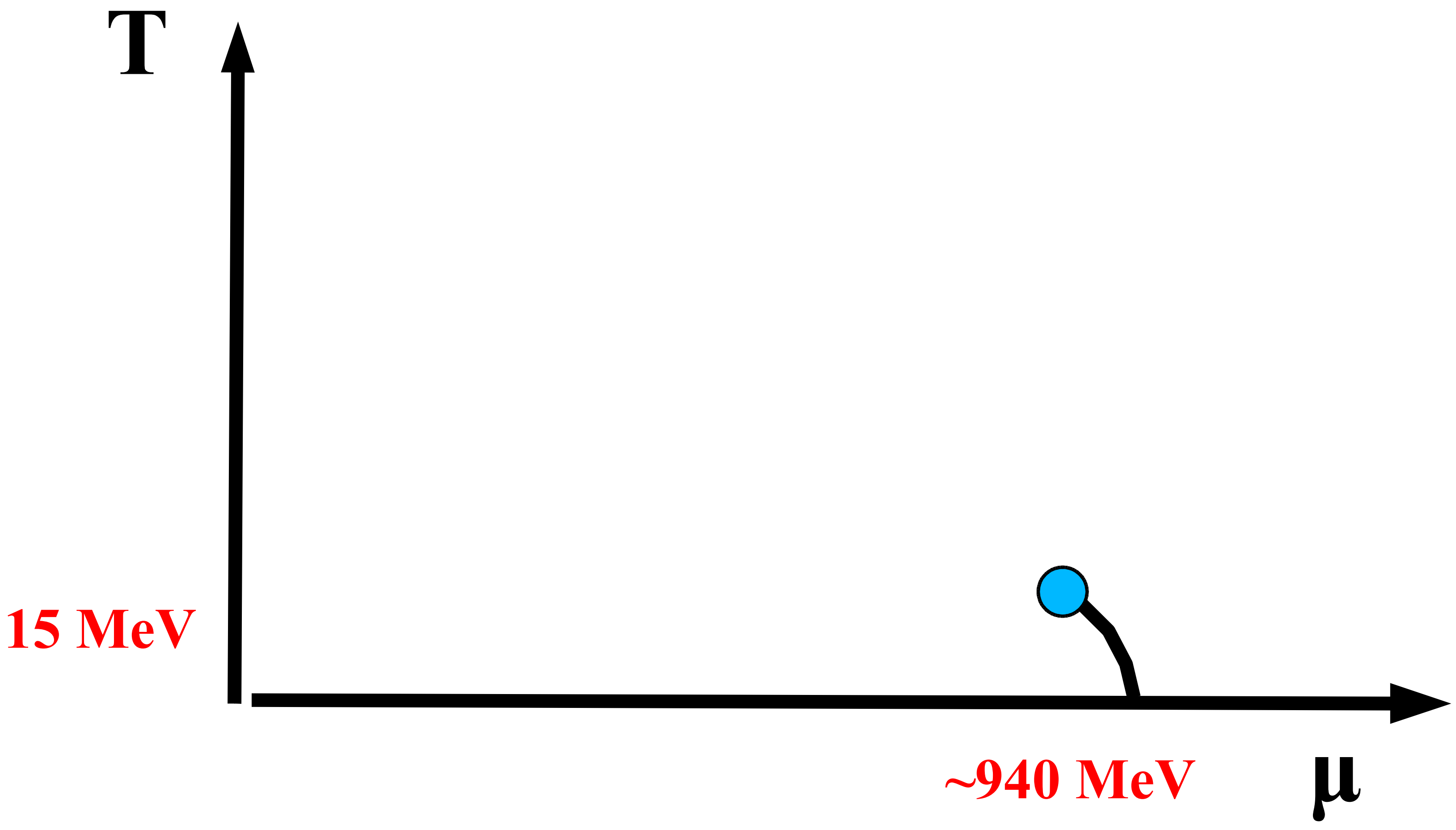
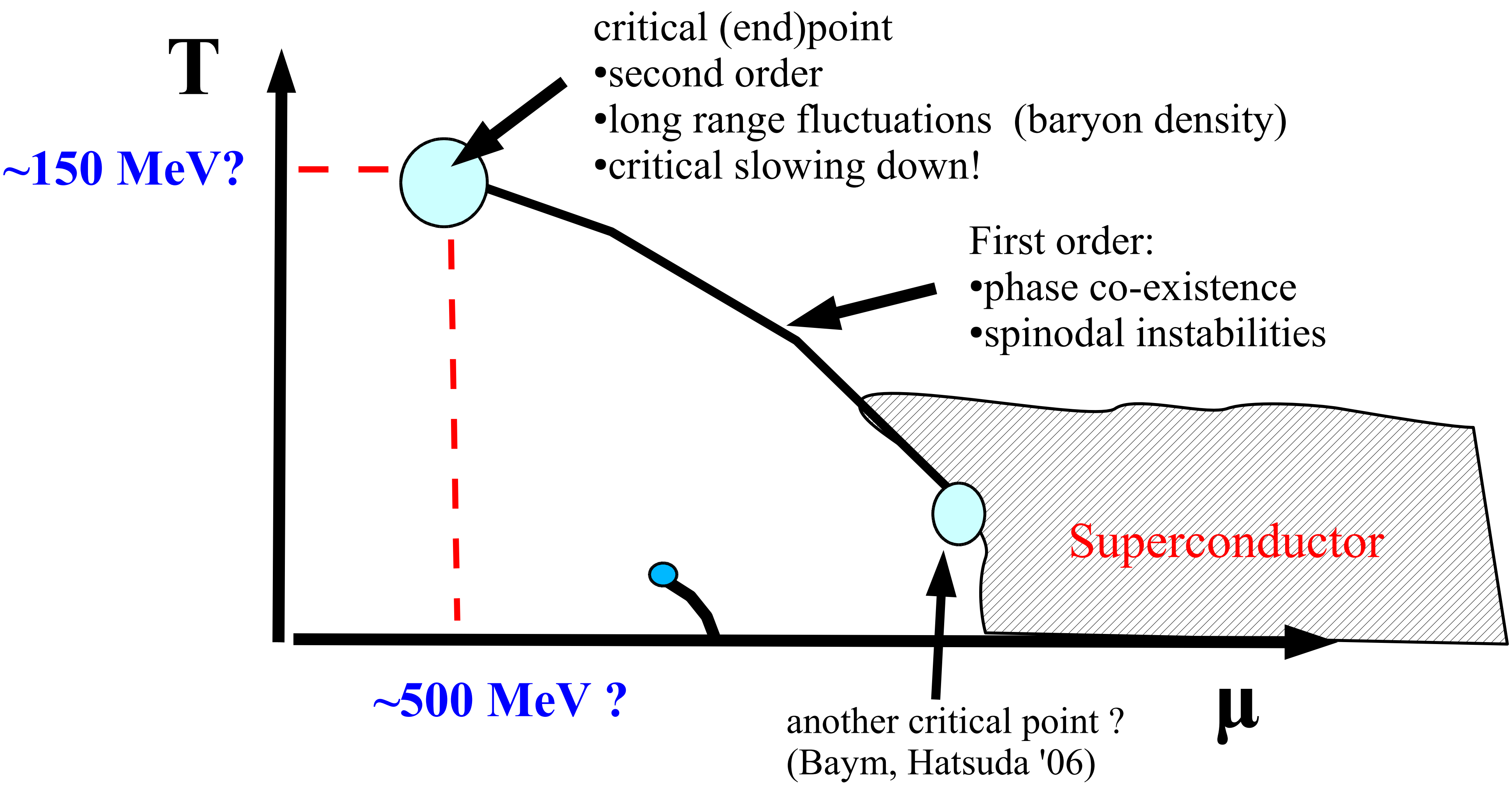
Phase Transition Phenomena in Nuclear Matter

