Tensor optimized shell model using bare interaction for light nuclei

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First EMMI-EFES workshop on neutron-rich exotic nuclei EENEN 09
- Realistic effective nuclear forces for neutron-rich nuclei -
GSI Darmstadt 2009. 2.9-11

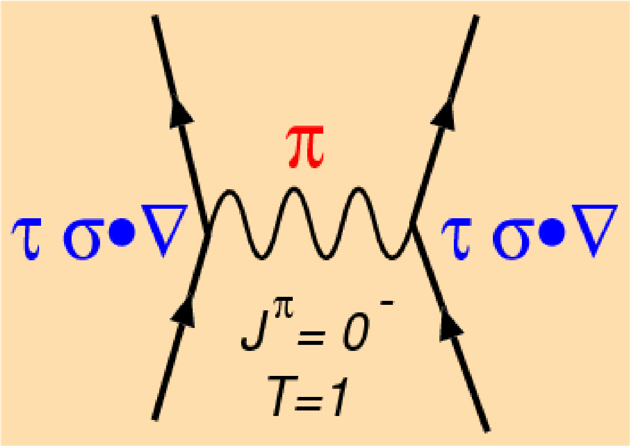
Outline

- ✓ Tensor Optimized Shell Model (TOSM)
- ✓ Unitary Correlation Operator Method (UCOM)
- ✓ TOSM + UCOM with bare interaction
- ✓ Application of TOSM to Li isotopes
  - Halo formation of ^{11}Li

- TM, K.Kato, H.Toki, K.Ikeda, PRC76(2007)024305
- TM, K.Kato, K.Ikeda, PRC76(2007)054309
- TM, Sugimoto, Kato, Toki, Ikeda, PTP117(2007)257
- TM. Y.Kikuchi, K.Kato, H.Toki, K.Ikeda, PTP119(2008)561
- TM, H. Toki, K. Ikeda, PTP121(2009) in press arXiv:0910.2978

Motivation for tensor force

- Tensor force (V_{tensor}) plays a significant role in the nuclear structure.
  - In ^{4}He, $\langle V_{tensor} \rangle \square \langle V_{central} \rangle$
  - $\frac{V_{\pi}}{V_{NN}}$ ~ 80% (GFMC)



R.B. Wiringa, S.C. Pieper, J. Carlson, V.R. Pandharipande, PRC62(2001)

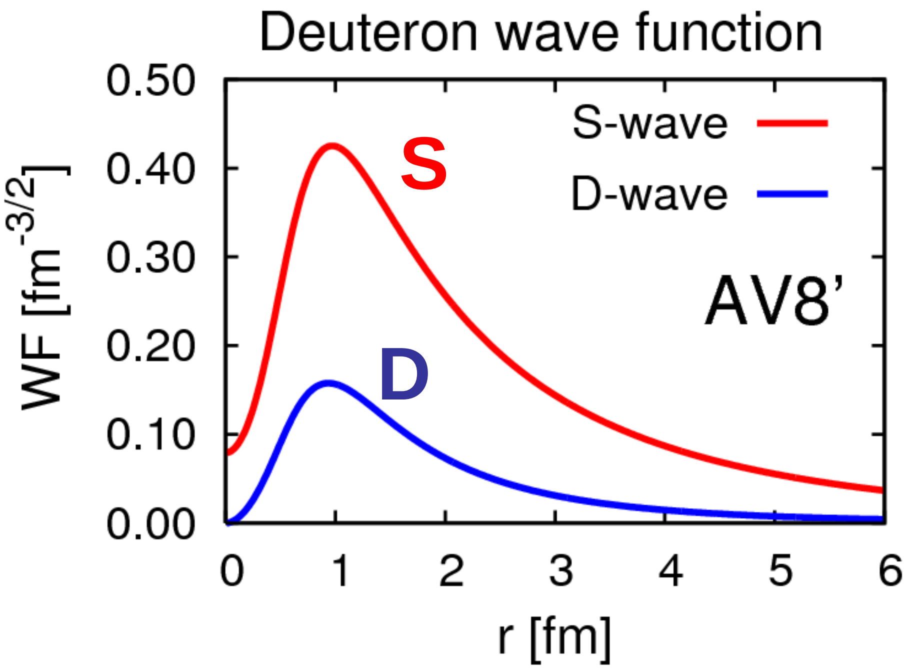
- We would like to understand the role of V_{tensor} in the nuclear structure by describing tensor correlation explicitly.
  - ✓ model wave function (shell model and cluster model)
  - ✓ He, Li isotopes (LS splitting, halo formation, level inversion)

- Structures of light nuclei with bare interaction
  - ✓ tensor correlation + short-range correlation

Tensor & Short-range correlations

- Tensor correlation in TOSM (long and intermediate)
  - $S_{12} \propto \left[Y_2(\hat{r}), [\vec{\sigma}_1, \vec{\sigma}_2]_2 \right]_0 \rightarrow \Delta L = \Delta S = 2$
  - 2p2h mixing optimizing the particle states (radial & high-L)

- Short-range correlation
  - Short-range repulsion in the bare NN force
  - Unitary Correlation Operator Method (UCOM)



H. Feldmeier, T. Neff, R. Roth, J. Schnack, NPA632(1998)61

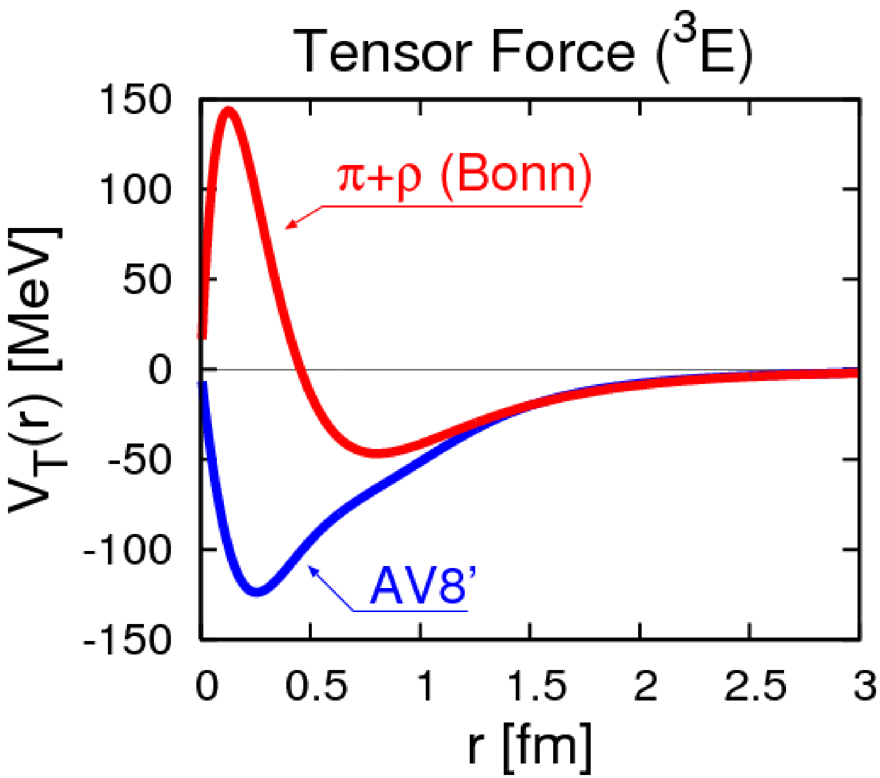
T. Neff, H. Feldmeier, NPA713(2003)311

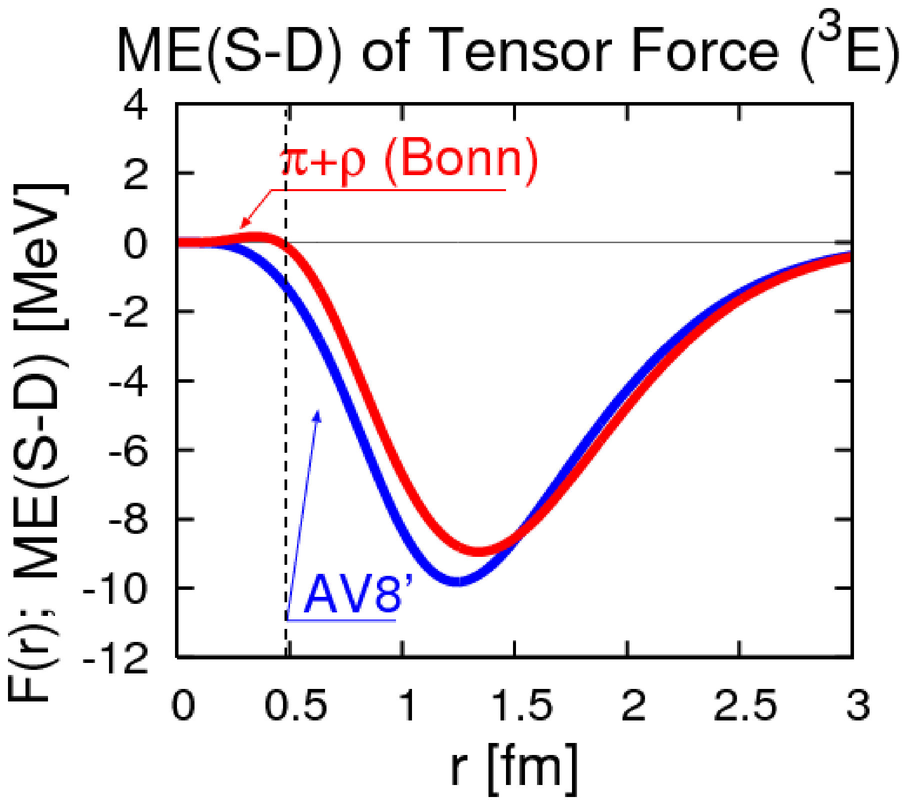
Property of the tensor force

$V_{tensor} = V_T(r) \cdot S_{12}$

$F(r) = r^2 \cdot \phi_{0s}(r, b_{0s}) \cdot V_T(r) \cdot \phi_{0d}(r, b_d)$

$b_s = 1.4 \cdot \sqrt{2}$ [fm] $b_d = b_s / 2$





Long and intermediate ranges

- Centrifugal potential (1GeV@0.5fm) pushes away the L=2 wave function.

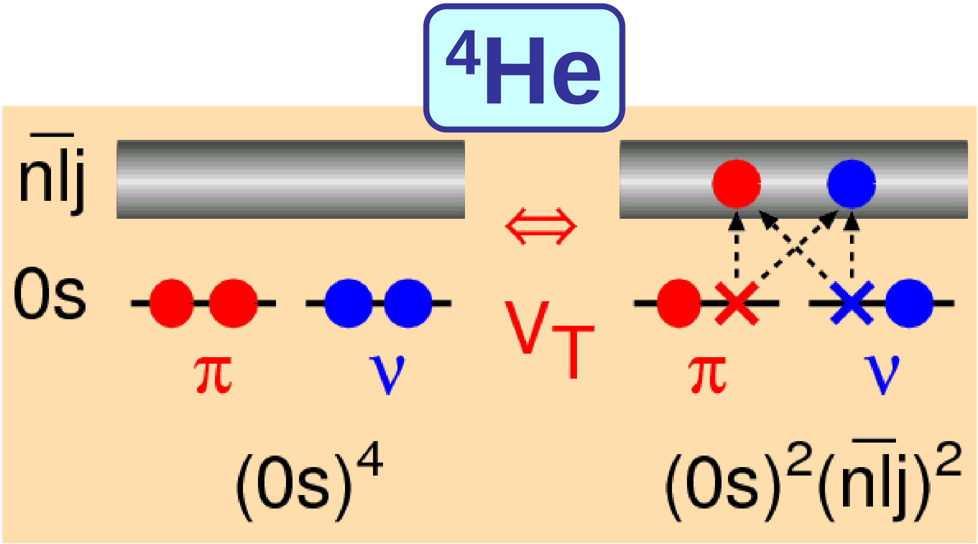
Tensor-optimized shell model (TOSM)

TM, Sugimoto, Kato, Toki, Ikeda
PTP117(2007)257

- Tensor correlation
  in the shell model type approach.

