The detector response simulation for the CBM Silicon Tracking System as a tool for **hit error estimation**

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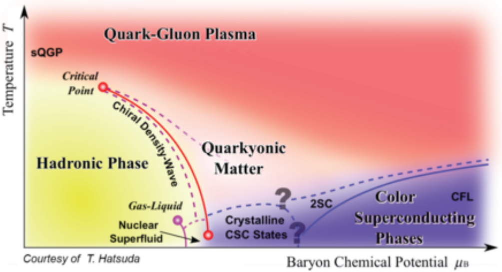
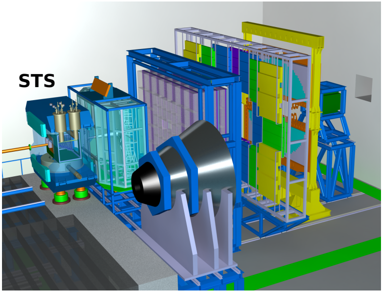
DPG Spring Meeting, Darmstadt, 14 - 18 March 2016

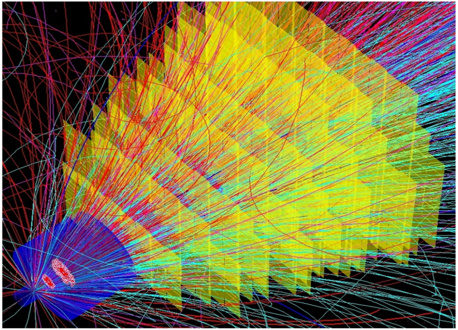
Compressed Baryonic Matter experiment @ FAIR





- QCD-diagram at moderate temperature and high density;
- Au + Au SIS100: 2-11 AGeV, $10^5 - 10^7$ interactions/s;
- measure rare physical probes;
- up to 1000 charged particles per central collision.

Silicon Tracking System (STS)



STS is a main tracking detector system:

- high efficiency;
- fast: hit rates up to $20\ \mathrm{MHz/cm^2}$;
- radiation hard: $10^{14}\ \mathrm{n_{eq}/cm^2}$;
- low mass: material budget per station $\sim 1\%\ \mathrm{X_0}$.

- 8 tracking stations downstream of the target, in a $1\ \mathrm{T}$ dipole magnet;
- double-sided micro-strip Si sensor: $\sim 300\ \mu m$ thickness, $58\ \mu m$ strip pitch, 7.5^0 stereo-angle;
- self-triggered read-out electronics.

Motivation

Why care: A reliable estimate of the hit position error $\Rightarrow$
get proper track χ^2 $\Rightarrow$
discard ghost track candidates $\Rightarrow$
improve the signal-to-background ratio and
keep the efficiency high.

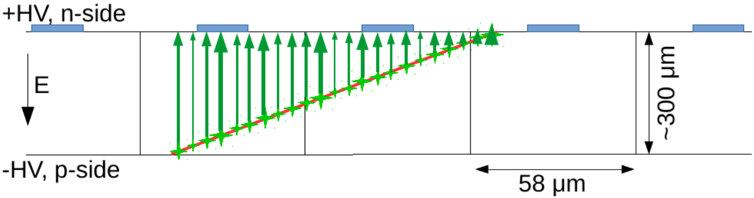
Method: Calculations from first principles and independent of:
measured spatial resolution;
simulated residuals.

Verification: Pull distribution: $\mathrm{pull} = \frac{\mathrm{residual}}{\mathrm{error}}$:
width must be ≈ 1;
shape must reproduce the shape of the residuals distribution.

Needs:
reliable detector response model;
good cluster position finding algorithm (CPFA).

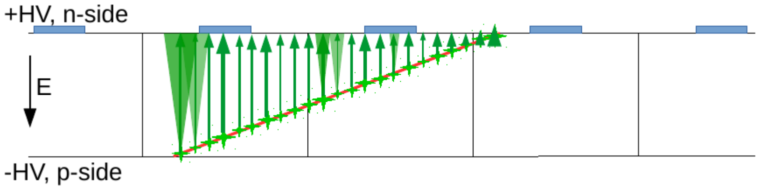
Detector responce model

- Divide trajectory into $3\ \mu m$-steps;
- Calculate energy loss in each using Urban method;



