Overview on Effective Interactions for Nuclear Structure

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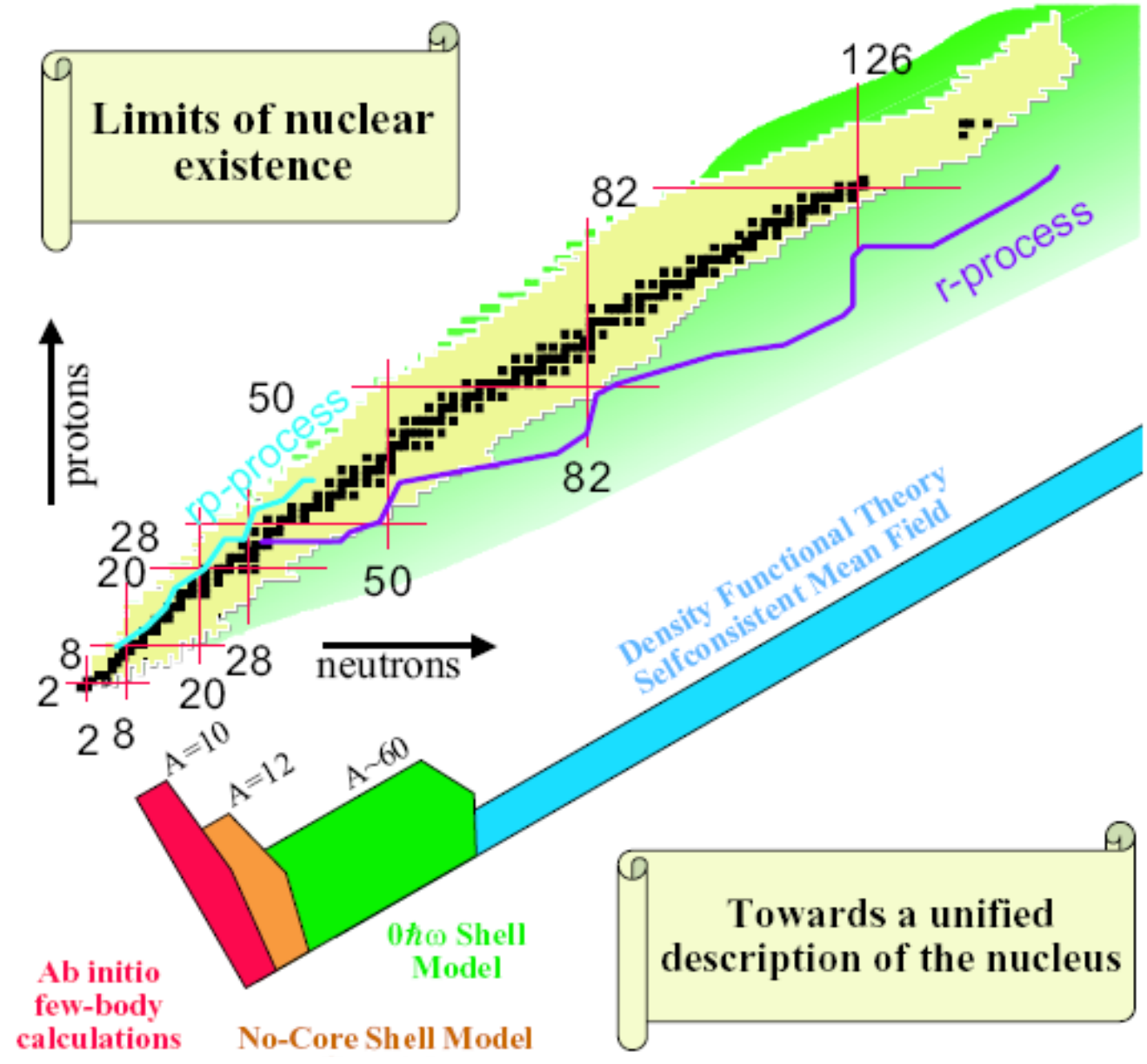
MICROSCOPIC NUCLEAR-STRUCTURE THEORY

1. Start with the bare interactions among the nucleons

2. Calculate nuclear properties using nuclear many-body theory


Limits of nuclear existence
126
82
82
50
50
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20
8
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2 8
protons
neutrons
rp-process
r-process
Density Functional Theory
Selfconsistent Mean Field
A=10
A=12
A~60
0ħω Shell Model
Ab initio few-body calculations
No-Core Shell Model
Towards a unified description of the nucleus

$$H\Psi = E\Psi$$

We cannot, in general, solve the full problem in the complete Hilbert space, so we must truncate to a finite model space

$\Longrightarrow$ We must use effective interactions and operators!

Some current shell-model references

1. E. Caurier, G. Martinez-Pinedo, F. Nowacki, A. Poves, and A. P. Zuker, "The Shell Model as a Unified View of Nuclear Structure," *Reviews of Modern Physics* **77**, 427 (2005)

2. B. A. Brown, "The Nuclear Shell Model towards the Drip Lines," *Progress in Particle and Nuclear Physics* **47**, 517 (2001)

3. I. Talmi, "Fifty Years of the Shell Model-The Quest for the Effective Interaction," *Advances in Nuclear Physics*, Vol. **27**, ed. J. W. Negele and E. Vogt (Plenum, NY, 2003)

4. B. R. B., "Effective Operators in Shell-Model Calculations," 10th Indian Summer School of Nuclear Physics:Theory of Many-Fermion Systems, *Czechoslovak Journal of Physics* **49**, 1 (1999)

5. 20th Chris Engelbrecht Summer School in Theoretical Physics, January 19-28, 2009 http://academic.sun.ac.za/somerskool/2009.html

III. EFFECTIVE INTERACTIONS

Except for approaches, such as the QMC, one usually needs to renormalize the N-N interaction:

