

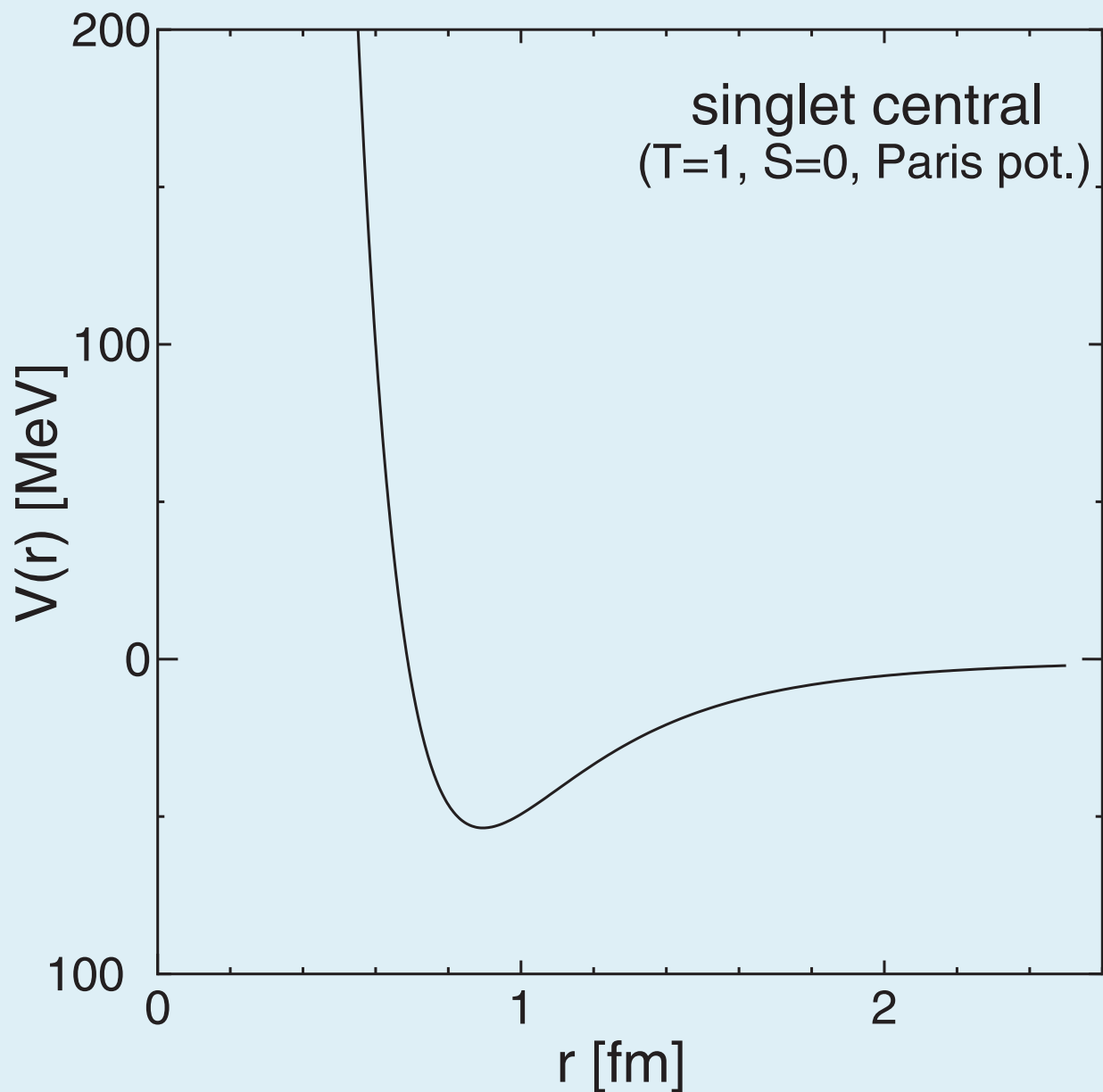
# Structure of neutron-rich carbon and oxygen isotopes in a restricted no-core shell-model space

**Shinichiro Fujii**  
**(Kyushu Univ.)**



九州大学  
KYUSHU UNIVERSITY

First EMMI-EFES Workshop on  
Neutron-Rich Exotic Nuclei EENEN09  
February 9-11, 2009, GSI Darmstadt



If we employ the bare NN force directly, extremely large-scale calculations are needed. ( $300\hbar\Omega$  space with h.o. basis states for the deuteron!)



To perform a realistic calculation in a smaller model space, we need to derive an effective interaction microscopically.



**Similarity / Unitary transformation**

# Derivation of effective interaction (Hamiltonian) by means of unitary transformation

## Hamiltonian

$$H = H_0 + V$$

## Unitary transformation of $H$

$$\tilde{H} = U^{-1}HU$$

$$U = e^S, \quad (S : \text{anti-Hermitian}, S^\dagger = -S)$$

## Decoupling equation

$$Q(e^{-S}He^S)P = 0$$

## Solution

$$S = \text{arctanh}(\omega - \omega^\dagger), \quad \omega = Q\omega P$$

(with the restrictive condition  $PSP = QSQ = 0$ )

K. Suzuki, Prog. Theor. Phys. **68** (1982), 246

## Effective Hamiltonian

$$\underline{H_{\text{eff}} = P\tilde{H}P}$$

## Effective interaction

$$\underline{V_{\text{eff}} = P\tilde{H}P - PH_0P}$$

## Unitary transformation operator $U$ in terms of $\omega$

$$U = (1 + \omega - \omega^\dagger)(1 + \omega^\dagger\omega + \omega\omega^\dagger)^{-1/2}$$

$$= \begin{pmatrix} P(1 + \omega^\dagger\omega)^{-1/2}P & -P\omega^\dagger(1 + \omega\omega^\dagger)^{-1/2}Q \\ Q\omega(1 + \omega^\dagger\omega)^{-1/2}P & Q(1 + \omega\omega^\dagger)^{-1/2}Q \end{pmatrix}$$

S. Ōkubo, Prog. Theor. Phys. **12** (1954), 603

# Our ongoing study

- **Unitary-model-operator approach (UMOA)**
  - Closed-shell nuclei, single-particle (-hole) states in nuclei up to the *pf*-shell region
  - $\Lambda$  hypernuclei