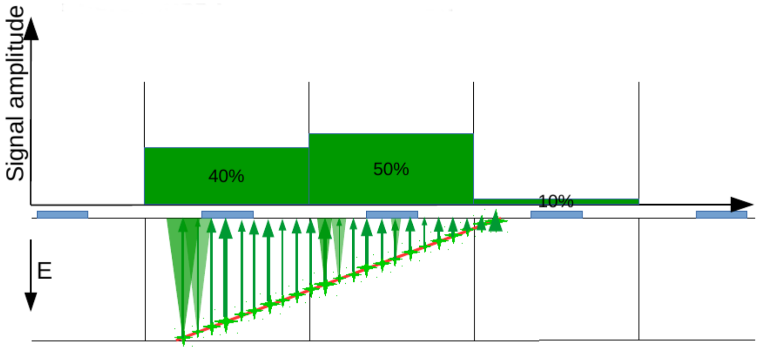
Detector responce model

- Charge carriers drift in uniform electric field;
- And diffuse in perpendicular direction;



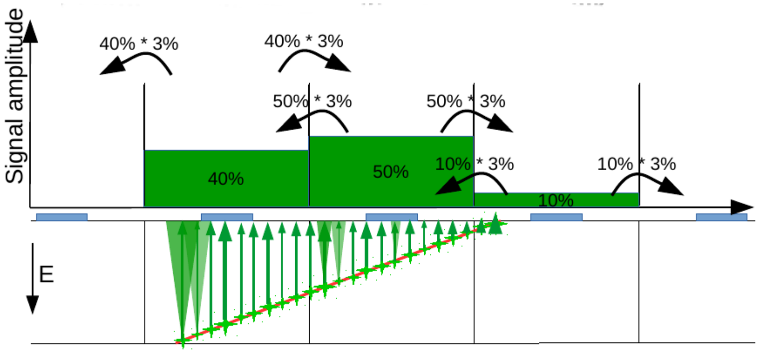
Detector responce model

- Register charge at both read-out planes;



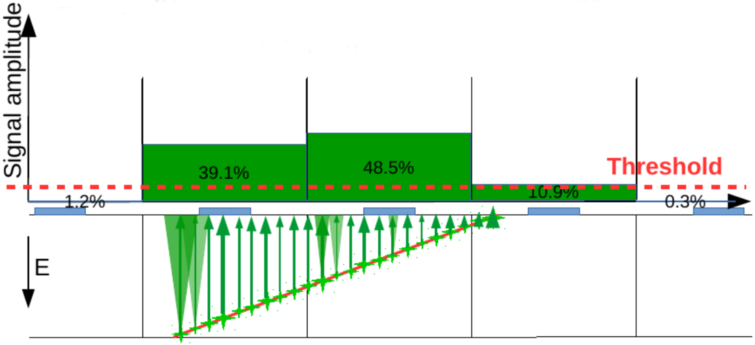
Detector responce model

- Cross-talk due to interstrip capacitance;



Detector responce model

- Noise (not shown);
- Apply threshold.



Unbiased cluster position finding algorithm (CPFA)

Center-Of-Gravity (COG):

$$x_{\rm rec} = \frac{\Sigma x_i q_i}{\Sigma q_i}$$

x_i – the coordinate of ith strip,
q_i – its charge,
$i = 1..n$ – the strip index in the n-strip cluster.

COG is biased: $\langle x_{\rm true} - x_{\rm rec}\rangle \equiv \langle \Delta x\rangle \neq 0$ for $n \geq 2$.

An **unbiased CPFA**:
2-strip clusters:

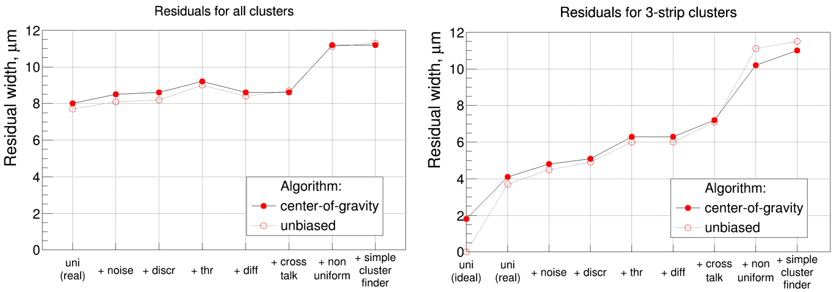
$$x_{\rm rec} = 0.5\,(x_1 + x_2) + \frac{p}{3}\frac{q_2 - q_1}{\max(q_1, q_2)}, \quad p \text{ – strip pitch;}$$

n-strip clusters (head-tail algorithm[1]):

$$x_{\rm rec} = 0.5\,(x_1 + x_n) + \frac{p}{2}\frac{q_n - q_1}{q}, \quad q = \frac{1}{n-2}\sum_{i=2}^{n-1} q_i$$