V. Koch, LBNL, Berkeley

The QCD Phase Diagram from experiment

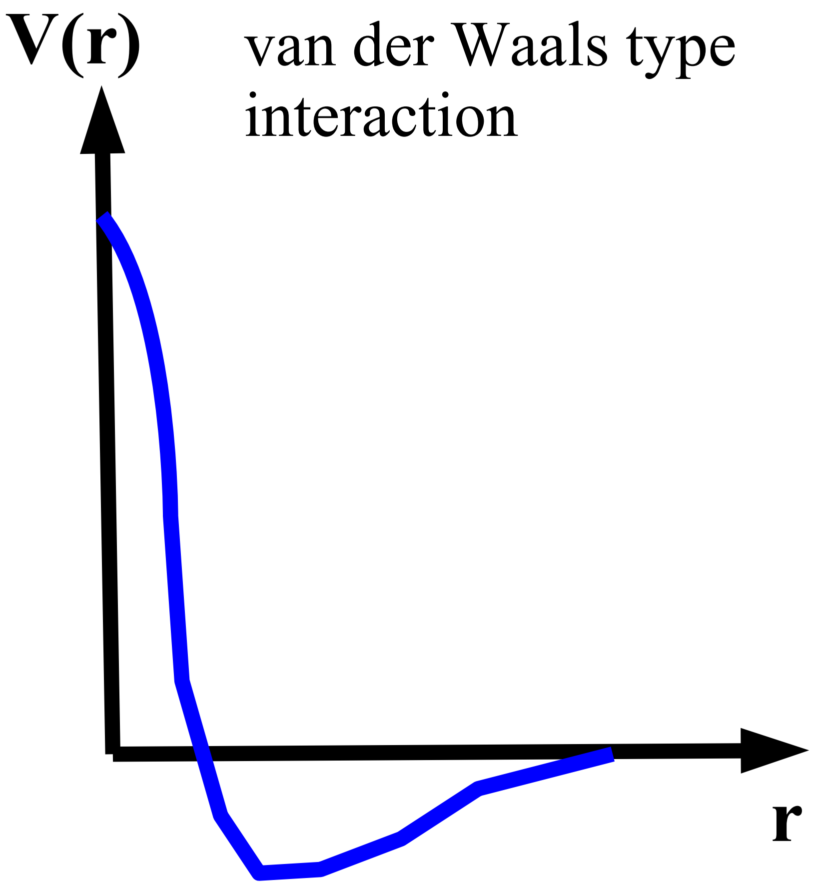


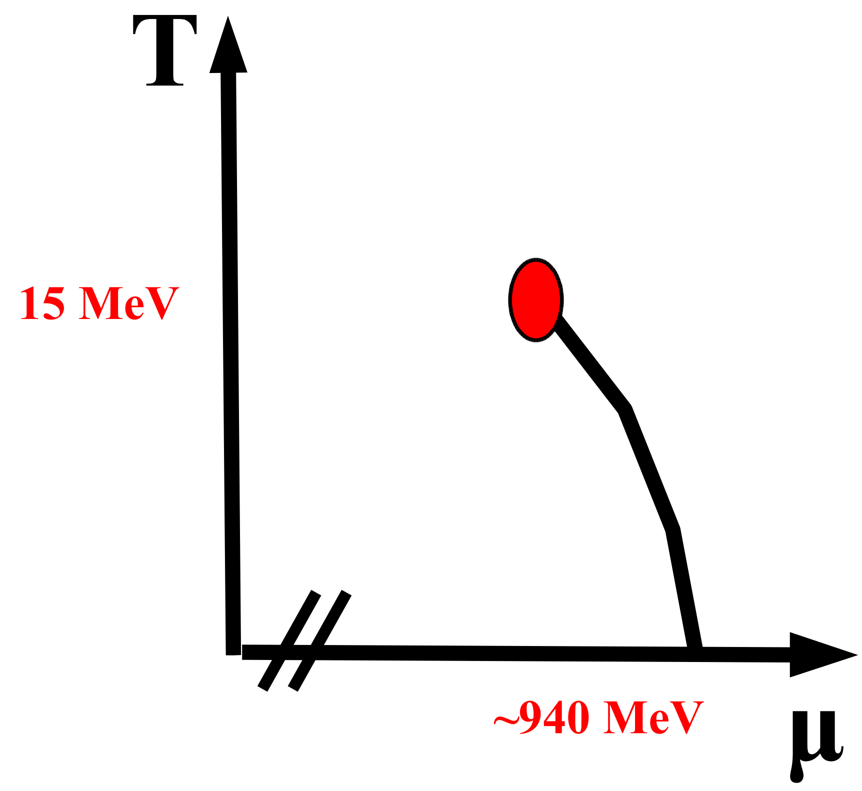
The QCD Phase Diagram
(from a theorist's perspective)



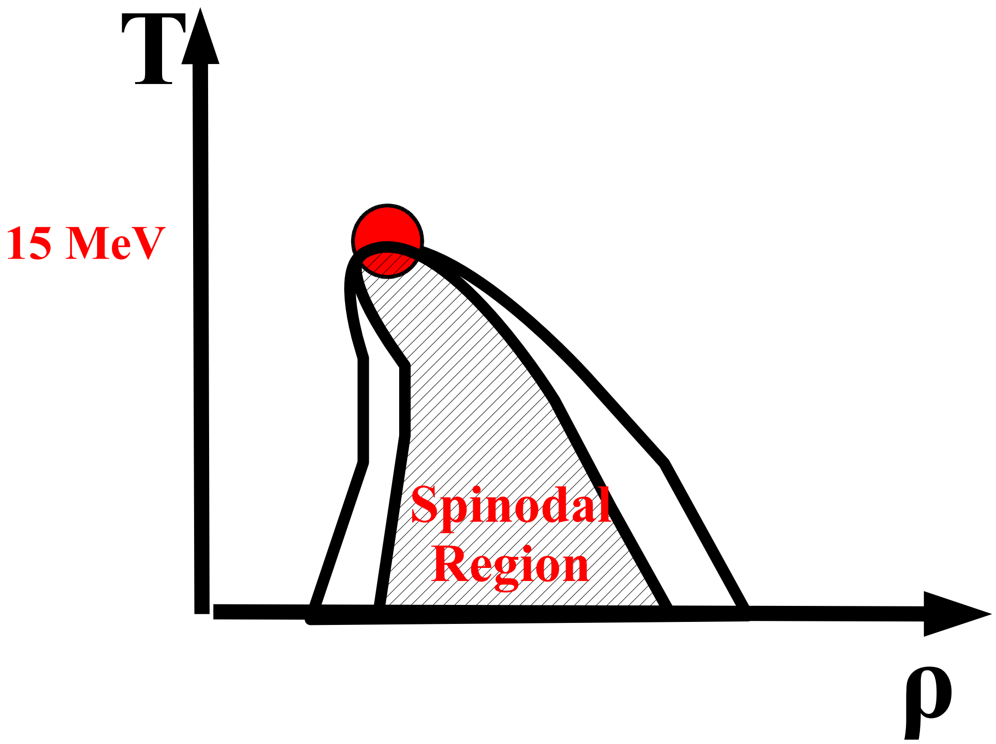
N.B.: Critical point of water: T_c=647.096 K, p_c = 22.064 MPa, ρ_c = 322 kg/m3

