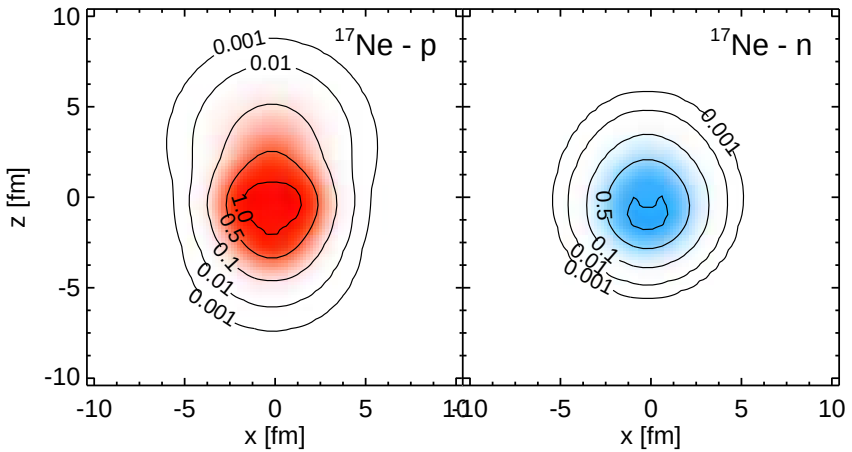
Clusters and Halos studied in Fermionic Molecular Dynamics



Thomas Neff

EENEN 09
First EMMI-EFES Workshop on Neutron-Rich Nuclei

GSI Darmstadt
February 10, 2009

HELMHOLTZ ASSOCIATION

GSI

Overview

Introduction

Unitary Correlation Operator Method

- **Short-range Correlations**
- **Correlated Interaction**

Fermionic Molecular Dynamics

- **Cluster States in ^{12}C**
- **Neon Isotopes – di-proton halo in ^{17}Ne**

UCOM

Central and Tensor Correlations

$$\underset{\sim}{C} = \underset{\sim}{C}_{\Omega}\underset{\sim}{C}_{r}$$

$$\mathbf{p} = \mathbf{p}_r + \mathbf{p}_\Omega$$

$$\mathbf{p}_r = \frac{1}{2}\left\{\frac{\mathbf{r}}{r}\left(\frac{\mathbf{r}}{r}\mathbf{p}\right) + \left(\mathbf{p}\frac{\mathbf{r}}{r}\right)\frac{\mathbf{r}}{r}\right\}, \qquad \mathbf{p}_\Omega = \frac{1}{2r}\left\{\mathbf{l}\times\frac{\mathbf{r}}{r} - \frac{\mathbf{r}}{r}\times\mathbf{l}\right\}$$

UCOM

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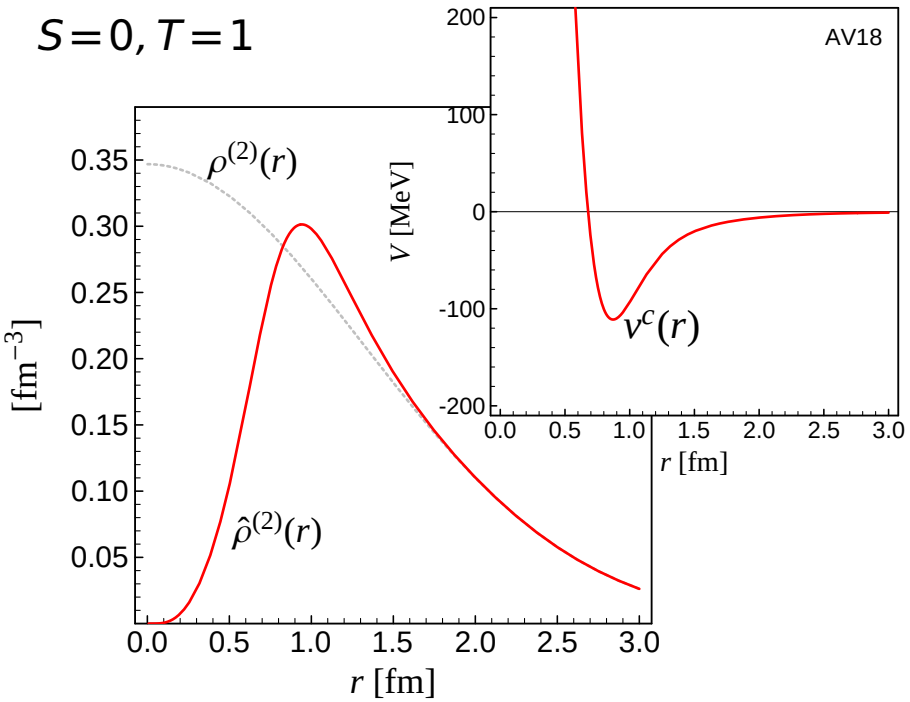
Central Correlations

$$\underset{\sim}{c}_r = \exp\left\{-\frac{i}{2}\{p_r s(r) + s(r) p_r\}\right\}$$

- probability density shifted out of the repulsive core



UCOM
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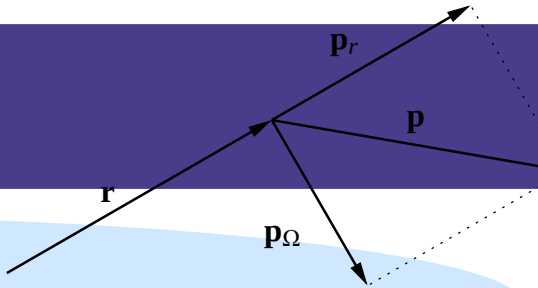

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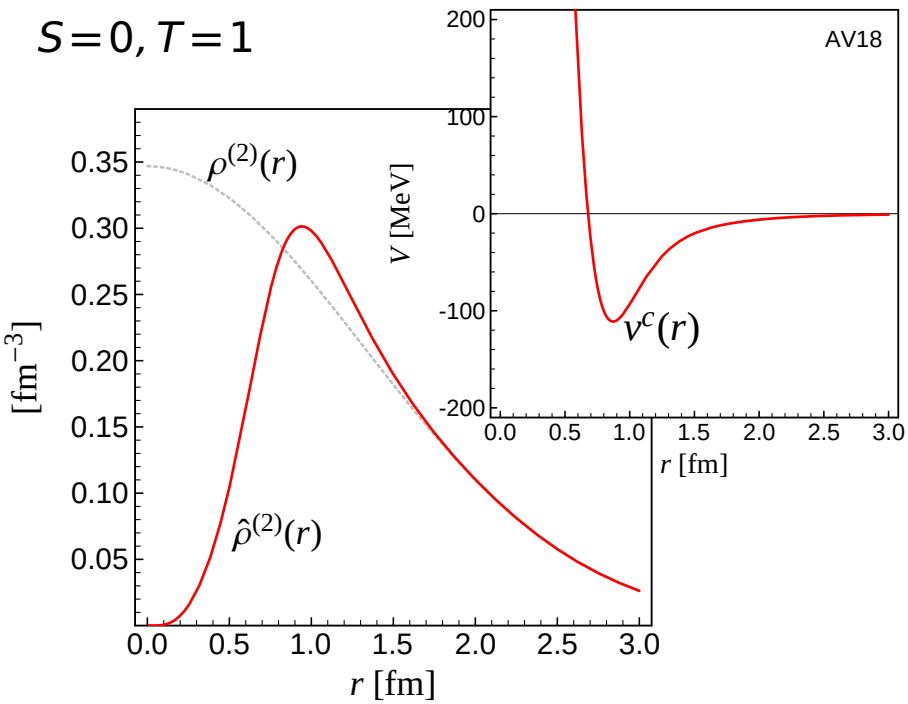
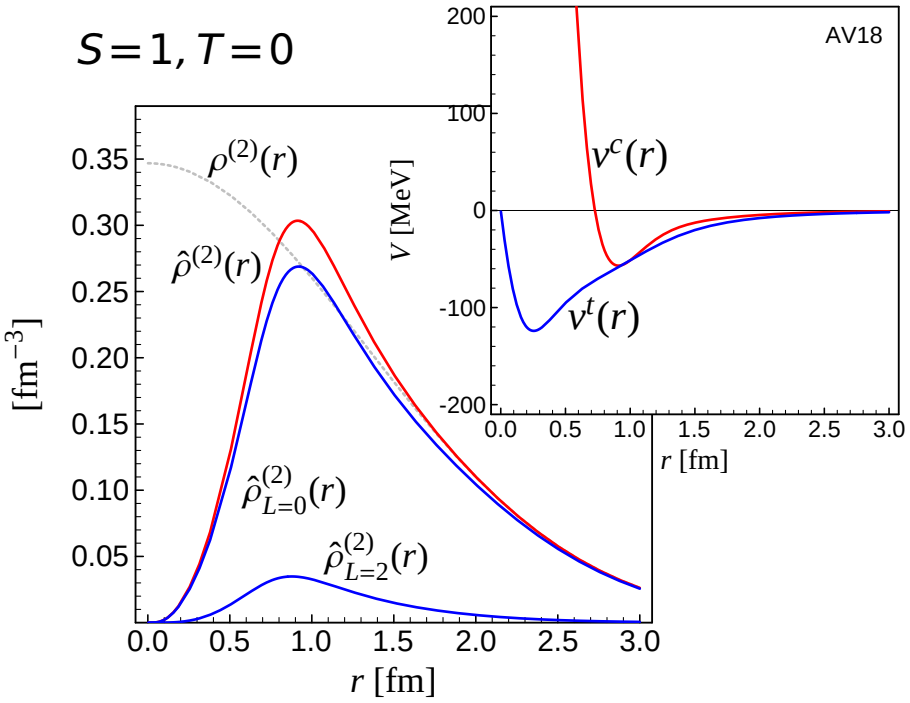


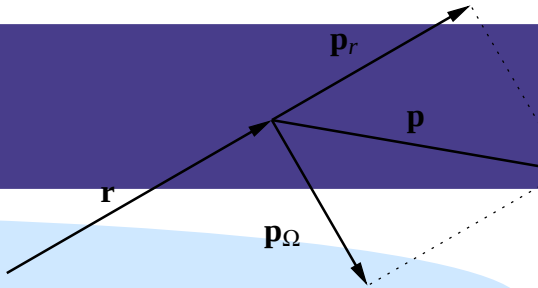
Tensor Correlations

$$\underset{\sim}{c}_\Omega = \exp\left\{-i\vartheta(r)\left\{\tfrac{3}{2}(\boldsymbol{\sigma}_1\cdot\mathbf{p}_\Omega)(\boldsymbol{\sigma}_2\cdot\mathbf{r}) + \tfrac{3}{2}(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{p}_\Omega)\right\}\right.$$

- tensor force admixes other angular momenta



UCOM
Central and Tensor Correlations



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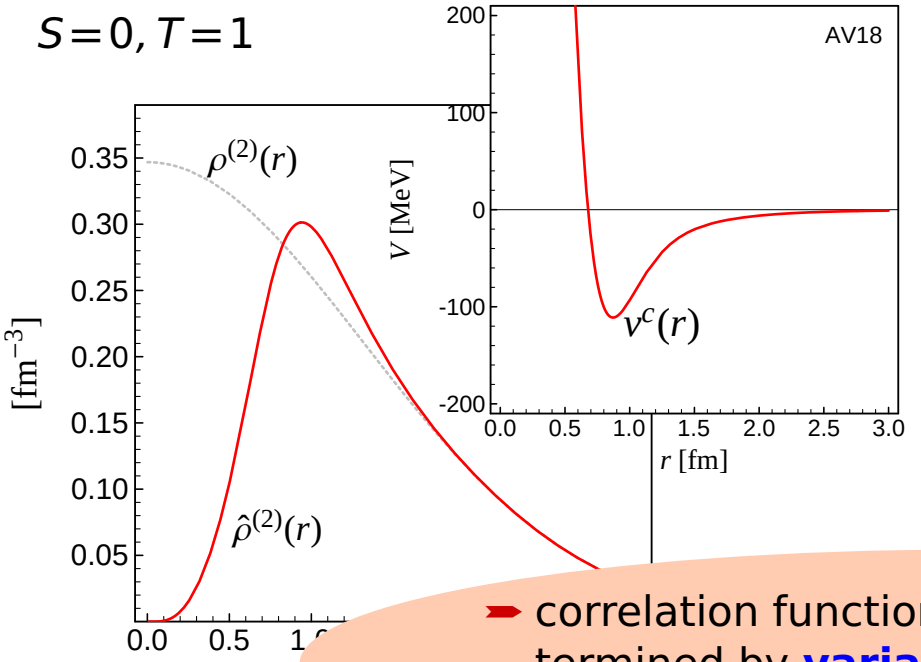
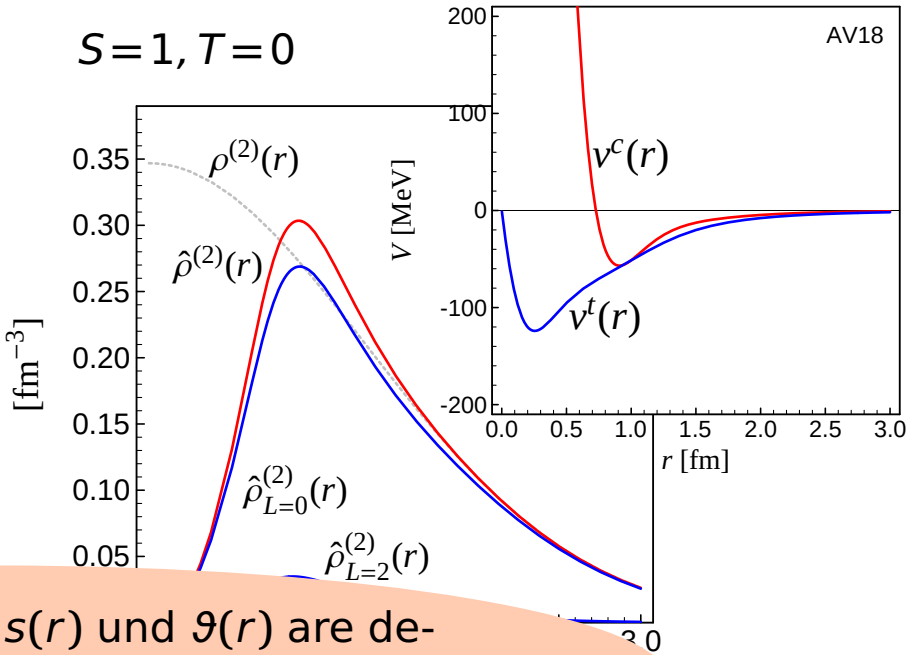
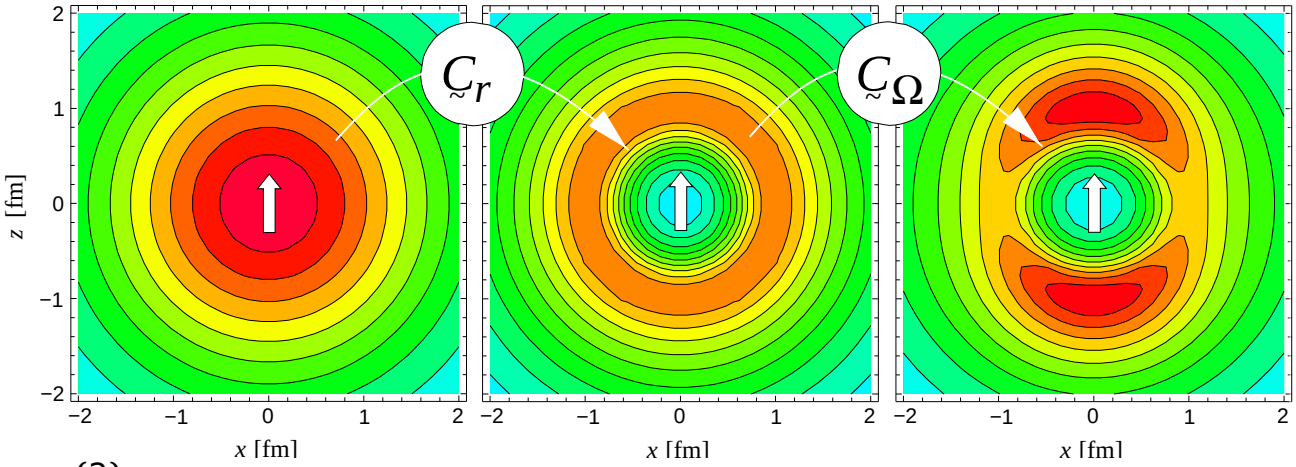




