Description of nuclear structures in light nuclei with Brueckner-AMD

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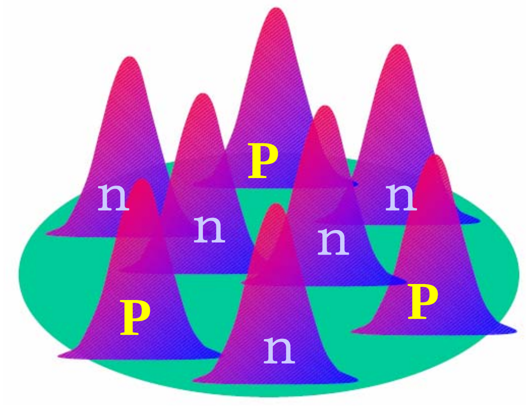


First EMMI-EFES workshop
@ GSI. Feb, 2009

Introduction (1)

One of the recent remarkable developments in theoretical nuclear physics;
***ab initio* calculations based on the realistic nuclear force**

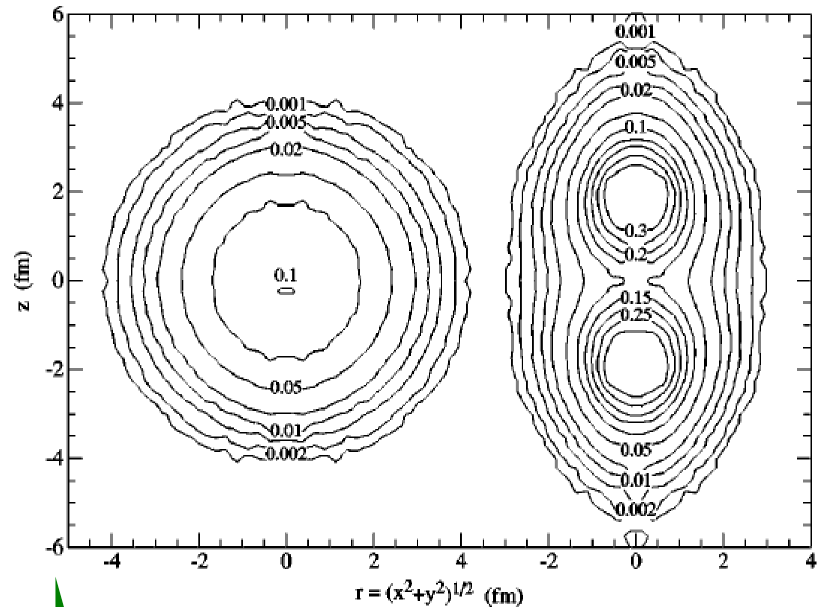
3,4-body systems can be solved strictly by Faddeev, GEM, and so on...

For heavier nuclei,

- Green's function Monte Carlo (GFMC)
- no-core shell model (NCSM)
- coupled-cluster (CC) method
- unitary-model operator approach (UMOA)
- Fermionic molecular dynamics (FMD) + unitary correlation operator method (UCOM)
- tensor optimized shell model (TOSM)

and so on...

R.B.Wiringa et al., PRC62 (2000) 014001



We can discuss cluster structures based on the realistic nuclear force.

Introduction (2)

Recently, the studies with the Antisymmetrized Molecular Dynamics (AMD) have been developed.

Advantages of AMD :

Y.Kanada-En'yo and H.Horiuchi, PTP Supple 142 (2001), 205

- **We can treat alpha-nuclei and non-alpha nuclei**
- **The wave function is written in a Slater determinant form**
- **Both states of shell model and cluster model can be described**
- **no assumption for configurations** ⇒ **one of *ab initio* calculation**

However...

The AMD have been carried out using the phenomenological potential.

Ex) A.B.Volkov, Nucl.Phys.74 (1965), 33.

⇓

We develop the new *ab initio* framework of AMD based on the realistic nuclear forces.

⇒ We apply the Brueckner theory to AMD and calculate *G* -matrix in AMD.

Brueckner-AMD; the Brueckner theory + AMD

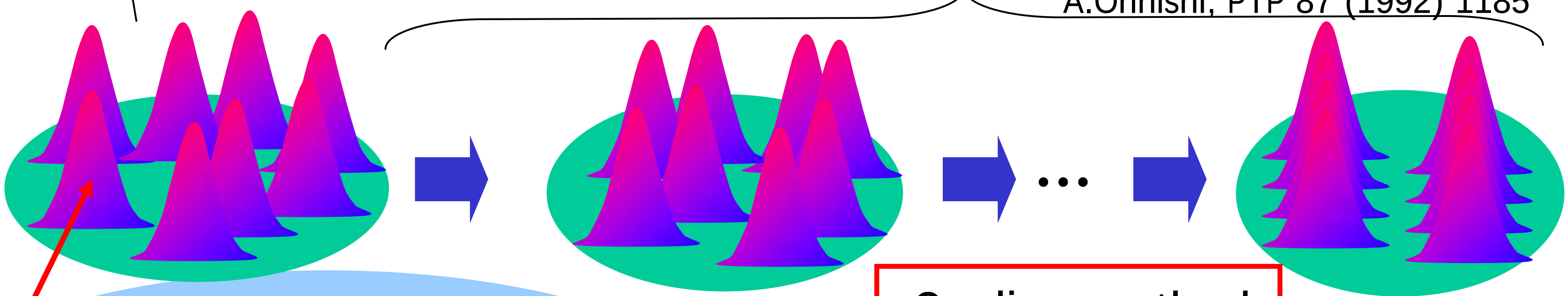
T.Togashi and K.Katō; Prog. Theor. Phys. 117 (2007) 189

Brueckner-AMD (1)

The initial configuration is chosen randomly.

The final configuration with minimum energy is determined by the frictional cooling method.

A.Ono, H.Horiuchi, T.Maruyama and A.Ohnishi, PTP 87 (1992) 1185



$$\varphi_i(\vec{r}) \propto \exp(-\nu(\vec{r} - \vec{Z}_i/\sqrt{\nu})^2)$$

(**Gaussian wave packet**)

Nucleon-nucleon (NN) correlations cannot be described sufficiently.