- Configuration mixing
  within 2p2h excitations
  with high-L orbit

  TM et al., PTP113(2005)
  TM et al., PTP117(2007)
  T.Terasawa, PTP22('59))



- Length parameters $\{b_\alpha\}$ such as $b_{0s}, b_{0p1/2}, \cdots$ are determined independently and variationally.
  - Describe high momentum component from V_{tensor}
    CPP-HF by Sugimoto et al,(NPA740) / Akaishi (NPA738)
    CPP-RMF by Ogawa et al.(PRC73), CPP-AMD by Dote et al.(PTP115)

Hamiltonian and variational equations in TOSM

$$H = \sum_{i=1}^{A} t_i - T_G + \sum_{i<j}^{A} v_{ij}, \qquad v_{ij}\ \text{: central+tensor+LS+Coulomb}$$

$$\Phi = \sum_k C_k \cdot \psi_k \qquad \psi_k\ \text{: shell model type configuration}$$

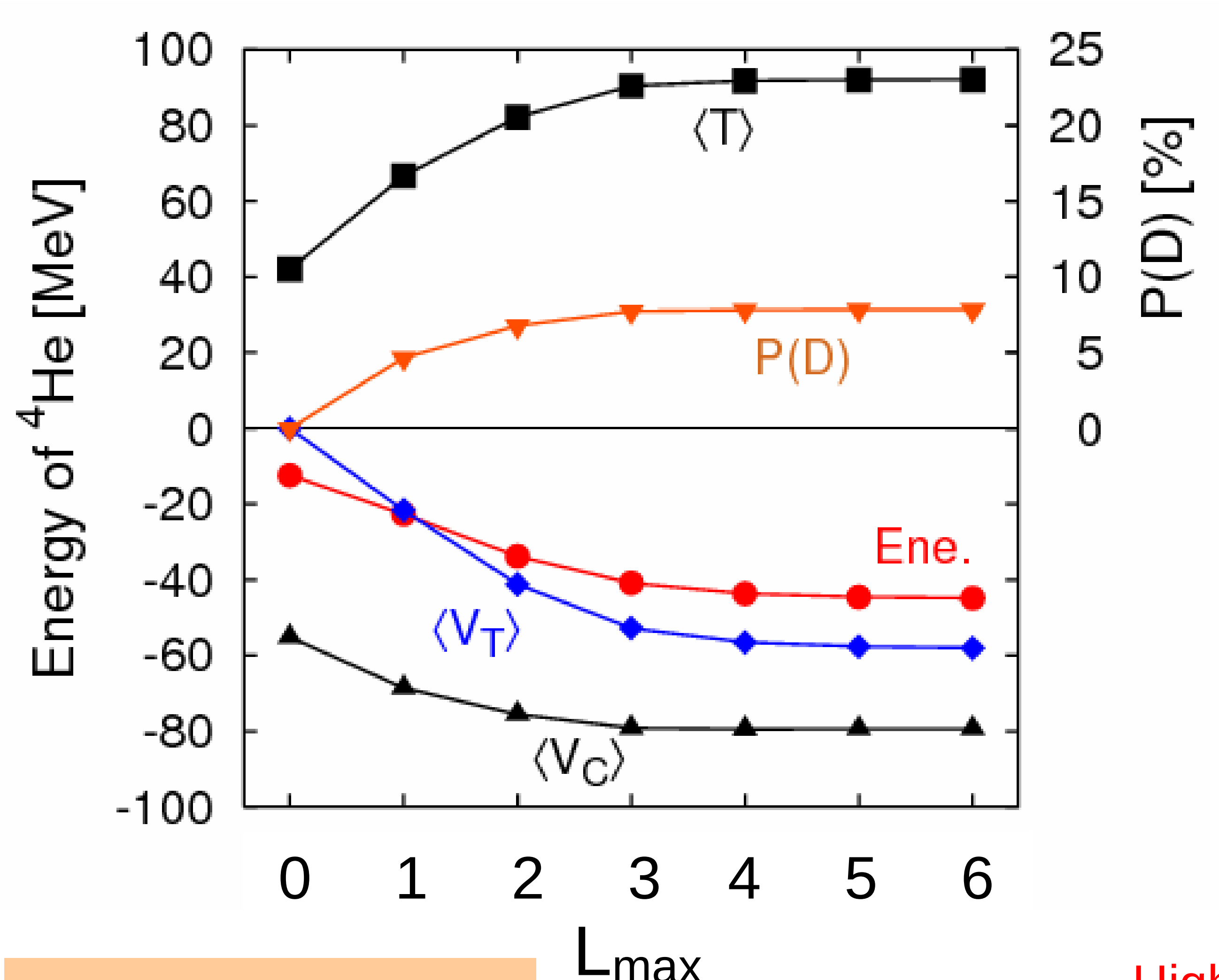
$$\delta \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0 \quad \Rightarrow \quad \frac{\partial \langle H - E \rangle}{\partial b_\alpha} = 0, \quad \frac{\partial \langle H - E \rangle}{\partial C_k} = 0$$

TM, Sugimoto, Kato, Toki, Ikeda, PTP117('07)257

- Effective interaction : Akaishi force (NPA738)
  - G-matrix from AV8' with k_Q=2.8 fm^{-1}
  - Long and intermediate ranges of V_{tensor} survive.
  - Adjust $V_{central}$ to reproduce B.E. and radius of ^4He

^{4}He in TOSM

v_{nn}: G-matrix



good convergence

Shrink

Length parameters

Orbit	$b_{particle}/b_{hole}$
$0p_{1/2}$	0.65
$0p_{3/2}$	0.58
$1s_{1/2}$	0.63
$0d_{3/2}$	0.58
$0d_{5/2}$	0.53
$0f_{5/2}$	0.66
$0f_{7/2}$	0.55

Higher shell effect □ $16\hbar\omega$

$(\vec{\sigma}\cdot\vec{q})$ in V_π

Cf. K. Shimizu, M. Ichimura and A. Arima, NPA226(1973)282.

Configuration of ^{4}He in TOSM

Energy (MeV)	– 28.0
$\langle V_{tensor} \rangle$	– 51.0

$(0s_{1/2})^4$	85.0 %
$(0s_{1/2})^2_{JT}(0p_{1/2})^2_{JT}$ JT=10	5.0
JT=01	0.3
$(0s_{1/2})^2_{10}(1s_{1/2})(0d_{3/2})_{10}$	2.4
$(0s_{1/2})^2_{10}(0p_{3/2})(0f_{5/2})_{10}$	2.0
P[D]	9.6

4 Gaussians instead of HO

$$\langle T \rangle = \ 71.2 \ \text{MeV}$$

$$\langle V_{central} \rangle = -48.6 \ \text{MeV}$$

c.m. excitation = 0.6 MeV

- 0^- of pion nature.
- deuteron correlation with (J,T)=(1,0)

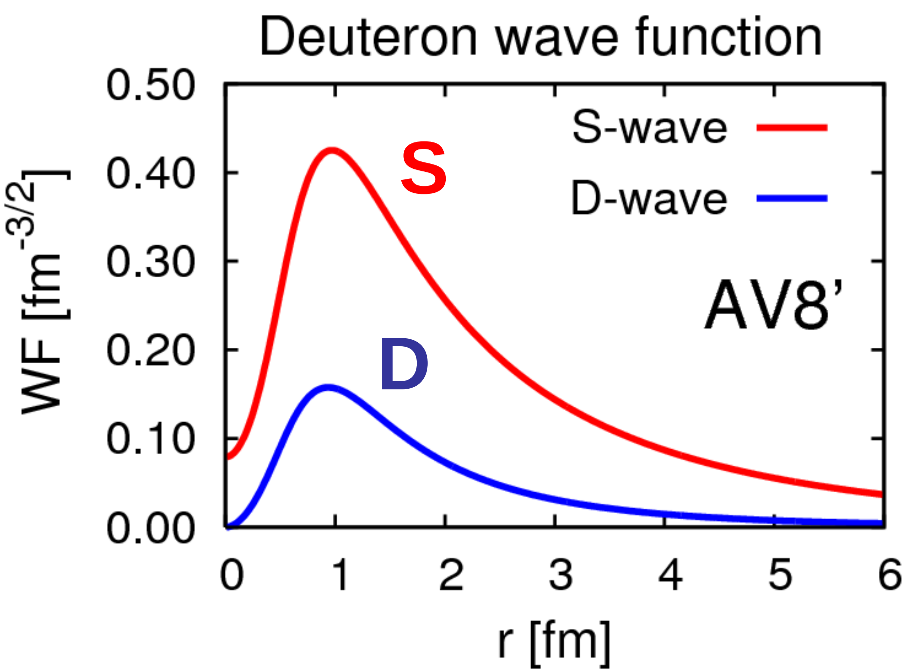
Cf. R.Schiavilla et al. (GFMC) PRL98('07)132501

Tensor & Short-range correlations

- Tensor correlation in TOSM (long and intermediate)
  - $S_{12} \propto \left[Y_2(\hat{r}),[\vec{\sigma}_1,\vec{\sigma}_2]_2\right]_0 \rightarrow \Delta L = \Delta S = 2$
  - 2p2h mixing optimizing the particle states (radial & high-L)

- Short-range correlation
  - Short-range repulsion in the bare NN force
  - Unitary Correlation Operator Method (UCOM)

→ **TOSM+UCOM**



H. Feldmeier, T. Neff, R. Roth, J. Schnack, NPA632(1998)61

T. Neff, H. Feldmeier NPA713(2003)311

Unitary Correlation Operator Method

$$\Psi_{\text{corr.}} = C \cdot \Phi_{\text{uncorr.}}$$

TOSM

short-range correlator

$C^\dagger = C^{-1}$ (Unitary trans.)

$$H\Psi = E\Psi \;\rightarrow\; C^\dagger H C \Phi \equiv \tilde{H}\Phi = E\Phi$$