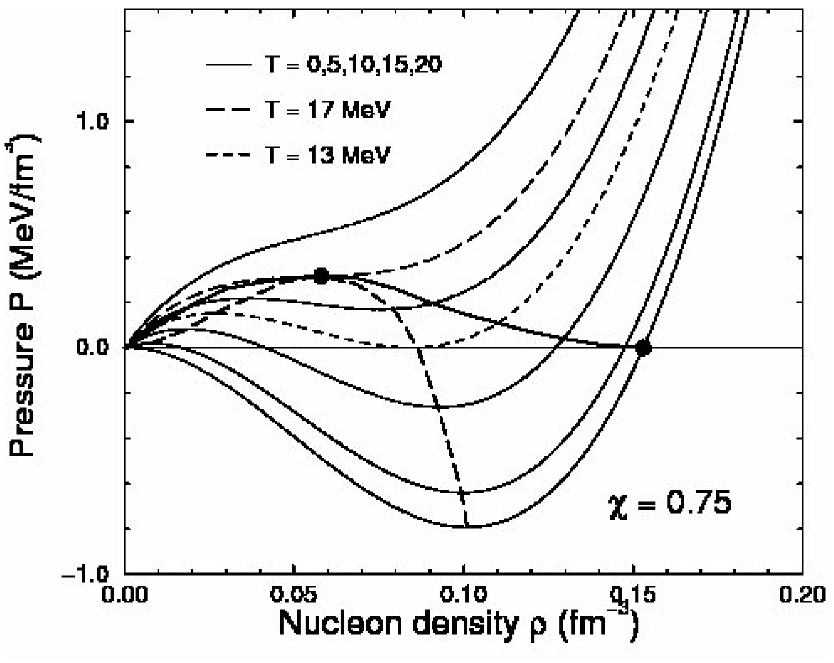
The Nuclear Liquid Gas Phase Transition





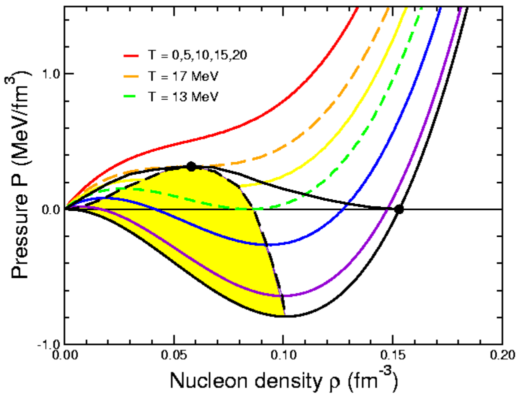
Nuclear Liquid-Gas Transition





Spinodal Multifragmentation

Nuclear EoS:

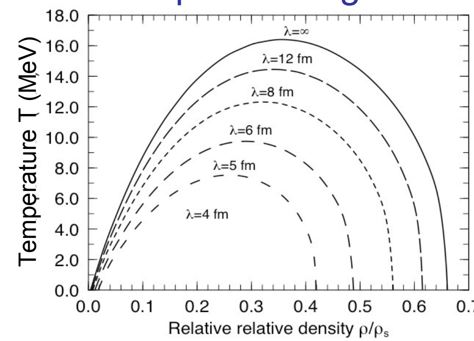


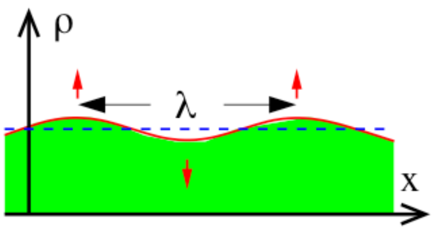
1st order phase transtion

⇕

Spinodal instability

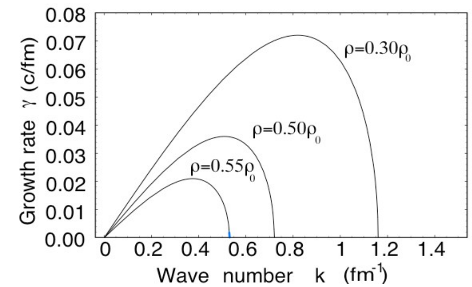
Spinodal region:



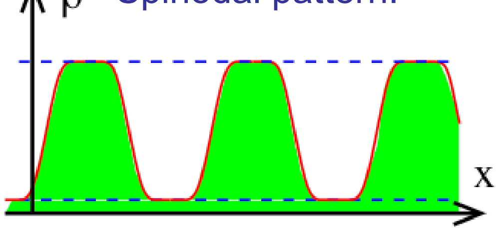


Density undulations may be amplified

Growth rates:



Spinodal pattern:



Ph Chomaz, M Colonna, J Randrup
Nuclear Spinodal Fragmentation
Physics Reports 389 (2004) 263

