



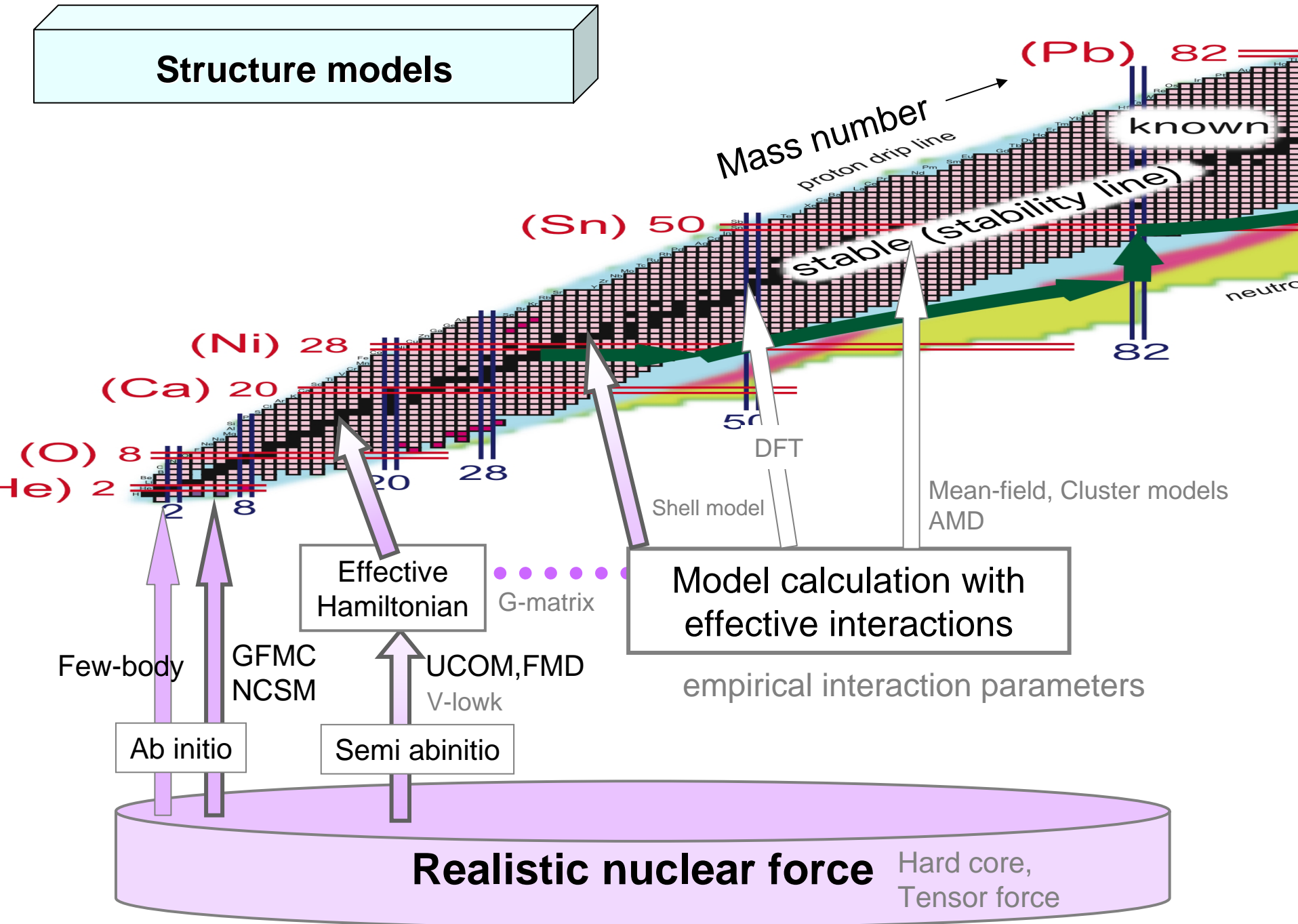
Structure of light unstable nuclei studied with effective interactions

Yoshiko Kanada-En'yo
YITP, Kyoto University

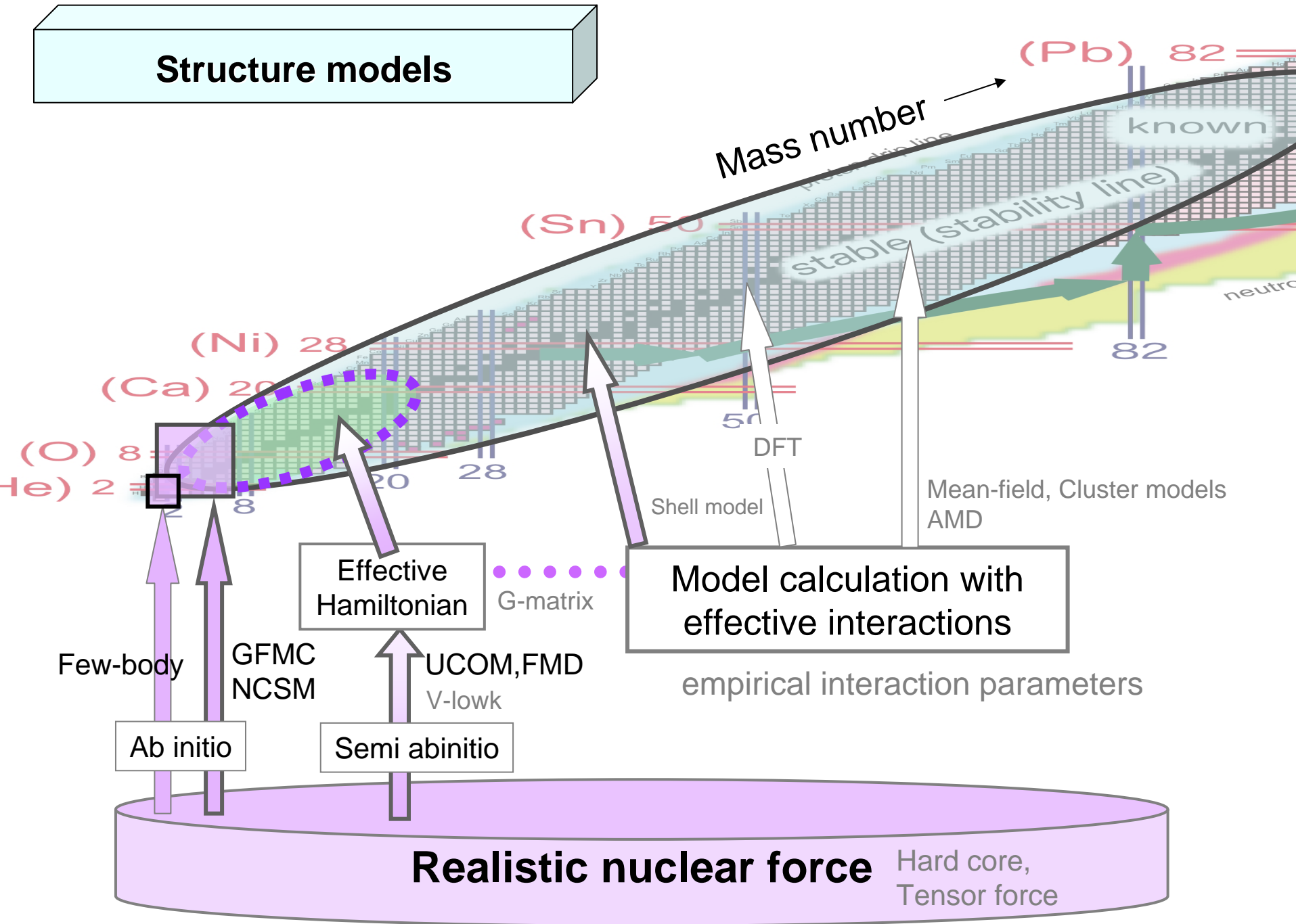


1. Introduction

Structure models



Structure models



phenomenological effective interactions

$$V_{ij} = \text{central} + \text{Is force}$$

2-body force

$$H = \sum_i t_i - T_G + \sum_{i<j} v(r_{ij})$$

Volkov, Minnesota (density independent)

$$v(r) = \sum_k (w_k + b_k P_\sigma - h_k P_\tau - m_k P_\sigma P_\tau) \exp\left(-\frac{r^2}{a_k^2}\right)$$

Gogny (density dependent)

$$v(r) = \sum_{k=1,2} (w_k + b_k P_\sigma - h_k P_\tau - m_k P_\sigma P_\tau) \exp\left(-\frac{r^2}{a_k^2}\right) + v_d (1 + x P_\sigma) \rho^\sigma(r) \cdot \delta(r)$$

2-body+3-body force

$$H = \sum_i t_i - T_G + \sum_{i<j} v(r_{ij}) + \sum_{i<j<k} V_3$$

MV1 force

$$V_3 = t_3 \delta(r_{12}) \delta(r_{23})$$

Enyo force

$$V_3 = t_3 \exp\left[-\frac{r_{12}^2 + r_{23}^2 + r_{31}^2}{a^2}\right]$$

1. Introduction

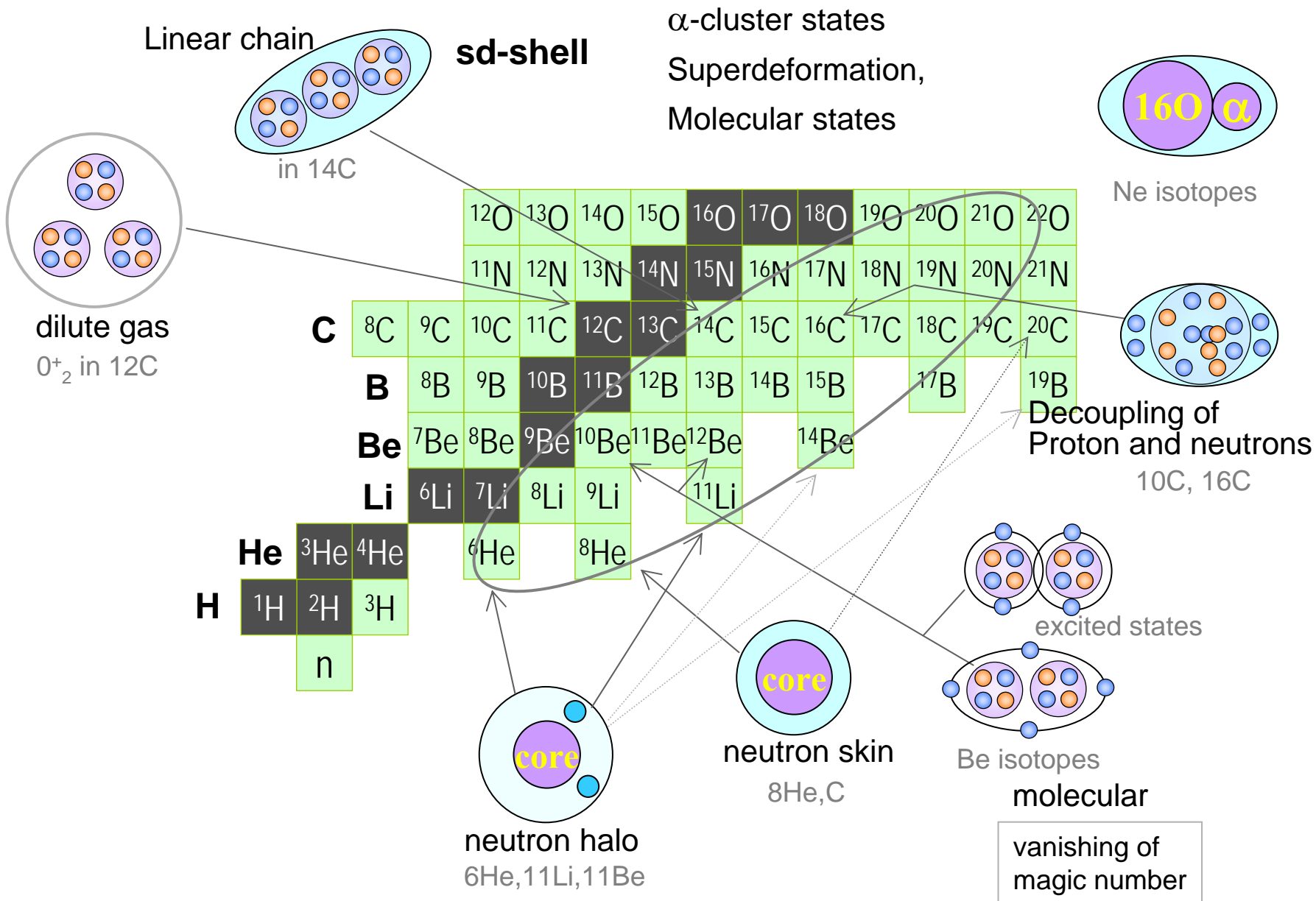
What are problems in present interactions ?