.
[1] R. Turchetta, "Spatial resolution of silicon microstrip detectors", 1993

Residuals comparison for 2 CPFAs



"Uni" – unifrom energy loss & ideal detector.
"Uni (real)" suffers from the trajectory discretization in Geant ⇒ artificial increase of the residuals.

- The unbiased algorithm yields $x_{\mathrm{rec}} = x_{\mathrm{true}}$ in case of a perfect detector;
- Non-ideal effects make the performance comparible;
- The unbiased CPFA has a computational advantage of the hit position error estimation.

Estimation of hit position error

Hit position error:

$$\sigma^2 = \sigma_{\rm alg}^2 + \sum_i \left(\frac{\partial x_{\rm rec}}{\partial q_i}\right)^2 \sum_{\rm sources} \sigma_j^2,$$

$\sigma_{\rm alg}$ – an error of the unbiased CPFA:

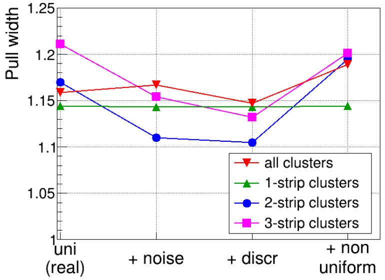
$$\sigma_1 = \frac{p}{\sqrt{24}}, \qquad \sigma_2 = \frac{p}{\sqrt{72}} \frac{|q_2 - q_1|}{\max(q_1, q_2)}, \qquad \sigma_{n>2} = 0.$$

σ_j – errors of the charge registration at one strip, among them already included:

- $\sigma_{\rm noise} = \text{Equivalent Noise Charge};$
- $\sigma_{\rm discr} = \dfrac{\text{dynamic range}}{\sqrt{12}\ \text{number of ADC}};$
- $\sigma_{\rm non-uni}$ is estimated assuming:
  - registered charge corresponds to the most probable value of the energy loss;
  - incident particle is ultrarelativistic ($\beta\gamma \gtrsim 100$).

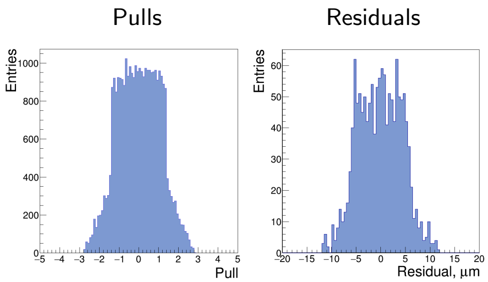
Verification: pull distribution

Width



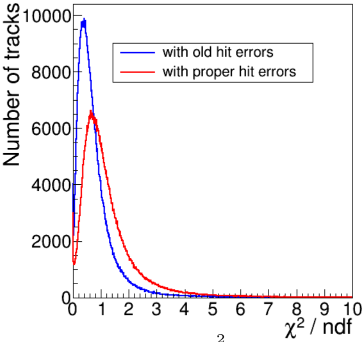
Trajectory discretization in Geant $\Rightarrow$
$\Rightarrow$ artificial increase of residuals $\Rightarrow$
$\Rightarrow$ pull width $\gtrsim 1$.

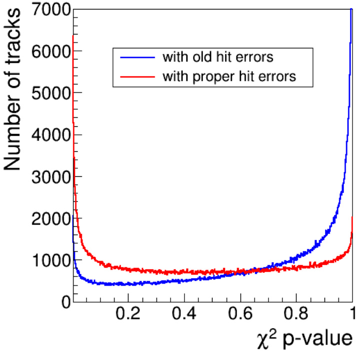
Shape



- ideal detector;
- 2-strip clusters;
- residuals at fixed: $\dfrac{|q_2 - q_1|}{\max(q_1, q_2)}$.