Bare Hamiltonian

Shift operator depending on the relative distance **r**

$$C = \exp(-i\sum_{i<j} g_{ij}), \quad g_{ij} = \tfrac{1}{2}\left\{p_r s(r_{ij}) + s(r_{ij}) p_r\right\} \quad \vec{p} = \vec{p}_r + \vec{p}_\Omega$$

$$R'_+(r) = \frac{s(R_+(r))}{s(r)} \qquad R_+(r) \simeq r + s(r)$$

2-body cluster expansion of Hamiltonian

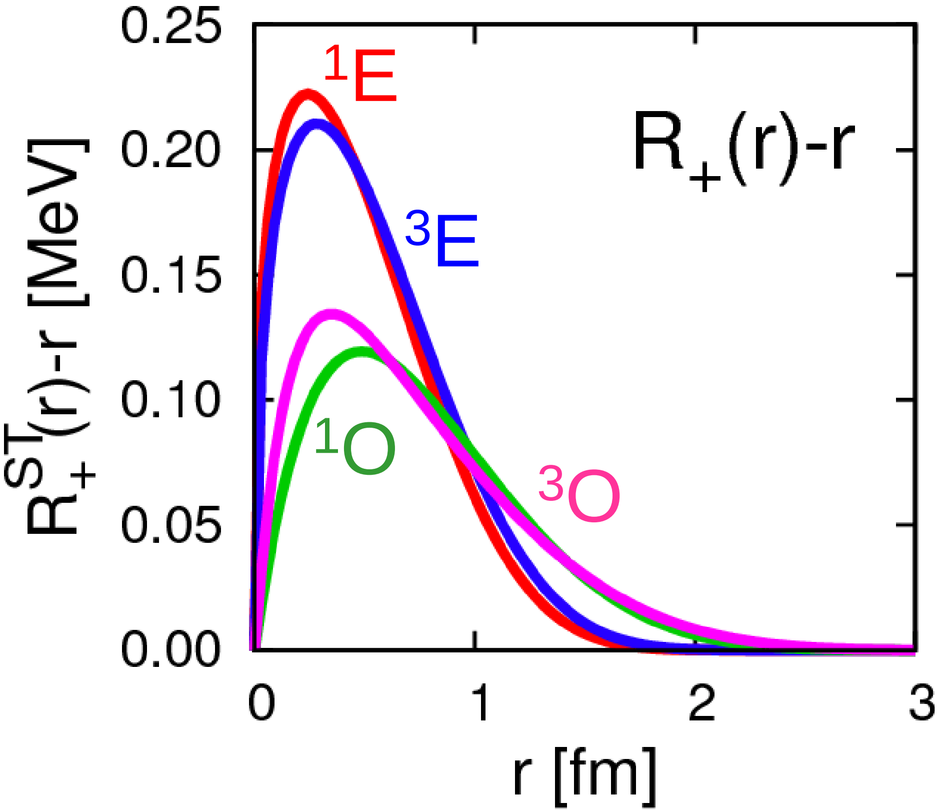
H. Feldmeier, T. Neff, R. Roth, J. Schnack, NPA632(1998)61

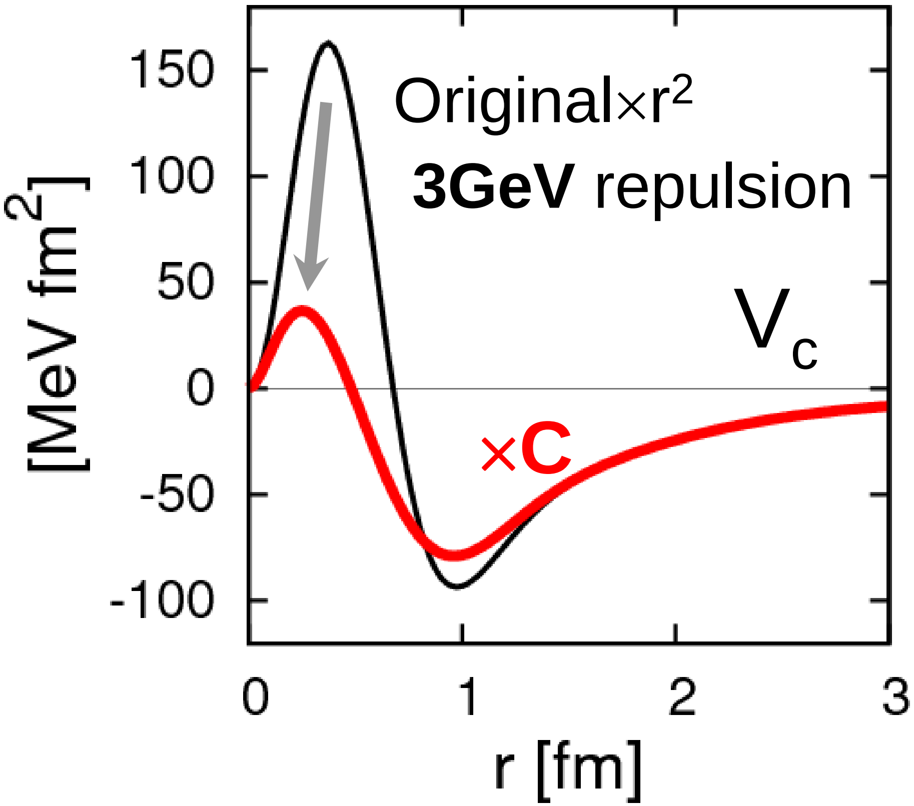
Short-range correlator : **C** (or **C$_r$**)

$C:\ r \rightarrow R_+(r)$ for Hamiltonian

$$C^\dagger r\ C = R_+(r)$$

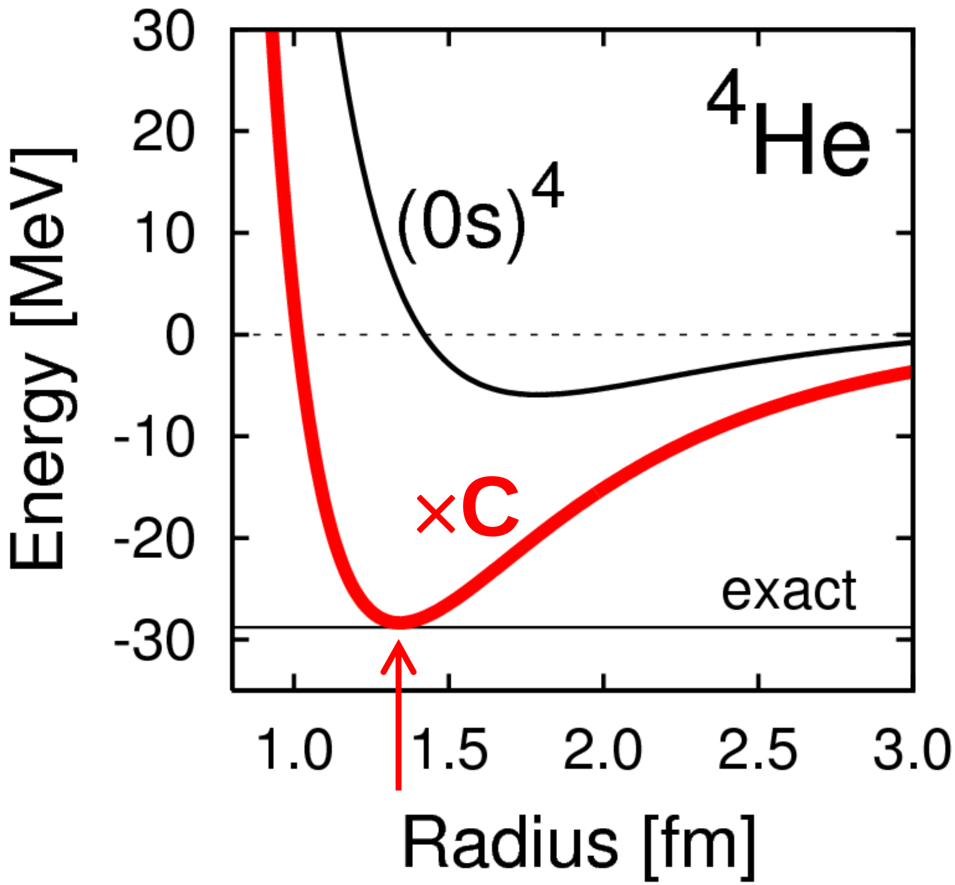
$$C^\dagger \vec{l} C = \vec{l} \qquad C^\dagger \vec{s} C = \vec{s}$$