	N-N scattering	α - α pha-s	α radius/B.E.	p-shell radii/B.E	sd-shell radii/B.E	Matter saturation
+density dependence Gauss 2-body	Minnesota S-wave	✓	✓ / ✓	✓ / ✓		
	Volkov		✓ / ✓	✓ / ✓		
+density dependence	MV1		□ / ✓	✓ / ✓	✓ /	✓
	Gogny, Skyrn		□ / ✓	✓ / ✓	✓ / ✓	✓
+finite-rang -3-body	Tohsaki's		✓ / ✓	✓ / ✓	✓ / ✓	✓
	Enyo	←	✓	✓ / ✓	✓ / ✓	✓

Effective forces used in recent AMD calculations

- Volkov, Minnesota (density independent)
Light nuclei $A < 20$
 - MV1 force
p-shell, sd-shell nuclei
- Mass number dependent parameters
- Gogny and Skyrme (density dependent)
sd-shell, fp-shell nuclei up to $A=50$
 - Enyo force (3-body)
sd-shell nuclei up to $A=40$

Exotic structure in light nuclei





2. Formulation of AMD

Formulation of AMD: wave function

Wave function

$$\Phi = c\Phi_{\text{AMD}} + c'\Phi'_{\text{AMD}} + c''\Phi''_{\text{AMD}} + \dots$$

$$\Phi_{\text{AMD}} = \mathcal{A}\{\varphi_1, \varphi_2, \dots, \varphi_A\}$$

Slater det.

$$\varphi_i = \phi_{\mathbf{Z}_i} \chi_i \begin{cases} \phi_{\mathbf{Z}_i}(\mathbf{r}_j) \propto \exp\left[-\nu(\mathbf{r} - \frac{\mathbf{Z}_i}{\sqrt{\nu}})^2\right] \\ \chi_i = \begin{pmatrix} \frac{1}{2} + \xi_i \\ \frac{1}{2} - \xi_i \end{pmatrix} \times \begin{matrix} \text{spin} \\ \text{isospin} \end{matrix} \end{cases} \quad \begin{matrix} \text{spatial} \\ \text{Gaussian} \end{matrix}$$

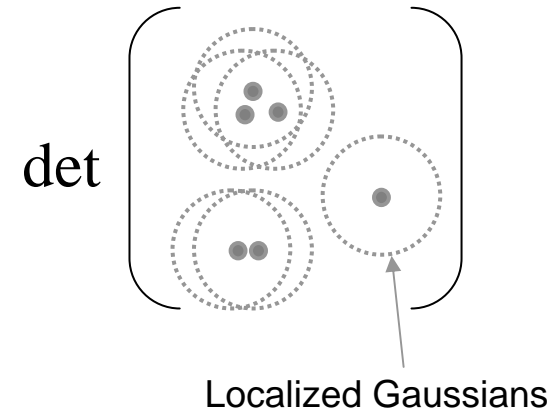
p or *n*

Complex parameter $\mathbf{Z} = \{ \mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_A, \xi_1, \dots, \xi_A \}$

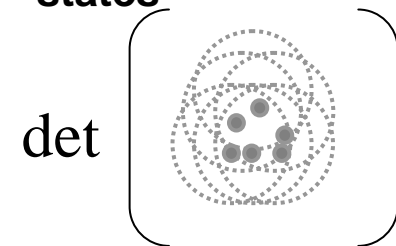
Variational method

$$\delta \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0$$

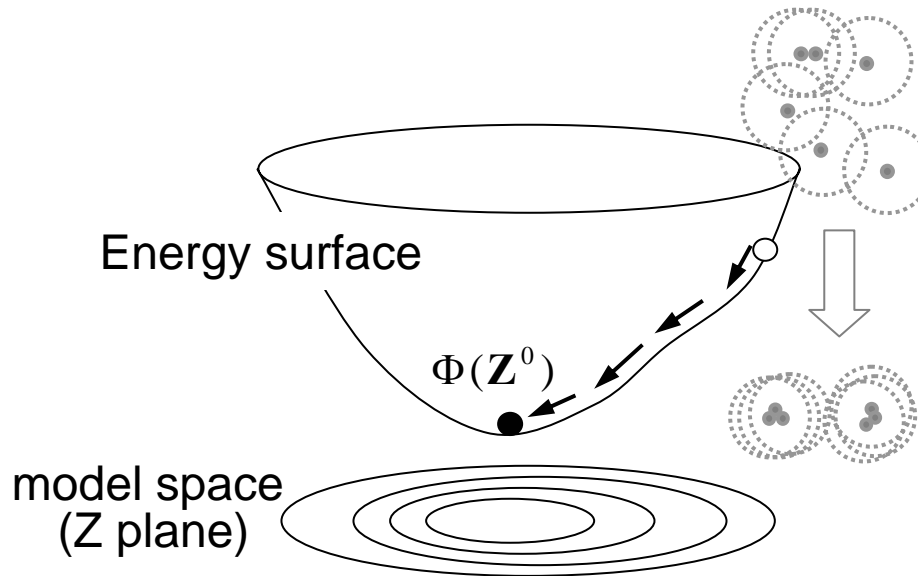
Cluster structure



Shell-model-like states



Formulation of AMD: energy variation



$$\delta \frac{\langle \Phi(\mathbf{Z}) | H | \Phi(\mathbf{Z}) \rangle}{\langle \Phi(\mathbf{Z}) | \Phi(\mathbf{Z}) \rangle} \equiv \delta E = 0$$

frictional cooling method

$$\frac{d\mathbf{Z}}{dt} = (\lambda + i\mu) \frac{1}{i\hbar} \frac{\partial E}{\partial \mathbf{Z}^*}$$

Simple AMD

Variation after parity projection before spin pro. (VBP)

VAP

Variation after spin-parity projection

$$\delta \frac{\langle P\Phi(\mathbf{Z}) | H | P\Phi(\mathbf{Z}) \rangle}{\langle P\Phi(\mathbf{Z}) | P\Phi(\mathbf{Z}) \rangle} \equiv \delta E = 0$$

Constraint AMD & superposition

≈

AMD + GCM

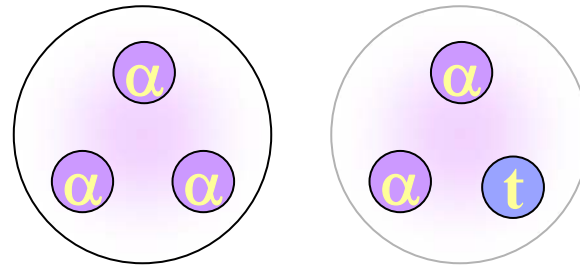
Variation with Constraints

Then superposition of basis wave functions.

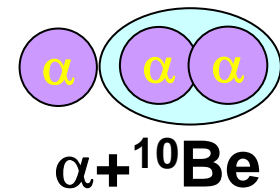
Multi-configuration AMD

3. Structure of unstable nuclei

3-1. cluster gas in $^{12}\text{C}^*$, $^{11}\text{B}^*$ ($^{11}\text{C}^*$), $^8\text{He}^*$



3-2. linear chain in C isotopes

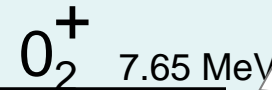
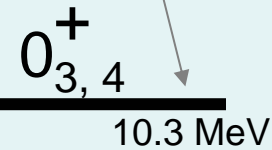
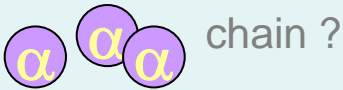


3-1. three-center cluster states:

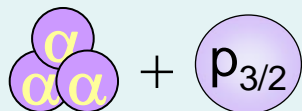
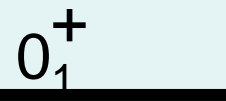


