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Application of chiral and low-momentum nuclear forces

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EMMI-EFES Workshop on Neutron-Rich Nuclei, GSI Darmstadt

Outline

- *Motivation*
- *NN and 3N interactions*
- *QCD & ChEFT approach to nuclear forces*
- *s-shell nuclei ^3H,^3He and ^4He*
- *p-shell nuclei and signatures of 3NF's*
- *Soft nuclear forces and 3N interactions*
- *3NF contributions in nuclear matter*

- *Conclusions & Outlook*

February 09, 2009

Nuclear many-body problem

A

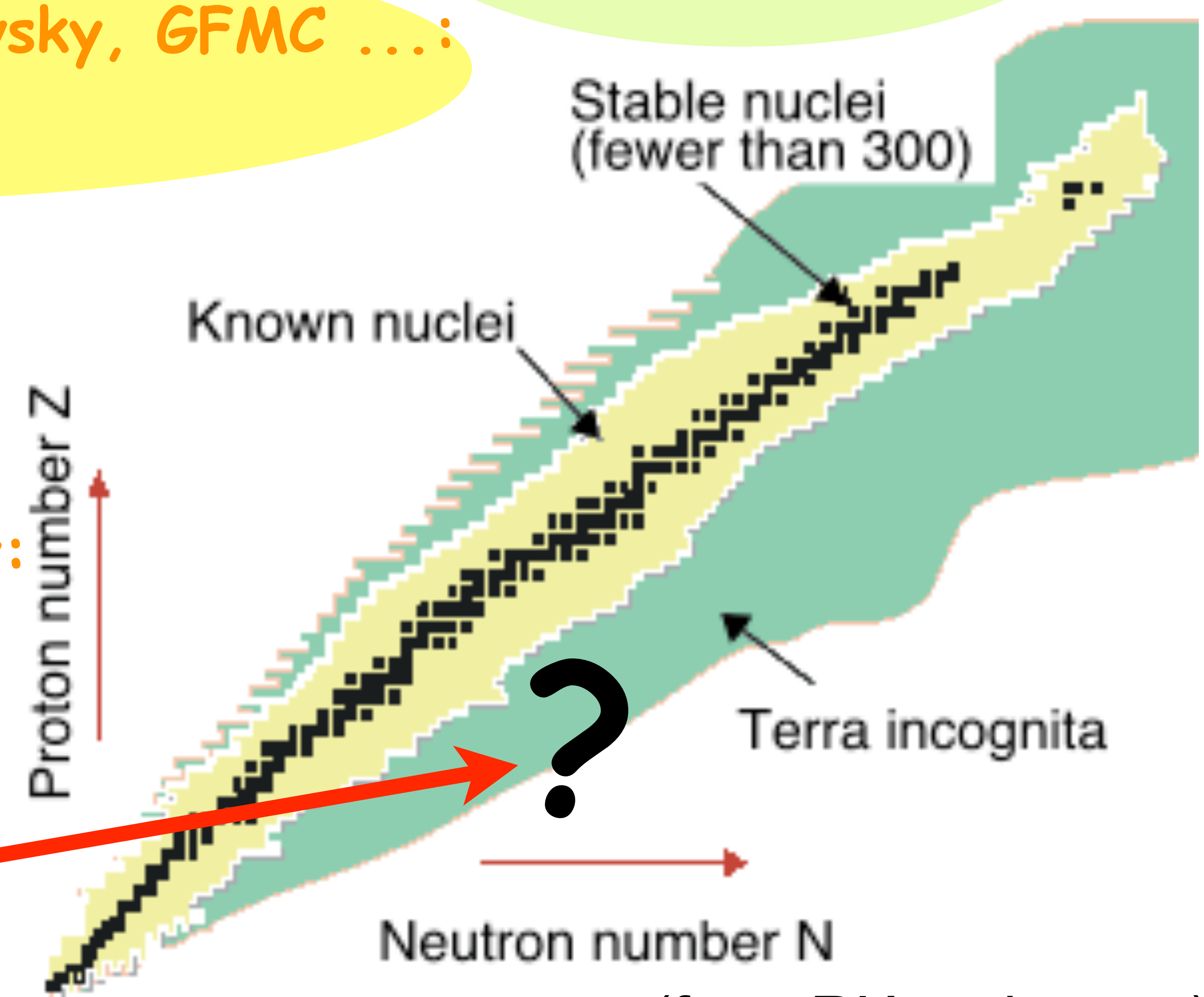
Lattice QCD:
A=0,1,2,...

NCSM, Faddeev-Yakubovsky, GFMC ...:
A=3-16

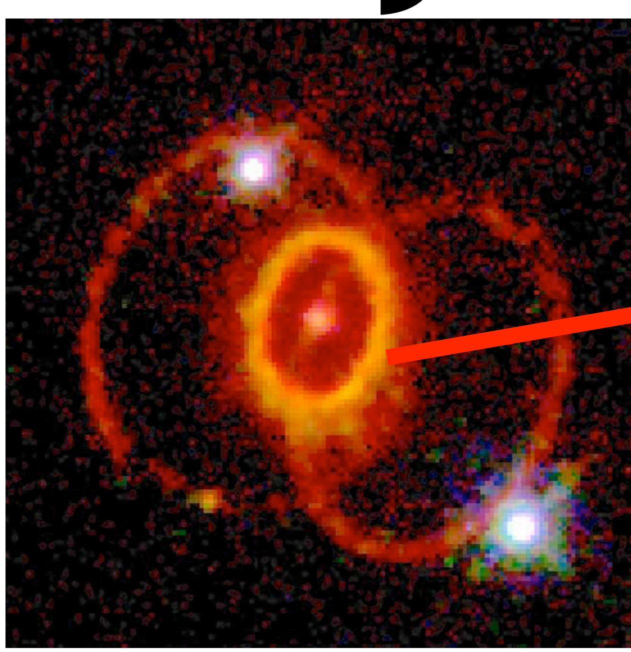
coupled cluster:
A=16-100

Density Functional Theory:
A>100

ChEFT interactions,
3N interactions



(from RIA web page)

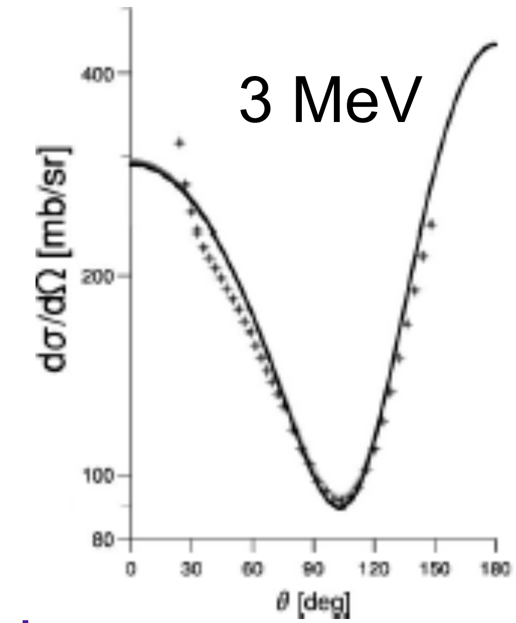


(SN1987a)

Experimental programs: Isolde, ISAC, FAIR, RIA, ...
Theoretical predictions?

Phenomenological approach

Several NN force models describe the data (~ 4000 data) up to the pion production threshold perfectly using ~ 40 parameters



Predictions based on NN forces:

Many low energy few-nucleon observables are well & model independently described !

Scattering observables at higher energies & binding energies are **model dependent** and not well described

(see e.g. Witała et al., 2001)

	^3H	^4He
CD-Bonn	-8.013	-26.23
AV18	-7.628	-24.25
Nijm I	-7.741	-24.99
Nijm II	-7.659	-24.55
Expt	-8.482	-28.30

(see e.g. A.N. et al., 2002)

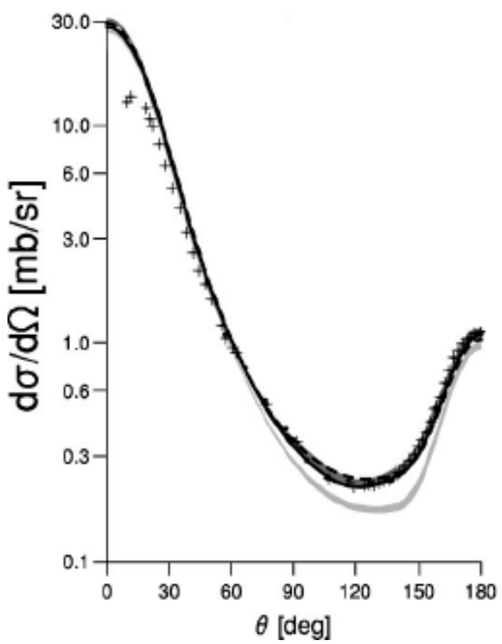
→ **3NF's are quantitatively important…**

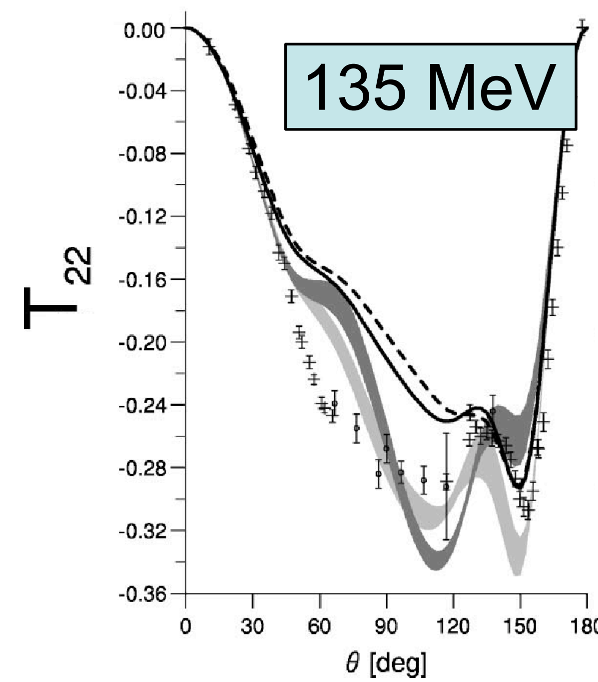
… but have not been understood yet

Phenomenological 3NF's

NN interactions can be augmented by phenomenological 3N interactions (2π-exchange)
usually the 3N force is adjusted so that the ^{3}H binding energy is described correctly

These phenomenological combinations are very useful to identify signatures of 3NF's
triggered a lot of experiments for pd scattering (RIKEN, KVI, IUCF, ...)





none of the phenomenological models describes all the data!