- correlation functions $s(r)$ und $\vartheta(r)$ are determined by **variation** of the energy in the **two-body system** for each S, T channel

Correlated Two-Body Densities and Energies

two-body densities



$\rho^{(2)}_{S,T}(\mathbf{r}_1 - \mathbf{r}_2) \quad S = 1, M_S = 1, T = 0$

central correlator $\hat{C}_\mathbf{r}$ shifts density out of the repulsive core

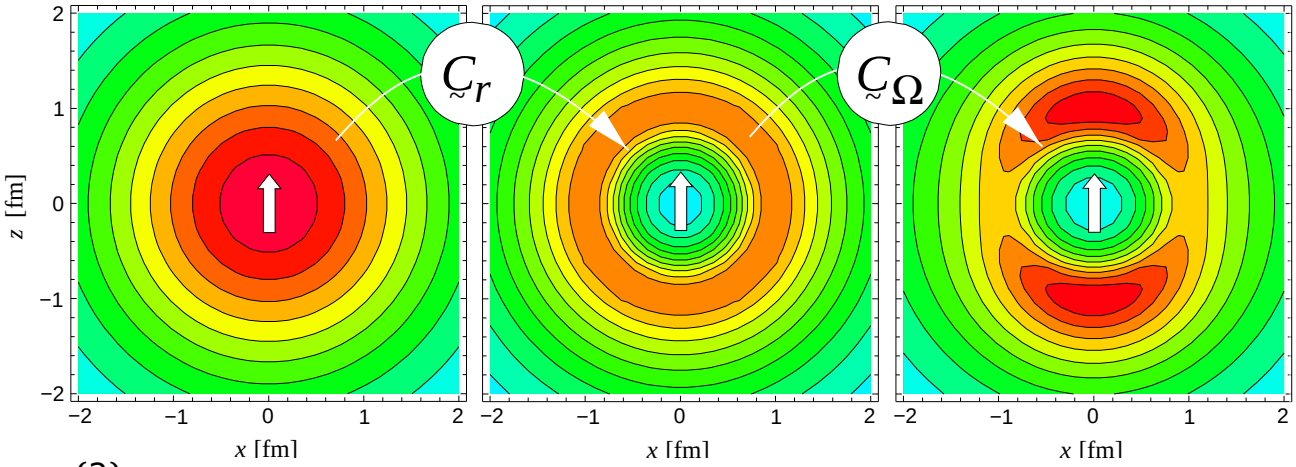
tensor correlator $\hat{C}_\mathbf{\Omega}$ aligns density with spin orientation

T. Neff and H. Feldmeier, Nucl. Phys. **A713** (2003) 311

UCOM

Correlated Two-Body Densities and Energies

two-body densities



$\rho^{(2)}_{S,T}(\mathbf{r}_1 - \mathbf{r}_2) \quad S = 1, M_S = 1, T = 0$

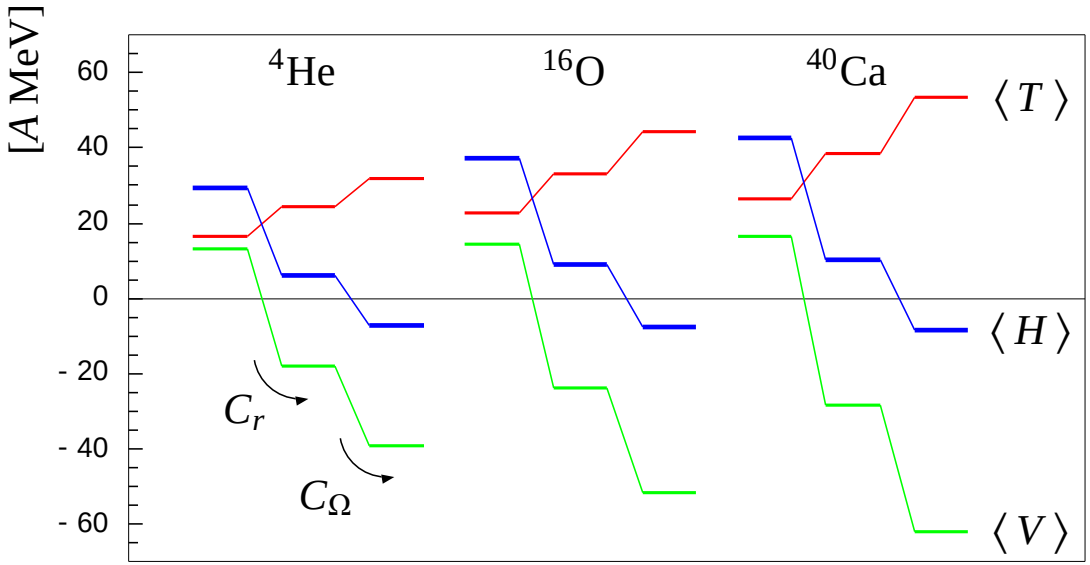
central correlator C_r shifts density out of the repulsive core

tensor correlator C_Ω aligns density with spin orientation

both central and tensor correlations are essential for binding

energies

$0\hbar\omega$ Harmonic Oscillator



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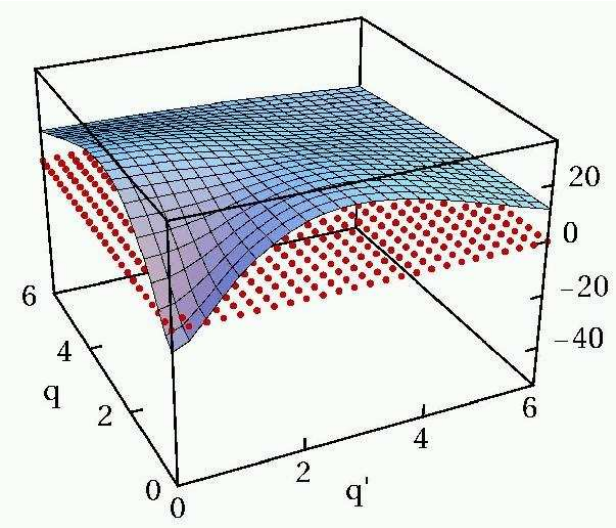
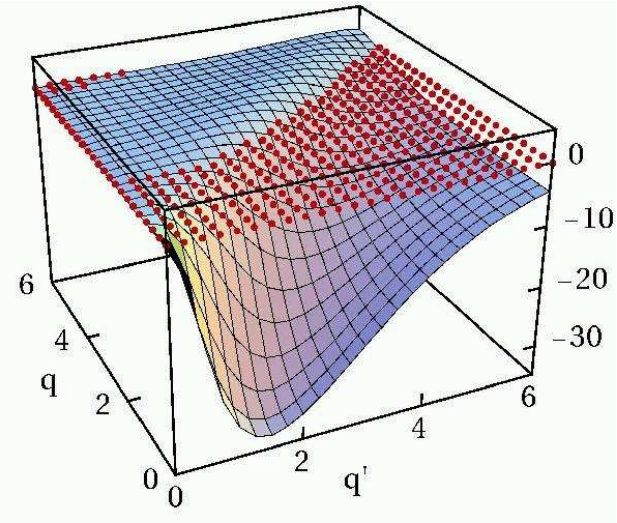
UCOM

Correlated Interaction in Momentum Space

$^3S_1$ bare



$^3S_1$ - $^3D_1$ bare



Phys. Rev. C **72**, 034002 (2005)

UCOM

Correlated Interaction in Momentum Space

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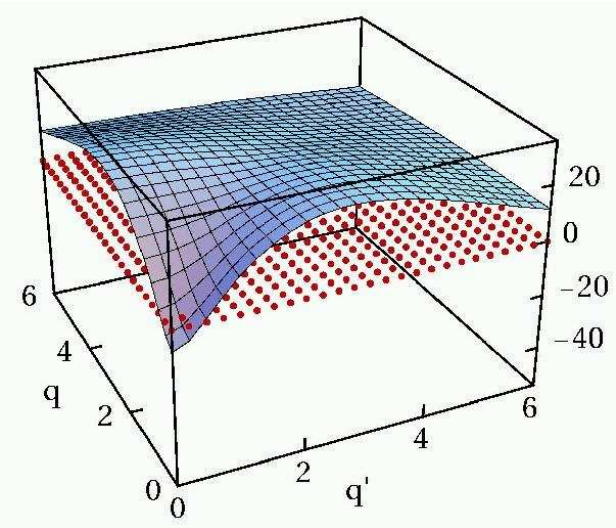
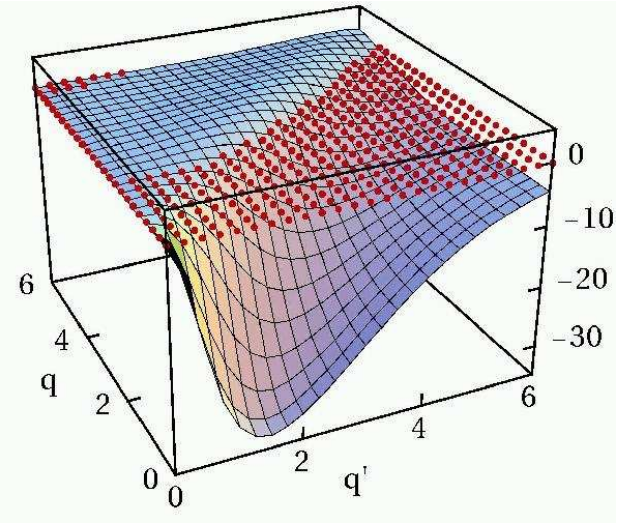


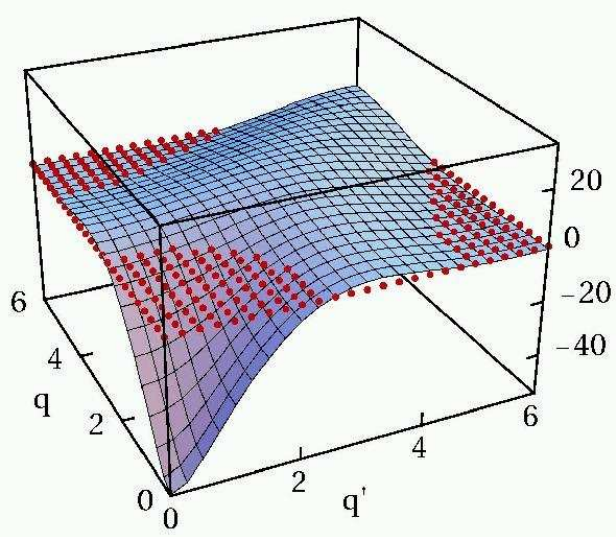
$^3S_1$ - $^3D_1$ bare



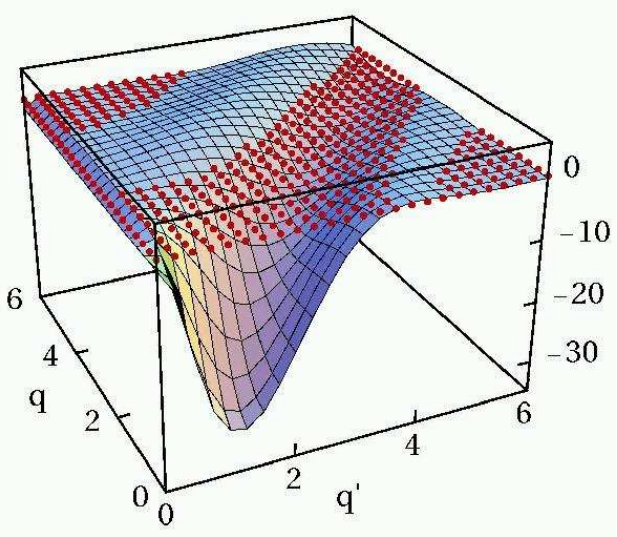
correlated interaction is **more attractive** at low momenta

off-diagonal matrix elements connecting low- and high- momentum states are **strongly reduced**

