The physics and astrophysics of merging neutron-star binaries

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Plan of the talk

* Numerical relativity as a theoretical laboratory
* Anatomy of the GW signal
* Role of B-fields and EM counterparts
* Ejected matter and nucleosynthesis

The goals of numerical relativity

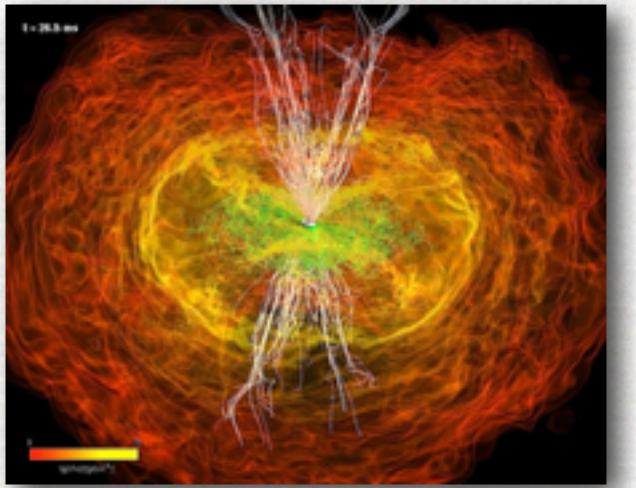
Einstein's theory is as beautiful as intractable analytically

Numerical relativity solves Einstein/HD/MHD eqs. in regimes in which no approximation is expected to hold.

To do this we build codes: our **"theoretical laboratories"**.

Theoretical laboratory

$$G_{\mu\nu} = 8\pi G T_{\mu\nu},$$
$$\nabla_\mu T^{\mu\nu} = 0$$



Think of them as a "*factory*" of "gedanken experiments"

The equations of numerical relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu} \,, \qquad \text{(field equations)}$$

$$\nabla_\mu T^{\mu\nu} = 0 \,, \qquad \text{(cons. energy/momentum)}$$

$$\nabla_\mu (\rho u^\mu) = 0 \,, \qquad \text{(cons. rest mass)}$$

$$p = p(\rho, \epsilon, Y_e, \ldots) \,, \qquad \text{(equation of state)}$$

$$\nabla_\nu F^{\mu\nu} = I^\mu \,, \quad \nabla^*_\nu F^{\mu\nu} = 0 \,, \qquad \text{(Maxwell equations)}$$

$$T_{\mu\nu} = T^{\text{fluid}}_{\mu\nu} + T^{\text{EM}}_{\mu\nu} + \ldots \qquad \text{(energy – momentum tensor)}$$

In vacuum space times the theory is complete and the truncation error is the only error **made**: "CALCULATION"

The equations of numerical relativity

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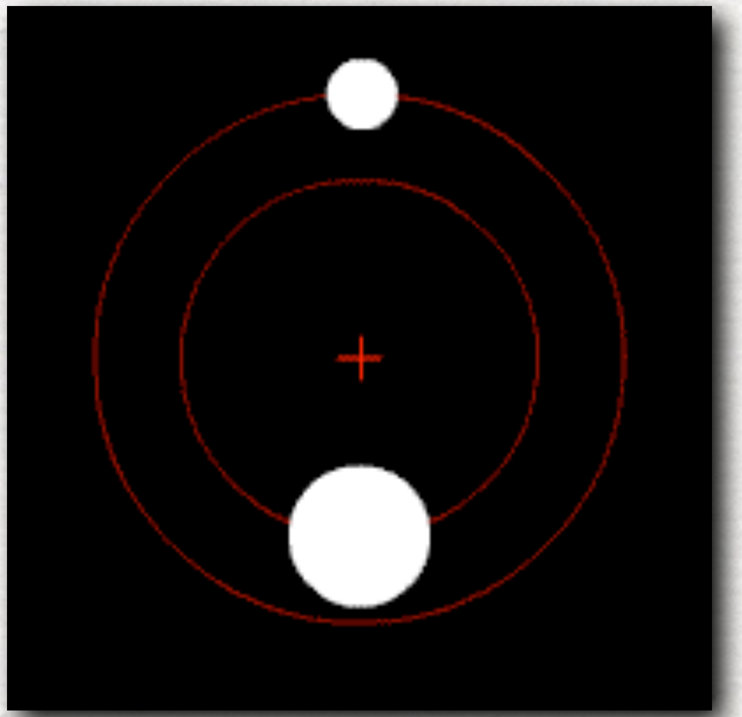
$$T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{EM}} + \ldots \qquad \text{(energy - momentum tensor)}$$

In non-vacuum space times the truncation error is the only error that is **measurable**: **"SIMULATION"**

It's our approximation to *"reality"*: improvable via microphysics, magnetic fields, viscosity, radiation transport, ...

The two-body problem: Newton vs Einstein

Take two objects of mass m_1 and m_2 interacting only gravitationally



In **Newtonian gravity** solution is analytic: there exist closed orbits (circular/elliptic) with

$$\ddot{\boldsymbol{r}} = -\frac{GM}{d_{12}^3}\boldsymbol{r}$$

where $M \equiv m_1 + m_2\,, \boldsymbol{r} \equiv \boldsymbol{r}_1 - \boldsymbol{r}_2\,, d_{12} \equiv |\boldsymbol{r}_1 - \boldsymbol{r}_2|\,.$

In **Einstein's gravity** no analytic solution! No closed orbits: the system loses energy/angular momentum via gravitational waves.

Catastrophic events…

Back-of-the-envelope calculation (Newtonian quadrupole approx.) shows the energy emitted in GWs per unit time is

$$L_{\rm GW} \simeq \left(\frac{G}{c^5}\right)\left(\frac{M\langle v^2\rangle}{\tau}\right)^2 \simeq \left(\frac{c^5}{G}\right)\left(\frac{R_{\rm Schw.}}{R}\right)^2\left(\frac{\langle v\rangle}{c}\right)^6$$

Near merger the binary is very compact ($R_{\rm Schw.}=2GM/c^2$) and moving at fraction of speed of light: GR is indispensable

$$R \simeq 10\,R_{\rm Schw.} \qquad \langle v\rangle \simeq 0.1\,c$$

As a result, the GW luminosity is:

$$L_{\rm GW} \simeq 10^{-8}\left(\frac{c^5}{G}\right) \simeq 10^{50}\,{\rm erg\ s^{-1}} \simeq 10^{17}\,L_\odot$$

This is roughly the combined luminosity of 1 million galaxies!

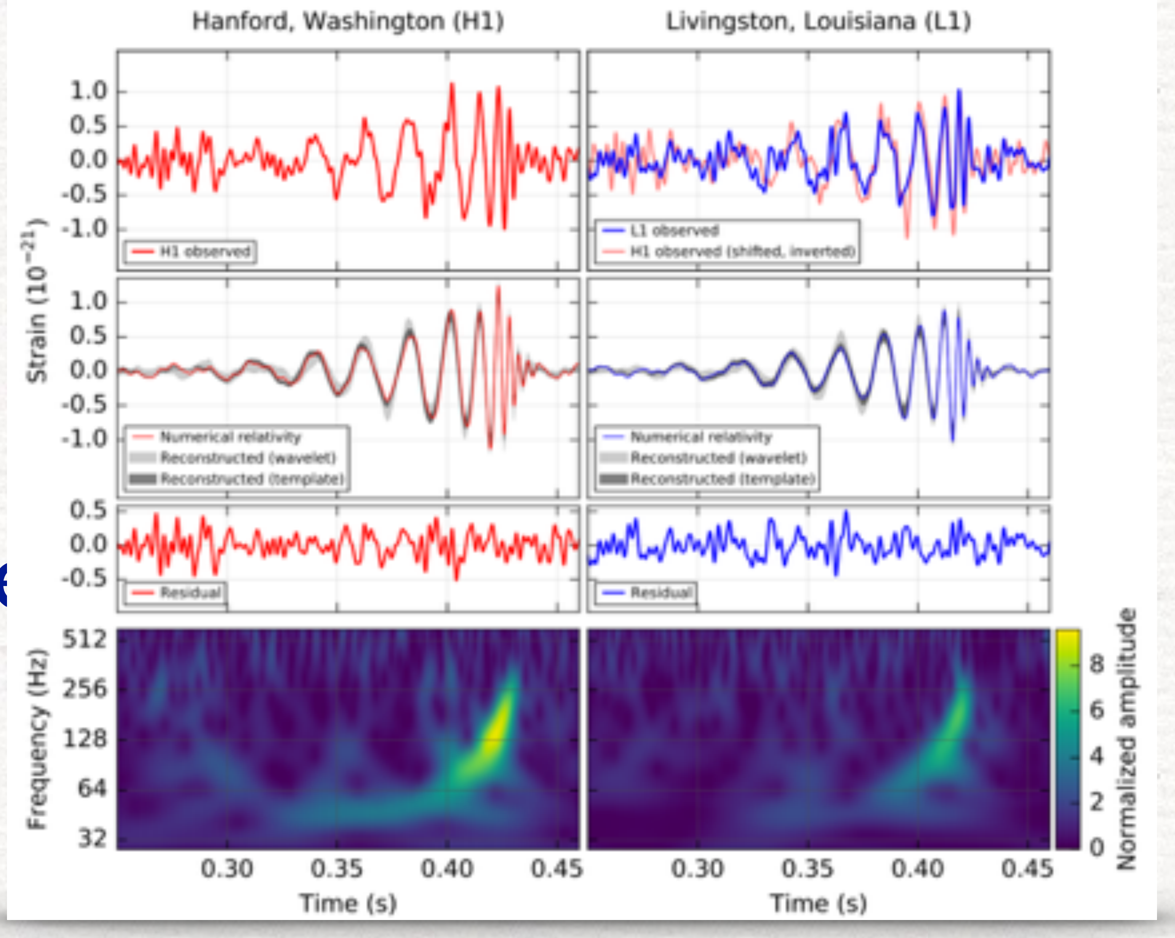
The two-body problem in GR

- For BHs we know what to **expect**:

BH + BH ⟶ BH + GWs

- For NSs the question is more **subtl**
hyper-massive neutron star (HMNS),

NS + NS ⟶ HMNS + ... ? ⟶



Abbott+ 2016

- **HMNS** phase can provide strong and clear information on **EOS**

- **BH+torus** system may tell us on the central engine of **GRBs**

Animations: Breu, Radice, LR

$$M = 2 \times 1.35\, M_{\odot}$$

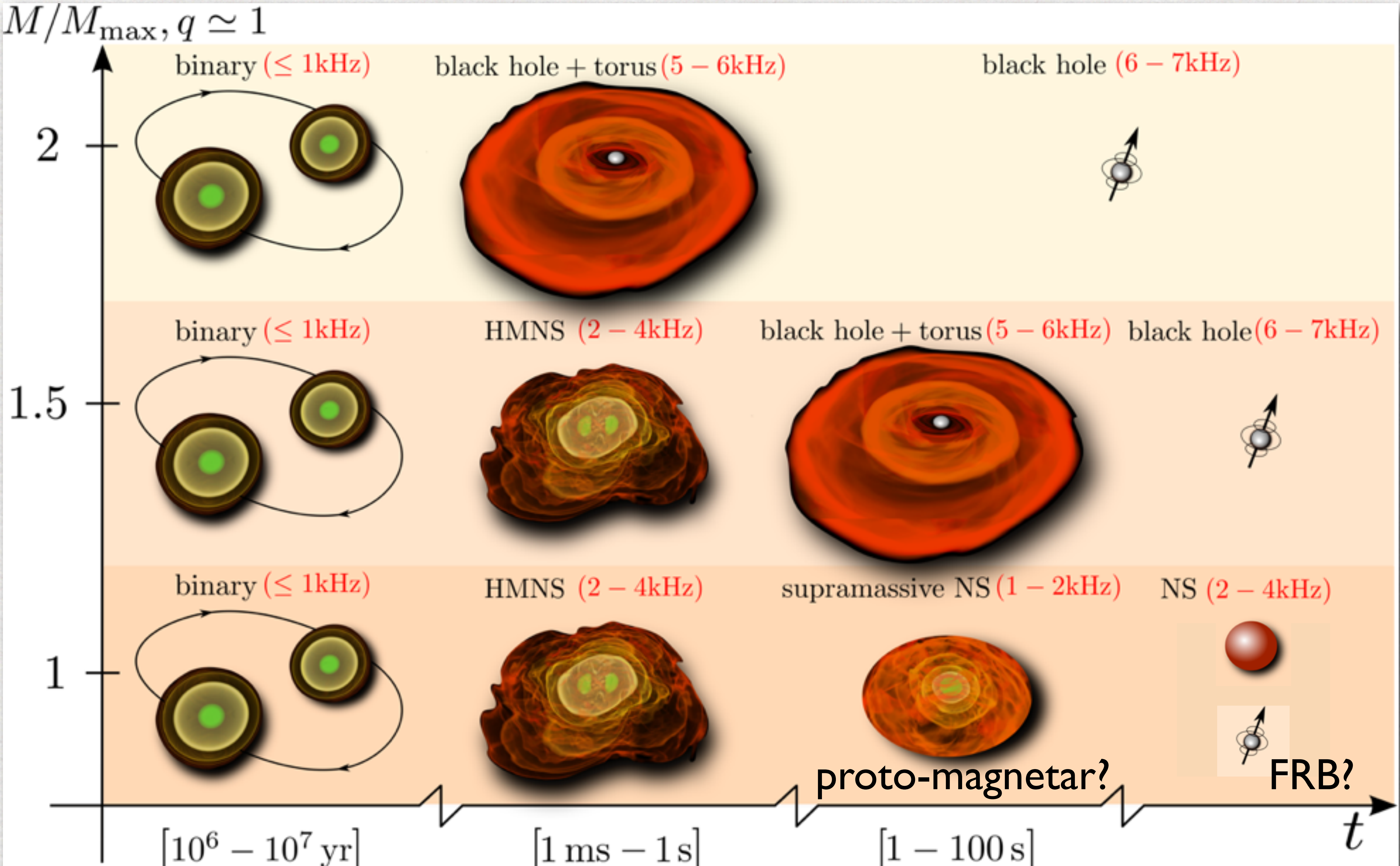
LS220 EOS

"merger → HMNS → BH + torus"

Quantitative differences are produced by:

- **differences induced by the gravitational MASS:**
  a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time

Broadbrush picture



“merger → HMNS → BH + torus”

Quantitative differences are produced by:

- **differences induced by the gravitational MASS:**
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- **differences induced by MASS ASYMMETRIES:**
  tidal disruption before merger; may lead to prompt BH

Animations: Giacomazzo, Koppitz, LR

Total mass : 3.37 $M_\odot$;   mass ratio :0.80;

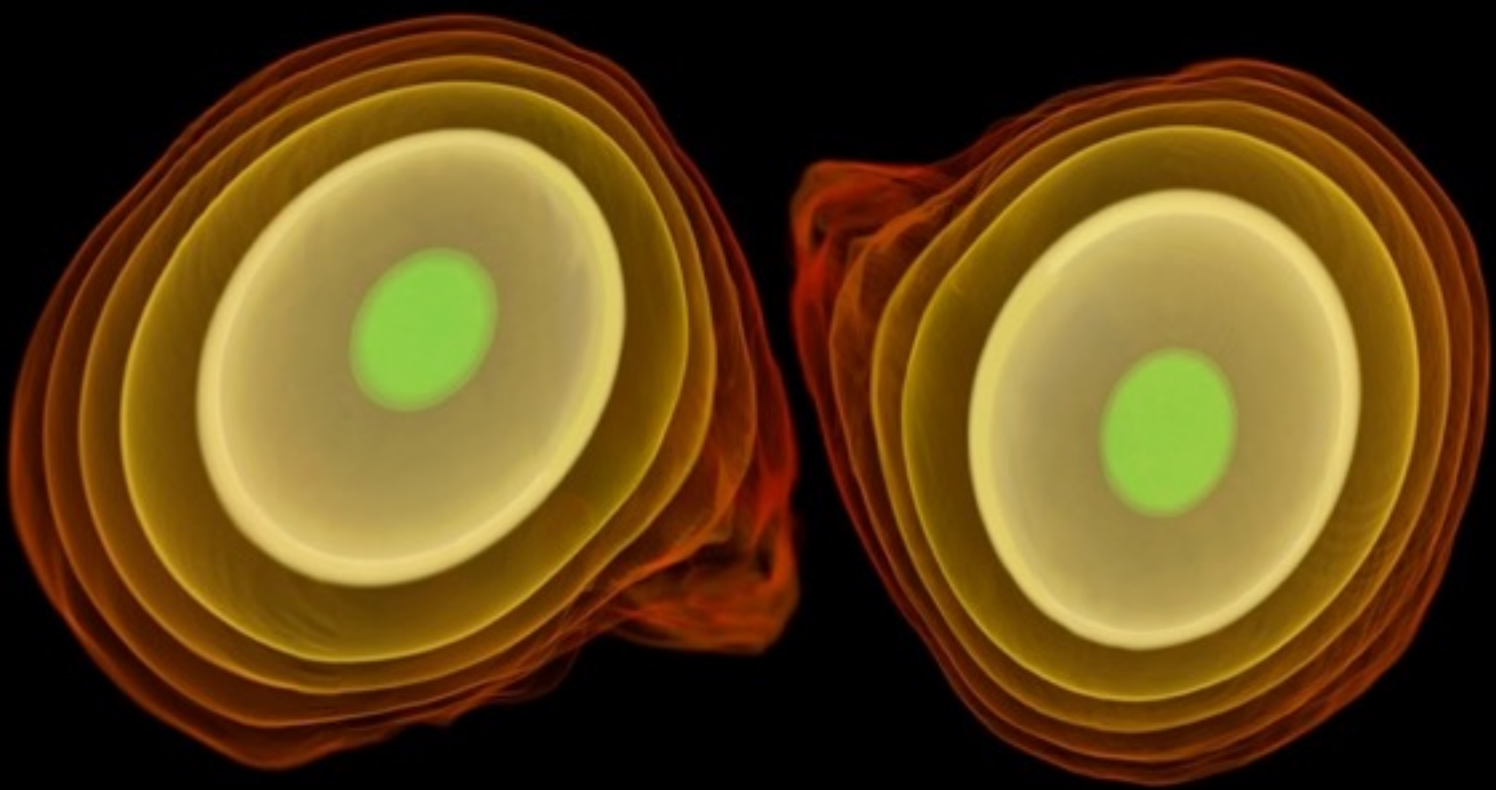
- the torii are generically more massive
- the torii are generically more extended
- the torii tend to stable quasi-Keplerian configurations
- overall unequal-mass systems have all the ingredients needed to create a GRB

"merger → HMNS → BH + torus"

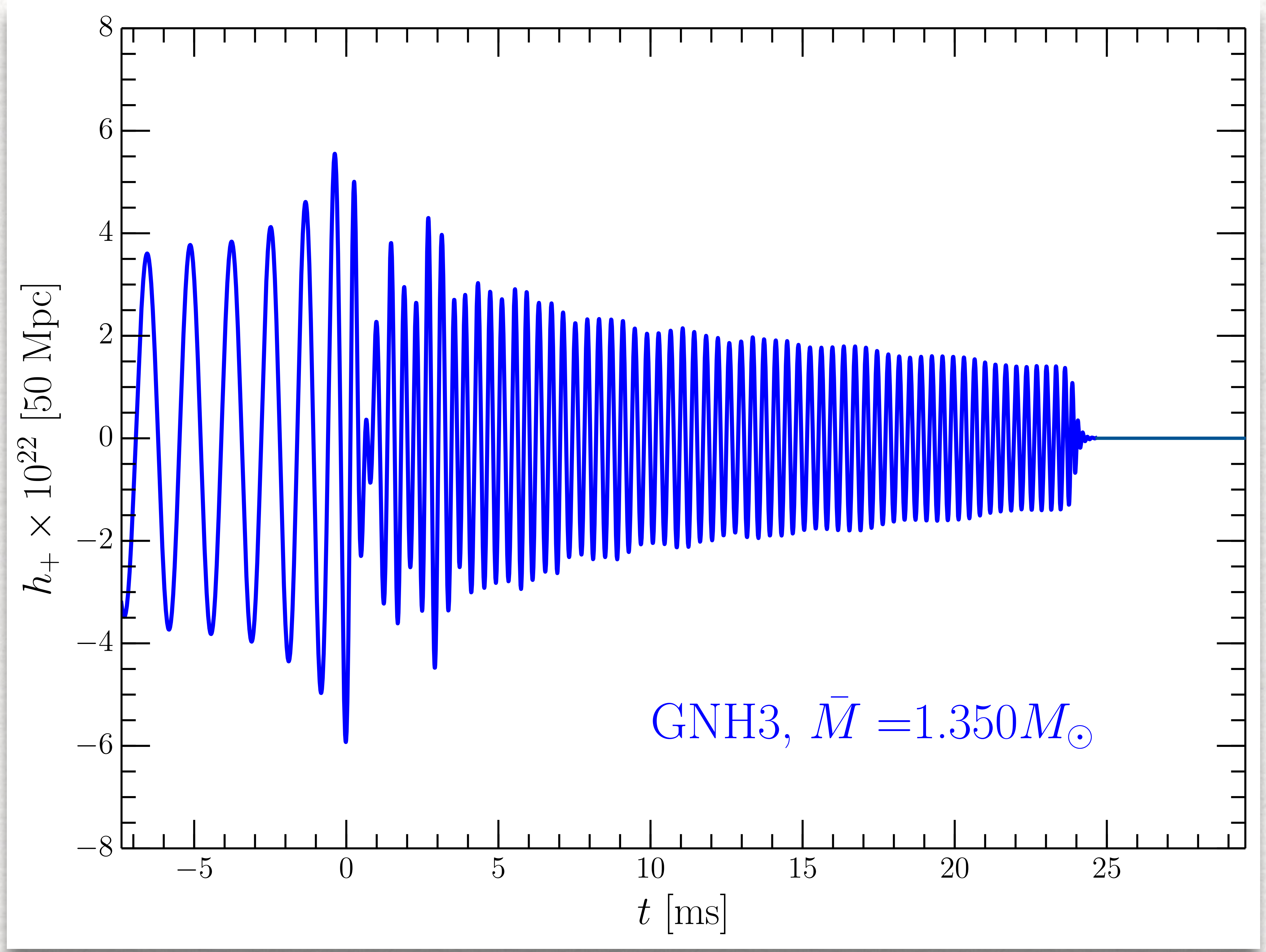
Quantitative differences are produced by:

- **differences induced by the gravitational MASS:**
  a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time
- **differences induced by MASS ASYMMETRIES:**
  tidal disruption before merger; may lead to prompt BH
- **differences induced by the EOS:**
  stiff/soft EOSs will have different compressibility and deformability, imprinting on the GW signal
- **differences induced by MAGNETIC FIELDS:**
  the angular momentum redistribution via magnetic braking or MRI can increase/decrease time to collapse; EM counterparts!
- **differences induced by RADIATIVE PROCESSES:**
  radiative losses will alter the equilibrium of the HMNS