⇒

Fragments
≈ *equal!*

★ Highly non-statistical => Good candidate signature

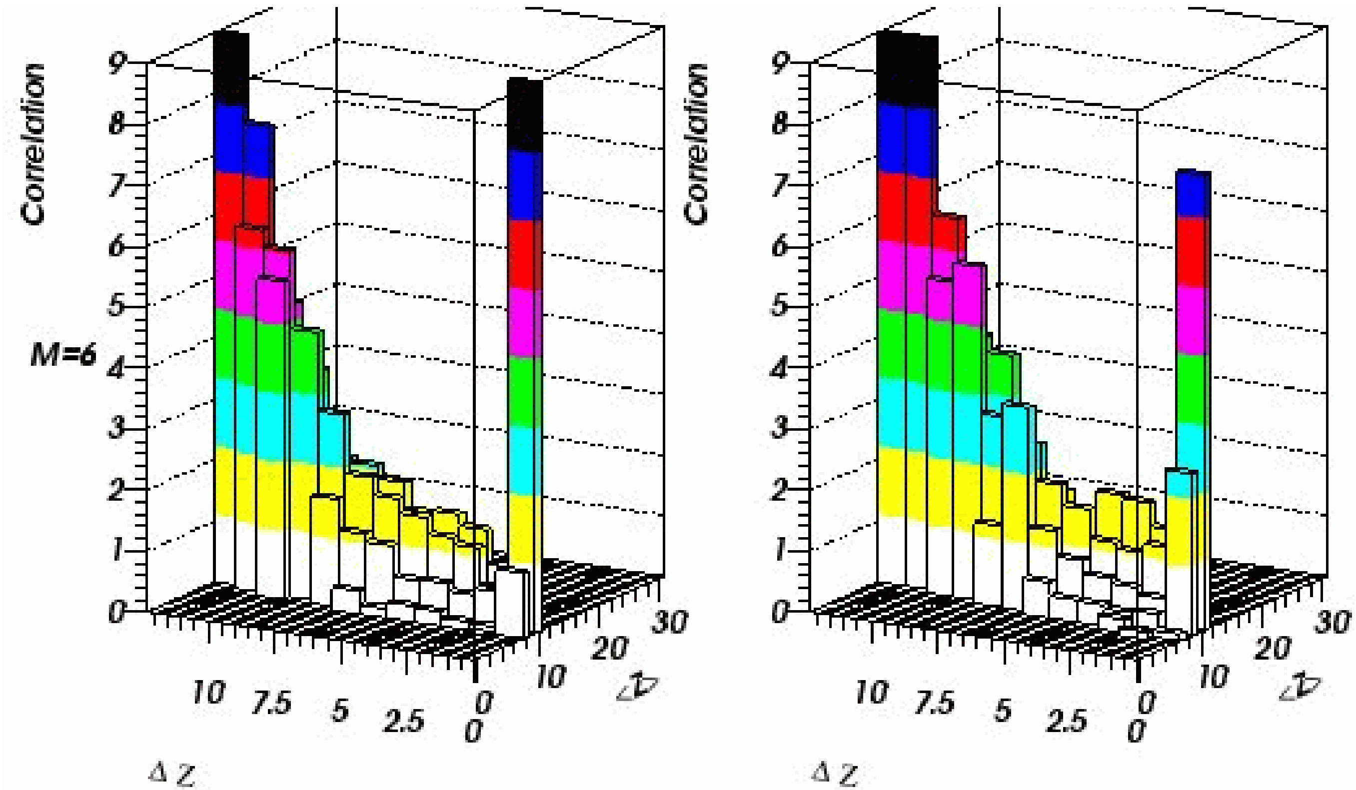
J. Randrup

Spinodal decomposition in nuclear multifragmentation

32 MeV/A Xe + Sn (b=0)

(select events with 6 IMFs)

Bin wrt $\begin{cases} \langle Z \rangle : & \text{average IMF charge} \\ \Delta Z : & \text{dispersion in IMF charge} \end{cases}$



Experiment (*INDRA @ GANIL*)

Borderie *et al*, PRL 86 (2001) 3252

Theory (*Boltzmann-Langevin*)