A systematic approach that leads to consistent NN and 3N forces is necessary

Nuclear structure & phenomenological 3NF's

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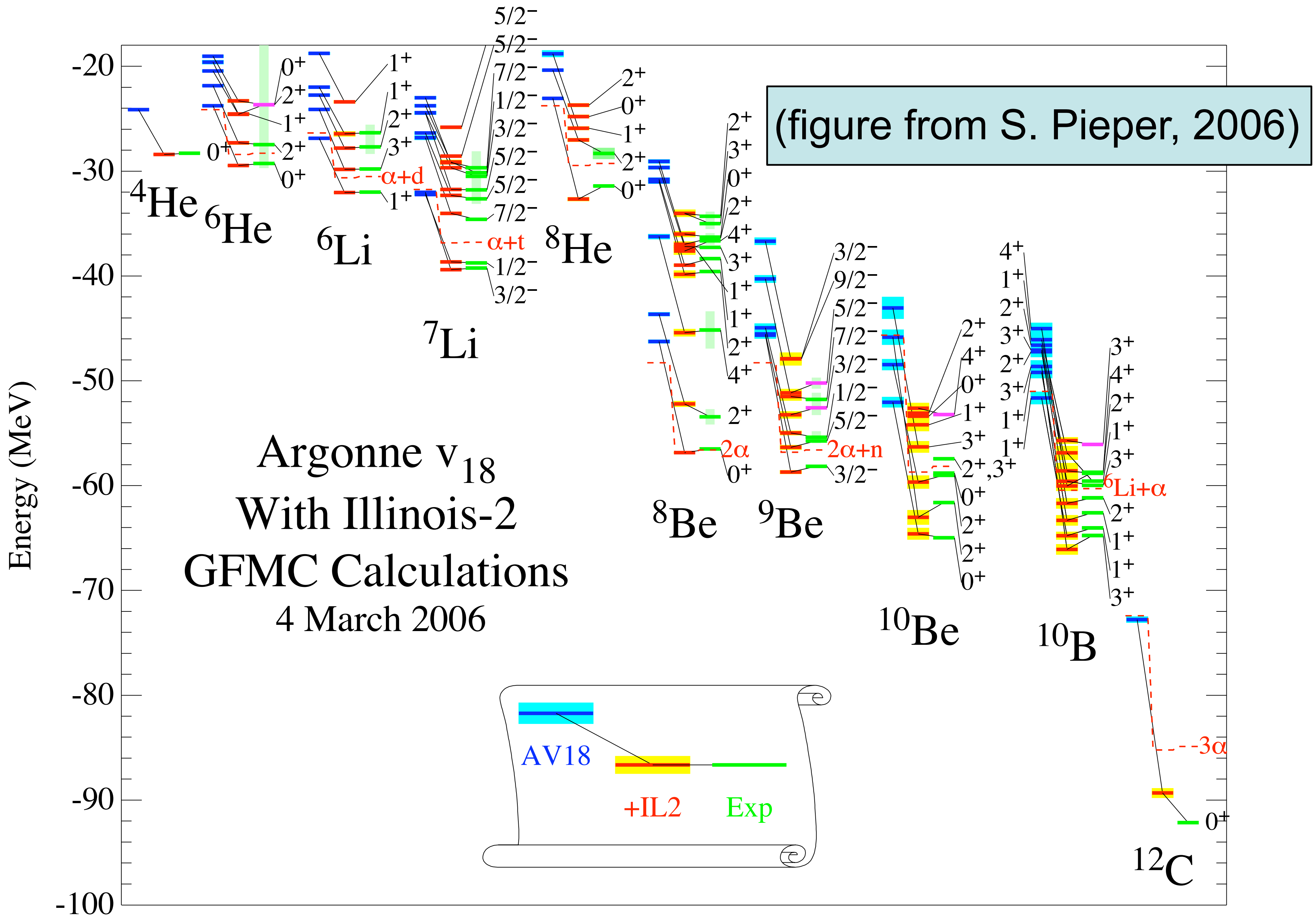
3NF important for nuclei - improvement of 3NF's is required !



Guidance is necessary to improve 3NF's (many structures!)

Extension to more complex nuclei becomes more and more involved!

EFT of QCD: chiral perturbation theory

QCD & approximate chiral symmetry

symmetries →

Effective Field Theory of QCD:
relevant degrees of freedom
nucleons & pions

$$\mathcal{L}_{QCD} = \bar{q}\; i\not{D}\; q - \frac{1}{2}\;\mathrm{Tr}\; G_{\mu\nu}G^{\mu\nu} - \bar{q}\; m\; q$$

expansion in $\frac{Q}{\Lambda_\chi}$

$Q \approx m_\pi$, typical momentum

spontaneously & explicitly broken chiral symmetry

Goldstone bosons: pions → **Chiral Perturbation Theory** (ChPT)

$$m_\pi \ll \Lambda_\chi \approx 1\ \mathrm{GeV}$$

„power counting“
a systematic scheme to identify
a finite numbers of diagrams
contributing at a given order

EFT of QCD: chiral perturbation theory

EFT allows to understand pion mass dependence of nuclear observables

→ connections to lattice QCD results

EFT can be applied to different strong interaction reactions

→ reveals connections of different processes,
connects NN, **3N**, 4N ... interactions

QCD → ChEFT involving π,N,...

→ πN

→ π production

....

→ nuclei

↑ pion mass dependence

↑ symmetries (chiral symmetry)

Naive estimate of 1π- & 2π-exchange

„power counting“:
The infinite number of possible diagrams can be ordered, so that only a **finite** number contributes at each order.

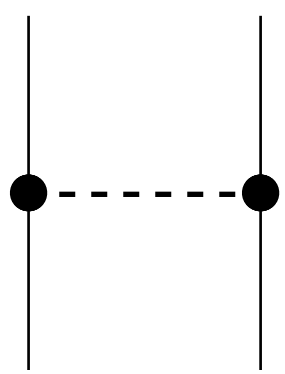
If this was strictly true, we would **not have bound states** within this framework.

E.g chiral order of well-known π-exchange

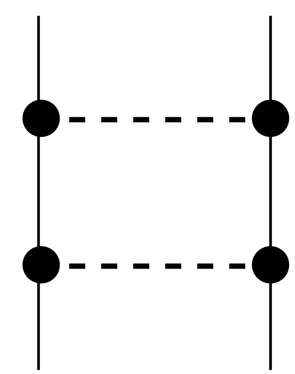
$$v = -4 + 2N + 2L + \sum_i \left(d_i + \frac{n_i}{2} - 2\right)$$ ➡ diagram contributes $\propto \left(\frac{Q}{\Lambda}\right)^{\nu}$

$$\Delta_i = d_i + \frac{n_i}{2} - 2 = 0$$

for πNN vertices



$N = 2, \quad L = 0$
$\nu = -4 + 2 \cdot 2 + 2 \cdot 0 + 2 \cdot 0 = 0$



$N = 2, \quad L = 1$
$\nu = -4 + 2 \cdot 2 + 2 \cdot 1 + 2 \cdot 0 = 2$

We naively find the 1π-exchange in leading order, and the 2π-exchange in subleading, etc.

Weinberg's observation

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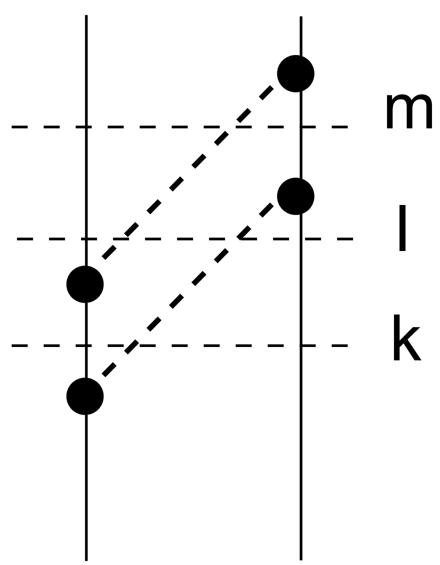
Time-ordered perturbation theory for 2π-exchange

1) **irreducible** diagram

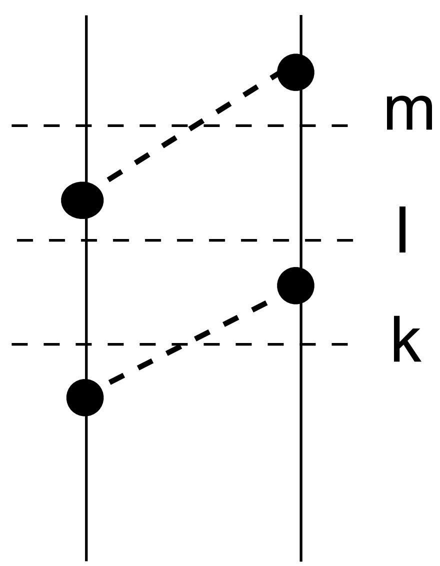


without purely two nucleon intermediate states

propagators contribute $$\frac{1}{E_j + i\epsilon - E_{k,l,m}} \propto \frac{1}{\frac{Q^2}{2m} + m_\pi} \propto \frac{1}{Q}$$

1/Q is in agreement with the power counting estimate

2) **reducible** diagram



with purely two nucleon intermediate states

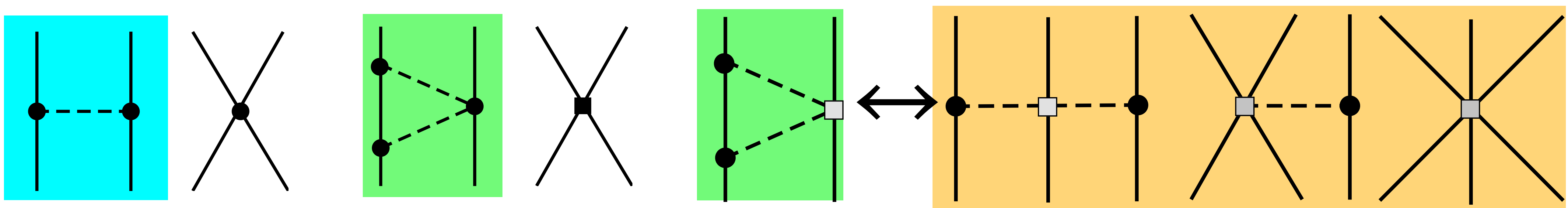
„l" propagator contributes $$\frac{1}{E_j + i\epsilon - E_l} \propto \frac{1}{\frac{Q^2}{2m}} \propto \frac{1}{Q}\,\frac{2m}{Q}$$

There is an enhancement of order m/Q (m nucleon mass)!

Solution: apply counting to **irreducible diagrams** which then **define a potential**

Chiral forces in LO,NLO and N2LO

LO: $\left(\frac{Q}{\Lambda}\right)^0$ NLO: $\left(\frac{Q}{\Lambda}\right)^2$ NNLO: $\left(\frac{Q}{\Lambda}\right)^3$



1π exchange determines the long range part of the interaction
(confirmed by, e.g., extraction of m_π from NN data, see Stoks et al., 1993)

Including 2π exchange reduce the # of boundary conditions in new PSA
(Rentemeester et al., 1999)

3NF's are **quantitatively** important to describe nuclei
many observables are sensitive to 3NF structure ← !