Cooling method

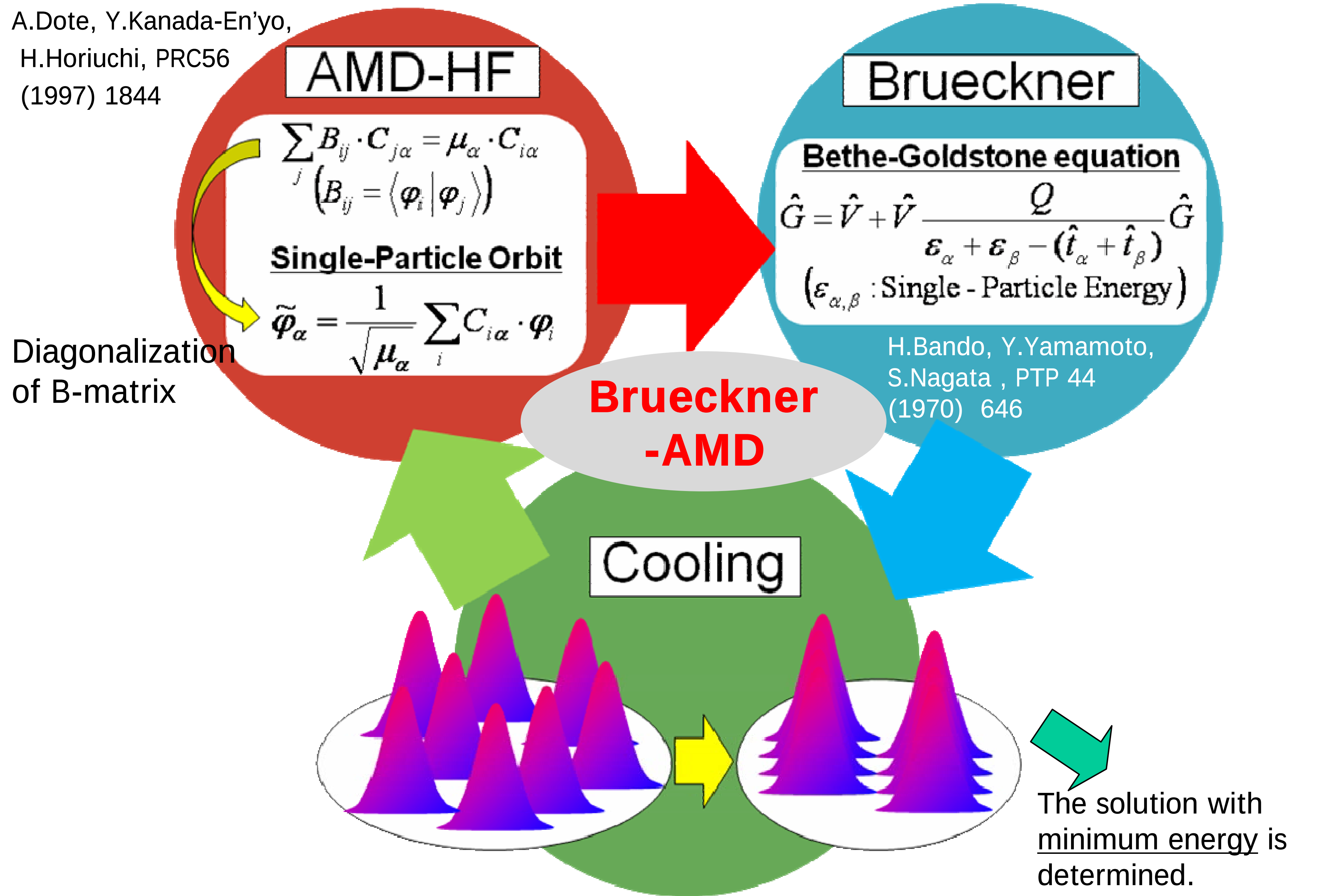
$$\begin{cases} \dfrac{d\vec{Z}_i}{dt} = -\dfrac{\partial H}{\partial \vec{Z}_i^*} \\ \dfrac{d\vec{Z}_i^*}{dt} = -\dfrac{\partial H}{\partial \vec{Z}_i} \end{cases}$$

→ Energy Variation

Basic concept of the Brueckner-AMD

- model wave function: AMD wave function
- NN correlations: Brueckner theory

Brueckner-AMD (2)



G -matrix in Brueckner-AMD (1)

Bethe-Goldstone Equation

$$\hat{G} = \hat{V} + \hat{V}\frac{Q}{\varepsilon_\alpha + \varepsilon_\beta - (\hat{t}_\alpha + \hat{t}_\beta)}\hat{G}$$

self-consistent

Single-particle energy

$$\varepsilon_\alpha = \langle \tilde{\varphi}_\alpha | \hat{t} | \tilde{\varphi}_\alpha \rangle + \sum_\beta \langle \tilde{\varphi}_\alpha \tilde{\varphi}_\beta | \hat{G} | A\{\tilde{\varphi}_\alpha \tilde{\varphi}_\beta\} \rangle$$

single-particle orbits in AMD

1st step

$$\hat{G}^0 = \hat{V} + \hat{V}\frac{1}{\varepsilon_\alpha + \varepsilon_\beta - (\hat{t}_\alpha + \hat{t}_\beta)}\hat{G}^0$$

→ solving the correlated w.f. for every pair

2nd step

$$\hat{G} = \hat{G}^0 + \hat{G}^0\frac{Q-1}{\varepsilon_\alpha + \varepsilon_\beta - (\hat{t}_\alpha + \hat{t}_\beta)}\hat{G}$$

→ taking into account Q -operator

Q – operator : $Q = 1 - \sum_{\alpha<\beta} |\tilde{\varphi}_\alpha \tilde{\varphi}_\beta\rangle\langle\tilde{\varphi}_\alpha \tilde{\varphi}_\beta|$

H.Bando, Y.Yamamoto, S.Nagata, PTP 44 (1970) 646

G -matrix in Brueckner-AMD (2)

$$\hat{G}^0 = \hat{V} + \hat{V} \frac{1}{\varepsilon_\alpha + \varepsilon_\beta - (\hat{t}_\alpha + \hat{t}_\beta)} \hat{G}^0$$

solving the correlated w.f. for every pair

Differential equation for the 2-body correlated w.f. ψ

$$\left[-\frac{\hbar^2}{M}\nabla^2 + T_C - (\varepsilon_\alpha + \varepsilon_\beta) \right] \left(\left| Z_{r_{kl}} \right\rangle - \left| \psi \right\rangle \right) = \hat{V} \left| \psi \right\rangle$$

2-body c.m. kinetic energy

$$T_C = \frac{\left\langle Z_{C_{ij}} \middle| \hat{T}_C \middle| Z_{C_{kl}} \right\rangle}{\left\langle Z_{C_{ij}} \middle| Z_{C_{kl}} \right\rangle}$$

2-body AMD relative w.f. expanded as partial waves

Correlation functions in Brueckner-AMD

Bethe-Goldstone Equation

$$\psi(ij) = \left(1 + \frac{Q}{e}\hat{G}\right)\phi(ij) \equiv \hat{F}_{ij}\cdot\phi(ij)$$

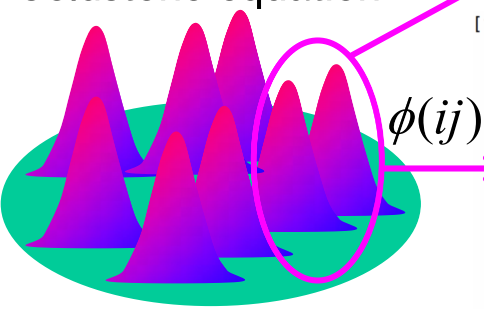
$$\hat{v}\cdot\psi(ij) = \hat{G}\cdot\phi(ij)$$

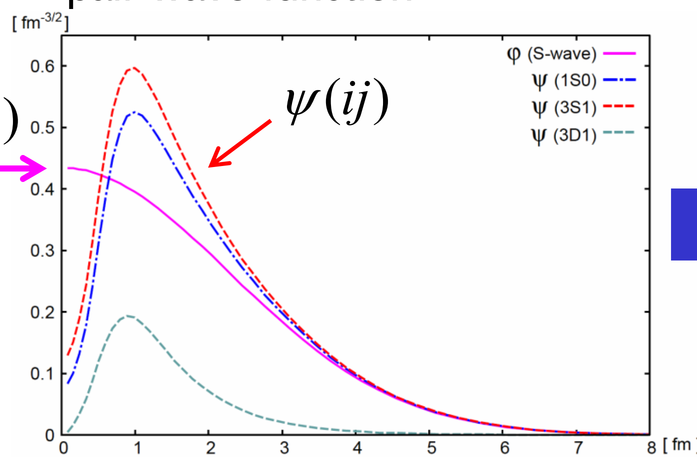
e : energy denominator

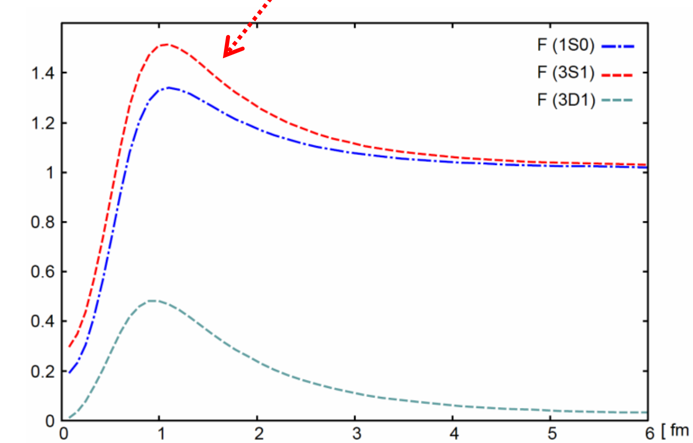
Solution of the Bethe-Goldstone equation