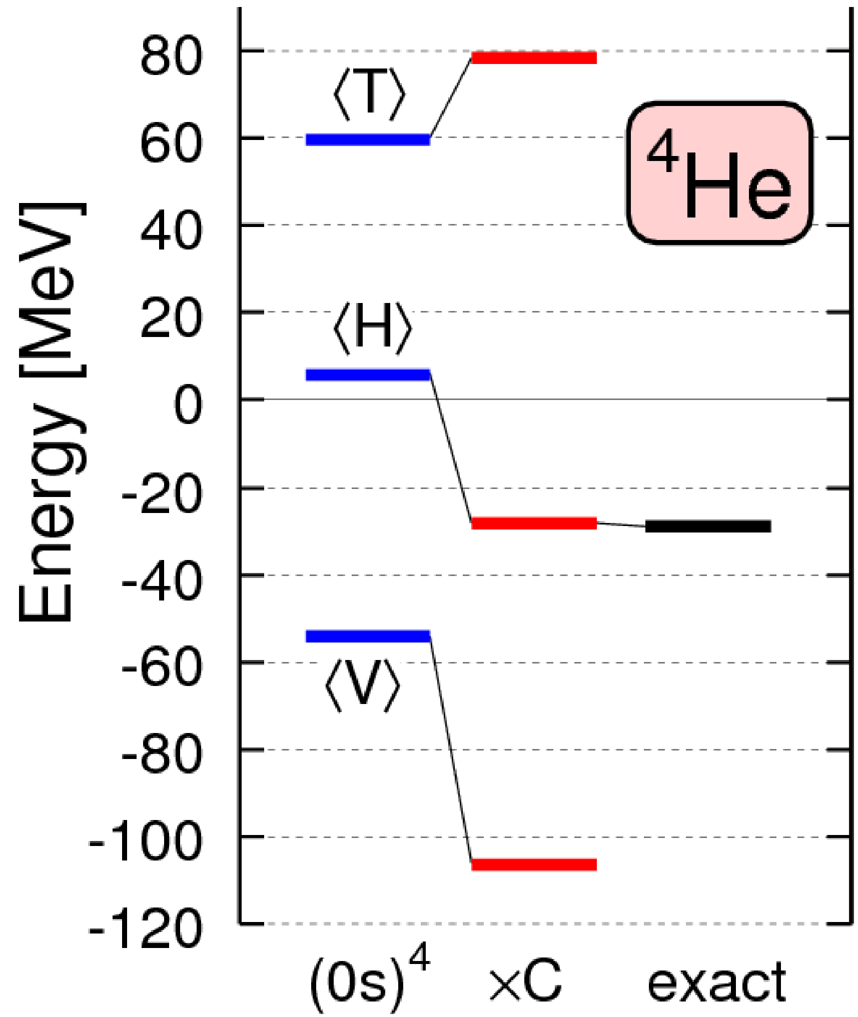




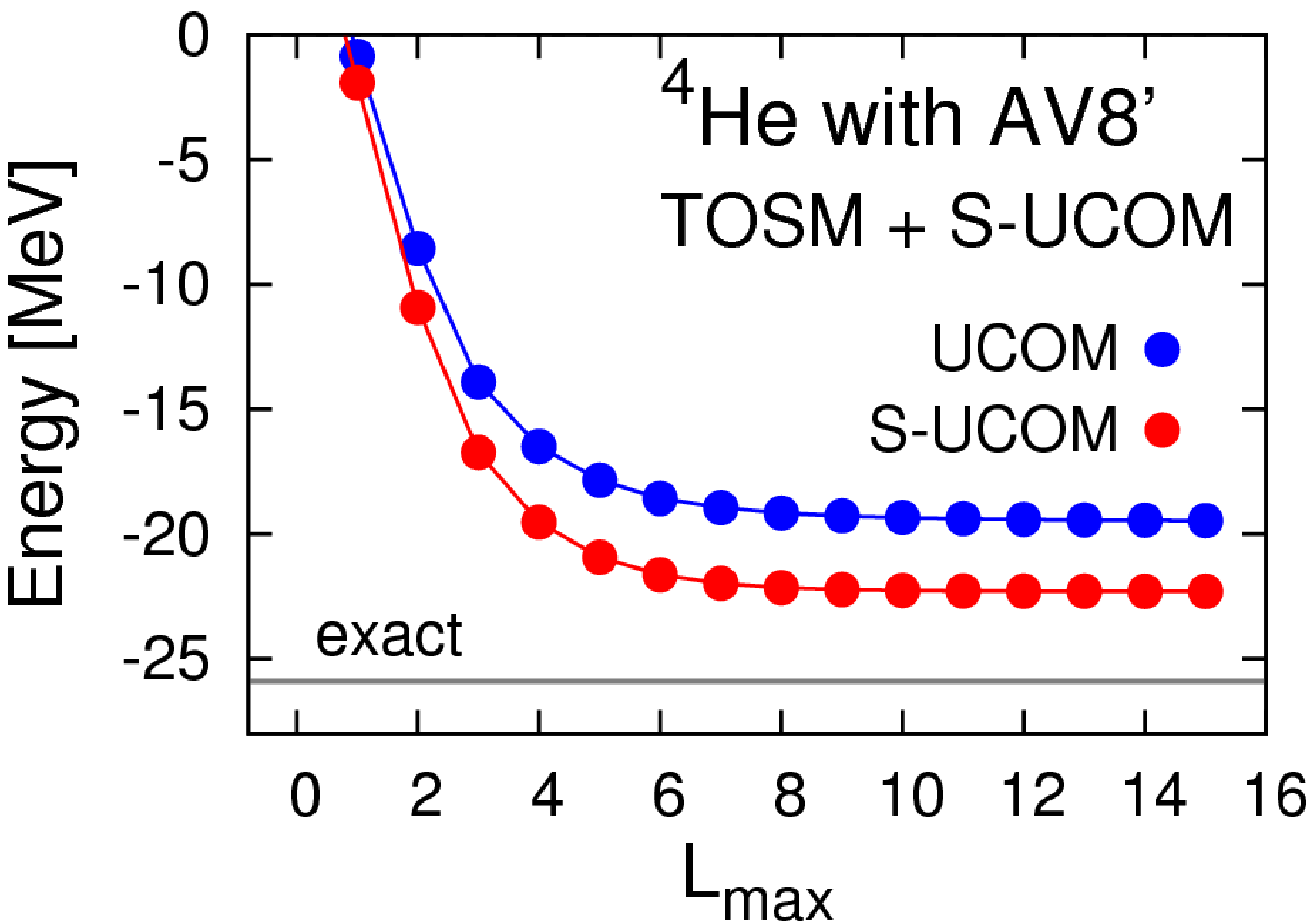
AV8' : Central+LS+Tensor

^{4}He in UCOM (Afnan-Tang, V_c only)





^{4}He with AV8' in **TOSM+UCOM**



AV8' :
Central+
LS+Tensor

exact

Kamada et al.
PRC64
(Jacobi)

- Gaussian expansion for particle states (6 Gaussians)
- Two-body cluster expansion of Hamiltonian

Extension of UCOM : S-wave UCOM

$C_S,\ R_+^S(r)$ for only **relative S-wave wave function**

– minimal effect of UCOM

$$C_S^\dagger T C_S = T + \Delta T \qquad \text{for s-wave}$$

$$C_S^\dagger V_{\text{central}}(r) C_S = \hat{V}_{\text{central}}(r) \qquad \text{for s-wave}$$

$$C_S^\dagger V_{\text{LS}}(r) C_S = V_{\text{LS}}(r) \qquad \text{No change}$$

$$\left\langle \psi_S^{\text{rel}} \middle| V_{\text{tensor}}(r) \middle| \psi_D^{\text{rel}} \right\rangle = \left\langle C_S \phi_S^{\text{rel}} \middle| V_{\text{tensor}}(r) \middle| \phi_D^{\text{rel}} \right\rangle$$

SD coupling

PTP121(2009) in press
arXiv:0901.2978

$\langle V_{\text{tensor}} \rangle$ □ 5 MeV gain

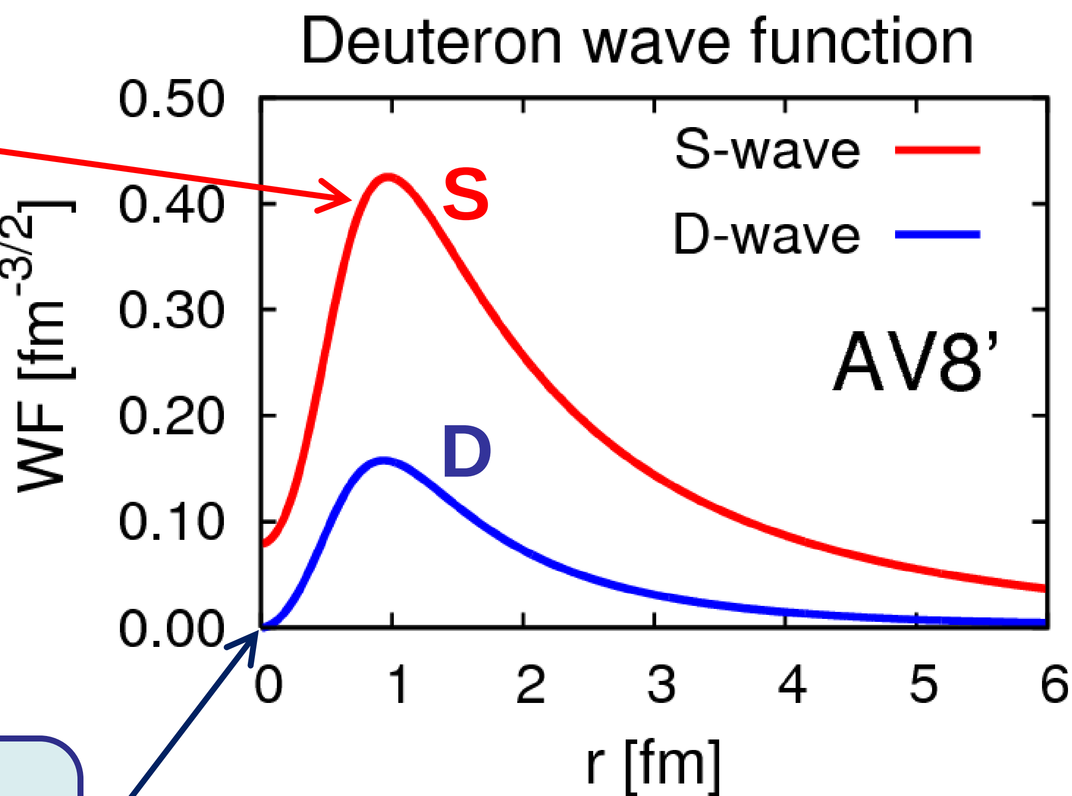
Different effects of correlation function