1. **for the strong short-range correlations in the N-N interaction**

and

2. **for the truncation of the Hilbert space.**

There are two basic approaches to obtaining effective interactions:

1. **PHENOMENOLOGICAL**

2. **MICROSCOPIC**

General Comments on the Effective Interaction

Many-body Schrödinger Equation (e.g., A nucleons)

$$H\Psi_\alpha = E_\alpha \Psi_\alpha \quad \text{in full Hilbert space } \mathcal{S}$$

Truncate to smaller Hilbert space $\mathcal{S}$ of dimension d

Effective Hamiltonian $\mathcal{H}$ in $\mathcal{S}$

$$\mathcal{H}\Phi_\beta = E_\beta \Phi_\beta \quad , \quad \Phi_\beta = P\Psi_\beta \text{ into } \mathcal{S}$$

The projections Φ_β are, in general, not orthogonal $\Rightarrow$ construct the biorthogonals $\tilde{\Phi}_\gamma$, i.e., $\langle \tilde{\Phi}_\gamma | \Phi_\beta \rangle = \delta_{\gamma\beta}$

$\therefore$ $\mathcal{H}$ always exists $\Rightarrow$ $\mathcal{H} = \sum_{\beta\in\mathcal{S}} |\Phi_\beta\rangle E_\beta \langle \tilde{\Phi}_\beta|$ $\Rightarrow$ non-Hermitian

Kirson*

The question is <u>not</u> whether $\mathcal{H}$ exists (it always does), but whether it has a <u>simple</u> enough form, so as to be <u>useful</u>.

*in "Nuclear Shell Models," M. Vallieres and B.H. Wildenthal, ed., (World Scientific, Singapore, 1985) p. 290.

Effective Hamiltonian for NCSM

Solving $H^{\Omega}_{A,\,a=2}\,\Psi_{a=2} = E^{\Omega}_{A,\,a=2}\,\Psi_{a=2}$

in "infinite space" 2n+l = 450 relative coordinates

P + Q = 1; P – model space; Q – excluded space;

$$E^{\Omega}_{A,2} = U_2 H^{\Omega}_{A,2} U_2^{\dagger} \qquad U_2 = \begin{pmatrix} U_{2,P} & U_{2,PQ} \\ U_{2,QP} & U_{2,Q} \end{pmatrix} \qquad E^{\Omega}_{A,2} = \begin{pmatrix} E^{\Omega}_{A,2,P} & 0 \\ 0 & E^{\Omega}_{A,2,Q} \end{pmatrix}$$

$$H^{N_{\max},\Omega,\mathrm{eff}}_{A,2} = \frac{U^{\dagger}_{2,P}}{\sqrt{U^{\dagger}_{2,P} U_{2,P}}} E^{\Omega}_{A,2,P} \frac{U_{2,P}}{\sqrt{U^{\dagger}_{2,P} U_{2,P}}}$$

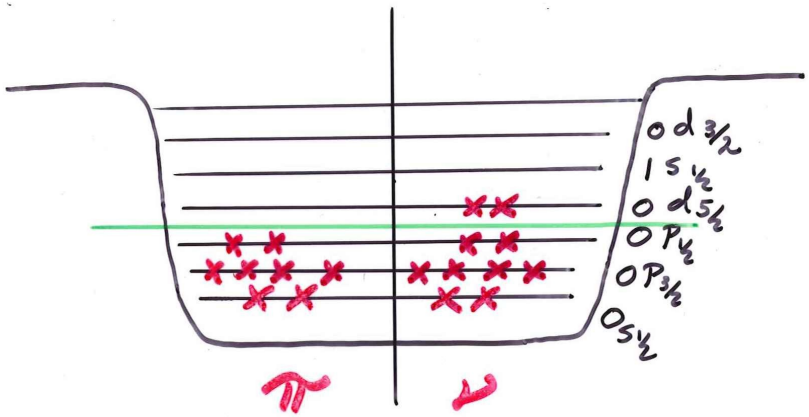
Two ways of convergence:

1) For $P \to 1$ and fixed a: $H^{\mathrm{eff}}_{A,a=2} \to H_A$

2) For $a \to A$ and fixed P: $H^{\mathrm{eff}}_{A,a} \to H_A$

Standard Example:

^{18}O S.M. $\Longrightarrow$ $\times n$ $\times n$ ^{16}O



$$\Psi^{J^\pi T}(^{18}O) = \Phi^{0^+0}(^{16}O) \times \Phi^{J^\pi T}(2\nu)$$

$$\Phi^{J^\pi T}(j^2) = \sum_{m_1 m_2} (j m_1 j m_2 | J M) \sum_{\tau_1 \tau_2} (\tfrac{1}{2}\tau_1 \tfrac{1}{2}\tau_2 | T M) \phi_{j m_1 \frac{1}{2}\tau_1}(1) \phi_{j m_2 \frac{1}{2}\tau_2}(2)$$

$$P_{1\leftrightarrow 2}\, \Phi^{J^\pi T} = -\Phi^{J^\pi T} = (-1)^{2j-J}(-1)^{2\times\frac{1}{2}-T}\, \Phi^{J^\pi T}$$

$\Rightarrow J+T =$ odd integer

For 2 neutrons $T=1 \Rightarrow J =$ even integer

PHENOMENOLOGICAL EFFECTIVE INTERACTIONS

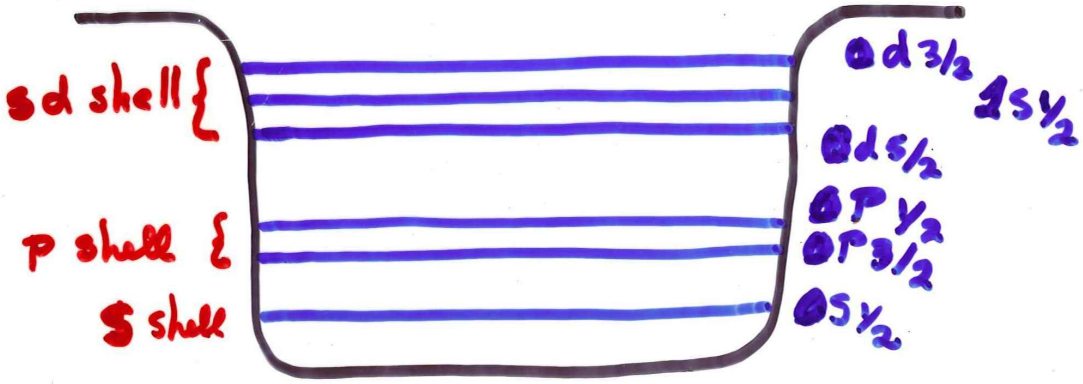
1. Usually constructed for a single major shell