Model (AMD) pair wave function

correlation function: $\hat{F}_{ij} = \psi(ij)/\phi(ij)$





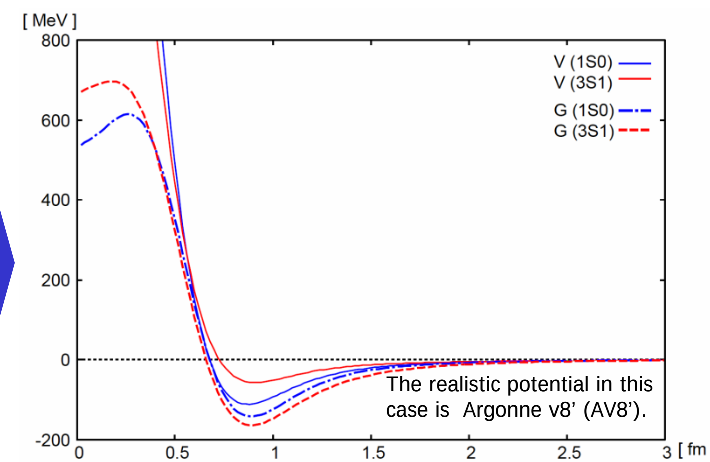


G-matrix element

$$\langle\phi(ij)|\hat{G}|\phi(ij)\rangle = \langle\phi(ij)|\hat{v}|\psi(ij)\rangle$$

$$= \langle\phi(ij)|\hat{v}\cdot\hat{F}_{ij}|\phi(ij)\rangle$$

Correlation function



Spin-parity projection in Brueckner-AMD

Ex) Parity Projection

Parity-projected state : $|\Phi^{\pm}\rangle \equiv \frac{1}{\sqrt{2}}(1 \pm \mathcal{P})|\Phi\rangle$

parity

Space-reflection operator

⇒ the liner combination of two Slater determinants

The G-matrix between the different configurations in bra and ket states is necessary.

$$\frac{\langle \Phi^a | \hat{G} | \Phi^b \rangle}{\langle \Phi^a | \Phi^b \rangle}$$

The G-matrix is calculated with the correlation functions.

$$\langle \phi_i^a \phi_j^a | \hat{G} | \phi_k^b \phi_l^b \rangle \equiv \frac{1}{2} \langle \phi_i^a \phi_j^a | \hat{F}_{ij}^{(a)*} \cdot \hat{v} + \hat{v} \cdot \hat{F}_{kl}^{(b)} | \phi_k^b \phi_l^b \rangle$$

Correlation functions derived from bra and ket states

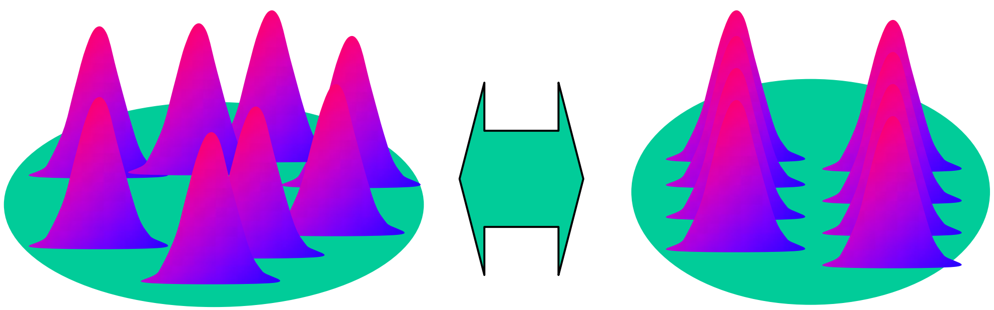
T.Togashi, T.Murakami and K.Katō; Prog. Theor. Phys. 121 (2009) in press.

Features of Brueckner-AMD

①. <u>The *G* -matrix and correlation functions can be solved strictly in Brueckner-AMD</u> because the single-particle orbits can be defined and applied to the Brueckner theory.

$$\hat{G} = \hat{V} + \hat{V}\frac{Q}{\varepsilon_{\alpha} + \varepsilon_{\beta} - (\hat{t}_{\alpha} + \hat{t}_{\beta})}\hat{G} \quad \Longleftrightarrow \quad \left(\begin{array}{l} \varepsilon_{\alpha} = \langle \tilde{\varphi}_{\alpha} | \hat{t} | \tilde{\varphi}_{\alpha} \rangle + \sum_{\beta} \langle \tilde{\varphi}_{\alpha}\tilde{\varphi}_{\beta} | \hat{G} | A\{\tilde{\varphi}_{\alpha}\tilde{\varphi}_{\beta}\} \rangle \\ Q = 1 - \sum_{\alpha<\beta} | \tilde{\varphi}_{\alpha}\tilde{\varphi}_{\beta} \rangle \langle \tilde{\varphi}_{\alpha}\tilde{\varphi}_{\beta} | \end{array} \right)$$

②. The *G* -matrix and correlation functions in Brueckner-AMD are <u>changed with the state of configurations</u>.



➡ The strong state dependence of nuclear force can be considered in Brueckner-AMD.

Application to light nuclei

Interactions

We use Argonne v8' (AV8') as a realistic nuclear interaction.

Av8'; P.R.Wringa and S.C.Pieper, PRL89 (2002), 182501.

Projection

Parity $(\pi = \pm)$: variation after projection (VAP)

➡ We describe the intrinsic w.f. of each parity eigenstate.

Spin (J) : projection after variation (PAV)

$$\left| \Phi^{J^{\pi=\pm}}_{MK} \right\rangle = P^J_{MK} \left| \Phi^{\pi=\pm} \right\rangle \quad (P^J_{MK} : \text{Spin projection operator})$$