2-2. Cluster gas-like states

^{12}C



$^8\text{Be} + \alpha$

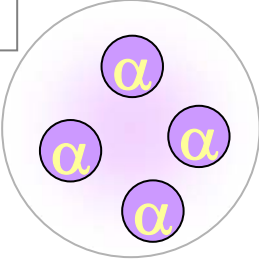


$3\alpha + p_{3/2}$ closed

A. Tohsaki et al., (2001)
 Funaki et al. (2003)

Dilute cluster gas
Bosonic behavior

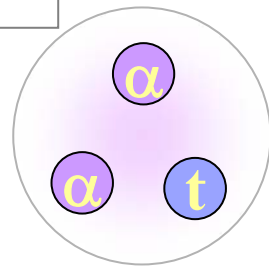
^{16}O



Tohsaki et al., Yamada et al.,
 Funaki et al. Wakasa et al.,

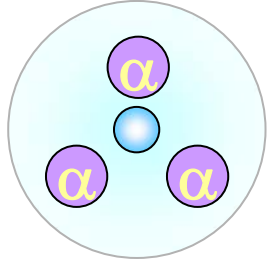
$^{16}\text{O}(0^+_{5}) ?$

$^{11}\text{C}, ^{11}\text{B}$



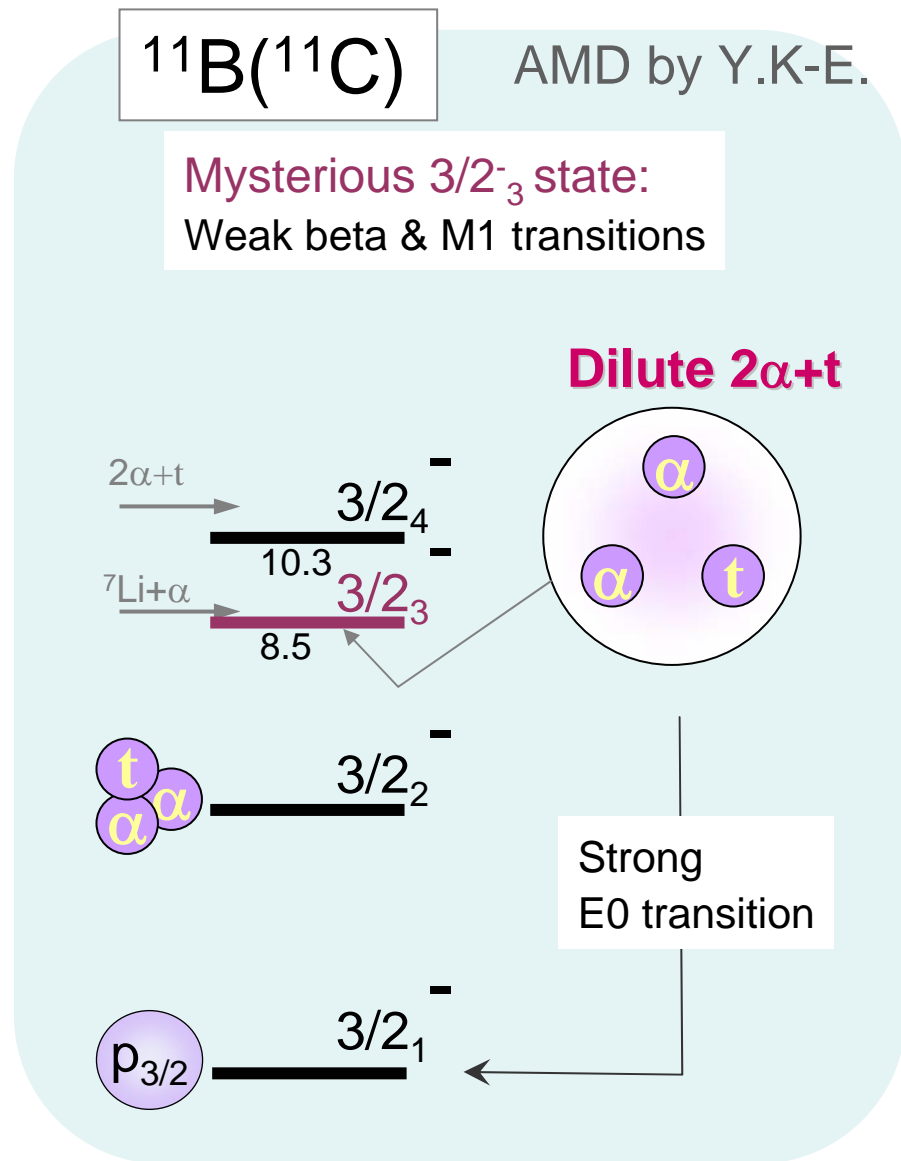
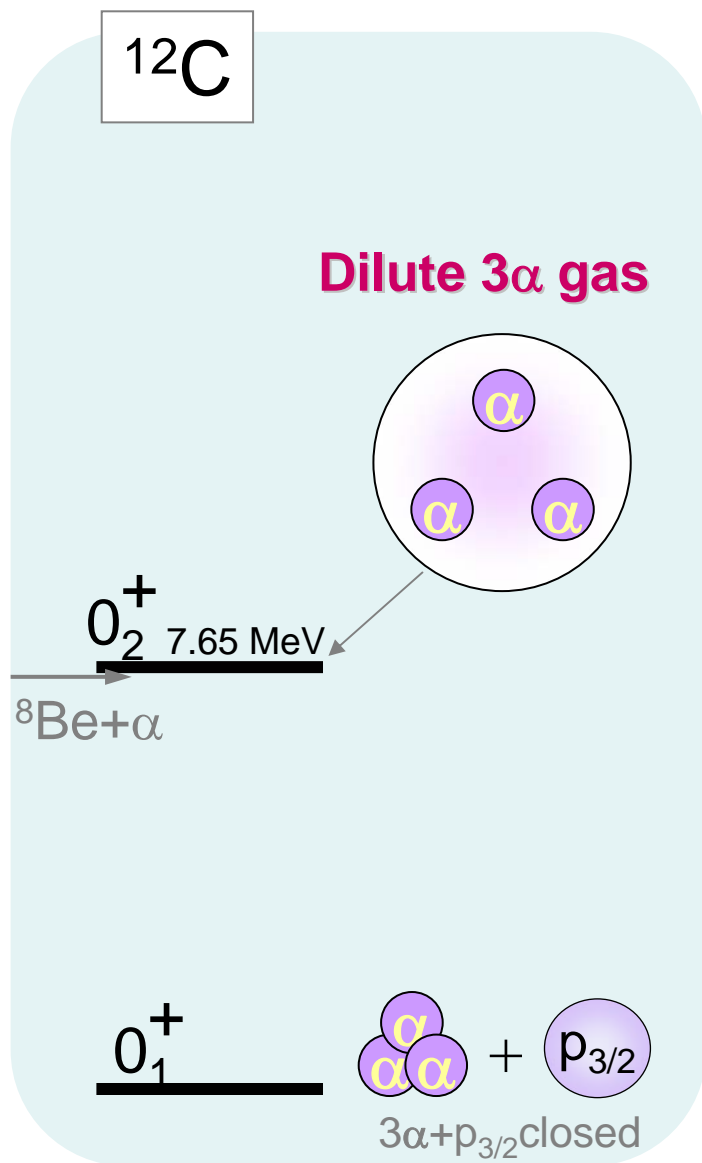
K-E., Kawabata et

$^{13}\text{C}, ^{14}\text{C}$



Itagaki, K-E.,
 Kawabata et

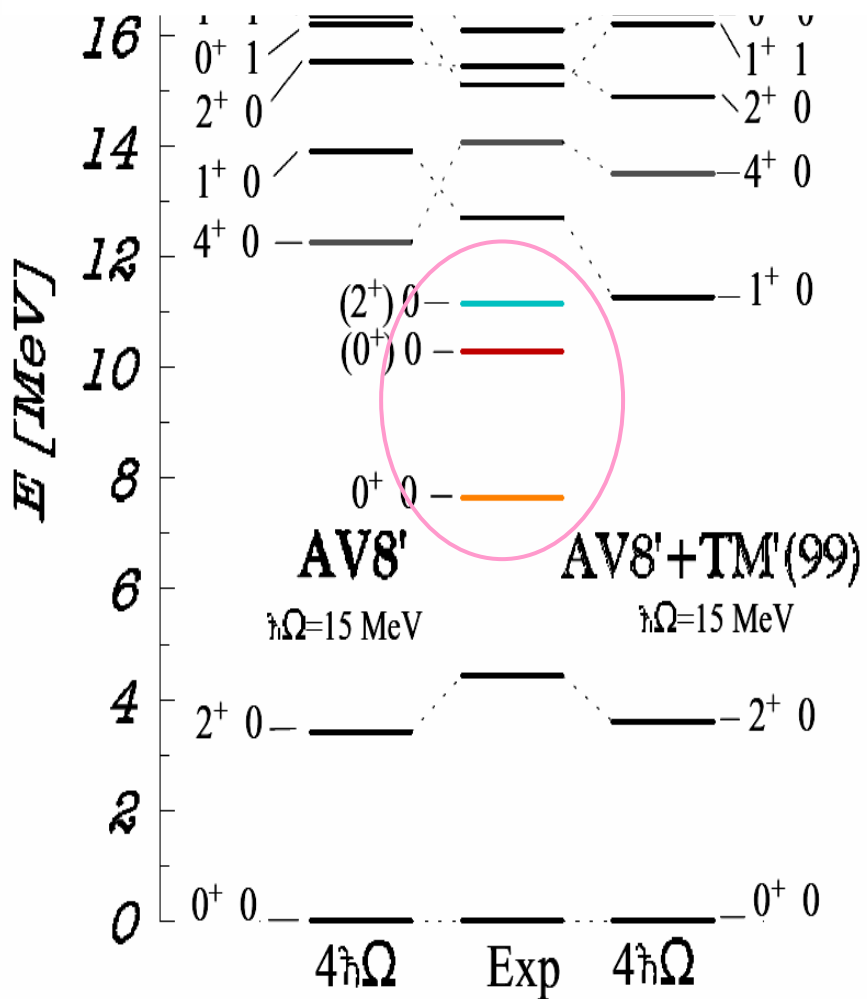
Three-center cluster states of ^{11}B and ^{11}C : analogy with $^{12}\text{C}(0^+_2)$



Missing in shell model calculations

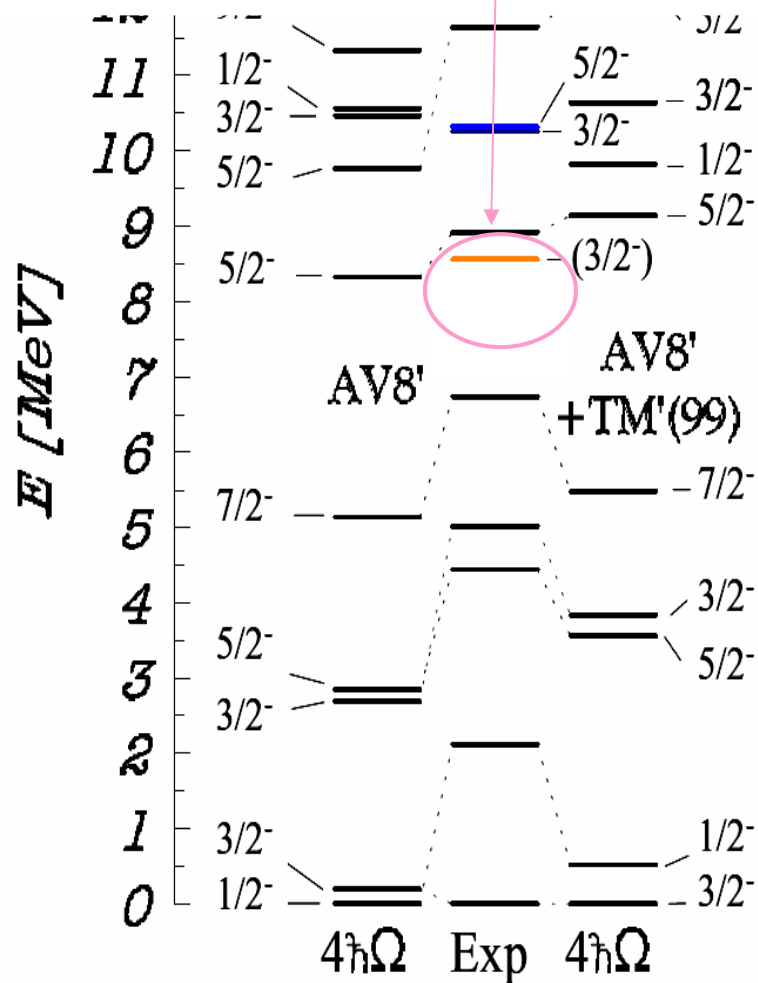
^{12}C

NCSM by Navratil et al. (2003)