4NF's appear in N3LO (Epelbaum 2006,2007)

→ **3NF structure challenges chiral approach**
& 3NF required for nuclear structure/reactions

Regularization

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The ChPT potentials need regularization to obtain a well define LS equation

$$V(p,p') \quad \rightarrow \quad e^{-\left(\frac{p}{\Lambda}\right)^n} \; V(p,p') \; e^{-\left(\frac{p'}{\Lambda}\right)^n}$$ or correspondingly for 3NF

In practice: Λ ~ 500 - 600 MeV

→ ChPT potentials are very soft / low momentum

→ fast convergence in many numerical calculations ✔

Higher Λ are possible, but require additional contact interactions

→ on-going controversy, problem of singular interactions

Do small Λ remove physically relevant degrees of freedom for nuclear physics ?

Binding energies for ^3H (NN forces only)

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^3H binding energies are based on NN forces only
(LO from AN, Timmermans, van Kolck, 2005
NLO and N2LO from Epelbaum, Glöckle, Meißner, 2005,
N3LO from Entem, Machleidt, 2003)

	$\Lambda/\tilde{\Lambda}$ [MeV]	E_b [MeV]	V [MeV]	ΔE_b [keV]	$\lvert\Delta E_b/V\rvert$ [%]
LO	500 / no loops	-7.50	-51.8	1430	3.0 (7.0)
	600 / no loops	-6.07			
NLO	400 / 700	-8.46		650	1.6 (0.5)
	550 / 700	-7.81	-41.1		
N2LO	450 / 700	-8.42	-38.3	530	1.3 (0.5)
	600 / 700	-7.89			
N3LO	500 / DR	-7.84	-42.3	40	0.1 (0.03)
	600 / DR	-7.80			

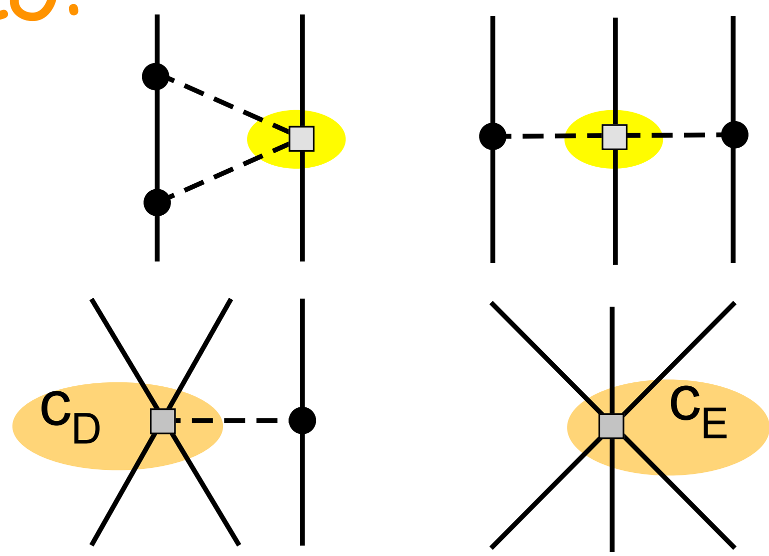
"power counting" estimates in brackets qualitatively agree ✓

N3LO leads to a very small cutoff dependence already for **$\Lambda \approx$ 500 MeV** !

Adding N2LO 3N force

Leading 3NF appears in N2LO (van Kolck, 1994)

N2LO:



subleading 2π exchange (c_1, c_3, c_4)
coefficients linked to NN force (✓)

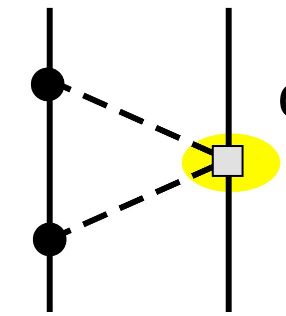
1π exchange + contact 3NF term ($\longrightarrow c_D$)
3N contact interaction ($\longrightarrow c_E$)

N3LO 3NF's are partly formulated and are being implemented (Bernard et al.,2008)

In the following we augment the N3LO NN interaction by only N2LO 3N forces!
(omitted contact interactions are of order $(Q/\Lambda)^5$)

Status c_i constants

How well do we know the strength of the subleading πN vertices



c_i constants link 2π-exchange NN-force and πN scattering

	c_1	c_3	c_4		
Rentmeester et al. PRC 67, 044001	-0.76	-4.78	3.96	NN	✓
Büttiker et al. NPA 668, 97	-0.81	-4.70	3.40	πN	
Fettes et al. NPA 640, 199	-1.23	-5.94	3.47	πN	
Meißner, talk at TRIUMF	-0.9	-4.7	3.5	πN	
Entem et al. PRC 66,014002	-0.81	-3.40	3.40	NN	
Entem et al. PRC 68,041001(R)	-0.81	-3.20	5.40	NN	← used here!

(red=input to analysis)

Note: there are sizable error bars !

Determination from πN scattering and fit to NN data agree qualitativly:

→ connection of subleading 2π-exchange and πN supported!