$^3S_1$ correlated



$^3S_1$ - $^3D_1$ correlated



Phys. Rev. C **72**, 034002 (2005)

Fermionic Molecular Dynamics

Motivation

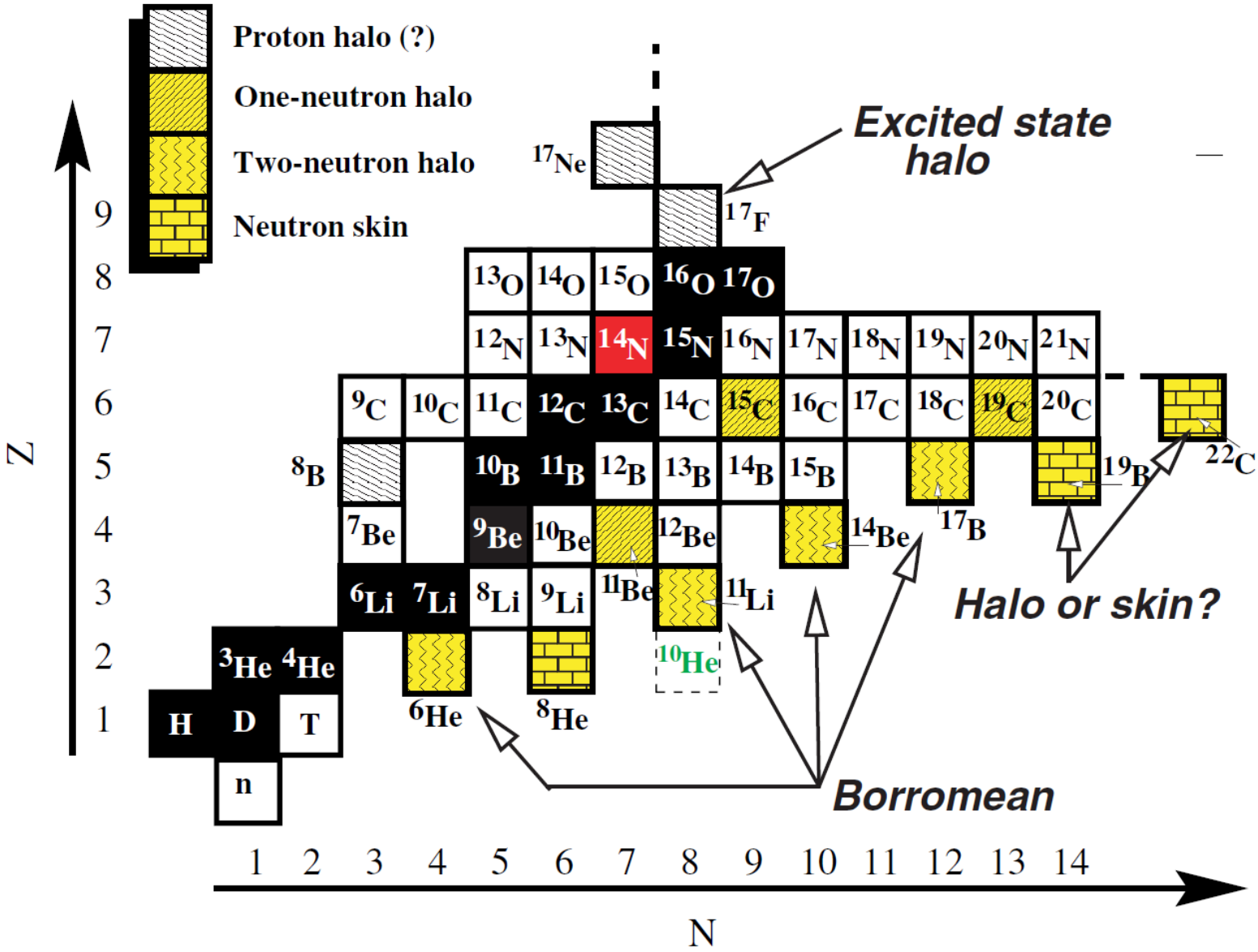
FMD Wave Functions

Nucleon-Nucleon Interaction

Mean-Field Calculations

**Projection After Variation,
Variation After Projection
and Multiconfiguration**

Exotica: Special Challenges



Al-Khalili, Nunes, J. Phys. G **29**, R89 (2003)

Fermionic Molecular Dynamics

Fermionic

Slater determinant

$$\left| Q \right\rangle = \underset{\sim}{\mathcal{A}} \left(\left| q_1 \right\rangle \otimes \cdots \otimes \left| q_A \right\rangle \right)$$

- antisymmetrized A-body state

Rev. Mod. Phys. **72** (2000) 655

Nucl. Phys. **A745** (2004) 3

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Molecular

single-particle states

$$\left\langle \mathbf{x} \middle| q \right\rangle = \sum_i c_i \exp\left\{ -\frac{(\mathbf{x} - \mathbf{b}_i)^2}{2a_i} \right\} \otimes \left| \chi^{\uparrow}_i, \chi^{\downarrow}_i \right\rangle \otimes \left| \xi \right\rangle$$

- Gaussian wave-packets in phase-space (complex parameter $\mathbf{b}_i$ encodes mean position and mean momentum), spin is free, isospin is fixed
- width a_i is an independent variational parameter for each wave packet
- superposition of two wave packets for each single particle state

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Fermionic Molecular Dynamics

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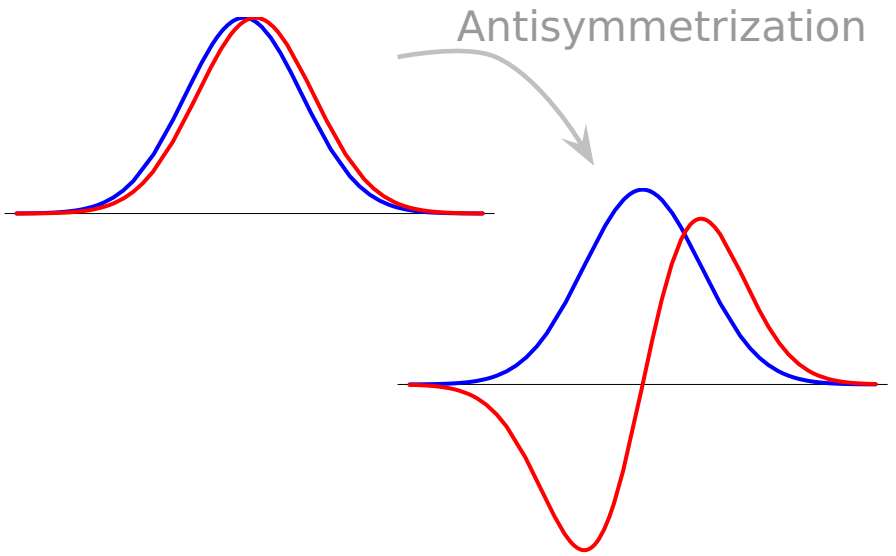

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Interaction Matrix Elements

(One-body) Kinetic Energy

$$\langle q_k \left| \underset{\sim}{T} \right| q_l \rangle = \langle a_k \mathbf{b}_k \left| \underset{\sim}{T} \right| a_l \mathbf{b}_l \rangle \langle \chi_k | \chi_l \rangle \langle \xi_k | \xi_l \rangle$$

$$\langle a_k \mathbf{b}_k \left| \underset{\sim}{T} \right| a_l \mathbf{b}_l \rangle = \frac{1}{2m} \left(\frac{3}{a_k^\star + a_l} - \frac{(\mathbf{b}_k^\star - \mathbf{b}_l)^2}{(a_k^\star + a_l)^2} \right) R_{kl}$$

(Two-body) Potential

- fit radial dependencies by (a sum of) Gaussians

$$G(\mathbf{x}_1 - \mathbf{x}_2) = \exp\left\{ -\frac{(\mathbf{x}_1 - \mathbf{x}_2)^2}{2\kappa} \right\}$$

- Gaussian integrals

$$\alpha_{klmn} = \frac{a_k{}^\star a_m}{a_k{}^\star + a_m} + \frac{a_l{}^\star a_n}{a_l{}^\star + a_n}$$

$$\boldsymbol{\rho}_{klmn} = \frac{a_m \mathbf{b}_k{}^\star + a_k{}^\star \mathbf{b}_m}{a_k{}^\star + a_m} - \frac{a_n \mathbf{b}_l{}^\star + a_l{}^\star \mathbf{b}}{a_l{}^\star + a_n}$$

$$R_{km} = \langle a_k \mathbf{b}_k \left| a_m \mathbf{b}_m \right\rangle$$

$$\langle a_k \mathbf{b}_k, a_l \mathbf{b}_l \left| \underset{\sim}{G} \right| a_m \mathbf{b}_m, a_n \mathbf{b}_n \rangle = R_{km} R_{ln} \left(\frac{\kappa}{\alpha_{klmn} + \kappa} \right)^{3/2} \exp\left\{ -\frac{\boldsymbol{\rho}_{klmn}^2}{2(\alpha_{klmn} + \kappa)} \right\}$$

- analytical formulas for matrix elements

Operator Representation of V_{UCOM}

$$\underset{\sim}{C}^{\dagger}(\underset{\sim}{T}+\underset{\sim}{V})\underset{\sim}{C} \;=\; \underset{\sim}{T}$$

one-body kinetic energy

$$+\sum_{ST}\hat{V}_{c}^{ST}(r)+\frac{1}{2}(\underset{\sim}{p_r}^2\,\hat{V}_{p^2}^{ST}(r)+\hat{V}_{p^2}^{ST}(r)\,\underset{\sim}{p_r}^2)+\hat{V}_{l^2}^{ST}(r)\,\underset{\sim}{\mathbf{l}}^2$$

central potentials

$$+\sum_{T}\hat{V}_{ls}^{T}(r)\,\underset{\sim}{\mathbf{l}}\cdot\underset{\sim}{\mathbf{s}}+\hat{V}_{l^2ls}^{T}(r)\,\underset{\sim}{\mathbf{l}}^2\underset{\sim}{\mathbf{l}}\cdot\underset{\sim}{\mathbf{s}}$$

spin-orbit potentials

$$+\sum_{T}\hat{V}_{t}^{T}(r)\,\underset{\sim}{S}_{12}(\mathbf{r},\mathbf{r})+\hat{V}_{trp_\Omega}^{T}(r)\,\underset{\sim}{p_r}\,\underset{\sim}{S}_{12}(\mathbf{r},\mathbf{p_\Omega})+\hat{V}_{tll}^{T}(r)\,\underset{\sim}{S}_{12}(\mathbf{l},\mathbf{l})+$$
$$\hat{V}_{tp_\Omega p_\Omega}^{T}(r)\,\underset{\sim}{S}_{12}(\mathbf{p_\Omega},\mathbf{p_\Omega})+\hat{V}_{l^2tp_\Omega p_\Omega}^{T}(r)\,\underset{\sim}{\mathbf{l}}^2\underset{\sim}{S}_{12}(\mathbf{p_\Omega},\mathbf{p_\Omega})$$

tensor potentials

bulk of tensor force mapped onto central part of correlated interaction

tensor correlations also change the spin-orbit part of the interaction

Nucl. Phys. **A745** (2004) 3

Phenomenological Correction to V_{UCOM}

Effective two-body interaction

- FMD model space can't describe correlations induced by residual medium-long ranged tensor forces

➨ use **longer ranged tensor correlator** to partly account for that

- no three-body forces, saturation with UCOM force not correct

➨ add phenomenological two-body correction term with a **momentum-dependend** central and (isospin-dependend) **spin-orbit** part (about 15% contribution to potential)

➨ fit correction term to binding energies and radii of "closed-shell" nuclei (^{4}He, ^{16}O, ^{40}Cα), (^{24}O, ^{34}Si, ^{48}Cα)

➨ **Todo:**
use **three-body** or **density dependent two-body force** instead of two-body correction term

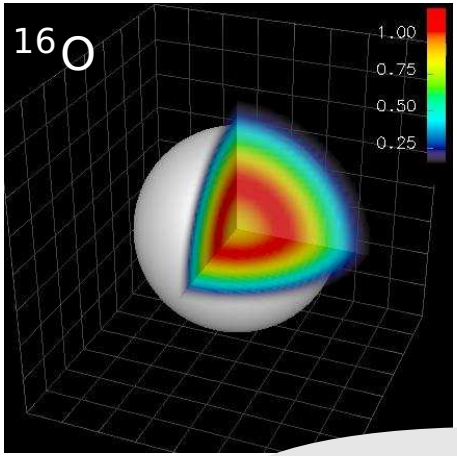
Perform Variation

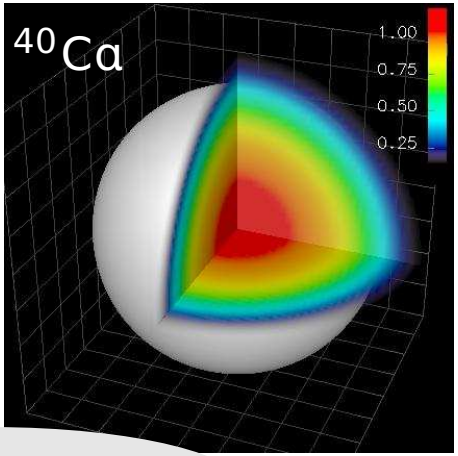
Minimization

- minimize Hamiltonian expectation value with respect to all single-particle parameters q_k

