Phase transitions in the Early Universe

Mikko Laine

(University of Bielefeld, Germany)

Cosmological background

Master equation:

$$t_{\rm univ} \approx \frac{3}{4}\sqrt{\frac{5}{\pi^3 g_*}}\frac{m_{\sf Pl}}{T_{\rm univ}^2} \quad \Leftrightarrow \quad \frac{t_{\rm univ}}{\sf s} \approx \left(\frac{\sf MeV}{T_{\rm univ}}\right)^2 \; ,$$

where $g_* = \#$ of species and $m_{\sf Pl} = 1.2 \times 10^{19}$ GeV.

Dynamical time scales are much smaller than $t_{\rm univ}$,

$$t_{\rm dyn} \sim \frac{1}{T} \ll t_{\rm univ} \sim \frac{m_{\sf Pl}}{T^2} \; , \quad \text{e.g. } 10^{-23}\,{\sf s} \ll 10^{-5}\,{\sf s} \; .$$

So, in contrast to heavy ion collisions, the bulk of the system is in perfect thermal equilibrium and very **ideal**.

General challenges

Experimental high energy physics

$\Rightarrow$ Which is the correct theory at a given energy scale?

Quantum equilibrium statistical physics

$\Rightarrow$ Does it possess any phase transitions?

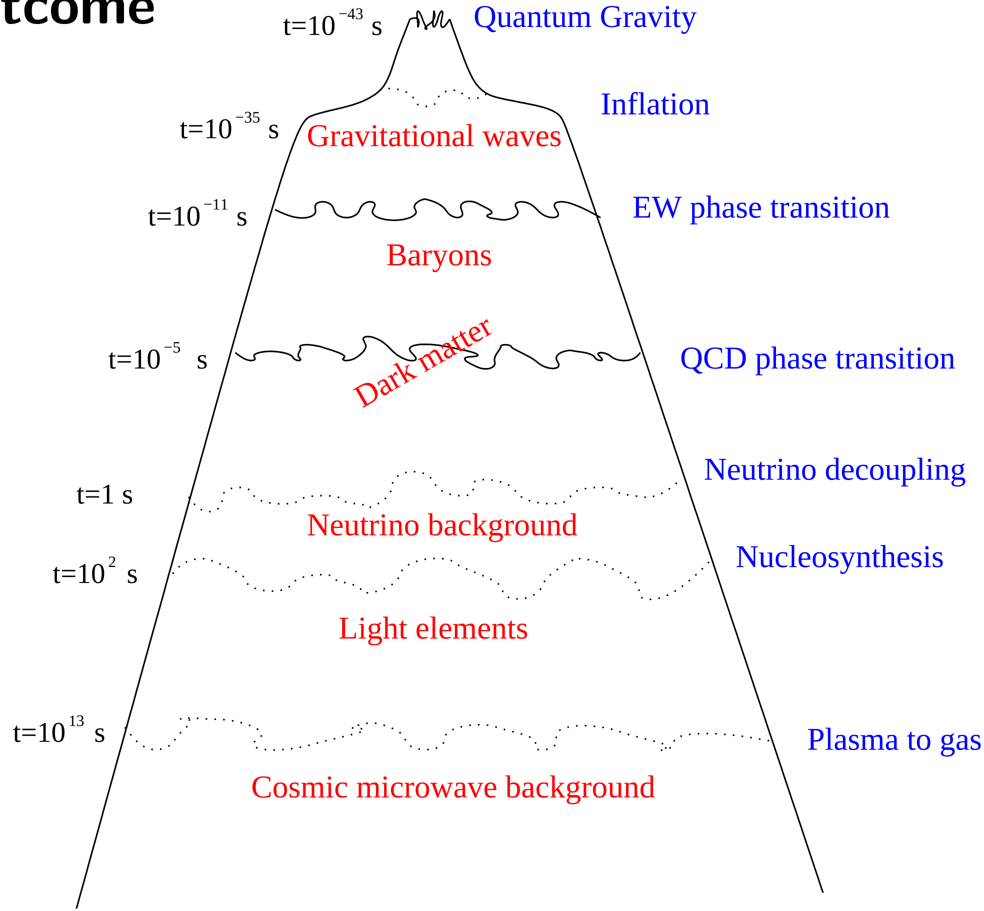
Cosmology

$\Rightarrow$ **Do they leave any signatures? (Signature $\equiv$ particle/field which is not in equilibrium.)**

Astronomy

$\Rightarrow$ Do we see the signatures?