Verification: track fit performance





- p-value $= 1 - \int_{-\infty}^{\chi^2} f_{\chi^2}(t)dt = 1 - \mathrm{CDF}$ (cumulative distribution function) – must be flat;
- with new error estimate p-value distribution is flat.

Conclusions

- The unbiased **cluster position finding algorithm** was implemented for the STS:
  - it gives similar residuals as the Center-Of-Gravity algorithm;
  - and simplifies error estimation.
- Developed method of **hit position error estimation** yields correct errors, that was verified with:
  - correct pulls (width and shape);
  - expected χ^2/ndf distribution;
  - flat p-value (prob) distribution.

Thank you for your attention!

Error due to non-uniform energy loss

The contribution from the non-uniformity of energy loss is more difficult to take into account because the actual energy deposit along the track is not known. The following approximations allow a straightforward solution:

- the registered charge corresponds to the most probable value (MPV) of energy loss;
- the incident particle is ultrarelativistic ($\beta\gamma \gtrsim 100$).

The second assumption is very strong but it uniquely relates the MPV and the distribution width (Particle Data Group)

$$MPV = \xi[\mathrm{eV}] \times \left(\ln\left(1.057 \times 10^6 \xi[\mathrm{eV}]\right) + 0.2 \right).$$

Solving this with respect to ξ gives the estimate for the FWHM (S. Merolli, D. Passeri and L. Servoli, Journal of Instrumentation, Volume 6, 2011)

$$\sigma_{\mathrm{non}} = w/2 = 4.018\xi/2.$$

1-strip clusters: why not $\sigma_{method} = p/\sqrt{12}$?

In general, for **all** track inclinations:

- $N = \int\limits_{x_{in}} \int\limits_{x_{out}} P_1(x_{in}, x_{out}) dx_{in} dx_{out} = p^2;$
- $\sigma^2 = \frac{1}{N} \int\limits_{x_{in}} \int\limits_{x_{out}} P_1(x_{in}, x_{out}) dx_{in} dx_{out} \Delta x^2 = \frac{p^2}{24}.$

Particullary, for **perpendicular** tracks: $x_{in} = x_{out}$

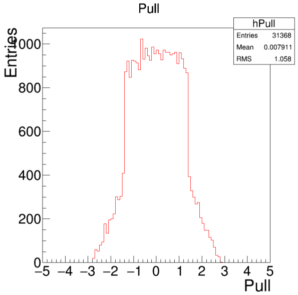
- $N = \int\limits_{x_{in}} P_1(x_{in}, x_{out}) dx_{in} = p;$
- $\sigma^2 = \frac{1}{N} \int\limits_{x_{in}} P_1(x_{in}, x_{out}) dx_{in} \Delta x^2 = \frac{p^2}{12}$

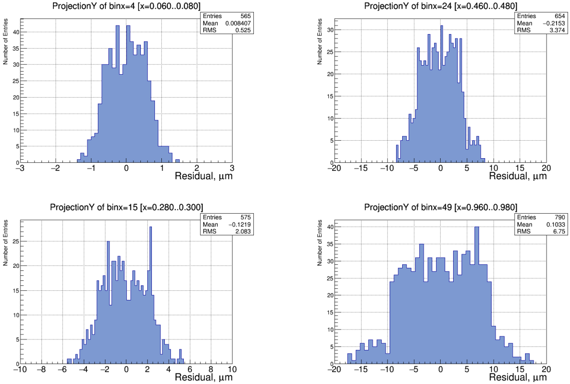
Error verification

How we can be sure, that we estimate errors correclty?

Shape of pulls distribution must reproduce shape of residuals.