How to use gravitational waves to constrain the EOS



Anatomy of the GW signal



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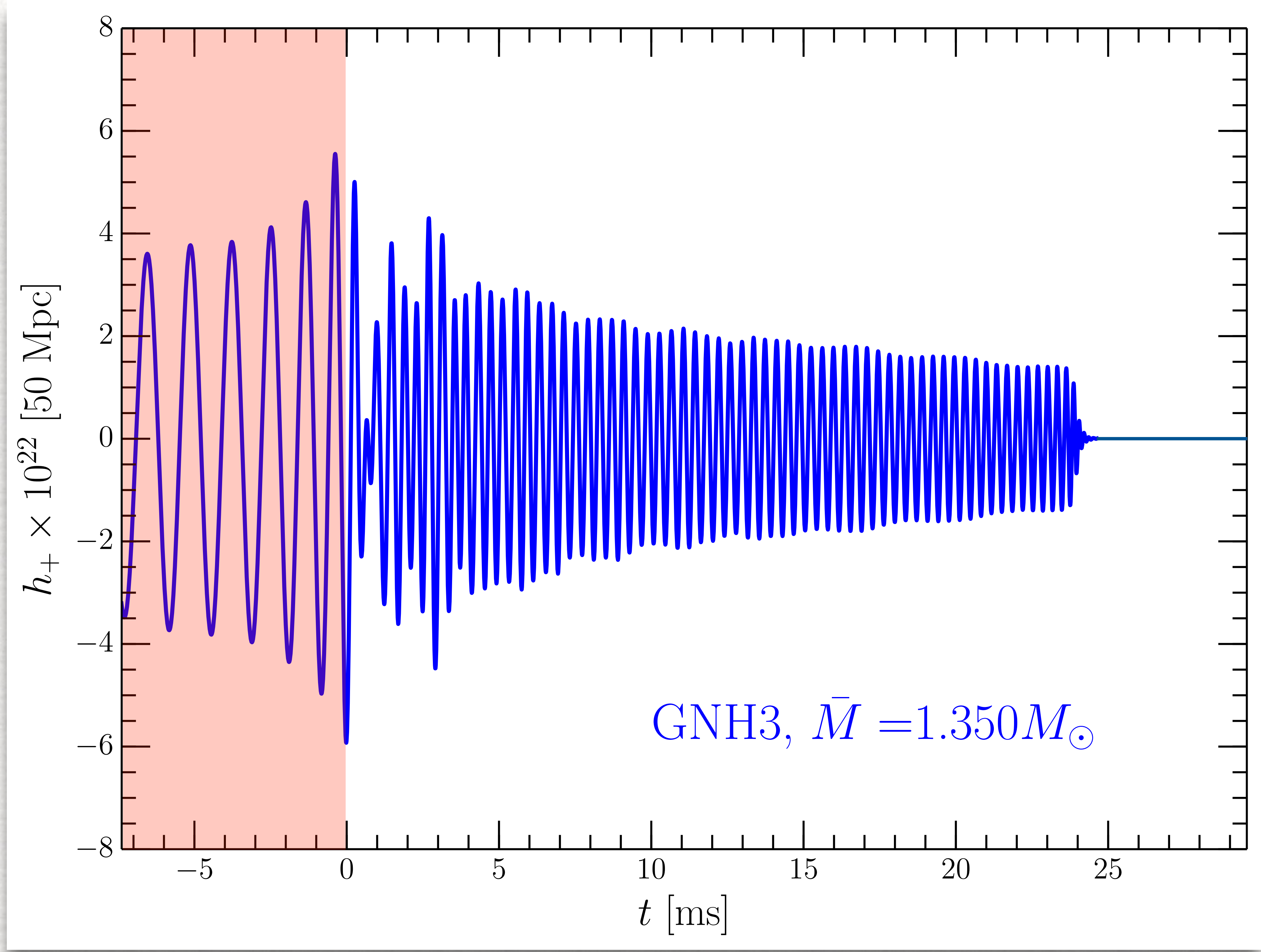
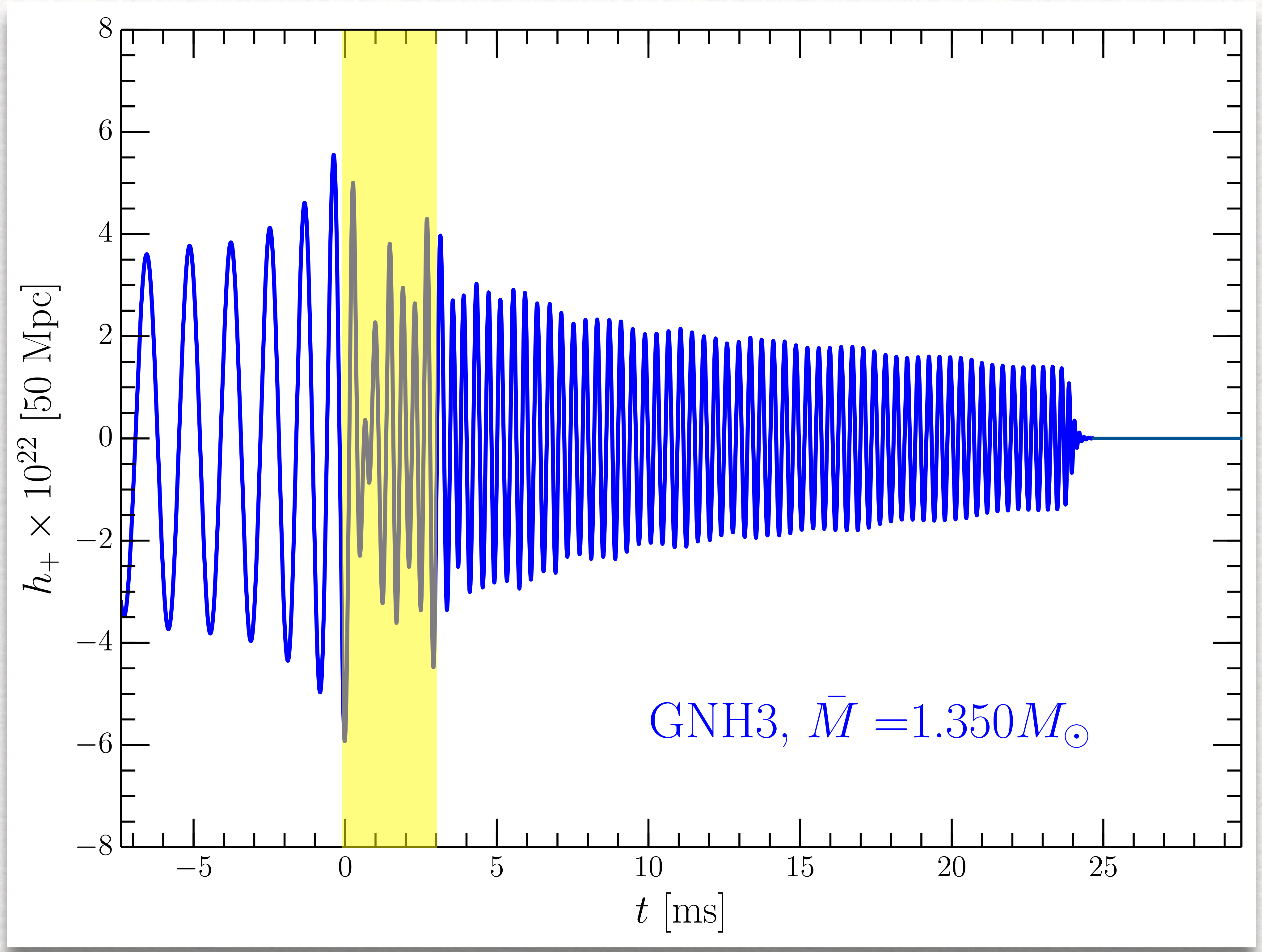


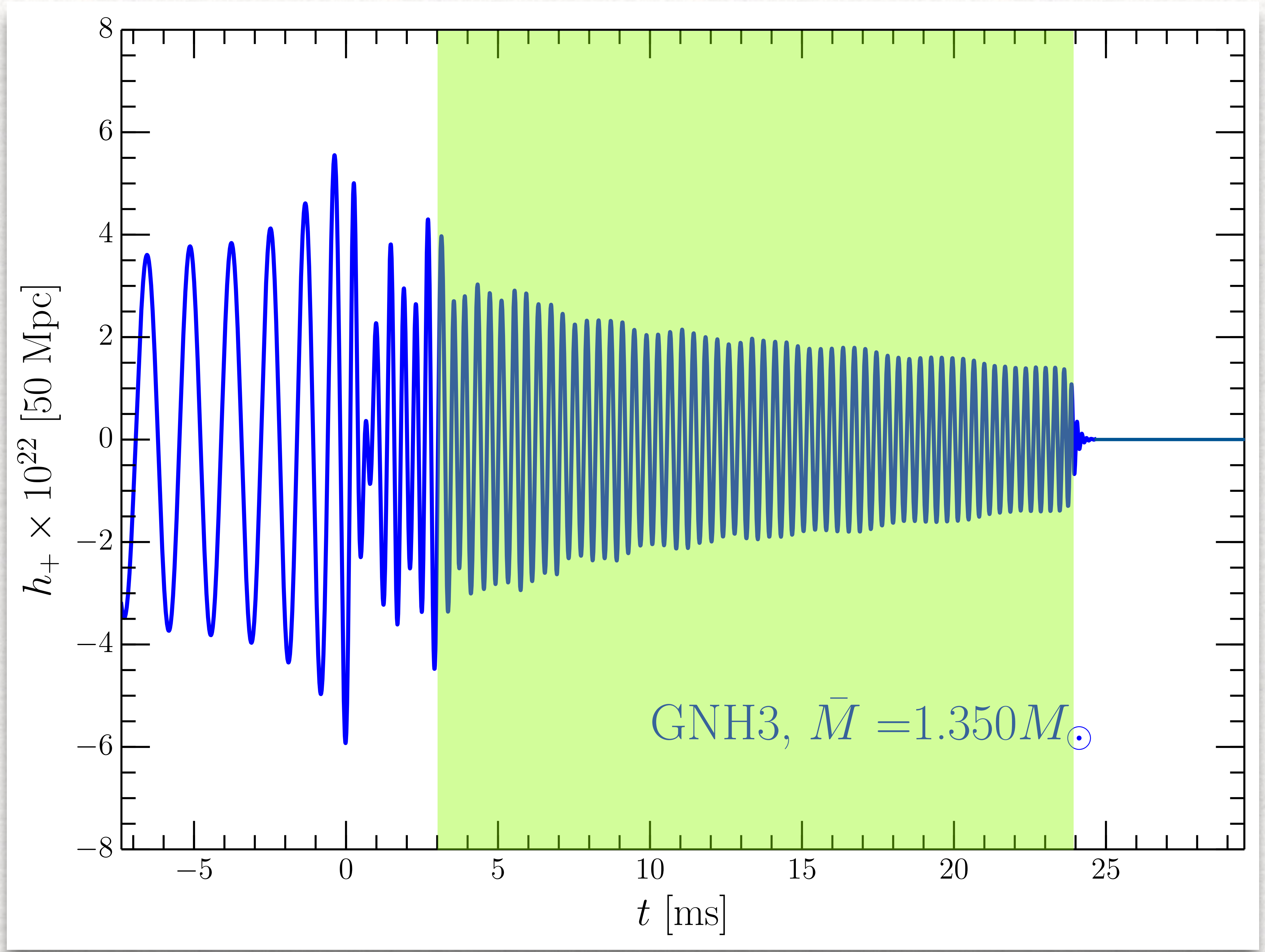
Inspiral: well approximated by PN/EOB; tidal effects important

Anatomy of the GW signal



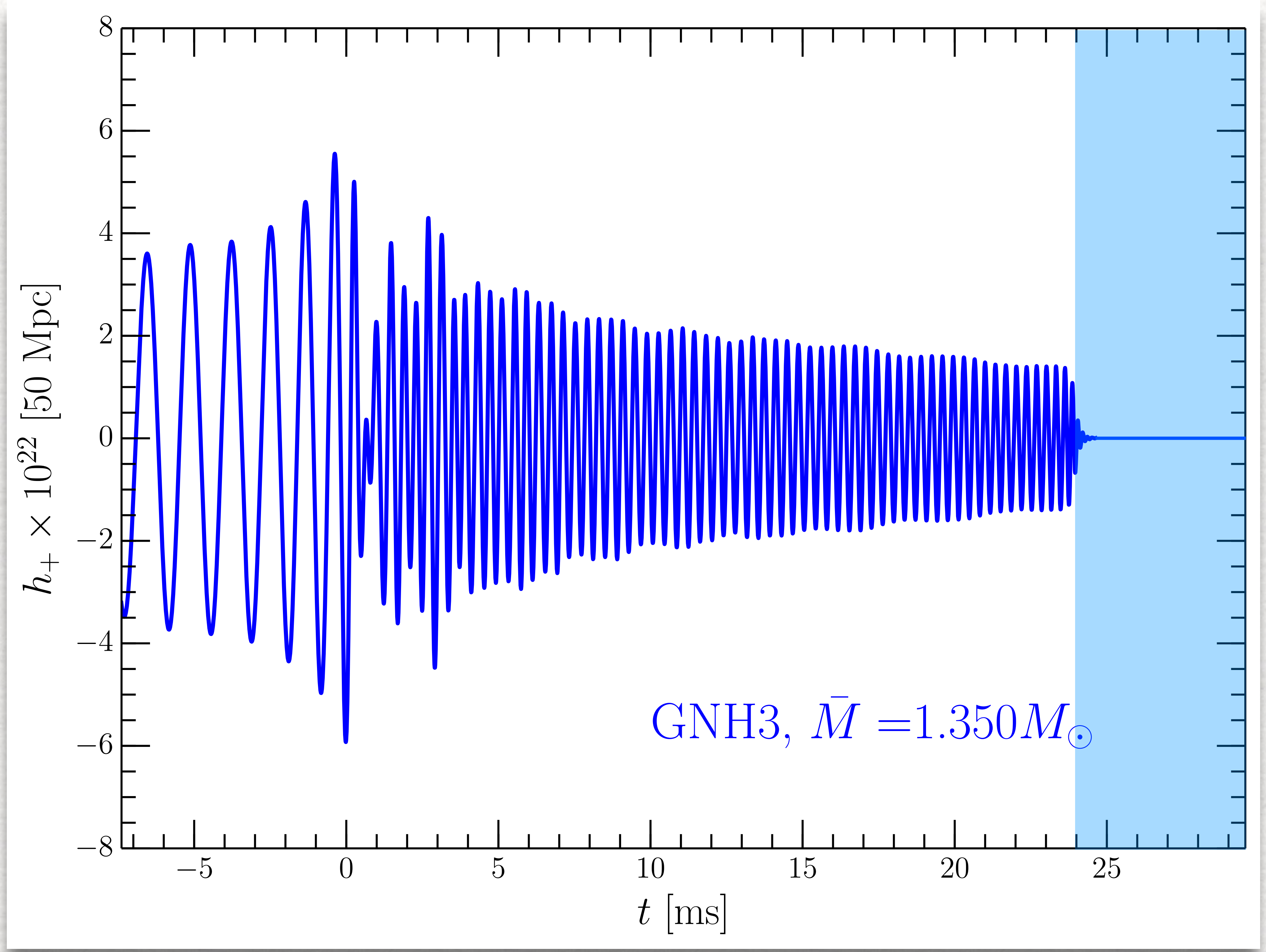
Merger: highly nonlinear but analytic description possible

Anatomy of the GW signal



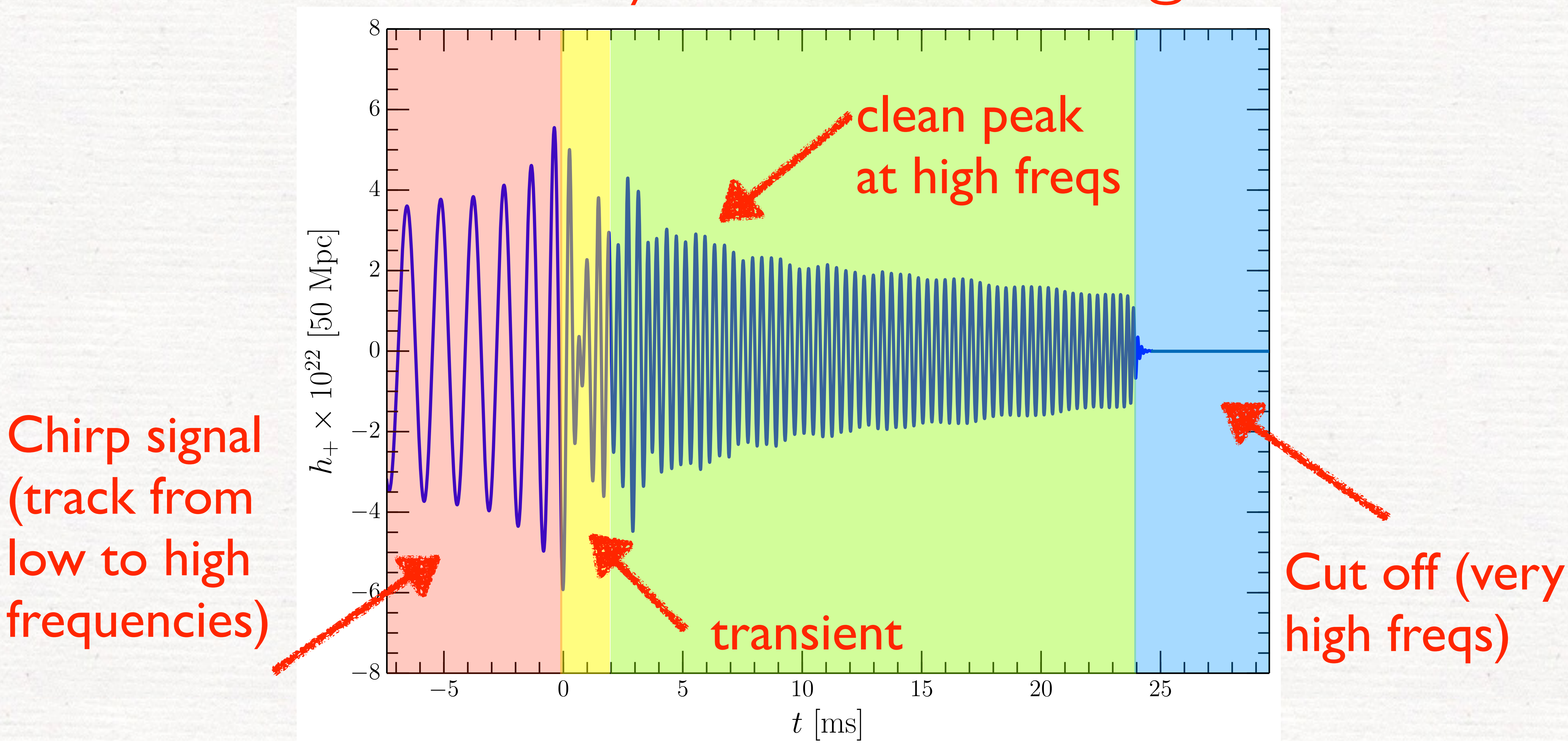
post-merger: quasi-periodic emission of bar-deformed HMNS

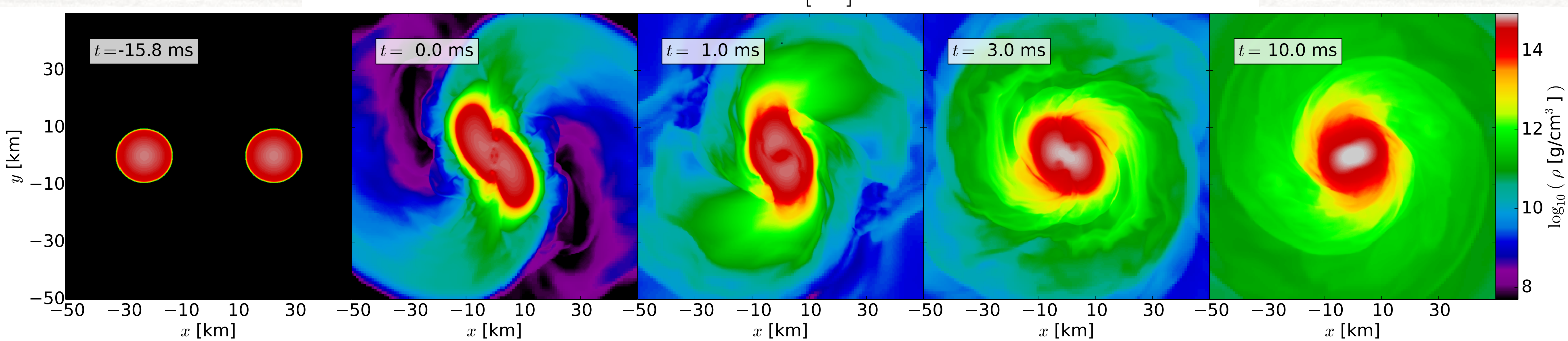
Anatomy of the GW signal



Collapse-ringdown: signal essentially shuts off.

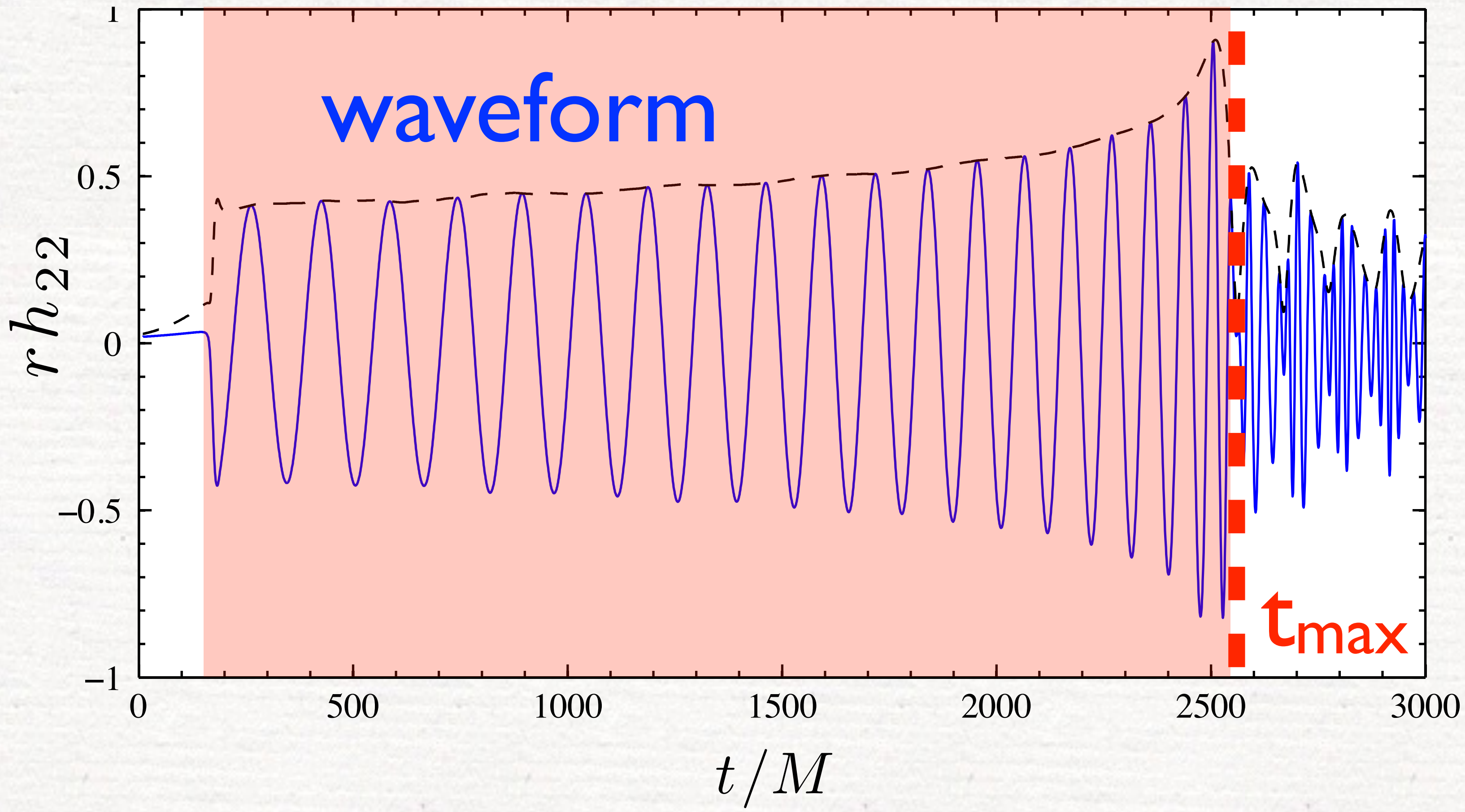
Anatomy of the GW signal



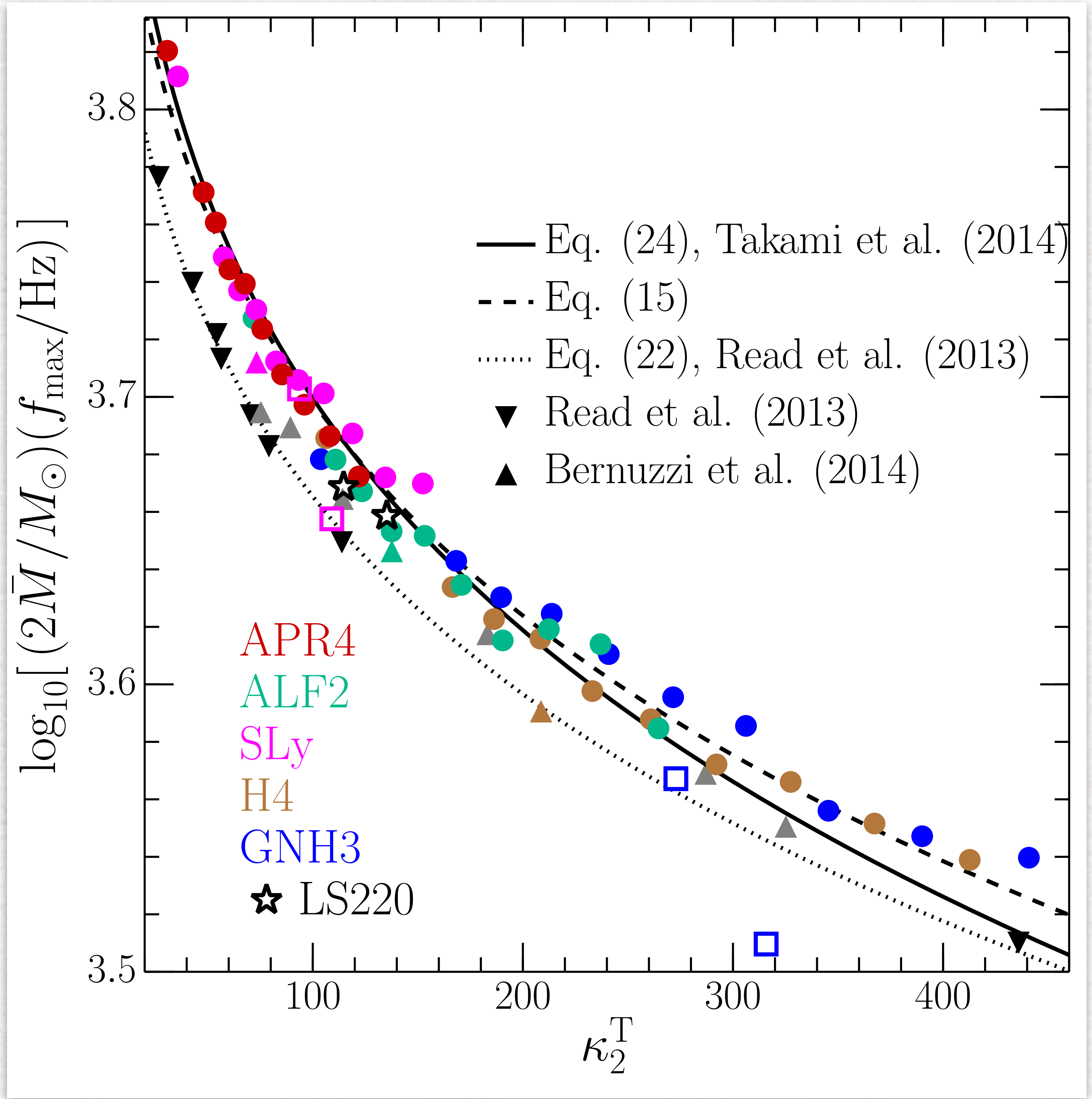


Anatomy of the GW signal

Inspiral



Hints of quasi-universality



Read+, 2013, found rather "surprising" result: quasi-universal behaviour of GW frequency at amplitude peak