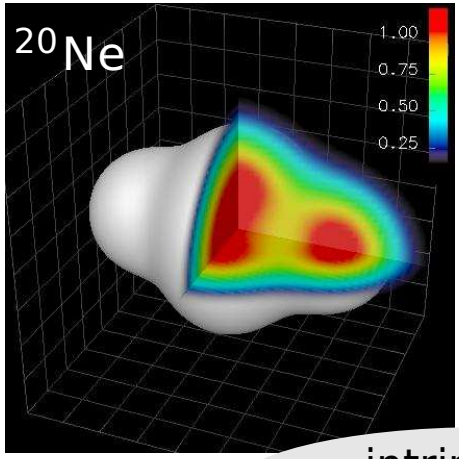
$$\min_{\{q_k\}} \frac{\langle Q \left| \underset{\sim}{H} - \underset{\sim}{T}_{cm} \right| Q \rangle}{\langle Q \left| Q \right. \rangle}$$

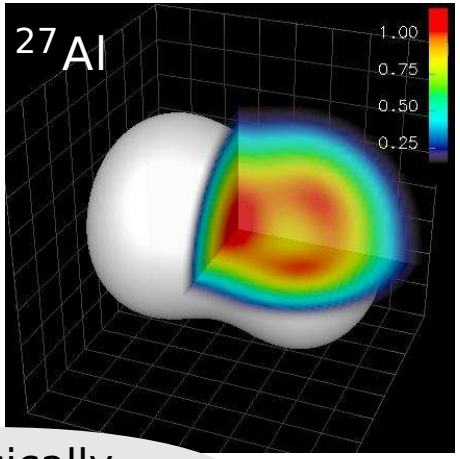
- this is a Hartree-Fock calculation in our particular single-particle basis
- the mean-field may break the symmetries of the Hamiltonian





spherical nuclei





intrinsically deformed nuclei

PAV, VAP and Multiconfiguration

Projection After Variation (PAV)

- mean-field may break symmetries of Hamiltonian
- restore inversion, translational and rotational symmetry by projection on parity, linear and angular momentum

$$\underset{\sim}{P}^{\pi} = \frac{1}{2}(1 + \pi \underset{\sim}{\Pi})$$

$$\underset{\sim}{P}^{J}_{MK} = \frac{2J+1}{8\pi^2} \int \mathrm{d}^3\Omega \, {D^{J}_{MK}}^{\star}(\Omega) \, \underset{\sim}{R}(\Omega)$$

$$\underset{\sim}{P}^{\mathbf{P}} = \frac{1}{(2\pi)^3} \int \mathrm{d}^3X \, \exp\{-i(\underset{\sim}{\mathbf{P}} - \mathbf{P}) \cdot \mathbf{X}\}$$

PAV, VAP and Multiconfiguration

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- effect of projection can be large
- perform Variation after Parity Projection PAV$^{\pi}$
- full Variation after Angular Momentum Projection (VAP)
- perform VAP in GCM sense by applying **constraints** on radius, dipole moment, quadrupole moment or octupole moment and minimizing the energy in the projected energy surface

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Multiconfiguration Calculations

- **diagonalize** Hamiltonian in a set of projected intrinsic states

$$\left\{ \left| Q^{(\alpha)} \right\rangle, \quad \alpha = 1, \ldots, N \right\}$$

$$\sum_{K'b} \left\langle Q^{(a)} \left| \underset{\sim}{H} \underset{\sim}{P}^{J^{\pi}}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \right| Q^{(b)} \right\rangle \cdot c^{\alpha}_{K'b} =$$

$$E^{J^{\pi}\alpha} \sum_{K'b} \left\langle Q^{(a)} \left| \underset{\sim}{P}^{J^{\pi}}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \right| Q^{(b)} \right\rangle \cdot c^{\alpha}_{K'b}$$

Cluster States in ^{12}C

Astrophysical Motivation

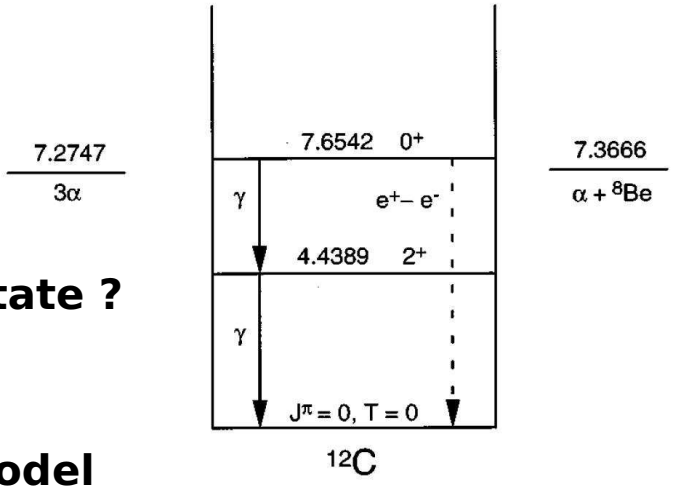
Structure

- **Is the Hoyle state a pure α-cluster state ?**
- **Other excited 0^+ and 2^+ states**

➨ **Compare FMD results to α-cluster model**

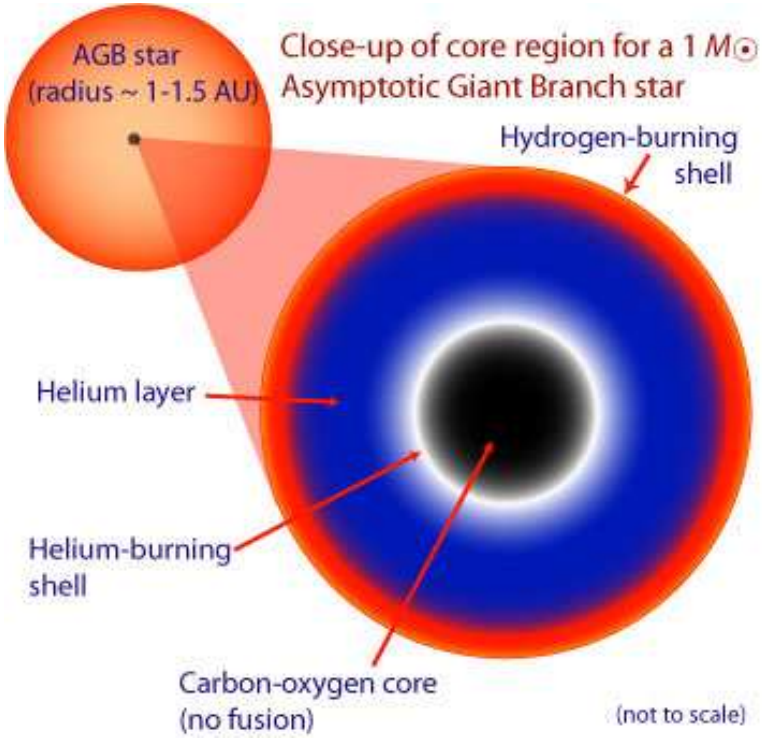
➨ **Analyze wave functions in harmonic oscillator basis**

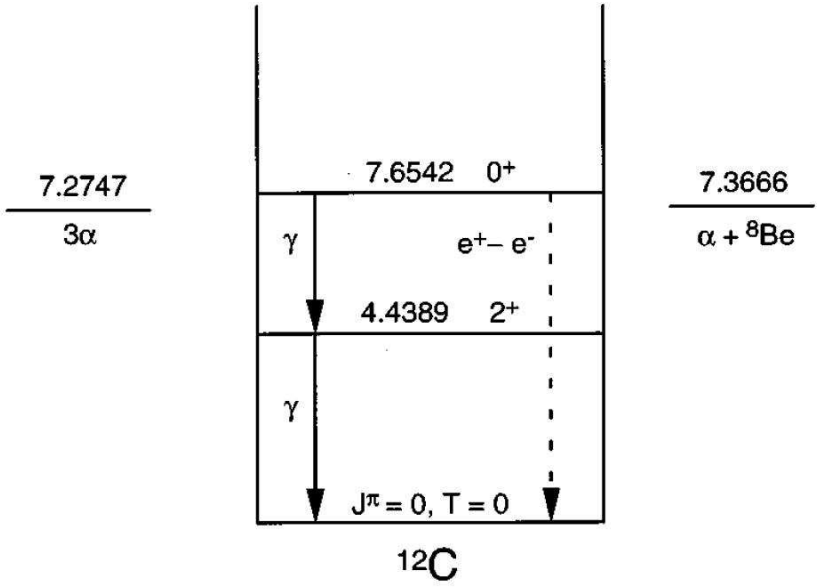
➨ **No-Core Shell Model Calculations ?**

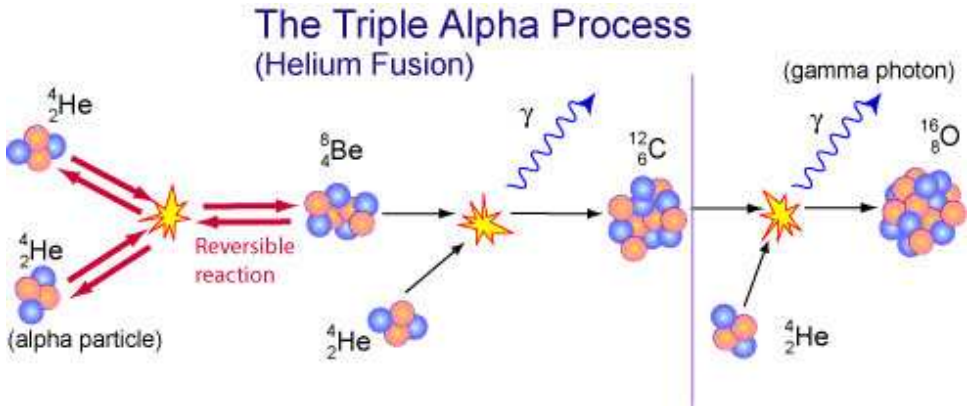


Cluster States in ^{12}C

Triple α Reaction

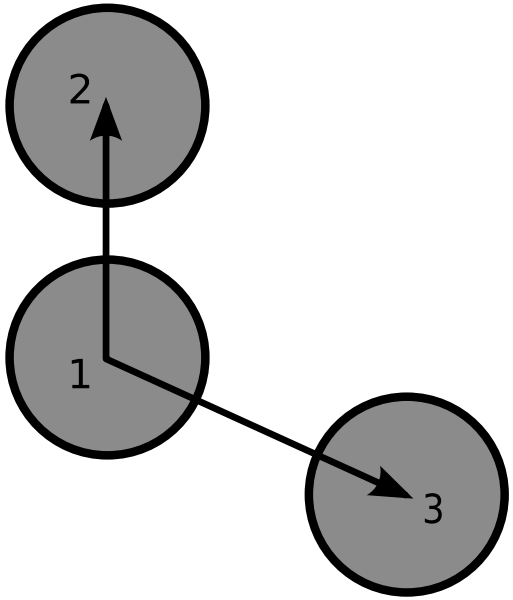






http://outreach.atnf.csiro.au/education/senior/astrophysics/stellarevolution_postmain.html

Cluster States in ^{12}C
Microscopic α-Cluster Model



$$R_{12} = (2, 4, \ldots, 10)\,\text{fm}$$
$$R_{13} = (2, 4, \ldots, 10)\,\text{fm}$$
$$\cos(\vartheta) = (1.0, 0.8, \ldots, -1.0)$$

alltogether 165 configurations

Basis States

- describe Hoyle State as a system of 3 ^{4}He nuclei

$$\left|\Psi_{3\alpha}(\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3); JMK\pi\right\rangle = P^J_{MK} P^\pi \mathcal{A}\left\{\left|\psi_\alpha(\mathbf{R}_1)\right\rangle \otimes \left|\psi_\alpha(\mathbf{R}_2)\right\rangle \otimes \left|\psi_\alpha(\mathbf{R}_3)\right\rangle\right\}$$

Volkov Interaction

- simple central interaction
- parameters adjusted to reproduce α binding energy and radius, $\alpha - \alpha$ scattering data and C12 ground state energy

✘ only reasonable for ^{4}He, ^{8}Be and ^{12}C nuclei

'BEC' wave functions

- same interaction and α-cluster parameters used by Funaki et al.