2. Take experimental single-particle energies

3. Determine two-body matrix elements

$$\langle (j_1 j_2) J T | V | (j_3 j_4) J T \rangle$$

by a least-squares fit to some subset of the experimental data



EXAMPLES OF PHENOMENOLOGICAL EFFECTIVE INTERACTIONS

RESTRICTED:

1. p-shell: Cohen-Kurath (CKI) 1965
2. sd-shell: Universal SD (USD): Wildenthal, et al. 1984
3. pf-shell: FPD6; Brown, Richter, et al. 1990
4. pf-shell: Universal PF (UPF): Honma, Otsuka, Brown and Mizusaki, PRC 65, R061301 (2002)
5. 1hw s-p-d model space: Millener-Kurath 1975

GLOBAL:

Skyrme, Gogny, M3Y

HYBRID:

Universal SM Interaction; Duflo-Zuker 1996

CONCEALED CONFIGURATION MIXING AND THE SPECTROSCOPY OF THE PSEUDONIUM NUCLEI *

S. COHEN and R.D. LAWSON
Argonne National Laboratory, Argonne, Illinois

and

J.M. SOPER **
Atomic Energy Research Establishment, Harwell, Berkshire

Received 1 April 1966

The calculation shows that many nuclear properties are extremely insensitive to configuration mixing arising from the excitation of zero-coupled pairs out of a closed shell.

The nuclear shell model has been very successful in predicting and correlating information on low-lying states of nuclei. Not only has it been successful in predicting binding energies of nuclei relative to closed shells, but also it has been able to interpret the excitation energies of low-lying nuclear states, to predict selection rules for beta and gamma decay, and to provide an understanding of certain static multipole moments of nuclei. If the consequences of assuming a simple, pure configuration are quantitatively borne out by experiment, it is tempting to say that this confirms the hypothesis that the shell is fairly pure. We show in this note that this conclusion is not warranted; this has consequences both for the shell model itself and for such things as the shell-model effective interaction.

In this paper we consider an exactly soluble problem. We take a model in which only neutrons are filling the $1d_{\frac{3}{2}}$ and $1f_{\frac{7}{2}}$ single-particle levels, which are assumed isolated from all other single-particle states. We take the residual neutron-neutron interaction to be

$$V = (V_S P_S + V_T P_T) \frac{\exp(-r_{12}/a)}{r_{12}/a} , \qquad (1)$$

where P_S and P_T are spin-singlet and spin-triplet projection operators and V_S and V_T are the corresponding potential strengths (taken as -30 MeV and -10MeV, respectively). In evaluating matrix elements of the residual two-body force, the single-particle eigenfunctions used are the harmonic-oscillator functions $\varphi \propto \exp -\frac{1}{2}(r/b)^2$. The range parameter that comes into the calculation is

$$\lambda = a/\sqrt{2}b = 0.665.$$

The Hamiltonian of this model can be set up and diagonalized exactly (numerically) †. We have done this for all values of J and all neutron numbers N between two and twelve. The amount of configuration mixing between the two single-particle levels depends, of course, on the spacing be-

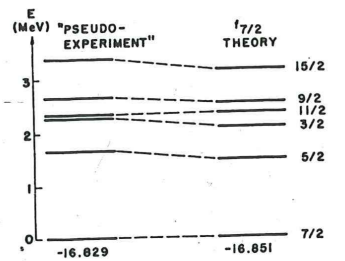


Fig. 1. Energy-level diagram of ^{43}Ps. "Pseudo-experiment" is the result of the exact calculation and "theory" is the least-squares fit using the pure $f_{\frac{7}{2}}$ model for the levels. The pseudo-experimental binding energy of ^{43}Ps relative to ^{40}Ps is -16.829 MeV; the theory gives -16.851 MeV.

* Work performed at Argonne National Laboratory under the auspices of the U.S. Atomic Energy Commission and the United Kingdom Atomic Energy Authority.

** On leave of absence at Argonne National Laboratory.

† This was done with the aid of the Argonne shell-model programs developed in collaboration with D. Kurath, M.H. Macfarlane and M. Soga.

40Ps g.s. is only ~9% $(1d_{3/2})^4$

MICROSCOPIC EFFECTIVE INTERACTIONS

BASIC FORMALISM:

1. Bloch and Horowitz, Nucl. Phys. **8**, 91 (1958)

2. Bertsch, Nucl. Phys. **74**, 234 (1965).

3. Brandow, Rev. Mod. Phys. **39**, 771 (1967)

4. Johnson and Baranger, Ann. Phys. (NY) **62**. 172 (1971)

5. Suzuki and Lee, Prog. Theor. Phys. **64**, 2091 (1980)

6. Hjorth-Jensen, Kuo and Osnes, Phys. Repts. **261**, 125 (1995)

⋮

APPLICATION OF THE FORMALISM:

1. Arima and Horie, Prog. Theor. Phys. **12**, 623 (1954)

2. Dawson, Talmi and Walecka, Ann. Phys. **18**, 339 (1962)

3. Kuo and Brown, Nucl. Phys. **85**, 40 (1966)

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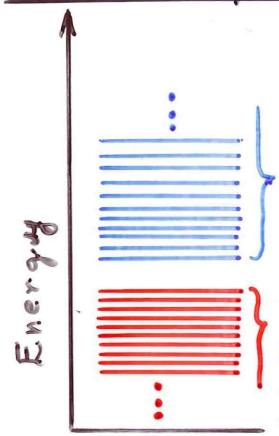
Time Independent Perturbation Theory Formalism

Bloch & Horowitz
Brandow



$P\Psi_\alpha = \Psi_\alpha$, dim. d

$$P + Q = 1$$
$$PQ = 0$$
$$P^2 = P$$
$$Q^2 = Q$$

H in S (infinite)	$\mathcal{H}$ in $\mathcal{S}$ (dimen. d)
$H\Psi_\alpha = E_\alpha \Psi_\alpha$	$\mathcal{H} P\Psi_\alpha = E_\alpha P\Psi_\alpha$
$H = \sum_{i=1}^{A} t_i + \sum_{i<j}^{A} v_{ij} = T + V$	$\mathcal{H} = H_0 + v_{eff}(E)$
$H = (T+U) + (V-U)$	$v_{eff} = P\bar{V}P + P\bar{V}Q\frac{1}{E-QHQ}Q\bar{V}P$
$H = H_0 + \bar{V}$	$E = E_0 + v_{eff}(E)$

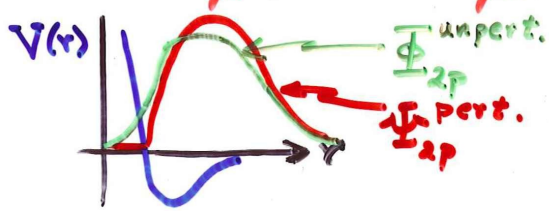
Note: In general, P, Q, $\mathcal{H}$ and $v_{eff}(E)$ are all A-nucleon operators.
E = one of the eigenenergies of the A-nucleon system = $E_0 + \Delta E$.

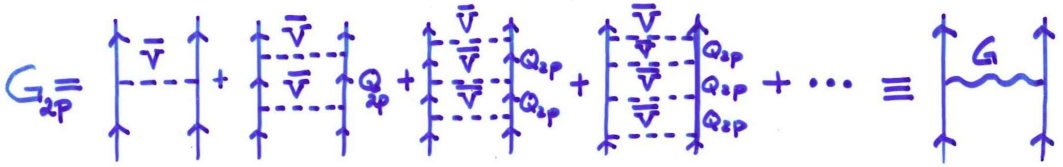
$$v_{eff}^A(E) = \bar{V} + \bar{V}\frac{Q}{E-H_0}v_{eff}^A(E) = \bar{V} + \bar{V}\frac{Q}{E_0-H_0}v_{eff}^A + \text{folded diagrams}$$

$$v_{eff}^A(E) \longrightarrow v_{eff}^2(E)$$

Replace V with G to treat strong, short-range correlations in V.

$$V\Psi_{2p}^{pert.} \equiv G_{2p}\Phi_{2p}^{unpert.}$$





$$G = \bar{V} + \bar{V}\frac{Q_{2p}}{\omega - H}\bar{V} = \bar{V} + \bar{V}\frac{Q_{2p}}{\omega - H_0}G(\omega)$$

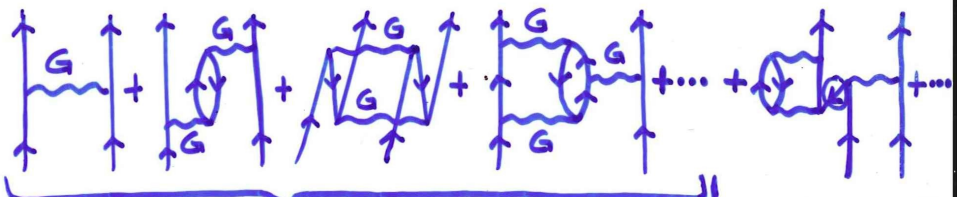
↑ starting energy

$$v_{eff}^{2b}(E) = G + G\frac{Q'}{E - H_0}v_{eff}(E), \text{ where } Q' = Q - Q_{2p}$$

or

$$v_{eff}^{2b}(E) = G + G\frac{Q'}{E_0 - H_0}v_{eff}(E_0) + \text{"folded diagrams"}$$

$v_{eff}^{2b}(E) =$



In general, the perturbation-theory expansion for $v_{eff}^{2b}(E)$ diverges due to <u>intruder states</u>. (Schucan and Weidenmüller)

Kuo and Brown, Nucl. Phys. 85, 40 (1966)

did is acceptable. For simplicity, we choose to use the perturbation method, although it would be worthwhile to look into this problem more carefully.

Another point which should be noticed is the spurious states. As is well-known, some components of the p-h excitation of the ^{16}O core merely correspond to the centre-of-mass motion of the core and therefore are spurious [1]). Hence we should prohibit them to enter into our calculation. But since only the $2\hbar\omega$ p-h excitations of the core enter into our calculation, the contamination of the spurious components in

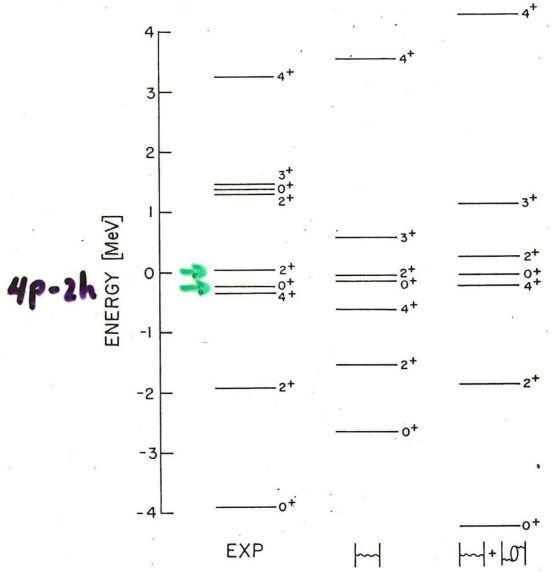


Fig. 13. The spectra of ^{18}O.

what we did is rather slight. The reason is that the $2\hbar\omega$ spurious state consists of predominantly two-particle-two-hole excitations of the core, as shown recently by Giraud [47]). It would be necessary, however, to handle this problem more carefully if the whole problem is to be treated more carefully instead of using the perturbation method as we have chosen.