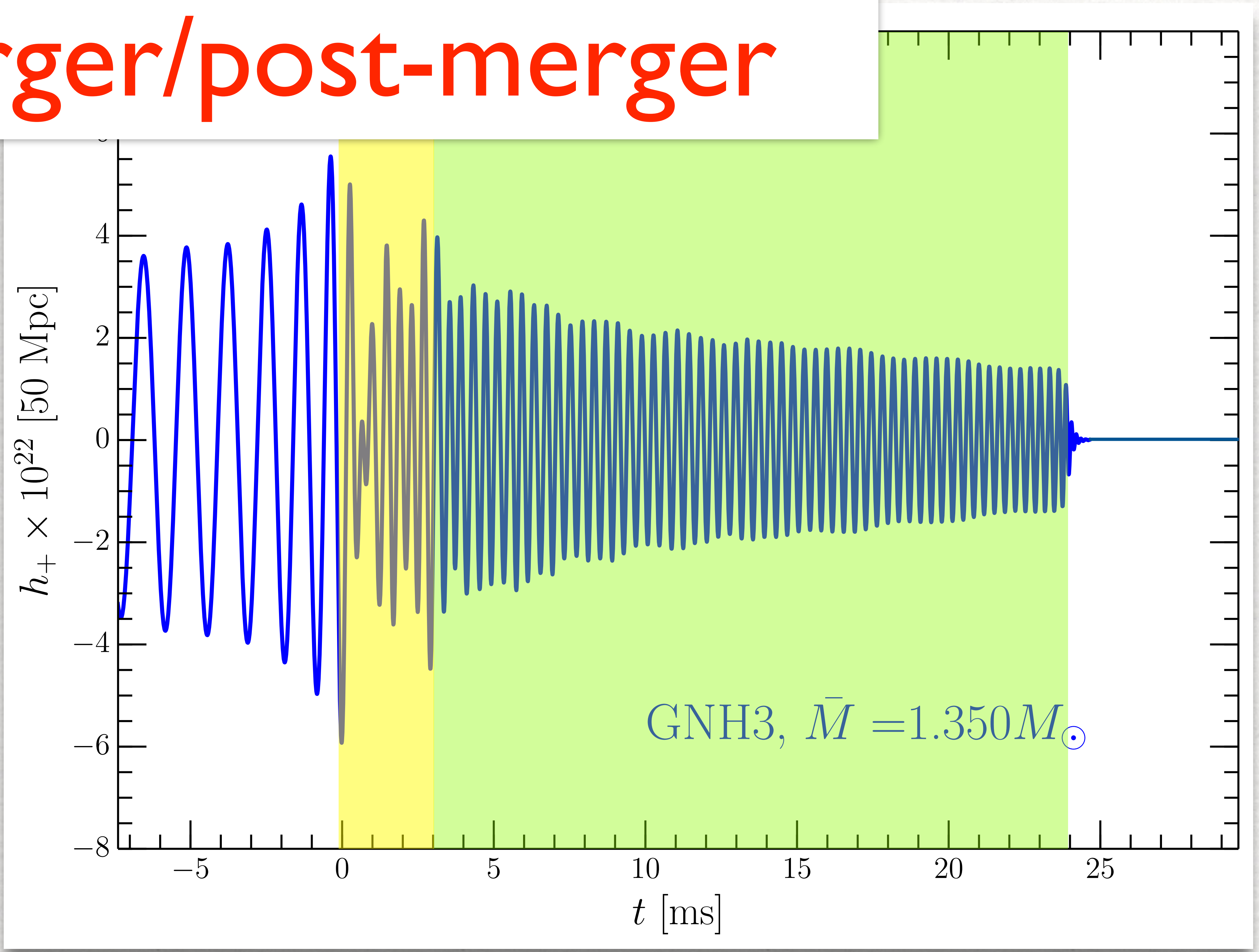
Bernuzzi+, 2014, Takami+, 2015, LR+2016 confirmed with new simulations.

Quasi-universal behaviour in the **inspiral** implies that once $\mathbf{f_{max}}$ is measured, so is tidal deformability, hence $I,\ Q,\ M/R$

$$\Lambda = \frac{\lambda}{\bar{M}^5} = \frac{16}{3}\kappa_2^T$$ tidal deformability or Love number

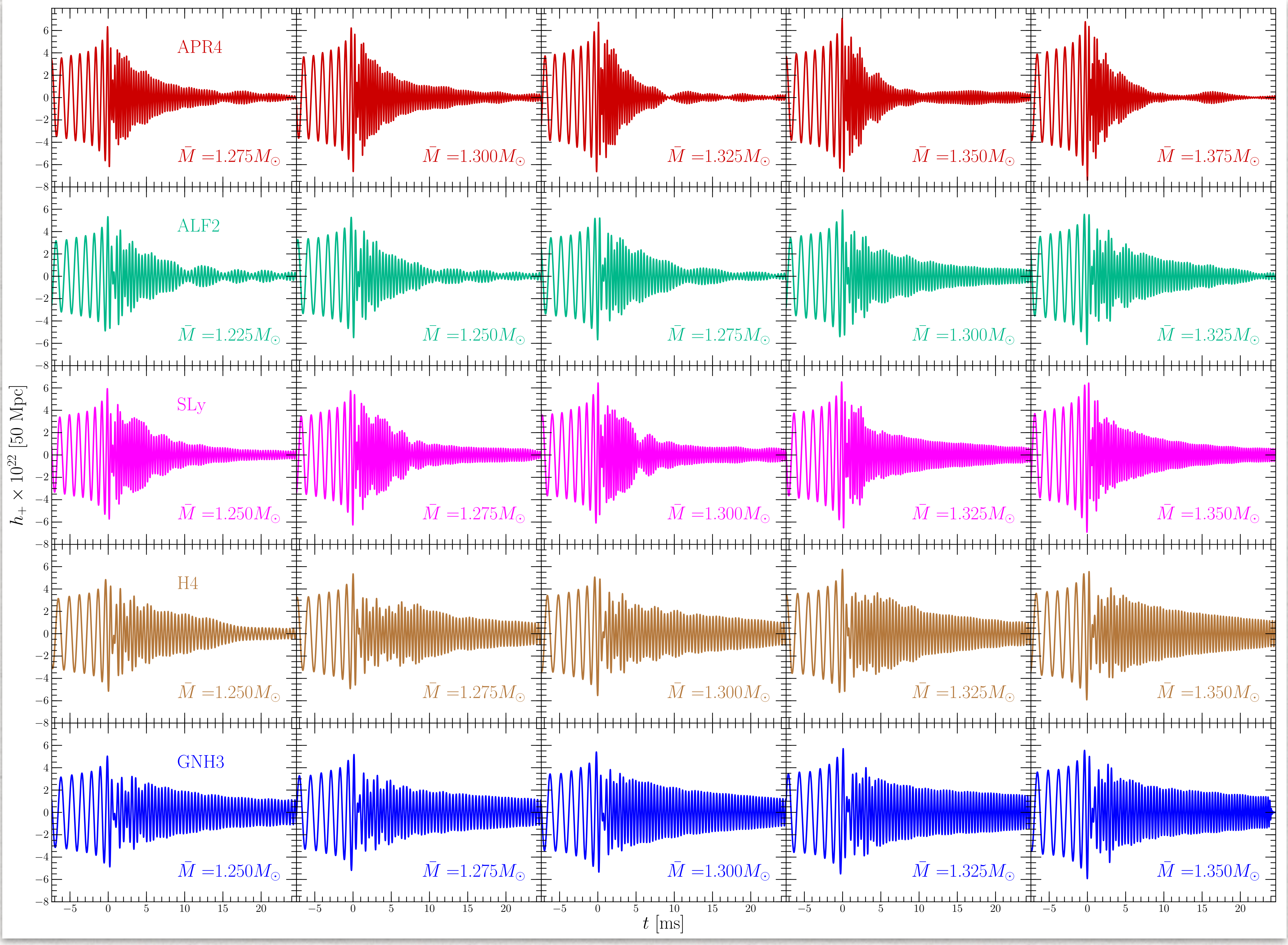
Anatomy of the GW signal

merger/post-merger



Extracting information from the EOS

Takami, LR, Baiotti (2014, 2015), LR+ (2016)



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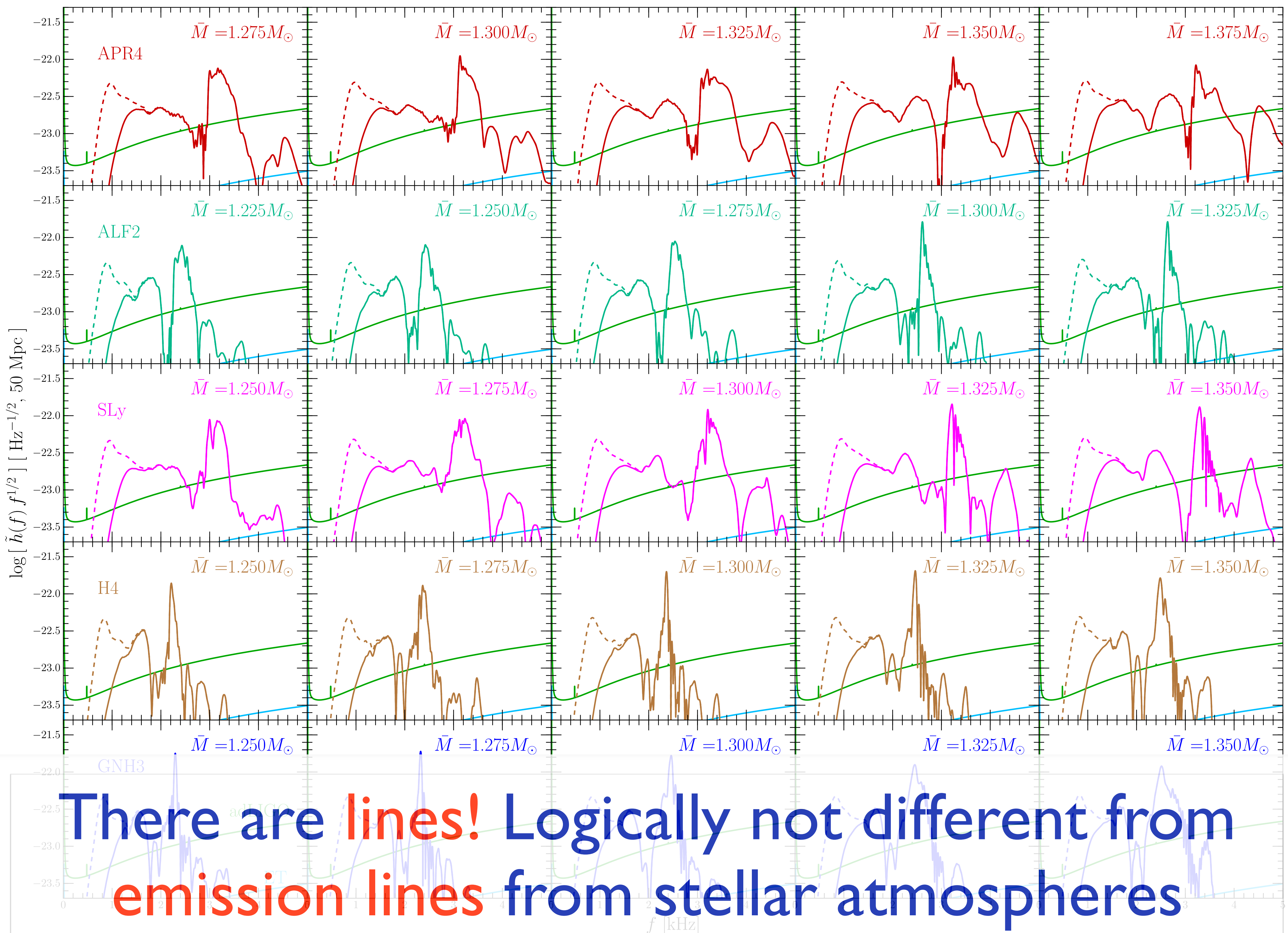
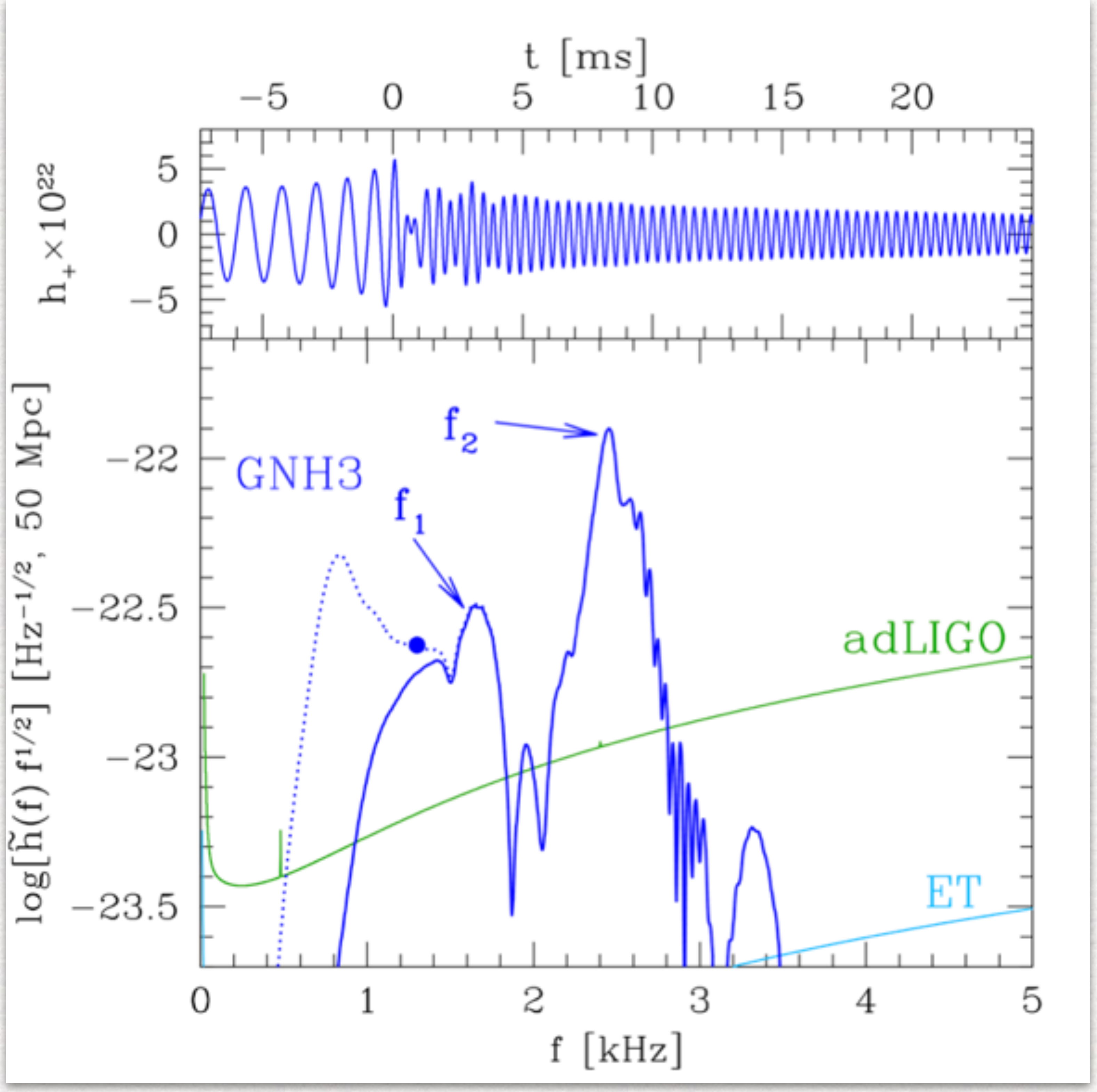


There are lines! Logically not different from emission lines from stellar atmospheres

A new approach to constrain the EOS

Oechslin+2007, Baiotti+2008, Bauswein+ 2011, 2012, Stergioulas+ 2011, Hotokezaka+ 2013, Takami 2014, 2015, Bernuzzi 2014, 2015, Bauswein+ 2015, LR+2016…



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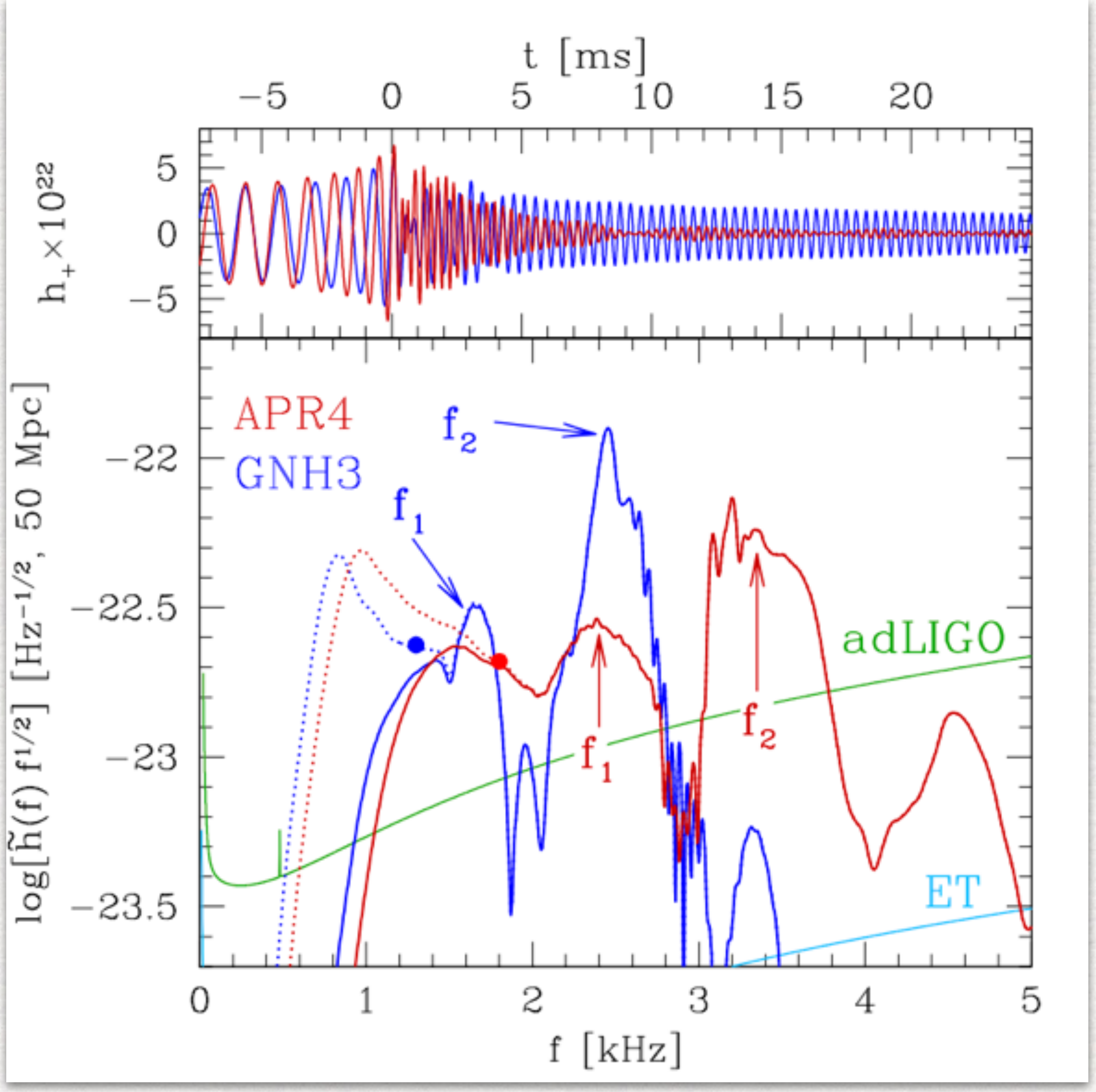
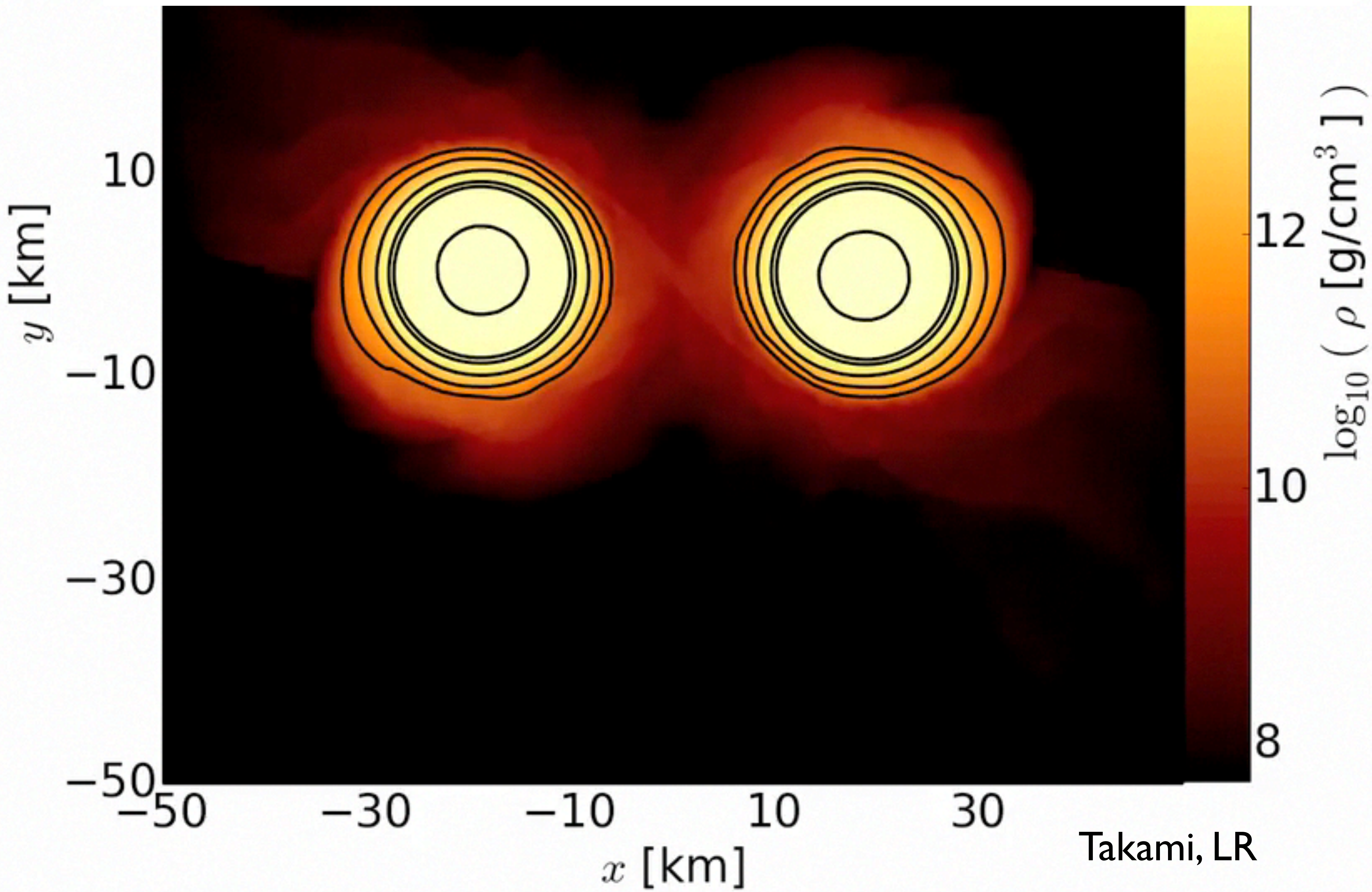
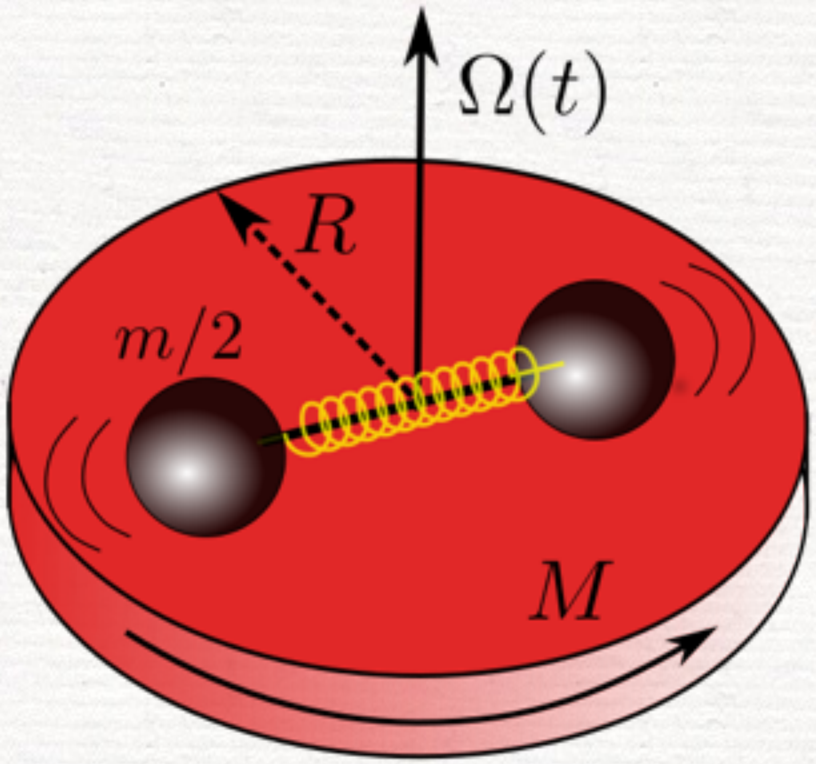