Dream outcome



More precisely

Thermal transition	Cosmological signature?
QCD crossover	⋆ imprint on gravity background ⋆ imprint on dark matter
EW crossover in SM	⋆ imprint on baryon asymmetry
EW in MSSM and more exotic theories	⋆ gravity background ⋆ baryon asymmetry
(ISS model hep-th/0602239,...	⋆ SUSY breaking) hep-th/0610334,...
(GUT, . . .	⋆ topological defects)

QCD

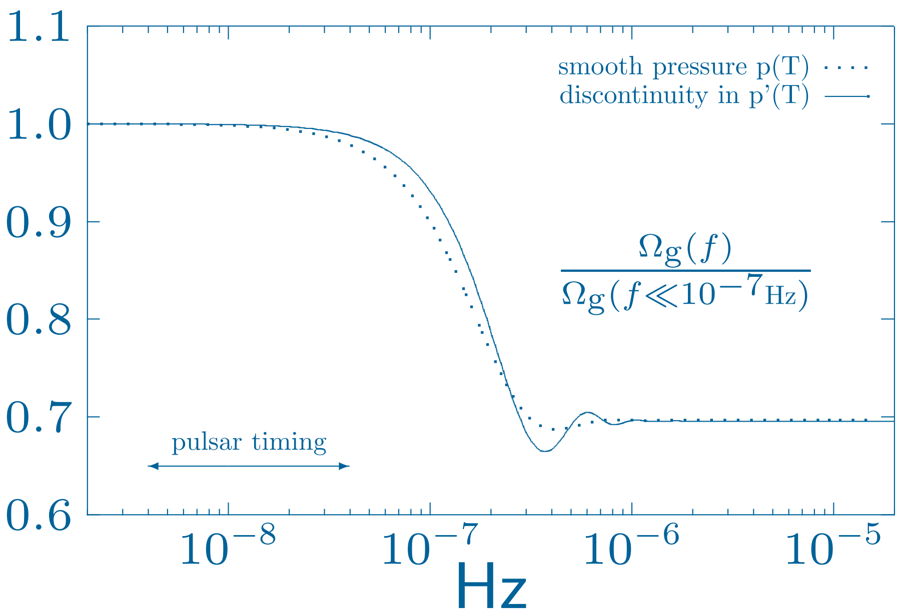
$++$ theory known

$+$ answer more or less known ($\mu_B \ll T$)

$-$ cosmological signatures feeble

QCD could leave an imprint on gravity background

Inflation generates a flat spectrum of gravitational waves, but the amplitude decreases once a mode is within the horizon ($\lambda \ll \ell_H$):



Schwarz gr-qc/9709027; Seto Yokoyama gr-qc/0305096; Boyle Steinhardt astro-ph/0512014

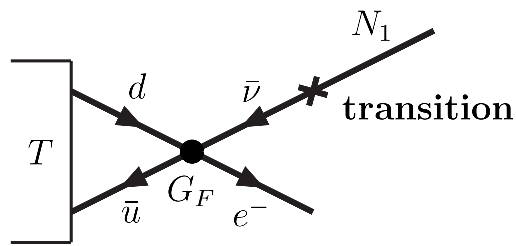
QCD has background effect on Cold Dark Matter (CDM)

Srednicki Watkins Olive NPB 310 (1988) 693; Hindmarsh Philipsen hep-ph/0501232

Effect is more significant for Warm Dark Matter (WDM)

Dodelson Widrow hep-ph/9303287; Shi Fuller Abazajian astro-ph/9810076, astro-ph/0204293