4.3. SPECTRA OF ^{18}O AND ^{18}F

The spectra of ^{18}O and ^{18}F are obtained by diagonalizing $G\Omega_{3p1h}$ given by eqs. (4.10) and (4.12) in the s-d shell two-nucleon subspace. Results are shown in figs. 13

$\bar{V} + \bar{V}\frac{Q_{2P}}{\omega - H_0}G(\omega) \Rightarrow$ "pairing-like"

~~$\Rightarrow$~~ dominated by $L=2$ multipole $\Rightarrow$ quadrupole-like

Gatlinburg Conf. Sept 12-17, 1966

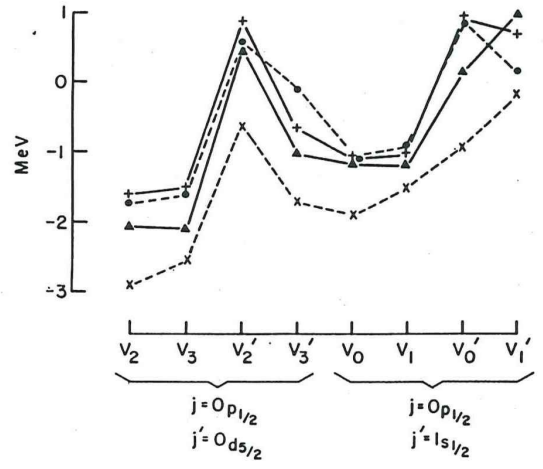


Fig. 8. Plotted matrix elements are $V'_J = (jj' J|G_{eff}|jj' J)_{E1}$; $V_J = \frac{1}{2}\{(jj' J|G_{eff}|jj' J)_{E0} + (jj' J|G_{eff}|jj' J)_{E1}\}$. (●) empirical values of Unna and Talmi;[3](×) bare Hamada-Johnston force; (▲) renormalized HJ force calculated with the C^{12} core; (+) renormalized HJ force calculated with the O^{16} core.

In conclusion, calculation of effective matrix elements starting from the Brueckner-Bethe reaction-matrix techniques seems to give surprisingly good results. We consider that the main job of connecting effective forces with the nucleon-nucleon force has been done. Remaining inaccuracies can be diminished by well-prescribed techniques but will require extensive computer time. We now feel that one should try to extend these techniques to incorporate realistic forces in the Hartree-Fock calculations and in nuclear reaction studies.

REFERENCES

1. Cohen, Lawson, Macfarlane, and Soga, Phys. Letters 10, 195 (1964).
2. Auerbach, N., Nucl. Phys. 76, 321 (1966).
3. Talmi, I., and Unna, I., Ann. Rev. of Nucl. Sci. 10, 353 (1960).

DISCUSSION

Malik: It is perhaps worthwhile to go back and ask a question—From where do we get the reaction matrix?—because the foundation of this calculation is the reaction matrix. In the old papers of Brueckner, he clearly says: Let us define the reaction matrix in nuclear matter to be the same as the reaction matrix that

describes the scattering of two particles. And of course then one puts in a projection operator to take care of the Pauli principle. But it is by no means clear whether that reaction matrix holds for a finite system. Often it cannot hold because, with the projection operator that is usually used, it is nonhermitian. Or else it is made hermitian in an ad hoc fashion. It doesn't take much more than a senior course in quantum mechanics to realize that one cannot get a unique solution of the reaction matrix equation if the operator is nonhermitian.

Well, one can say we are getting good results. But how good are these results? If we look into the nickel isotopes, and if my recollection is correct, the other nuclear properties cannot be obtained in this fashion. For example, the transition rates cannot by explained so easily without effective charges. So I do not think that the question of the interaction is entirely settled.

G. E. Brown: Well, I am not going to answer in much detail. The reaction matrix that we use is hermitian. There is nothing ad hoc about the way we get from A to Z. I think Professor Bethe would have given a somewhat stronger answer along these lines. I don't think you have read the papers by the Cornell group or by our group.

The transitions in the nickel isotopes are in pretty good shape. The stripping and pickup reactions are not too well described by these wave functions. It's clear that there must be admixtures of other things which however we believe to be small and we have included them through perturbation theory, which probably is not adequate, but will do until we look at the situation.

We have produced semiquantitatively every peculiar result that Talmi has pointed out to us throughout the periodic system, usually to within 10 to 20%, and of course agreement with experiment, or agreement with Talmi even, I wouldn't say is the be all and end all. There are many cases where one has agreement, and yet the theories used are wrong. However, one starts here and solves things in a rather systematic way, and can evaluate higher order corrections and has control over them. So that I think the situation is quite different than you portray.

Now of course, this may be because I never had a senior course in quantum mechanics! (Laughter.)

Malik: I'm afraid I read your paper, but I just kept on wondering from where did you get those reaction matrices? The situation is somewhat different in Bethe's calculation, which treats nuclear matter, where the wave functions are asymptotically plane waves.