Prototypical simulation: ALF2 EOS, M=1.325M$_\odot$

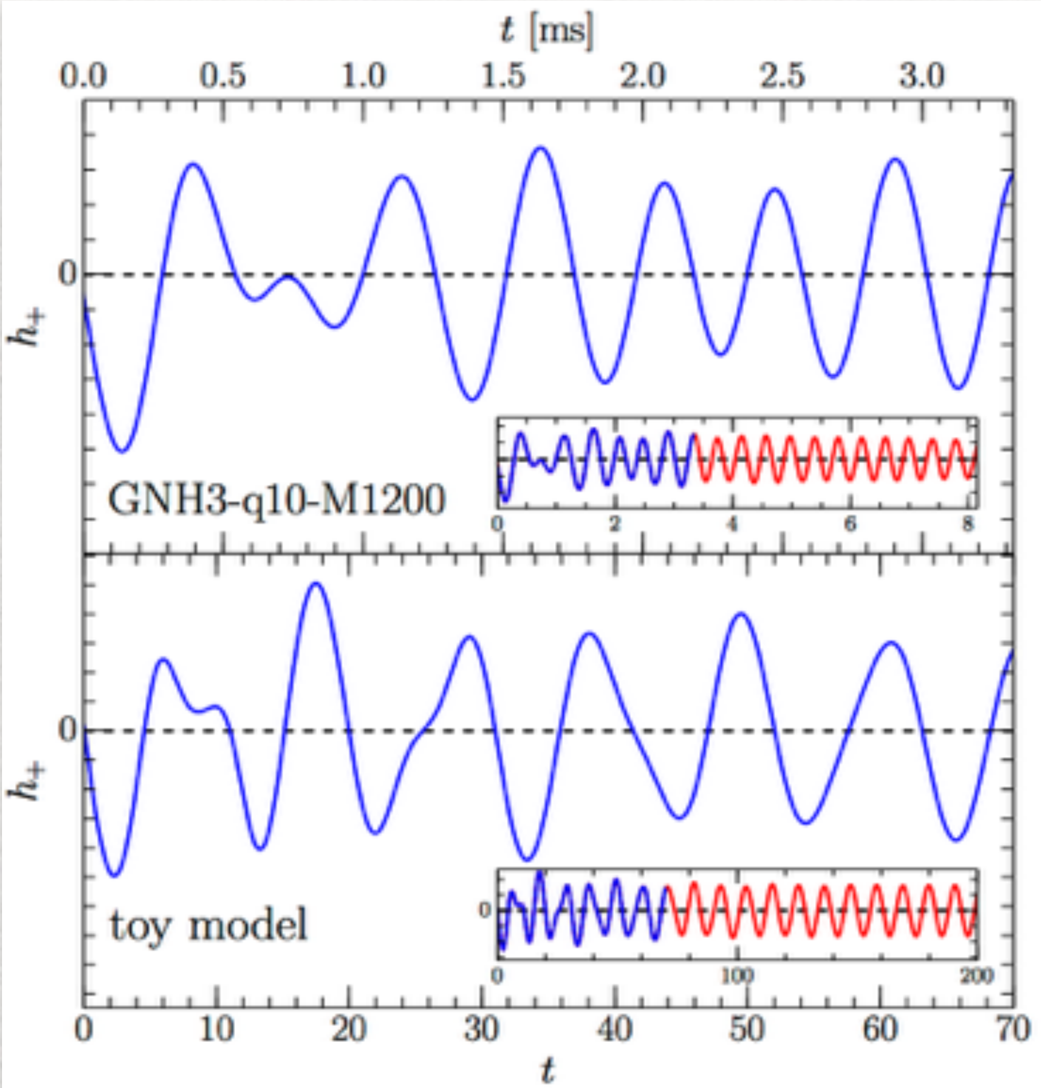


Takami, LR

A mechanical toy model



- Consider disk with 2 masses moving along a shaft and connected via a spring ~ HMNS with 2 stellar cores
- Let disk rotate and mass oscillate while conserving angular momentum

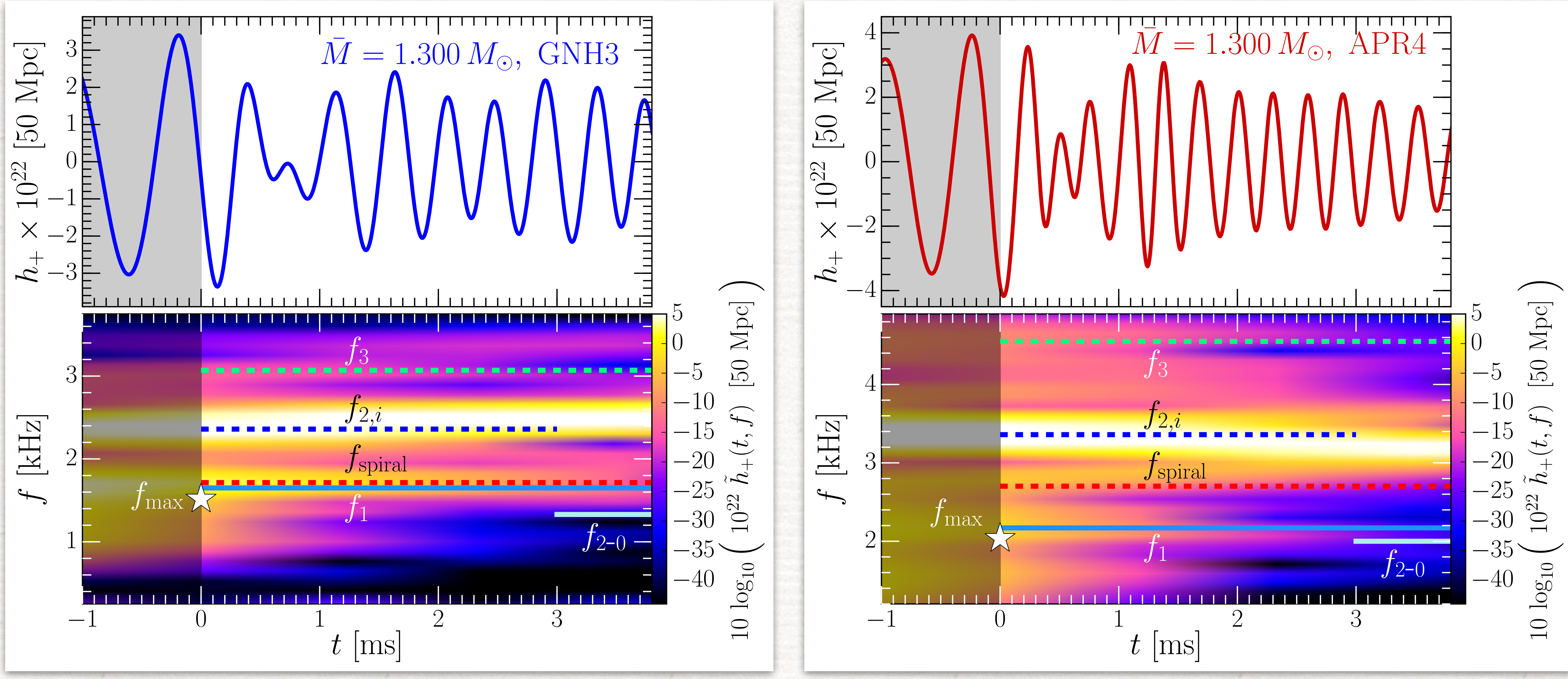


- If there is no friction, system will spin between: low freq (**f_1**, masses are far apart) and high (**f_3**, masses are close).
- If friction is present, system will spin asymptotically at **$f_2 \sim (f_1+f_3)/2$**.
- analytic model possible of post merger (see later).

Understanding mode evolution

On a **short** timescale after the merger, it is possible to see the emergence of **f_1**, **f_2**, and **f_3**.

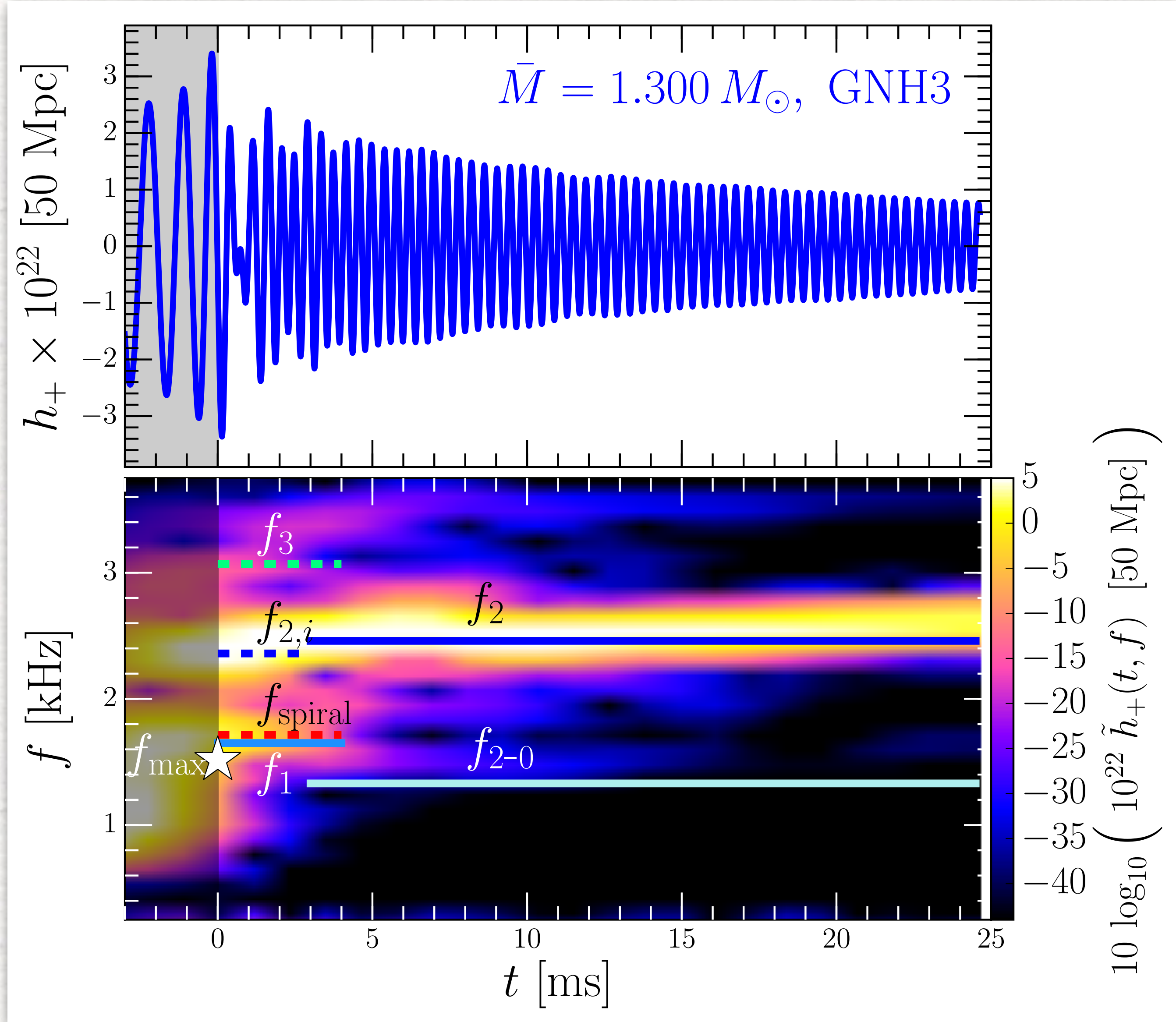
Note that it is easy to confuse **f_1** and **f_{spiral}**

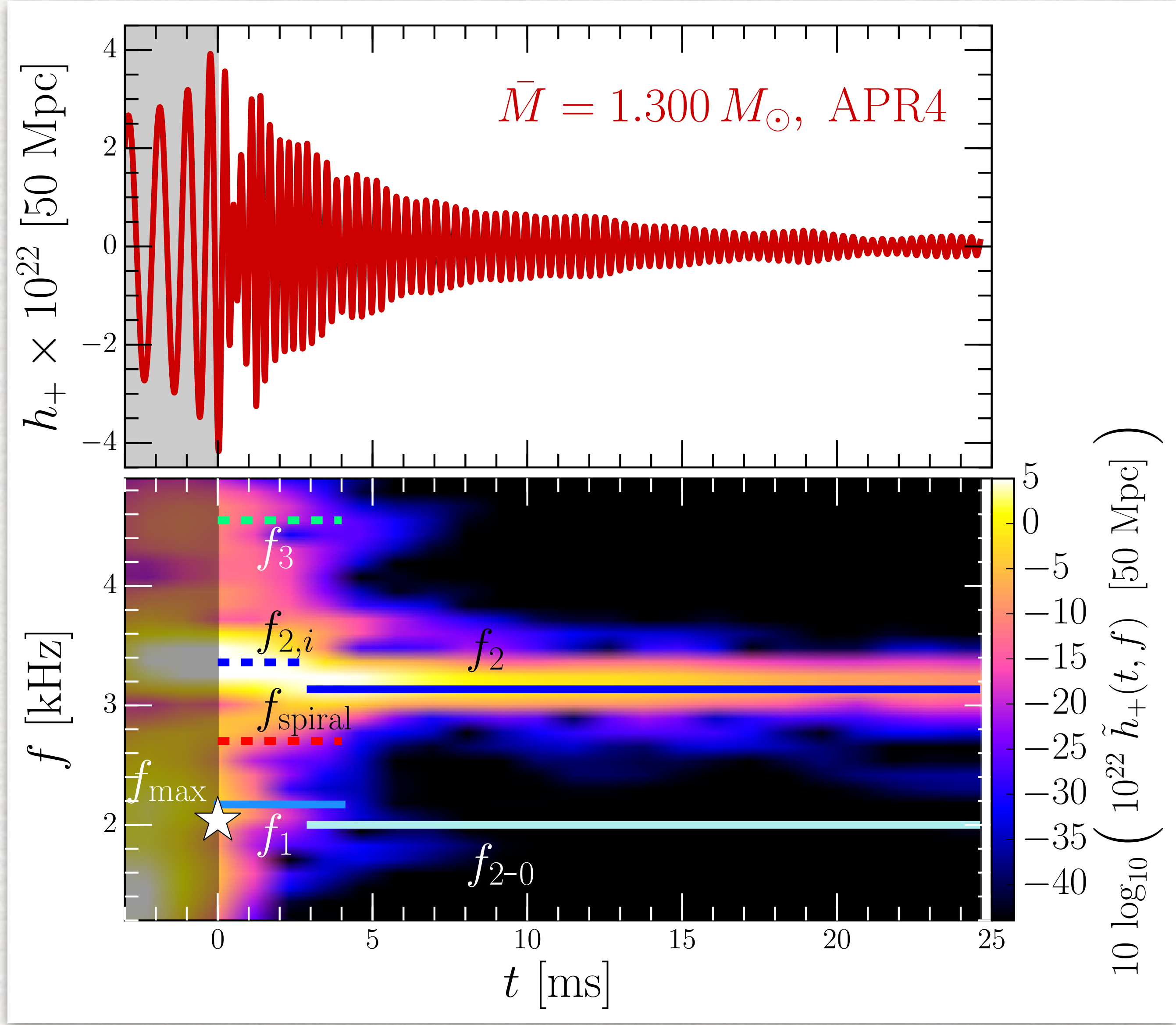


Understanding mode evolution

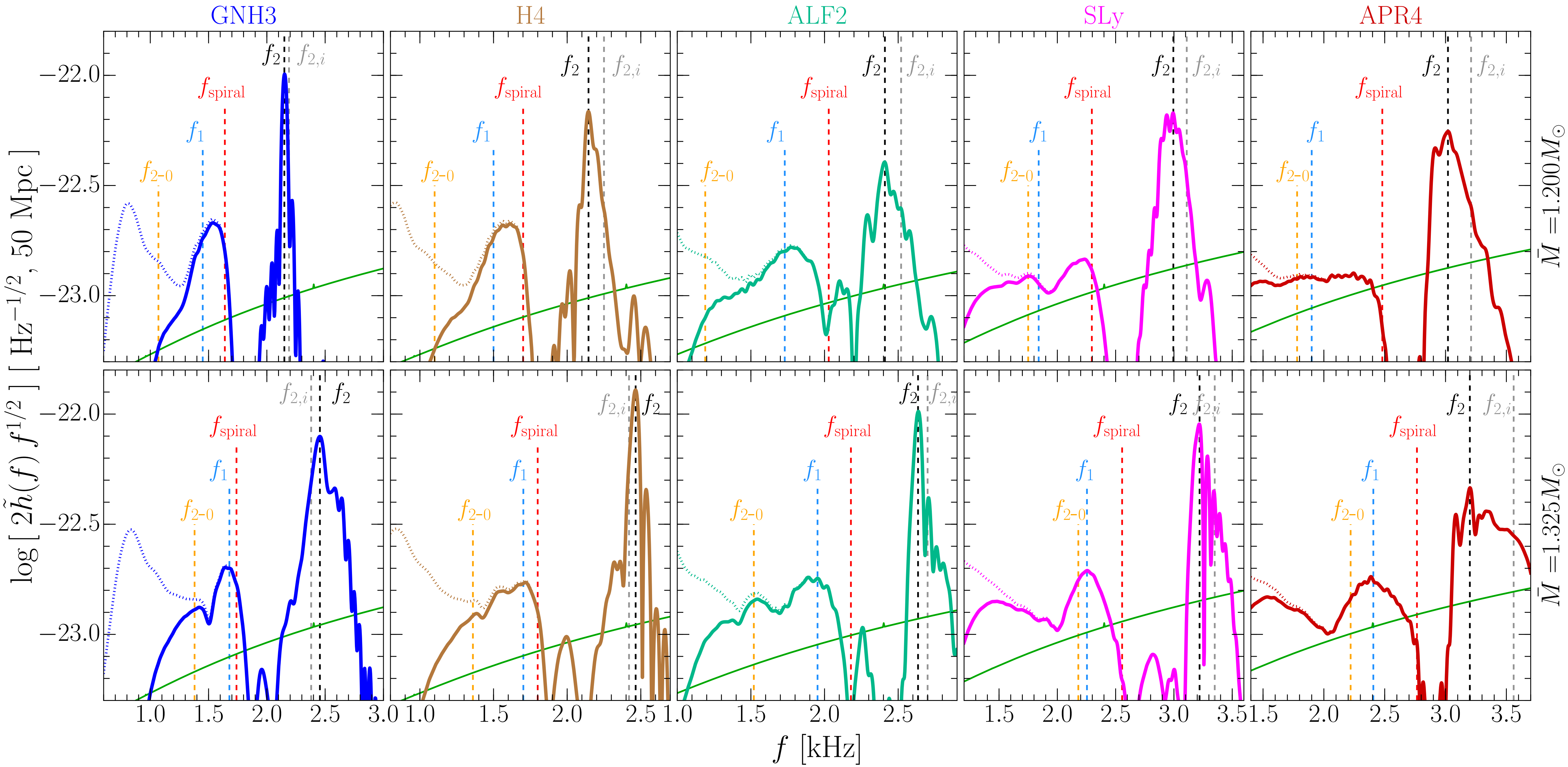
On a **long** timescale after the merger, only **f_2** survives.

Note that **f_{20}** is present but very weak.





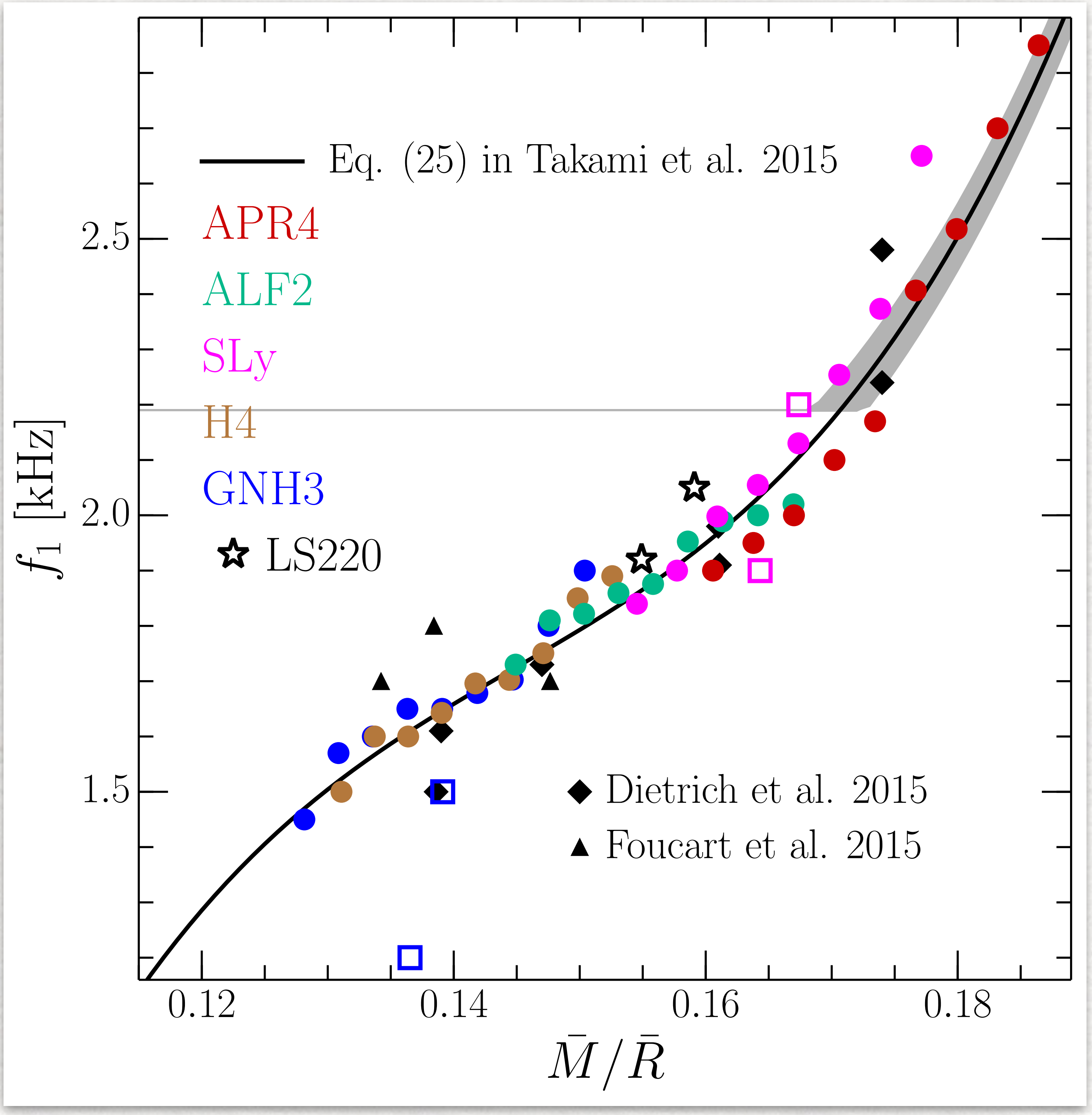
Some representative PSDs



$\mathbf{f_1}$, $\mathbf{f_2}$, and $\mathbf{f_3}$ frequencies are robust features of the spectra.

It's easy confuse $\mathbf{f_1}$ and $\mathbf{f_{spiral}}$ but latter is ill defined at times.

Quasi-universal or not?