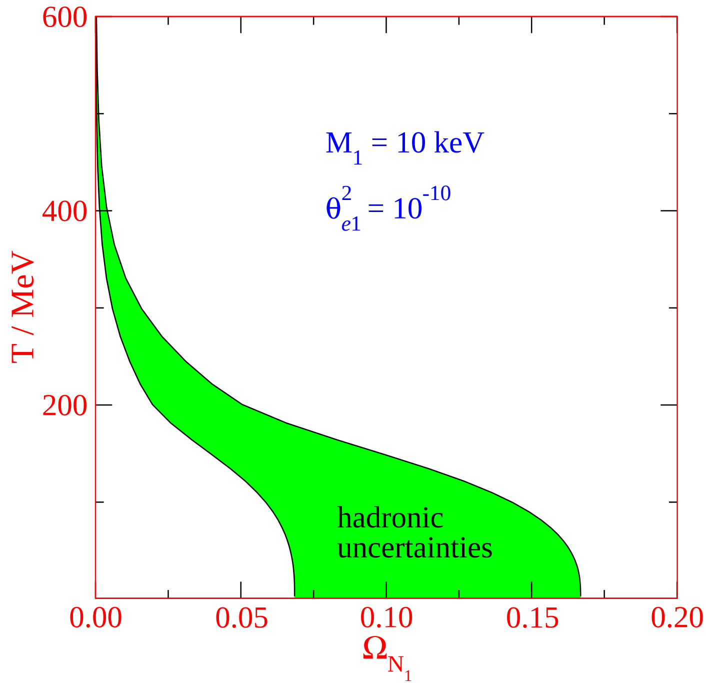
The observed neutrino masses suggest the existence of right-handed "sterile" neutrinos, but do not fix their masses M. If $M \sim 1 \ldots 50$ keV, they could act as WDM, produced through active-sterile oscillations.



Production peaks at $T \sim \left(\frac{M}{10\,G_F}\right)^{\frac{1}{3}} \sim 200$ MeV $\left(\frac{M}{1\,\mathrm{keV}}\right)^{\frac{1}{3}}$.

A concrete example:

Asaka Laine Shaposhnikov hep-ph/0612182



$\Rightarrow$ QCD plays an important role, but it is indirect: observation of $\Omega_{N_1} \neq 0$ would **not** prove the existence of any transition!

Comprehensive review: Boyarsky Ruchayskiy Shaposhnikov 0901.0011

EW

$-$ theory not known

$++$ answer known for many possibilities

$+$ cosmology could be exciting

Standard Model

Light degrees of freedom are Matsubara zero-modes of SU(2)×U(1) gauge fields and the Higgs boson.

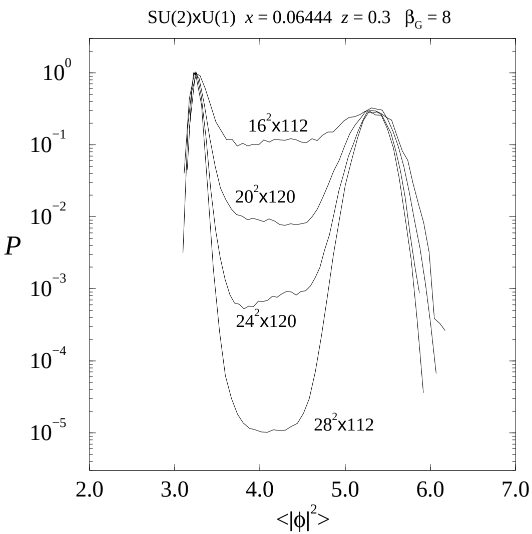
$$\mathcal{H}_{\rm cl} \; = \; \frac{1}{2}\,\mathrm{Tr}\,F_{ij}^2 + \frac{1}{4}B_{ij}^2 + (D_i\phi)^\dagger D_i\phi + m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2,$$

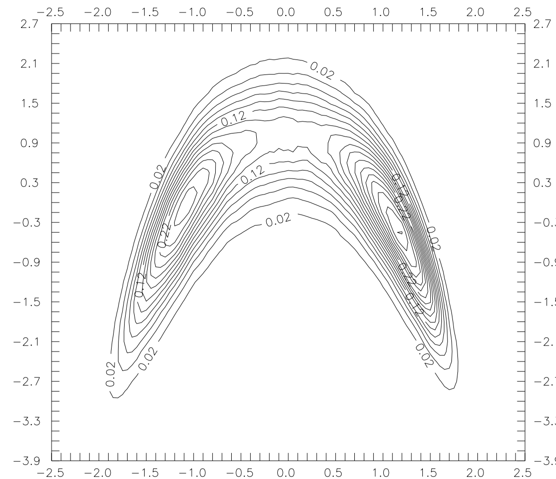
$$Z \; = \; \mathrm{Tr}\exp(-\beta\hat{H}) = \int \mathcal{D}\Phi\,\exp[-\beta\int \mathrm{d}^3\mathbf{x}\,\mathcal{H}_{\rm cl}(\Phi)].$$

