Extraction of moments of net-particle event-by-event fluctuations in the CBM experiment

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Outline

• Higher-order fluctuations on phase diagram
• Rate of statistical convergence of different moments
• Efficiency corrections
• GEANT simulation and reconstruction of fluctuations
• Summary
Introduction

- A future fixed target experiment at FAIR facility.
- Up to $10^7$ Au+Au collisions per second at 4-11A GeV (SIS100) and 11-35A GeV (SIS300).
- Measurement of bulk and rare probes.

Physics programme

- Equation of state at high baryonic densities
- Phase transitions at high $\mu_B$
- QCD critical point, probed by e-by-e fluctuations
- Subthreshold production of hadrons
- Hypernuclei production
Let $N$ be a random variable and $P(N)$ its probability distribution.

$k$-th moment: \[ \langle N^k \rangle = \sum_N N^k P(N) \]

Variance: \[ \sigma^2 = \langle (\Delta N)^2 \rangle = \langle (N - \langle N \rangle)^2 \rangle \]

Scaled variance: \[ \frac{\sigma^2}{\langle N \rangle} = \frac{\kappa_2}{\kappa_1} = \frac{\sigma^2}{\langle N \rangle} \quad \text{width} \]

Skewness: \[ S\sigma = \frac{\kappa_3}{\kappa_2} = \frac{\langle (\Delta N)^3 \rangle}{\sigma^2} \quad \text{asymmetry} \]

Kurtosis: \[ \kappa\sigma^2 = \frac{\kappa_4}{\kappa_2} = \frac{\langle (\Delta N)^4 \rangle - 3\langle (\Delta N)^2 \rangle^2}{\sigma^2} \quad \text{peakedness} \]

and so on...

In heavy-ion collisions $N$ can be conserved charge (baryon, electric, strangeness) or some particle number in a specific phase-space region
Fluctuations in thermodynamics

Why are fluctuations interesting?

In thermodynamics fluctuations are related to susceptibilities \( \chi^{(n)} \)

\[
\chi^{(n)} = \frac{\partial^n (p/T^4)}{\partial (\mu/T)^n}
\]

\[
\frac{\sigma^2}{M} = \frac{\chi^{(2)}}{\chi^{(1)}}, \quad S\sigma = \frac{\chi^{(3)}}{\chi^{(2)}}, \quad \kappa\sigma^2 = \frac{\chi^{(4)}}{\chi^{(2)}},
\]

Fluctuations are very sensitive to QCD equation of state and can be used to study QCD phase transitions.

Near CP \( \sim \) increasing powers of \( \xi \)

\[
\chi^{(2)} \sim \xi^2
\]

\[
\chi^{(3)} \sim \xi^{4.5}
\]

\[
\chi^{(4)} \sim \xi^7
\]

Infinite system: \( \xi \to \infty \) at CP

In HIC \( \xi \lesssim 2 - 3 \) fm
Fluctuations in $T - \mu$ plane: VDW nuclear matter

Nuclear matter as van der Waals system of nucleons

Vovchenko et al., PRC 91, 064314 (2015) and PRC 92, 054901 (2015)
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Beam energy dependence

Can be measured in different acceptance windows at different energies

For small window fluctuations approach ideal gas

For large window global charge conservation plays role

Measurements should be performed in different windows

Peculiarities in energy dependence may signal criticality
Needed statistics to measure higher moments

How much statistics are needed for accurate estimation of higher moments? For a large sample of Gaussian distributed variables

\[
\Delta(S\sigma) = \sqrt{\frac{6\sigma^2}{n}}, \quad \Delta(\kappa\sigma^2) = \sqrt{\frac{24\sigma^4}{n}}, \quad \Delta(\kappa_6/\kappa_2) = \sqrt{\frac{720\sigma^8}{n}}.
\]


Simulation result
Monte Carlo simulation: Poisson statistics

$10^{12}$ Poisson-distributed numbers with $\bar{N} = 5$

Expected values: $\sigma^2 / M = S\sigma = \kappa\sigma^2 = \kappa_6 / \kappa_2 = 1$
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Higher fluctuation moments require higher statistics.
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Higher fluctuation moments require higher statistics.
Monte Carlo simulation: Poisson statistics

$10^{12}$ Poisson-distributed numbers with $\tilde{N} = 15$

Expected values: $\sigma^2/M = S\sigma = \kappa \sigma^2 = \kappa_6/\kappa_2 = 1$

Statistical error grows with $\tilde{N}$.
Convergence rate will depend on kinematic window.
Monte Carlo simulation: Poisson statistics

$10^{12}$ Poisson-distributed numbers with $\bar{N} = 40$

Expected values: $\sigma^2/M = S\sigma = \kappa\sigma^2 = \kappa_6/\kappa_2 = 1$

Statistical error grows with $\bar{N}$.