Kamimura, Nuc. Phys. **A351** (1981) 456

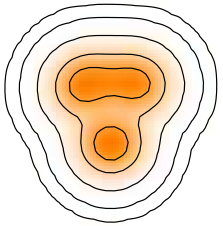
Funaki et al., Phys. Rev. C **67** (2003) 051306(R)

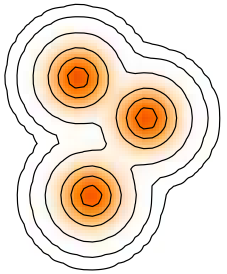
FMD

Basis States

- 20 FMD states obtained in Variation after Projection on 0^+ and 2^+ with constraints on the radius
- 42 FMD states obtained in Variation after Projection on parity with constraints on radius and quadrupole deformation
- 165 α-cluster configurations

➨ projected on angular momentum and linear momentum



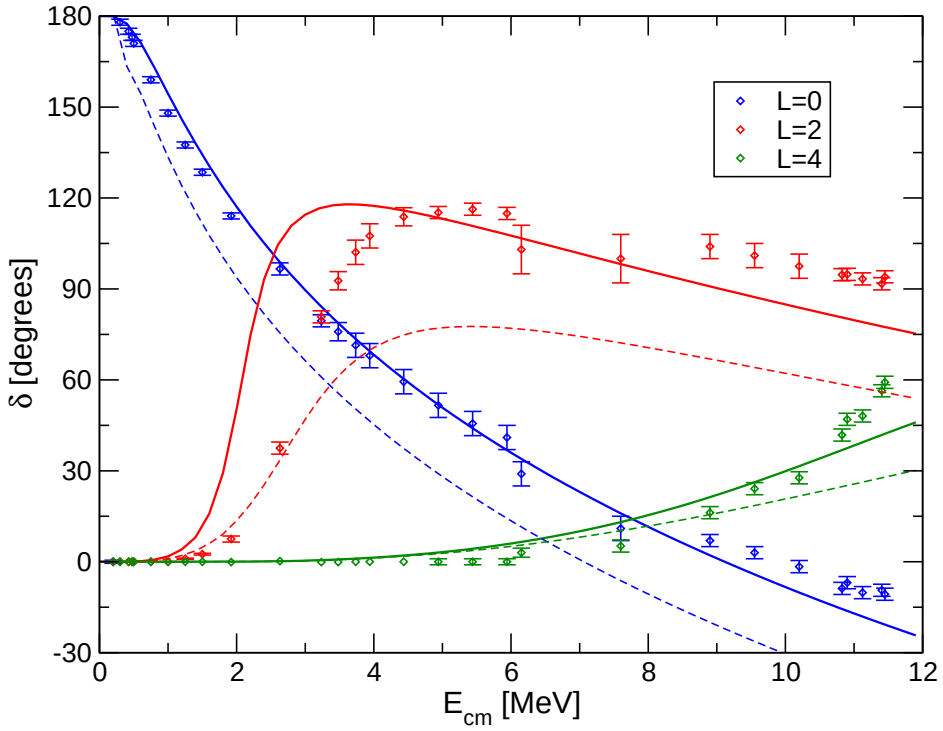


Interaction

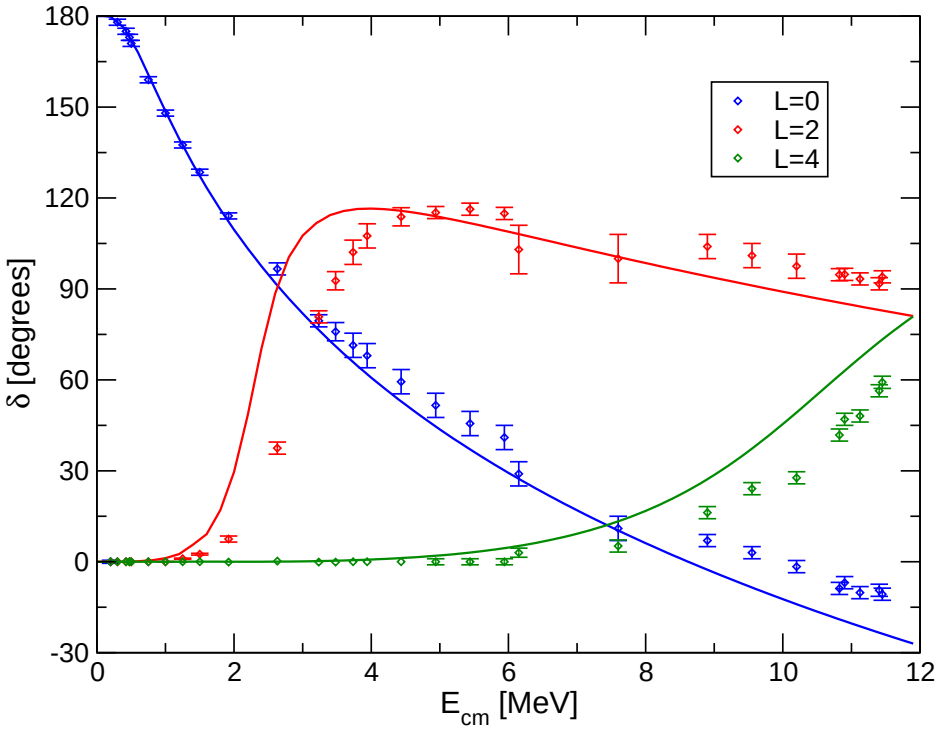
- not tuned for α-α scattering or ^{12}C properties

Cluster States in ^{12}C
α-α Phaseshifts

FMD



Cluster Model

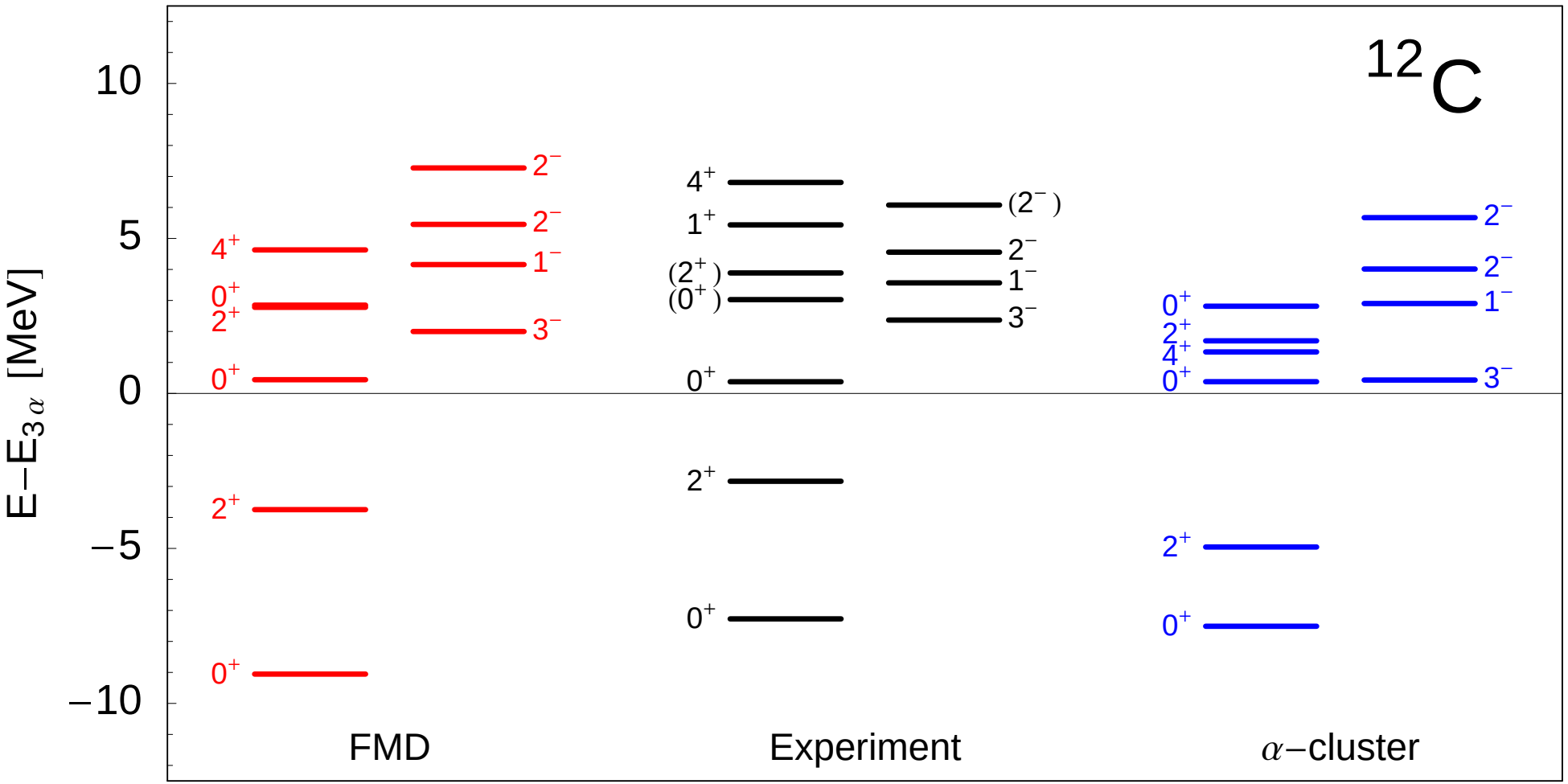


- Phaseshifts calculated with cluster configurations only (dashed lines)
- Phaseshifts calculated with additional FMD VAP configurations in the interaction region (solid lines)

- only cluster configurations included

➨ similar quality for description of α-α-scattering

Cluster States in ^{12}C
Comparison



Cluster States in ^{12}C
Comparison

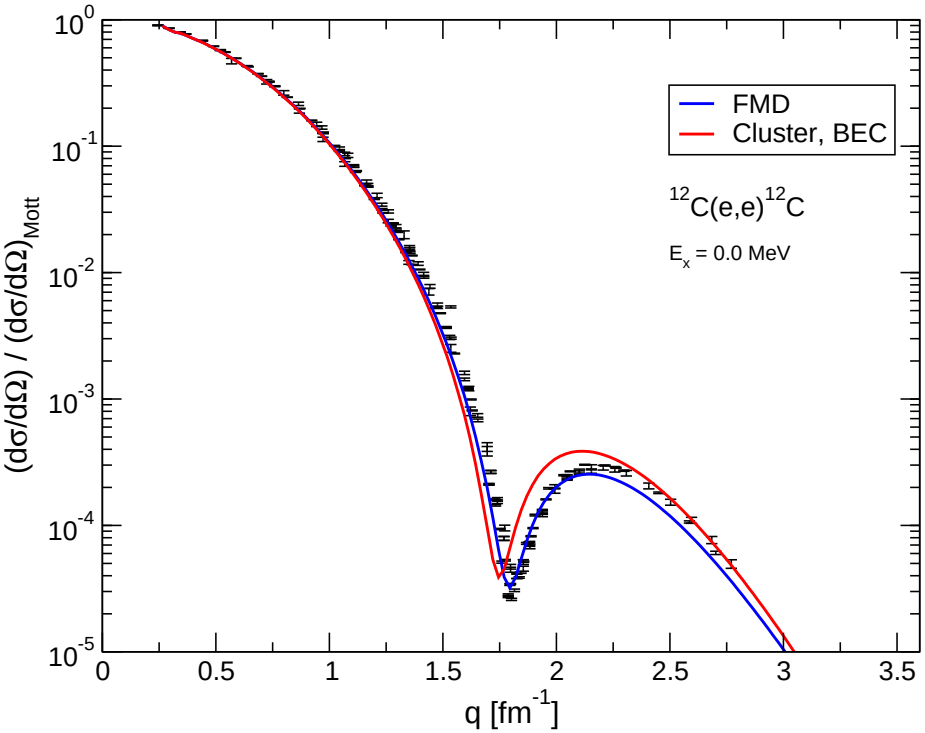
	Exp[1]	Exp[2]	Exp[3]	FMD	α-cluster	'BEC'[4]
$E(0_1^+)$	-92.16			-92.64	-89.56	-89.52
$E^*(2_1^+)$	4.44			5.31	2.56	2.81
$E(3\alpha)$	-84.89			-83.59	-82.05	-82.05
$E(0_2^+) - E(3\alpha)$	0.38			0.43	0.38	0.26
$E(0_3^+) - E(3\alpha)$	(3.0)	2.7(3)	3.96(5)	2.84	2.81	
$E(2_2^+) - E(3\alpha)$	(3.89)	2.6(3)	6.63(3)	2.77	1.70	
$r_{\text{charge}}(0_1^+)$	2.47(2)			2.53	2.54	
$r(0_1^+)$				2.39	2.40	2.40
$r(0_2^+)$				3.38	3.71	3.83
$r(0_3^+)$				4.62	4.75	
$r(2_1^+)$				2.50	2.37	2.38
$r(2_2^+)$				4.43	4.02	
$M(E0, 0_1^+ \to 0_2^+)$	5.4(2)			6.53	6.52	6.45
$B(E2, 2_1^+ \to 0_1^+)$	7.6(4)			8.69	9.16	
$B(E2, 2_1^+ \to 0_2^+)$	2.6(4)			3.83	0.84	

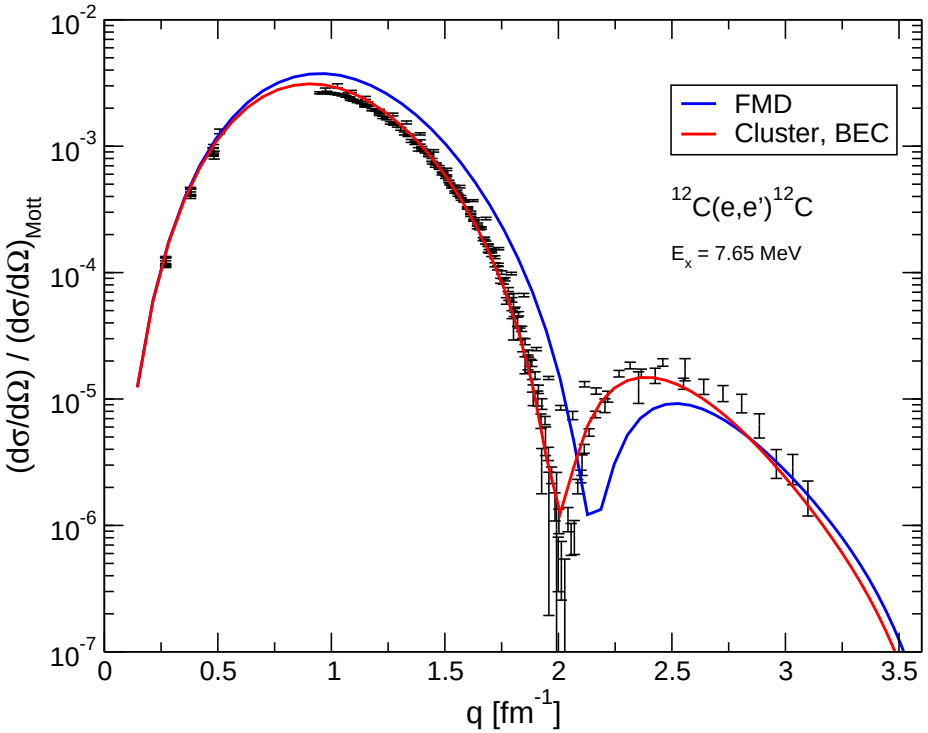
experimental situation for 0_3^+ and 2_2^+ states still unsettled