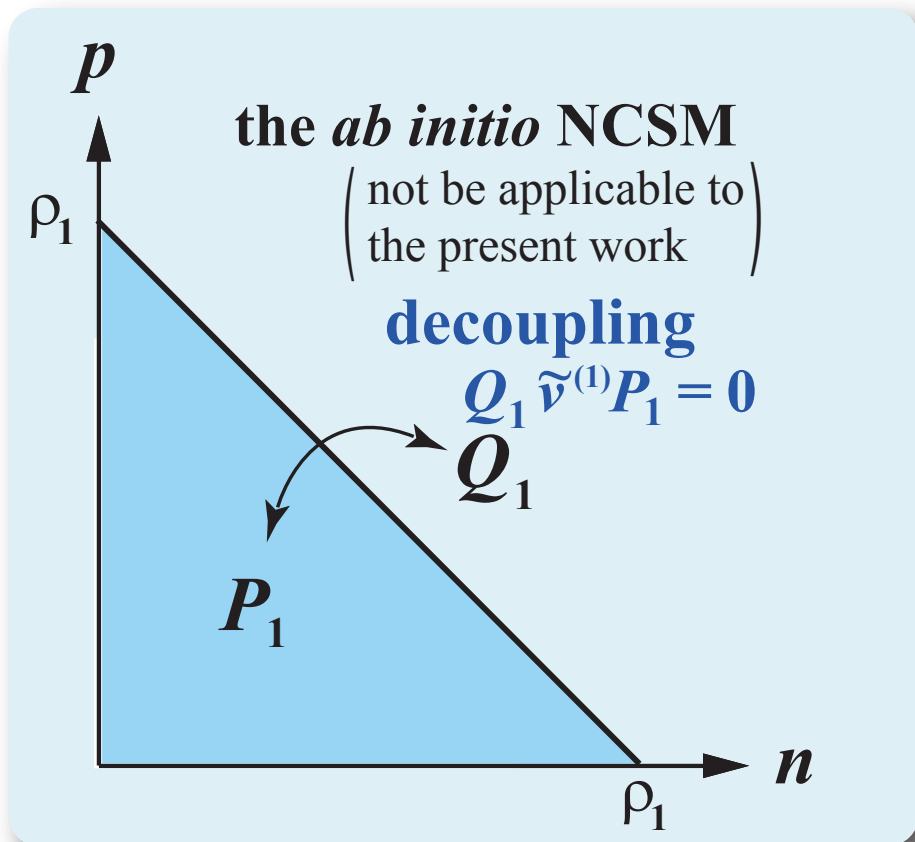
S. Fujii, R. Okamoto, and K. Suzuki, Phys. Rev. C **69**, 034328 (2004)

- **"No-core" shell model (in a restricted model space)**  
Hybrid method combining a no-core type of shell model with single-particle information obtained by the UMOA
  - Neutron-rich carbon isotopes, oxygen isotopes

S. Fujii, T. Mizusaki, T. Otsuka, T. Sebe, and A. Arima, Phys. Lett. **B650**, 9 (2007)

# Derivation of effective interaction

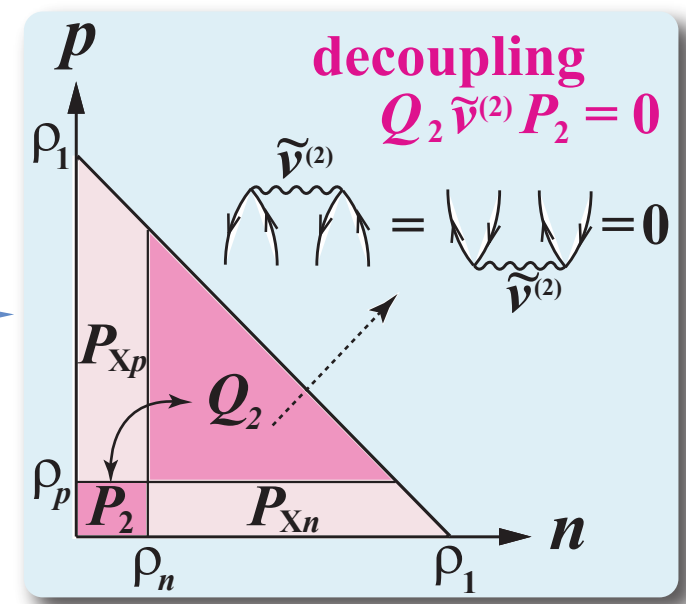
## • Eff. int. in a huge model space



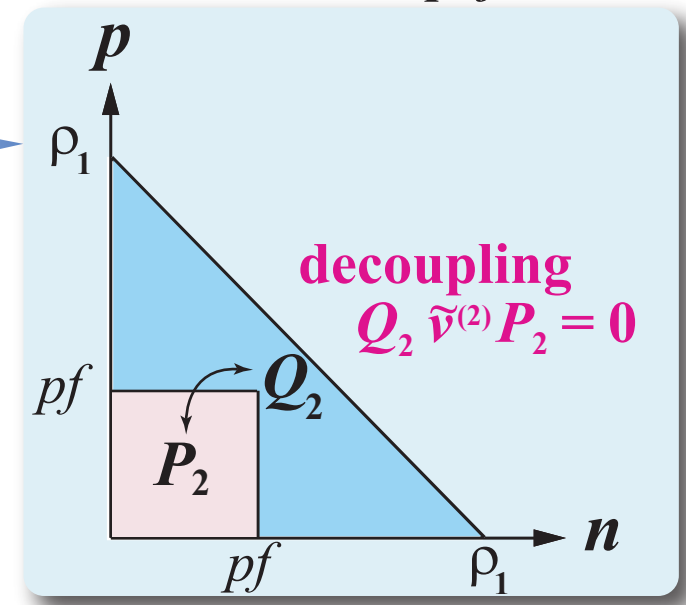
$\rho_1 = 2n_a + l_a + 2n_b + l_b$  ( $\{n_a, l_a\}$  and  $\{n_b, l_b\}$ : sets of h.o. quantum numbers of the two-body states)

- ( For details,
- S. F., T. Mizusaki, T. Otsuka, T. Sebe, and A. Arima, Phys. Lett. **B650**, 9 (2007).
  - S. F., R. Okamoto, and K. Suzuki, Phys. Rev. C **69**, 034328 (2004).