Information about other modes is in effective couplings.

$$m_3^2 \sim -\frac{1}{2}m_H^2 + g^2T^2, \quad \frac{\lambda_3}{g_3^2} \approx \frac{1}{8}\frac{m_H^2}{m_W^2} + \mathcal{O}\left(\frac{g^2}{(4\pi)^2}\frac{m_{\rm top}^4}{m_W^4}\right).$$

Remaining dynamics can be studied with lattice simulations. Signals for a 1st order transition / 2nd order transition:

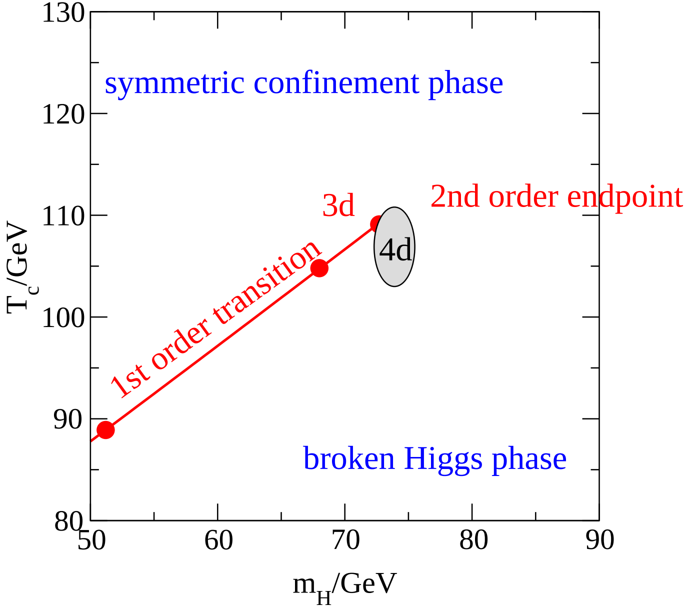




hep-lat/9612006

hep-lat/9805013

Phase diagram after infinite volume ($V \sim 20^3 \ldots 80^3$) and continuum ($g_3^2 a \sim 1 \ldots 0.2$) extrapolations:



3d lattice results:
[SU(2)×U(1)+Higgs+fermions]
Kajantie et al hep-ph/9605288, hep-lat/9805013, hep-lat/9809045

4d lattice results:
[SU(2)+Higgs; relative endpoint position conserved]
Csikor et al hep-ph/9809291

For physical $m_H \gtrsim 114$ GeV just a crossover.

Crossover leaves an imprint on existing baryon asymmetry

Baryon-number violating anomalous “sphaleron” transitions do become very slow, and one **can** find a temperature T_{ew} with

$$\frac{1}{t_{\rm dyn}} \sim \Gamma_{B+L}\left(\frac{v_{\text{ew}}}{T_{\text{ew}}}\right) \quad = \quad \frac{1}{t_{\rm univ}} \sim \frac{T^2_{\rm univ}}{m_{\rm Pl}} \, ,$$

where v_{ew} is the Higgs expectation value.

Burnier et al hep-ph/0511246

Consequently pre-existing baryon number “freezes”,

$$B_{\text{present}} \simeq 4\Big(\frac{77T^2_{\text{ew}} + 27v^2_{\text{ew}}}{869T^2_{\text{ew}} + 333v^2_{\text{ew}}}\Big)(B-L)_{T_{\text{ew}}} \, ,$$