but: some determinations are highly controversial

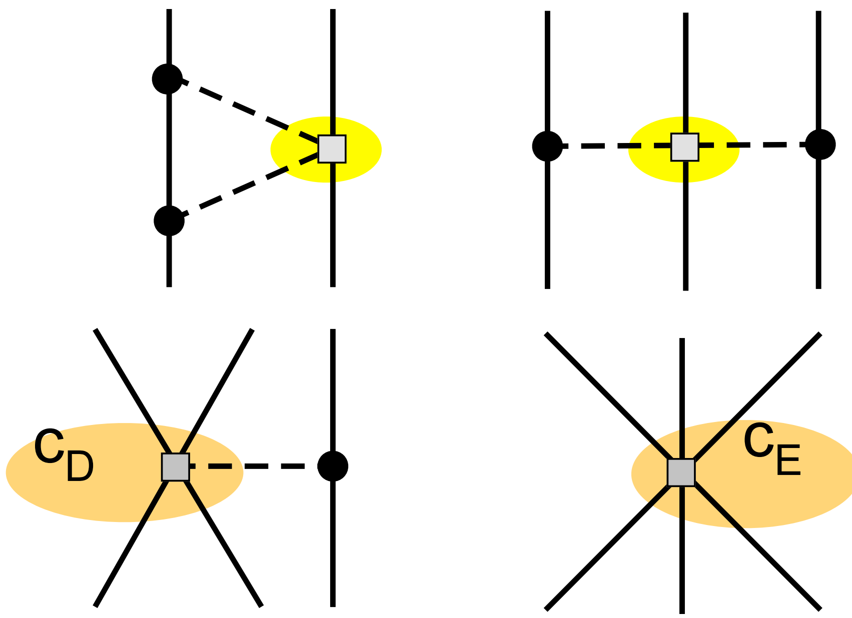
sensitivity of the NN data is small

c_1 is usually not extracted from NN data, but input to the analysis

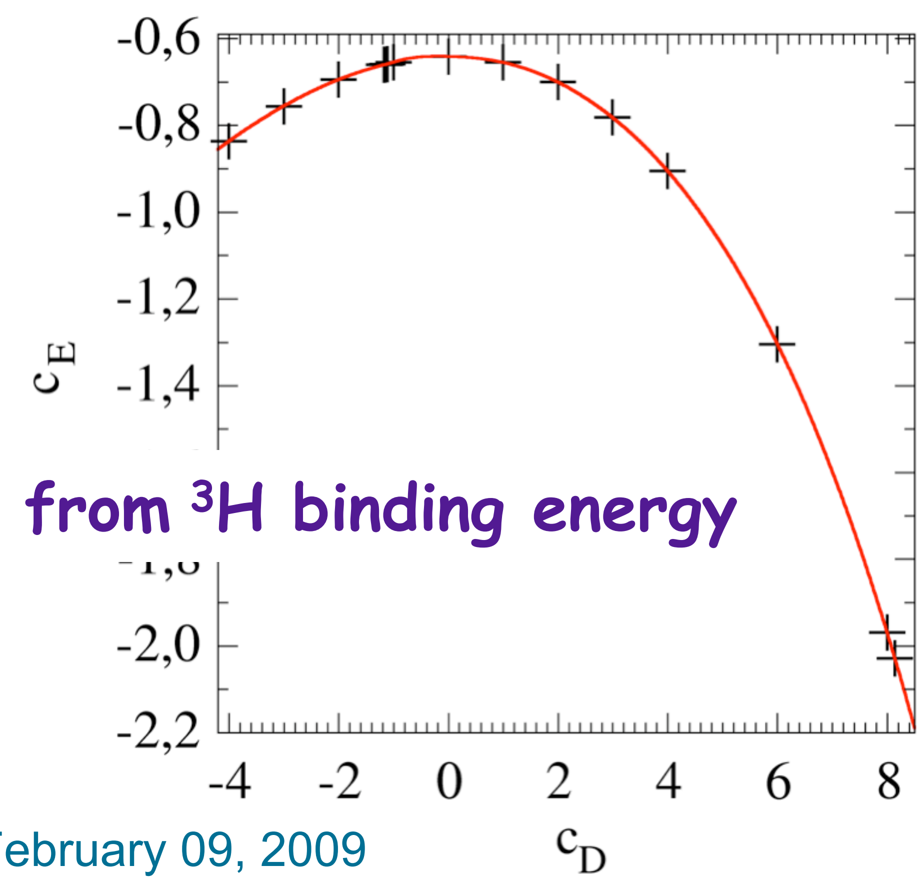
Determination of c_D and c_E

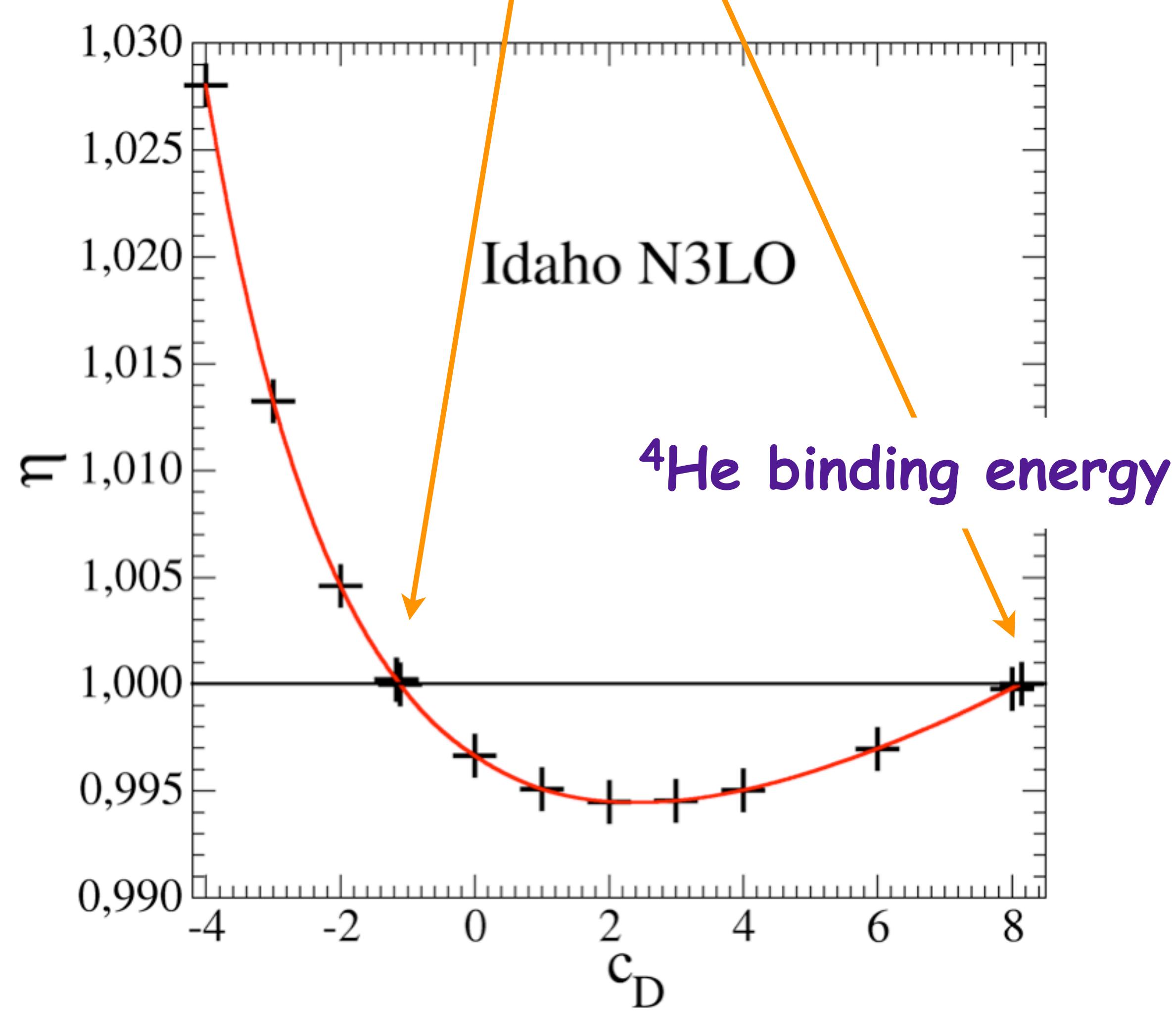
Here, we use the **^3H and ^4He binding energies** to fix the strength of the 3NF counter terms (other methods are possible, e.g. using ^3H and $^2a_{nd}$)

we find two solutions that describe the 3N and 4N binding energies equally well



	c_D	c_E
3NF-A	-1.11	-0.66
3NF-B	8.14	-2.02





3NF contributions

3NF and NN expectation values for ^{4}He

3NF contribution is or the order of the **power counting estimate of 2 %** ✓

	$\Lambda/\tilde{\Lambda}$ [MeV]	E_b [MeV]	V_{NN} [MeV]	V_{123} [MeV]	$\lvert V_{123} / V_{NN} \rvert$ [%]
N2LO	450 / 700	-27.65	-84.56	-1.11	1.3
	600 / 700	-28.57	-93.73	-6.83	7.3
N3LO	500 / DR-3NF-A	-28.27	-99.45	-4.06	4.1
	500 / DR-3NF-B	-28.24	-98.92	-7.10	7.2

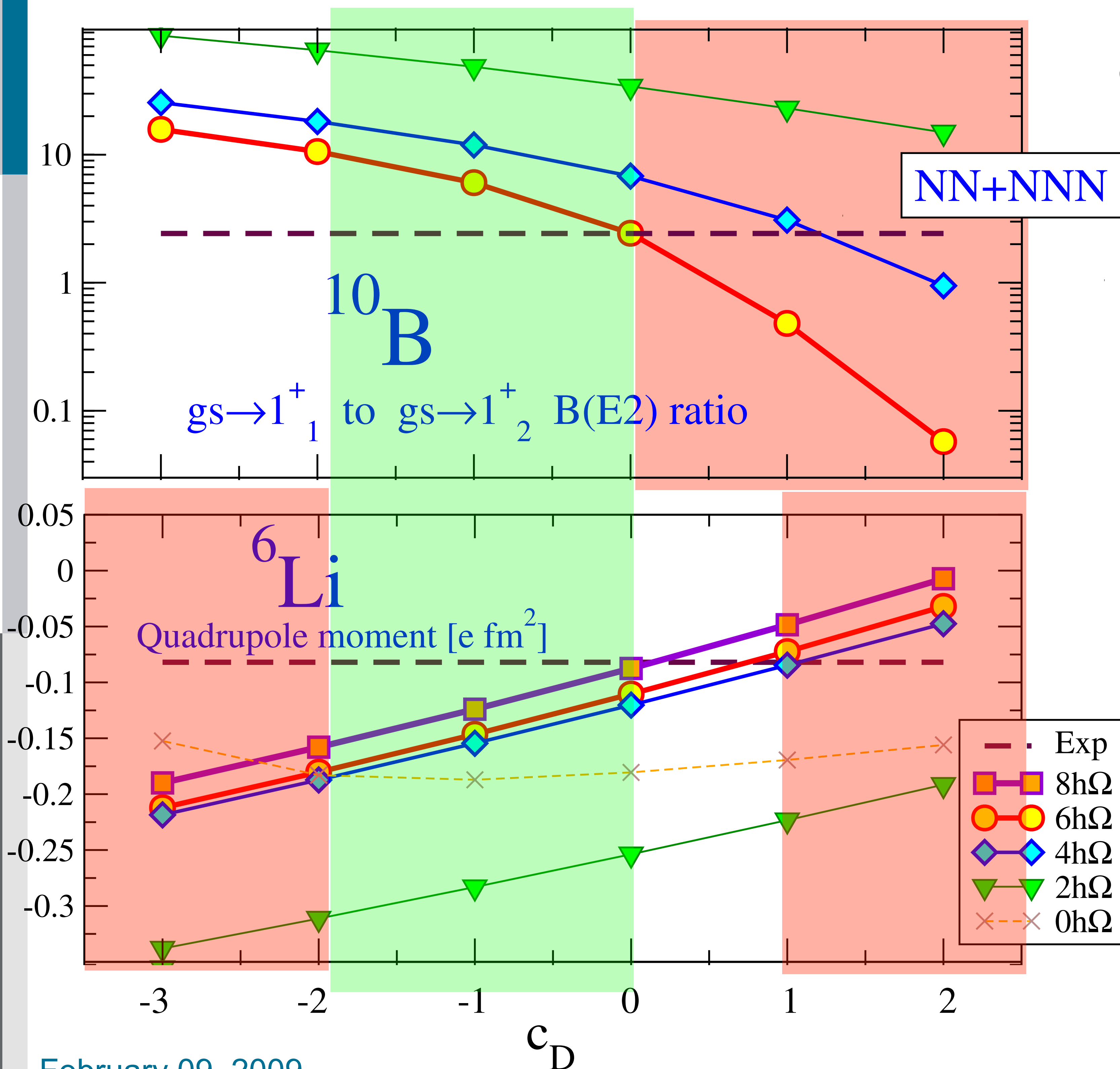
Summary:

Expectation values of chiral NN and 3NF's are consistent with the power counting

Especially for N3LO, the cutoff values seem to be large enough to predict binding energies

→ Ready to explore p-shell nuclei to identify possible sensitivities to 3NF's

p-shell nuclei (NCSM)



Survey of A=6,10-13 nuclei revealed a **few** observables that are sensitive to c_D/c_E
(Navrátil et al., PRL (2007))

- ^{3}H described for all c_D

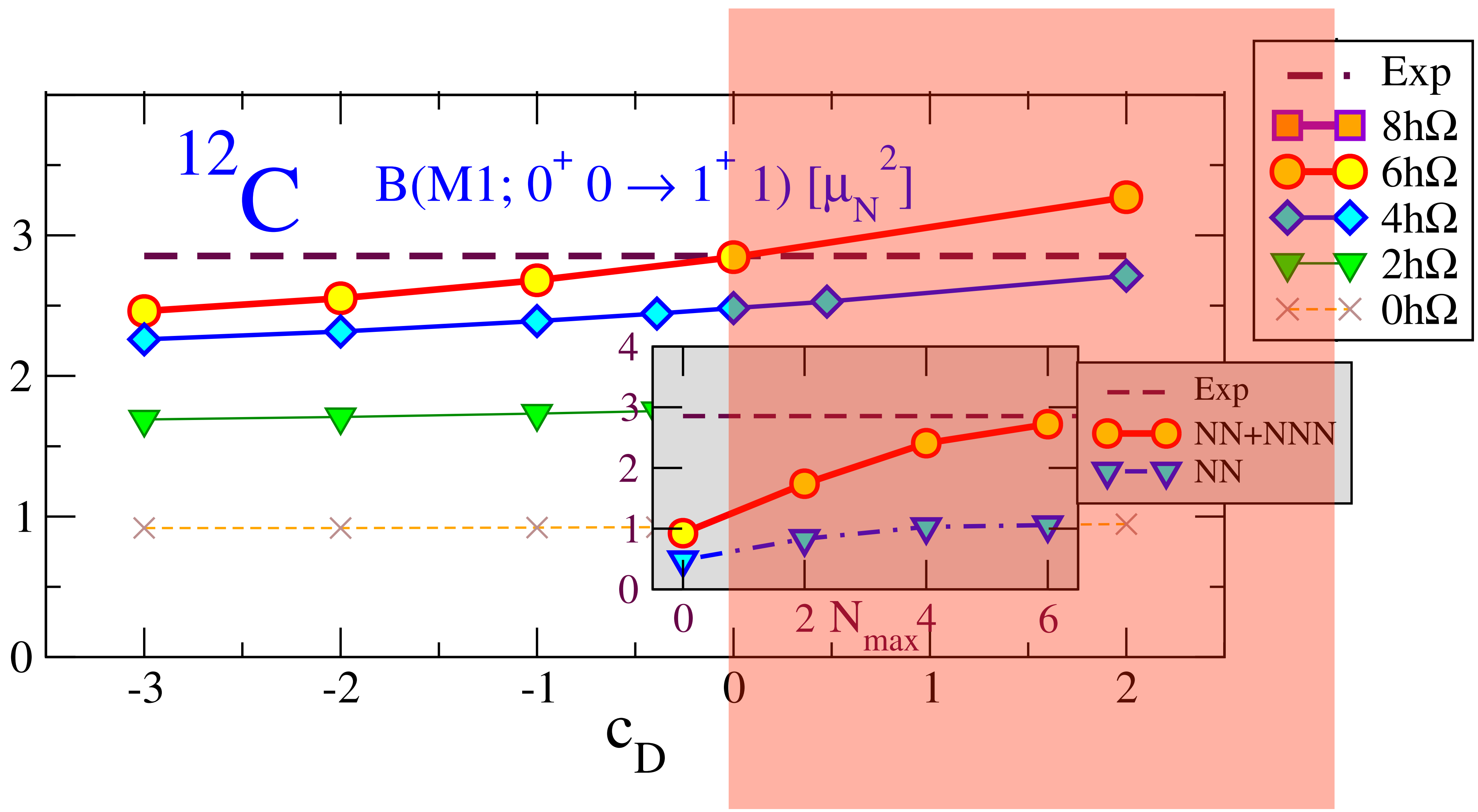
- green area accommodates B(E2,^{10}B) and Q(^{6}Li)

- other observables are either insensitive to variation of c_D or are consistently described

(Note: these calculations have been regulated restricting the momentum transfer!)