- **S-wave**

$$\phi_S^{\text{rel}} \neq 0 \text{ at } r = 0$$

No Centrifugal Barrier

Short-range repulsion

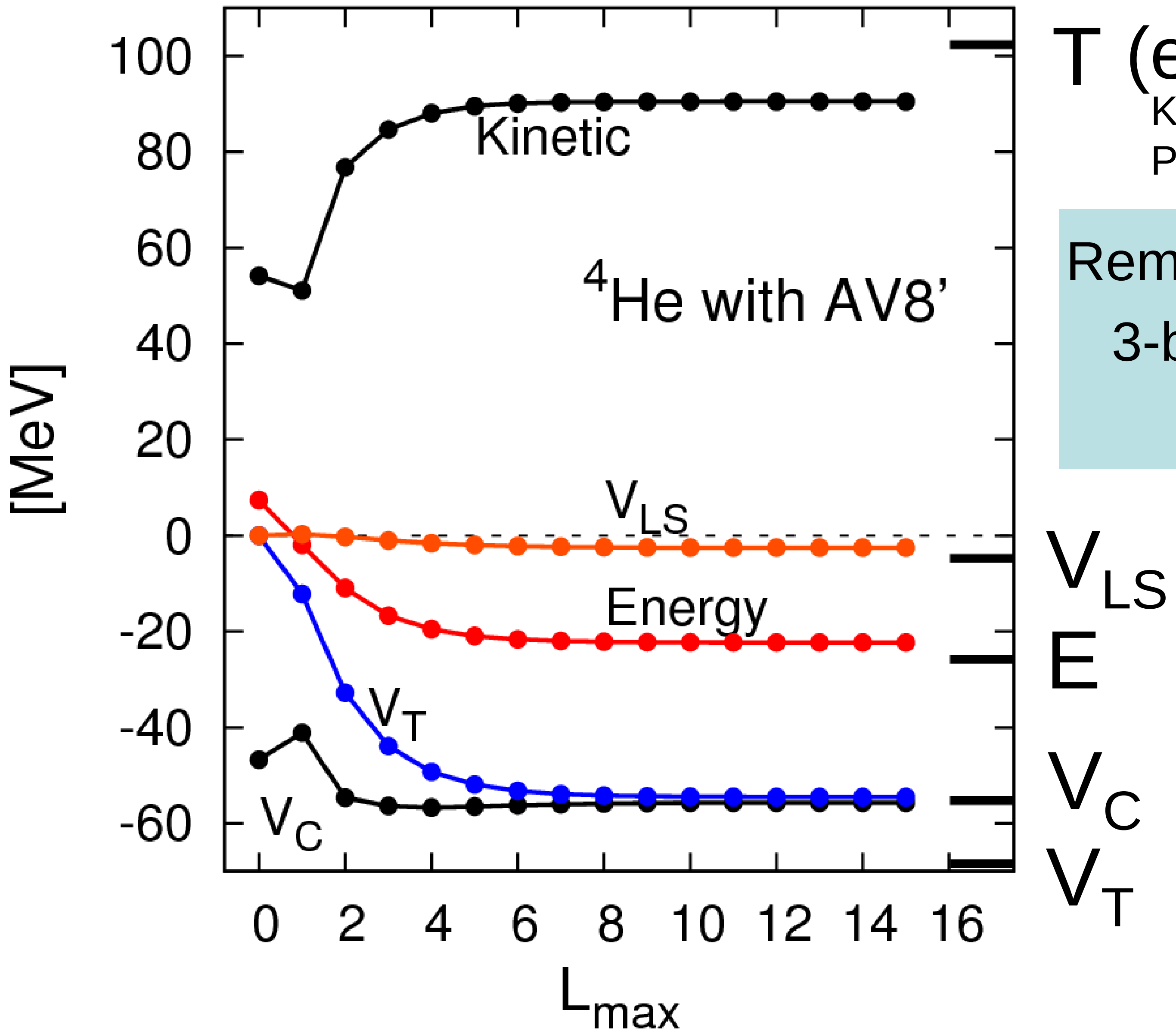


- **D-wave**

$$\phi_D^{\text{rel}} = 0 \text{ at } r = 0$$

due to Centrifugal Barrier

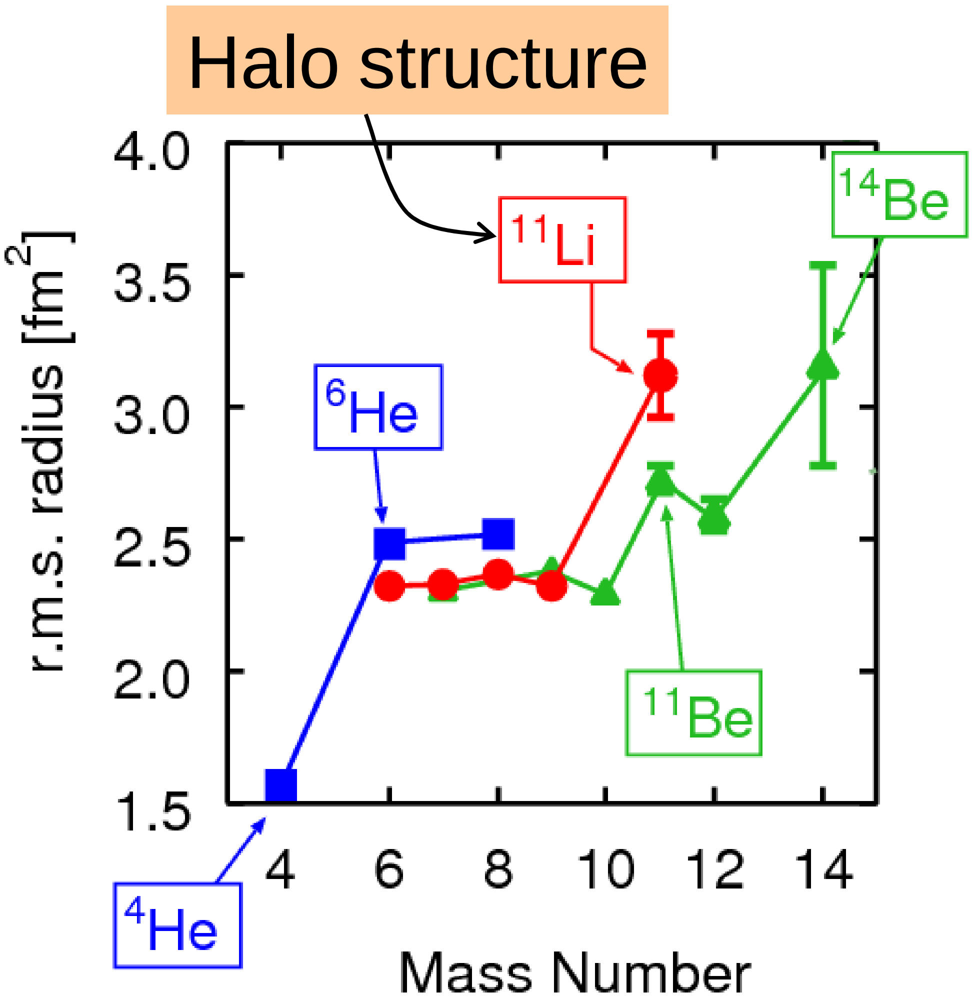
^{4}He in TOSM + S-wave UCOM



Kamada et al.
PRC64 (Jacobi)

Remaining effect :
3-body cluster term
in UCOM

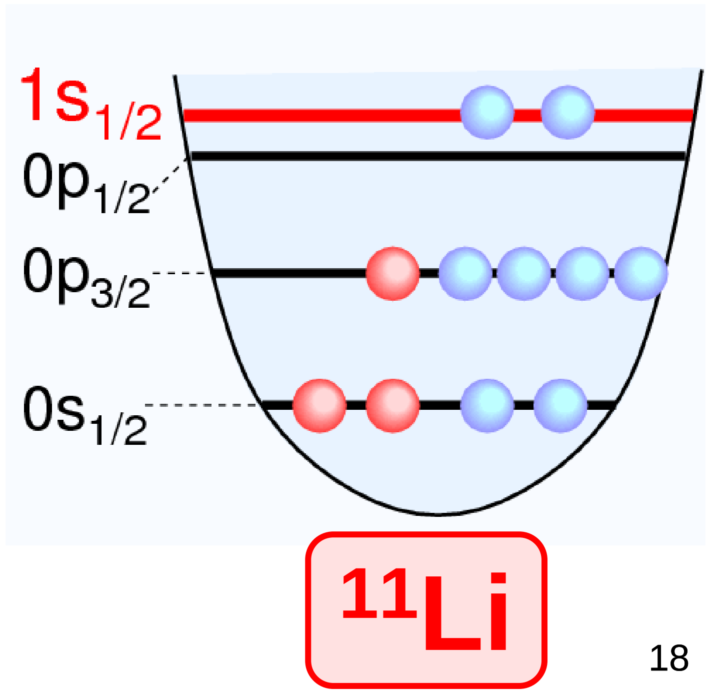
Characteristics of Li-isotopes



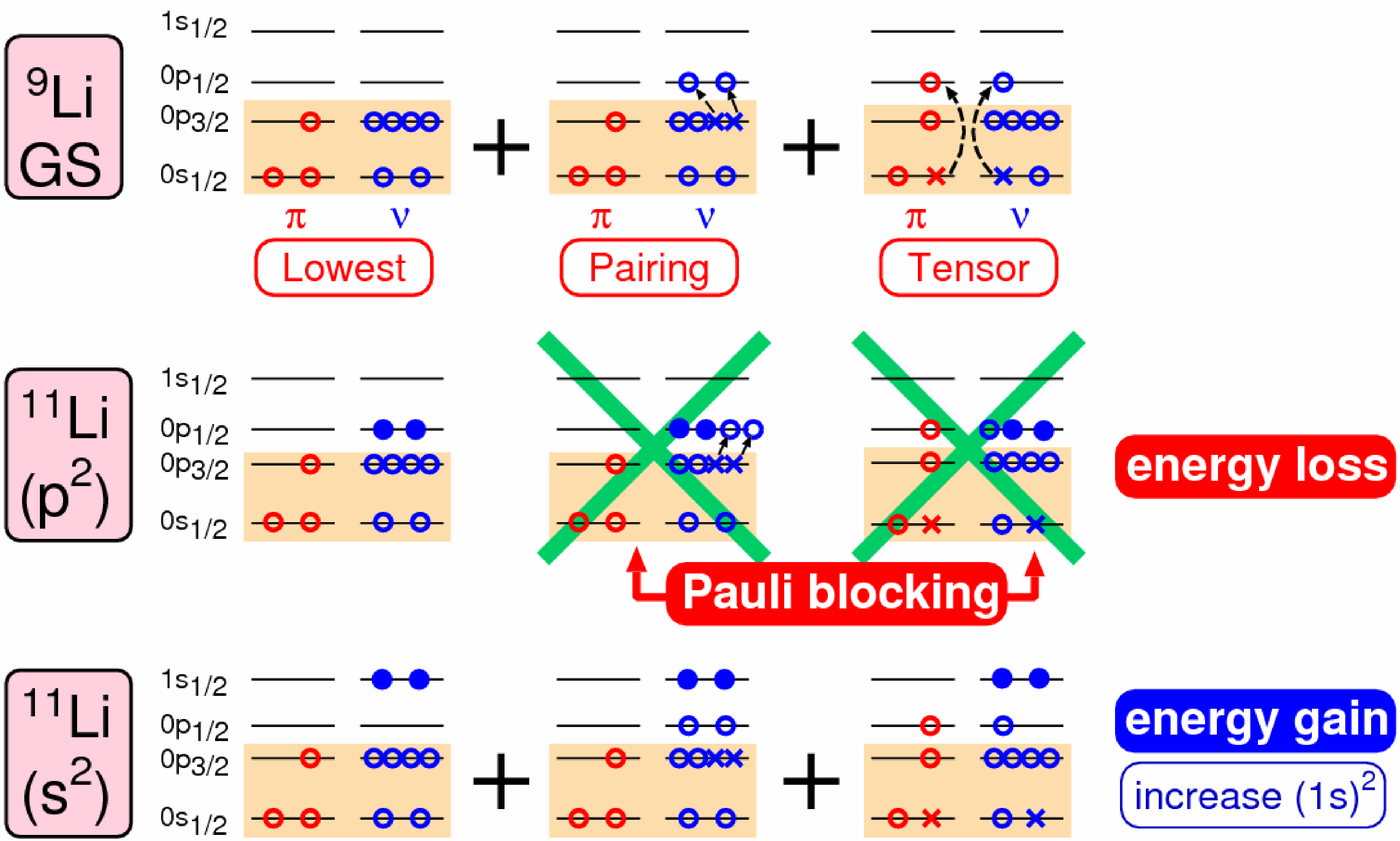
I. Tanihata et. al
PLB206(1988)592

- Breaking of magicity N=8
  - $^{10\text{-}11}Li$, $^{11\text{-}12}Be$
  - ^{11}Li … $(1s)^2 \sim 50\%$.

    (Expt by Simon et al.,PRL83)
  - Mechanism is unclear



Expected effects of pairing and tensor correlations in ^{11}Li



Pairing-blocking :

K.Kato,T.Yamada,K.Ikeda,PTP101('99)119, Masui,S.Aoyama,TM,K.Kato,K.Ikeda,NPA673('00)207.
TM,S.Aoyama,K.Kato,K.Ikeda,PTP108('02)133, H.Sagawa,B.A.Brown,H.Esbensen,PLB309('93)1.

^{11}Li in coupled ^{9}Li+n+n model

- System is solved based on RGM

$$H(^{11}\mathrm{Li}) = H(^{9}\mathrm{Li}) + H_{nn} \qquad \Phi(^{11}\mathrm{Li}) = A\left\{\sum_{i=1}^{N} \psi_i(^{9}\mathrm{Li}) \cdot \chi_i(nn)\right\}$$

$$\sum_{i=1}^{N} \left\langle \psi_j(^{9}\mathrm{Li}) \middle| H(^{11}\mathrm{Li}) - E \middle| A\left\{\psi_i(^{9}\mathrm{Li}) \cdot \chi_i(nn)\right\}\right\rangle = 0$$

$\psi_i(^{9}\mathrm{Li})$: shell model type configuration $\rightarrow$ **TOSM**

- Orthogonality Condition Model (OCM) is applied.

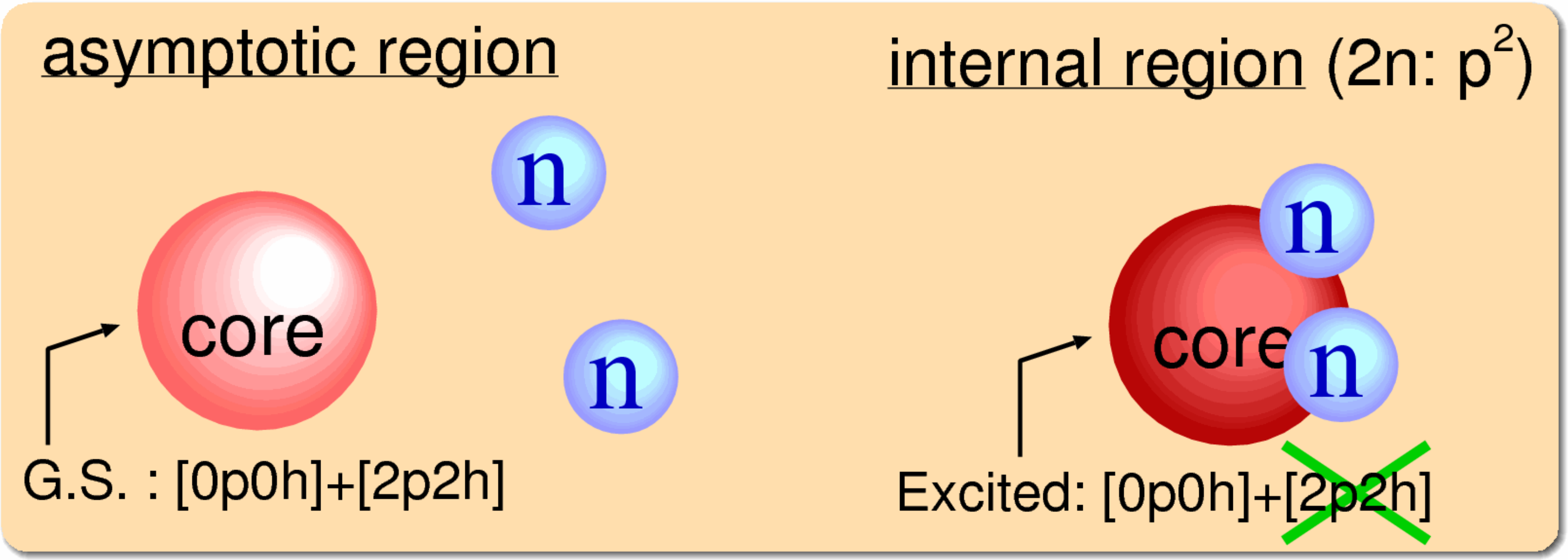
$$\sum_{i=1}^{N} \left[H_{ij}(^{9}\mathrm{Li}) + (T_1 + T_2 + V_{c1} + V_{c2} + V_{12}) \cdot \delta_{ij} \right] \chi_j(nn) = E \chi_i(nn)$$