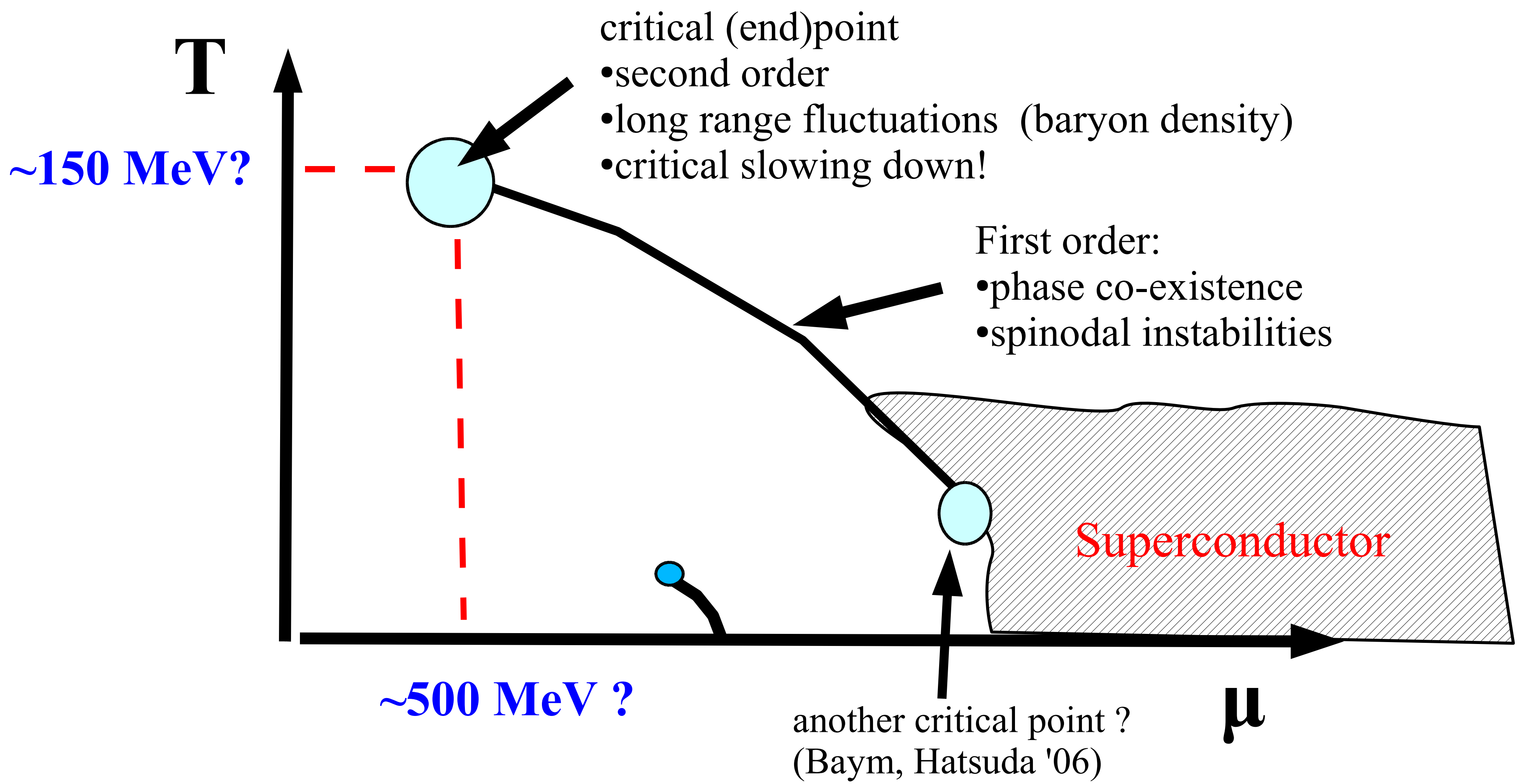
Chomaz, Colonna, Randrup, …

J. Randrup

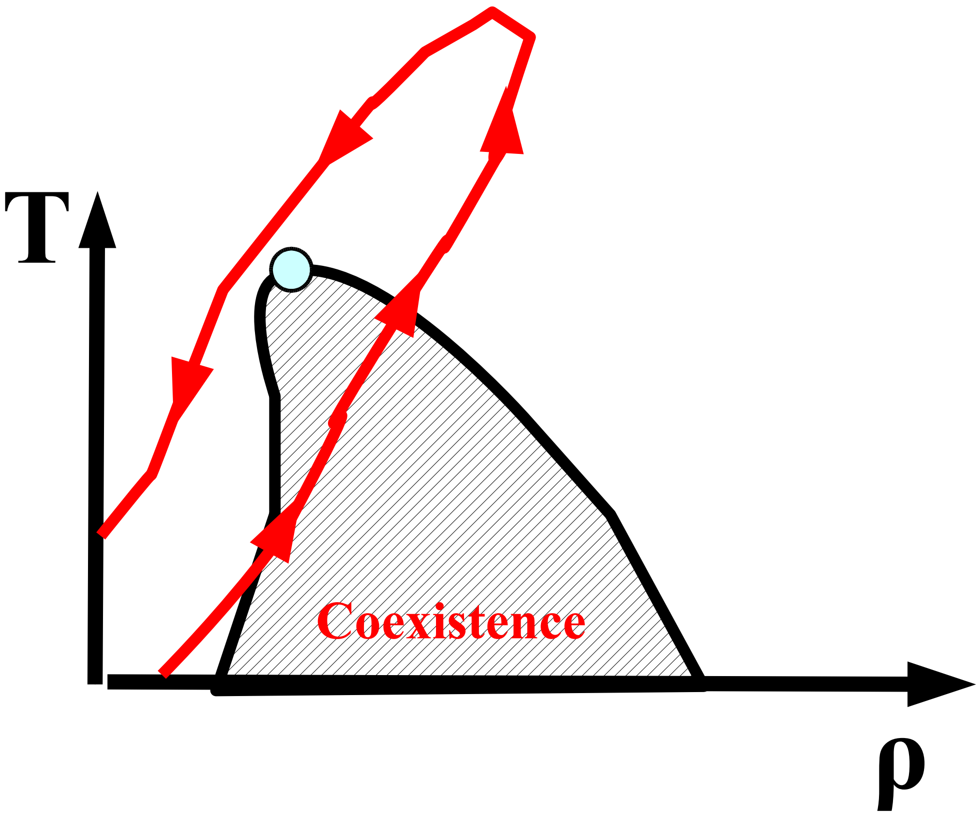
Summary Nuclear Liquid Gas

- Conceptually very straightforward
  - Force of van der Waals type
- Signs for co-existence has been found
  - Spinodal
  - Systematics of fragment distribution follows Fisher model
    - Extrapolate to critical point
- Phases are rather well defined
- >20 years of work !

The QCD Phase Diagram
(from theory)



Critical point vs co-existence



- Difficult to “hit” a point!

- Lesson learned from nuclear Liquid gas:
  - Establish co-existence and extrapolate to CP
  - Carefully chose energy such that system stalls in co-existence region

Fluctuations and Correlations in thermal system

e.g. Lattice QCD

$$Z = Tr[\exp(-\beta(H - \mu_Q Q - \mu_B B - \mu_S S))]$$

Mean : $\langle \alpha \rangle = T \frac{\partial}{\partial \mu_\alpha} \log(Z) = -\frac{\partial}{\partial \mu_\alpha} F$

$\alpha, \beta = Q, B, S$

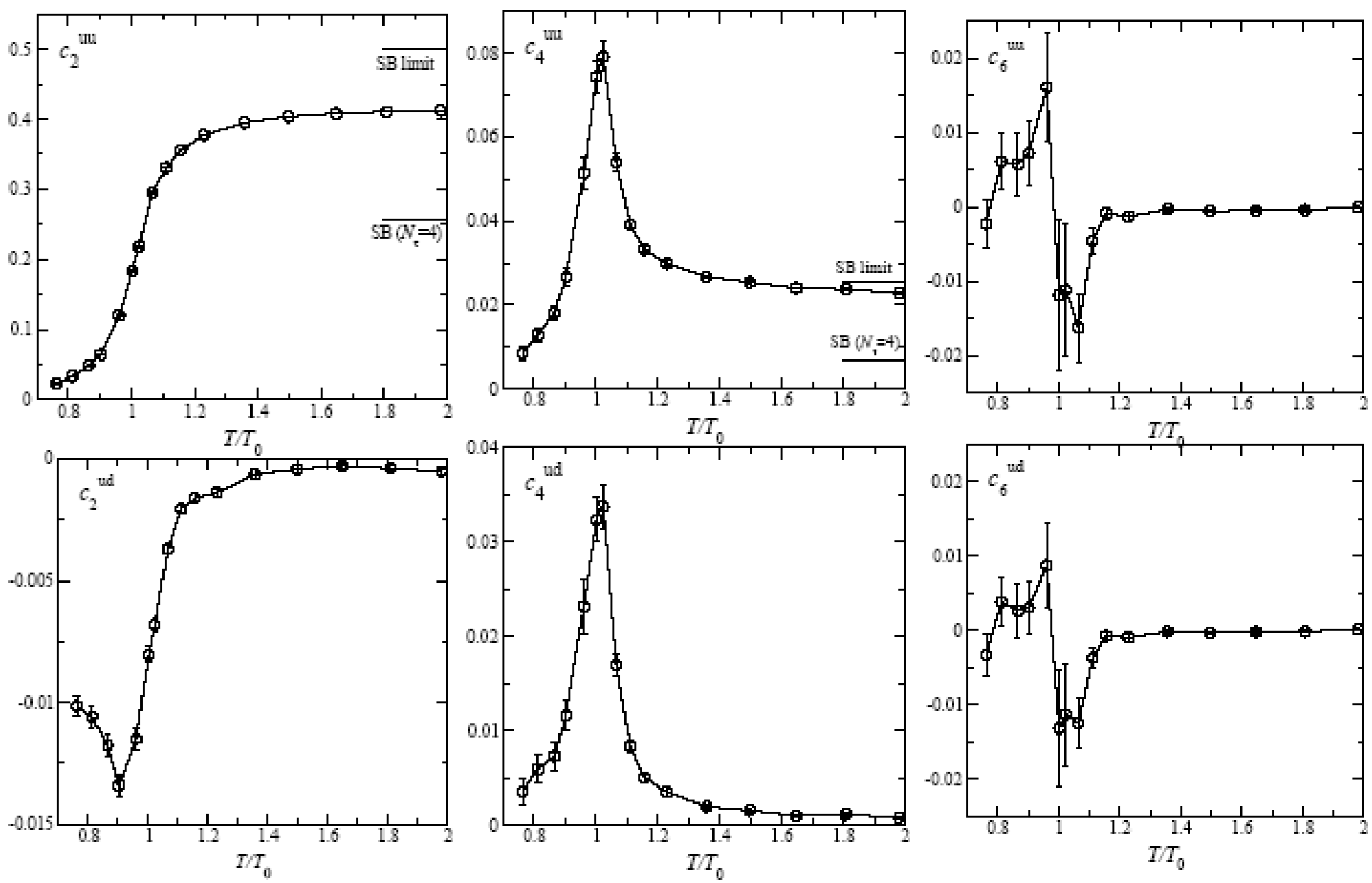
Variance: $\langle (\delta \alpha)^2 \rangle = T^2 \frac{\partial^2}{\partial \mu_\alpha^2} \log(Z) = -T \frac{\partial^2}{\partial \mu_\alpha^2} F$

Co-Variance: $\langle (\delta \alpha)(\delta \beta) \rangle = T^2 \frac{\partial^2}{\partial \mu_\alpha \partial \mu_\beta} \log(Z) = -T \frac{\partial^2}{\partial \mu_\alpha \partial \mu_\beta} F$