Khlebnikov, Shaposhnikov hep-ph/9607386

Minimal Supersymmetric Standard Model

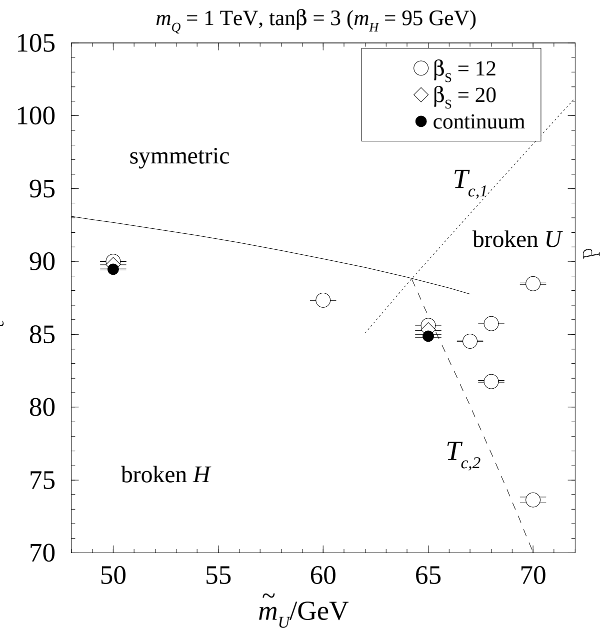
Has new strongly interacting light bosonic particles, the squarks. The prime example is a dominantly right-handed stop, with $m_{\tilde{t}_R} \sim \sqrt{m_U^2 + m_{\rm top}^2} < m_{\rm top}$, i.e. $m_U^2 \equiv -\tilde{m}_U^2 < 0$.

$$\delta\mathcal{H}_{\rm cl} \sim h_t^2\, U^\dagger U \phi^\dagger \phi + (D_i^s U)^\dagger D_i^s U + \frac{1}{2}\,{\rm Tr}\, G_{ij}^s G_{ij}^s + \ldots$$

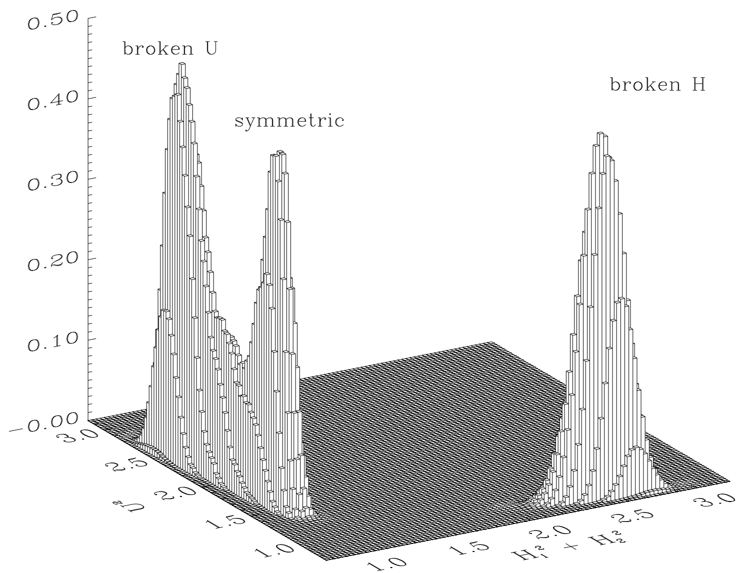
The dynamics related to U makes the transition stronger.

The most extreme case is a two-stage transition through a color-breaking phase

Bödeker et al hep-ph/9612364



hep-lat/9804019



Simulation at the triple point [hep-lat/0009025]

A strong transition could do a lot for cosmology!

Bubbles nucleate (ℓ_B), expand, and release latent heat (L).

Bubble collisions can lead to gravitational waves:

$$\Omega_{\mathsf{GW}} h^2 \lesssim 10^{-7} \left(\frac{L}{e}\right)^2 \left(\frac{\ell_B}{\ell_H}\right)^2 .$$

Witten PRD 30 (1984) 272;
Hogan MNRAS 218 (1986) 629;
Kamionkowski et al astro-ph/9310044;
Caprino et al 0901.1661.