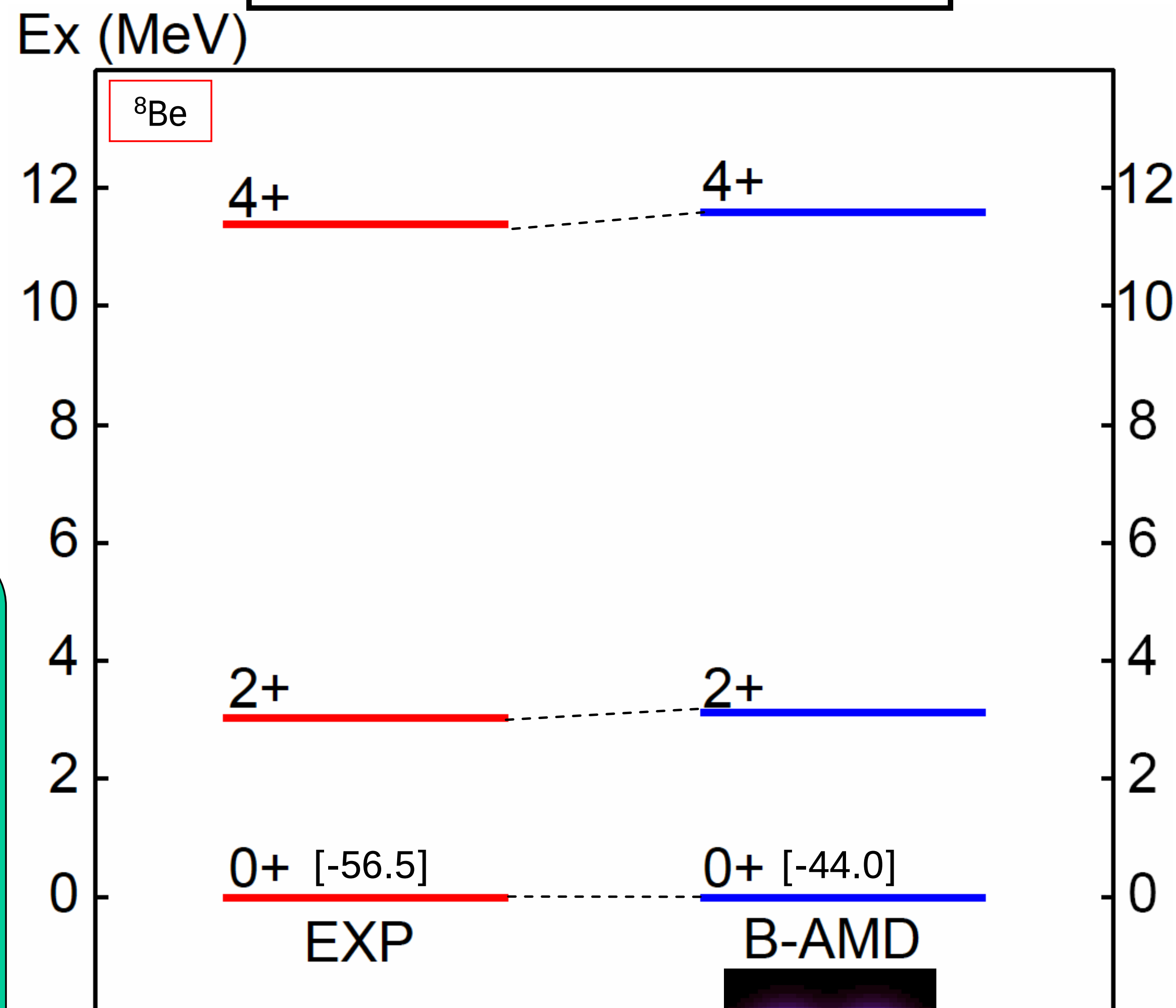
Parity eigenstate obtained with VAP

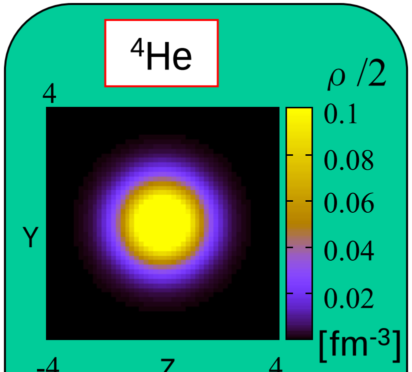
➡ We calculate the binding energy of the ground state and the energy levels of some excited states.

⬇

We apply the Brueckner-AMD (B-AMD) to some light nuclei, ^{4}He, ^{8}Be, ^{7}Li, ^{9}Be, ^{11}B, and ^{12}C.

Results of ^{4}He and ^{8}Be





binding energy (0^+)

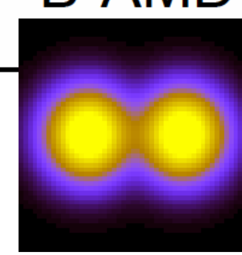
-24.6(MeV)

Few-body cal†

-25.9(MeV)

EXP

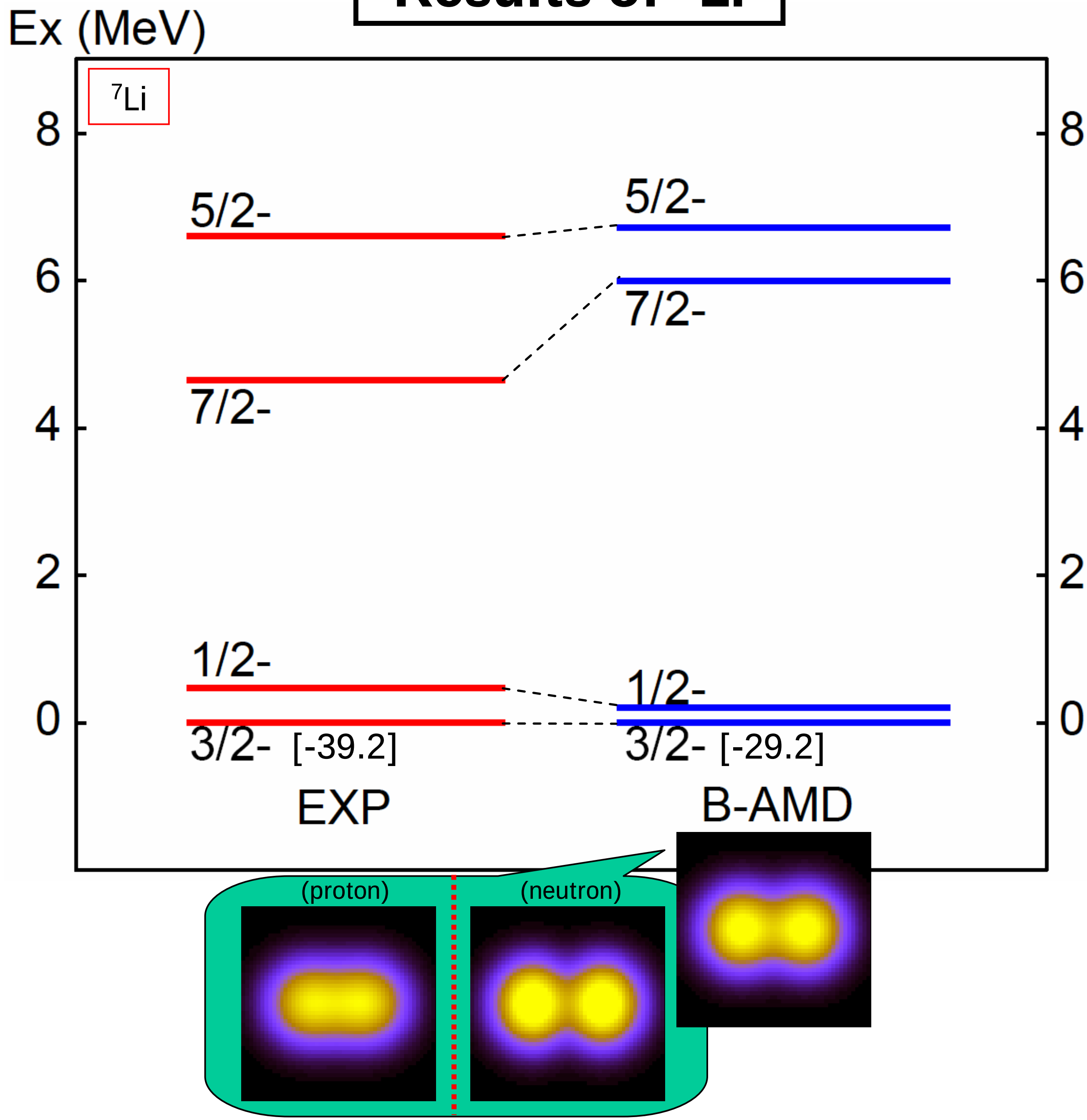
-28.3(MeV)