Susceptibility: $\chi_{\alpha\beta} = -\frac{1}{V} \frac{\partial^2}{\partial \mu_\alpha \partial \mu_\beta} F = -\frac{1}{V} \frac{\partial}{\partial \mu_\alpha} \langle \beta \rangle$

Lattice-QCD susceptibilities

$$\frac{\chi(T,\mu_q)}{T^2} = 2c_2 + 12c_4\left(\frac{\mu_q}{T}\right)^2 + 30c_6\left(\frac{\mu_q}{T}\right)^4 + \ldots$$



Rule of thumb:

$$c_n \sim \langle X^n \rangle$$
$$X = B, Q, S, \ldots$$

Alton et al, PRD 66 074507 (2002)

Susceptibilities and Phasetransitions

$$Z = Tr[\exp(-\beta(H - \mu N))]$$

Susceptibility: $\chi \sim \frac{1}{V}\frac{\partial^2}{\partial \mu^2}\log(Z) = \frac{1}{V}(\langle N^2\rangle - \langle N\rangle^2)$

Poisson: $\chi \sim \frac{\langle N\rangle}{V}$ independent of volume $\Rightarrow$ $\langle(\delta N)^2\rangle = N \sim V$

In general: $\chi \sim \frac{1}{V}\int d^3x\, d^3y \langle \rho(x)\rho(y)\rangle_{connected} = \int d^3r \langle \rho(r)\rho(0)\rangle_{connected} \sim \xi^2$

$$\langle \rho(r)\rho(0)\rangle_{connected} \sim \frac{e^{(-r/\xi)}}{r} \qquad \xi = correlation\ length$$

Cross-over: $\xi = const \rightarrow \chi = const \rightarrow \langle(\delta N)^2\rangle \sim V$

Second Order: $\xi \sim V^{(1/3)} \rightarrow \chi \sim V^{(2/3)} \rightarrow \langle(\delta N)^2\rangle \sim V^{(5/3)}$

First Order: $\langle \rho(r)\rho(0)\rangle = const \rightarrow \chi \sim V \rightarrow \langle(\delta N)^2\rangle \sim V^2$

Susceptibilities and Observables

Susceptibility: $\chi \sim \frac{1}{V} \frac{\partial^2}{\partial \mu^2} \log(Z) = \frac{1}{V} (\langle N^2 \rangle - \langle N \rangle^2)$

Fluctuations of some sort!

Cross-over: $\xi = const \rightarrow \chi = const \rightarrow \langle (\delta N)^2 \rangle \sim V$

Second Order: $\xi \sim V^{(1/3)} \rightarrow \chi \sim V^{(2/3)} \rightarrow \langle (\delta N)^2 \rangle \sim V^{(5/3)}$

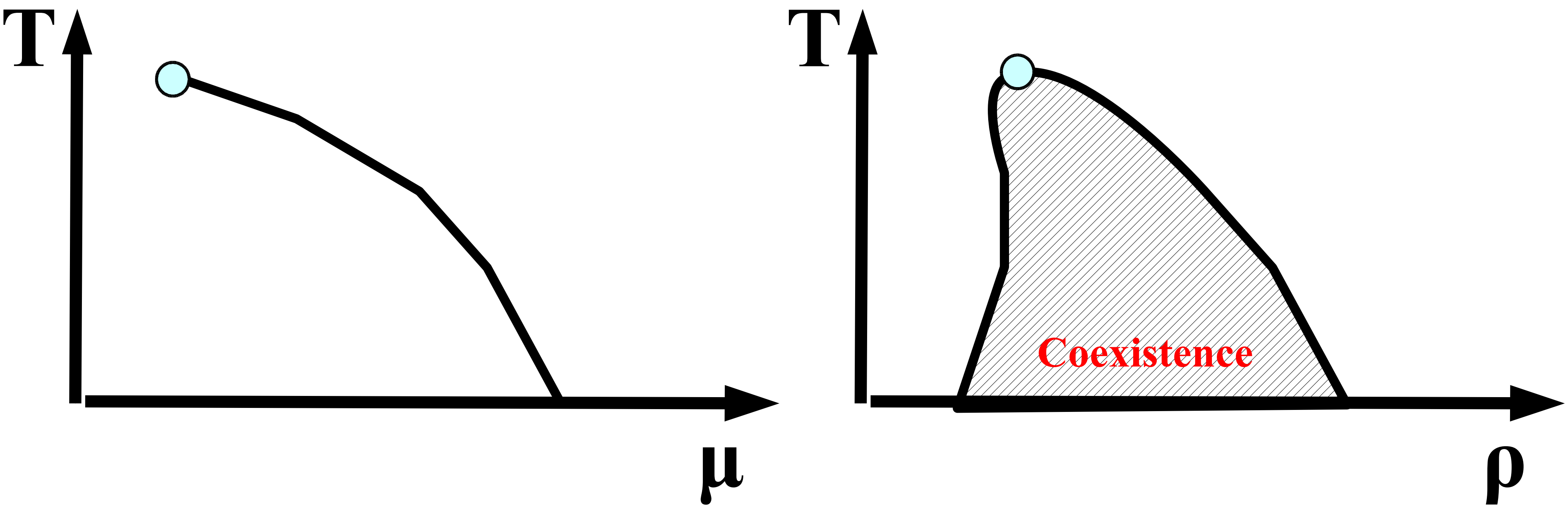
First Order: $\langle \rho(r) \rho(0) \rangle = const \rightarrow \chi \sim V \rightarrow \langle (\delta N)^2 \rangle \sim V^2$

Since fluctuations diverge at phase transition any sort will do!

System size dependence!

Note of caution: Co-variances also diverge; trigger?