B(M1,0+0 → 1+1) sensitivity

- B(M1,0+0 → 1+1) in ^{12}C is sensitive to the 3NF (Hayes et al. PRL, 2003) and specifically to the choice of c_D



^{10}B & ^{13}C spectra

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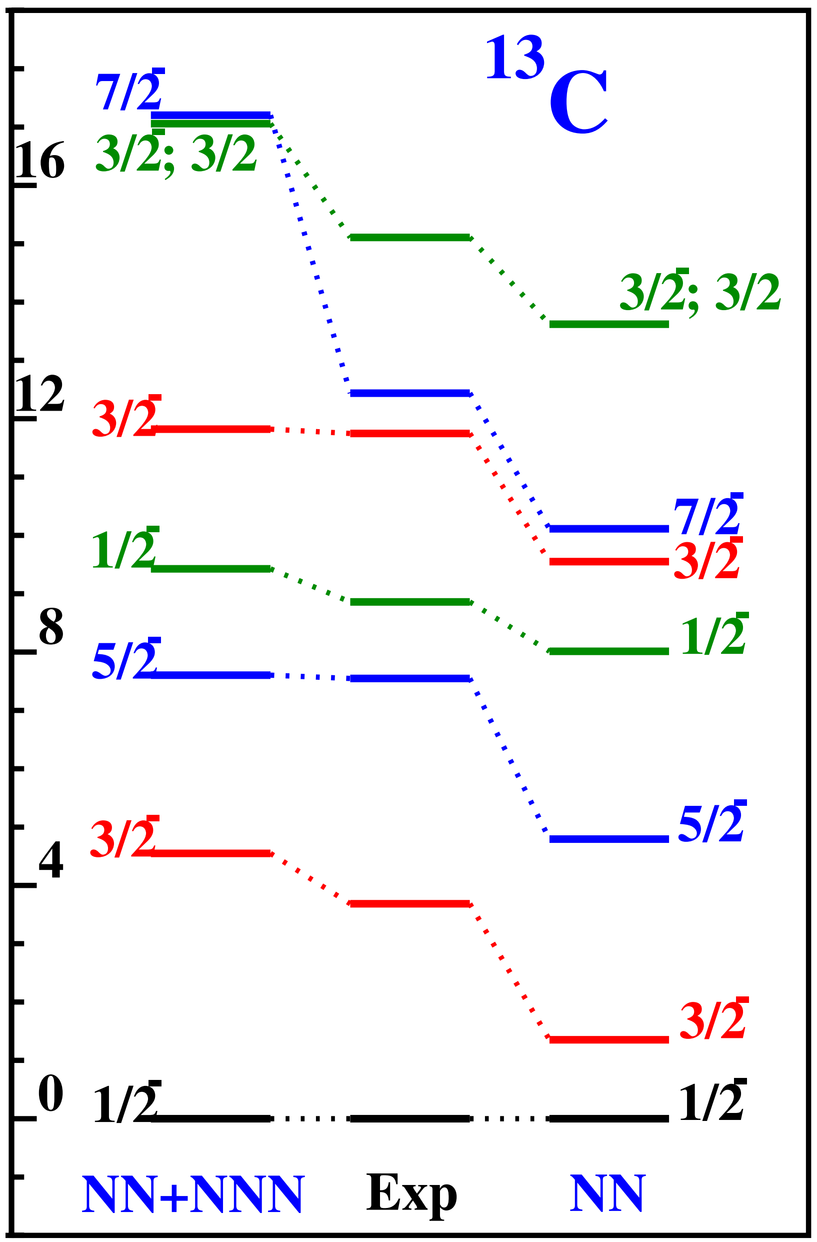
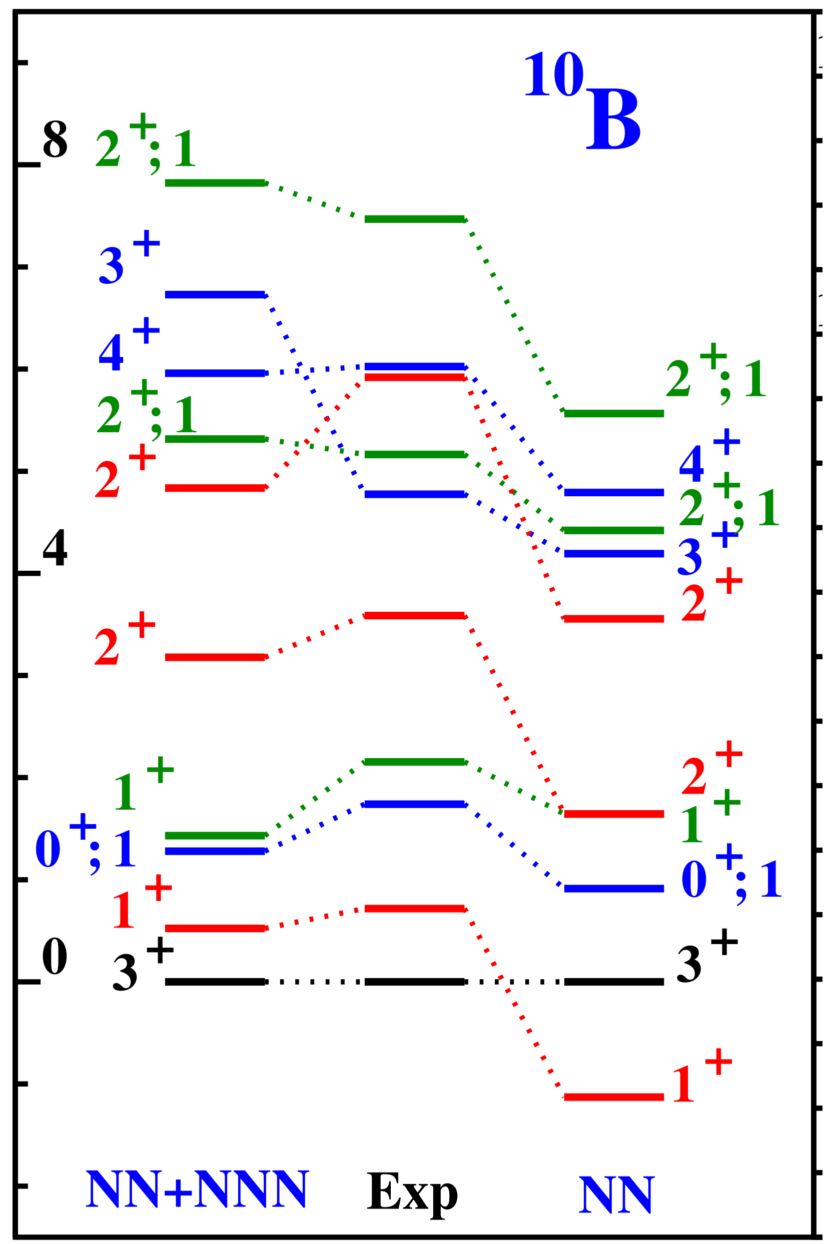
Choose c_D =-1 and obtain spectra and their sensitivity on the 3NF ....





(Navrátil et al., PRL (2007))

- Clear improvement of description compared to experiment.
- Some corrections are too strong
- Binding energies are OK (within our accuracy)

Nuclear structure with soft-interactions

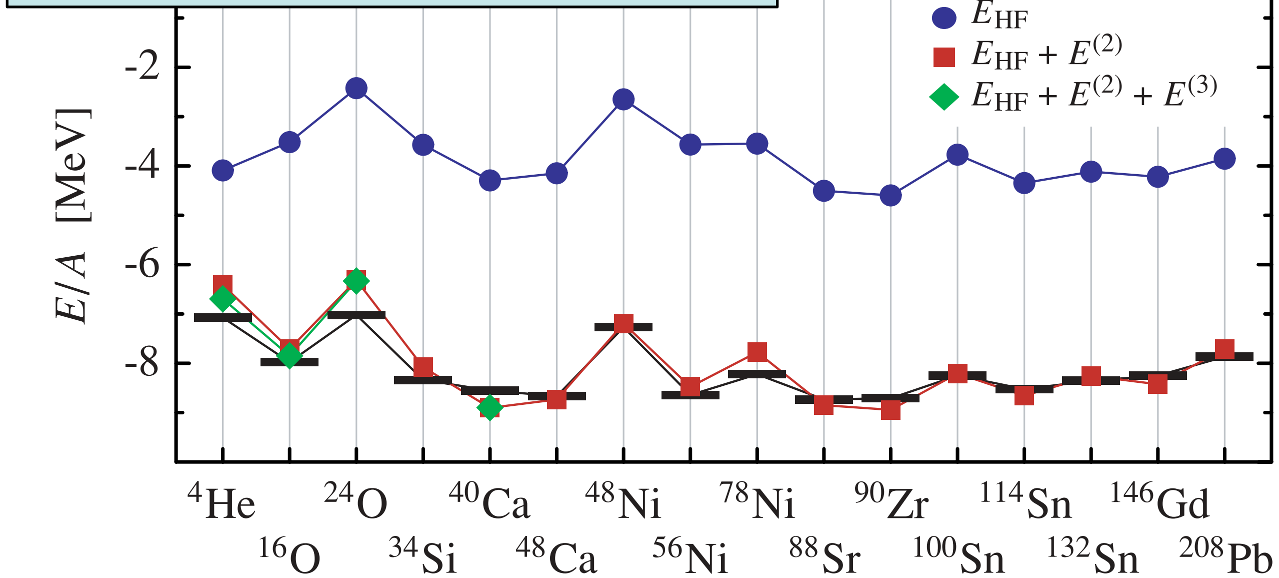
E.g. UCOM-approach (Roth, Feldmeier, Neff, Hergert, ...)