We compare 2-strip cluster pulls and residuals at **fixed** q_i: $\dfrac{|q_2 - q_1|}{max(q_1, q_2)}$

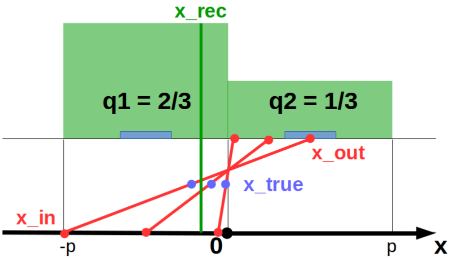


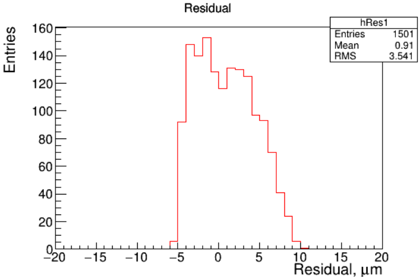


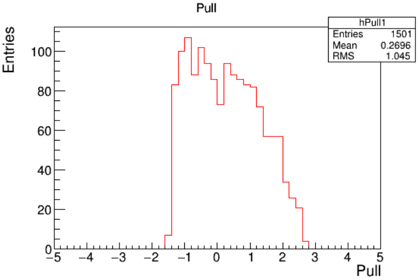
Error verification: Example

Consider:

- 2-strip clusters;
- fixed: $q_1 = 2/3$, $q_2 = 1/3$.







General ideas for error calculation

Reconstructed position: $x_{reconstructed} \equiv x_{rec} = f(q_i) \neq x_{true}$,
where q_i – charge at strip i in cluster.

- Error has 2 terms: $\sigma = \sqrt{\sigma^2_{method} + \sigma^2_{measurements}}$;
- σ_{method} comes from non-ideal algorithm of position finding;
- calculation of σ_{method} should be done for each algorithm separately using some assumptions for distributions of track parameters;
- for example, x_{in} and x_{out} coordinates define the track exactly;
- $\sigma_{measurements} \equiv \sigma_{meas}$ comes from non-ideal measurements of q_i;
- σ_{meas} is easy to define: $\sigma_{meas} = \sqrt{\sum \left(\frac{\partial x_{rec}}{\partial q_i} \Delta q_i \right)^2}$.

Measurement error: General ideas

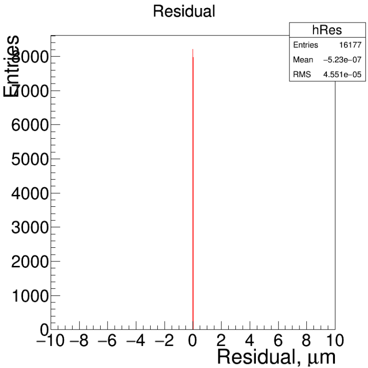
- $\sigma_{meas} = \sqrt{\sum \left(\frac{\partial x_{rec}}{\partial q_i} \Delta q_i \right)^2}$;
- Error of charge measurements has several terms:
  $\Delta q_i^2 = \sigma^2_{noise} + \sigma^2_{discretization} + \sigma^2_{non-uniform\ energy\ loss} + ...$;
- $\sigma_{noise} = ENC$ — Equivalent Noise Charge;
- $\sigma_{discretization} \equiv \sigma_{discr} = \frac{dynamic\ range}{\sqrt{12}\ number\ of\ ADC}$;
- $\sigma_{non-uniform\ energy\ loss} \equiv \sigma_{non}$ — will be explained later;
- In progress estimation of:
  - $\sigma_{\text{threshold}}$,
  - $\sigma_{\text{cross-talk}}$,
  - $\sigma_{\text{Lorentz shift}}$.

Residuals for 3-strip clusters with more details

Use cbmroot for: track simulation and determination x_{in} and x_{out}.

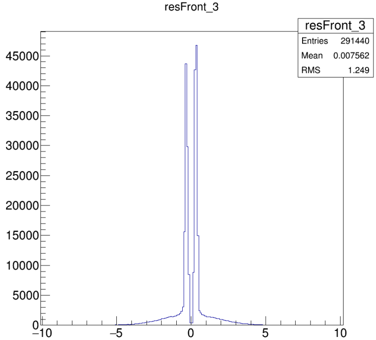
I case

Project trajectory to the read-out plane.
$x_{true} = (x_{in} + x_{out})/2$ – in the middle of **trajectory**.

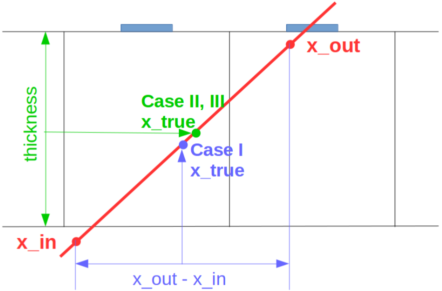


II case

Project trajectory to the read-out plane.
x_{true} – get from hit coordinate – in the middle of **sensor thickness**.