^{11}B

$3/2^-_3$ state is missing



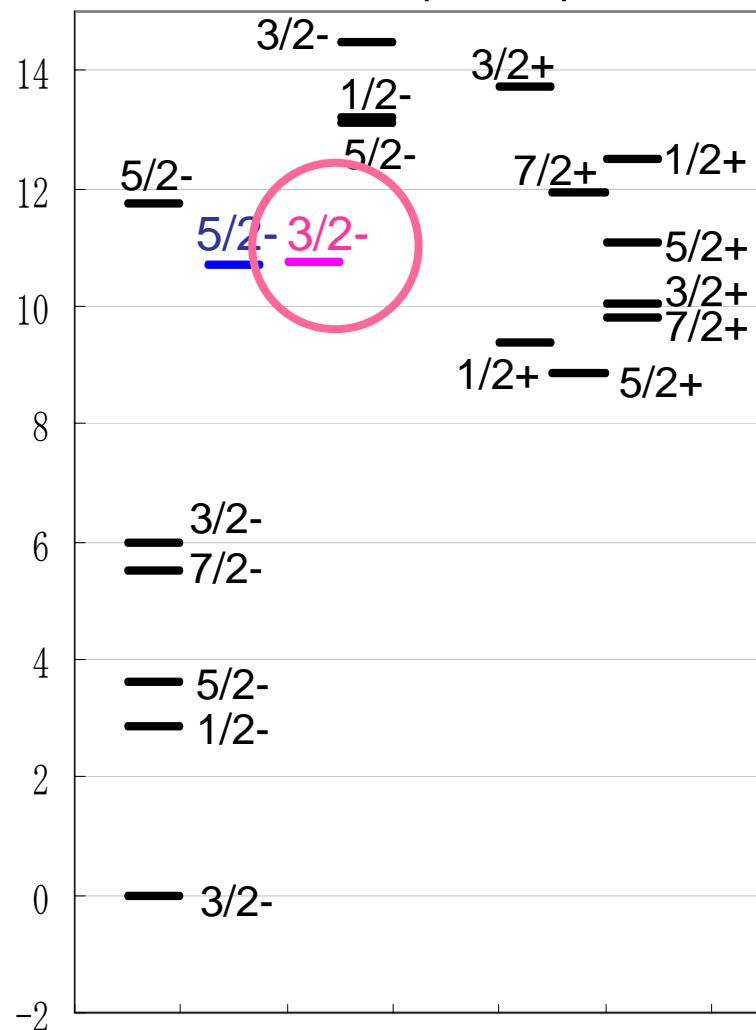
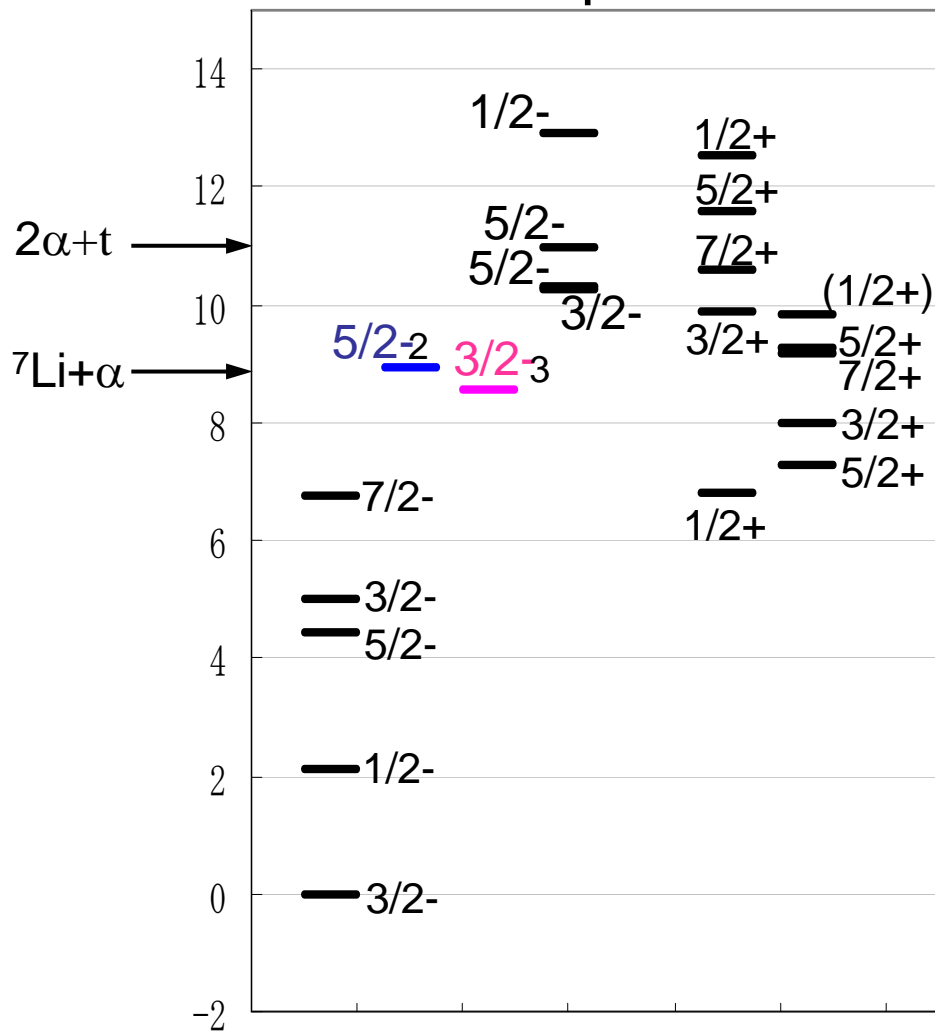
Present results by AMD

^{11}B

Energy levels

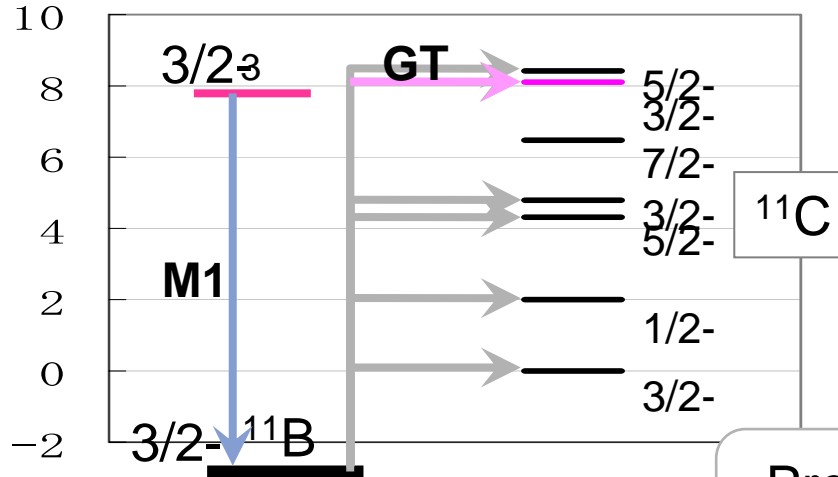
exp

AMD(VAP)



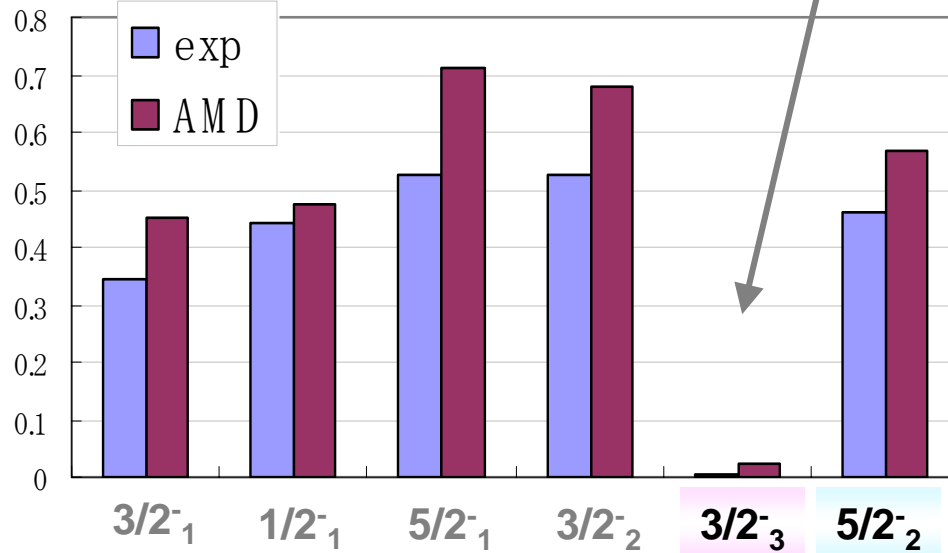
$^{11}\text{B} \rightarrow ^{11}\text{C}^*$ GT-transition strength

$^{11}\text{B} \rightarrow ^{11}\text{C}^*$



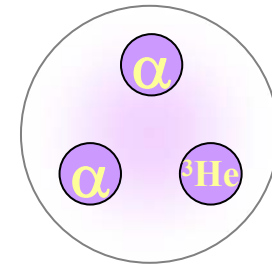
B(GT)

Present calc. reproduces weak GT and M1 transitions



exp: Y. Fujita, et al. PRC 70, 011306(R)(2004).