$H_{ij}(^{9}\mathrm{Li}) = \left\langle \psi_i \middle| H(^{9}\mathrm{Li}) \middle| \psi_j \right\rangle$: Hamiltonian for ^{9}Li

$\chi(nn) = A\{\varphi_1 \varphi_2\}$: 2 neutrons with Gaussian expansion method

$\langle \varphi_i | \phi_\alpha \rangle = 0, \ \{\phi_\alpha \in {}^{9}\mathrm{Li}\}$: Orthogonality to the Pauli-forbidden states

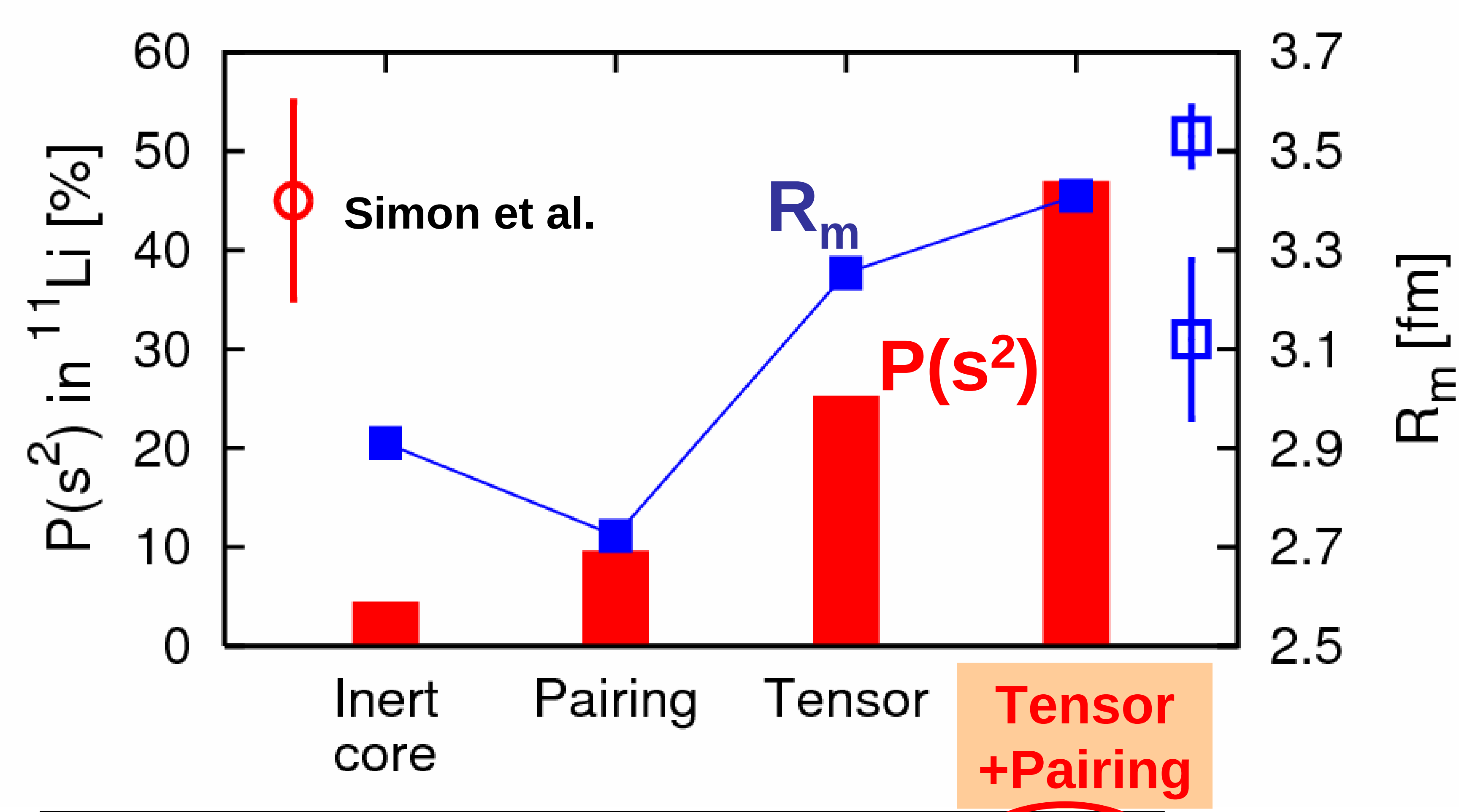
Boundary condition of the coupled ^{9}Li+n+n model



TM, K.Kato, H.Toki, K.Ikeda, PRC76('07)024305.

TM. Y.Kikuchi, K.Kato, H.Toki, K.Ikeda, PTP119('08)561.

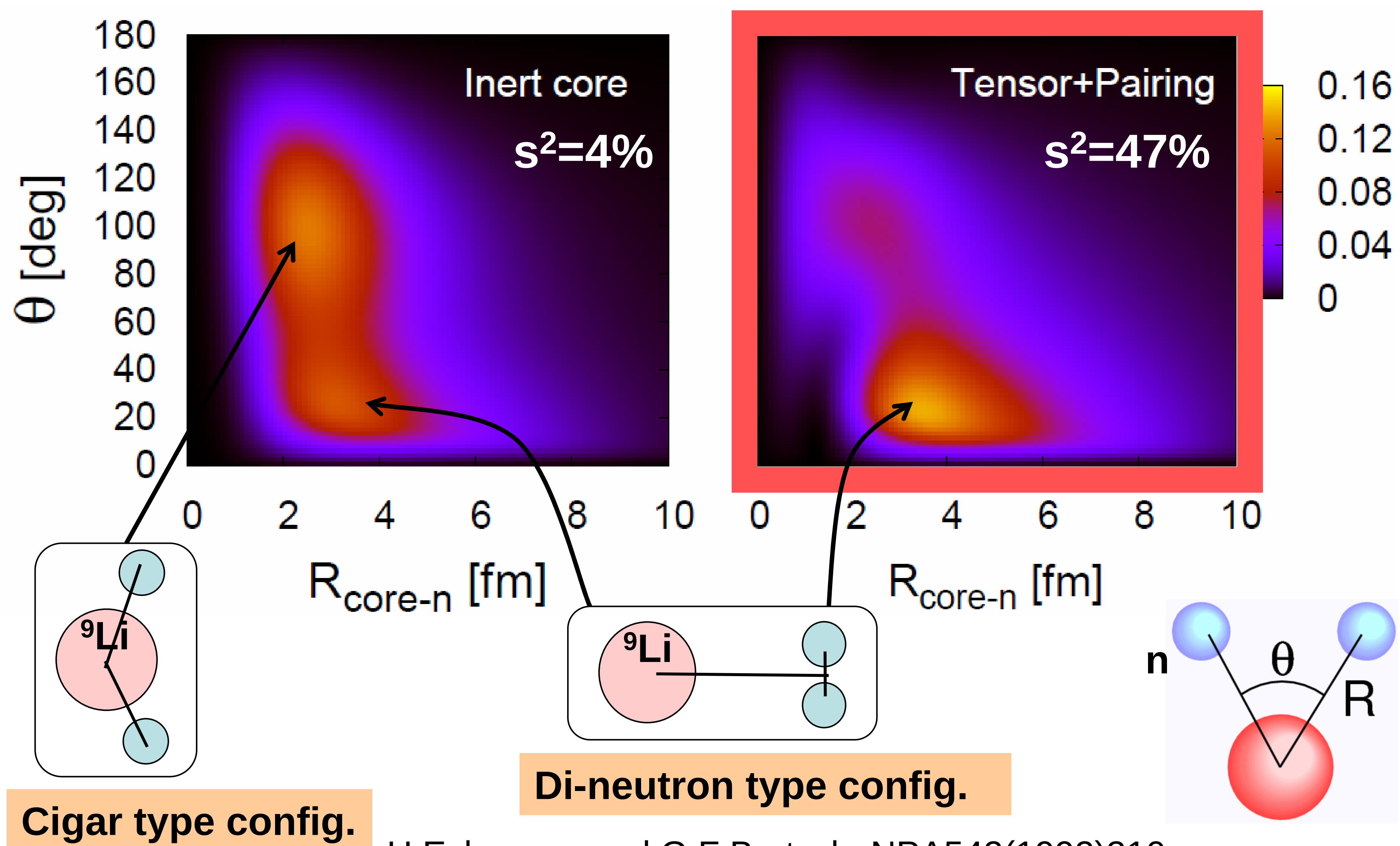
^{11}Li G.S. properties (S_{2n}=0.31 MeV)



$E(s^2)-E(p^2)$	2.1	1.4	0.5	-0.1	[MeV]