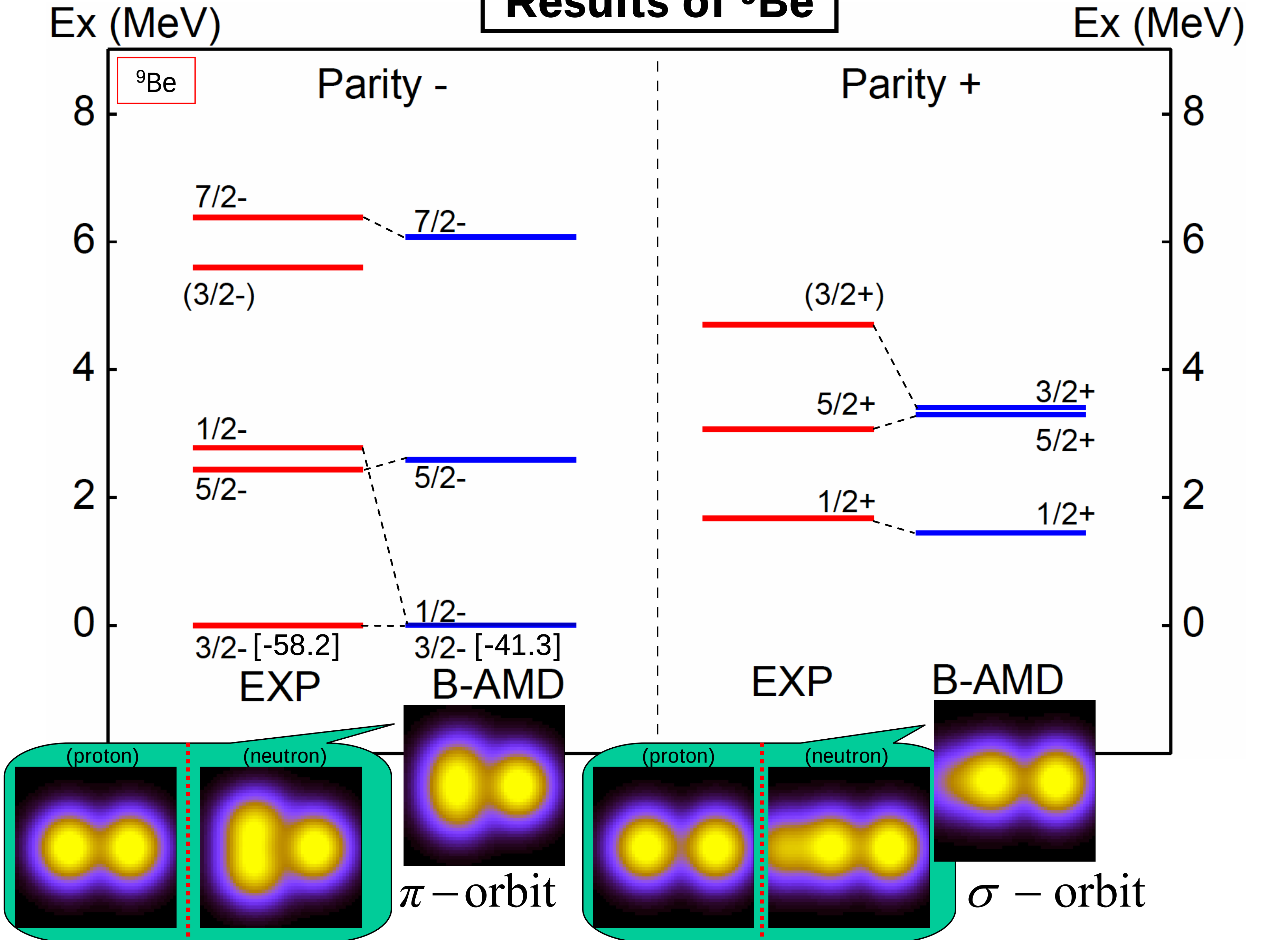
→ $\alpha + \alpha$ cluster

†Ref: H. Kamada et al. , PRC64 (2001) 044001

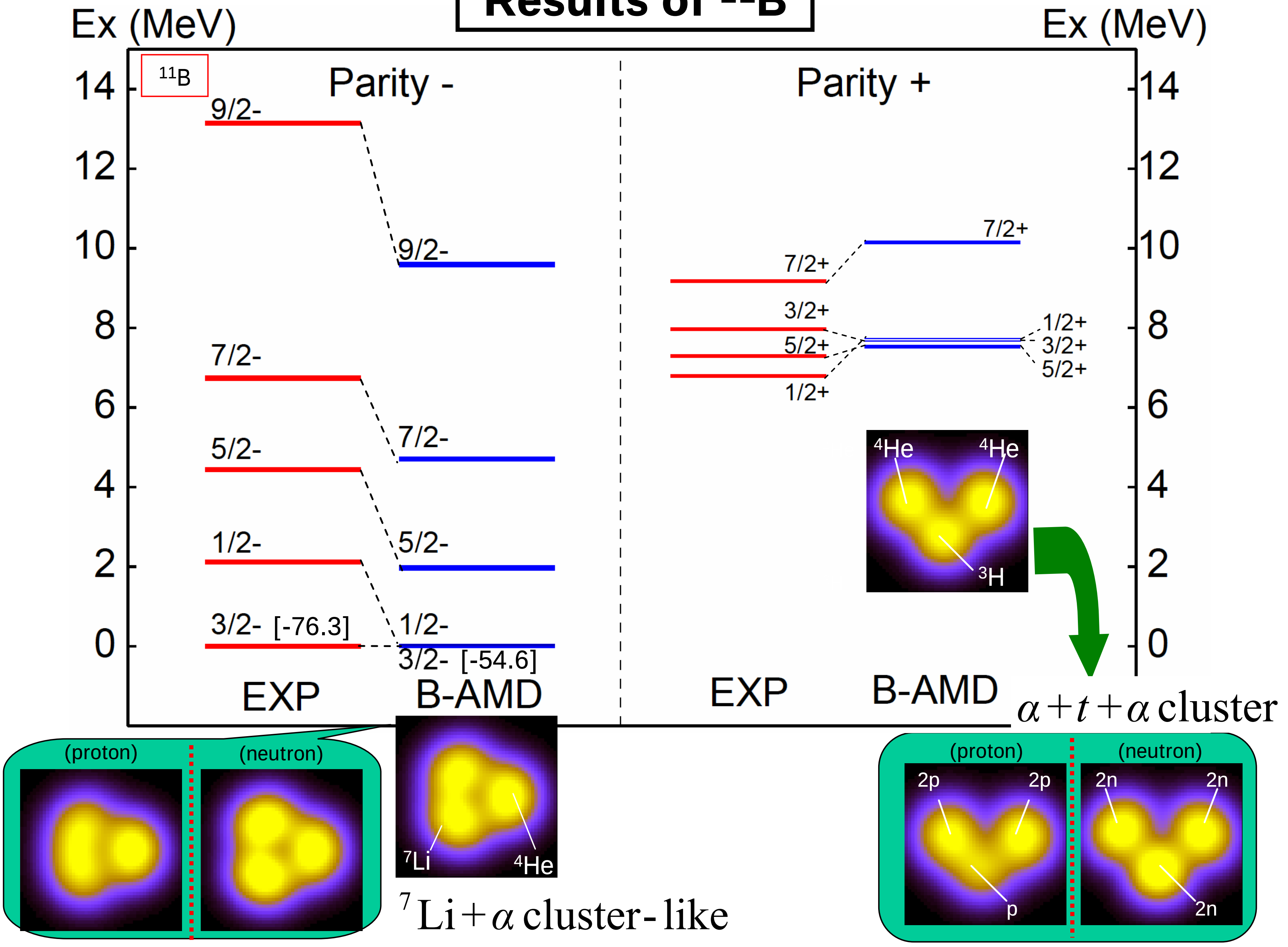
Results of ^7Li



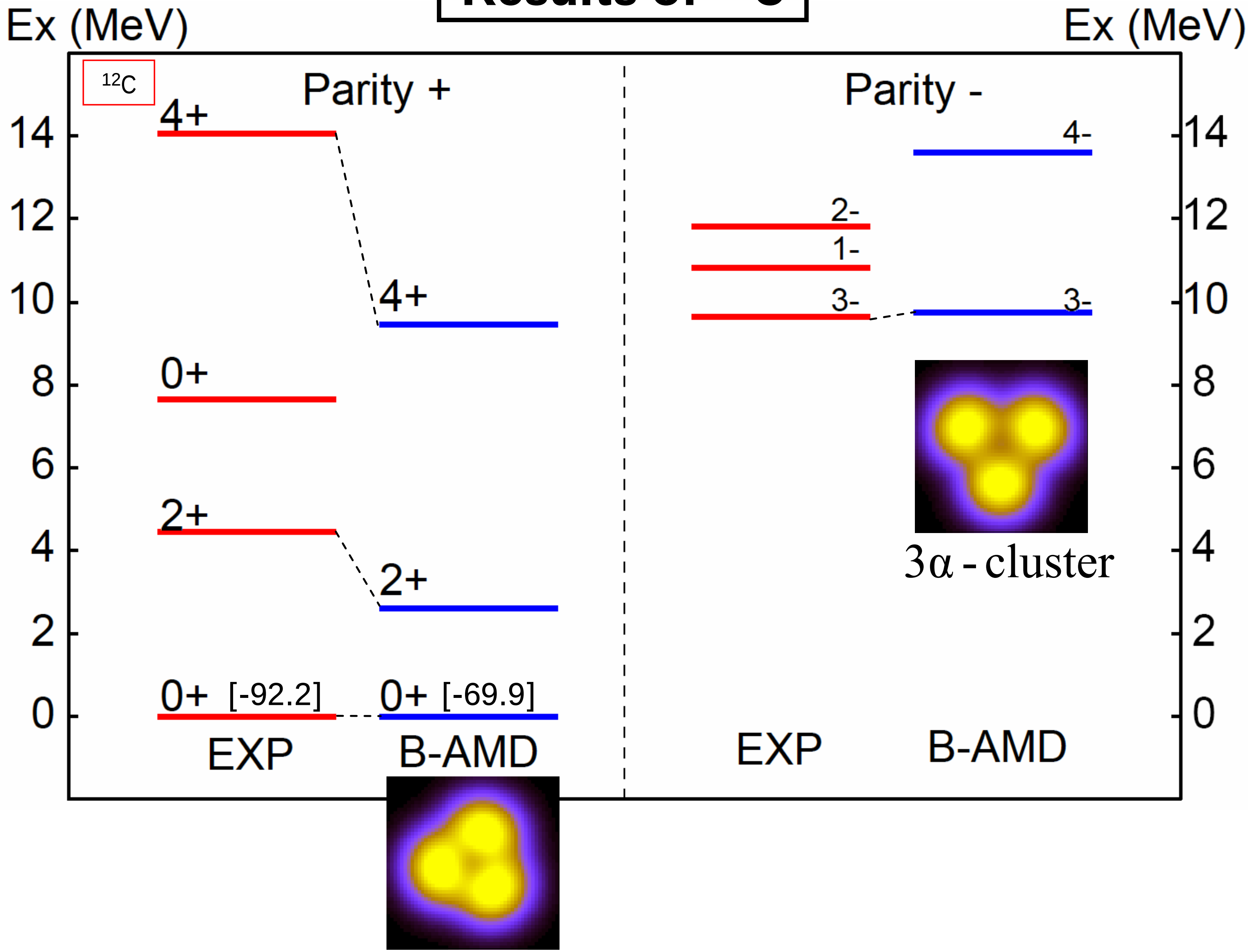
Results of ^{9}Be



Results of ^{11}B



Results of ^{12}C



Description of Higher J states

In order to describe higher J states, it is necessary to superpose the intrinsic configurations different from the lowest J state.

We perform the energy variation with the orthogonality to the lowest J state.

Y.Kanada-En'yo, PTP117 (2007) 655

$$\left|\Phi\right\rangle = \left|\Phi(Z)\right\rangle - \left|\Phi(g.s.)\right\rangle \cdot \frac{\left\langle\Phi(g.s.)\middle|\Phi(Z)\right\rangle}{\left\langle\Phi(g.s.)\middle|\Phi(g.s.)\right\rangle}$$

Intrinsic state of the excited state

$\left(\left|\Phi(g.s.)\right\rangle, \left|\Phi(Z)\right\rangle\right.$: parity-projected states$\left.\right)$

Diagonalization of Norm and Hamiltonian for J^{π} -projected states

We apply this method to the second 0^+ state of ^{4}He as the first example.

The 0_2^+ state of ^{4}He

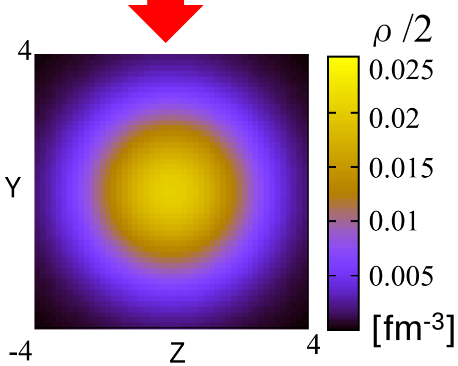
Argonne v8'

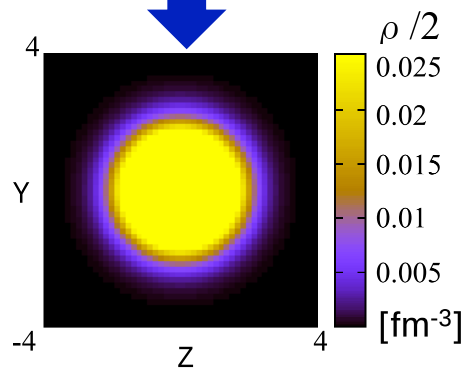
We optimize the Gaussian width of a wave packet: $\nu = 1/2b^2$

The excited state