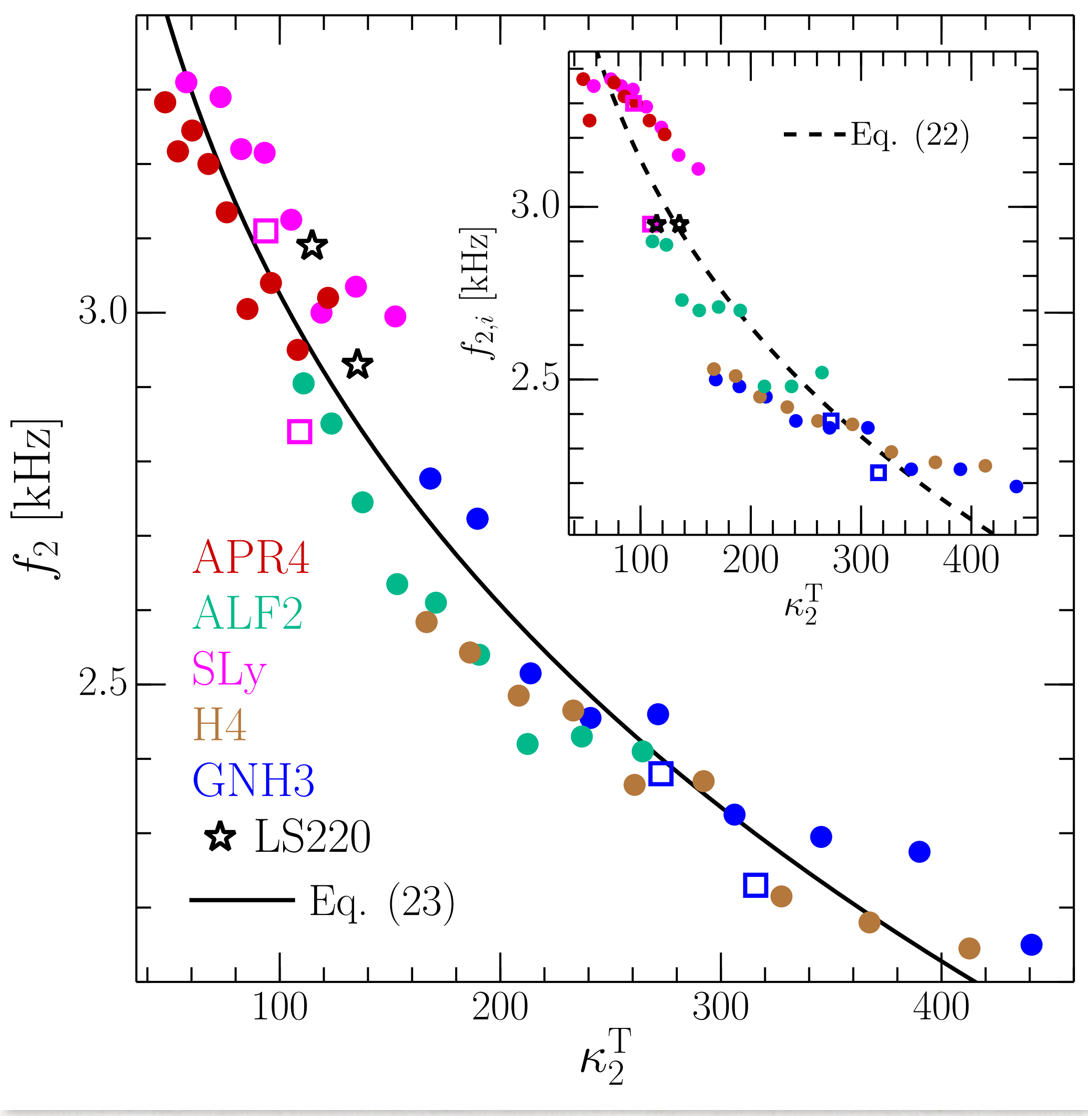


f_1 identification of PSDs is delicate, since created in short time window.

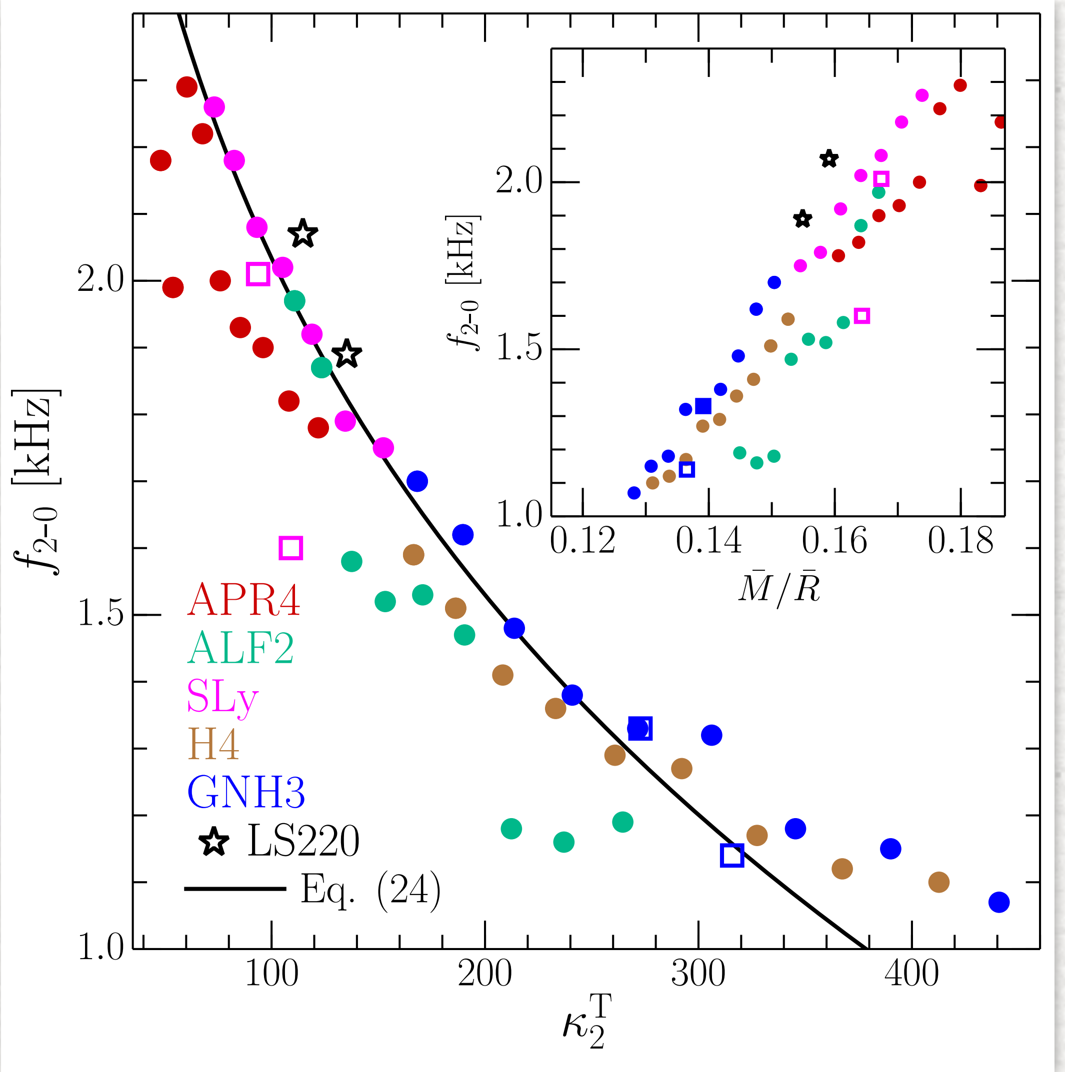
Spectrograms help the identification and results of other groups (Bernuzzi+ 2015, Foucart+ 2015) confirm quasi-universality.

Despite different claims, universality not lost at very low (1.2 $M_\odot$), very high (1.5 $M_\odot$) masses (LR+ 2016)

Quasi-universal or not? The case for **f_2, f_{20}**



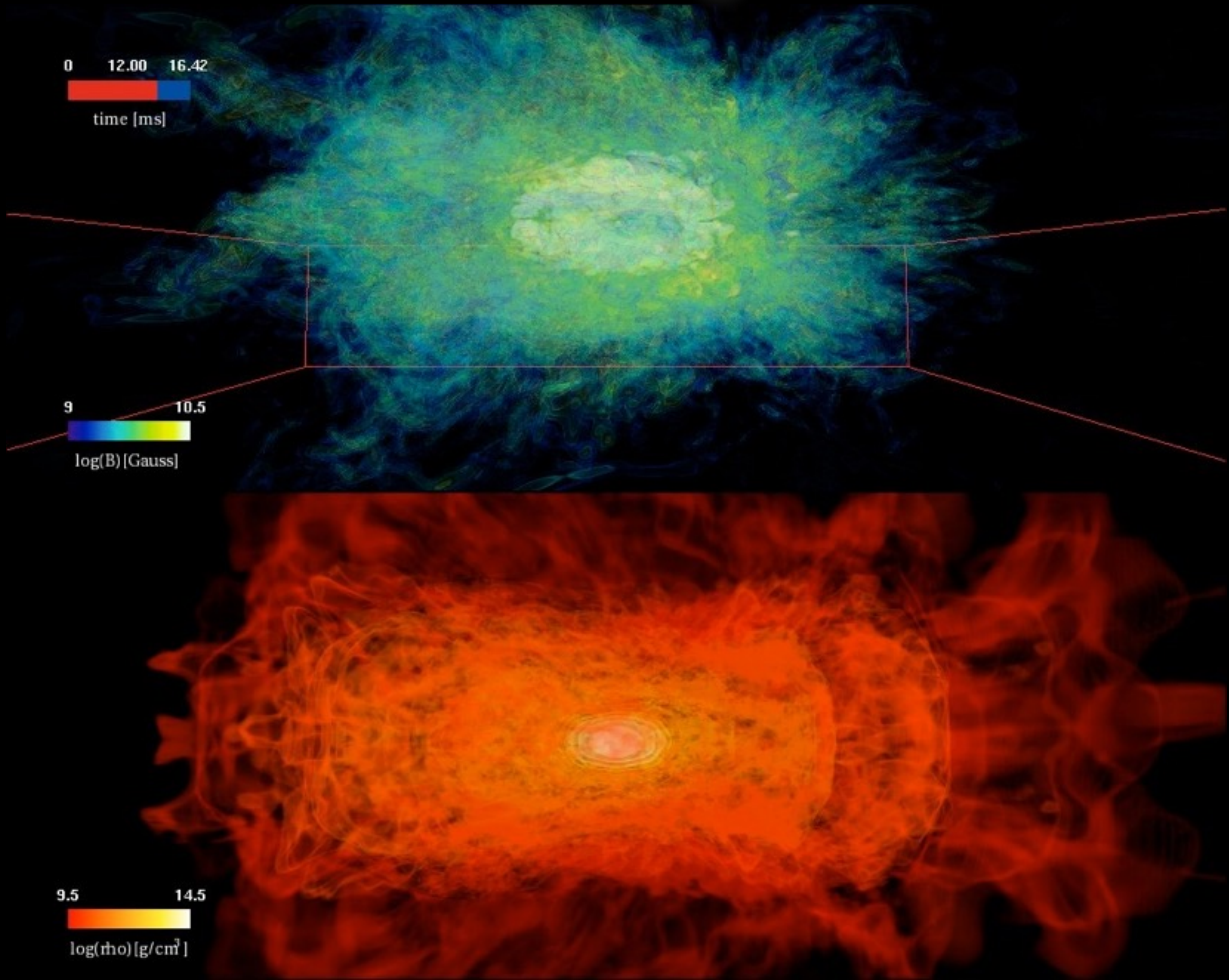
Correlations with stellar properties (Love number) have been found also for **f_2** and **$f_{2\text{-}0}$** peak (Takami+ 2015, Bernuzzi+ 2015, LR+2016)



These correlations are weaker but equally important.

Despite its complexity, a complete analytical description of pre- and post-merger signal is **possible**.

The role of magnetic fields



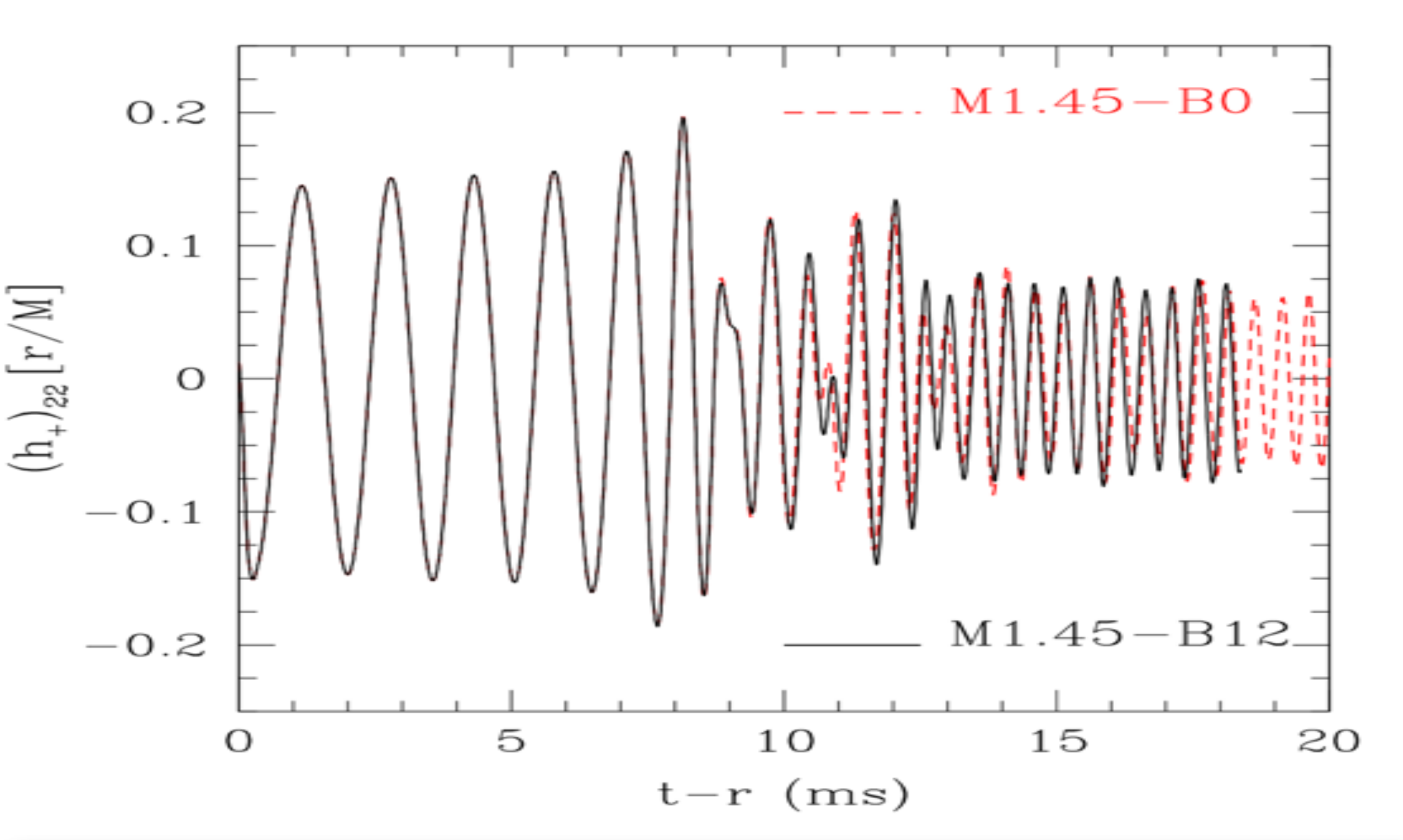
Ideal Magnetohydrodynamics

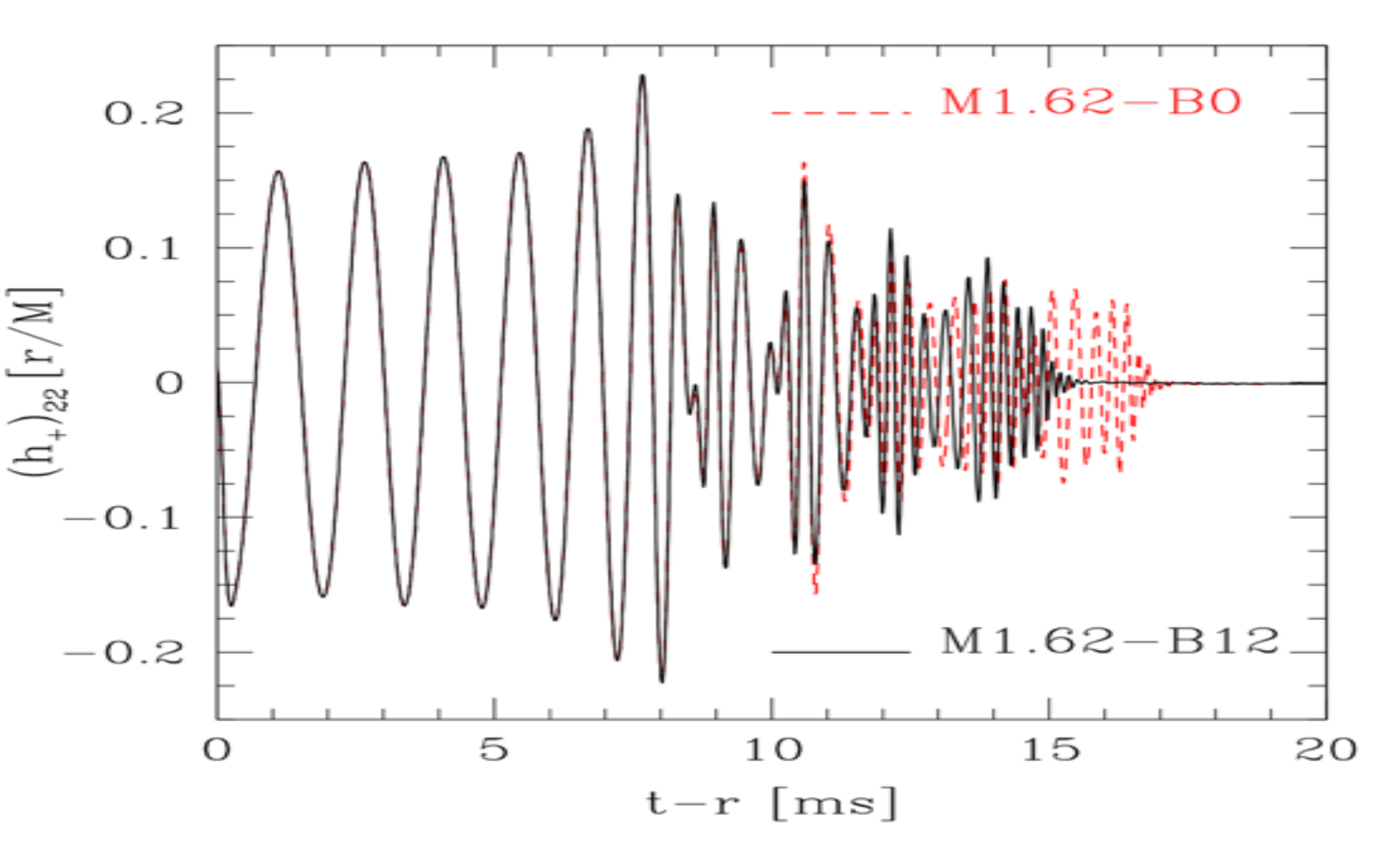
Most simulations to date make use of **ideal MHD**: conductivity is infinite and magnetic field simply advected.

You can ask some simple questions.

- can B-fields be detected during the inspiral?
- can B-fields be detected in the HMNS?
- can B-fields grow after BH formation?

Waveforms: comparing against magnetic fields





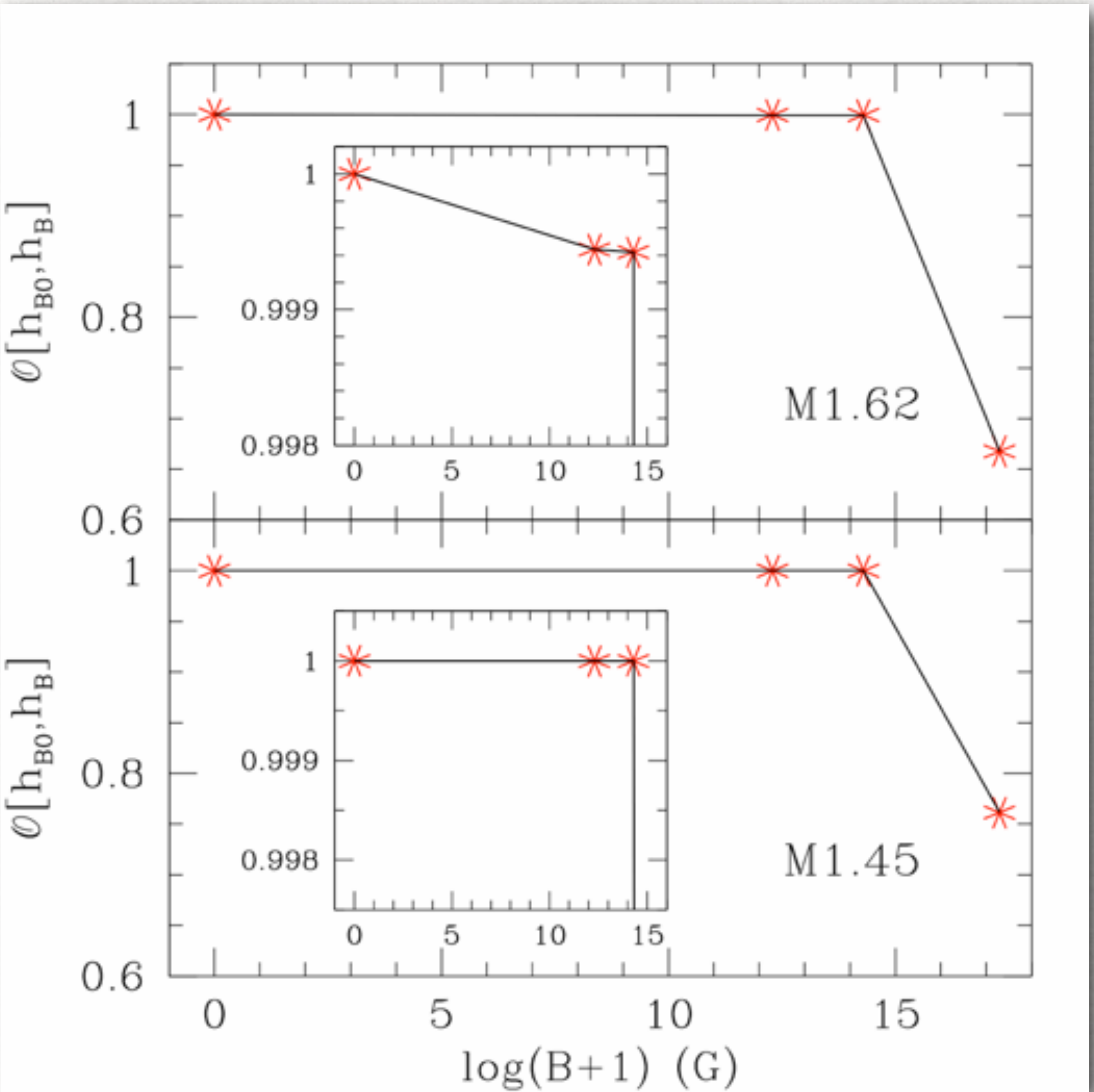
Compare B/no-B field:

- the evolution in the **inspiral** is different but only for ultra large B-fields (i.e. B~10^{17} G). For realistic fields the difference is not significant.
- the **post-merger** evolution is different for all masses; strong B-fields delay the collapse to BH

However, **mismatch** must computed using detector sensitivity

Can we detect B-fields in the inspiral?