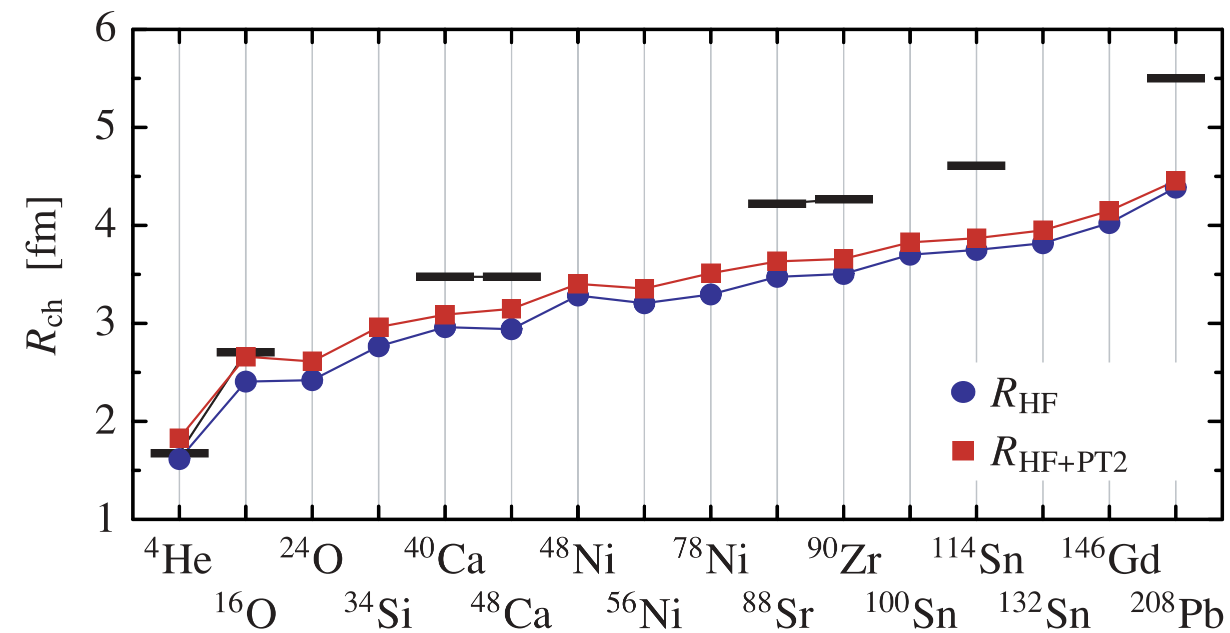
Calculations for complex nuclei can been performed based on soft interactions (although softer than typical chiral forces) !

Are soft interactions physically sensible?

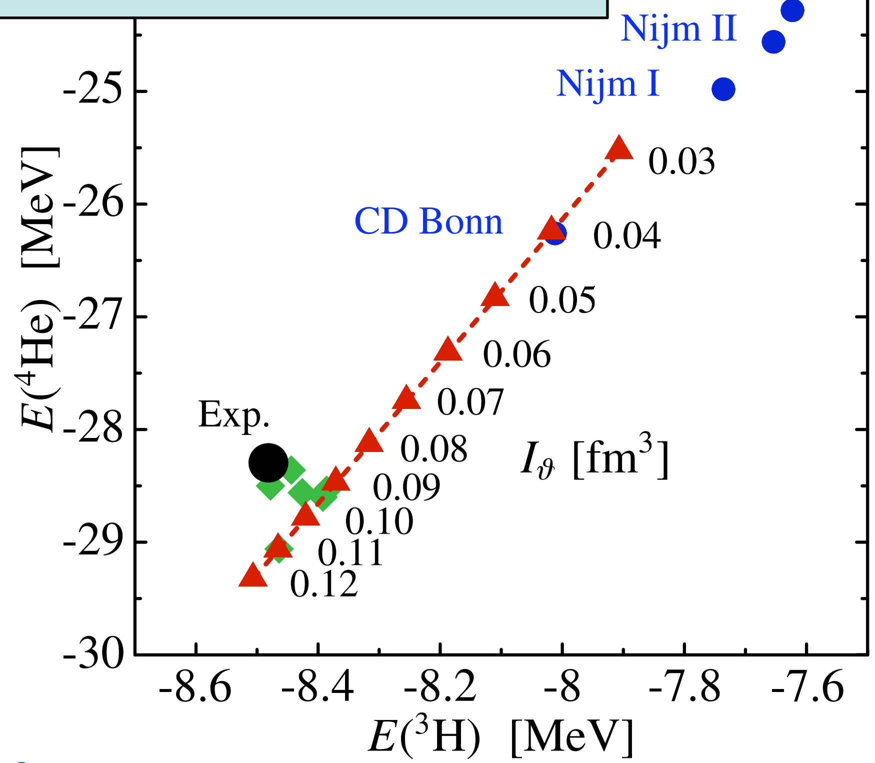
3NF contribution?

Roth et al. PRC (2006)



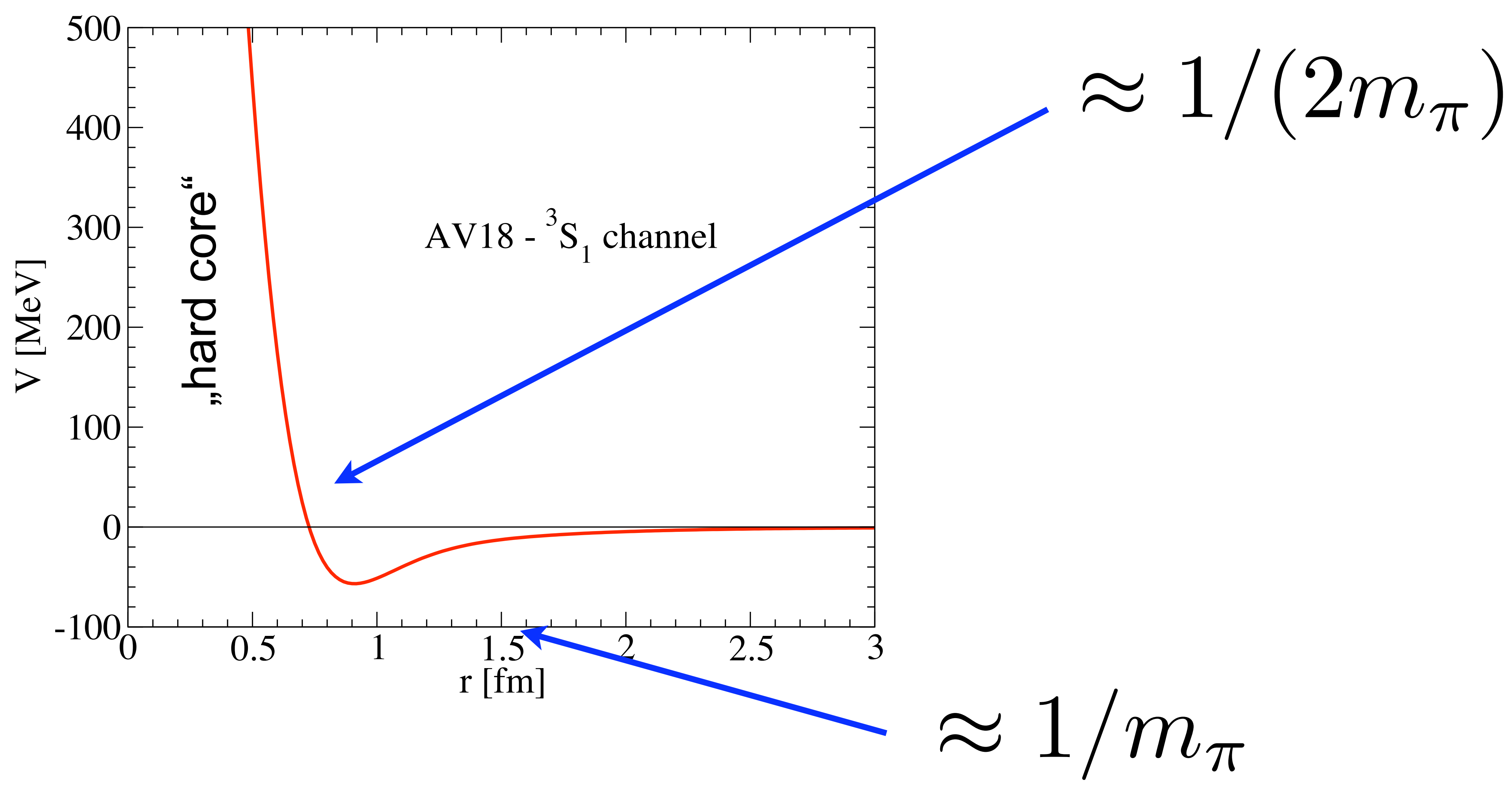


Roth et al., NPA (2007)



Typical phenomenological NN force

E.g. for a typical NN interaction: AV18 in the $^3S_1$ channel



Long range part is given by the one-pion exchange

2π-exchange is usually not included, instead there is the repulsive core !

Repulsive core induces high momentum components in the Fourier-transformed potential

Low momentum nuclear interactions (Vlowk)

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Systematical reduction of high momentum components (removal of hard core)

- Models have been adjusted only to low energy data (below 350 MeV lab energy)
- wave functions at high momenta (> 300 MeV/c) are strongly model dependent
- high momentum components are difficult to handle in many-nucleon systems

Idea of vlowk:

RG equation to decouple low and high momentum components

Description of data at low momenta is exactly preserved

RG for NN forces

Lippmann-Schwinger equation for finite cutoff Λ

$$K(p,p') = V_\Lambda(p,p') + \int_0^\Lambda dp'' \; p''^2 \; \frac{m V_\Lambda(p,p'') \, K(p'',p')}{p'^2 - p''^2}$$

Start with large Λ and decrease infinitesimally keeping "half-off-shell" K-matrix invariant

→ Differential equation for potential depending on Λ

$$\frac{d}{d\Lambda} \, V_\Lambda(p,p') \equiv \beta(V_\Lambda,\Lambda) = -\frac{m\Lambda^2}{p'^2 - \Lambda^2} \, V_\Lambda(p,\Lambda) K(\Lambda,p')$$

Hermitization required, but possible

→ low momentum interaction (vlowk) for a specific starting interaction

NN data up to p=Λ are described exactly

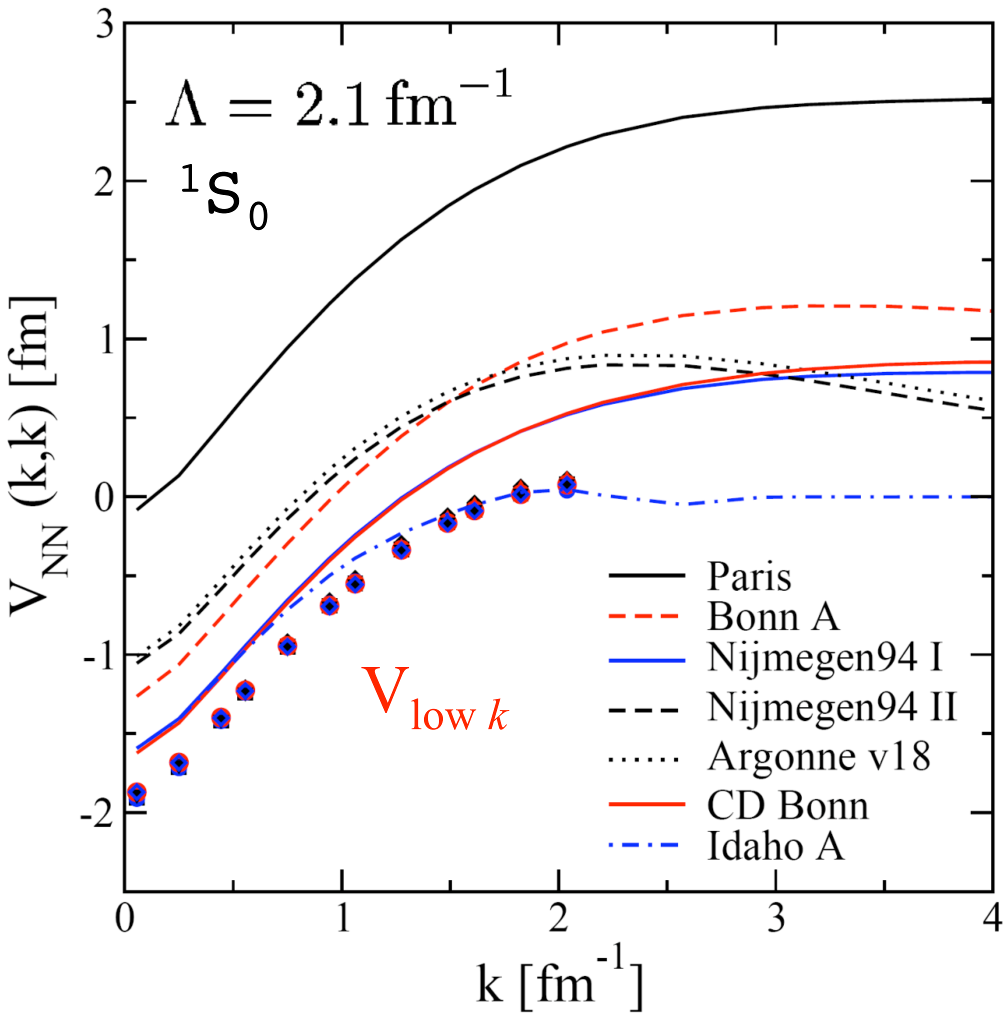
(Bogner, Kuo, Schwenk, 2003)