AMD:



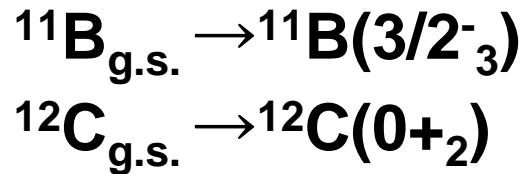
$2\alpha + ^3\text{He}$
loosely bound 3 clusters

Dilute $2\alpha + ^3\text{He}(t)$

Isoscalar E0 and radius of $^{11}\text{B}(3/2^-_3)$ state

Large $B(E0; IS) \equiv |M_p + M_n|^2$ (fm⁴)

Inelastic scattering data (d,d')
Kawabata et al. nucl-ex/051204(2005)



Exp.	AMD
94 ± 16	94
121 ± 9	137

Large radius (AMD)

Dilute
Cluster states

^{11}B	r.m.s.r (fm)
$3/2^-_1$	2.5
$3/2^-_2$	2.7
$3/2^-_3$	3.1

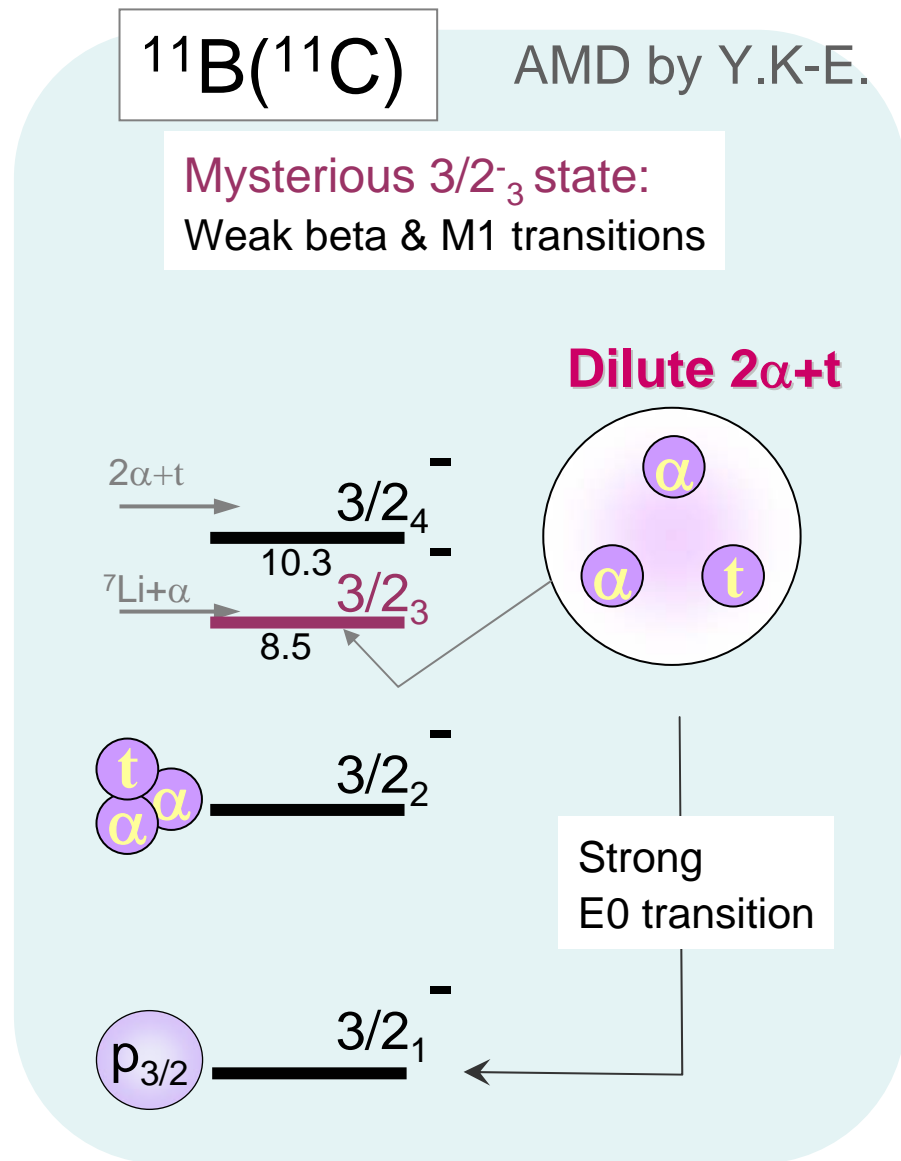
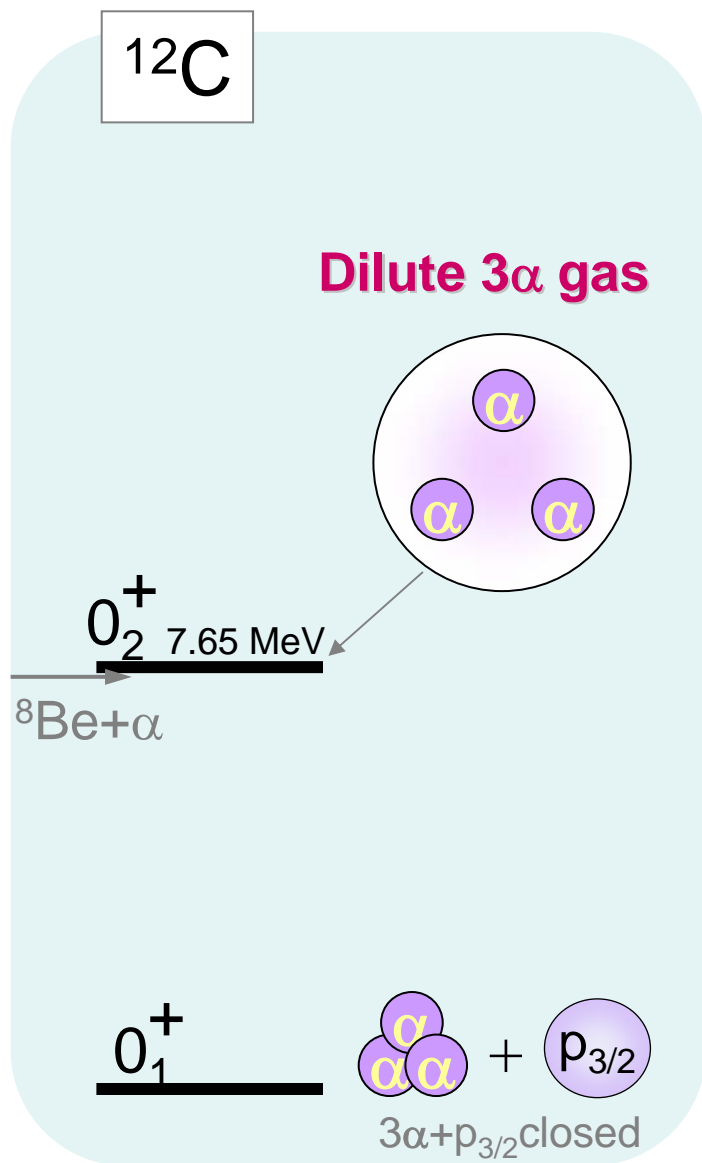
$2\alpha+t$

^{12}C	r.m.s.r (fm)
0^+_1	2.5
0^+_2	3.3

3α

3.5 (RGM)
3.8 (ACW)

Three-center cluster states of ^{11}B and ^{11}C : analogy with $^{12}\text{C}(0^+_2)$

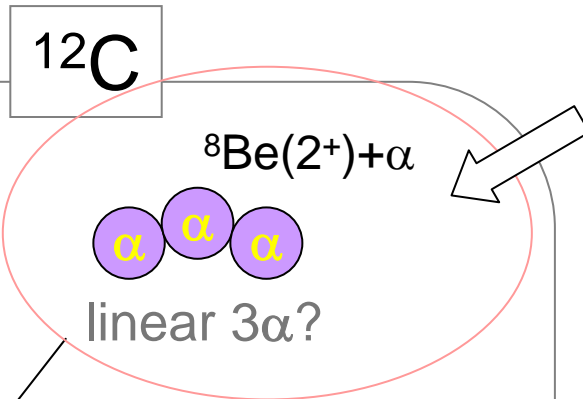




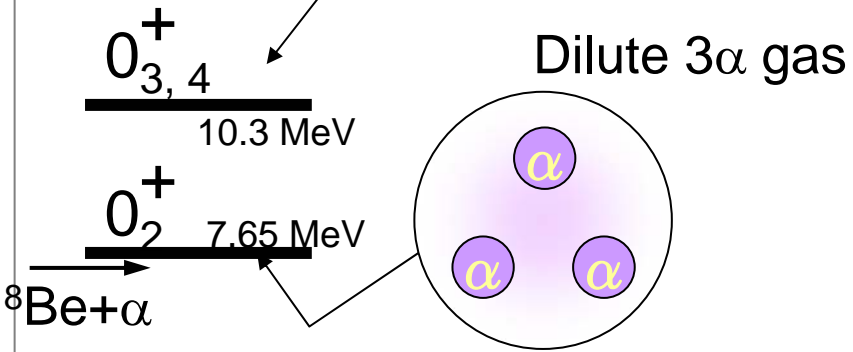
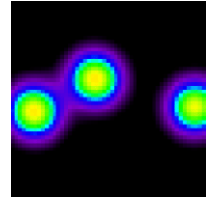
**3-2. Linear chain structure
in C isotopes**

Linear chain-like structure

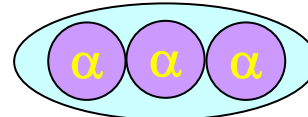
Uegaki et al.
 Kamimura et al.
 Tohsaki et al.
 Funaki et al.
 Kurokawa et al.
 Kanada-En'yo et al.



theoretically suggested
 in $3\alpha\text{GCM}$, AMD, FMD and
 $3\alpha\text{OCM}$ calc.



In neutron-rich C

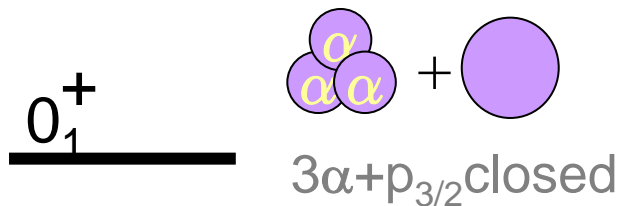
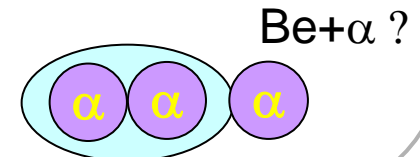
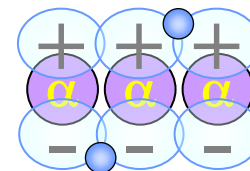


^{14}C ?

can be stabilized by
 additional neutrons ?