$$|\Phi\rangle = |\Phi(Z)\rangle - |\Phi(g.s.)\rangle \cdot \frac{\langle \Phi(g.s.)|\Phi(Z)\rangle}{\langle \Phi(g.s.)|\Phi(g.s.)\rangle}$$

$b_{ex} = 3.0\,[\mathrm{fm}]$ $\qquad b_{g.s.} = 1.5\,[\mathrm{fm}]$





Diagonalization of Norm and Hamiltonian for J^π -projected states

The results of this work

B.E. (0_1^+) : -25.4 (MeV)

B.E. (0_2^+) : -7.84 (MeV)

(Few-body calculation)

B.E. (0_1^+) : -25.9 (MeV)

B.E. (0_2^+) : -7.86 (MeV)

PRC70 (2004) 031001(R), E.Hiyama et.al.

Role of tensor force in clusterization

It is considered that tensor force has strong state dependence and importance in clusterization.

H.Bando, Y.Yamamoto, S.Nagata, PTP 44 (1970) 646

However...

In the Brueckner theory, tensor correlations are renormalized into *G* -matrix and tensor contributions are not discussed directly.

Recently, we proposed the method to analyze contributions of the tensor force in Brueckner-AMD.

How to Analyze Renormalized Components

Bethe-Goldstone Equation

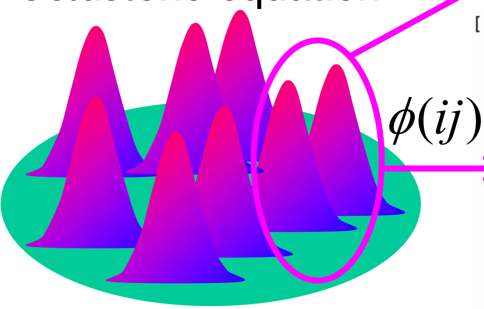
$$\psi(ij) = \left(1 + \frac{Q}{e}\hat{G}\right)\phi(ij) \equiv \hat{F}_{ij} \cdot \phi(ij)$$

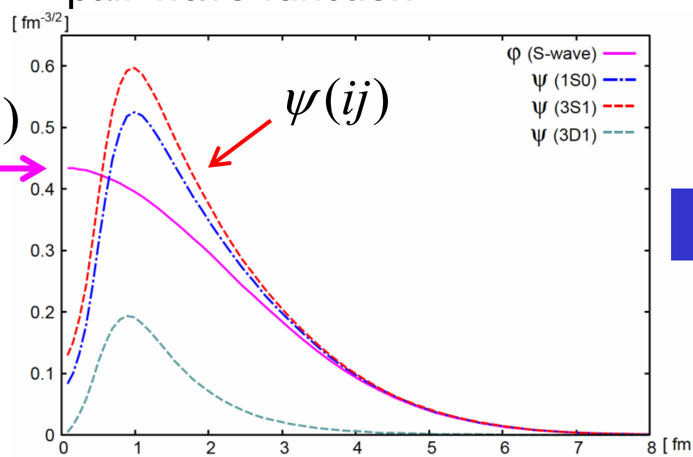
$$\hat{v} \cdot \psi(ij) = \hat{G} \cdot \phi(ij)$$
e : energy denominator

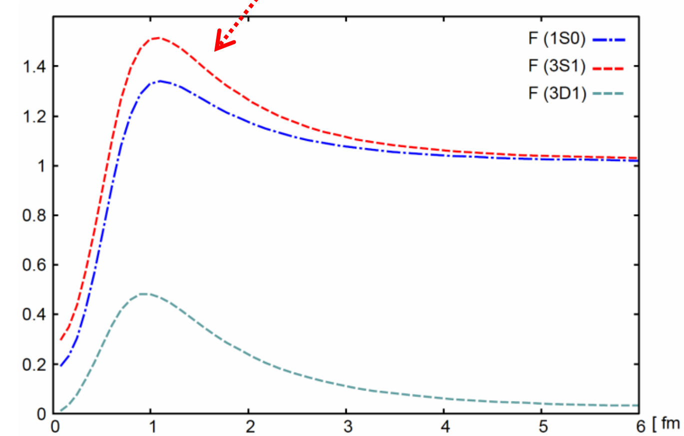
Solution of the Bethe-Goldstone equation