Pairing correlation of last 2n couples $(0p)^2$ and $(1s)^2$

2n correlation density in ^{11}Li

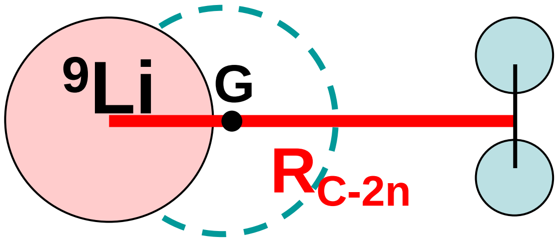


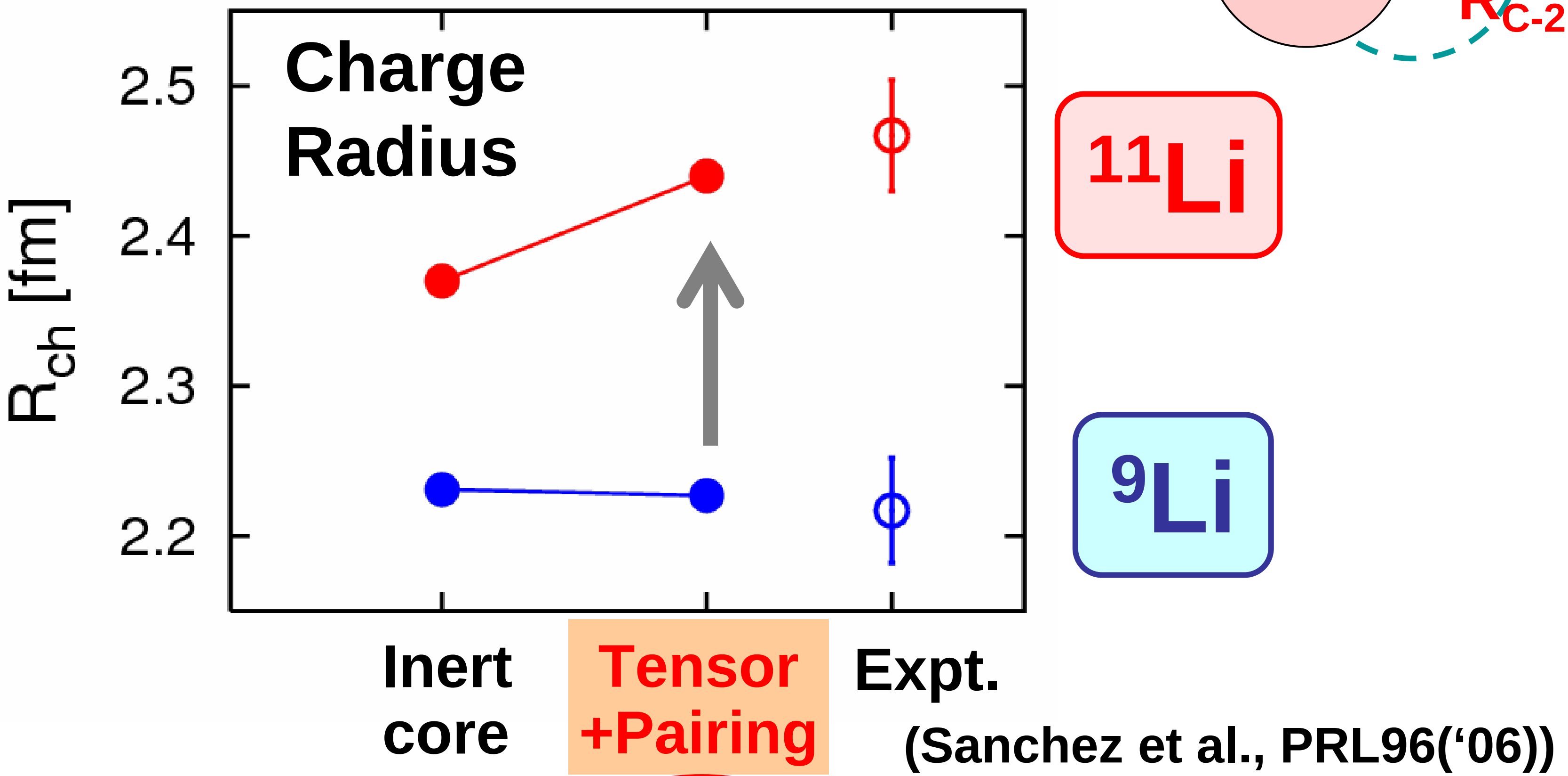
H.Esbensen and G.F.Bertsch, NPA542(1992)310

K. Hagino and H. Sagawa, PRC72(2005)044321

Charge Radii of Li isotopes

$$R^2_{\text{proton}}(^{11}\text{Li}) = R^2_{\text{proton}}(^{9}\text{Li}) + \left(\frac{2}{11}\right)^2 R^2_{\text{C-2n}}$$





	Inert core	Tensor +Pairing	
R_{C-2n}	4.67	5.69	[fm]
$P(s^2)$	4	47	%

Virtual s-wave states in ^{10}Li

- **$1s_{1/2}$ virtual state:**

$$(0p_{3/2})_\pi (1s_{1/2})_\nu \rightarrow 1^-, 2^-$$

a_s : scattering length of ^{9}Li+n

J^π	Inert core	Tensor +Pairing
1^-	+1.4 fm	-5.6 fm
2^-	+0.8 fm	-17.4 fm

TM. Y.Kikuchi, K.Kato, H.Toki, K.Ikeda
PTP119(2008)561
arXiv:0803.0590

Expt. M. Thoennessen et al., PRC59 (1999)111.
M. Chartier et al. PLB510(2001)24.
H.B. Jeppesen et al. PLB642(2006)449.

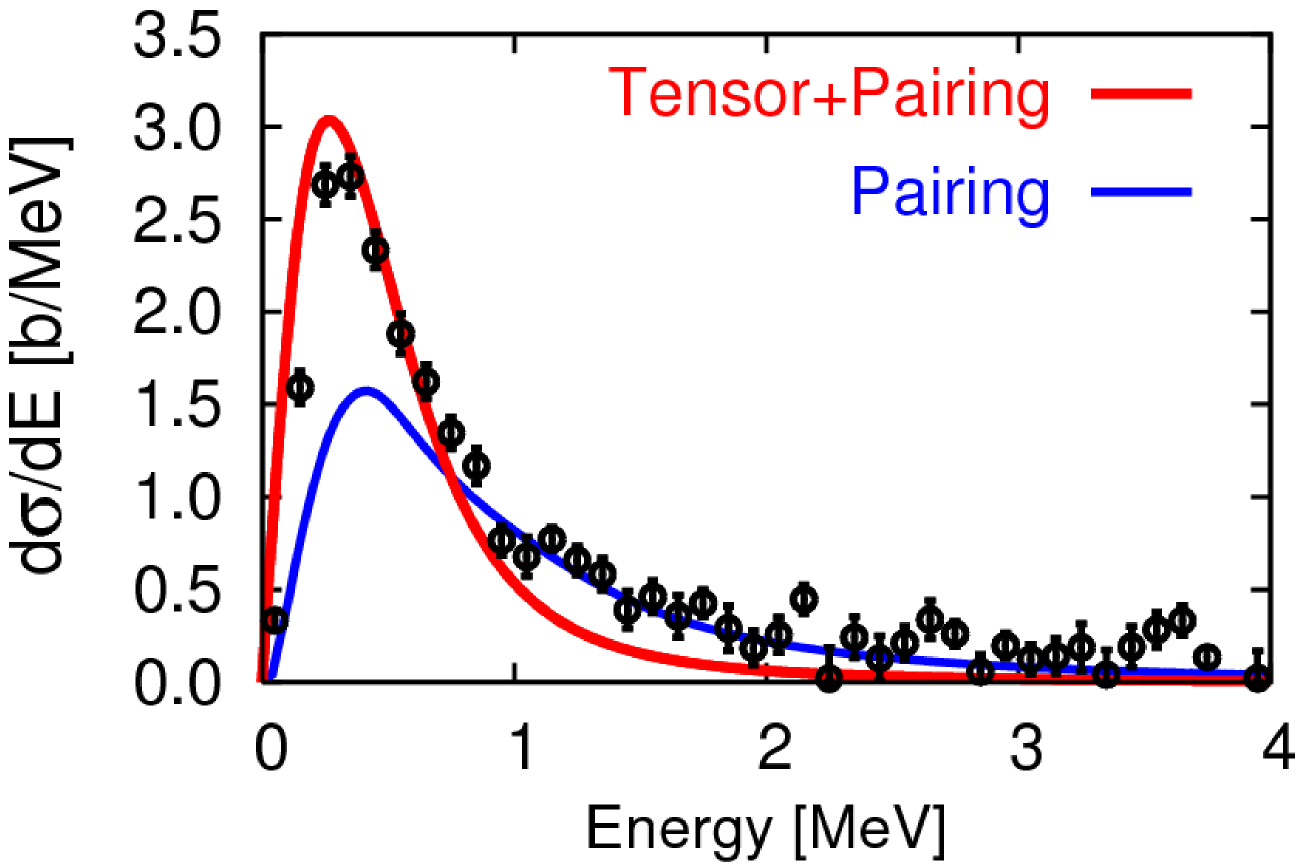
a_s = –10 ~ –25 fm

cf. $a_s(nn) : -18.5 \pm 0.5$ fm

Pauli-blocking naturally describes virtual s-state in ^{10}Li

Coulomb breakup strength of ^{11}Li

$$^{11}\mathrm{Li}\,(\mathrm{G.S.}) \rightarrow {}^{9}\mathrm{Li} + n + n$$



No three-body resonance

E1 strength by using the Green's function method

+Complex scaling method

+Equivalent photon method
(TM, Aoyama, Kato, Ikeda, PRC63('01)054313)

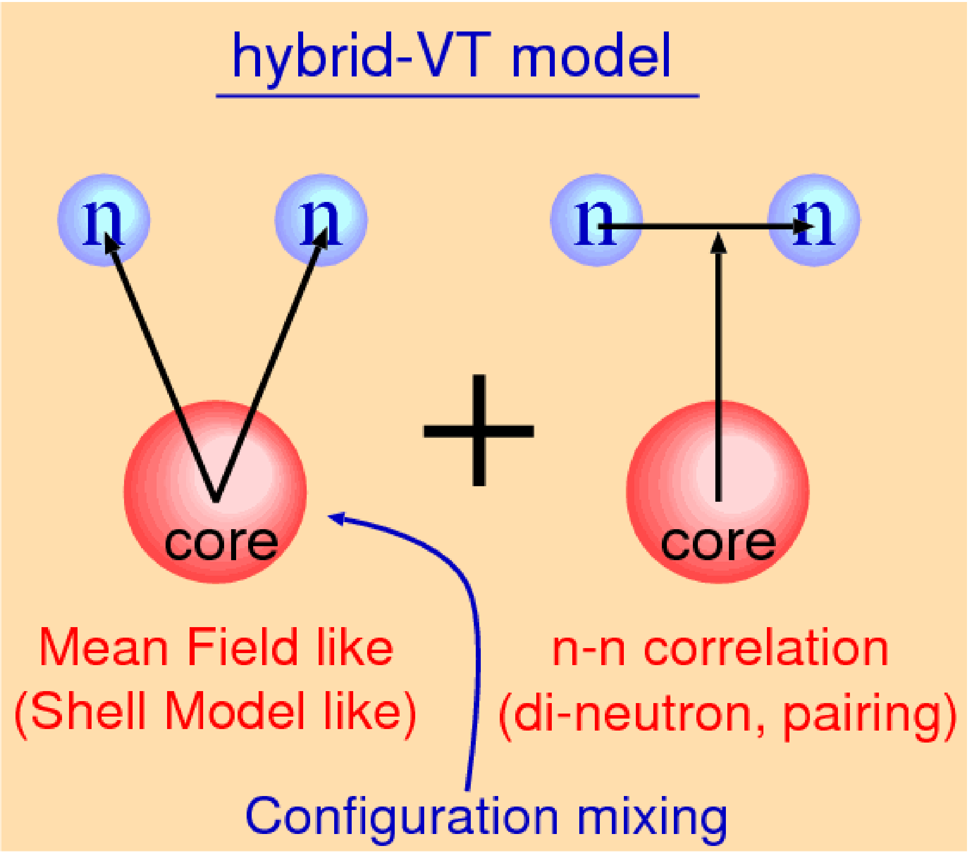
- Expt: T. Nakamura et al. , PRL96,252502(2006)
- Energy resolution with $\sqrt{E}$ =0.17 MeV.

Summary

- Tensor and short-range correlations
  - Tensor optimized shell model (TOSM)
  - Unitary Correlation Operator Method (UCOM)
    - Extended UCOM : S-wave UCOM
- Li isotopes with TOSM
  - Halo formation in ^{11}Li due to Pauli-blocking
- In TOSM+UCOM, we can study the nuclear structure starting from the bare interaction.
  - Spectroscopy of light nuclei (p-shell, sd-shell)

Hamiltonian of ^{11}Li

- $V_{9Li\text{-}n}$: folding potential
  - Same strength for s- and p-waves
  - Adjust to reproduce S_{2n}=0.31 MeV
- $V_{n\text{-}n}$: Argonne potential (AV8')
- 2n : Gaussian expansion



TM, K.Kato, H.Toki, K.Ikeda, PRC76('07)024305.