^{15}C K-E. et al,

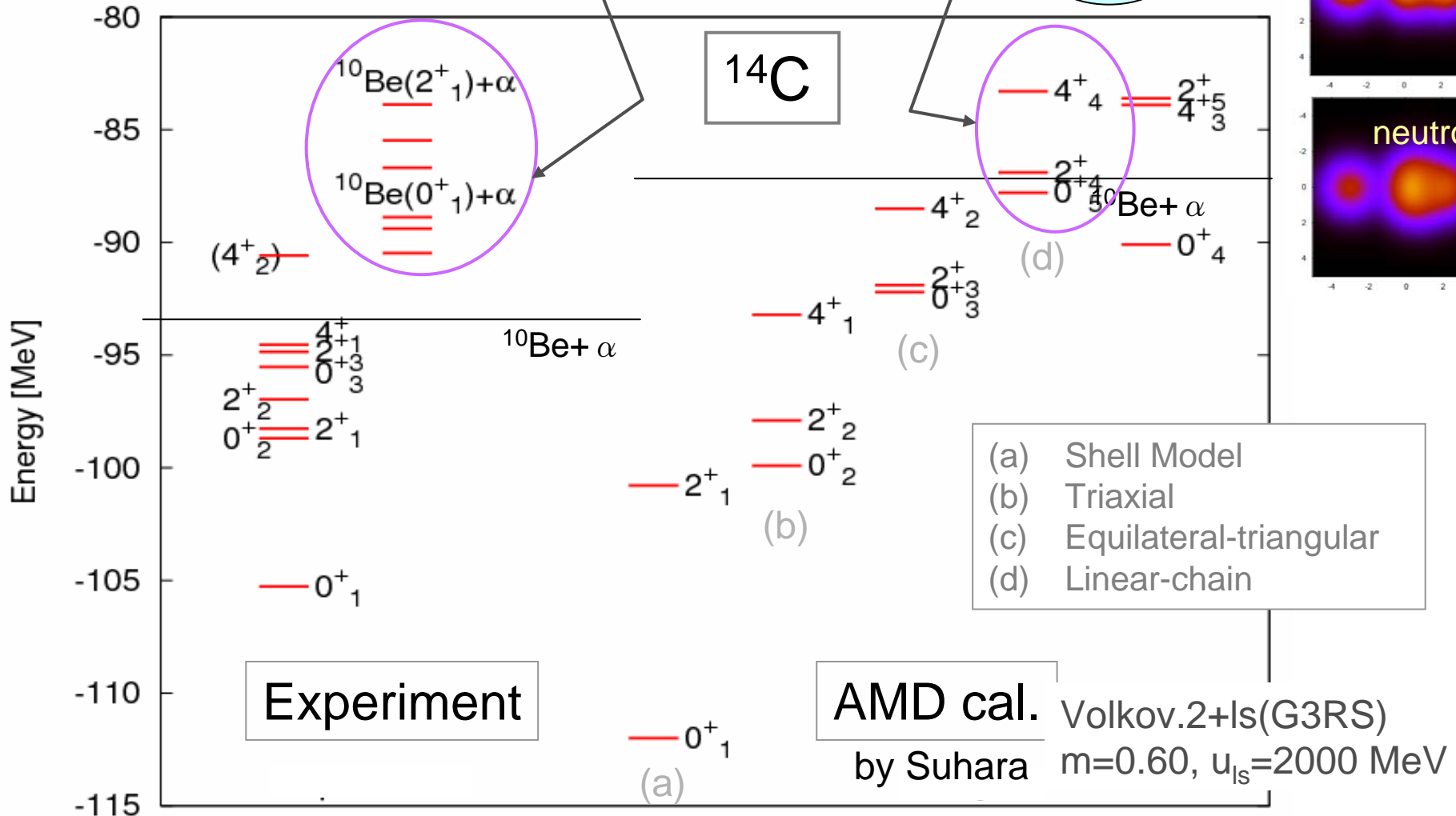
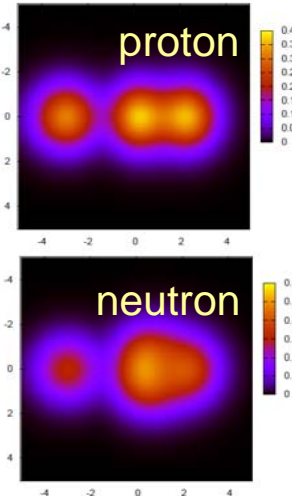
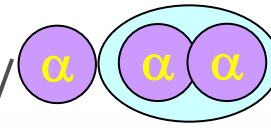
^{16}C Itagaki et al., von Oertzen et al.,



Linear chain-like state in ^{14}C

Recent observation
by price et al.(PRC75,2007)
no spin-parity assignment

Linear chain band



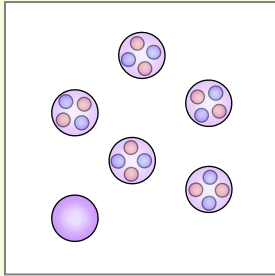
preliminary

3-3. dineutron cluster in $^8\text{He}^*$

Y. K-E., arXiv:0707.2120

Dineutron gas ?

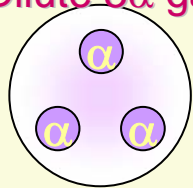
α -condensation



Roepche et al.(1998)

$^{12}\text{C}(0_2^+)$

Dilute 3α gas

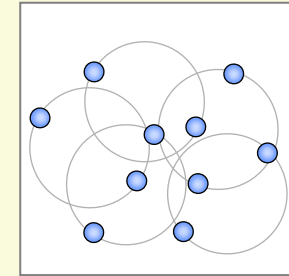
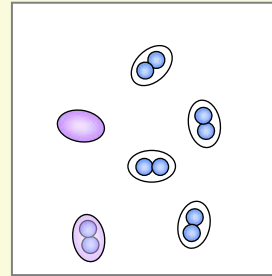


$^{16}\text{O}^*$

Tohsaki et al., Yamada et al.,
Funaki et al.

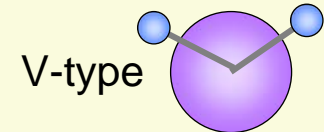
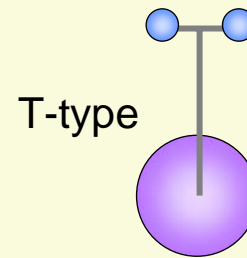


di-neutron in dilute neutron matter

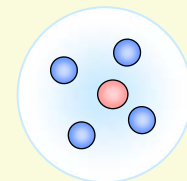
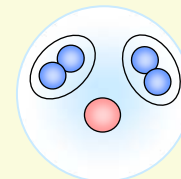


Matsuo et al.(2006)

di-neutrons in halo nuclei (^6He , ^{11}Li)



$5, 7\text{H}^*$? Aoyama, et al.

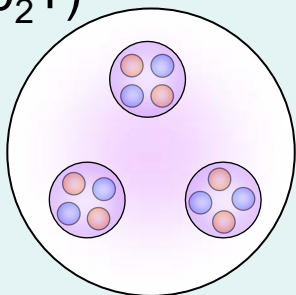


Two dineutrons in ^8He

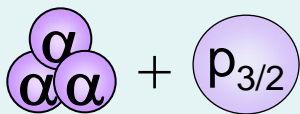
^{12}C

Dilute cluster gas

$^{12}\text{C}(0_2^+)$

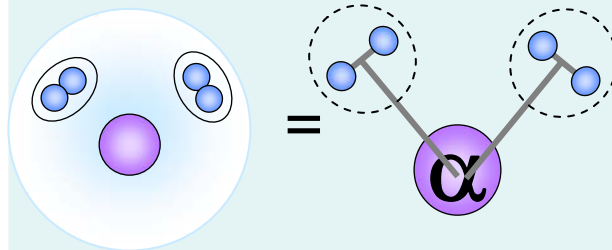


$^{12}\text{C}(0_1^+)$

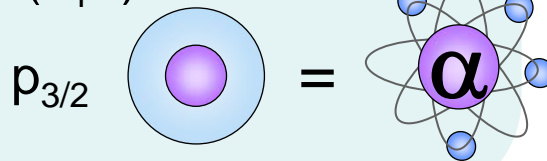


^8He

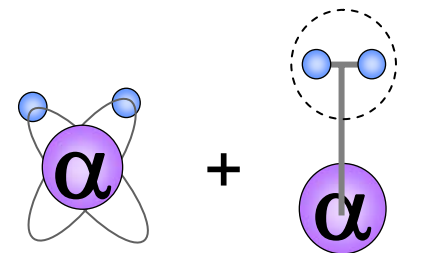
Excited States ?



$^8\text{He}(0_1^+)$



$^6\text{He}(0_1^+)$



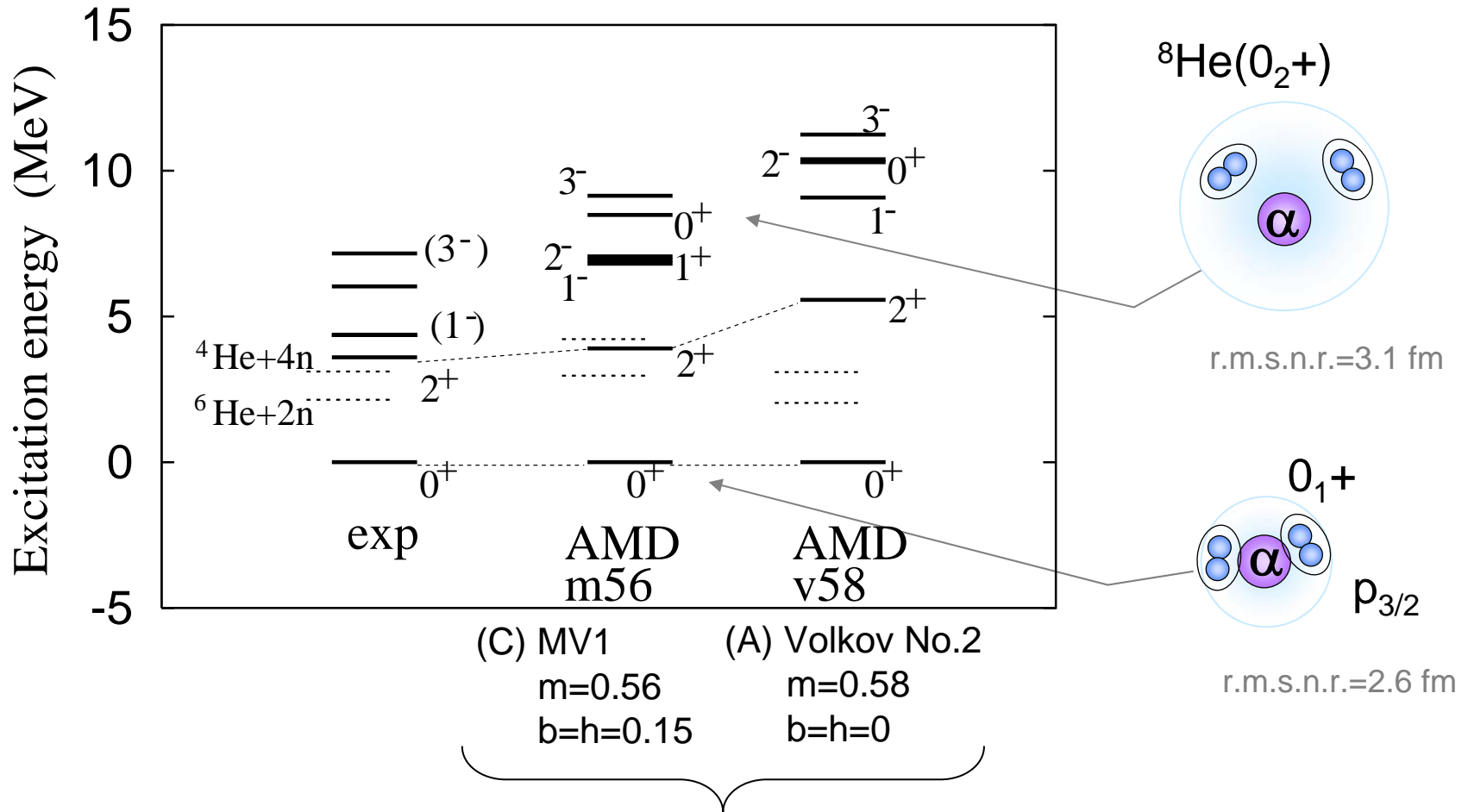
V-type

T-type

Possible dineutron gas-like state of ^8He

^8He

AMD+GCM calc. suggests dineutron gas-like state



AMD+GCM

with effective N-N interactions(MV1, Volkov)