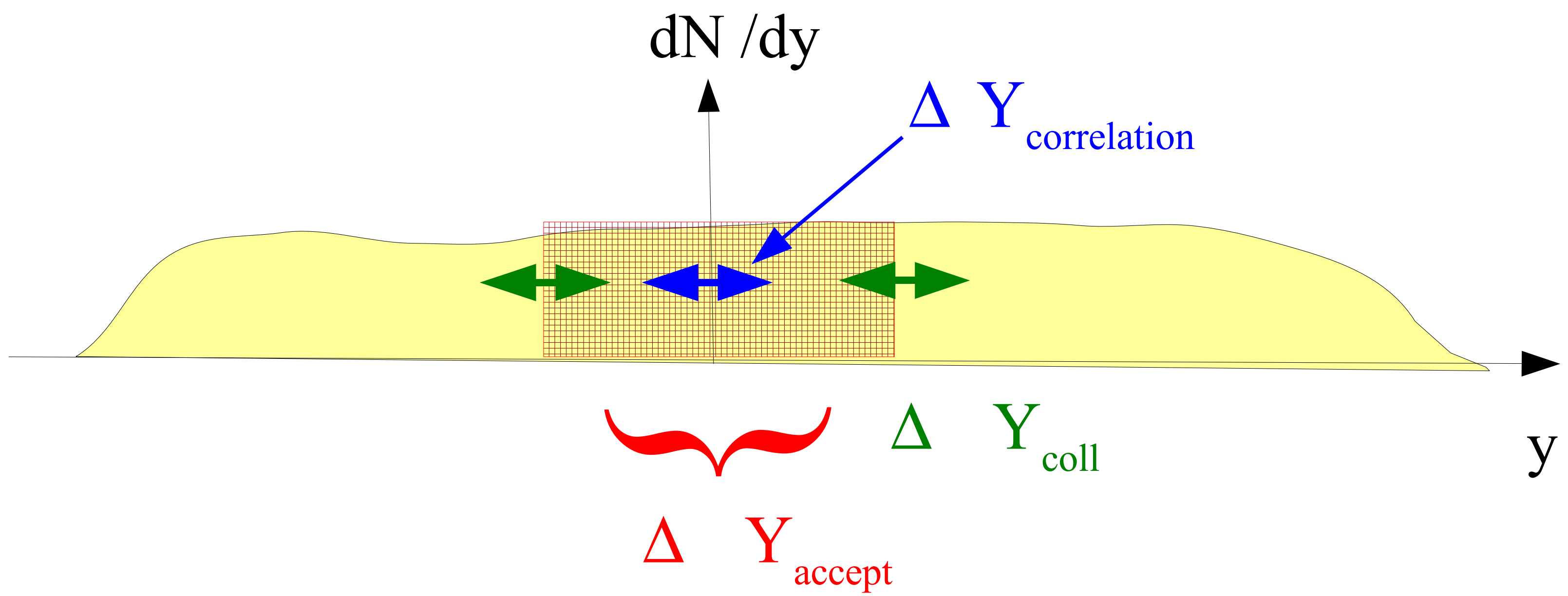
Order Parameter



Baryon density is a good order parameter
density fluctuations are a good observable (theoretically...)

→ **Baryon Number** fluctuations also good in principle, but
global baryon number conservation is an issue at low energies

“Charge” fluctuations

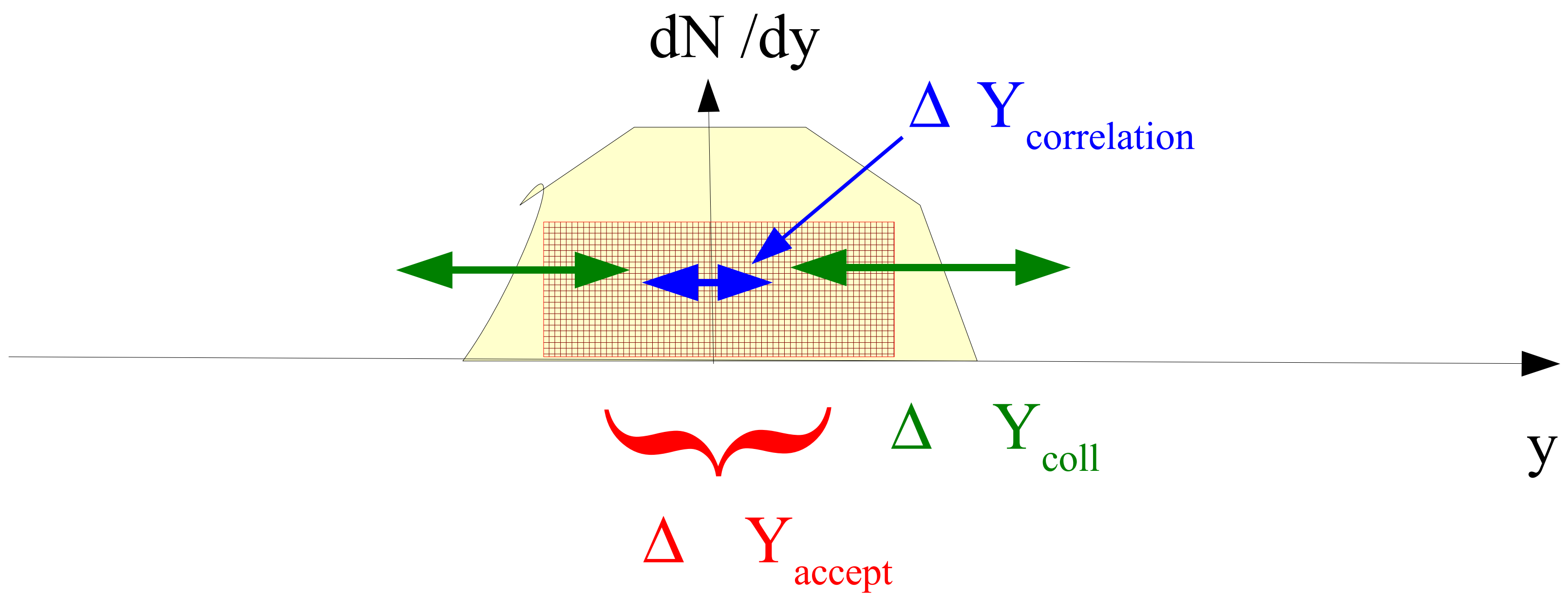


Condition for “charge” fluctuations:

1) $\Delta Y_{corrrelation} << \Delta Y_{accept}$ **(catch the physics)**

3) $\Delta Y_{total} >> \Delta Y_{accept} >> \Delta Y_{coll}$ **(keep the physics)**

“Charge” fluctuations at SPS and below



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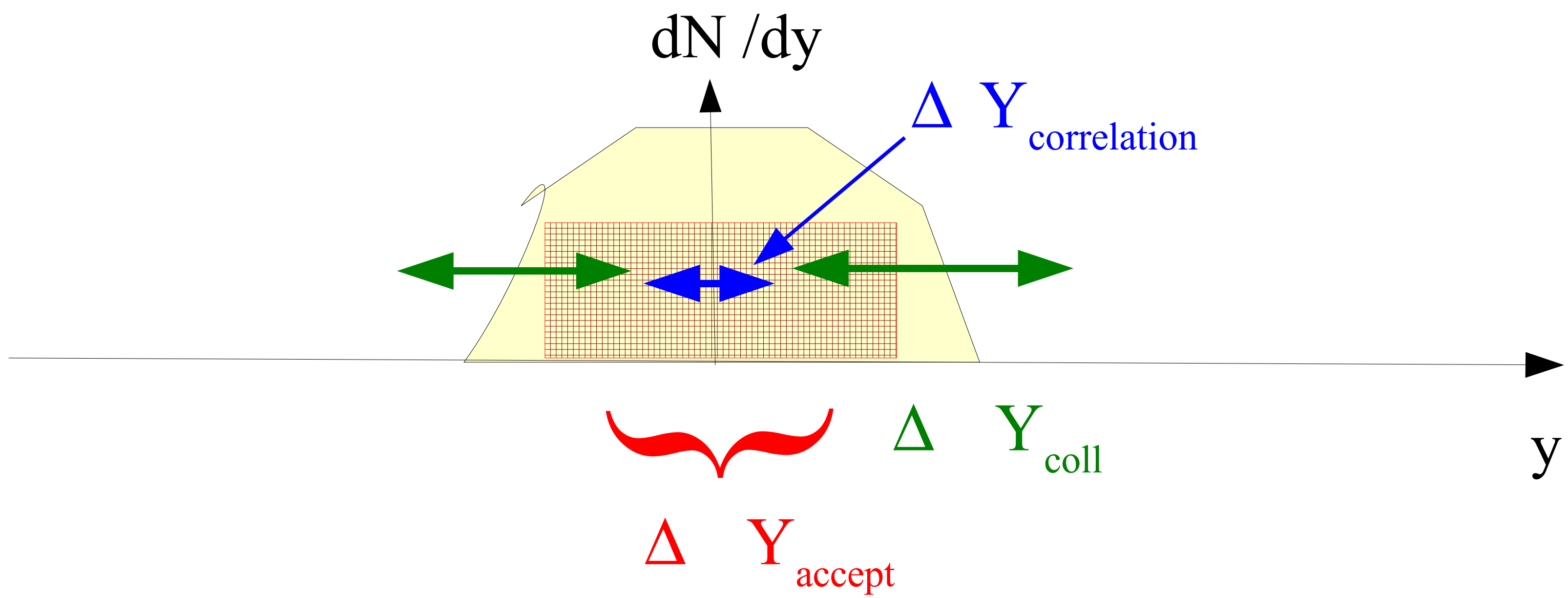