To quantify the differences and determine whether detectors will see a difference in the inspiral, we calculate the overlap



$$\mathcal{O}[h_{B1}, h_{B2}] \equiv \frac{\langle h_{B1} | h_{B2} \rangle}{\sqrt{\langle h_{B1} | h_{B1} \rangle \langle h_{B2} | h_{B2} \rangle}}$$

where the scalar product is

$$\langle h_{B1} | h_{B2} \rangle \equiv 4\Re \int_0^\infty df \, \frac{\tilde{h}_{B1}(f) \tilde{h}^*_{B2}(f)}{S_h(f)}$$

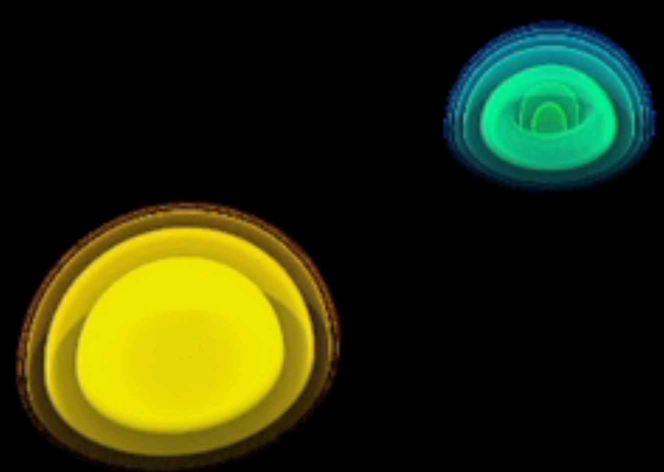
In essence, at these res:

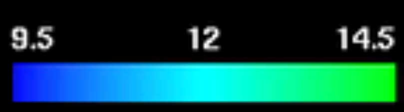
$$\mathcal{O}[h_{B0}, h_B] \gtrsim 0.999$$

for $\mathrm{B} \lesssim 10^{17}$ G

Influence of B-fields on inspiral is **unlikely to be detected**

Typical evolution for a magnetized binary (hot EOS) $M = 1.5\, M_{\odot}, B_0 = 10^{12}\, \mathrm{G}$

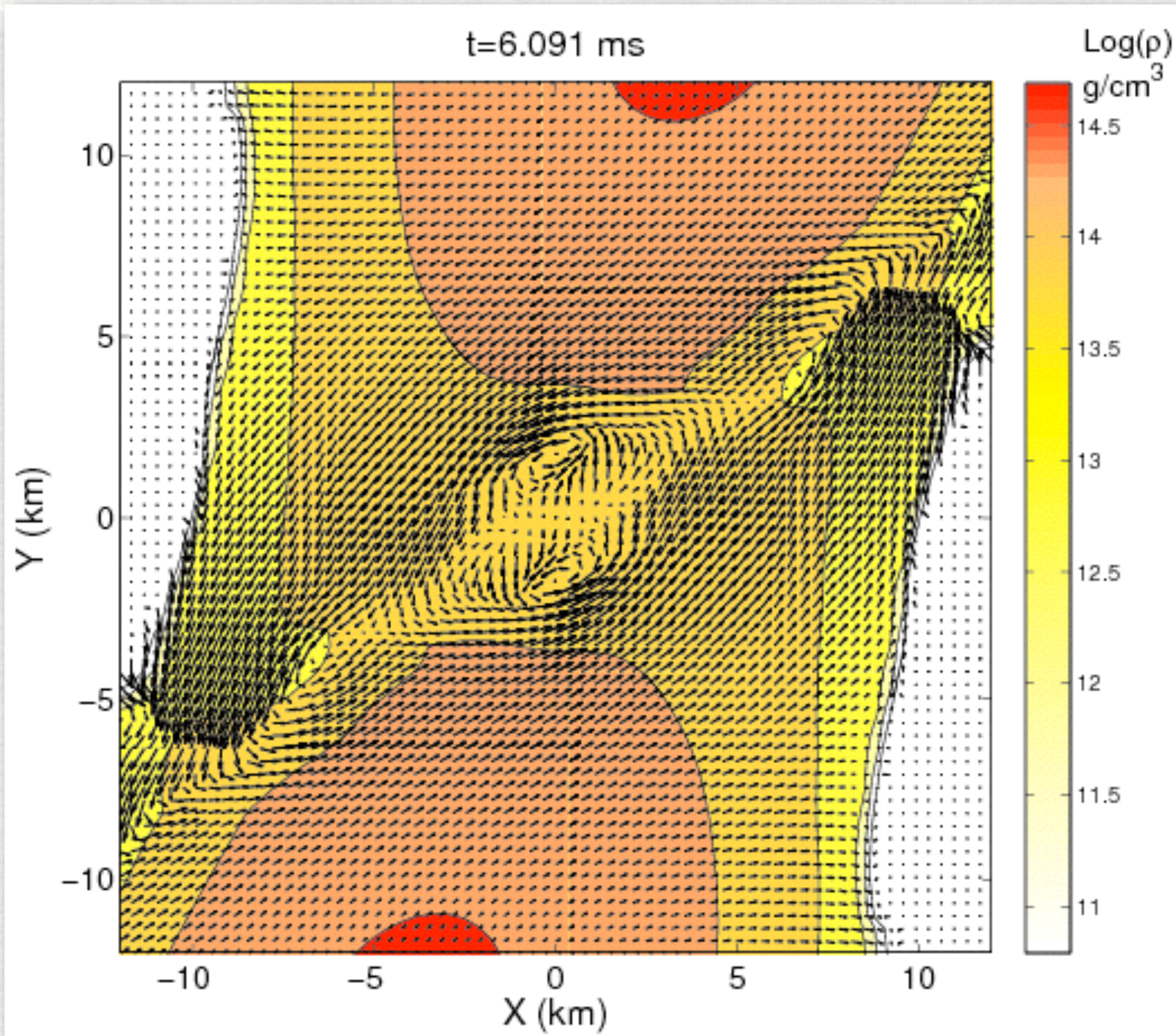




Animations:, LR, Koppitz

MHD instabilities and B-field amplifications

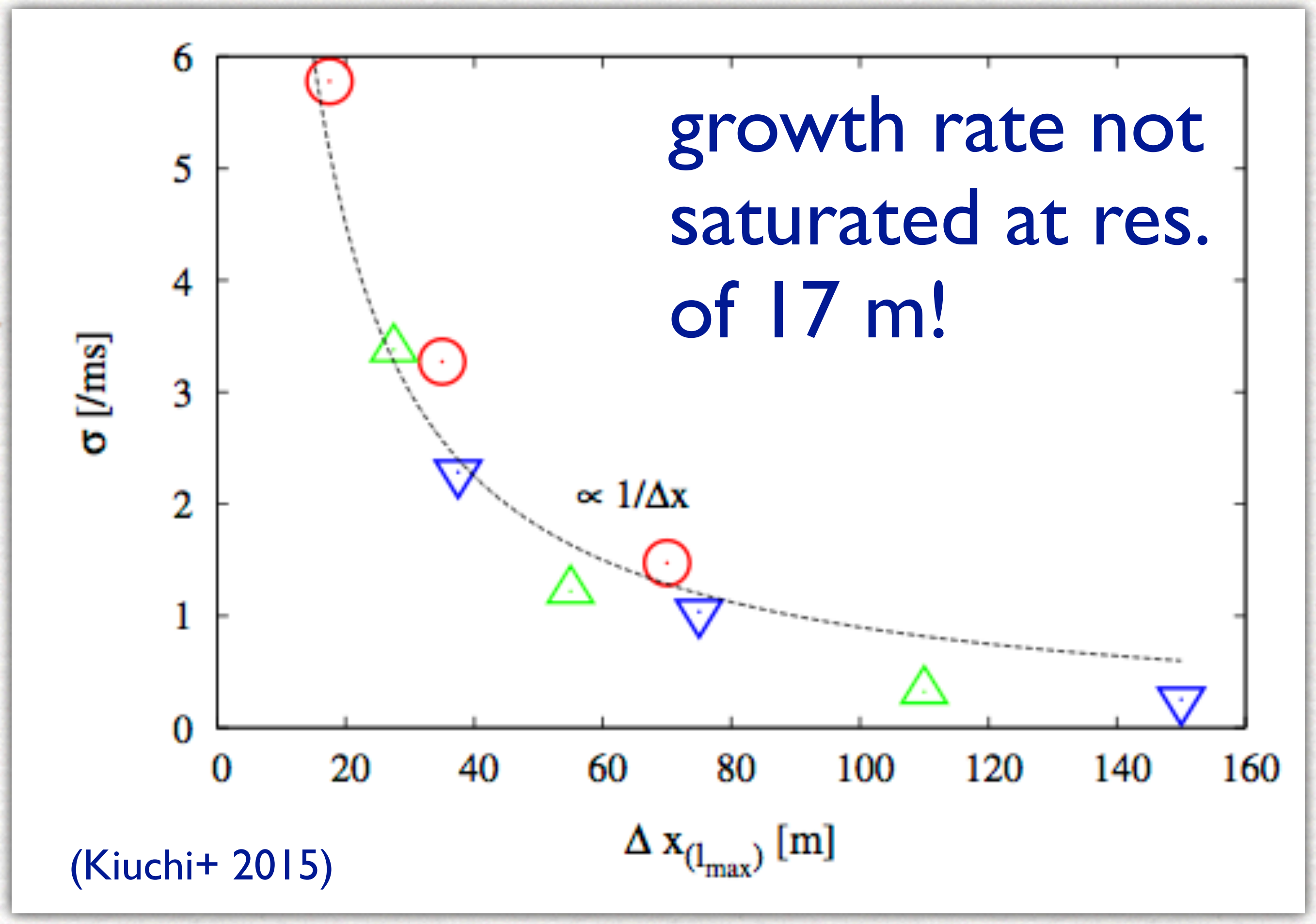
- at the merger, the NS create a strong shear layer which could lead to a **Kelvin-Helmholtz instability**; magnetic field can be amplified



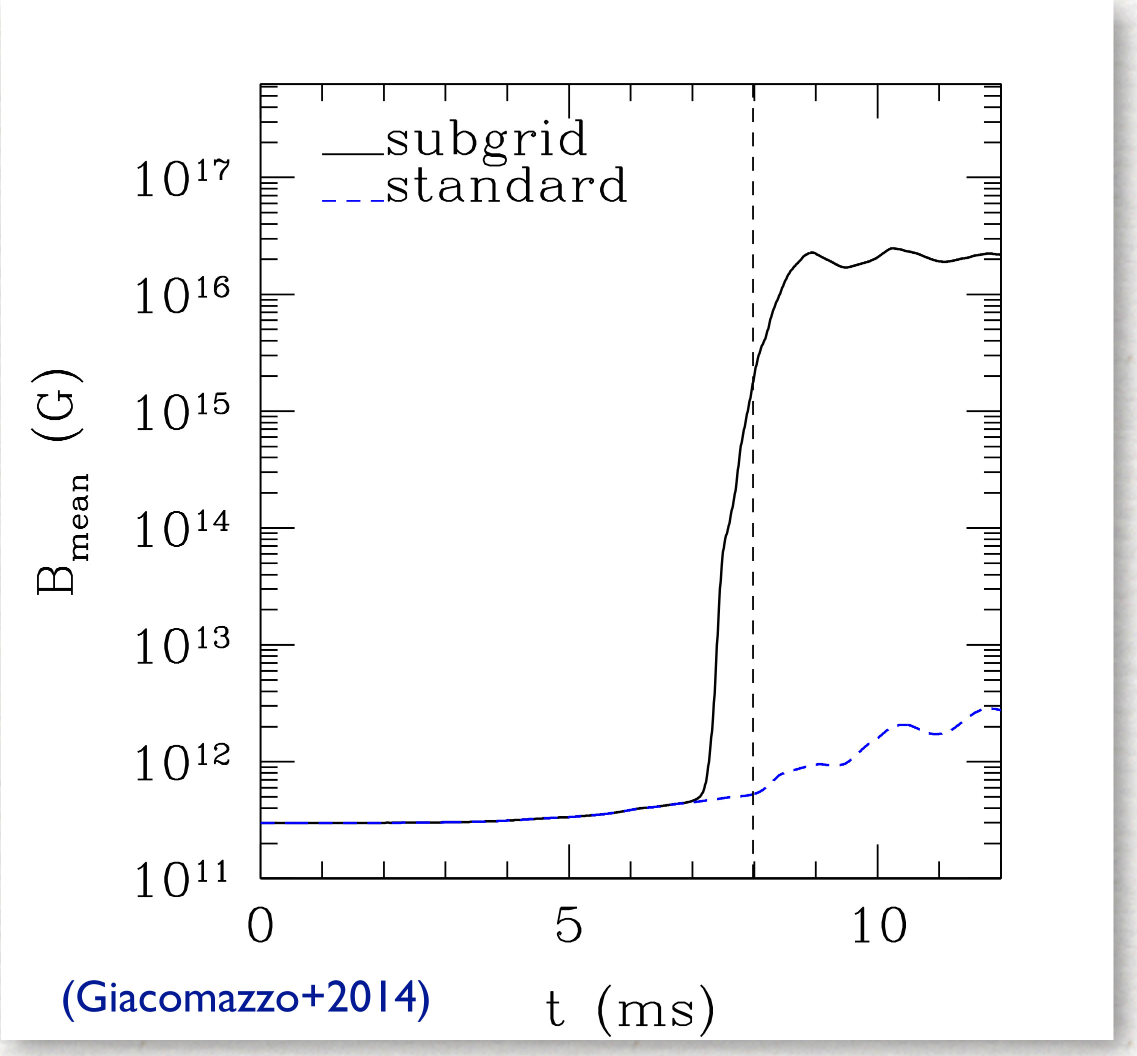
(Baiotti+2008)

MHD instabilities and B-field amplifications

- at the merger, the NS create a strong shear layer which could lead to a **Kelvin-Helmholtz instability**; magnetic field can be amplified
- low-res simulations don't show exponential growth (Giacomazzo+2011) high-res simulations show increase of ~ 3 orders of mag (Kiuchi+2015)
- sub-grid models suggest B-field grows to 10^{16} G (Giacomazzo+2014)



(Kiuchi+ 2015)



(Giacomazzo+2014)

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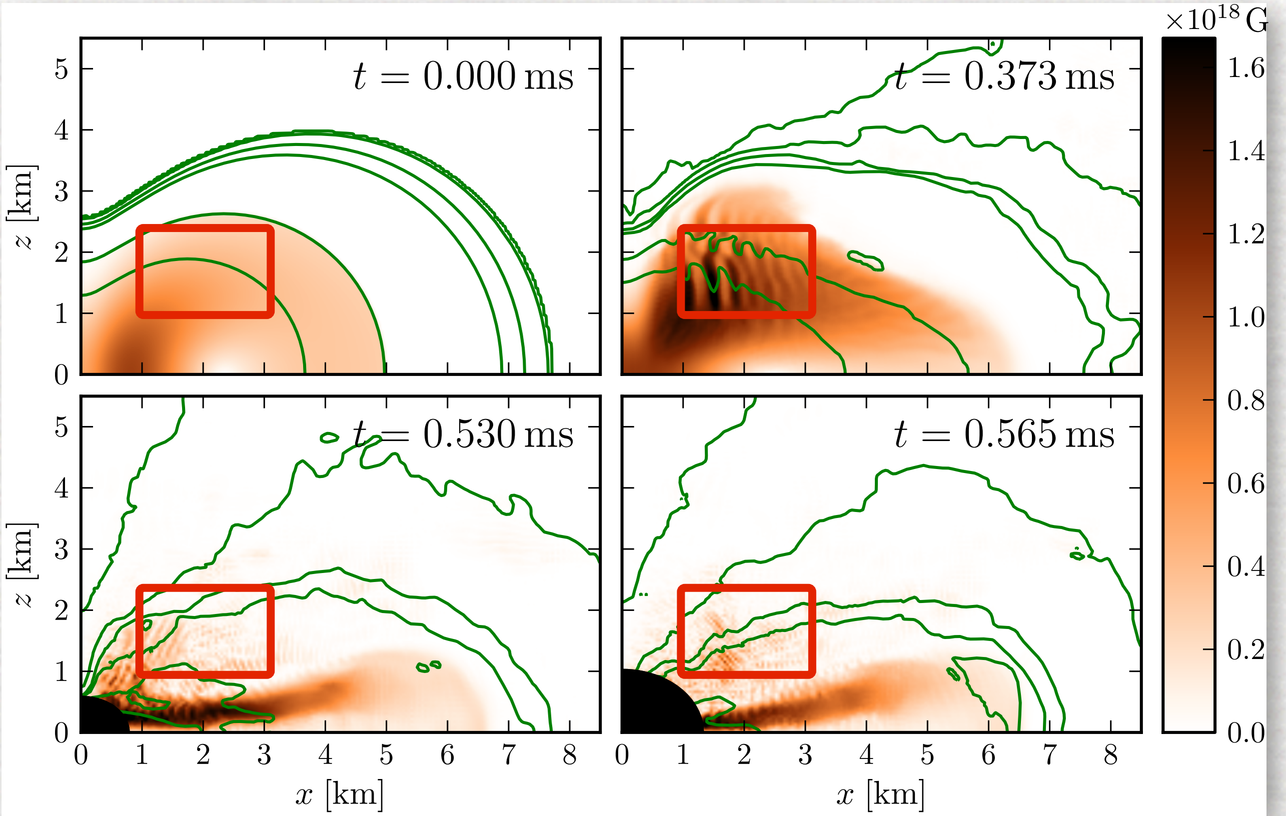
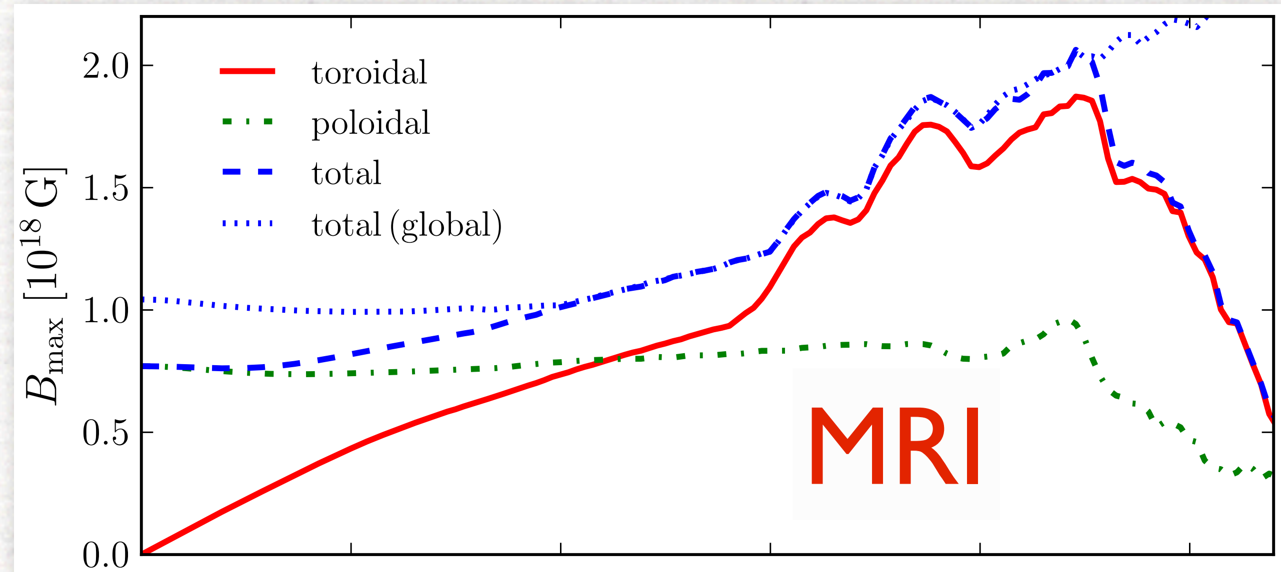
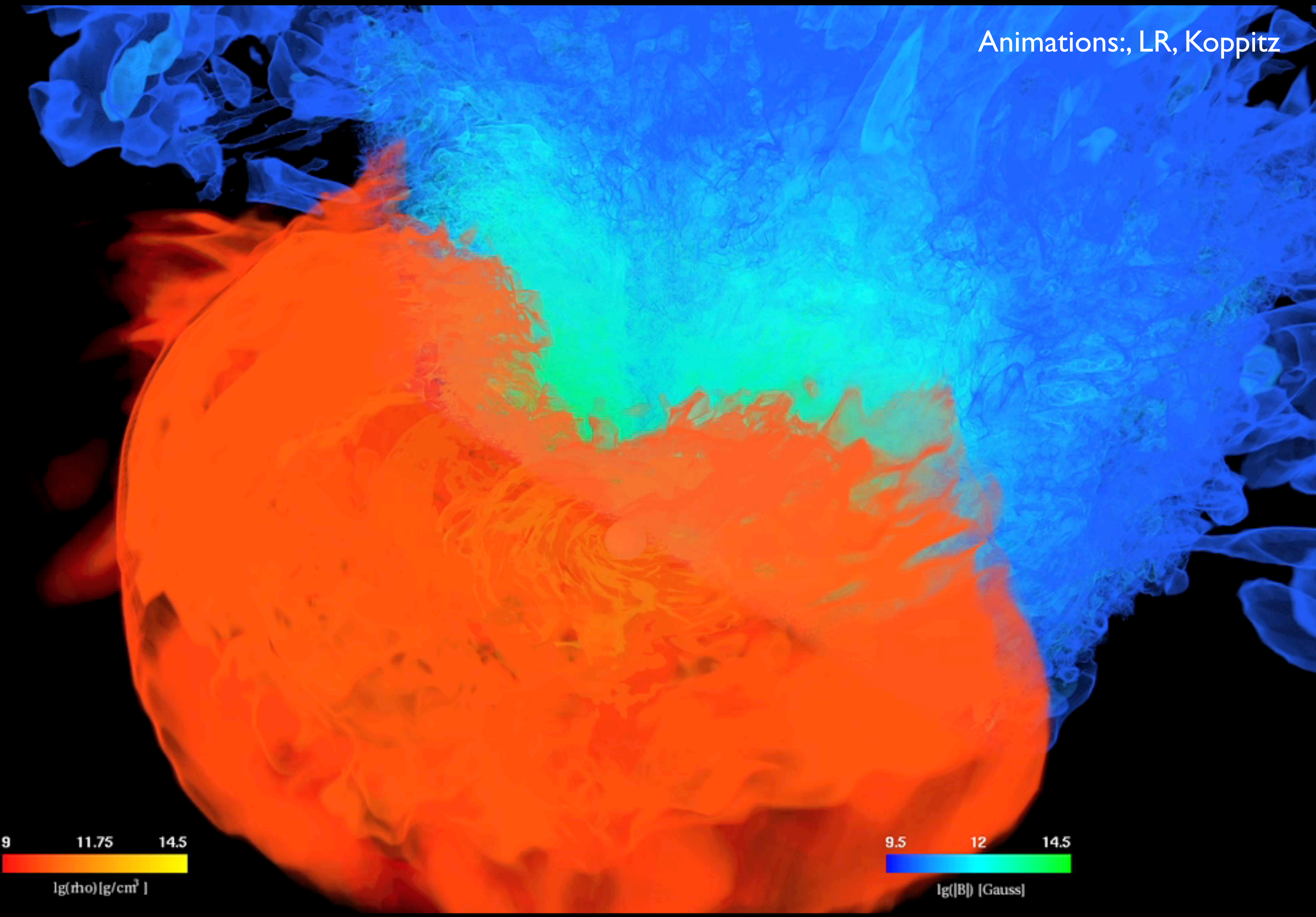


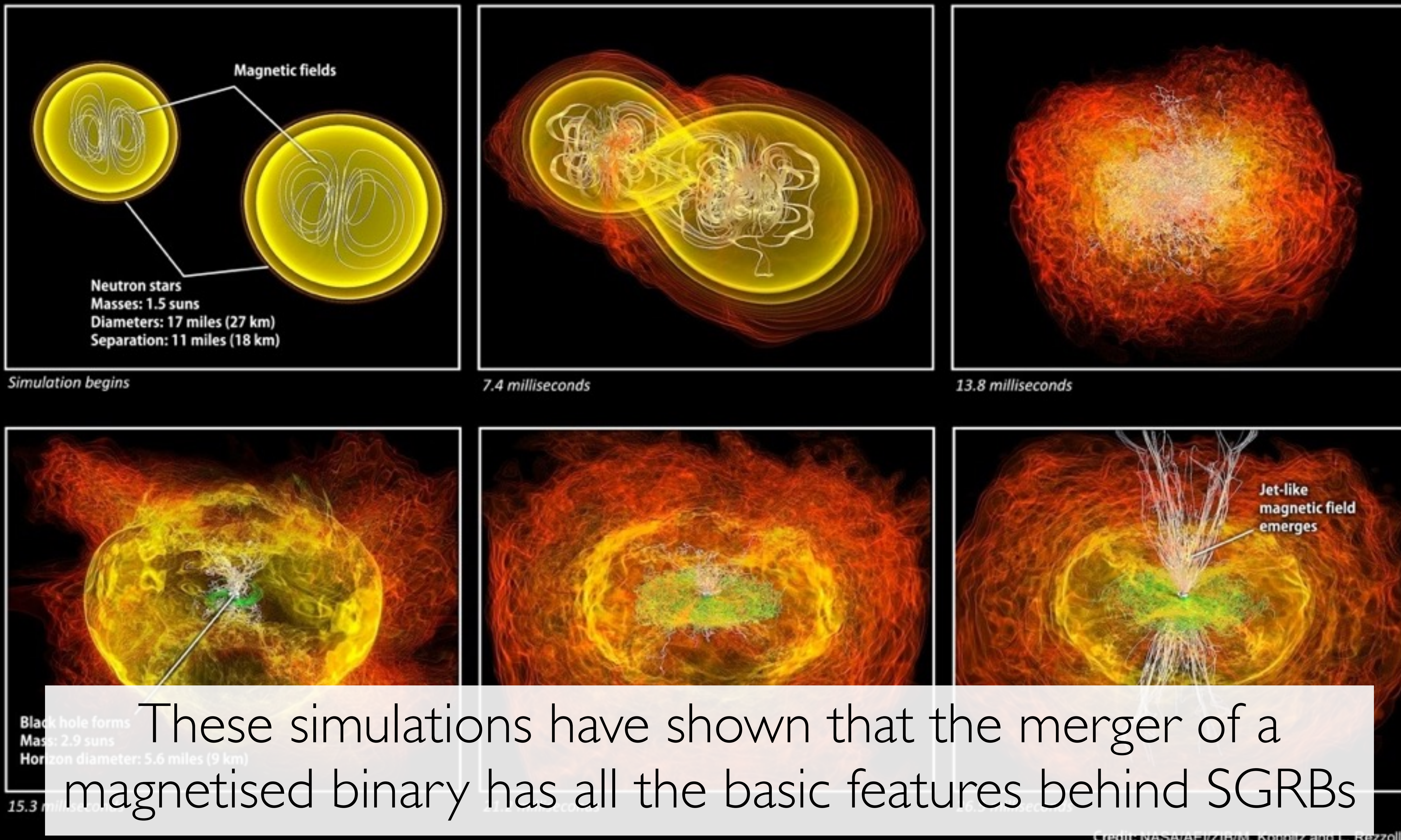


Siegel+2013

- differentially rotating magnetized fluids develop the **MRI (magnetorotational instability;** Velikhov 1959, Chandrasekhar 1960)
- the MRI leads to exponential growth of B-field and to an outward transfer of angular momentum: responsible for accretion in discs
- overall, consensus MRI **can develop** in HMNS (Siegel+2013,Kiuchi+2014)
- degree of amplification is unknown: 2-3 or 5-6 orders of magnitude? What about resistivity? (Kiuchi+2015, Obergaulinger+2015)