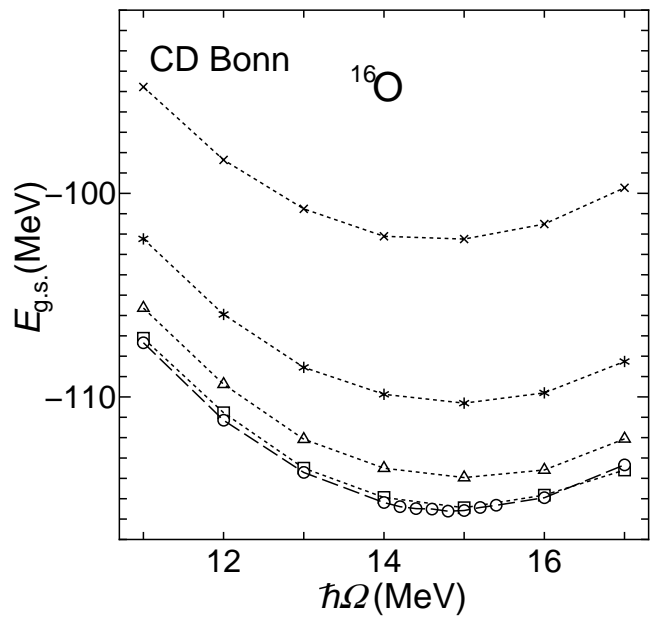
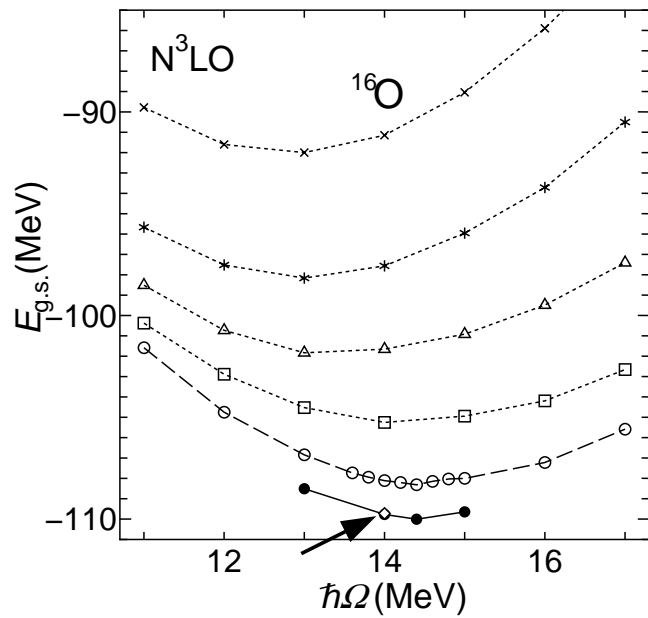
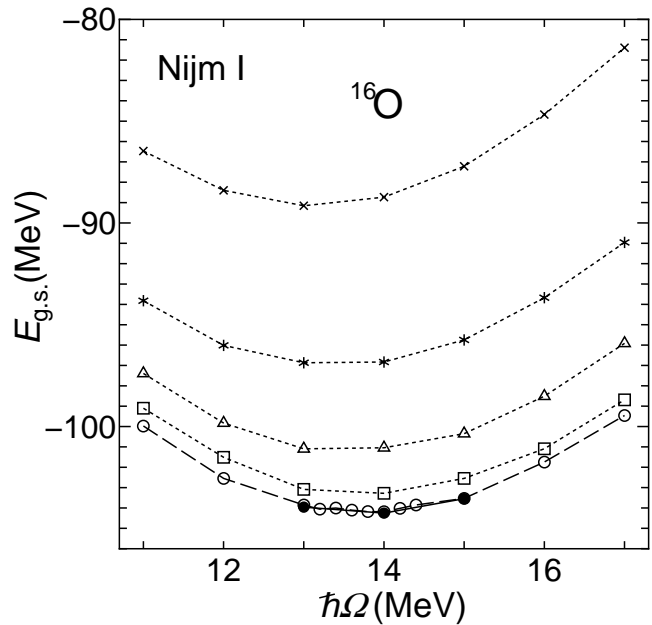
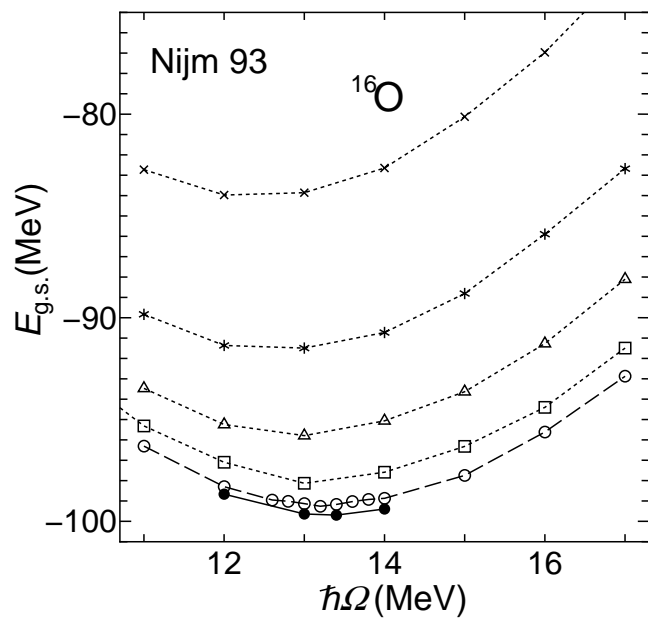
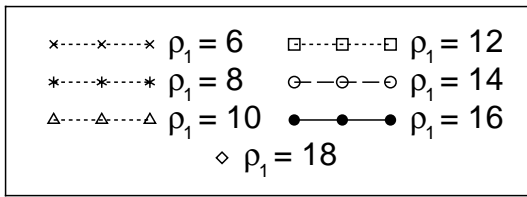
## • Eff. int. for the UMOA