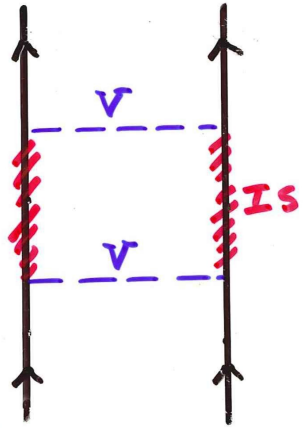
Danos: The force you use is actually the result of using configuration mixing of a certain kind of configurations and putting them into the force. This seems to be a very good thing as long as you then use a very restricted function space in configuration mixing. But as soon as you mention the word configuration mixing in addition to

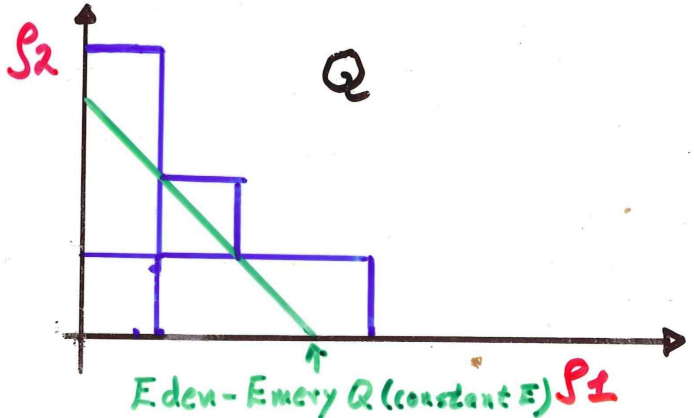
G-matrix Calculations

$$G(\omega) = V + VQ \frac{1}{\omega - H_0} QG(\omega)$$

Problems:

1. Starting energy dependent
2. Intermediate state problem
3. Treatment of the Pauli Operator
4. Spurious Center of Mass Motion





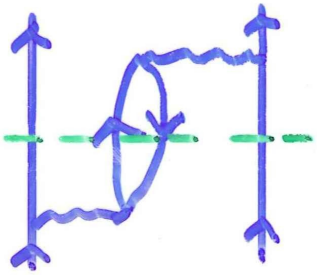
Perturbation Theory Expansion for V_{eff}

1. Hard to expand beyond 3rd order
   No systematic way of expanding
   Partial infinite resummation
   e.g. TDA $|0p0h\rangle$, RPA $|0p0h\rangle$

2. Spurious Center of Mass Motion problem

3. No clear signs of convergence
   in terms of G-matrix
   [Schucan and Weidenmüller (1972)]
   Intruder State Problem

4. Vary-Sauer-Wong effect



Intermediate summation to at least 2th Ω

RENORMALIZATION GROUP, SIMILARITY AND UNITARY TRANSFORMATION APPROACHES

1. V_low-k: Schwenk, Bogner, Kuo, Nogga, et al.

2. Similarity Renormalization Group Interactions: Furnstahl, et al.

3. Unitary Correlation Operator Method: Feldmeier, Neff, Roth, et al.

4. Similarity and Unitary Transformations:

   a. Okubo/Lee-Suzuki/etc.: e.g. as employed in the No Core Shell Model, etc.

   b. Unitary Model Operator Approach: Suzuki, Okamoto, Fujii

No Core Shell Model

"*Ab Initio*" approach to microscopic nuclear structure calculations, in which all A nucleons are treated as being active.

Want to solve the A-body Schrödinger equation

$$H_A \Psi^A = E_A \Psi^A$$

R P. Navrátil, J.P. Vary, B.R.B., PRC 62, 054311 (2000)

Effective Interaction

- Must truncate to a finite model space: V_{ij} --> $V_{ij}^{effective}$
- In general, V_{ij}^{eff} is an A-body interaction
- We want to make an a-body cluster approximation

$$\mathcal{H} = \mathcal{H}^{(I)} + \mathcal{H}^{(A)} \gtrapprox \mathcal{H}^{(I)} + \mathcal{H}^{(a)}$$
$$a < A$$

Two-body cluster approximation (a=2)

$$\mathcal{H} \approx \mathcal{H}^{(I)} + \mathcal{H}^{(2)}$$

$$H_2^\Omega = \underbrace{H_{0_2} + H_2^{CM}}_{h_1+h_2} + V_{12} = \frac{\vec{p}^{\,2}}{2m} + \frac{1}{2} m\Omega^2 \vec{r}^{\,2} + H_2^{CM} + V(\sqrt{2}\vec{r}) - \frac{m\Omega^2}{A}\vec{r}^{\,2}$$

Carry out a unitary transformation on H_2^Ω

$$\mathcal{H}_2 = e^{-S^{(2)}} H_2^\Omega e^{S^{(2)}} \quad \text{where } S^{(2)} \text{ is anti Hermitian}$$

$S^{(2)}$ is determined from the decoupling condition

$$Q_2 e^{-S^{(2)}} H_2^\Omega e^{S^{(2)}} P_2 = 0$$

P_2 = model space, Q_2 = excluded space, $P_2 + Q_2 = 1$

with the restrictions $P_2 S^{(2)} P_2 = Q_2 S^{(2)} Q_2 = 0$

Two-body cluster approximation (a=2)

It is convenient to rewrite S(2) in terms of a new operator

$$S^{(2)} = \operatorname{arctanh}(\omega - \omega^\dagger) \quad \text{with} \quad Q_2 \omega P_2 = \omega$$

Then the Hermitian effective operator in the P2 space can be expressed in the form

$$\mathcal{H}^{(2)}_{eff} = P_2 \mathcal{H}_2 P_2 = \frac{P_2 + P_2 \omega^\dagger Q_2}{\sqrt{P_2 + \omega^\dagger \omega}} H_2^\Omega \frac{P_2 + Q_2 \omega P_2}{\sqrt{P_2 + \omega^\dagger \omega}}$$

Analogously, any arbitrary operator can be written in the P_2 space

$$\mathcal{O}^{(2)}_{eff} = P_2 \mathcal{O}_2 P_2 = \frac{P_2 + P_2 \omega^\dagger Q_2}{\sqrt{P_2 + \omega^\dagger \omega}} O \frac{P_2 + Q_2 \omega P_2}{\sqrt{P_2 + \omega^\dagger \omega}}$$