Model (AMD) pair wave function

correlation function: $\hat{F}_{ij} = \psi(ij) / \phi(ij)$





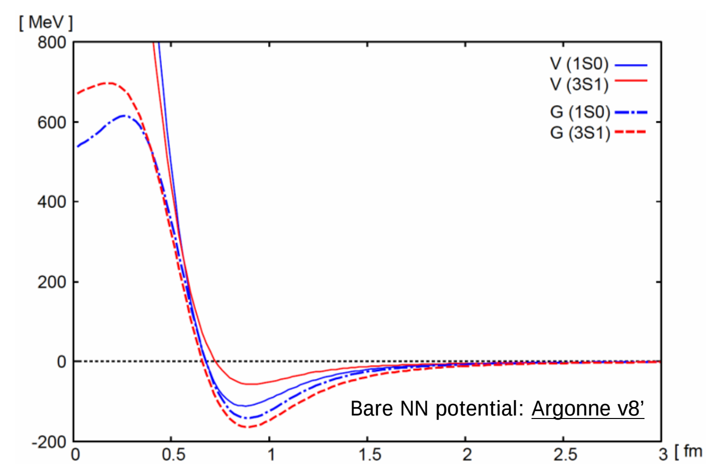


G-matrix element

$$\langle\phi(ij)|\hat{G}|\phi(ij)\rangle = \langle\phi(ij)|\hat{v}|\psi(ij)\rangle$$
$$= \langle\phi(ij)|\hat{v}\cdot\hat{F}_{ij}|\phi(ij)\rangle$$

For each potential term

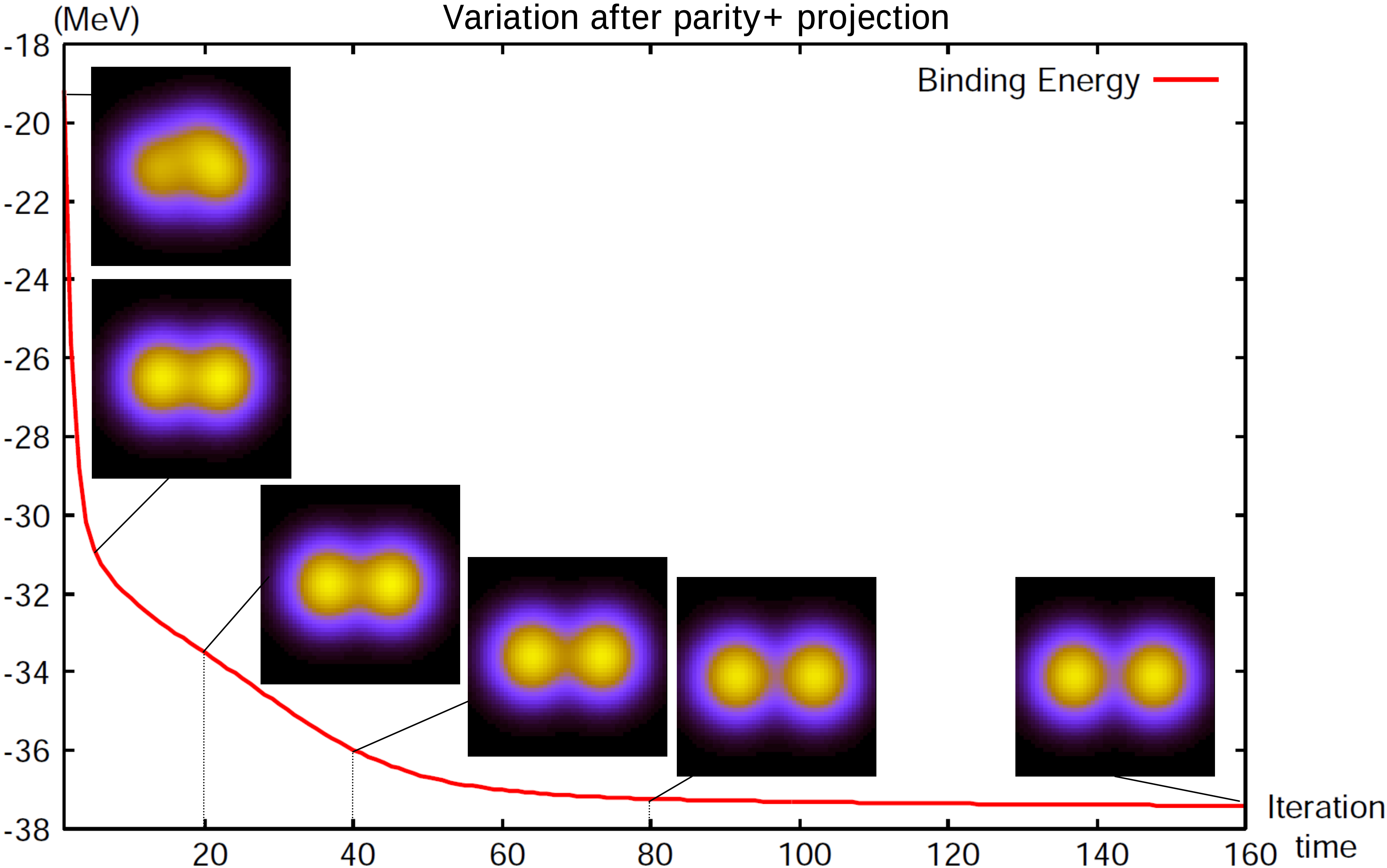
$$\langle\phi(ij)|(\hat{v}_c + \hat{v}_t + \hat{v}_{ls})\cdot\hat{F}_{ij}|\phi(ij)\rangle$$