Fine tuning of effective N-N interaction

Central (MV1 or Volkov)+LS (G3RS with $u_{\perp}=-u_{\parallel}=2000$ MeV)

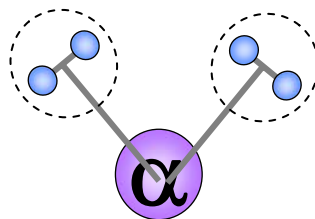
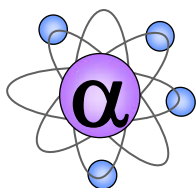
$$v(r) = ((1-m) + bP_{\sigma} - hP_{\tau} - mP_{\sigma}P_{\tau}) \sum_{k=1,2} V_k \exp\left(-\frac{r^2}{a_k^2}\right)$$

N-N properties

	(A) Volkov 2	(B) MV1	(C) MV1	(D) V2	exp.
	m=0.58 b=h=0	m=0.62 b=h=0	m=0.56 b=h=0.15	m=0.51, b=h=0.15	
n-n	bound	bound	almost unbound	unbound	unbound
S(1E) fm	9.7	6.4	>100	-24	-18
S(3E) fm	9.7	6.4	4.2	5.4	5.4

${}^8\text{He}(0_1^+)$

$p_{3/2}$



n-n

α -n: ${}^5\text{He}$

α -2n

2n-2n

α - α ?

Effective N-N interaction

Effective N-N Interaction

Central (MV1 or Volkov)

+LS (G3RS with $u_{\parallel} = -u_{\perp} = 2000$ MeV)

(A) Volkov 2

$m=0.58,$

$b=h=0$

(B) MV1

$m=0.62,$

$b=h=0$

(C) MV1

$m=0.56,$

$b=h=0.15$

(D) v2

$m=0.51,$

$b=h=0.15$

n-n

too strong

too strong

slightly strong

good

α -n: ^5He

good

good

slightly strong

too strong

α -2n: ^6He

good

strong

good

weak

α - α

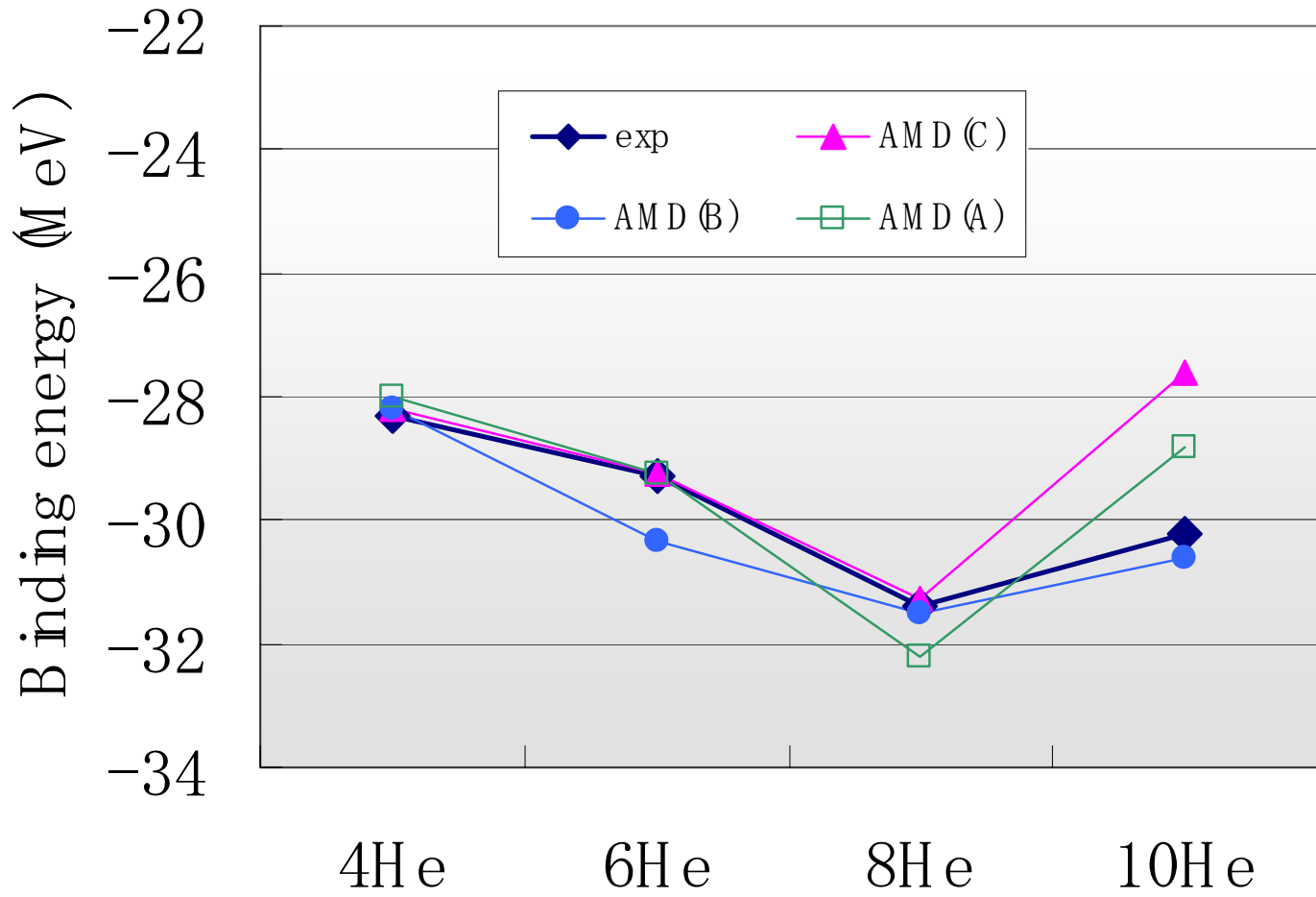
good

too weak

slightly weak

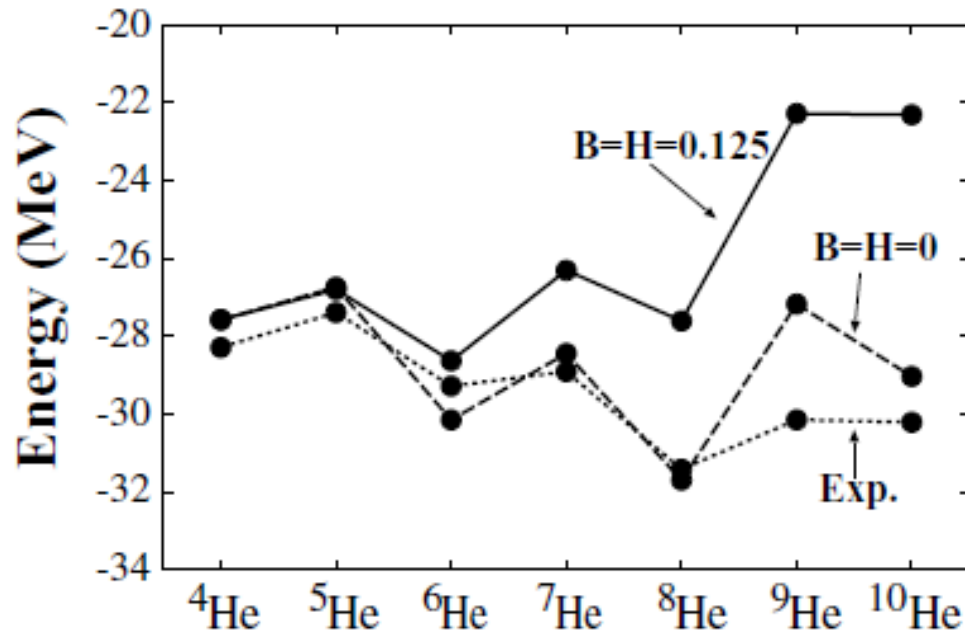
too strong

Binding energy



Other works with effective nuclear forces

S.Aoyama et al. ,PRC 74, 017307(2006)
Volkov No.2 (m=0.60)



nn(T=1) force
is too strong

FIG. 2. Systematics of the calculated binding energies of the He isotopes compared with experimental data. The solid line corresponds to the case of $B = H = 0.125$, whereas the dashed line to the case of $B = H = 0$. The experimental values are shown as the dotted line.



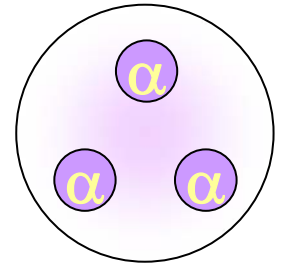
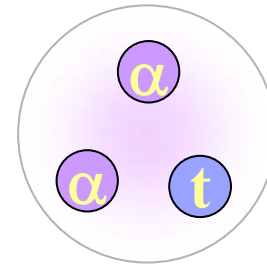
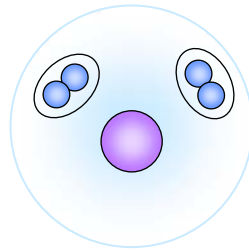
4. Summary

Various cluster states appear in excited states.

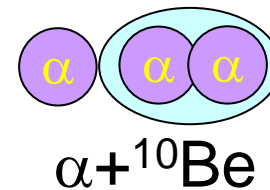
1. Cluster gas states similar to $^{12}\text{C}^*$

$^{11}\text{B}^*(^{11}\text{C}^*)$ with $2\alpha+t(^3\text{He})$

$^8\text{He}^*: \alpha+2n+2n$



2. linear chain in $^{14}\text{C}^*$



Problems in effective interactions

Effective interactions with a simple form (central and LS Forces) were used.

Global fit of experimental data with one parameter set is difficult.
Fine tuning of interaction parameters was done.
System depending parameters.

For more quantitative reproduction of energy levels and for more predictive power, we need to overcome problems of the present effective interactions.