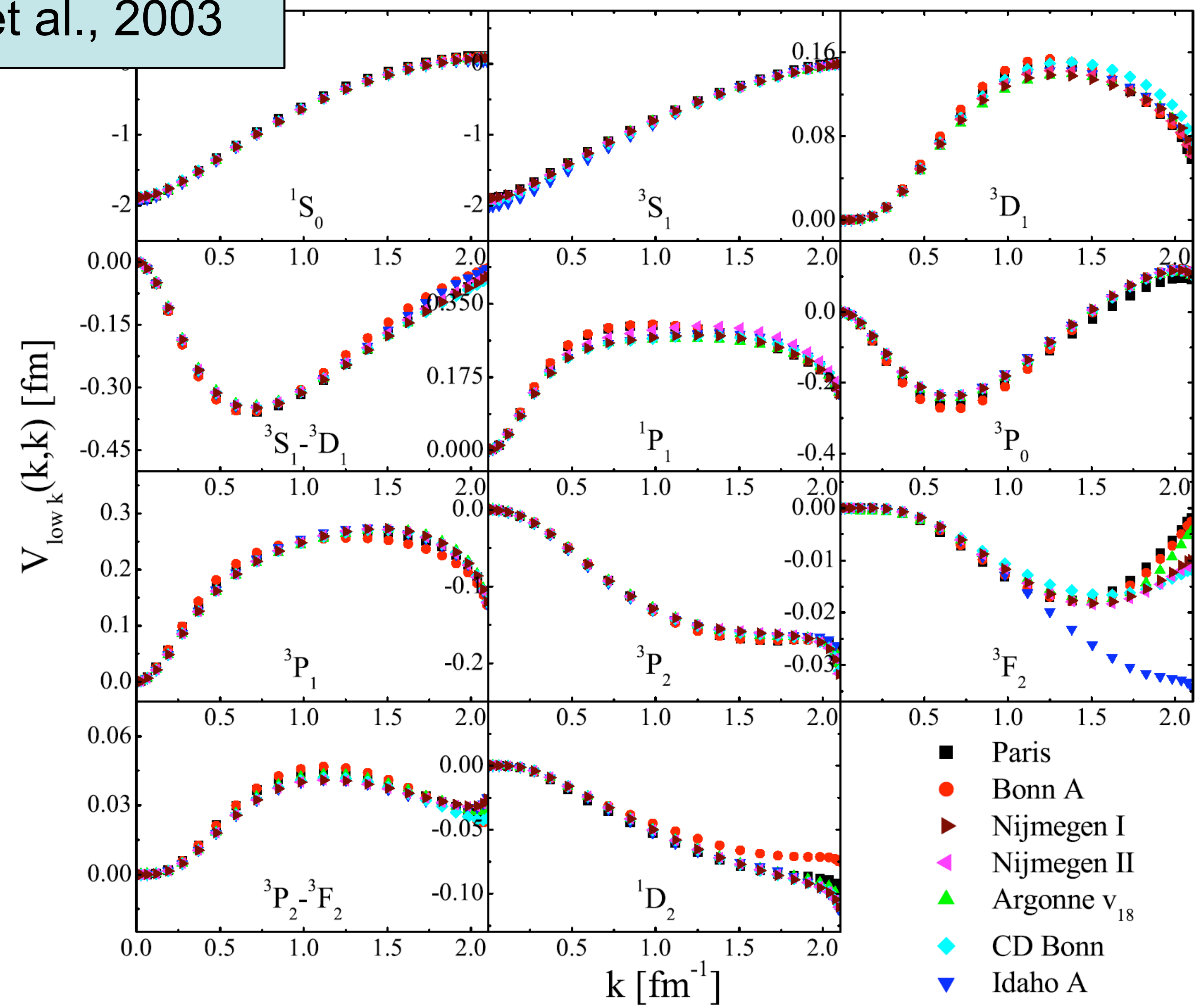
Vlowk is model independent

one observes for small cutoffs (< 400 MeV):
Vlowk is independent of starting model!

agrees with Vlowk of chiral NN interaction → connection to chiral interactions

Bogner et al., 2003



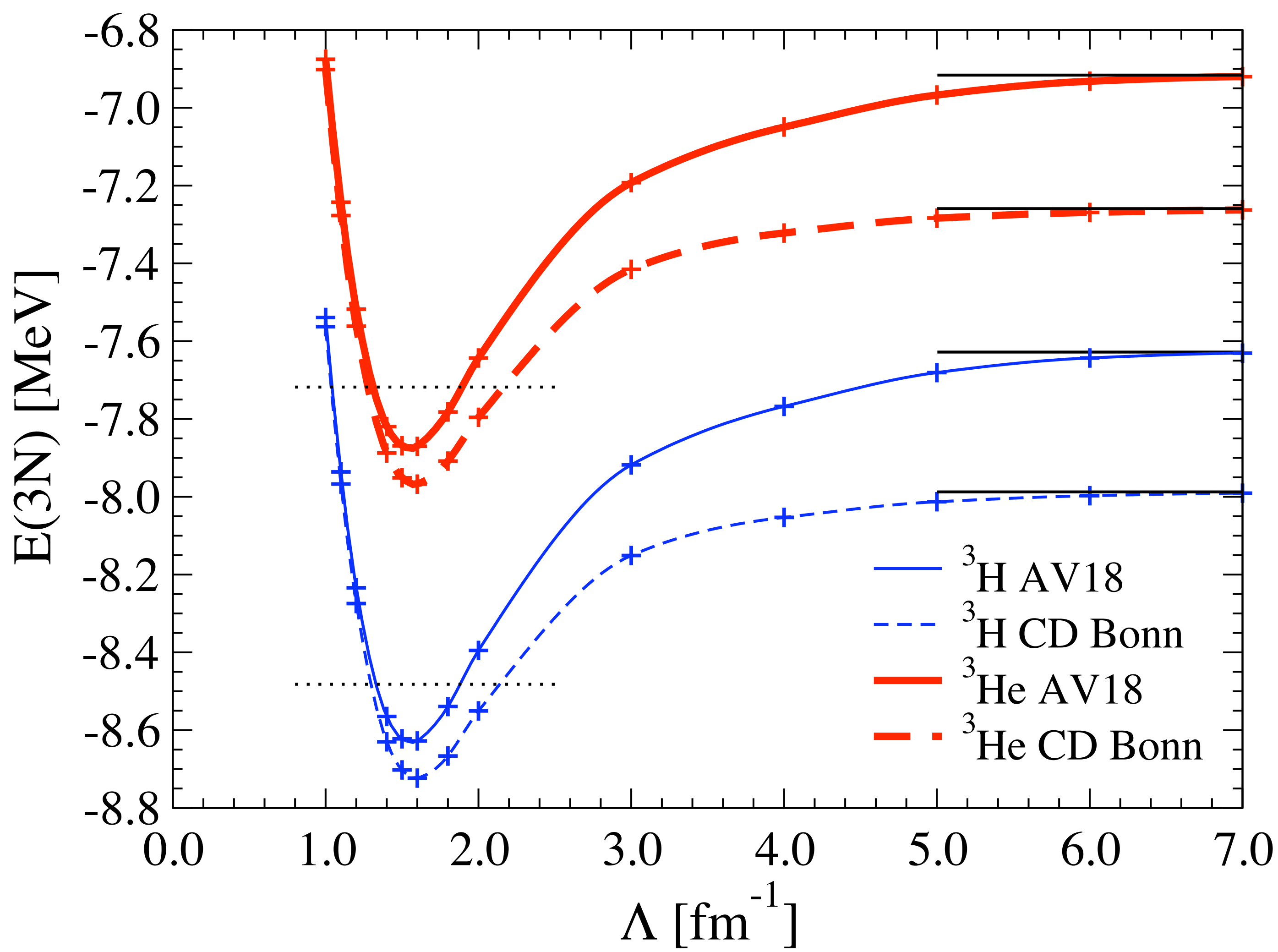


Vlowk for A=3

Λ dependence of ^{3}H binding energy is of the same order of magitude as 3NF contributions!

→ no indication that there are unnaturally large induced 3N interactions

this is even true for Λ < 300 MeV/c !



AN, Bogner, Schwenk (2004)

Also ^{4}He binding energies support this conclusion

Vlowk and chiral potentials

How to define a consistent 3NF?