## • Eff. int. for the 0s-1p0f shell model

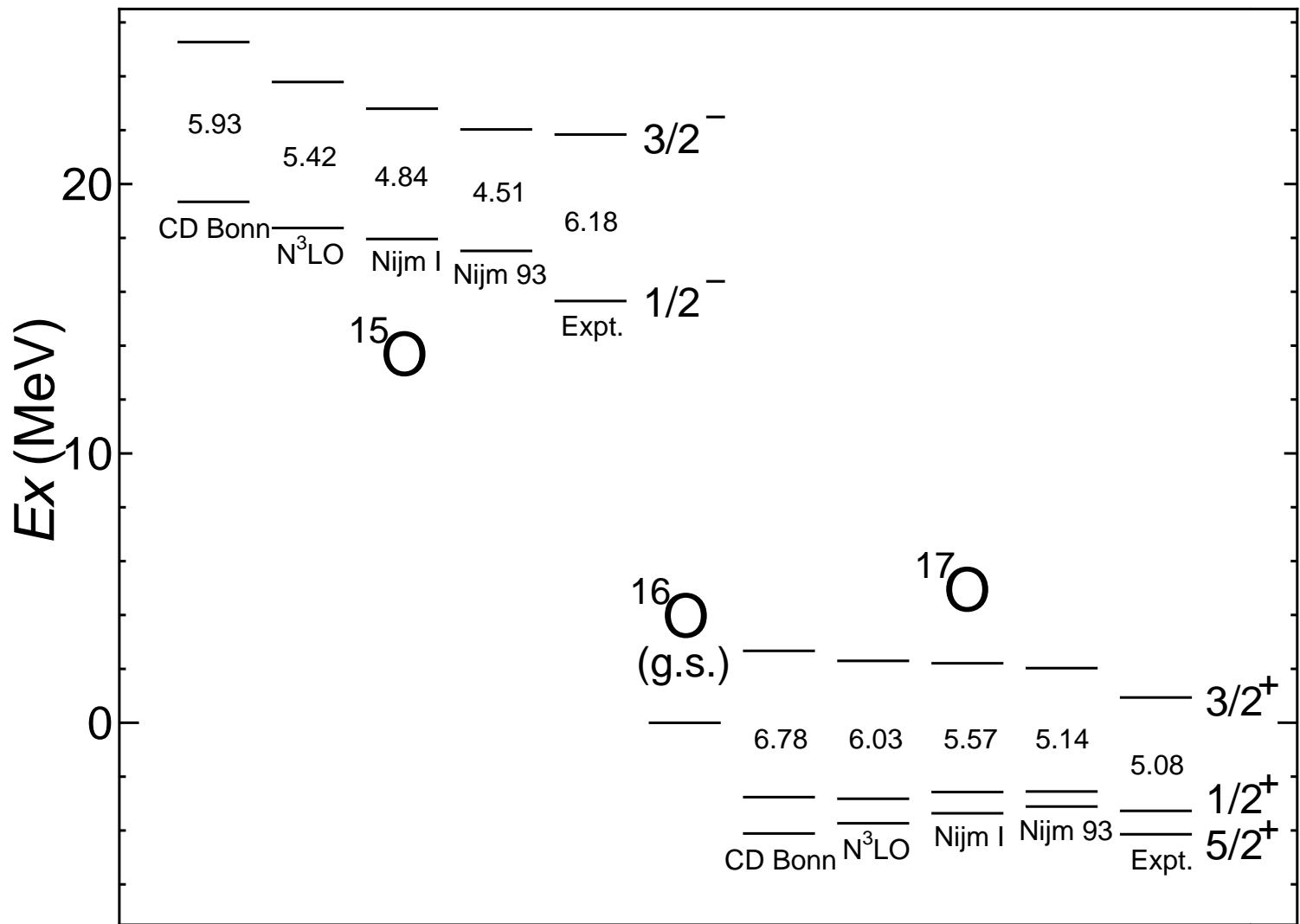


# Ground-state energies of $^{16}\text{O}$

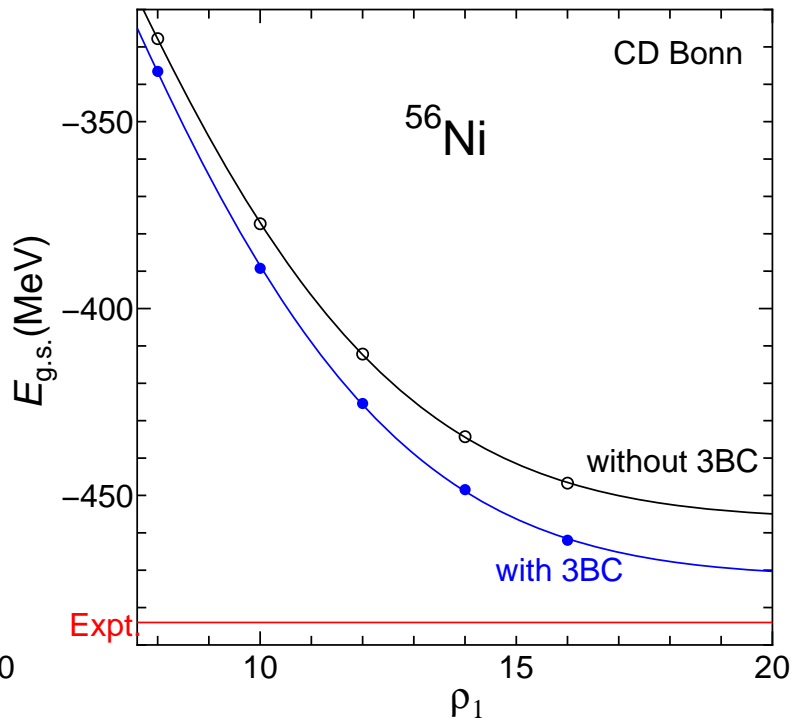
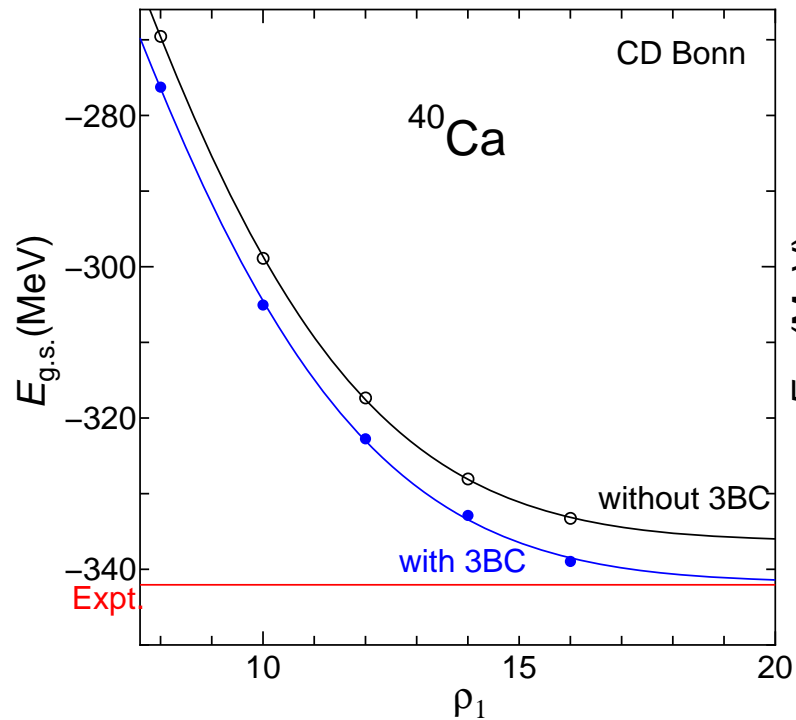


	Nijm 93	Nijm I	$\text{N}^3\text{LO}$	CD Bonn	Expt.	
$E_{g.s.}$	-99.69	-104.25	-110.00	-115.62	-127.62	
BE/A	6.23	6.52	6.88	7.23	7.98	(in MeV)