Exact solution for ω:

Let E_k and $|k\rangle$ be the eigensolutions

$$H\frac{\Omega}{2}|k\rangle = E_k|k\rangle$$

Let $|\alpha_P\rangle$ & $|\alpha_Q\rangle$ be HO states belonging to the model space P and the excluded space Q, respectively. Then ω is given by:

$$\langle\alpha_Q|k\rangle = \sum_{\alpha_P}\langle\alpha_Q|\omega|\alpha_P\rangle\langle\alpha_P|k\rangle$$

or

$$\langle\alpha_Q|\omega|\alpha_P\rangle = \sum_{k\in K}\langle\alpha_Q|k\rangle\langle\tilde{k}|\alpha_P\rangle$$

Exact solution for matrix elements of ω, i.e., $\langle \alpha_Q | \omega | \alpha_P \rangle$

$H|K\rangle = E_K|K\rangle$ { E_K is one of the eigenenergies that we want to reproduce. Subset K of full eigenspace.

$|\alpha_P\rangle$ = basis state in P space

$|\alpha_Q\rangle$ = basis state in Q space

As seen earlier $Q\omega P = \omega \Rightarrow$

$Q\omega P|K\rangle = \omega|K\rangle = Q|K\rangle$ so that

$\langle \alpha_Q | Q\omega P | K\rangle = \langle \alpha_Q | Q | K\rangle$

$P = \sum_{\alpha_P} |\alpha_P\rangle\langle\alpha_P|$

↑ Component of K in the Q space.

$$\sum_{\alpha_P} \langle \alpha_Q | \omega | \alpha_P \rangle \langle \alpha_P | K \rangle = \langle \alpha_Q | K \rangle$$

Dimension "d" of K and the model space are the same. Have "d" such equations.

Invert the $d \times d$ $\langle \alpha_P | K \rangle$ matrix. $\Rightarrow$

$$\langle \alpha_Q | \omega | \alpha_P \rangle = \sum_{K \in K} \langle \alpha_Q | K \rangle \widetilde{\langle K | \alpha_P \rangle}$$

"inverse"

Effective Hamiltonian for NCSM

Solving $H^{\Omega}_{A,\,a=2}\,\Psi_{a=2} = E^{\Omega}_{A,\,a=2}\,\Psi_{a=2}$

in "infinite space" 2n+l = 450 relative coordinates

P + Q = 1; P – model space; Q – excluded space;

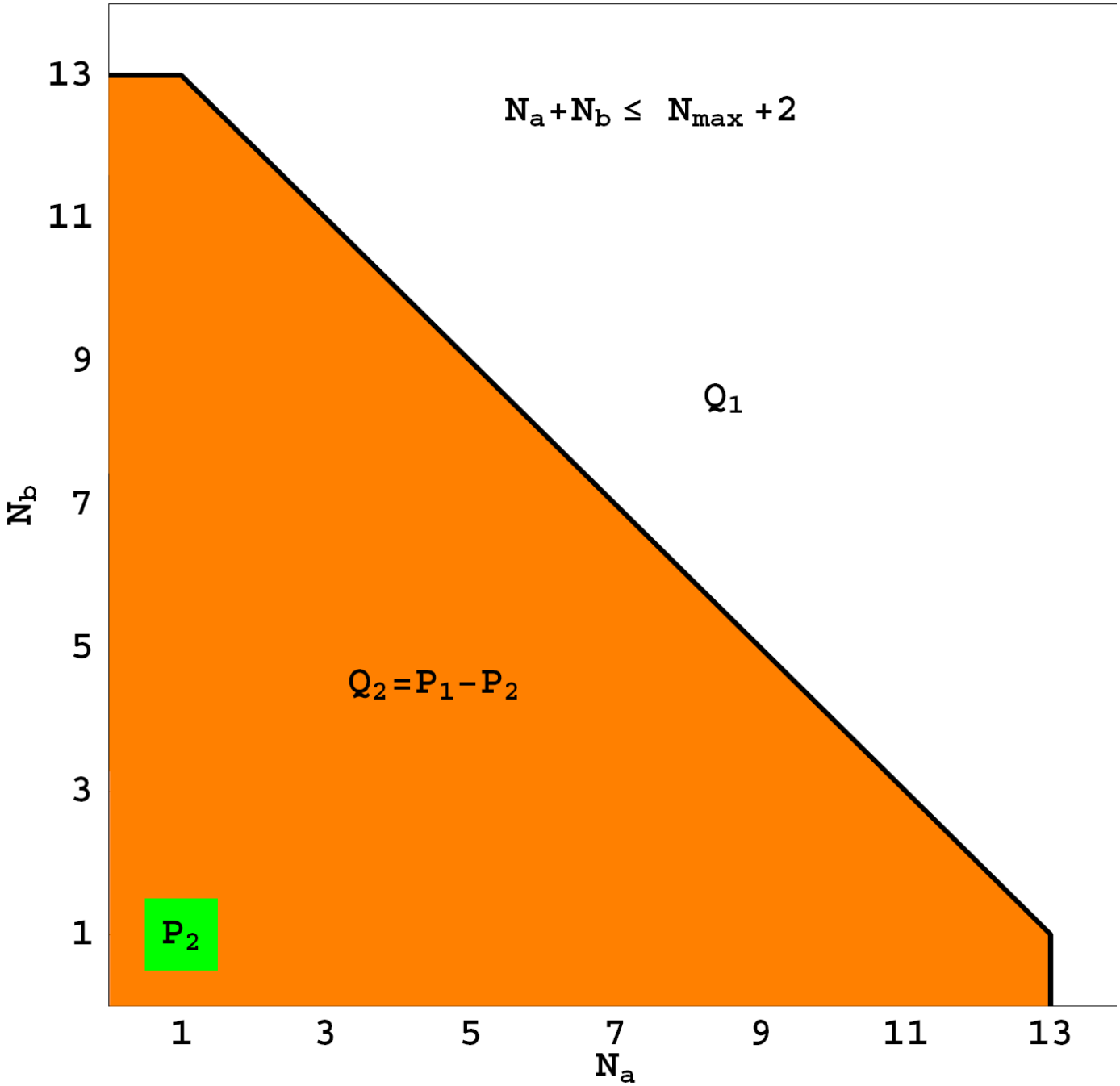
$$E^{\Omega}_{A,2} = U_2 H^{\Omega}_{A,2} U_2^{\dagger} \qquad U_2 = \begin{pmatrix} U_{2,P} & U_{2,PQ} \\ U_{2,QP} & U_{2,Q} \end{pmatrix} \qquad E^{\Omega}_{A,2} = \begin{pmatrix} E^{\Omega}_{A,2,P} & 0 \\ 0 & E^{\Omega}_{A,2,Q} \end{pmatrix}$$