Animations:, LR, Koppitz
9
11.75
14.5
lg(rho) [g/cm³]
9.5
12
14.5
lg(|B|) [Gauss]

LR+ 2011



These simulations have shown that the merger of a magnetised binary has all the basic features behind SGRBs

$J/M^2 = 0.83$ $\mathrm{M}_{\rm tor} = 0.063 M_\odot$ $\mathrm{t}_{\rm accr} \simeq M_{\rm tor}/\dot{M} \simeq 0.3\ \mathrm{s}$

Resistive Magnetohydrodynamics

Dionysopoulou, Alic, LR (2015)

- Ideal MHD is a good approximation in the inspiral, but not after the merger; match to **electro-vacuum** not possible.
- Main difference in resistive regime is the current, which is dictated by Ohm's law but microphysics is **poorly** known.
- We know conductivity σ is a **tensor** and proportional to density and inversely proportional to temperature.
- A simple prescription with scalar (isotropic) conductivity:

$$J^i = qv^i + W\sigma[E^i + \epsilon^{ijk}v_j B_k - (v_k E^k)v^i]\,,$$

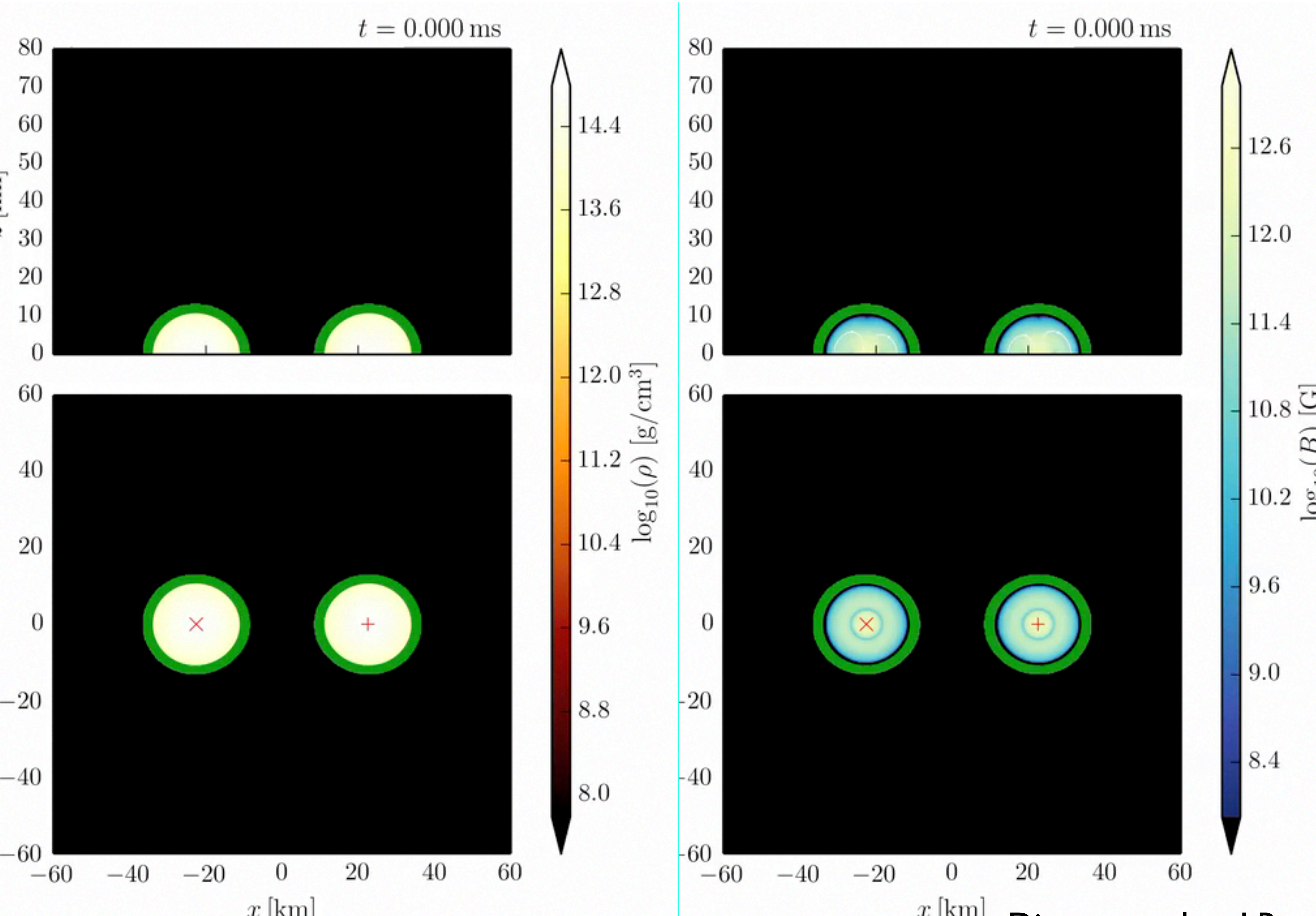
$\sigma \to \infty$ ideal-MHD (IMHD)

$\sigma \neq 0$ resistive-MHD (RMHD)

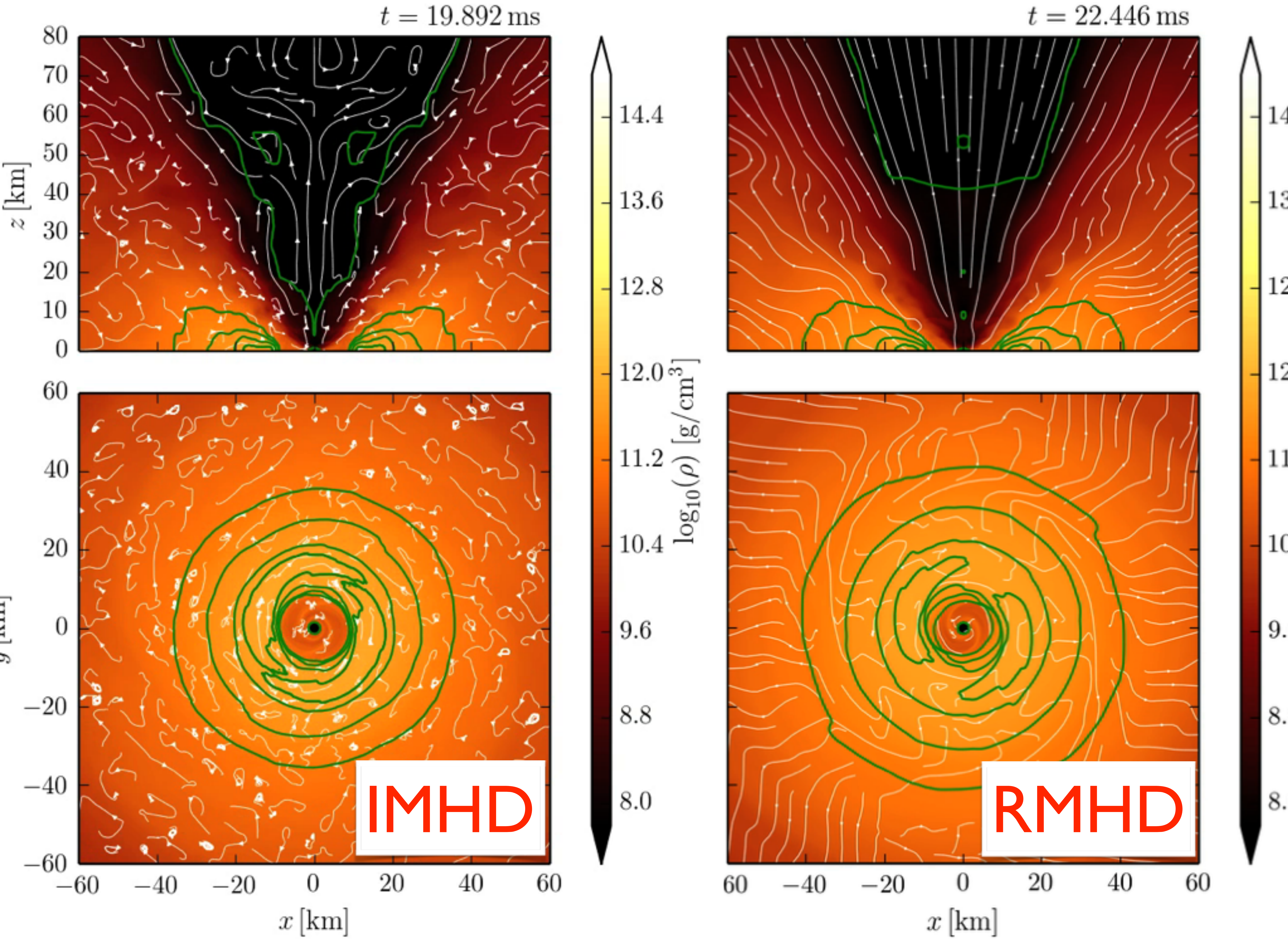
$\sigma \to 0$ electrovacuum

$$\sigma = f(\rho, \rho_{\min})$$

phenomenological prescription



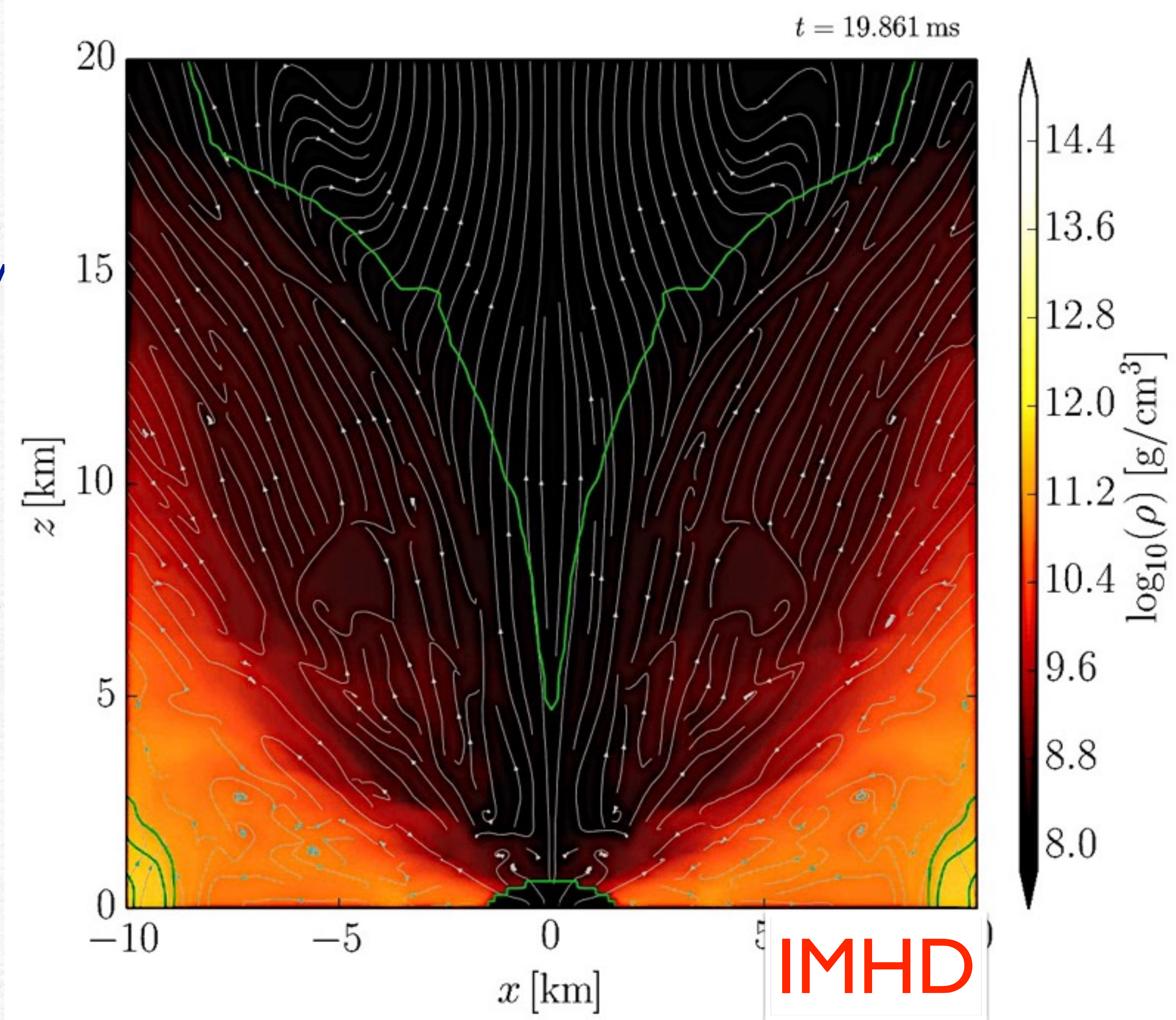
Dionysopoulou, LR



t = 19.892 ms
t = 22.446 ms
z [km]
y [km]
x [km]
x [km]
log10(ρ) [g/cm3]
14.4
13.6
12.8
12.0
11.2
10.4
9.6
8.8
8.0
IMHD
RMHD

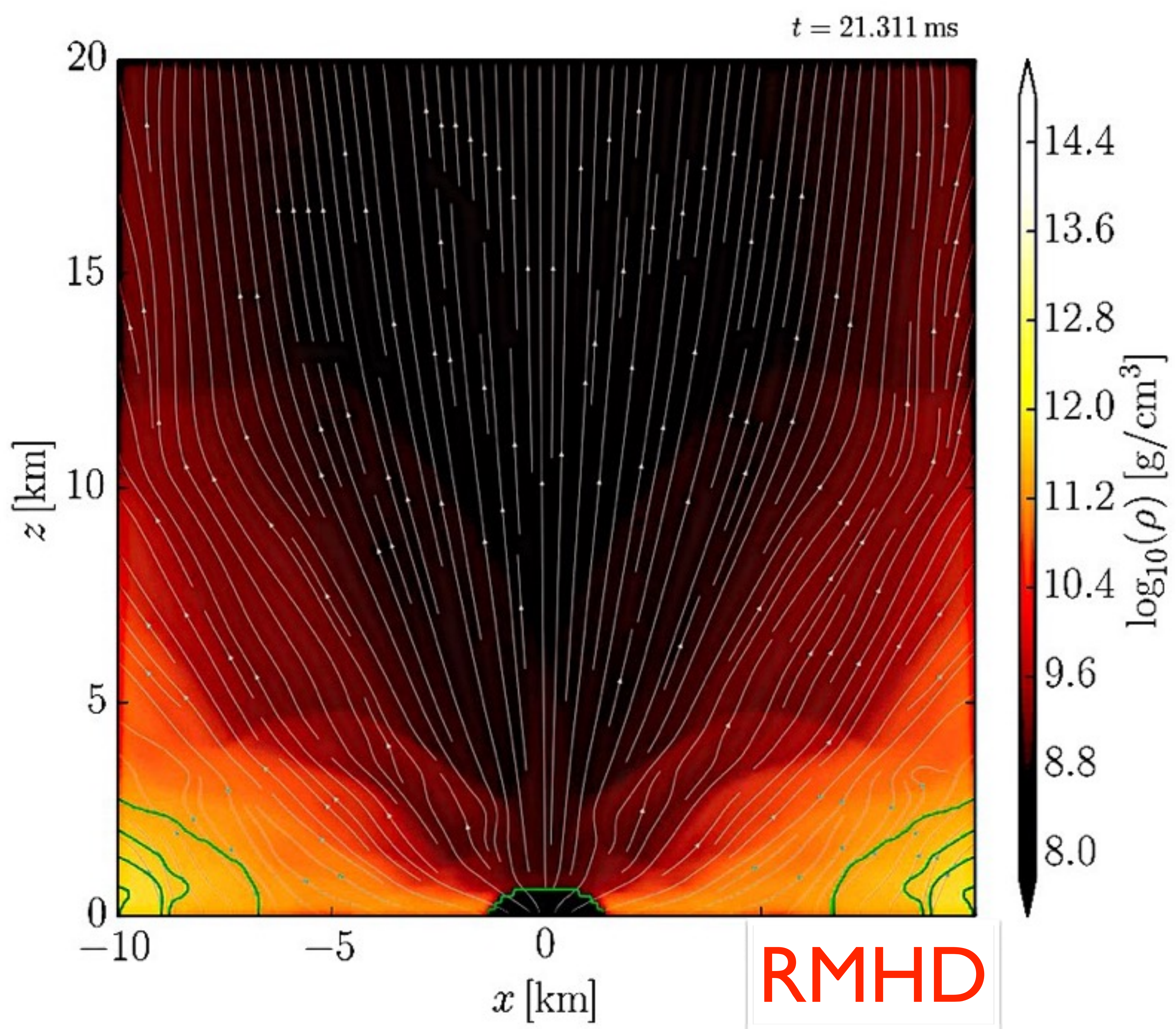
NOTE: the **magnetic jet structure** is **not** an **outflow** It's a plasma-confining structure.

In **IMHD** the magnetic jet structure is present but less regular.

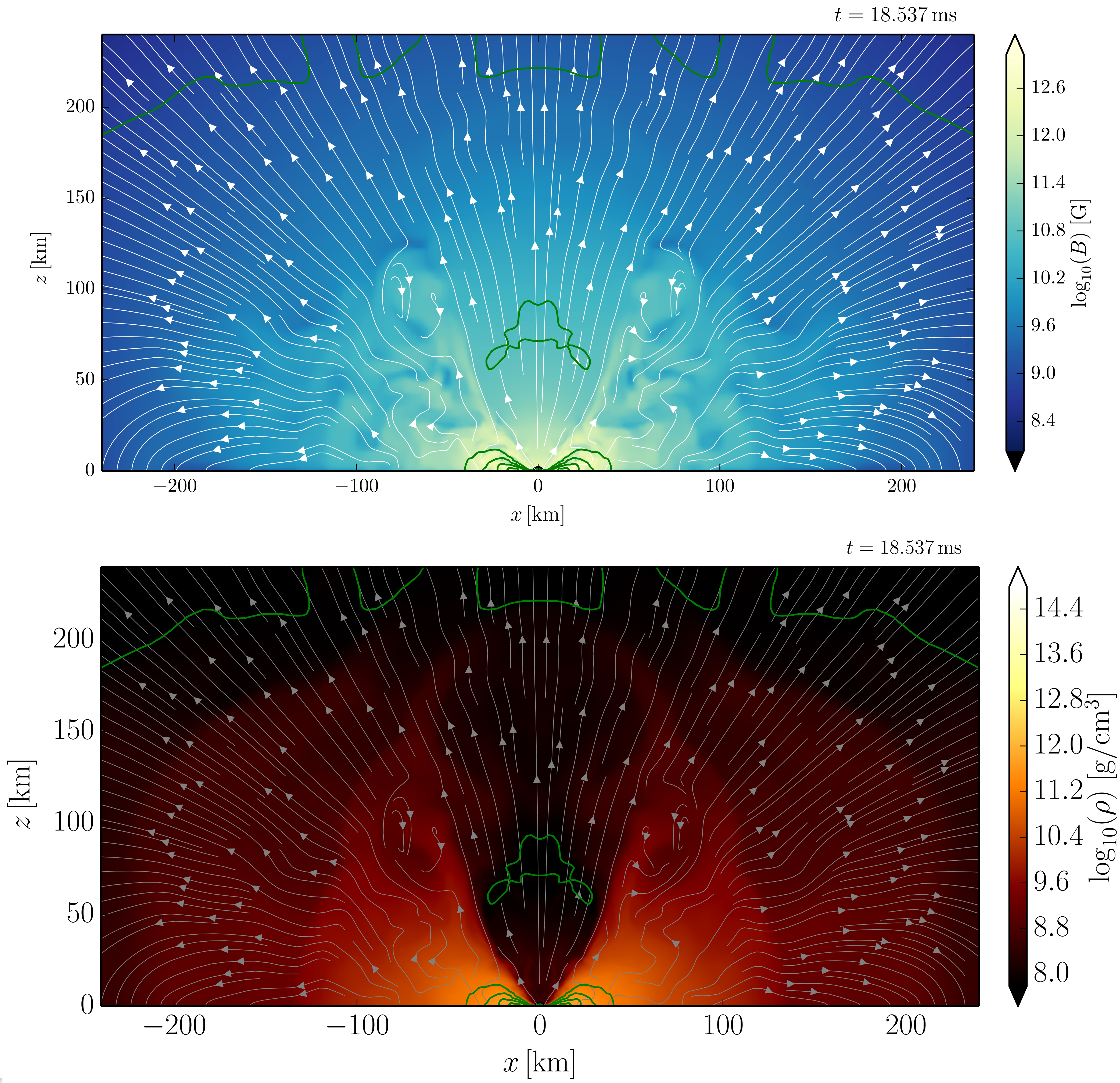


NOTE: the **magnetic jet structure** is **not** an **outflow**. It's a plasma-confining structure.

In **RMHD** the magnetic jet structure is present from the scale of the horizon (res.: h ~150m).



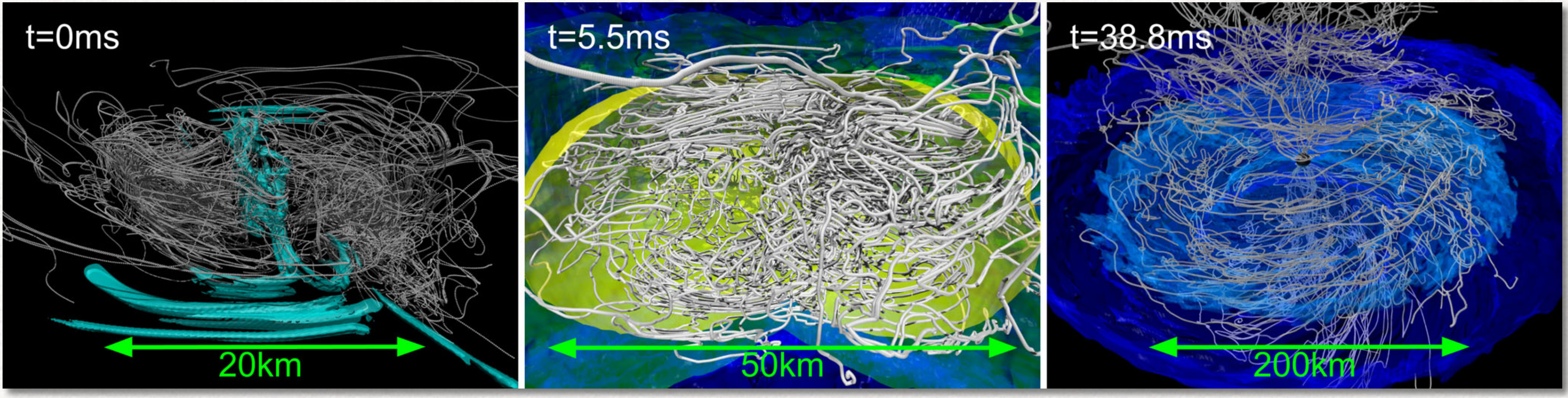
The magnetic jet structure maintains its coherence up to the largest scale of the system.



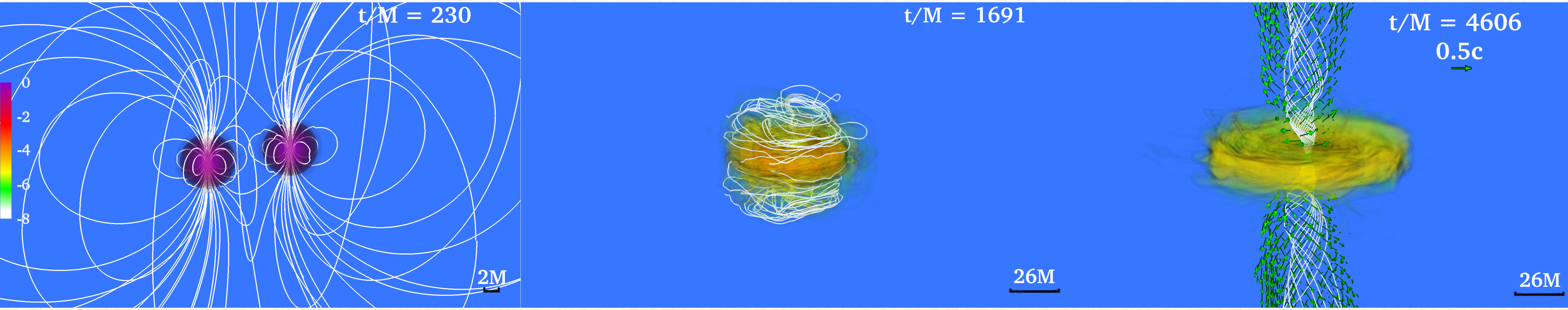
RMHD

Results from other groups (IMHD only)

With due differences, other groups confirm this picture.

