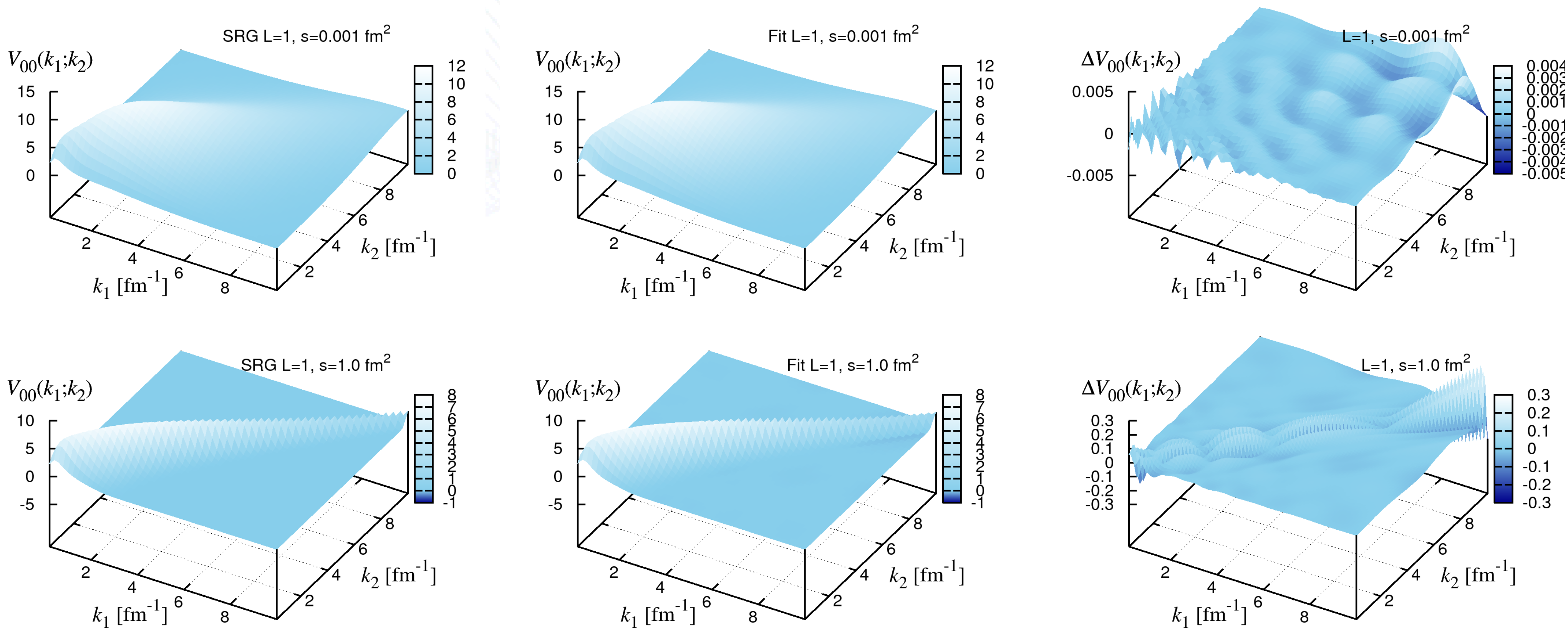
- $S=0,\ T=1$ and $L=0$
- Ansatz: $V=V(\boldsymbol{r},\boldsymbol{p})$