Residuals for 3-strip clusters: case I vs case II



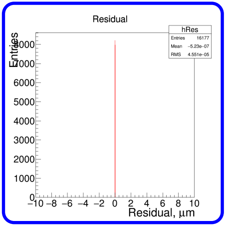
- Finite step-size in Geant;

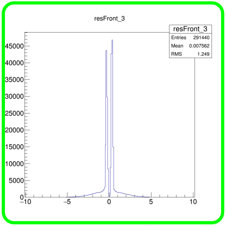
  $\Downarrow$

- x_{in} and x_{out} are not at edges of sensor (can be till $\pm 10\mu m$);

  $\Downarrow$

- middle of trajectory $\neq$ middle of sensor thickness!

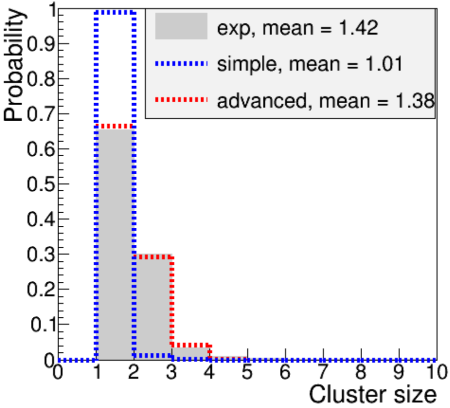




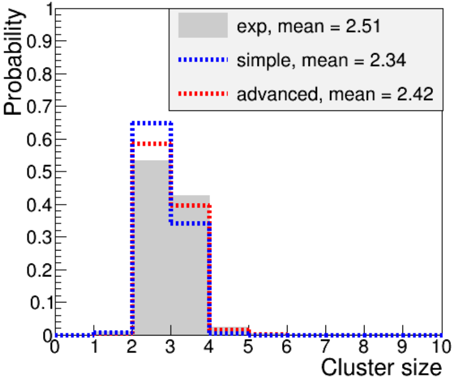
Digitizer VS experimental data from beamtime @ COSY

Experiment: 2 GeV protons.

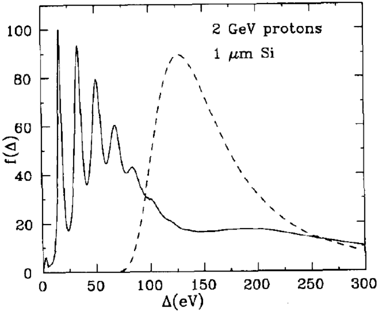
Cluster size distribution 0^0



Cluster size distribution 20^0



Energy loss process



The energy loss spectrum for 2 GeV protons transversing a Si absorber of thickness $1\mu m$ as calculated by [Bichsel,1990] (solid line): only inelastic interactions with plasmons and electrons in the detector material are included (no bremsstrahlung, no nuclear interactions).
The function $f(\Delta)$ extends to a maximum value $\Delta_M = 9\ MeV$. The separate peaks at 17, 34, 51 ... eV correspond to 1, 2, 3 ... plasmon excitations. The Landau function is shown as dashed line.

Energy loss models

A particle losses its energy non-uniformly along the path. Divide the track in a sensor into thin layers and calculate the deposited energy for each layer independently.

- **Landau theory**. However, it is not valid for a layer as thin as 1μm or even 10μm.
- **Urban method**. It is a Monte Carlo method that is applicable to thin layers. It is used in GEANT to compute the energy loss, when the Landau theory is not valid.
- **The photoabsorption ionization (PAI) model**.
- **"Bichsel's"-model**.

We use Urban model, because it is employed in Geant for thin layers by default and it doesn't require huge tables of cross-sections.

Urban method

Landau theory has 2 limits of validity:

- the number of collisions in which a particle loses an amount of energy close to the maximum value of the transferable energy in one collision should be small compared to the total number of collisions;
- the number of collisions in which a particle loses a small amount of energy should be large in the path length under consideration

For very thin layers second condition is violated.