# Comparison of Expt. and UMOA results from modern NN interactions



# The $\rho_1$ dependence of the calculated ground-state energies of $^{40}\text{Ca}$ and $^{56}\text{Ni}$





# New approach to neutron-rich C isotopes

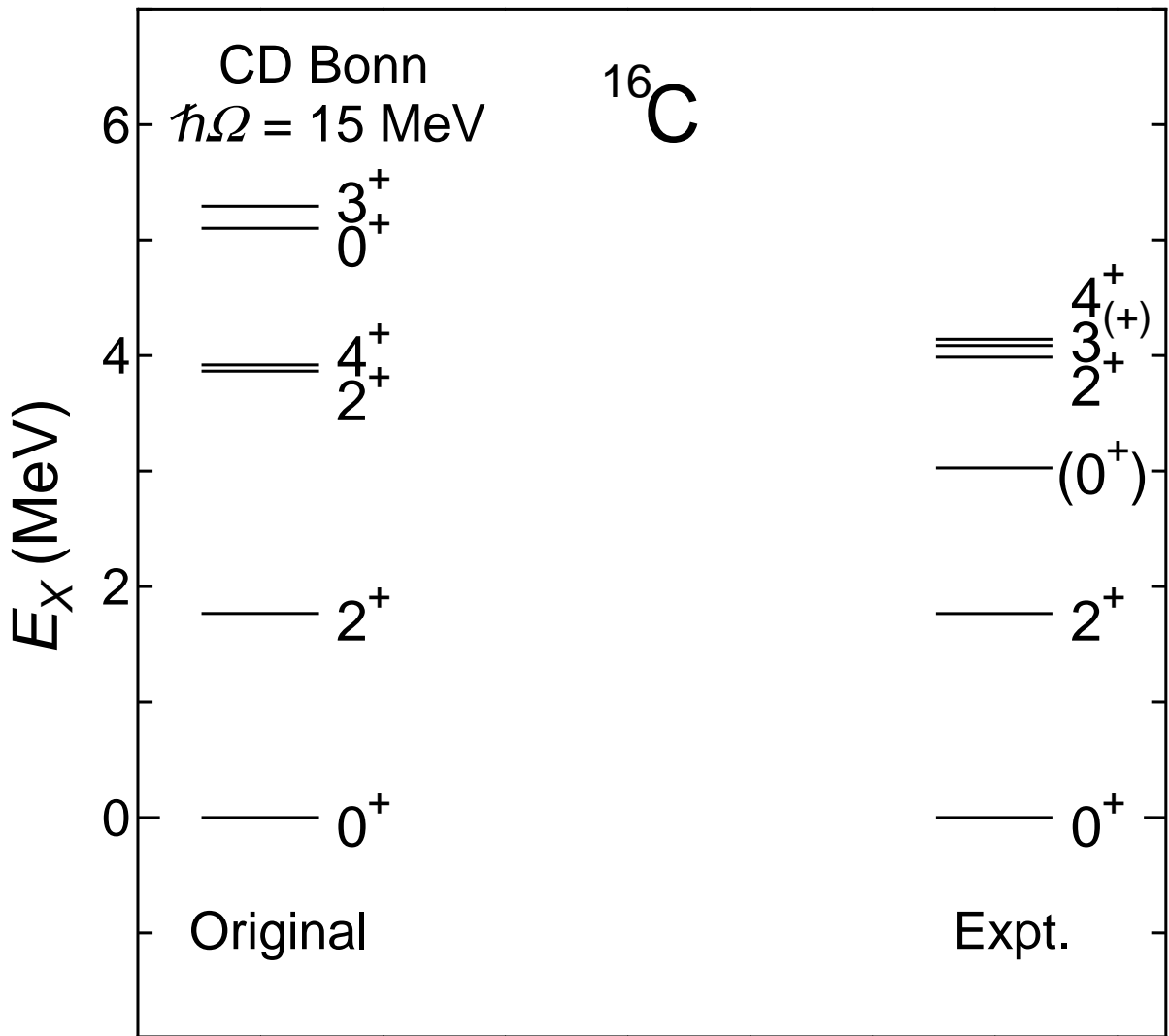
- **Large-scale shell model**

- **Code:** newly developed version of MSHELL
- **Model space:** the  $0s - 1p0f$  shells
- **Nucleon excitation:** up to 2 nucleons from the occupied shells for  $^{14}\text{C}$   
up to 2 nucleons to the  $1p0f$  shells
- **Bare transition operator**

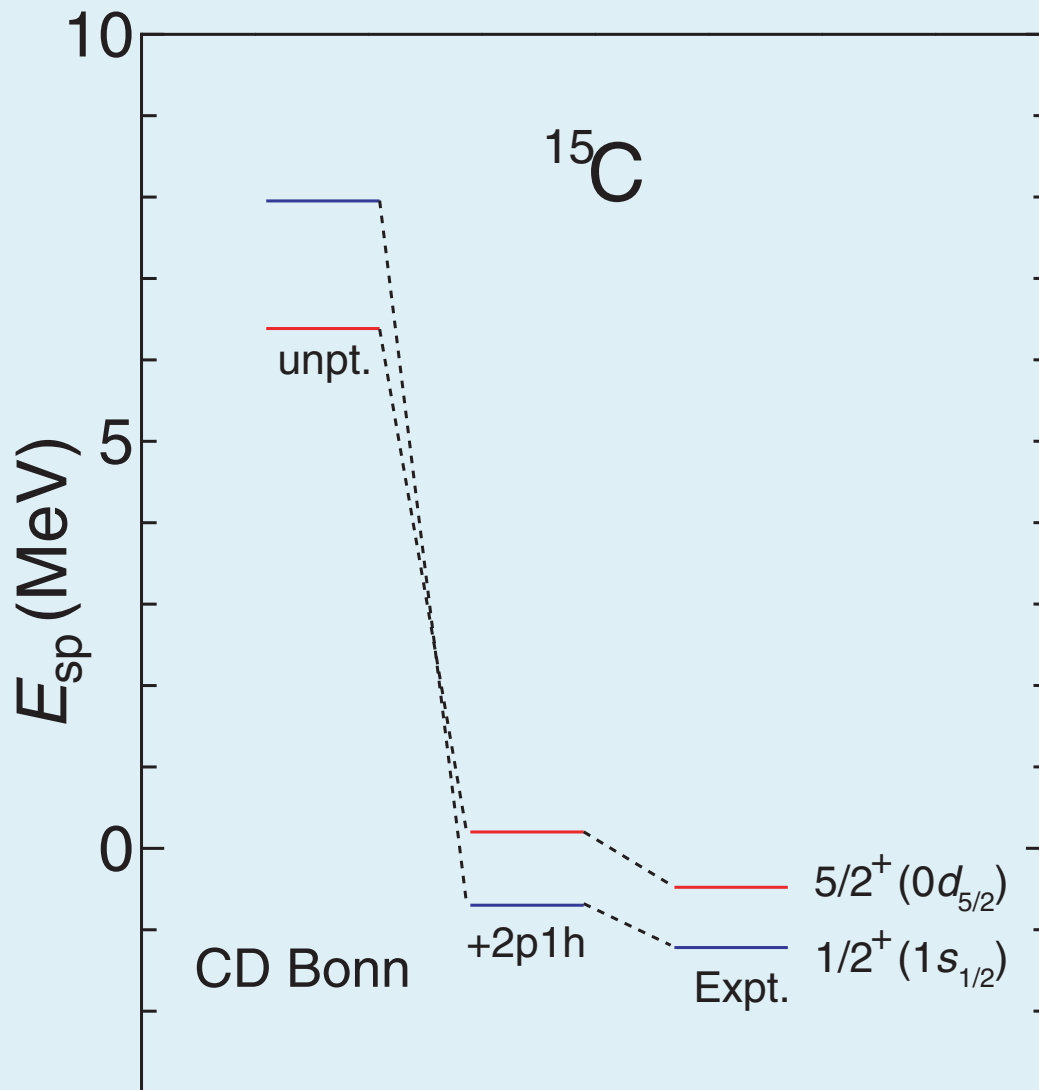
- **Microscopic effective interaction**

Derived from a high-precision NN interaction (CD Bonn, ...) and the Coulomb force in the neutron-proton formalism for the given model space through a unitary-transformation theory

# Low-lying energy levels in $^{16}\text{C}$



## Single-particle energies in $^{15}\text{C}$



Calculated results by  
the unitary-model-operator approach  
(UMOA)

In the present shell model without any adjustable parameters

→ wrong ordering for the  $1/2^+$  and  $5/2^+$  states in  $^{15}\text{C}$  due to the *small* model-space size