From partial wave matrix elements to operator representation - SRG

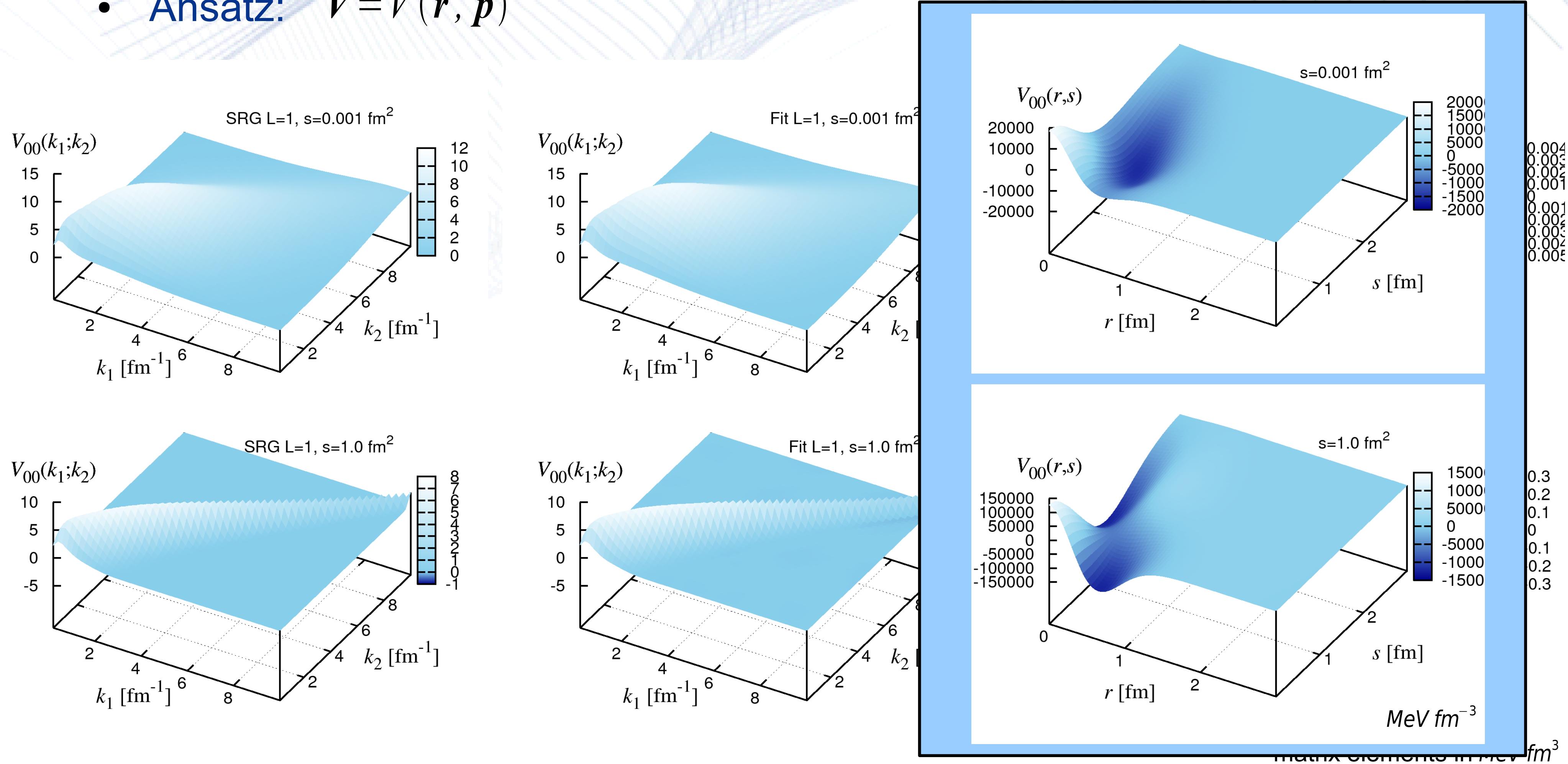
- $S=0,\ T=0$ and $L=1$
- Ansatz: $\boldsymbol{V}=V(\boldsymbol{r},\boldsymbol{p})$



matrix elements in *MeV fm*^{3}

From partial wave matrix elements to operator representation - SRG

- $S=0,\ T=0$ and $L=1$
- Ansatz: $V=V(\boldsymbol{r},\boldsymbol{p})$



From partial wave matrix elements to operator representation - SRG

- still to do: operator representation ?

$$
\begin{aligned}
\boldsymbol{V}_{SRG} \quad = \quad & \sum_{S,T} V^{Z}_{ST}(\boldsymbol{r},\boldsymbol{p})\boldsymbol{\Pi}_{ST} \\
+ \quad & \sum_{S,T} V^{L2}_{ST}(\boldsymbol{r},\boldsymbol{p})\boldsymbol{\Pi}_{ST}\vec{\boldsymbol{L}}^{2} \\
+ \quad & \sum_{T} V^{LS}_{1T}(\boldsymbol{r},\boldsymbol{p})\boldsymbol{\Pi}_{1T}\vec{\boldsymbol{L}}\cdot\vec{\boldsymbol{S}} \\
+ \quad & \sum_{T} V^{T}_{1T}(\boldsymbol{r},\boldsymbol{p})\boldsymbol{\Pi}_{1T}\boldsymbol{S}_{12} \\
+ \quad & \sum_{T} V^{TLL}_{1T}(\boldsymbol{r},\boldsymbol{p})\boldsymbol{\Pi}_{1T}\boldsymbol{s}_{12}(\vec{L},\vec{L})
\end{aligned}
$$

Summary and Outlook

- models (for example FMD) use operator representation of nuclear potentials
- determine operator representation starting from partial wave matrix elements

- UCOM: good reproduction by local and quadratic momentum terms
- SRG: complex momentum dependence, nonlocal potentials needed

- outlook:
  - how many operators are needed to describe UCOM potential
  - operator representation for SRG
  - Understanding nature of nonlocality