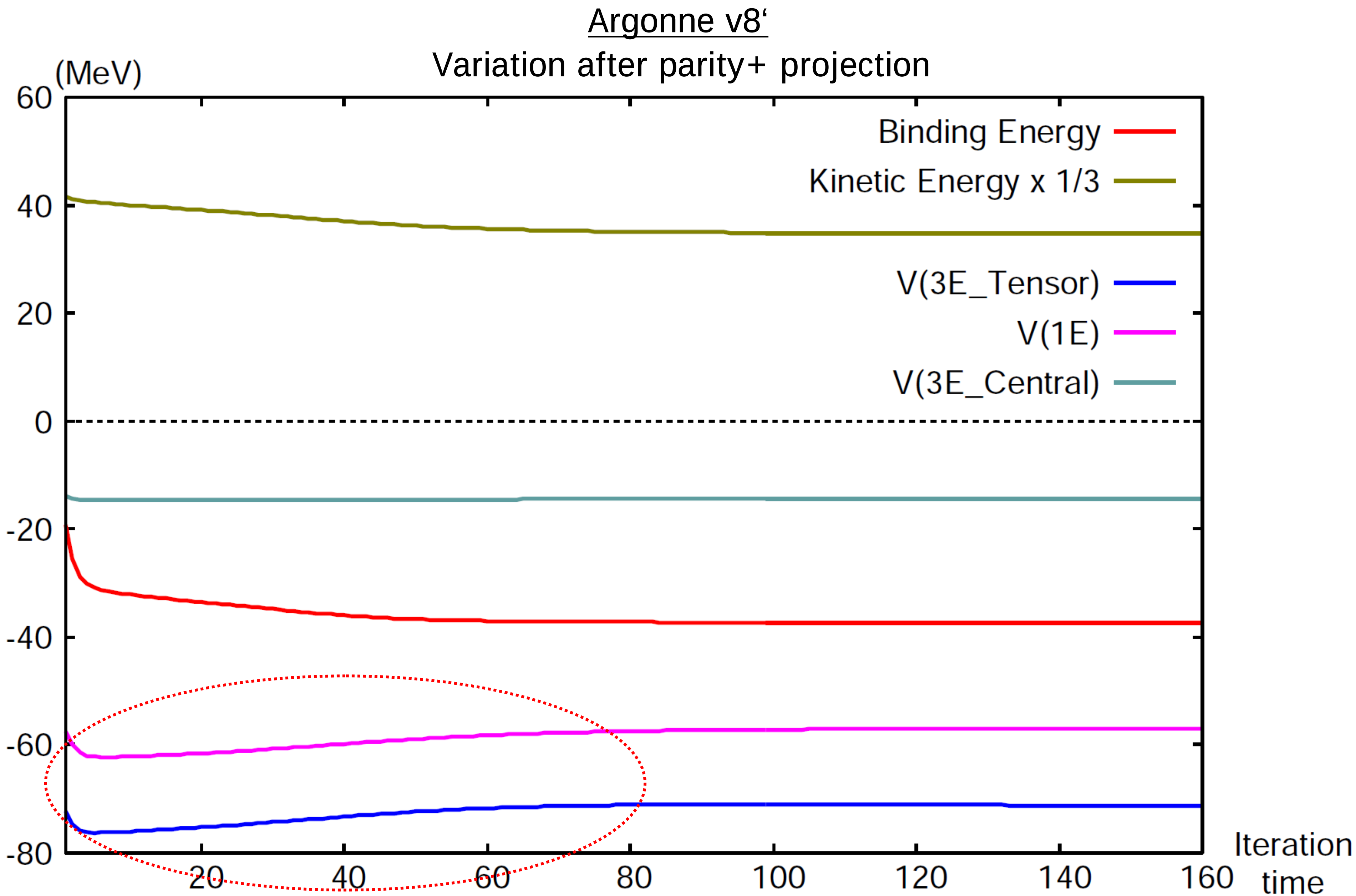
Clusterization in ^{8}Be

Argonne v8'

Variation after parity+ projection



Variation of Potential Components



Summary & Proceeding works

Summary

・ We developed the framework of AMD with realistic interactions based on the Brueckner theory.

・ We applied Brueckner-AMD to some light nuclei and succeeded to describe reasonable structures and energy-level schemes.

・ We evaluate tensor force contributions in Brueckner-AMD with the correlation functions on the basis of Bethe-Goldstone equation.

Proceeding works

・ Application of the Brueckner-AMD + Multiconfiguration calculation to higher 0^+ states in ^{12}C

・ Role of the tensor force and clusterization in Be-isotopes

・ Introduction of genuine 3-body forces

Appendix

The solutions of Bethe-Goldstone equation in Brueckner-AMD

$$\hat{G}^0 = \hat{V} + \hat{V}\frac{1}{\varepsilon_\alpha + \varepsilon_\beta - (\hat{t}_\alpha + \hat{t}_\beta)}\hat{G}^0$$

$$\left[-\frac{\hbar^2}{M}\nabla^2 + T_C - (\varepsilon_\alpha + \varepsilon_\beta)\right]\left(\left|Z_{r_{kl}}\right\rangle - \left|\psi^0\right\rangle\right) = \hat{V}\left|\psi^0\right\rangle$$

G^0-matrix element

$$\left\langle \tilde{\varphi}_\alpha \tilde{\varphi}_\beta \left| \hat{G}^0 \right| A\{\tilde{\varphi}_\alpha \tilde{\varphi}_\beta\} \right\rangle = \sum_{ijkl} \tilde{C}^*_{i\alpha} \tilde{C}^*_{j\beta} \tilde{C}_{k\alpha} \tilde{C}_{l\beta} \left\langle Z_{r_{ij}} \left| \hat{V} \right| \psi^0 \right\rangle \cdot \left\langle Z_{C_{ij}} \middle| Z_{C_{kl}} \right\rangle$$

Factorization of Q-operator effect

$$C_Q = \frac{\left\langle \tilde{\varphi}_\alpha \tilde{\varphi}_\beta \left| \hat{G} \right| A\{\tilde{\varphi}_\alpha \tilde{\varphi}_\beta\} \right\rangle}{\left\langle \tilde{\varphi}_\alpha \tilde{\varphi}_\beta \left| \hat{G}^0 \right| A\{\tilde{\varphi}_\alpha \tilde{\varphi}_\beta\} \right\rangle}$$

The solutions of BG.eq can be represented as

$$\left|\psi\right\rangle = C_Q \cdot \left|\psi^0\right\rangle$$

G -matrix in Brueckner-AMD (1)

1st step

$$\hat{G}^0 = \hat{V} + \hat{V}\frac{1}{\varepsilon_\alpha + \varepsilon_\beta - (\hat{t}_\alpha + \hat{t}_\beta)}\hat{G}^0$$

solving the correlated w.f. for every pair

Differential equation for the 2-body correlated w.f. ψ

$$\left[-\frac{\hbar^2}{M}\nabla^2 + T_C - (\varepsilon_\alpha + \varepsilon_\beta) \right]\left(\left| Z_{r_{kl}} \right\rangle - \left| \psi \right\rangle \right) = \hat{V}\left| \psi \right\rangle$$

2-body c.m. kinetic energy

$$T_C = \left\langle Z_{C_{ij}} \middle| \hat{T}_C \middle| Z_{C_{kl}} \right\rangle / \left\langle Z_{C_{ij}} \middle| Z_{C_{kl}} \right\rangle$$

2-body AMD relative w.f. expanded as partial waves

G^0-matrix element in AMD single-particle orbits

$$\left\langle \tilde{\varphi}_\alpha \tilde{\varphi}_\beta \middle| \hat{G}^0 \middle| A\{\tilde{\varphi}_\alpha \tilde{\varphi}_\beta\} \right\rangle = \sum_{ijkl} \tilde{C}^*_{i\alpha} \tilde{C}^*_{j\beta} \tilde{C}_{k\alpha} \tilde{C}_{l\beta} \left\langle Z_{r_{ij}} \middle| \hat{V} \middle| \psi \right\rangle \cdot \left\langle Z_{C_{ij}} \middle| Z_{C_{kl}} \right\rangle$$

G-matrix in Brueckner-AMD (2)

2nd step

$$\hat{G} = \hat{G}^0 + \hat{G}^0 \frac{Q-1}{\varepsilon_\alpha + \varepsilon_\beta - (\hat{t}_\alpha + \hat{t}_\beta)} \hat{G}$$

→ taking into account Q-operator

Q – operator : $Q = 1 - \sum_{\alpha<\beta} \left| \tilde{\varphi}_\alpha \tilde{\varphi}_\beta \right\rangle \left\langle \tilde{\varphi}_\alpha \tilde{\varphi}_\beta \right|$

↓

G^0-matrix element

$$\sum_{\alpha\beta} \left\{ \delta_{\gamma_1,\alpha}\, \delta_{\delta_1,\beta} + \frac{\left\langle \tilde{\varphi}_\alpha \tilde{\varphi}_\beta \left| \hat{G}^0 \right| A\{\tilde{\varphi}_\alpha \tilde{\varphi}_\beta\} \right\rangle}{e(\gamma_0\delta_0, \alpha\beta)} \right\} \left\langle \tilde{\varphi}_\alpha \tilde{\varphi}_\beta \left| \hat{G} \right| A\{\tilde{\varphi}_{\gamma_0} \tilde{\varphi}_{\delta_0}\} \right\rangle$$

G-matrix element

$$= \left\langle \tilde{\varphi}_{\gamma_1} \tilde{\varphi}_{\delta_1} \left| \hat{G}^0 \right| A\{\tilde{\varphi}_{\gamma_0} \tilde{\varphi}_{\delta_0}\} \right\rangle$$

$$\left(e(\gamma_0\delta_0, \alpha\beta) = \varepsilon_{\gamma_0} + \varepsilon_{\delta_0} - \left\langle \tilde{\varphi}_\alpha \left| \hat{t} \right| \tilde{\varphi}_\alpha \right\rangle - \left\langle \tilde{\varphi}_\beta \left| \hat{t} \right| \tilde{\varphi}_\beta \right\rangle \right)$$