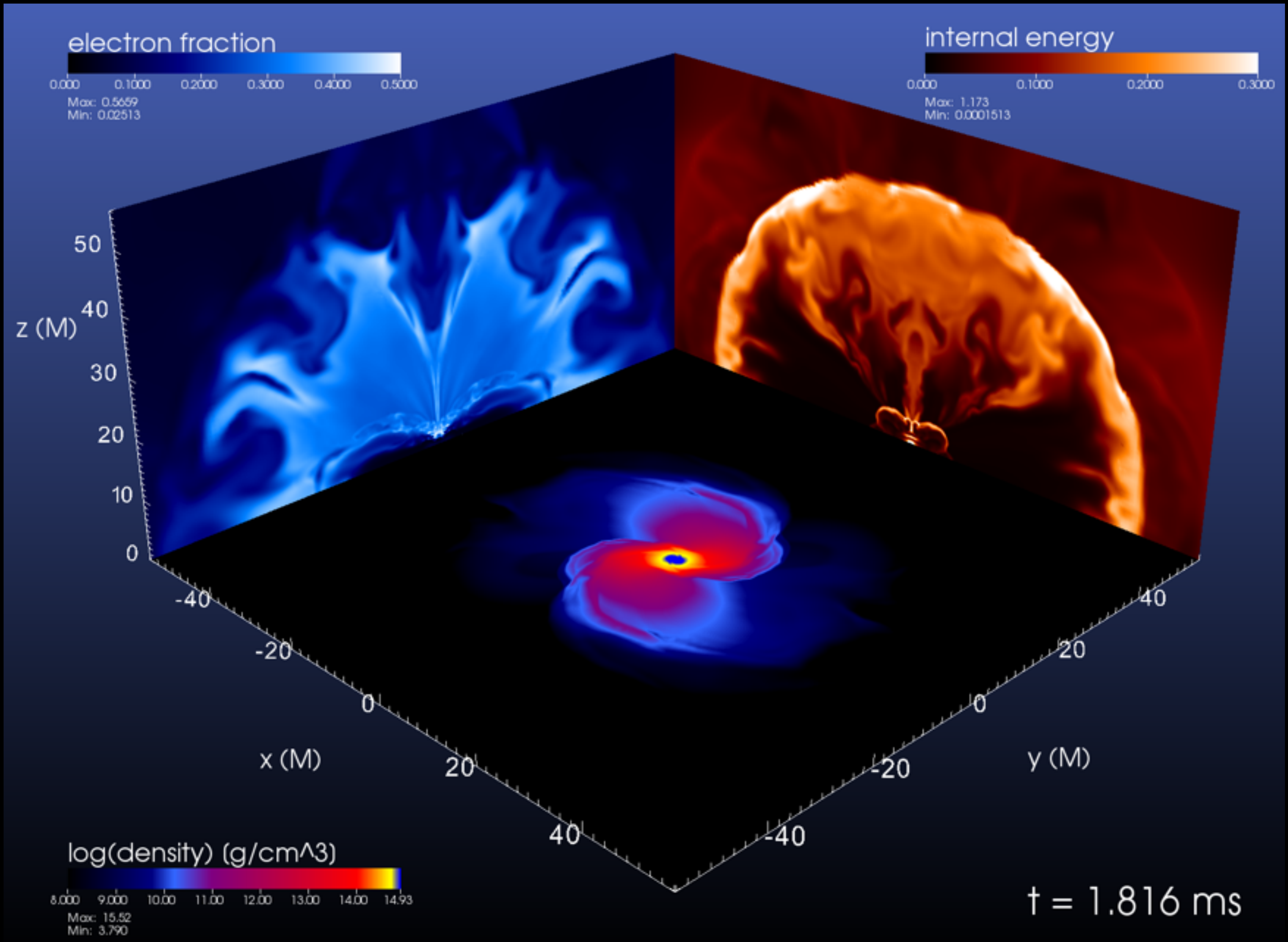


Kiuchi+ 2014

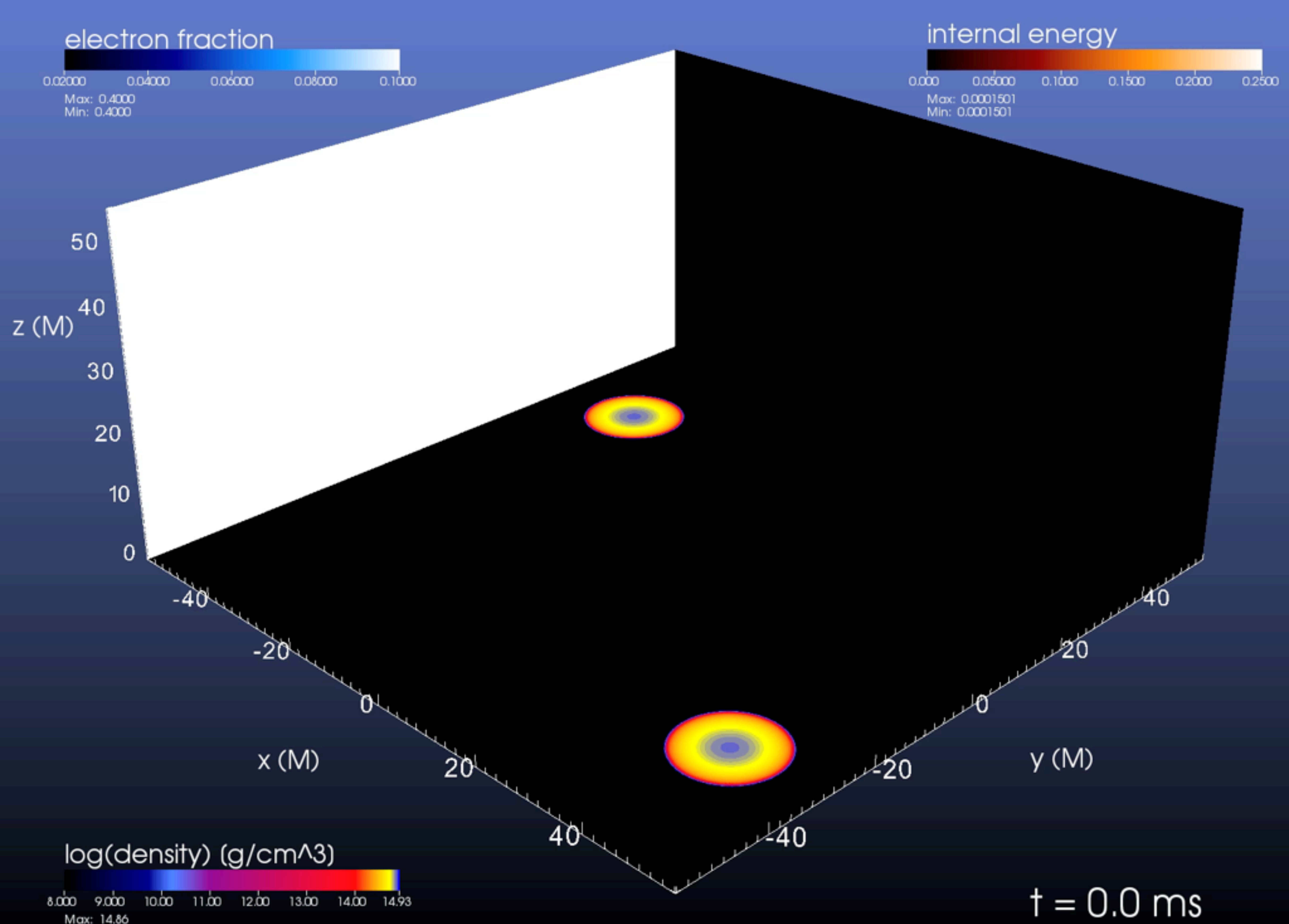


Ruiz+ 2016

Dynamically captured binaries

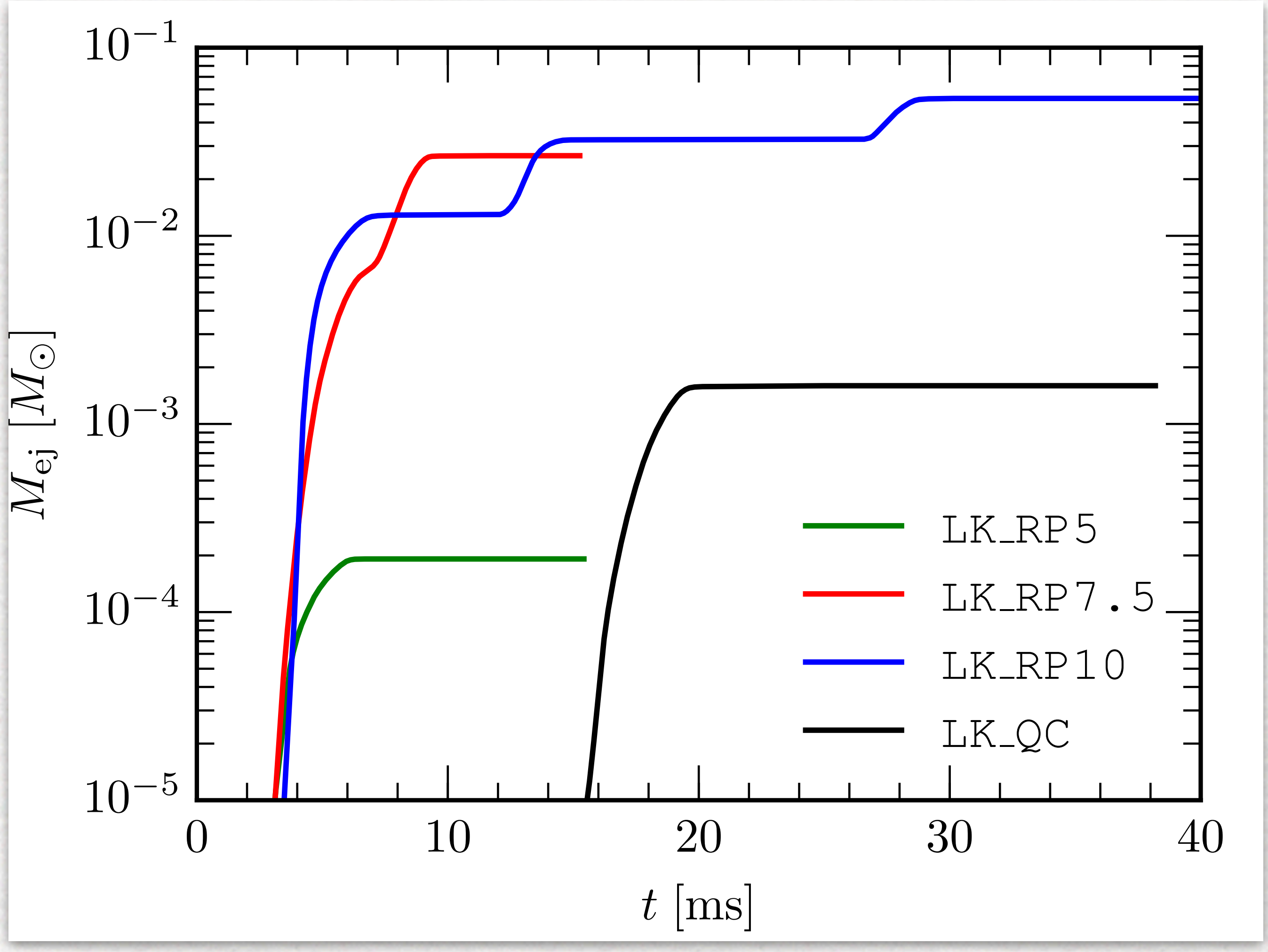


- High-eccentricity mergers can occur in dense stellar environments, e.g., globular clusters (GCs).
- About 10% of all SGRBs show significant offsets from the bulge of their host galaxies.
- Offsets could be due to kicks imparted to the binaries, or to binaries being in GCs around host galaxy.

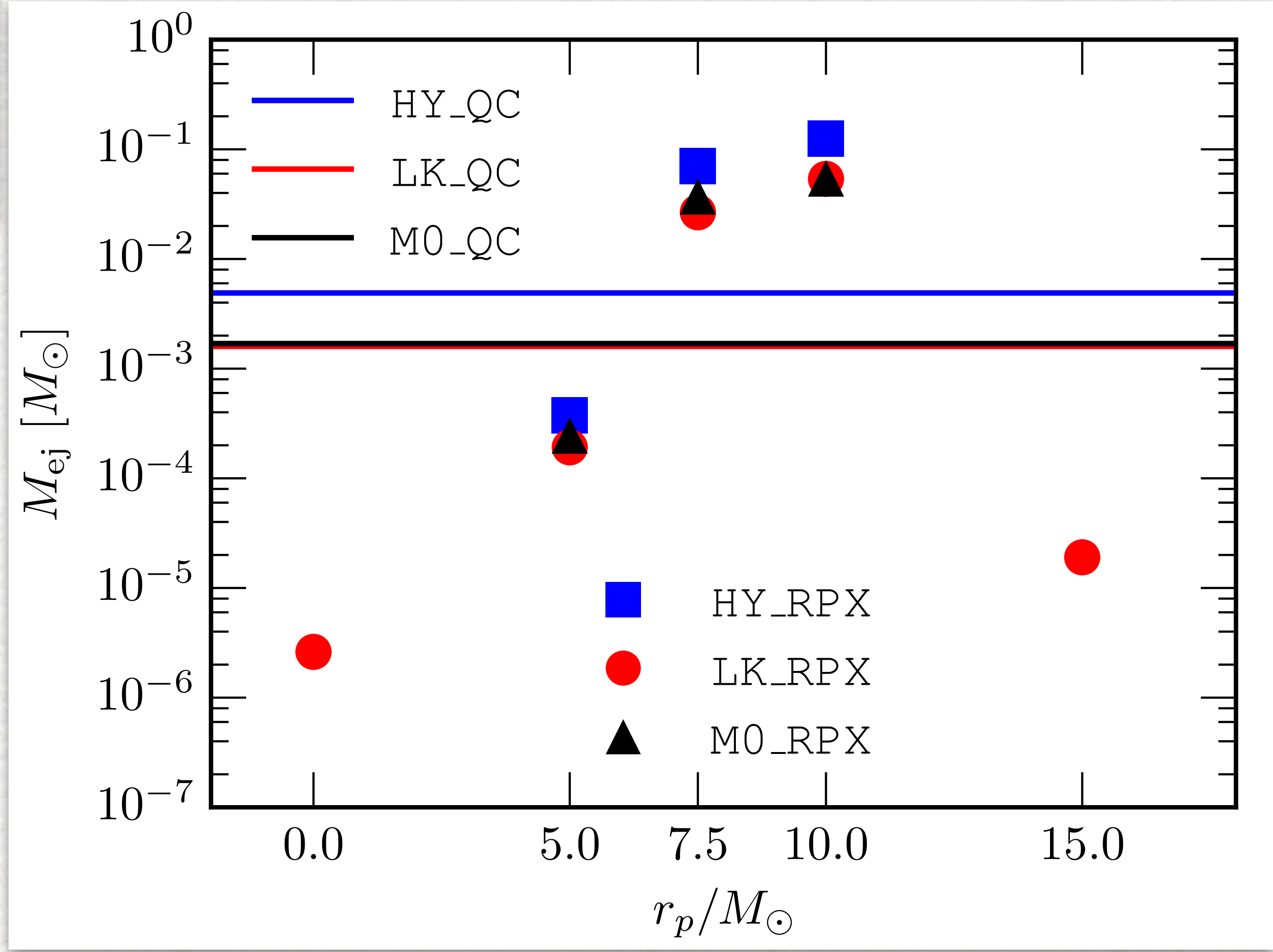


animations by J. Papenfort, L. Bovard, LR

Mass ejection

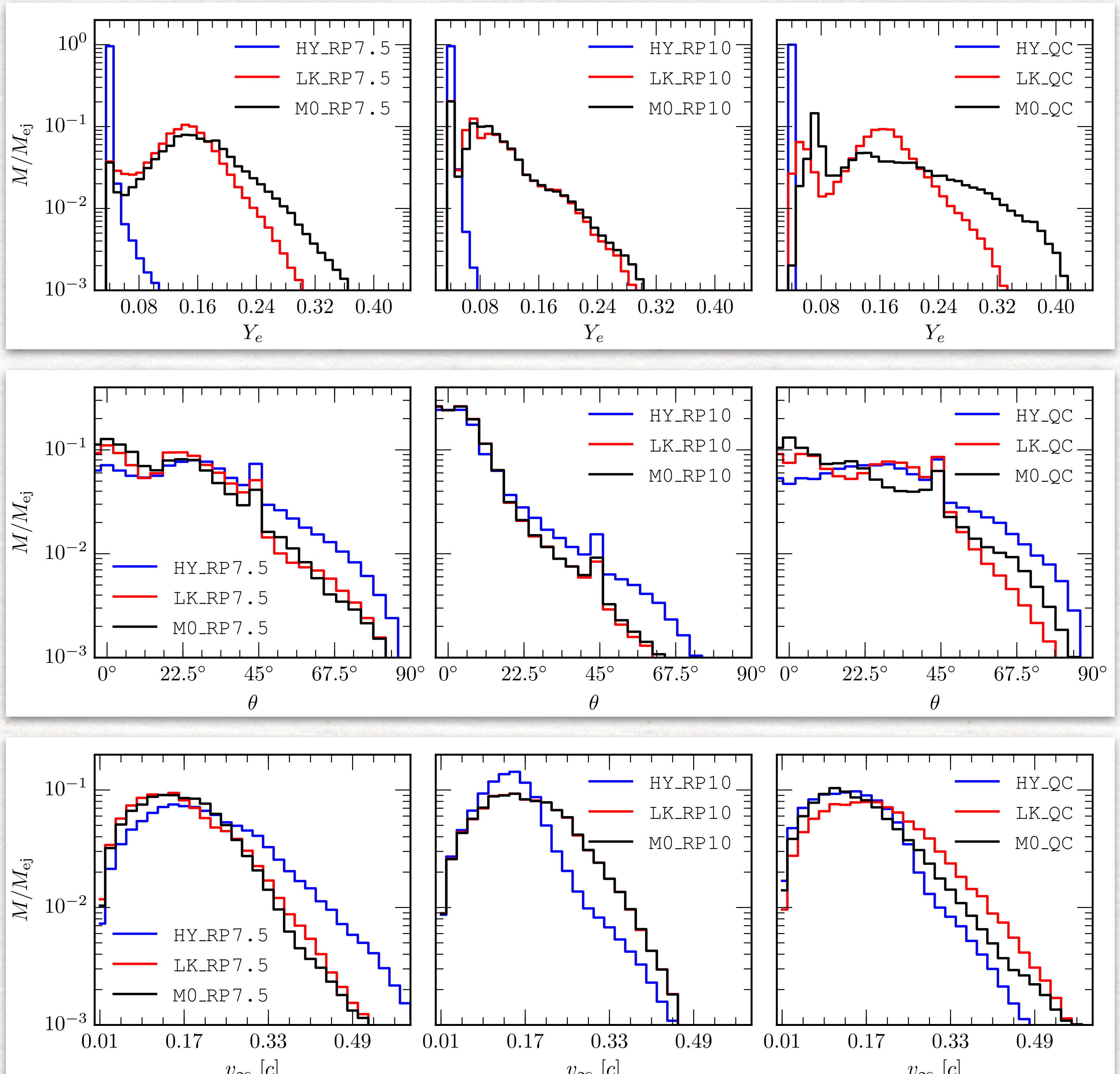


- Mass ejected depends on impact parameter and takes place at each encounter.
- Quasi-circular binaries have smaller ejected masses (1-2 orders of magnitude)



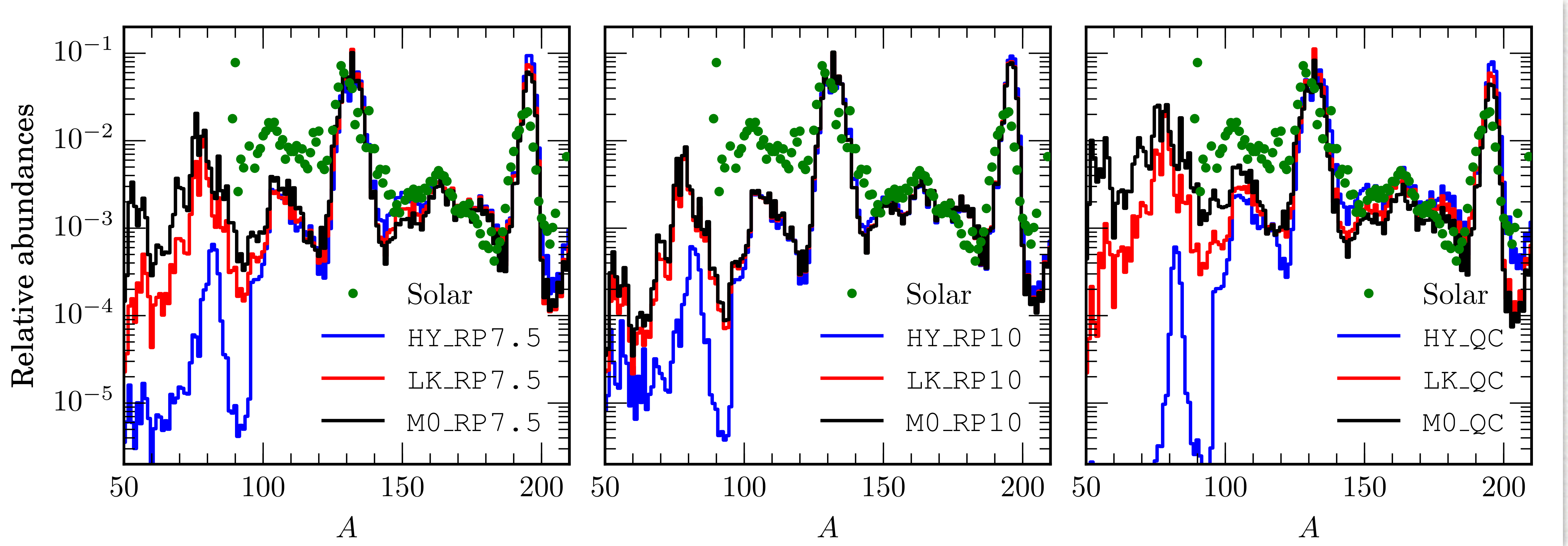
- Mass ejected depends on whether neutrino losses are taken into account (less ejected mass if neutrinos are taken into account)

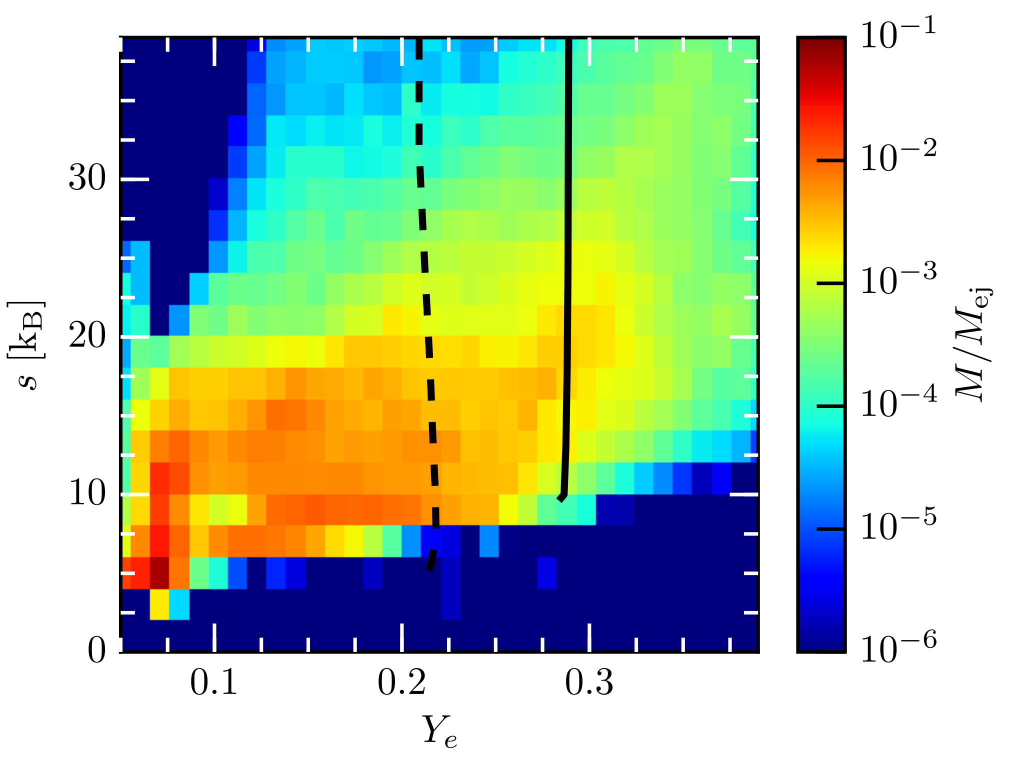
Distributions in electron fraction, entropy, velocity



- Broad distribution in Ye when neutrino losses are taken into account
- Mass ejected at all latitudes but predominantly at low elevations
- Broad distribution in *asymptotic* velocities independent of initial conditions

Nucleosynthesis





- Final abundances in the ejecta after synthesis of nuclear-reaction network
- Abundance pattern for **A>120** is robust and good agreement with solar
- Correlation entropy and Ye allows to distinguish 2nd and 3rd peak material

Conclusions

✴Modelling of binary NSs in full GR is **mature**: GWs from the inspiral can be computed with precision of binary BHs

✴Spectra of post-merger shows clear peaks: cf lines for stellar atmospheres. Some peaks are **"quasi-universal"**

✴If observed, post-merger signal will set tight constraints on EOS

✴Magnetic fields unlikely to be detected during the inspiral but **important** after the merger: instabilities and EM counterparts

✴ **Eccentric** binaries alternative to quasi-circular ones. GW signal is more complex, but ejected matter is much larger (factor 10-100) and "high-*A*" nucleosynthesis matches the observations.