Urban method assumes that the atoms have only two energy levels with binding energy E_1 and E_2. The particle-atom interaction will then be an excitation with energy loss E_1 or E_2, or an ionization with an energy loss distributed according to a function $g(E) \sim 1/E^2$

Charge carriers motion in silicon sensor

The charge carriers produced in the silicon sensor move in the electric (and magnetic) field and undergo thermal diffusion:

- in zero approximation e-h pairs move in **planar** electric field;
- the charge carriers **deviate** due to the Lorentz force by the angle:

$$tan\theta_{L,i} = \frac{\Delta x_i}{d} = \mu_i B \tag{1}$$

  where μ is the Hall mobility, Δx is the Lorentz shift, i denotes the carrier type;
- the spatial distribution of the charge carriers becomes **broader** after time t according to the Gaussian law with the standard deviation:

$$\sigma = \sqrt{2Dt}, \quad D = \frac{kT}{e}\mu \tag{2}$$

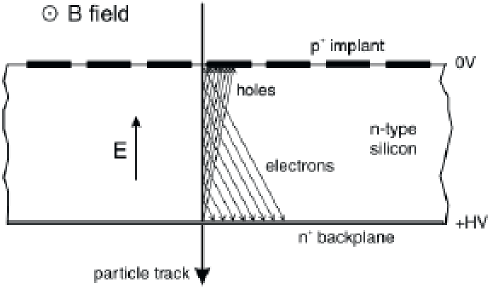
  D is the diffusion coefficient, T is the temperature, e is an elementary charge and μ is a mobility of the charge carrier.

Lorentz shift in magnetic field

If magnetic field is orthogonal to the electric field in the sensor, the charge carriers will no longer drift straight to the electrodes but will be deflected due to the Lorentz force on the angle:

$$tan\theta_{L,i} = \frac{\Delta x_i}{d} = \mu_i B \tag{3}$$

where μ – the Hall mobility, Δx – the Lorentz shift, i – denotes the carrier type (e – electrons or h – holes).



Thermal diffusion

After time t the distribution of the charge carriers becomes broader according to the Gaussian law with standard deviation:

$$\sigma = \sqrt{2Dt}, \quad D = \frac{kT}{e}\mu \tag{4}$$

D – diffusion coefficient, T – temperature, e – elementary charge and μ – mobility of the charge carrier. Since $D \sim \mu$ and the collection time $t \sim 1/\mu$, the resulting σ doesn't depend on the type of the charge carrier.

Possibilities for the electric field modelling

- To approximate the electric field with analytic expression for a field in a **planar** abrupt p-n junction:

$$E(z) = -\left(\frac{V_{bias} + V_{dep}}{d} - \frac{2z}{d^2}V_{dep}\right), \quad (5)$$

  where d – thickness of the sensor, V_{dep} – depletion voltage, V_{bias} – bias voltage. And then project produced charge onto read-out plane;

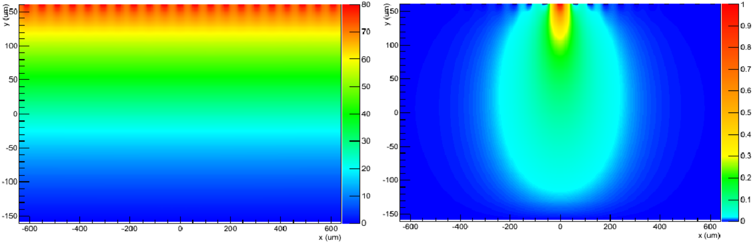
- To calculate the **detailed map** of the electric potential and to solve the equation of motion for each hole and electron: $\vec{v} = \mu\vec{E}$, where μ — mobility of the charge carrier.

Finally, to evaluate the current, induced at time t on the kth electrode by a moving carrier from the Shockley-Ramo theorem:

$$i_k(t) = -q\vec{v}\cdot\vec{E}_{wk} \quad (6)$$

where q – the charge of the carrier, $\vec{v}$ – its velocity and $\vec{E}_{wk}$ - the weighting field associated to the kth electrode which is determined by setting the electrode k to unit potential and all others to zero potential.