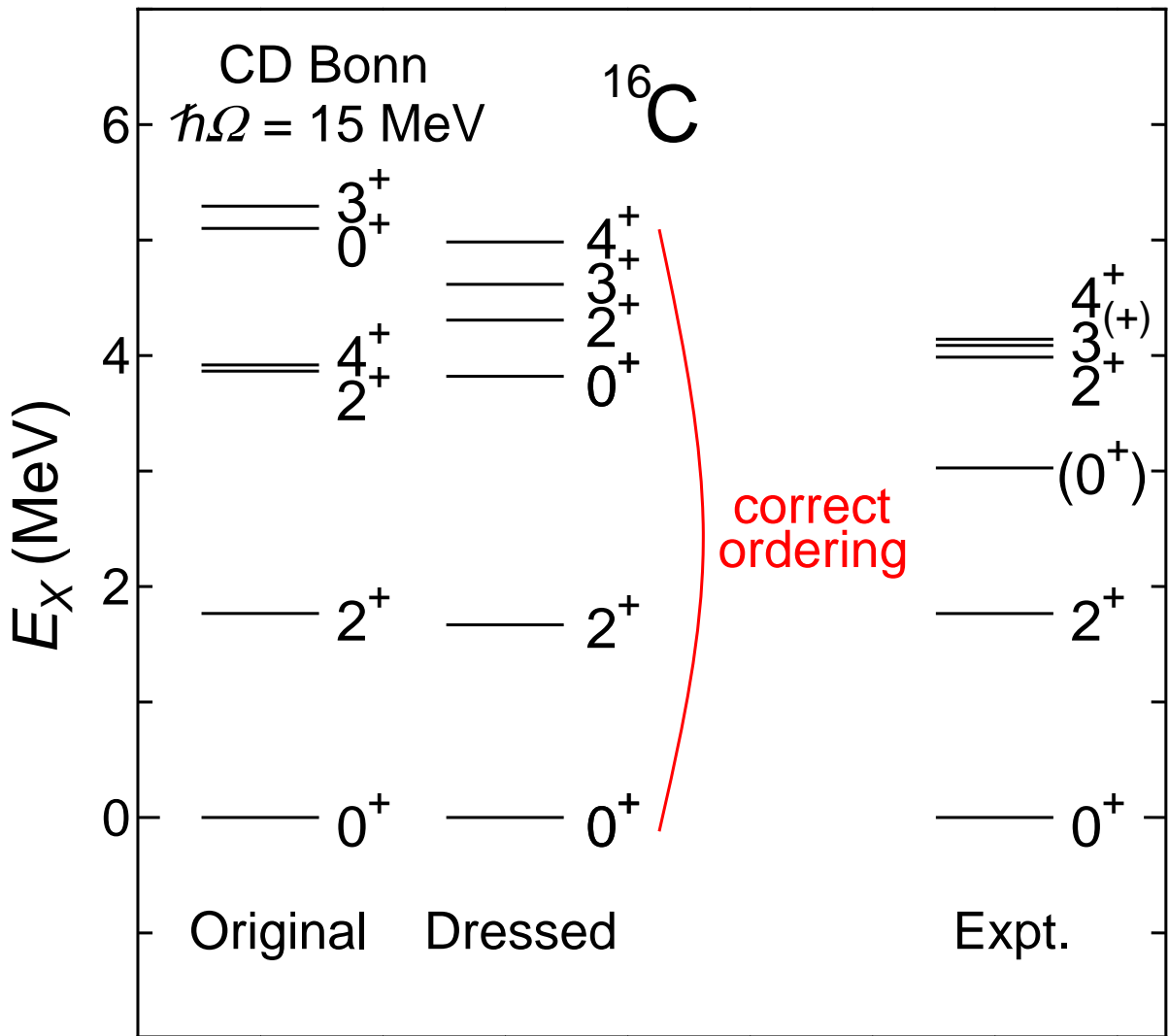
To remedy the wrong ordering and reproduce the binding energies for the  $1/2^+$  and  $5/2^+$  states of the UMOA results

→ introduce a minimal refinement of the one-body energies for the  $0d_{5/2}$  and  $1s_{1/2}$  orbits of the neutron