$$H^{N_{\max},\Omega,\mathrm{eff}}_{A,2} = \frac{U^{\dagger}_{2,P}}{\sqrt{U^{\dagger}_{2,P} U_{2,P}}} E^{\Omega}_{A,2,P} \frac{U_{2,P}}{\sqrt{U^{\dagger}_{2,P} U_{2,P}}}$$

Two ways of convergence:

1) For $P \to 1$ and fixed a: $H^{\mathrm{eff}}_{A,a=2} \to H_A$

2) For $a \to A$ and fixed P: $H^{\mathrm{eff}}_{A,a} \to H_A$


$N_a+N_b \leq N_{max}+2$
Q_1
$Q_2=P_1-P_2$
P_2
13
11
9
7
5
3
1
N_b
1
3
5
7
9
11
13
N_a

Effective Hamiltonian for SSM

Two ways of convergence:

1) For $P \to 1$ and fixed a: $H^{eff}_{A,a=2} \to H_A$: previous slide

2) For $a_1 \to A$ and fixed P_1: $H^{eff}_{A,a1} \to H_A$

$P_1 + Q_1 = P$; P_1 - small model space; Q_1 - excluded space;

$$\mathcal{H}^{N_{1,\max},N_{\max}}_{A,a_1} = \frac{U^{A,\dagger}_{a_1,P_1}}{\sqrt{U^{A,\dagger}_{a_1,P_1}U^{A}_{a_1,P_1}}} E^{N_{\max},\Omega}_{A,a_1,P_1} \frac{U^{A}_{a_1,P_1}}{\sqrt{U^{A,\dagger}_{a_1,P_1}U^{A}_{a_1,P_1}}}$$

Valence Cluster Expansion

$N_{1,max} = 0$ space (p-space) ; $a_1 = A_c + a_v$; a_1 - order of cluster;

A_c - number of nucleons in core; a_v - order of valence cluster;

$$\mathcal{H}^{0,N_{\max}}_{A,a_1} = \sum_{k}^{a_v} V^{A,A_c+k}_{k}$$

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No-core shell model in an effective-field-theory framework

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Abstract

We present a new approach to the construction of effective interactions suitable for many-body calculations by means of the no-core shell model (NCSM). We consider an effective field theory (EFT) with only nucleon fields directly in the NCSM model spaces. In leading order, we obtain the strengths of the three contact interactions from the condition that in each model space the experimental ground-state energies of ^{2}H, ^{3}H and ^{4}He be exactly reproduced. The first $(0^+;0)$ excited state of ^{4}He and the ground state of ^{6}Li are then obtained by means of NCSM calculations in several spaces and frequencies. After we remove the harmonic-oscillator frequency dependence, we predict for ^{4}He an energy level for the first $(0^+;0)$ excited state in remarkable agreement with the experimental value. The corresponding ^{6}Li binding energy is about 70% of the experimental value, consistent with the expansion parameter of the EFT.

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how matter came about, and they add a great deal of significance and importance to nuclear physics and to certain experiments in nuclear physics which would have only little importance to the problems we have discussed here. Perhaps in the next conference we should have a session where we discuss these things; it is not enough just to go to Mr. Cameron or Mr. Fowler and ask him what shall we measure, we ought to know why we do it.

The second and last point I would like to raise is this. To round up the conference I come back to the first remark of Peierls, when he opened up the conference and asked the question, why are we interested in nuclear structure. May I add my own little verse to this. I have heard many people say that Nuclear Structure is not a fundamental problem, the real thing is high energy physics; the object of nuclear structure is after all nothing else but solving a Schroedinger equation for A particles. I strongly disagree with this point of view. The discovery and the understanding of phenomena hidden in a many-body problem can be a task of fundamental importance, if the object itself is of central interest.

Physics inquires into the nature of things. The nucleus, our nucleus, is an essential part of nature, it is the centre of the atom. It is not just a little phenomenon, it is the most prominent constituent of matter. The understanding of the phenomena occurring in this nucleus is therefore of paramount importance. Hence Nuclear Physics is an essential part of physics. I found out that some theorists, both in the east and in the west, consider the only thing worth doing is elementary particle physics. Experimentalists usually don't say so because they work with real matter and hence they know that the nucleus is an important thing. These theorists, however, worship the theory of elementary particles, a theory which in fact doesn't even exist. They knock their heads daily against a wall of dispersion-relations, Mandelstam representations and the like. Let them do it. After all the proton and the meson are also an important part of nature. In fact we should give them all the moral support they need. They are a brave lot who fight a very difficult fight and some day they will find the theory. But don't let yourself be talked into believing that the nucleus is not interesting. It is so small and it has so few parts and still it shows a tremendous variety of phenomena. Its investigation requires the whole arsenal of presently available experimental techniques and its understanding makes use of almost all branches of theoretical physics. What a marvellous invention! It is worth devoting a lifetime to it.