Latent heat over energy density can be written as

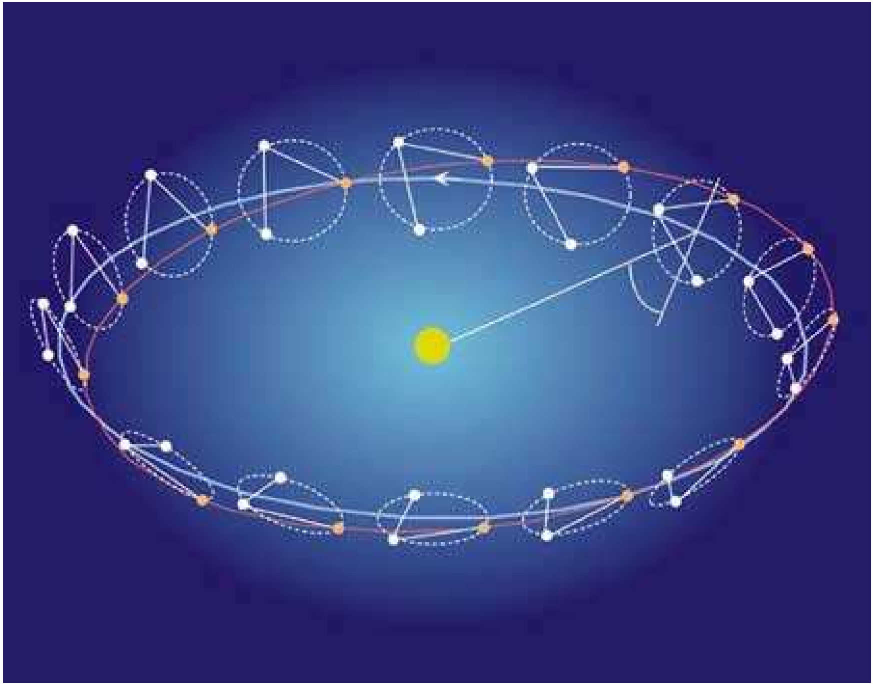
$$\frac{L}{e} \simeq \frac{30\ L}{\pi^2 g_* T_c^4} \simeq 0.03\ \frac{L}{T_c^4} \ ,$$

while the bubble distances become

Ignatius et al hep-ph/9405336

$$\frac{\ell_B}{\ell_H} \quad \simeq \quad 0.0035\ \frac{(\sigma/T_c^3)^{\frac{3}{2}}}{(L/T_c^4)}$$

Laser Interferometer Space Antenna [lisa.nasa.gov] ($\geq$ 2018?)



Threshold: $\Omega_{\mathsf{GW}} h^2 \sim 3 \times 10^{-12}$

Horizon radius ℓ_H corresponds to 1 AU today; subhorizon physics ℓ_B to shorter wavelengths. Matches $f_{\mathsf{LISA}} \sim 10^{-4} \ldots 10^{-2}$ Hz!

Or could generate a baryon asymmetry

The rate Γ_{B+L} is fast at high temperatures, exponentially small at low: $\Gamma_{B+L} \sim \exp(-45\, v_{\rm ew}/T_{\rm ew})$. As long as the rate is fast, we are in equilibrium, and

$$\begin{aligned} \langle [\hat{B}+\hat{L}](t) \rangle_T &\propto \mathrm{Tr}\left\{ e^{-\hat{H}/T} e^{i\hat{H}t/\hbar} [\hat{B}+\hat{L}] e^{-i\hat{H}t/\hbar} \right\} \\ &= \langle [\hat{B}+\hat{L}](0) \rangle_T = 0. \end{aligned}$$

However, if there is a sufficiently strong first order transition, taking the system momentarily out of equilibrium, and having $v_{\text{ew}}/T_{\text{eq}} \gtrsim 1$ afterwards such that the results remain put, processes could be "tilted" and something could be generated.

Shaposhnikov NPB 287 (1987) 757

Why are these consequences not there in QCD?

- No (strong) first order transition at $\mu_B/T \ll 1$.

- The horizon radius today is 1 ly $\gg$ 1 AU, and the corresponding frequency scale around 10^{-7} Hz, so not in the LISA window anyway.

- Baryon number is not violated by strong interactions, so no baryon asymmetry could be generated anyway.

Nevertheless, techniques developed for QCD may become valuable, if the correct EW theory is something like technicolor, with SU$(?)_L\times$SU$(?)_R$ restoration!

see e.g. Kikukawa et al 0709.2221.