Make use of model independence of vlowk:

For small cutoffs, vlowk is very similar for all models and the chiral interactions

→ chiral 3NF should be consistent to vlowk (at least for small enough Λ)

Vlowk(Λ) + chiral 3NF (adjusted to 3N and 4N observables)

Adjustment procedure is the same as for purely chiral interactions

Combination can be applied to more complex systems

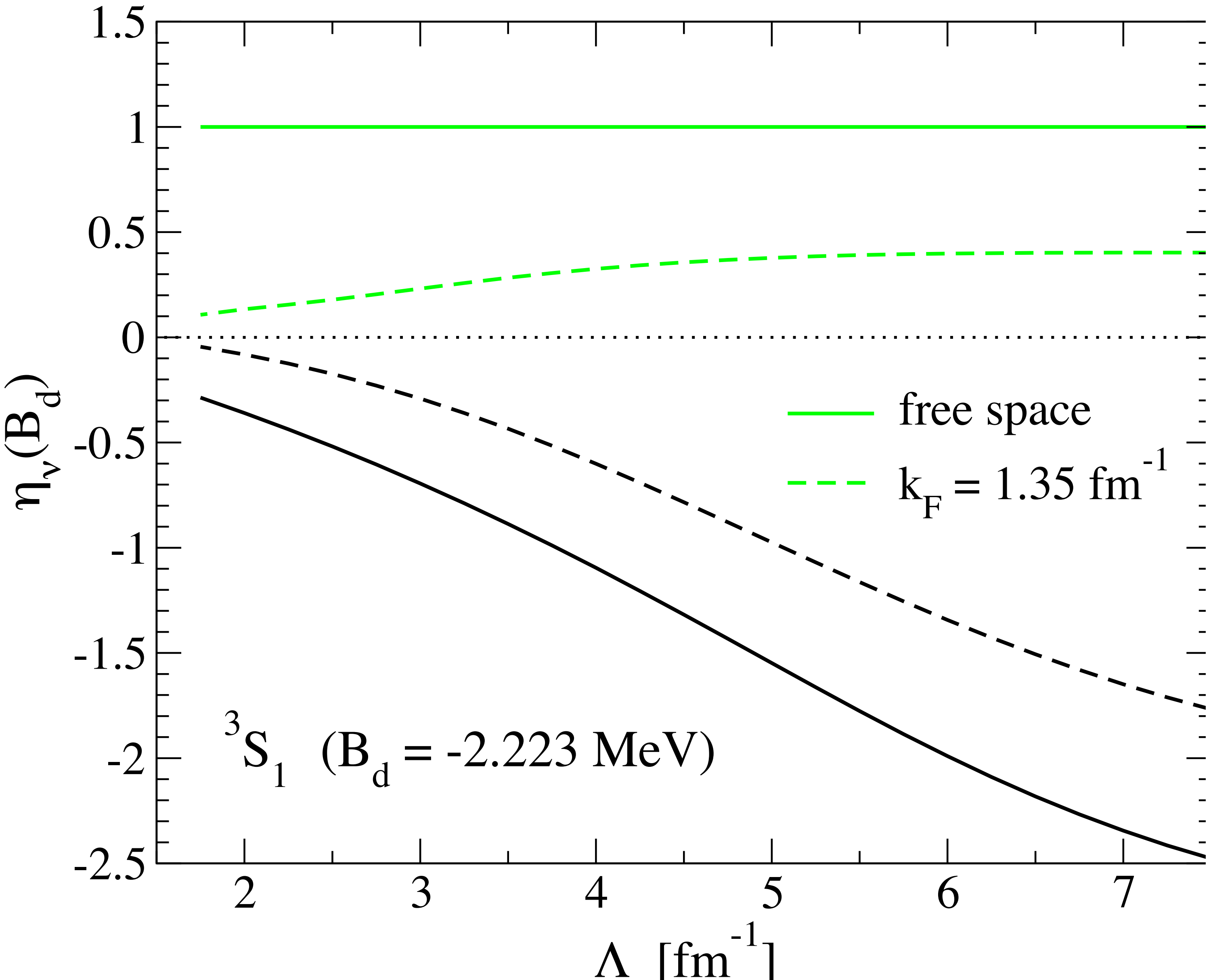
- Λ dependence? → Tool to get idea of possible uncertainties
- 3NF contribution? → Size at higher densities?

What is gained?

Perturbativity of NN interaction:

Many-body perturbation theory works for low momentum interactions (like for UCOM) !

Can be quantified: Weinberg eigenvalues of $G_0(E)\,V$

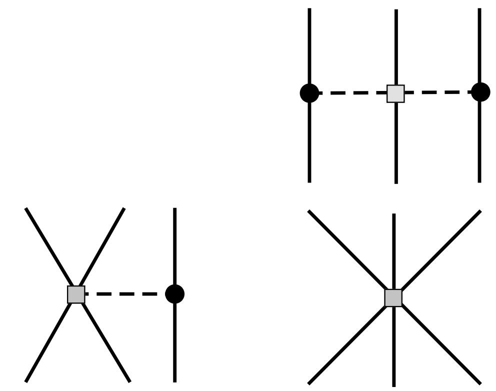


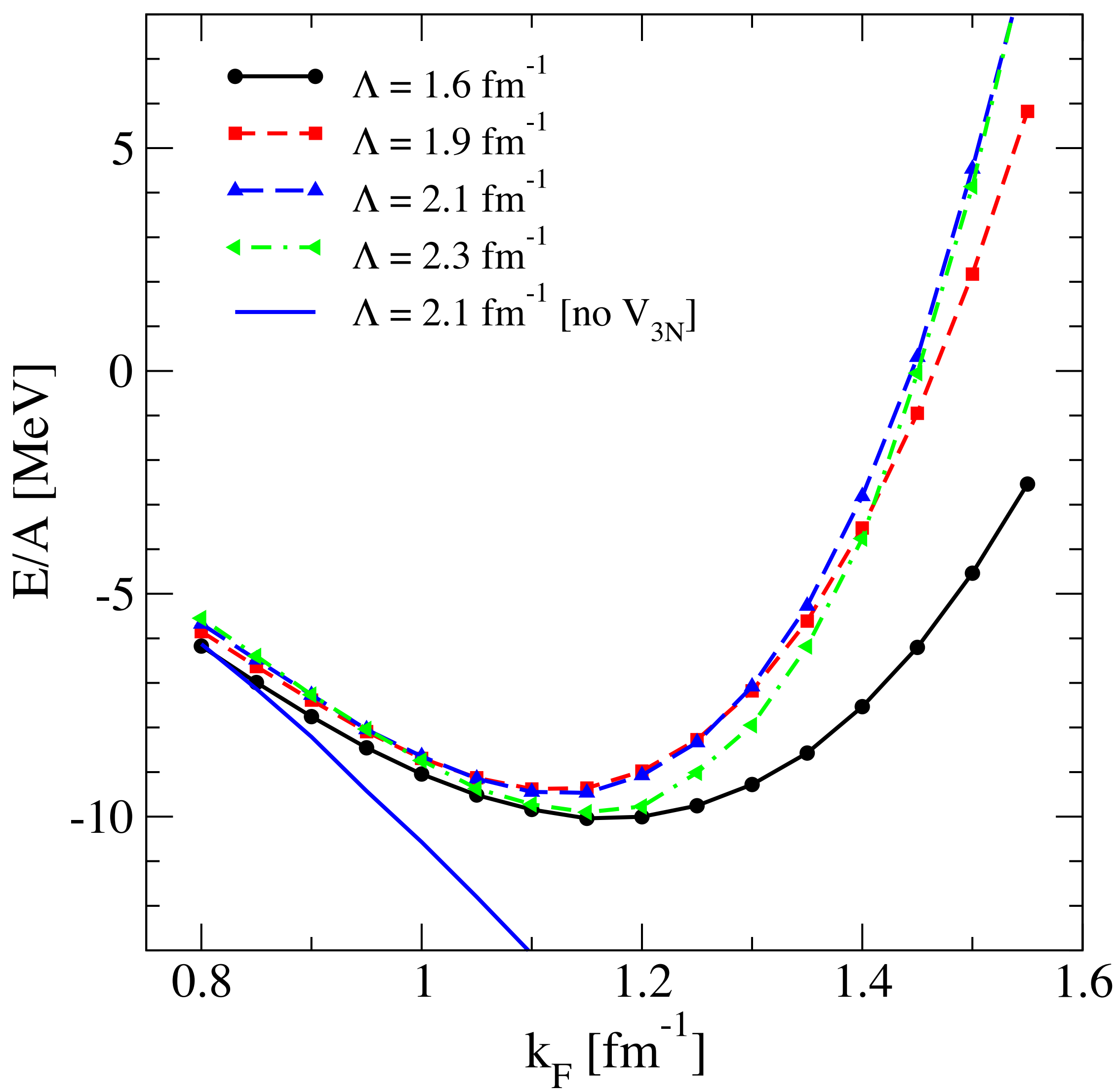
Bogner, Schwenk, Furnstahl, AN (2005)

Application to nuclear matter

non-complete study up to second order in perturbation theory

- Λ independence up to saturation density
- convergence of Goldstone-expansion
- 3NF contribution is important for saturation
- natural size of 3NF contribution





k_F	Λ	T	$V_{\mathrm{low}\,k}$	V_c	V_D	V_E
1.0	1.6	15.50	−26.58	1.49	0.34	−0.29
	1.9	16.29	−26.81	0.85	−0.09	0.55
	2.1	16.92	−27.04	0.11	0.05	0.79
	2.3	17.60	−27.27	−0.89	0.43	0.85
1.2	1.6	20.86	−37.66	4.59	1.03	−0.65
	1.9	21.80	−37.38	3.99	−0.50	1.28
	2.1	22.87	−37.53	2.27	−0.37	1.82
	2.3	24.32	−37.95	−0.38	0.51	1.78
1.35	1.6	26.09	−47.85	8.73	1.96	−1.12
	1.9	26.75	−46.72	9.14	−1.16	2.24
	2.1	28.05	−46.47	6.99	−1.33	3.22
	2.3	30.06	−46.45	3.10	−0.35	3.26

Conclusions

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- Chiral perturbation theory leads to consistent NN & 3N forces
  3N forces are an important part of nuclear forces

- Results for A=3 and A=4 indicate that chiral nuclear forces of high order (N3LO) give predictions for nuclear binding energies that are **cutoff independent and therefore useful**

- Predictions for p-shell nuclei lead to realistic binding energies and spectra
  3N forces have impact on the results

- Low momentum interactions are related to chiral forces

- 3NF's can be consistently added

- many body calculations are possible

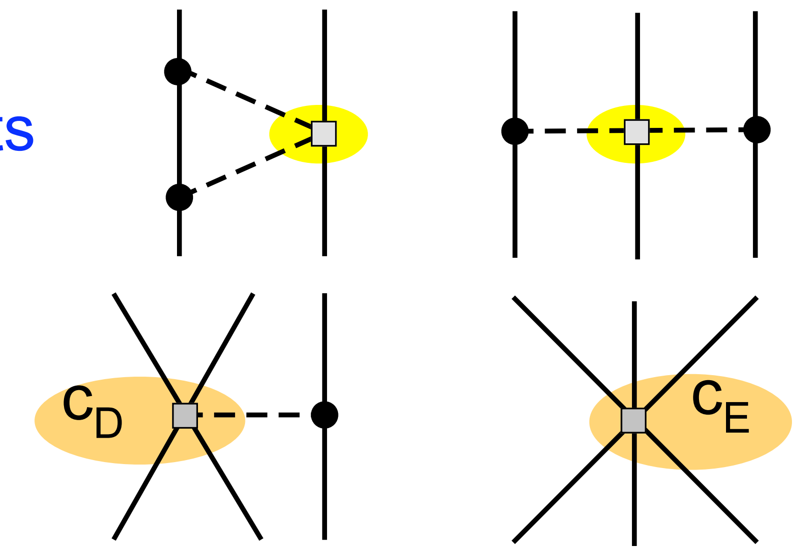
  - *e.g. reasonable description of nuclear matter*

- Λ dependence is a tool to identify missing contributions

Questions & Outlook

Can we improve the convergence of the chiral expansion in the NN sector ?

- importance of counter terms & cutoff dependence
- inclusion of Δ's in EFT
- independent constraint of the ππNN coupling constants



Chiral & vlowk few-body calculations to

- understand sensitivity of observables on c_i
- independent determinations of c_D/c_E to confirm consistency
- more quantitative relation of chiral forces and vlowk/UCOM
- external probes using ChPT

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