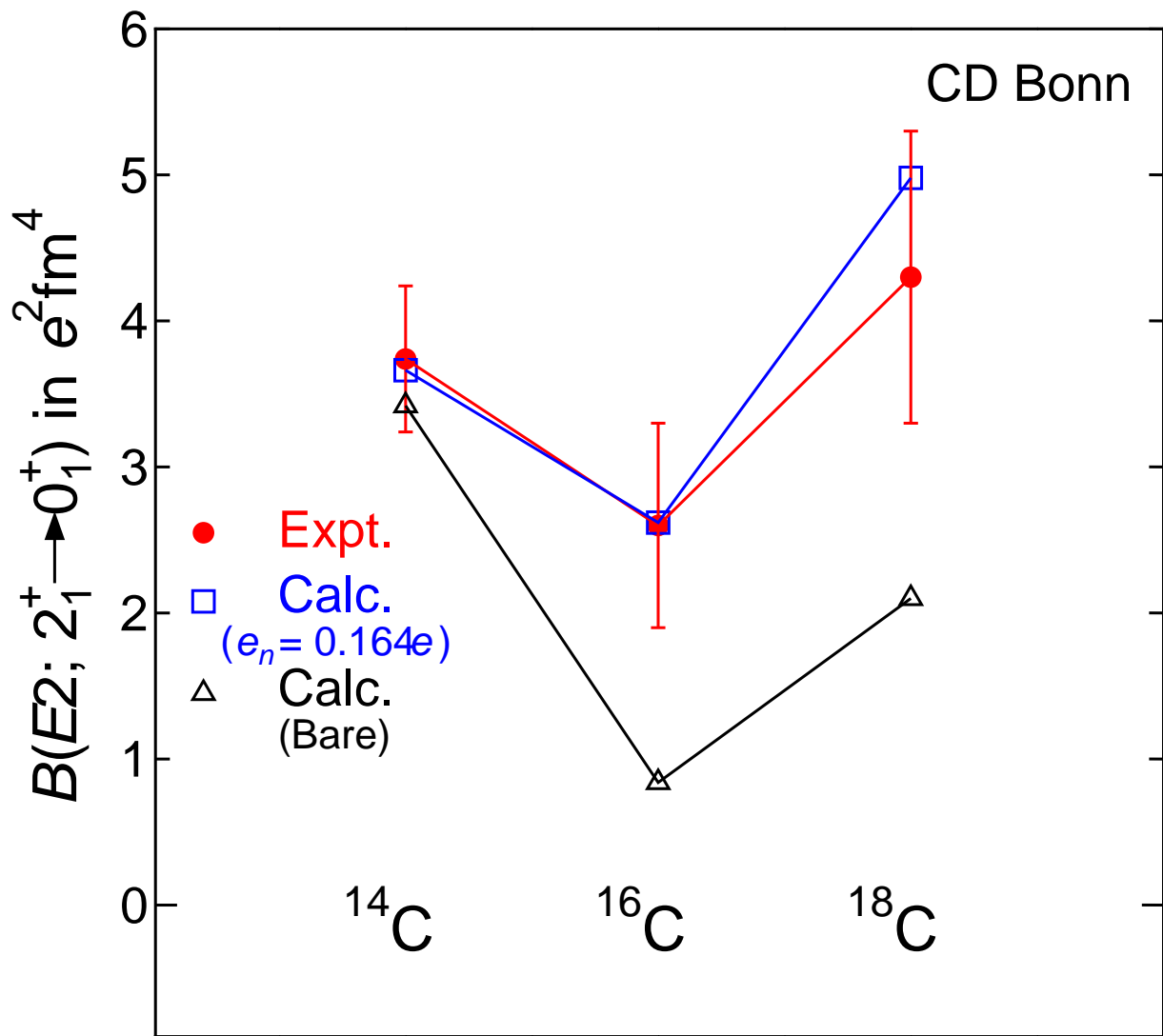


The calculated results are denoted by  
**"dressed"**

# Low-lying energy levels in $^{16}\text{C}$



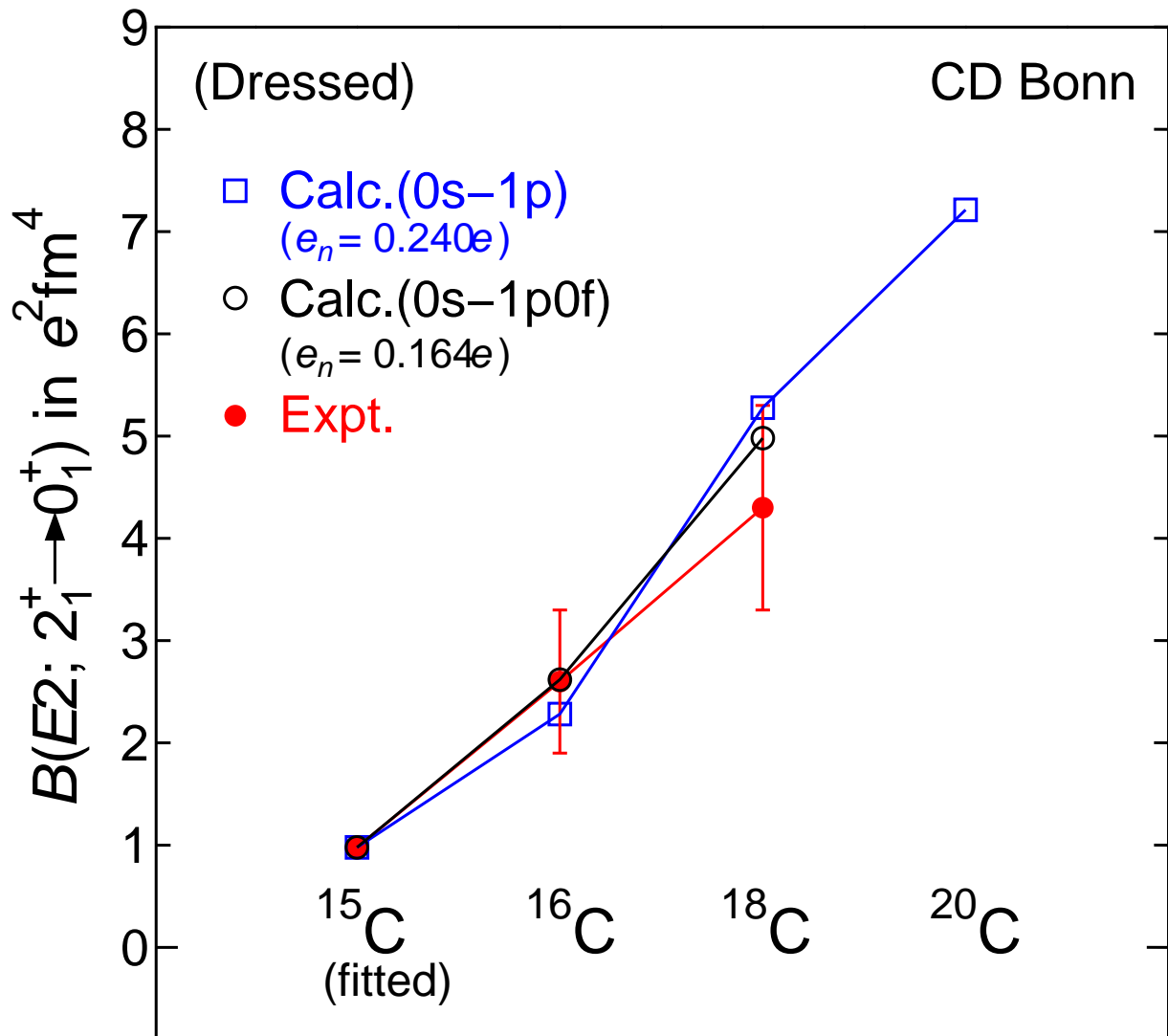
# $B(E2)$ of $^{14-18}\text{C}$



Expt. data for  $^{16}\text{C}$  and  $^{18}\text{C}$ :