Future plan: Model calculations based on realistic force

Structure dependence: explicit treatment of tensor force

Less structure dependent part:

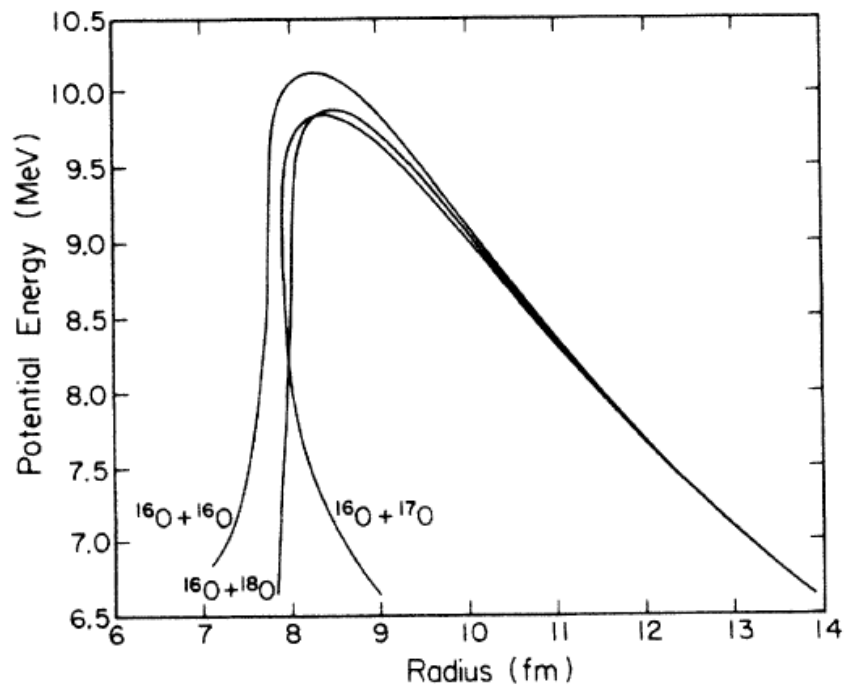
Hard core of central force with UCOM-like treatment

density-dependent l_s force, other operator terms ?

Link of effective interactions with realistic interactions: GCM

J.Thomas, Phys.Rev.C 33, 1679 (1986)

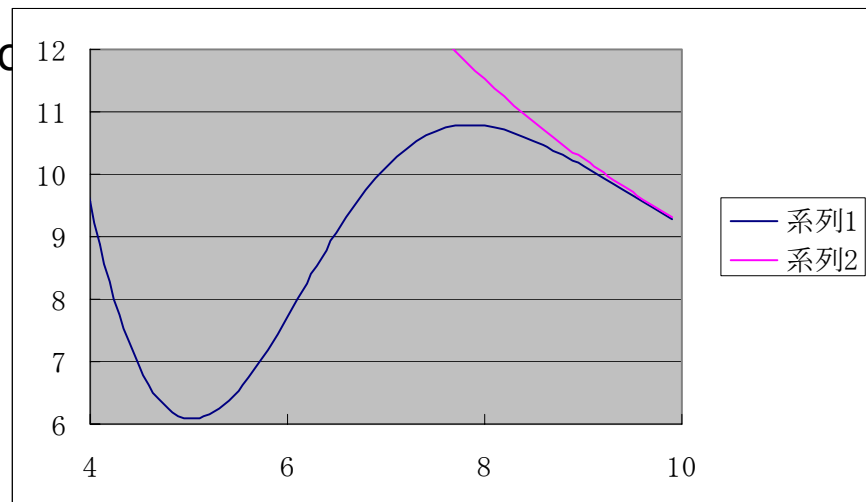
16O-16O potential reduced from fusion



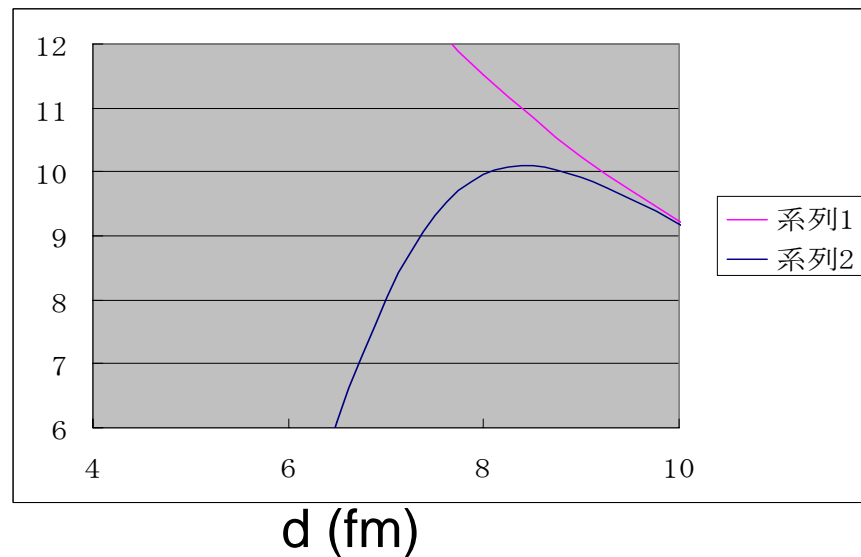
Barrier 10 MeV high
at 8 fm

Gogny(D1S)

V(d)



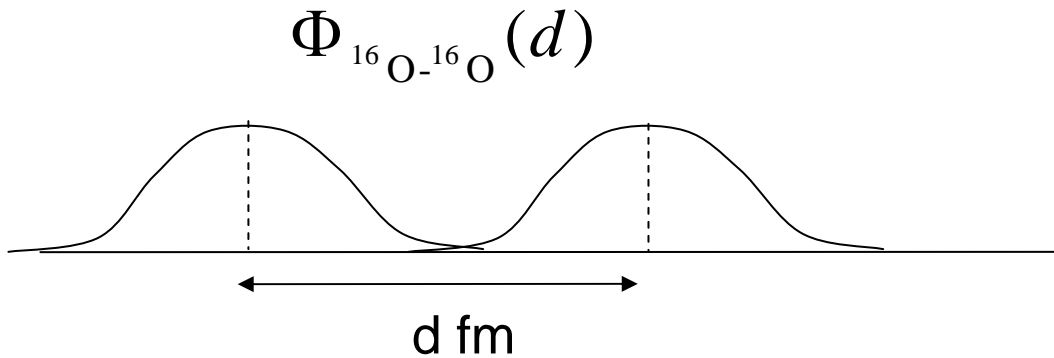
MV1(m=0.60)



16O-16O

Barrier height : 0.5~1 MeV larger, Fusion cross : 2-3 times smaller

Treatment of relative motion (coordinate)

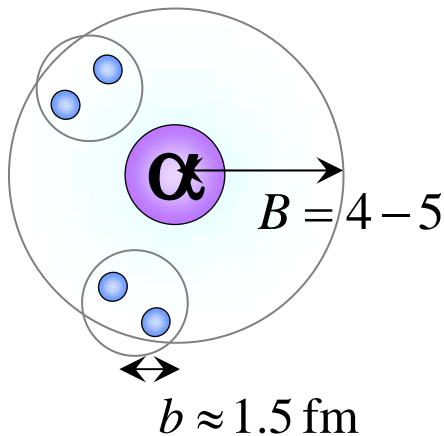


$$E(d) = \frac{\langle \Phi_{16\text{O}-16\text{O}}(d) | H | \Phi_{16\text{O}-16\text{O}}(d) \rangle}{\langle \Phi_{16\text{O}-16\text{O}}(d) | \Phi_{16\text{O}-16\text{O}}(d) \rangle}$$

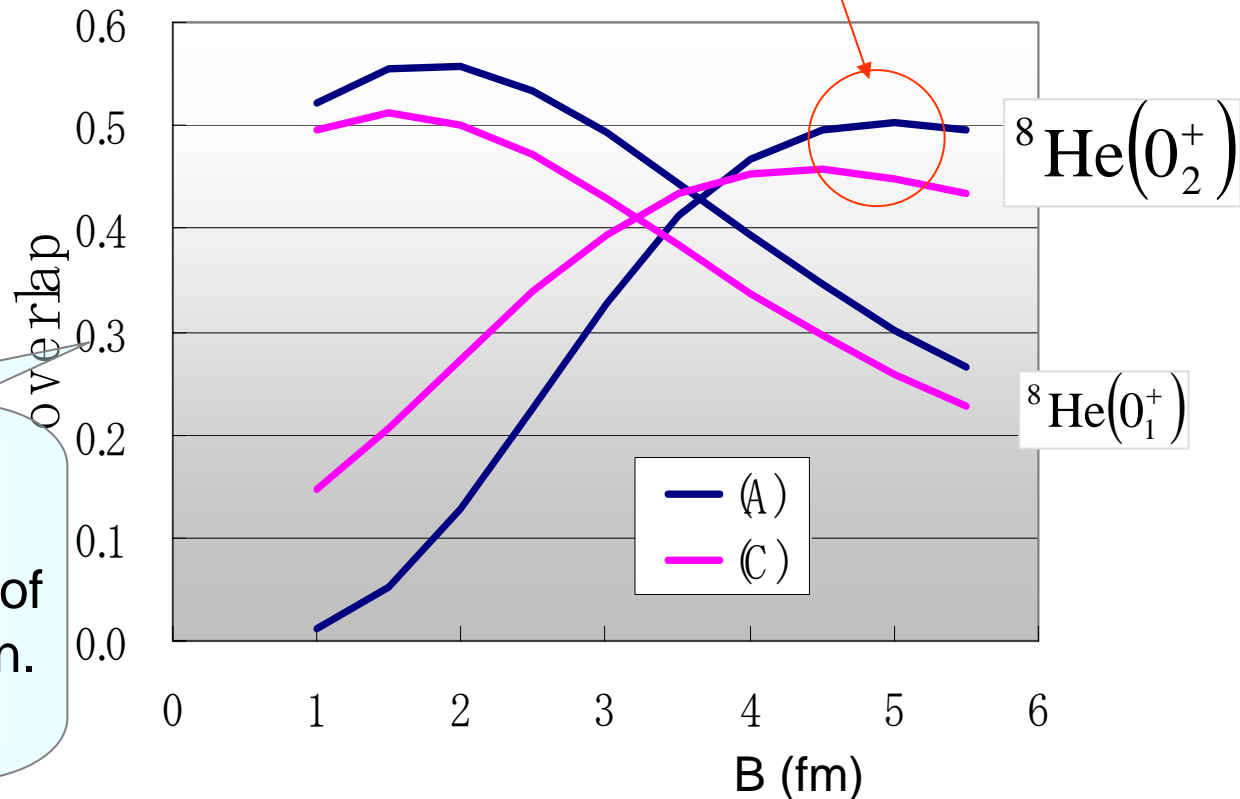
$$V(d) \approx E(d) - T_{rel}(\infty)$$

$2n$ condensate wave function

$$\Phi_{Cond}(B) = \int d^3\mathbf{R}_1 d^3\mathbf{R}_2 \exp\left(-\frac{(\mathbf{R}_1 - \mathbf{R}_\alpha)^2}{B^2}\right) \exp\left(-\frac{(\mathbf{R}_2 - \mathbf{R}_\alpha)^2}{B^2}\right) A[\phi_{2n}(\mathbf{R}_1)\phi_{2n}(\mathbf{R}_2)\phi_\alpha(\mathbf{R}_\alpha)]$$



$$\left| \langle {}^8\text{He}(0_2^+) | \Phi_{Cond}(B = 4-5 \text{ fm}) \rangle \right|^2 \approx 0.5$$



${}^8\text{He}(0_2^+)$

has 50 % component of
 $2n$ condensate wave fn.

$$\Phi_{Cond}(B = 4-5 \text{ fm})$$