2_2^+ resonance at 1.8 MeV above treshold included in NACRE compilation

[1] Ajzenberg-Selove, Nuc. Phys. **A506**, 1 (1990)
[2] Itoh et al., Nuc. Phys. **A738**, 268 (2004)
[3] Fynbo et al., Nature **433**, 137 (2005). Diget et al., Nuc. Phys. **A738**, 760 (2005)
[4] Funaki et al., Phys. Rev. C **67**, 051306(R) (2003)

Cluster States in ^{12}C
Electron Scattering Data





- compare with precise electron scattering data up to high momenta in Distorted Wave Born Approximation
- use intrinsic density

$$\rho(\mathbf{x}) = \sum_{k=1}^{A} \langle \Psi \left| \delta(\underset{\sim}{\mathbf{x}}_k - \underset{\sim}{\mathbf{X}} - \mathbf{x}) \right| \Psi \rangle$$

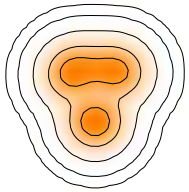
- elastic form factor described very well by FMD
- transition form factor better described by cluster model

M. Chernykh, H. Feldmeier, T. Neff, P. von Neumann-Cosel, and A. Richter, Phys. Rev. Lett. **98** (2007) 032501

Cluster States in ^{12}C

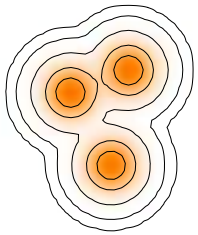
Important Configurations

- Calculate the overlap with FMD basis states to find the most important contributions to the Hoyle state

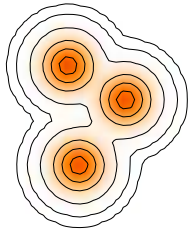


$\left|\left\langle \cdot \mid 0_1^+ \right\rangle\right| = 0.94$

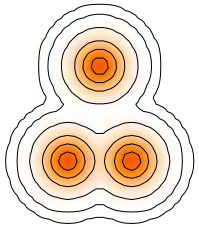
$\left|\left\langle \cdot \mid 2_1^+ \right\rangle\right| = 0.93$



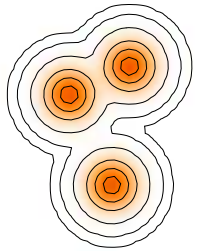
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.72$



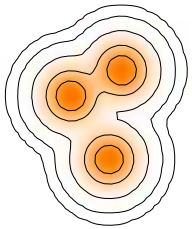
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.71$



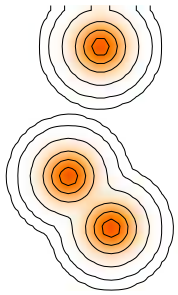
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.61$



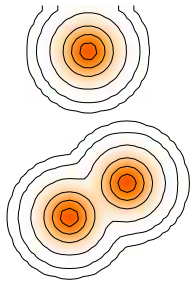
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.61$



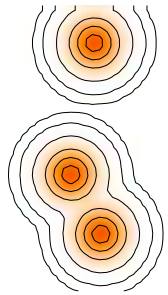
$\left|\left\langle \cdot \mid 3_1^- \right\rangle\right| = 0.83$



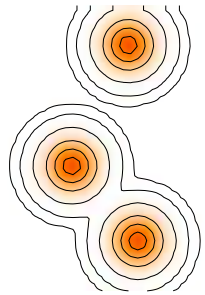
$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.50$



$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.49$



$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.44$



$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.41$

FMD basis states are not orthogonal!

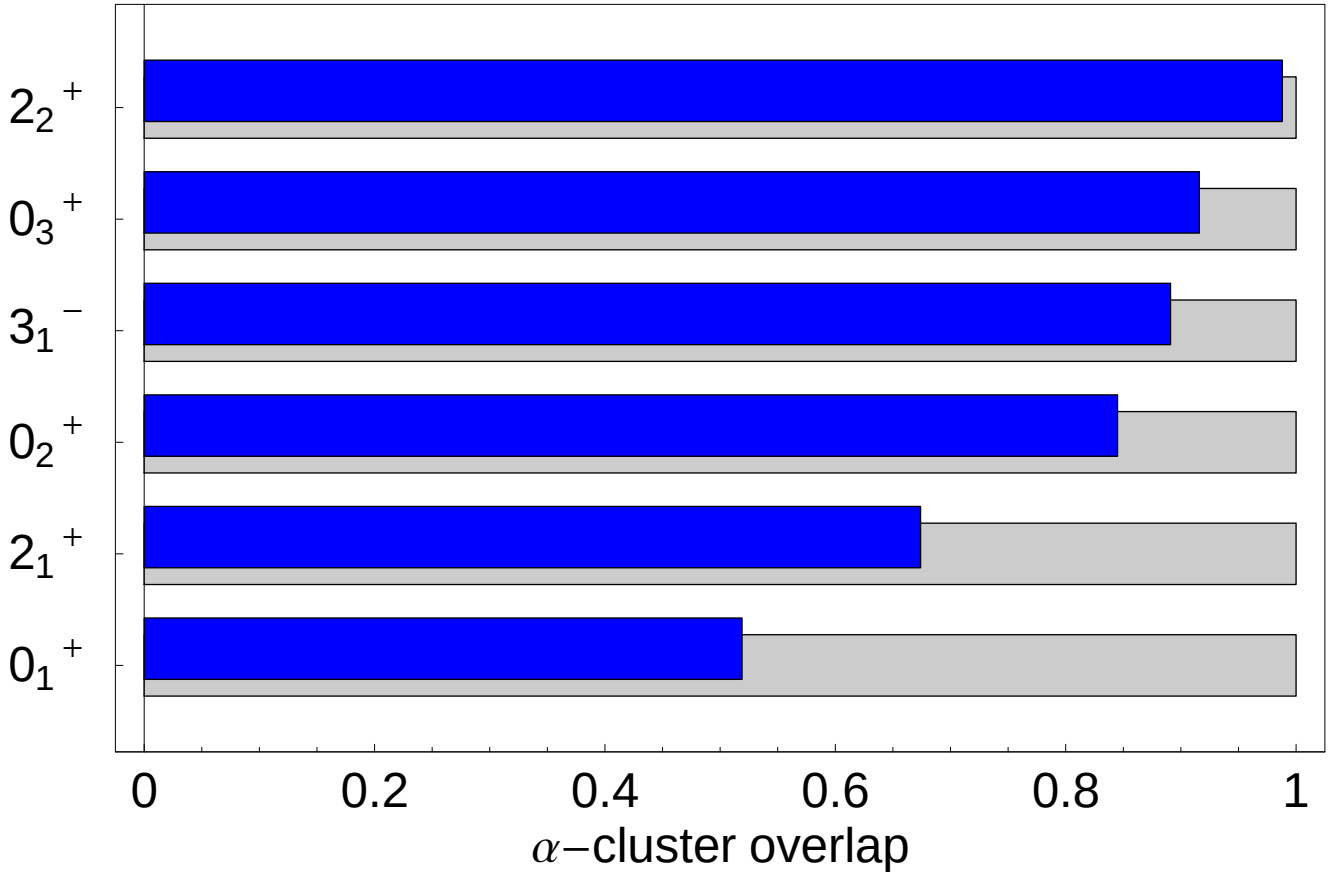
loosely bound, gas-like states

Cluster States in ^{12}C

Overlap with Cluster Model Space

Calculate the overlap of FMD wave functions with pure α-cluster model space

$$N_\alpha = \langle \Psi \left| \underset{\sim}{P}_{3\alpha} \right| \Psi \rangle$$



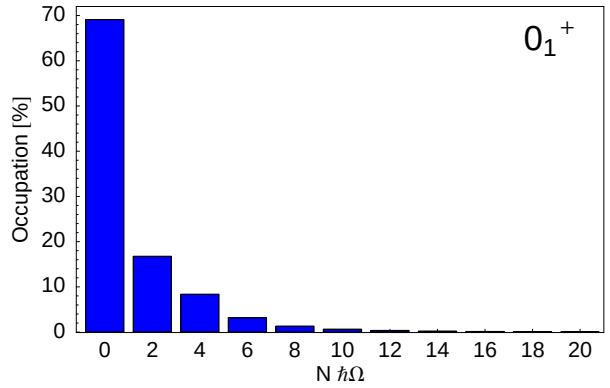
Hoyle state has 15% non-alpha admixtures

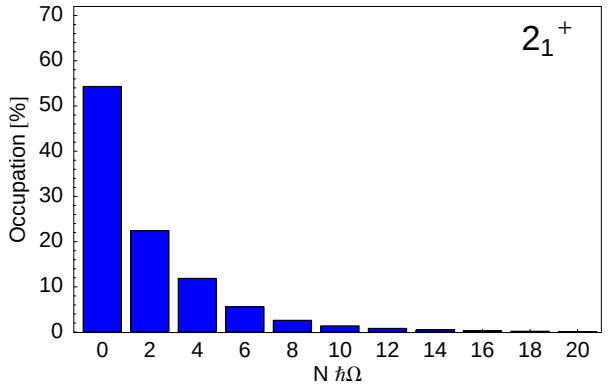
Cluster States in ^{12}C
Harmonic Oscillator $N\hbar\Omega$ Excitations

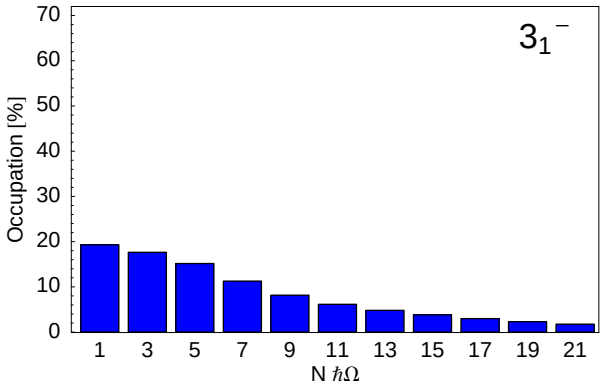
Y. Suzuki et al., Phys. Rev. C **54** (1996) 2073

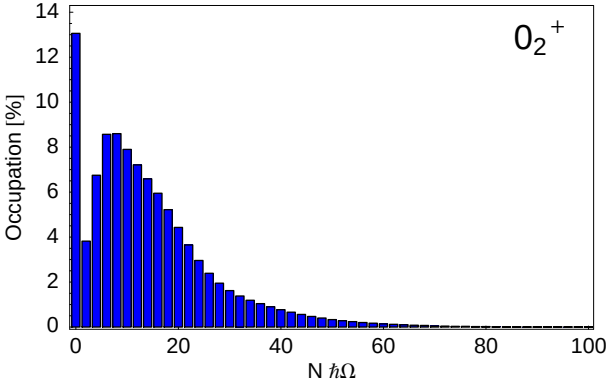
$$\mathrm{Occ}(N) = \langle \Psi \left| \delta \left(\sum_i (\underset{\sim}{H}_i^{HO}/\hbar\Omega - 3/2) - N \right) \right| \Psi \rangle$$

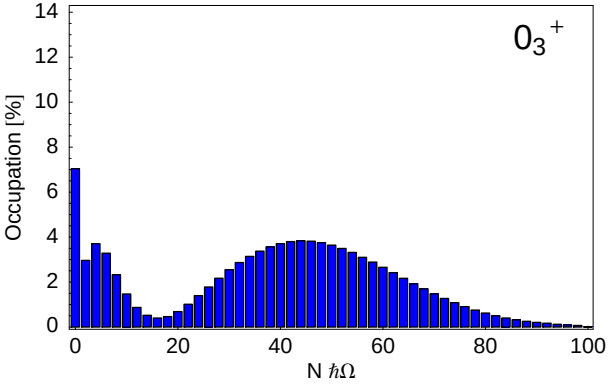
FMD

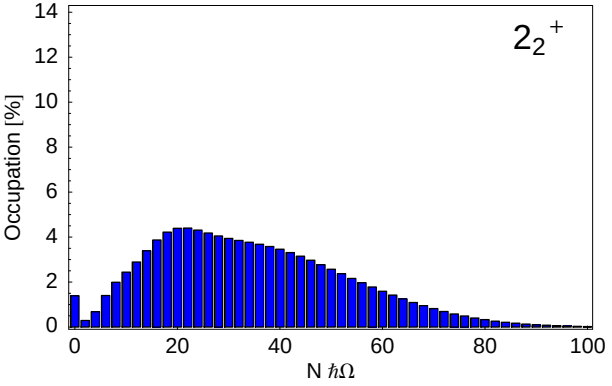










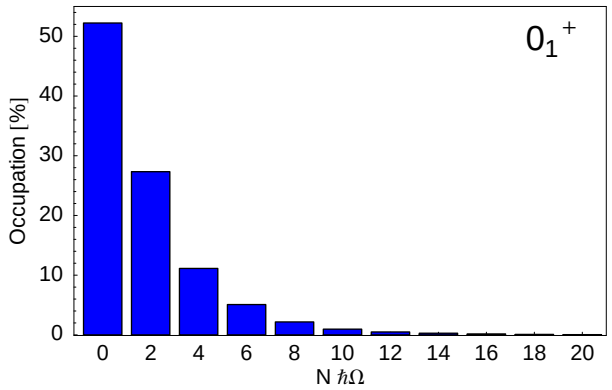


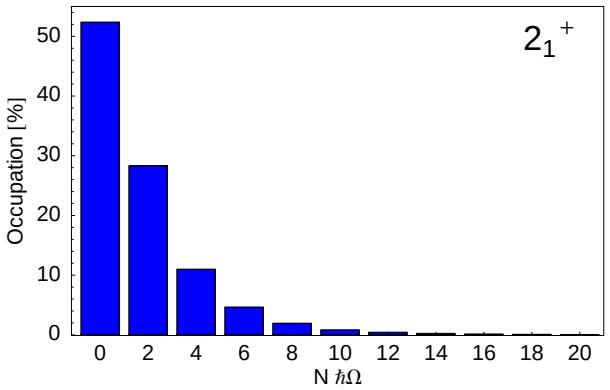
Cluster States in ^{12}C
Harmonic Oscillator $N\hbar\Omega$ Excitations

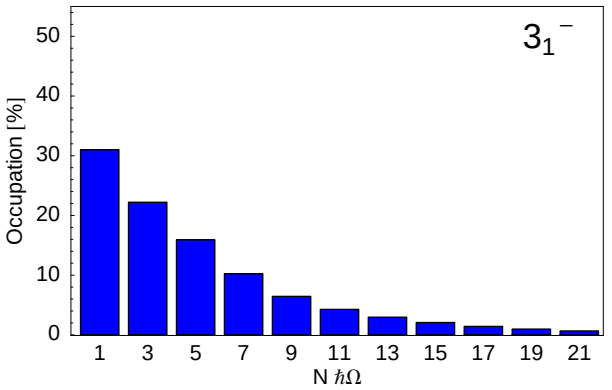
Y. Suzuki *et al*, Phys. Rev. C **54**, 2073 (1996).