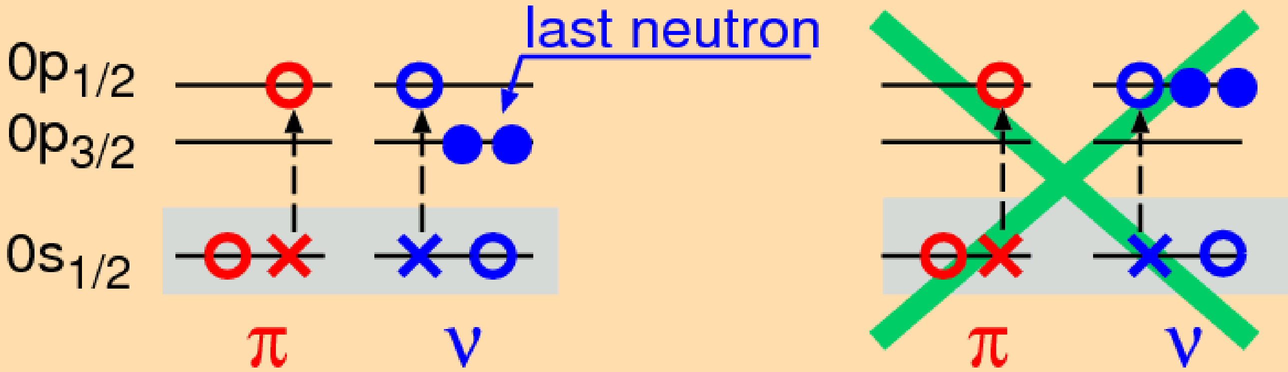
TM. Y.Kikuchi, K.Kato, H.Toki, K.Ikeda, PTP119('08)561.

^{11}Li 2n configuration

Config.	Inert	Present	Exp.
$(p_{1/2})^2$	90.6	42.7	
$(s_{1/2})^2$	4.3	46.9	45±10
$(p_{3/2})^2$	0.8	2.5	
$(d_{3/2})^2$	1.3	1.9	
$(d_{5/2})^2$	2.1	4.1	
$(f_{5/2})^2$	0.2	0.5	
$(f_{7/2})^2$	0.3	0.6	

Simon et al.,PRL83

Tensor correlation in ^{6}He



$(0p_{3/2})^2_\nu$

main configuration in G.S. (0^+)

$(0p_{1/2})^2_\nu$

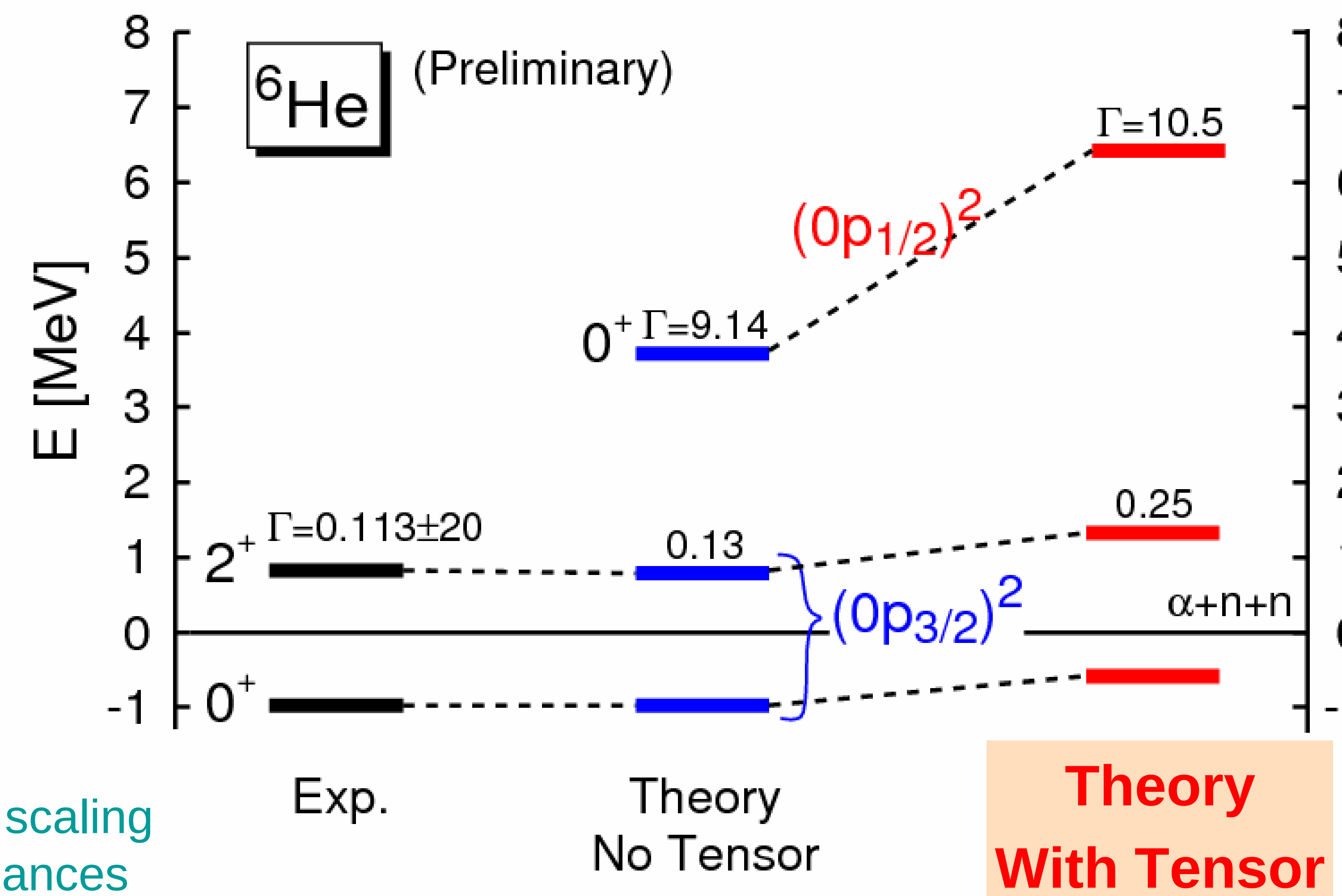
Tensor correlation is suppressed due to Pauli-Blocking

Ground state

Excited state

TM, K. Kato, K. Ikeda, J. Phys. G31 (2005) S1681

^{6}He results in coupled ^{4}He+n+n model



complex scaling for resonances

- $(0p_{3/2})^2$ can be described in Naive ^{4}He+n+n model
- $(0p_{1/2})^2$ loses the energy → Tensor suppression in 0^+_2

Pion exchange interaction vs. V_{tensor}

$$3(\vec{\sigma}_1 \cdot \hat{q})(\vec{\sigma}_2 \cdot \hat{q})\frac{q^2}{m^2+q^2} = (\vec{\sigma}_1 \cdot \vec{\sigma}_2)\frac{q^2}{m^2+q^2} + S_{12}\frac{q^2}{m^2+q^2}$$

$$= (\vec{\sigma}_1 \cdot \vec{\sigma}_2)\left[\frac{m^2+q^2}{m^2+q^2} - \frac{m^2}{m^2+q^2}\right] + S_{12}\frac{q^2}{m^2+q^2}$$

Delta interaction

Yukawa interaction

Involve large momentum

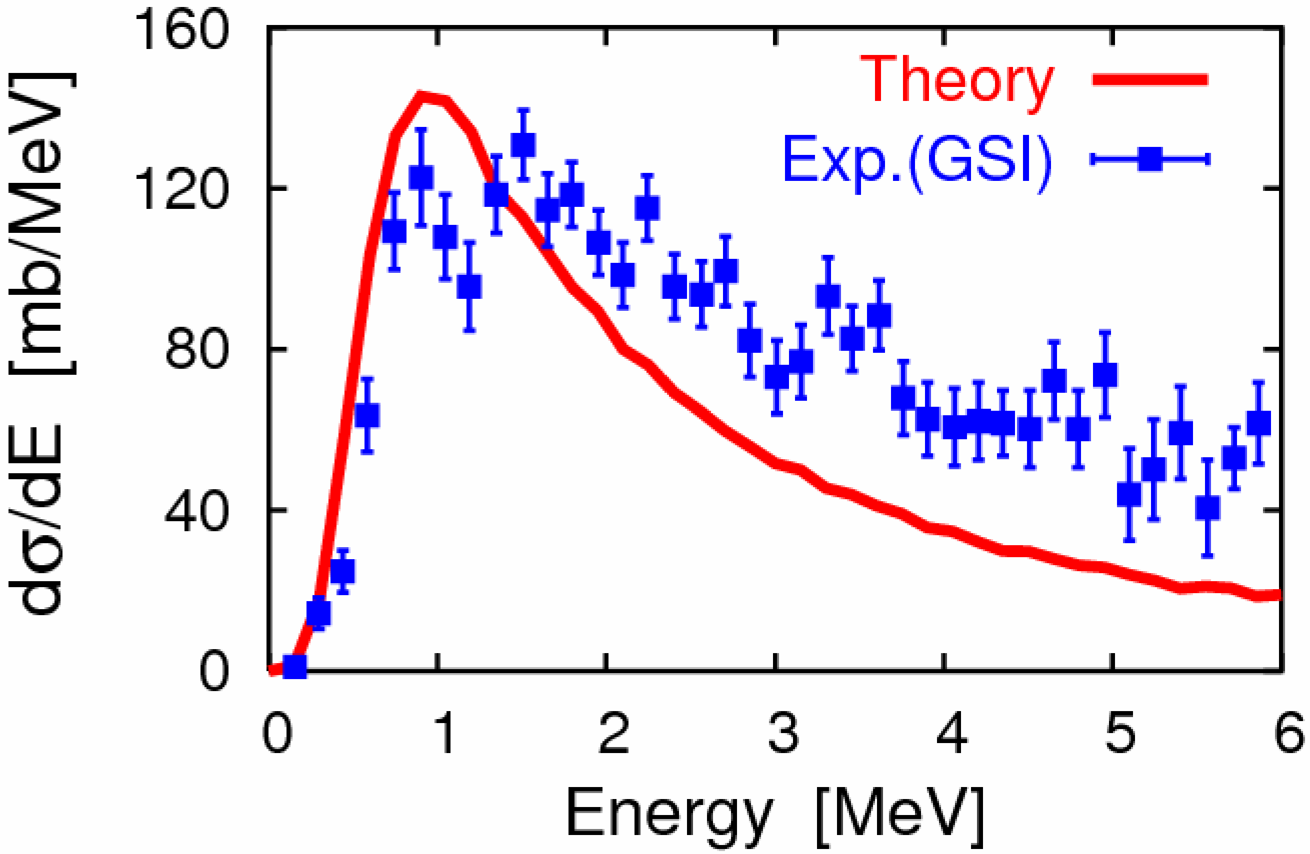
Tensor operator

$$S_{12} = 3(\vec{\sigma}_1 \cdot \hat{q})(\vec{\sigma}_2 \cdot \hat{q}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

- V_{tensor} produces the high momentum component.

Coulomb breakup strength of ^{6}He

$$^{6}\text{He (G.S.)} \rightarrow {}^{4}\text{He} + n + n$$



T.M, K. Kato, S. Aoyama and K. Ikeda PRC63(2001)054313.

^{6}He : 240MeV/A, Pb Target (T. Aumann et.al, PRC59(1999)1252)