Convergence rate will depend on kinematic window.
Efficiency corrections

Since not all particles are reconstructed and identified, the efficiency corrections are needed.

The simplest one is the binomial correction.

Binomial correction assumptions

- Detection of all particles is independent of each other.
- Probability to register particle is binomial.
- Only a single efficiency parameter $\varepsilon$ is needed.

Original cumulants $K_i$ reconstructed from measured $k_i$:

\[
K_1 = \frac{k_1}{\varepsilon}
\]

\[
K_2 = \frac{k_2 + (\varepsilon - 1)k_1}{\varepsilon^2}
\]

\[
K_3 = \frac{k_3 + 3(\varepsilon - 1)k_2 + (\varepsilon - 1)(\varepsilon - 2)k_1}{\varepsilon^3}
\]

\[
\quad \vdots
\]

For net-particle numbers ($N = N_+ - N_-$) more complicated correction involving factorial moments exists.
Monte Carlo simulation: Binomial efficiency

Test of efficiency correction on non-trivial (non-Poisson) fluctuations

Testing procedure

1. Take e-by-e proton yields from 5 million PHSD Au+Au events
2. Simulate detector response by performing Bernoulli trials on each proton in each event with given efficiency $\varepsilon$
3. Compare efficiency corrected cumulants with original ones

![Graph showing efficiency correction](image)
Monte Carlo simulation: Binomial efficiency

Test of efficiency correction on non-trivial (non-Poisson) fluctuations

In ideal case correction works properly for non-trivial initial fluctuations
Monte Carlo simulation: Fluctuating efficiency

In more realistic scenario efficiency is changes from event to event, e.g., due to fluctuations in number of tracks, momenta etc. Simulate efficiency fluctuations by Gaussian around \( \langle \varepsilon \rangle \) with particular \( \delta \varepsilon \):

- **Skewness**
  - 5 million PHSD central Au+Au events at 10A GeV
  - \( \sigma \) parameters:
    - \( \sigma < \langle \varepsilon \rangle = 0.80 \)
    - \( \delta \varepsilon = 0.01 \)

- **Kurtosis**
  - 5 million PHSD central Au+Au events at 10A GeV
  - \( \kappa \sigma^2 \) parameters:
    - \( \kappa \sigma^2 < \langle \varepsilon \rangle = 0.80 \)
    - \( \delta \varepsilon = 0.01 \)

Small efficiency fluctuations may destroy agreement! Especially higher moments more strongly affected.
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Simulate efficiency fluctuations by Gaussian around $\langle \varepsilon \rangle$ with particular $\delta \varepsilon$.

Skewness

Kurtosis

Small efficiency fluctuations may destroy agreement!
Especially higher moments more strongly affected.
First try with GEANT simulation

GEANT simulation
- Realistic CBM detector response
- UrQMD events put through GEANT
- Tracks and momenta rec. with STS
- Particle ID by TOF
- Implemented in KF Particle Finder

Problems
- Individual efficiency correlations e.g. track merging
- Particle misidentification
- Identification of primaries
The simple correction is not enough!
Becomes worse with increasing moments and/or kinematic window
Still preliminary, reconstruction can likely be improved
Summary

1. Fluctuations of conserved charges carry information about finer details of the equation of state and exhibit rich structures near critical point. Both NN interactions and chiral criticality may play role at SIS100/300 energies.

2. Interaction rate at CBM should be enough to measure the efficiency uncorrected moments up to sixth order.

3. The errors due to binomial correction increase with decreasing efficiency and increasing cumulant order. The validity of the correction is very sensitive to fluctuations and correlations of efficiencies of individual particles.

4. Higher moments cannot be properly corrected by the binomial correction in realistic situations. More elaborate procedure is likely needed.
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Thanks for your attention!