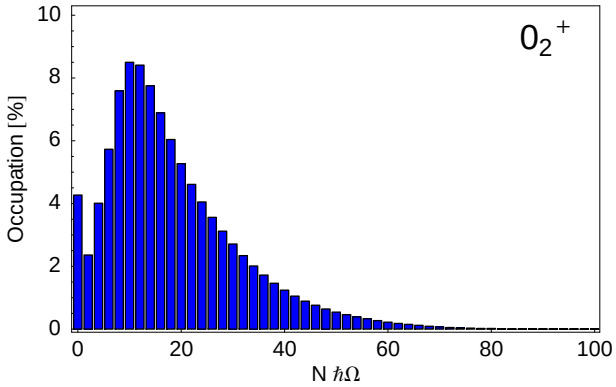
$$\mathrm{Occ}(N) = \langle \Psi \left| \delta \left(\sum_i (\underset{\sim}{H}_i^{HO}/\hbar\Omega - 3/2) - N \right) \right| \Psi \rangle$$

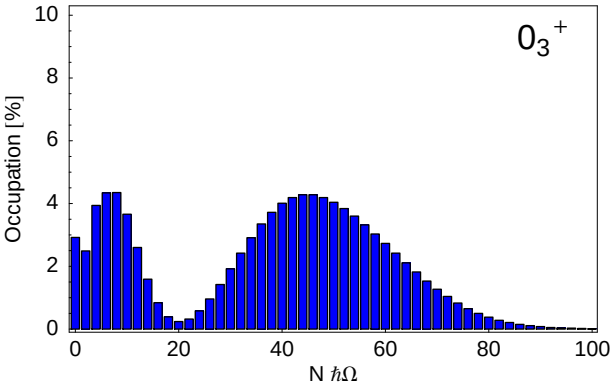
Cluster Model

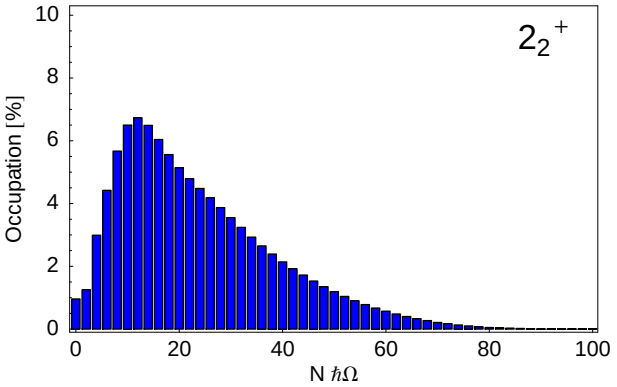












Hoyle State

α-cluster states in the No-Core Shell Model ?

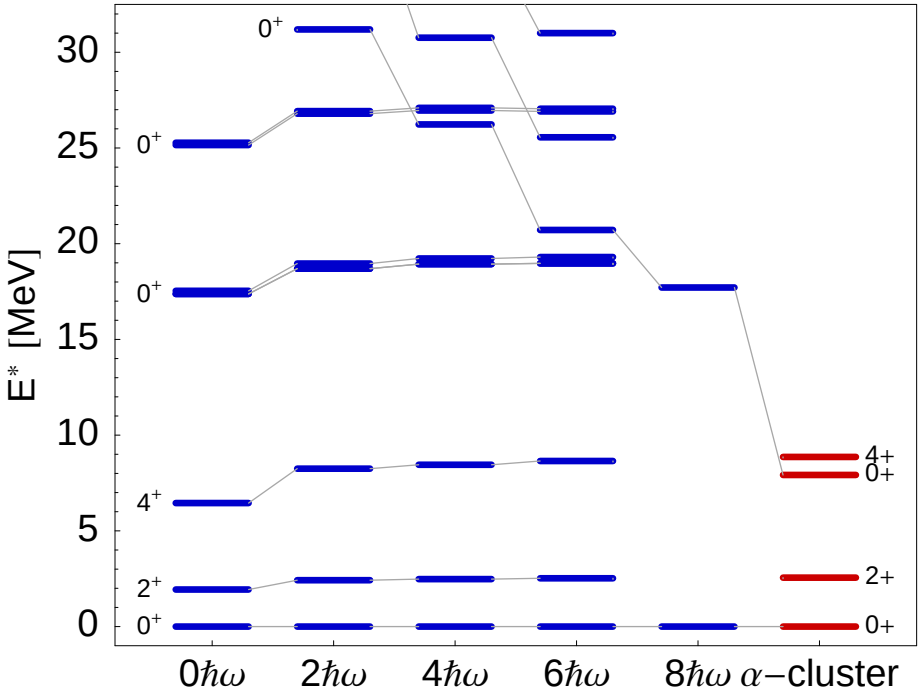
- compare spectra in NCSM and α-cluster model using the Volkov interaction
- bare interaction used in NCSM calculations

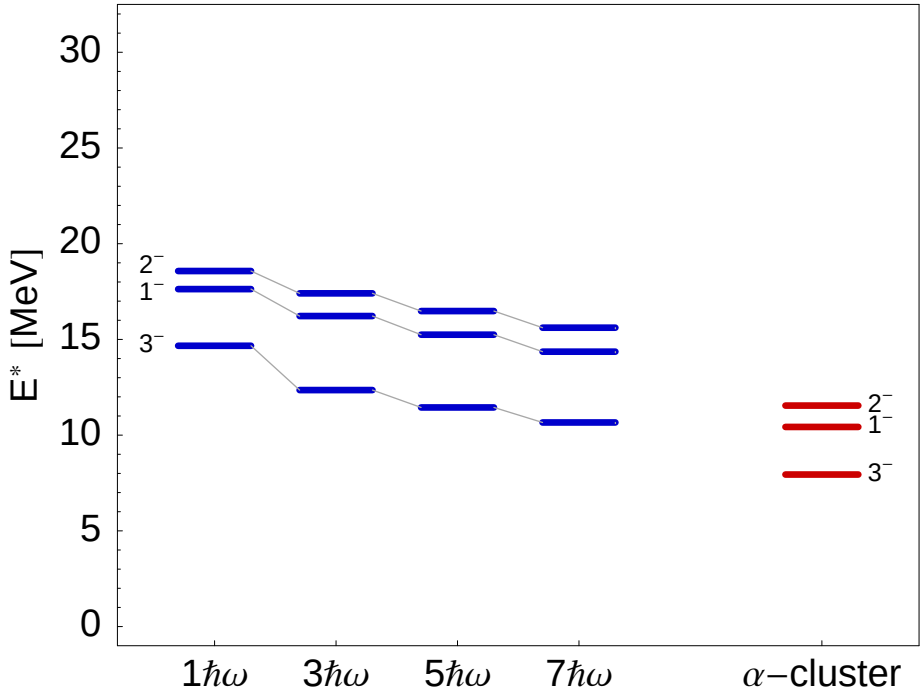
➨ good agreement for ground state band (0^+_1, 2^+_1, 4^+_1)

➨ very slow convergence for cluster states

Binding energies

	^{4}He	^{12}C
Cluster	-27.3 MeV	-89.6 MeV
NCSM	-28.3 MeV	-95.4 MeV





Neon Isotopes ^{17}Ne-^{22}Ne

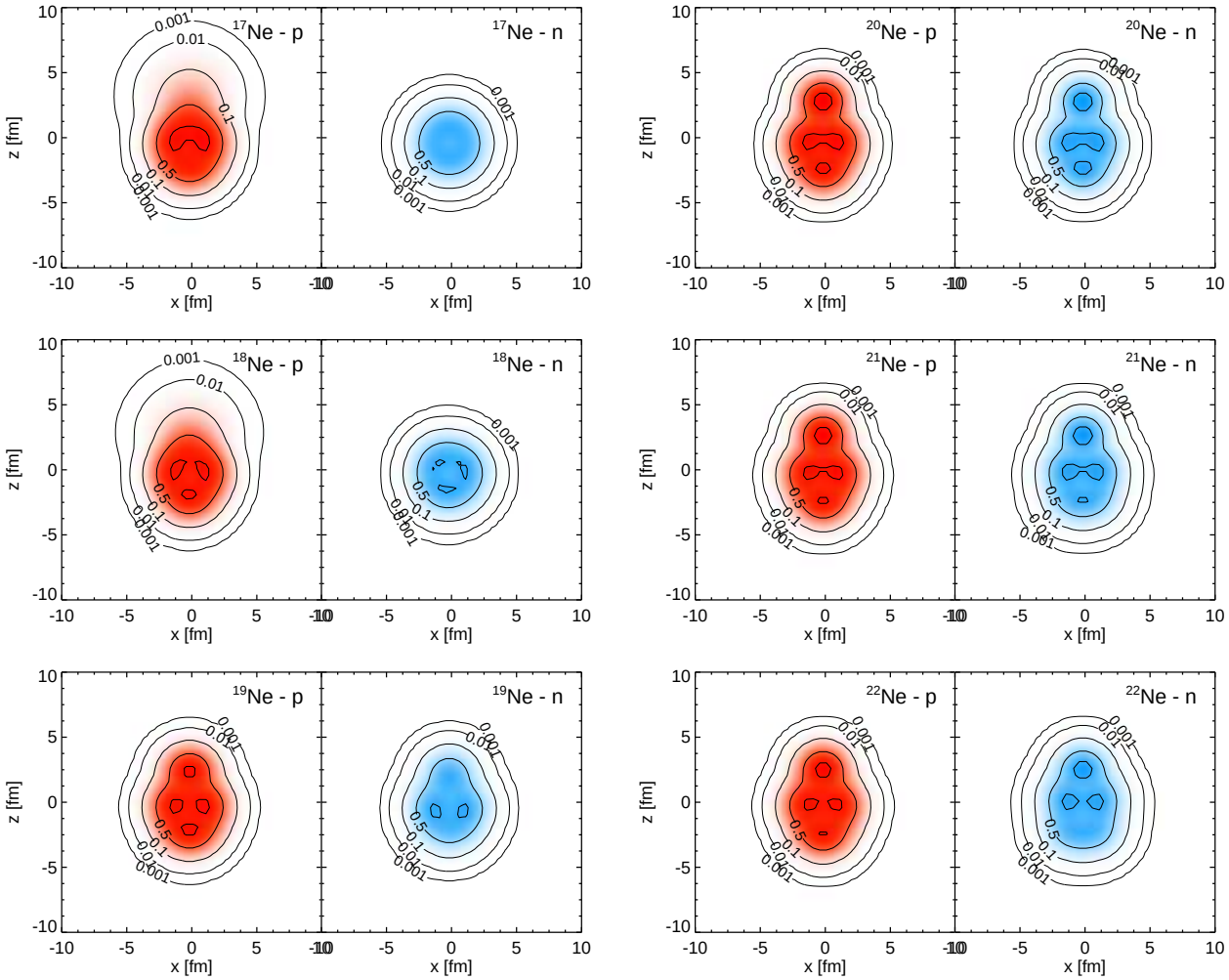
Structure

- s^2/d^2 **occupation in** ^{17}Ne **and** ^{18}Ne
- ^{3}He **and** ^{4}He **cluster admixtures**