Weighting field



(a) Drift potential (b) Weighting potential

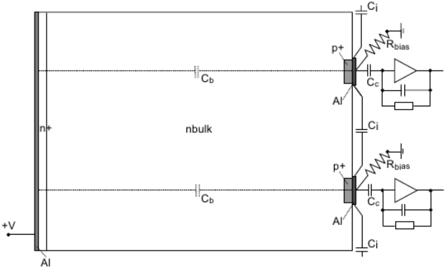
- When pre-amplifier integration time $>$ collection time of all charge carriers, the measured electrode gets only the current, induced by those charge carriers, which moving terminates on this electrode, while other electrodes get zero net current.
- STS-XYTER integration time in slow channel is $\sim 80\ ns$.
- At bias voltage of $100\ V$, depletion voltage of $60\ V$, detector thickness of $300\ \mu m$: electrons collection time $\leqslant 8\ ns$, holes collection time $\leqslant 22\ ns$.
- That is why the effect of the weighting field will be integrated out.

Effects of the read-out electronics

- **The cross-talk effect** — a redistribution of a signal among the strips due to the interstrip capacitances C_i.

$$i_{nstrip} = \frac{i_{strip} C_i}{C_b + C_c + C_i}, \qquad (7)$$

i_{strip} – the signal on the measured strip,
i_{nstrip} – the signal on one of its neighbour,
C_c – the coupling capacitance,
C_b – bulk capacitance.

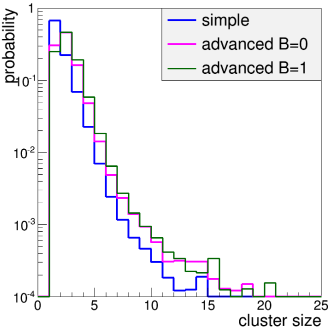


- Currently, the model is **time-independent**;
- **Random noise** on each strip can be modelled with a Gaussian with a standard deviation equals to the equivalent noise charge (ENC) of the detector system;
- **Digitization**: conversion the charge from number of electrons to ADC-value;
- A read-out **electronics threshold**.

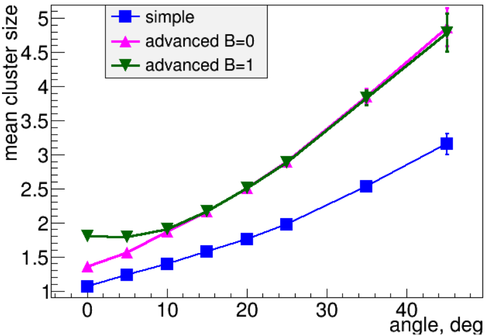
Clusters

Simulation: 500 min bias events Au+Au @ 10 GeV.

Cluster size distribution



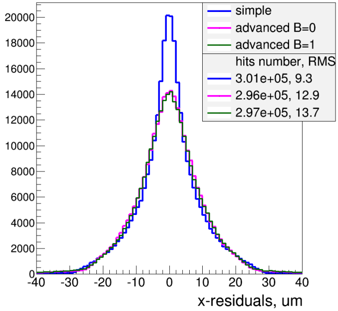
Mean cluster size VS track inclination



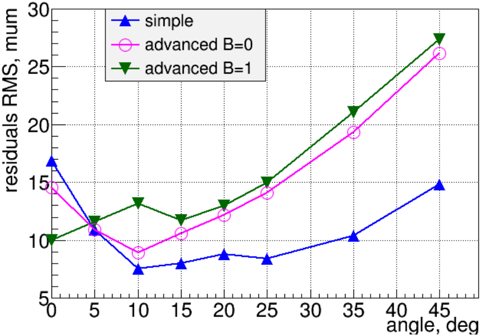
Hits

Simulation: 500 min bias events Au+Au @ 10 GeV.

X-position residuals



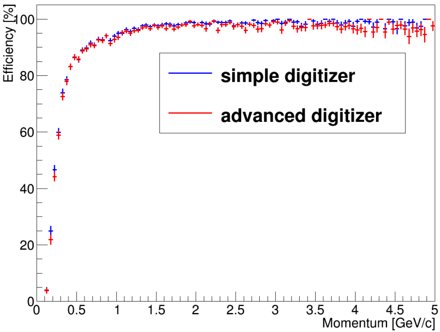
Residuals VS track inclination



Tracks

Simulation: 500 min bias events Au+Au @ 10 GeV.

Reconstruction efficiency



Momentum resolution