H. J. Ong *et al.*, Phys. Rev. C **78**, 014308 (2008)

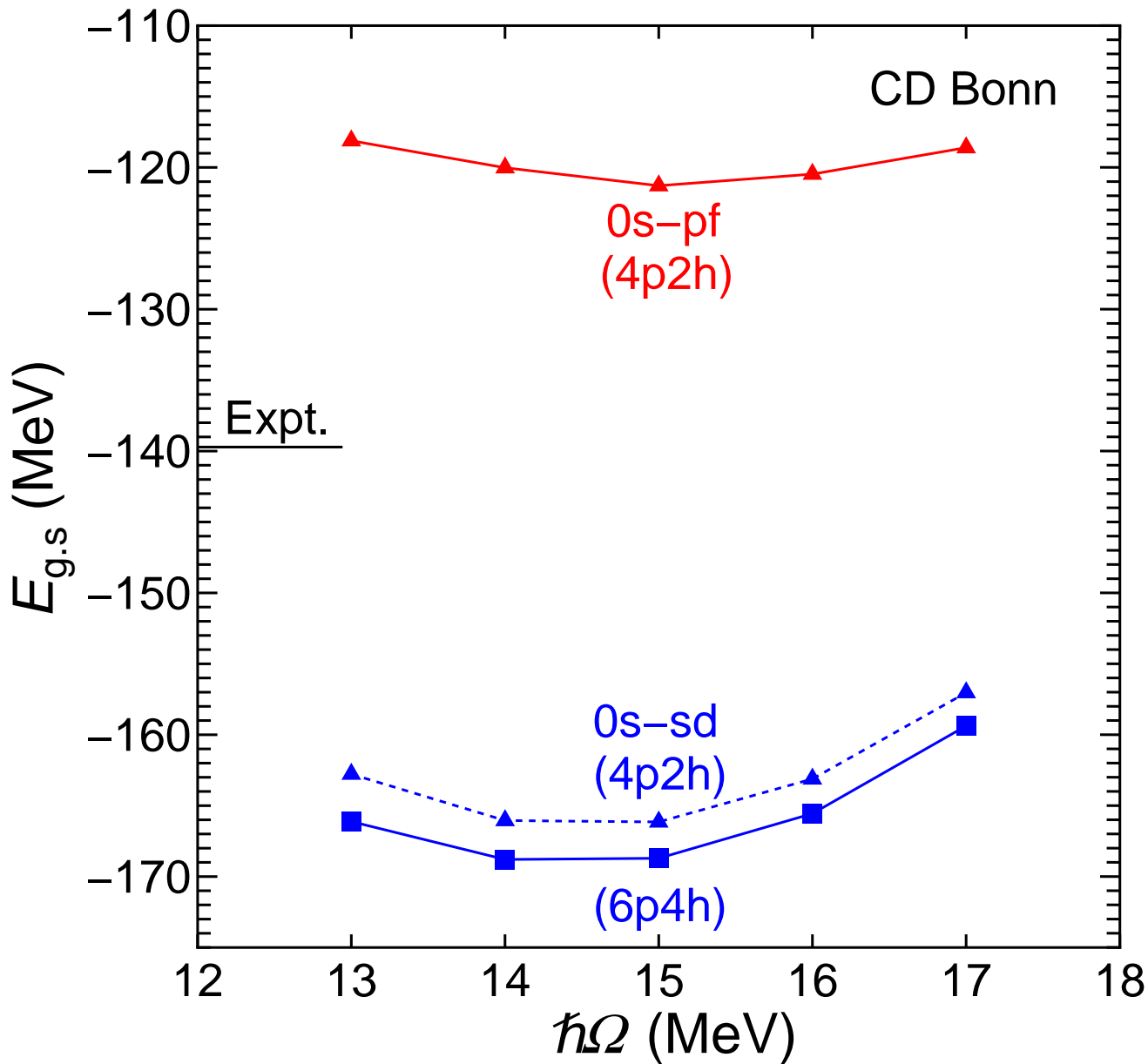
# $B(E2)$ of $^{15-20}\text{C}$



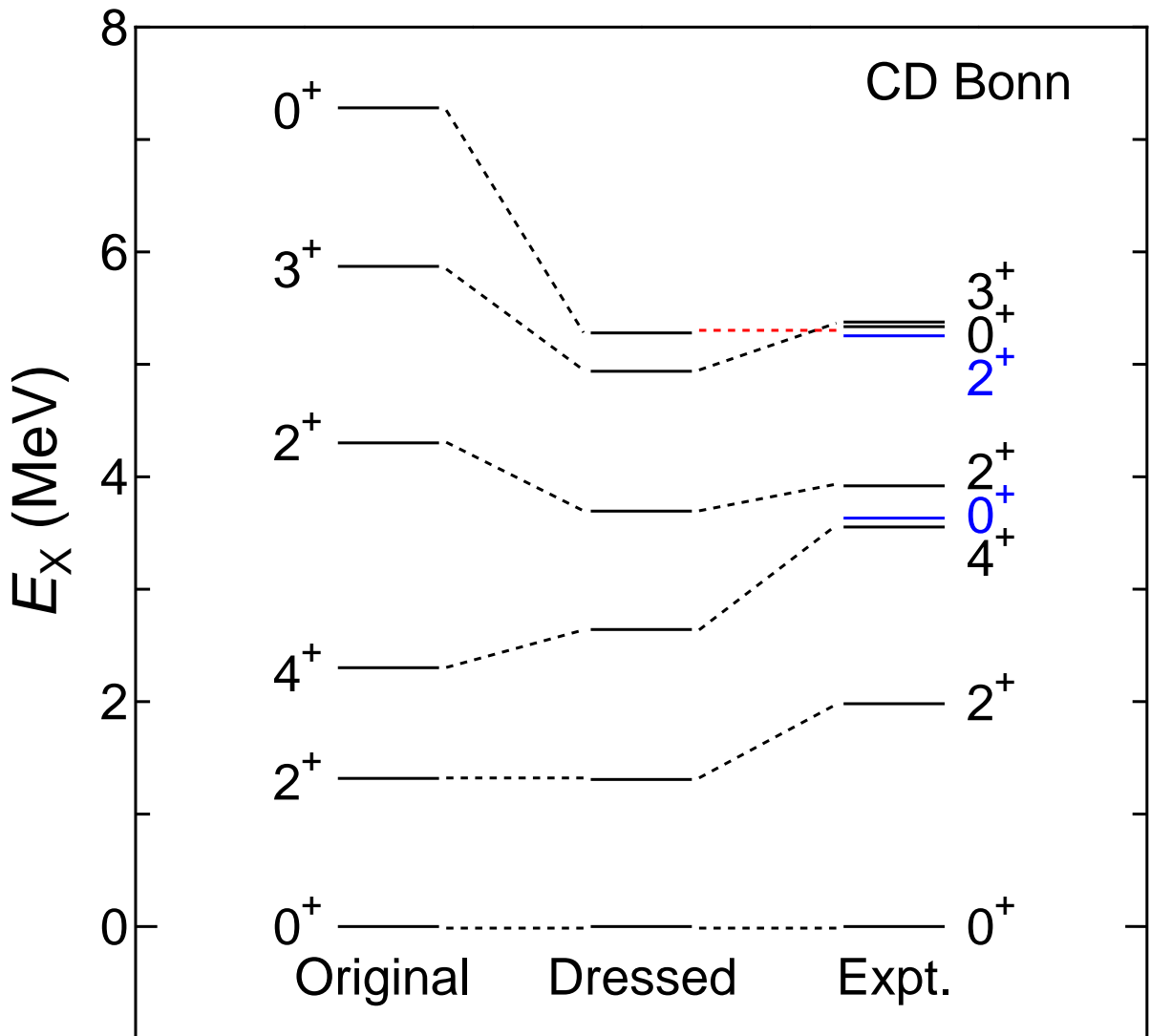
Expt. data for  $^{16}\text{C}$  and  $^{18}\text{C}$ :

H. J. Ong *et al.*, Phys. Rev. C **78**, 014308 (2008)

The  $\hbar\Omega$  dependence of calculated ground-state energies of  $^{18}\text{O}$



# Energy levels in $^{18}\text{O}$





**Excitation energies and probabilities of  
the 4p2h configuration for the lowest  
4p2h dominant  $0^+$  state in  $^{18}\text{O}$**

<b>Configuration (<i>spsd</i>)</b>	<b>4p2h</b>	<b>6p4h</b>	<b>8p6h</b>
<b><i>E<sub>x</sub></i> (MeV)</b>	<b>41.75</b>	<b>31.50</b>	<b>30.29</b>
<b><i>P</i><sub>4p2h</sub></b>	<b>0.996</b>	<b>0.855</b>	<b>0.816</b>

# Summary

- We have developed two methods for the microscopic description of nuclei beyond  $p$  shell with a realistic NN force in free space.
  - Unitary-model-operator approach (UMOA)
  - “No-core” shell model in a restricted model space
- In both methods, the microscopic effective interaction derived through a unitary transformation is the key ingredient.
- The “no-core” shell model has been applied to neutron-rich carbon isotopes and  $^{18}\text{O}$ .
- The structures of low-lying states are well described, except for the experimental  $0_2^+$  and  $2_3^+$  states in  $^{18}\text{O}$ .
- For those intruder states, we need a larger model space to reveal the real structure.

# Collaborators

## UMOA

**Ryoji Okamoto (Kyushu Inst. of Tech.)**

**Kenji Suzuki (Kyushu Inst. of Tech.)**

## **“No-core” shell model**

**Takahiro Mizusaki (Senshu Univ.)**

**Takaharu Otsuka (Univ. of Tokyo)**

**Takashi Sebe (Hosei Univ.)**

**Akito Arima (Japan Science Foundation)**

**Application of the “No-core” shell model to  $^{18}\text{O}$**

**Bruce R. Barrett (Univ. of Arizona)**