Observables

- **Charge Radii**
- **Matter Radii**
- **Is** ^{17}Ne **a Halo nucleus ?**

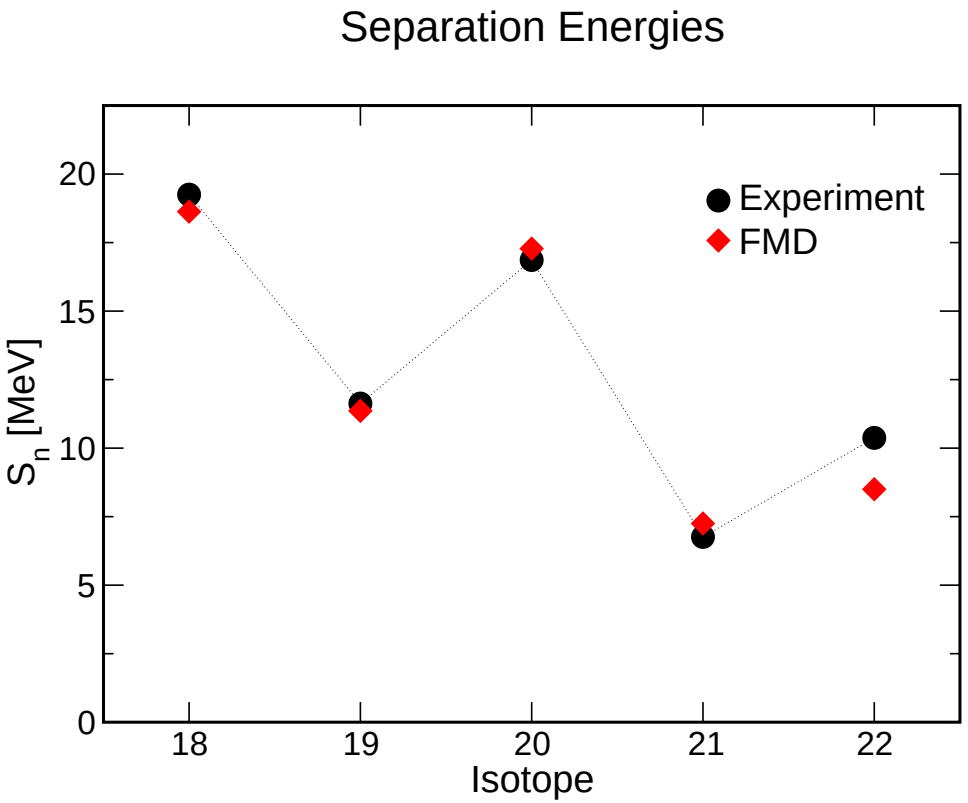
Calculation



Intrinsic proton/neutron densities of dominant FMD state

- Variation after parity projection on positive and negative parity
- Crank strength of spin-orbit force, changes properties of single-particle orbits and their occupations
- "s^2" and "d^2" minima in ^{17,18}Ne
- explicit cluster configurations:
  ^{17}Ne: ^{14}O-^{3}He
  ^{18}Ne: ^{14}O-^{4}He
  ^{19}Ne: ^{16}O-^{3}He and ^{15}O-^{4}He
  ^{20}Ne: ^{16}O-^{4}He
  ^{21}Ne: "^{17}O"-^{4}He
  ^{22}Ne: "^{18}O"-^{4}He

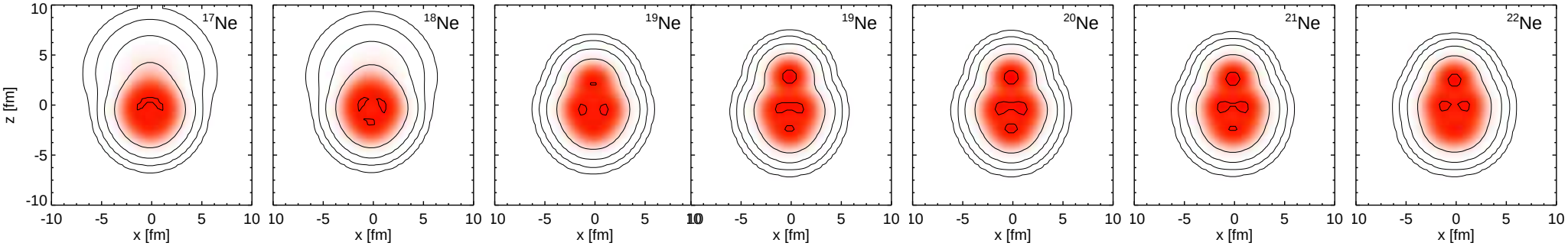
Separation energies



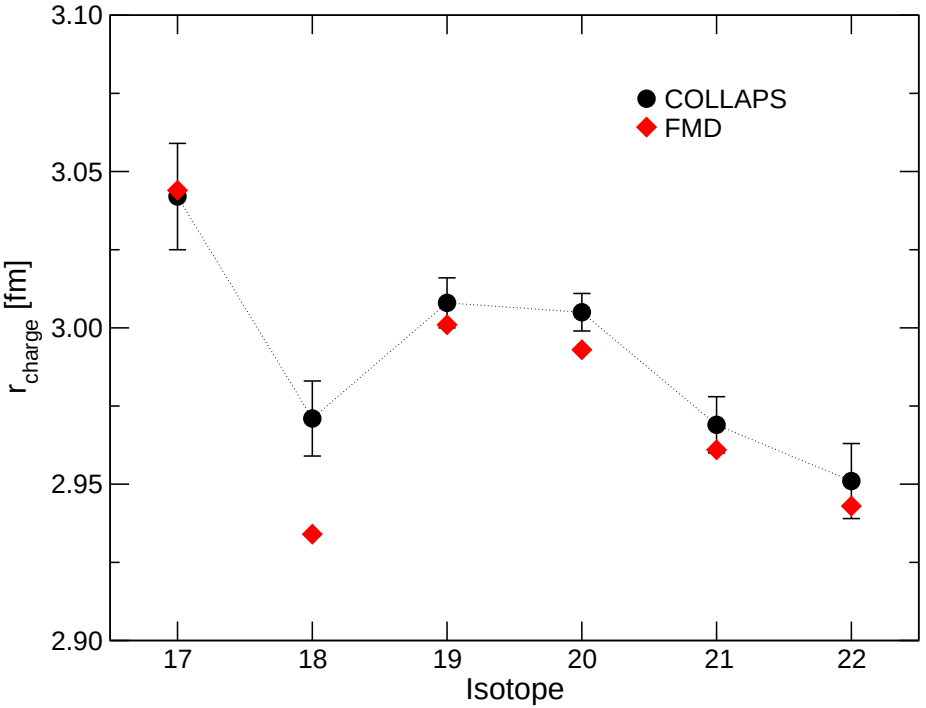
^{17,18}Ne: s^2/d^2 admixture

^{19}Ne: ^{3}He, α clustering

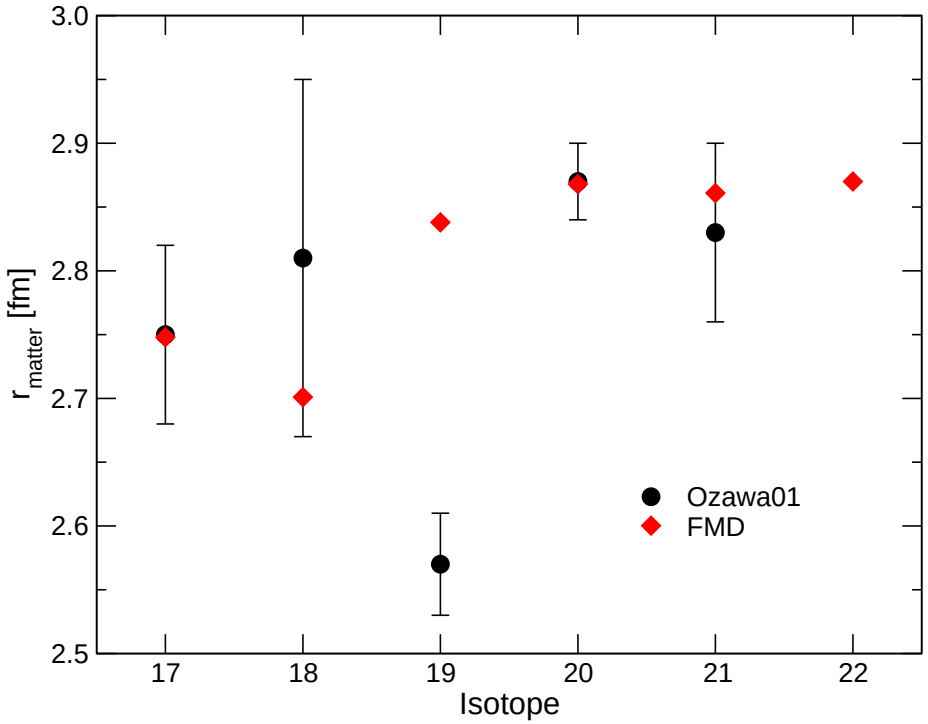
$^{20-22}$Ne: α clustering



Charge and Matter Radii



- charge radii of ^{17,18}Ne depend strongly on s^2/d^2 occupations
- cluster admixtures responsible for large charge radii in $^{19-22}$Ne
- measurements of charge radii by COLLAPS@ISOLDE



- matter radii from interaction cross sections

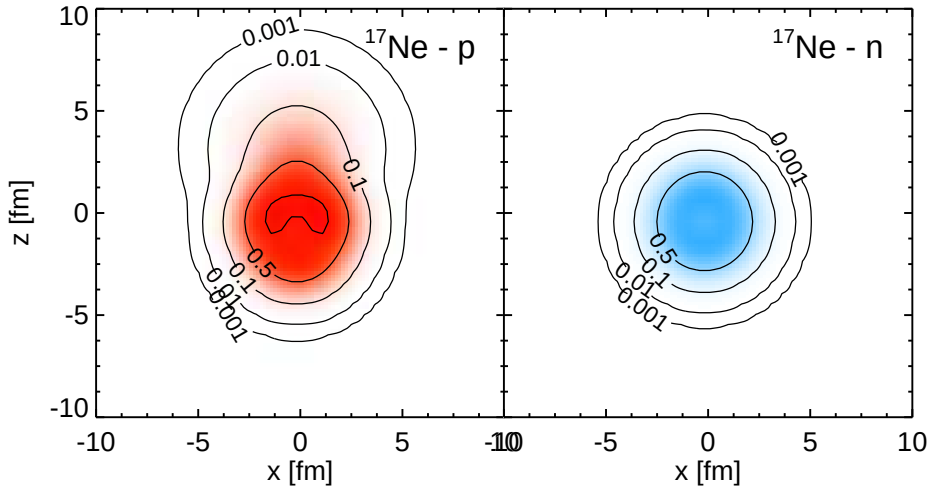
  A. Ozawa *et al.*, Nuc. Phys. **A693** (2001) 32

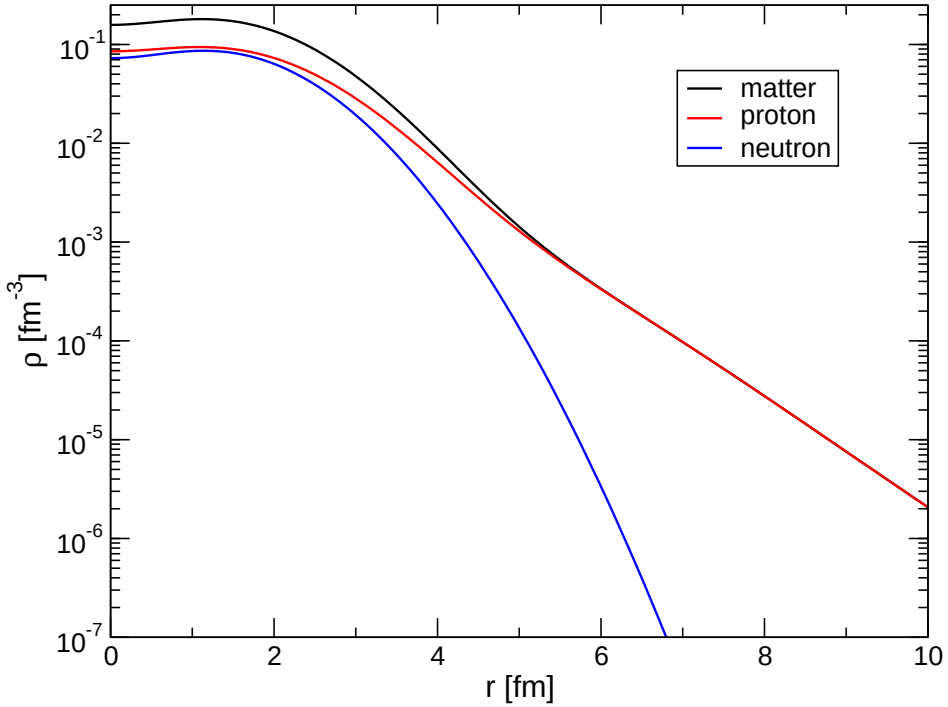
- good agreement with expection of ^{19}Ne

W. Geithner, T. Neff *et al.*, Phys. Rev. Lett. **101**, 252502 (2008)

Neon Isotopes

^{17}Ne Halo ?





	FMD	Experiment
r_{ch}[fm]	3.03	3.042(17)
r_{mat}[fm]	2.75	2.75(7)
$B(E2; \frac{1}{2}^- \rightarrow \frac{3}{2}^-)[e^2\text{fm}^4]$	76.7	66^{+18}_{-25}
$B(E2; \frac{1}{2}^- \rightarrow \frac{5}{2}^-)[e^2\text{fm}^4]$	119.8	124(18)
occupancy s^2	40%	
occupancy d^2	55%	

- proton skin $r_p - r_n = 0.45$ fm
- 40% probability to find a proton at $r > 5$ fm

Summary

Unitary Correlation Operator Method

- Explicit description of short-range central and tensor correlations
- Phase-shift equivalent correlated interaction V_{UCOM}
- Interaction in momentum-space
- *ab initio* calculations with few- and many-body methods

Fermionic Molecular Dynamics

- Microscopic many-body approach using Gaussian wave-packets
- Consistent description of well bound states with shell structure and loosely bound states of cluster or halo nature
- ^{12}C spectrum, Hoyle state and other high-lying 0^+ and 2^+ states, monopole transition form factor, analysis of FMD wave functions in harmonic oscillator basis, comparison with no-core shell model calculations
- Neon isotopes, separation energies, charge and matter radii, halo structure in ^{17}Ne, importance of cluster admixtures in $^{19-22}$Ne ground states

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