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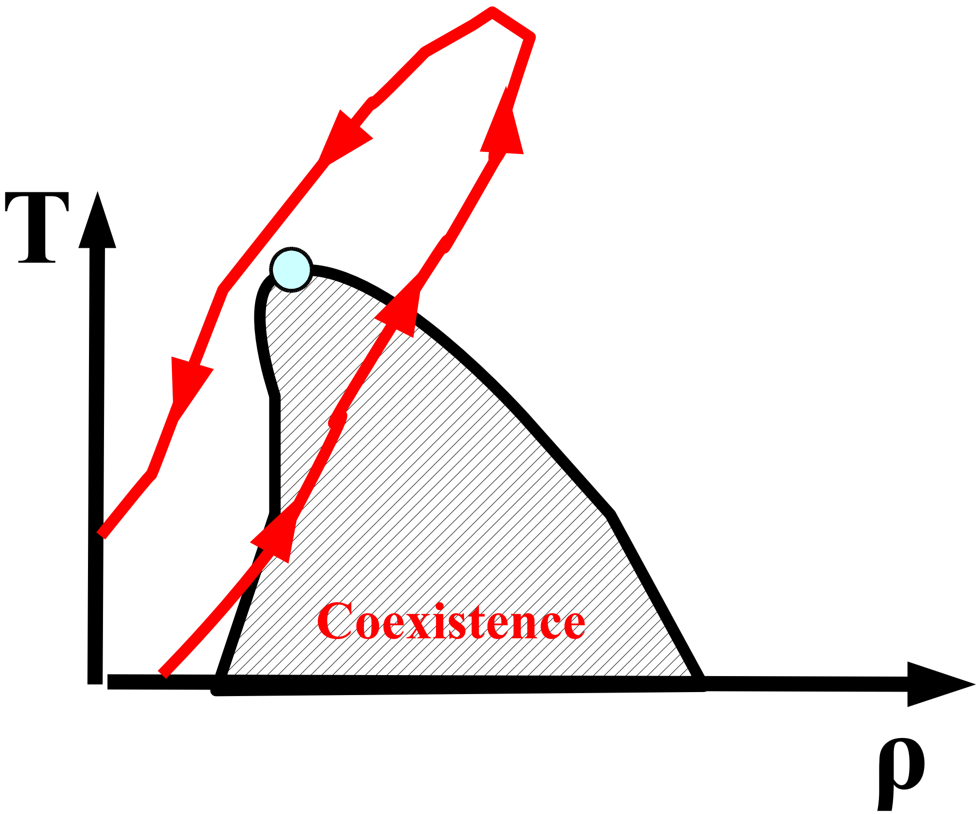
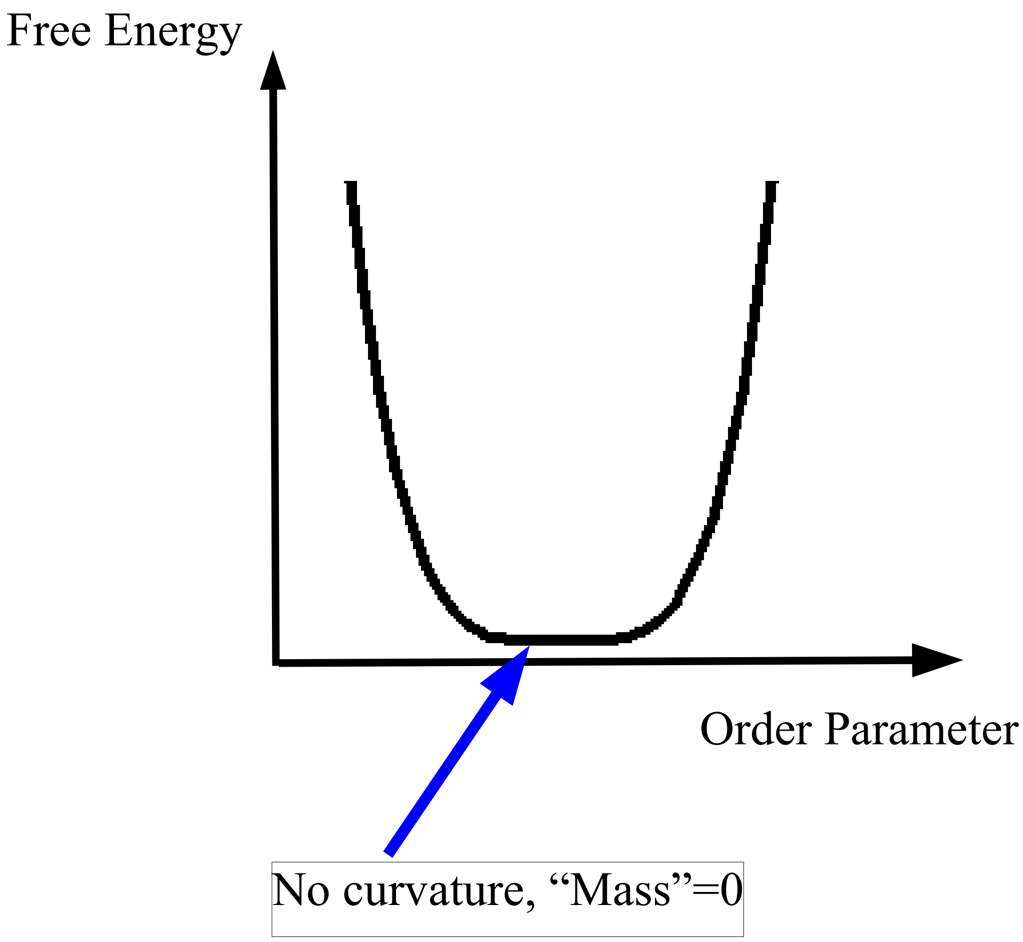
Critical point vs co-existence



•Difficult to “hit” a point!

•Lesson learned from nuclear Liquid gas:
- Establish co-existence and extrapolate to CP
- Carefully chose energy such that system stalls in co-existence region

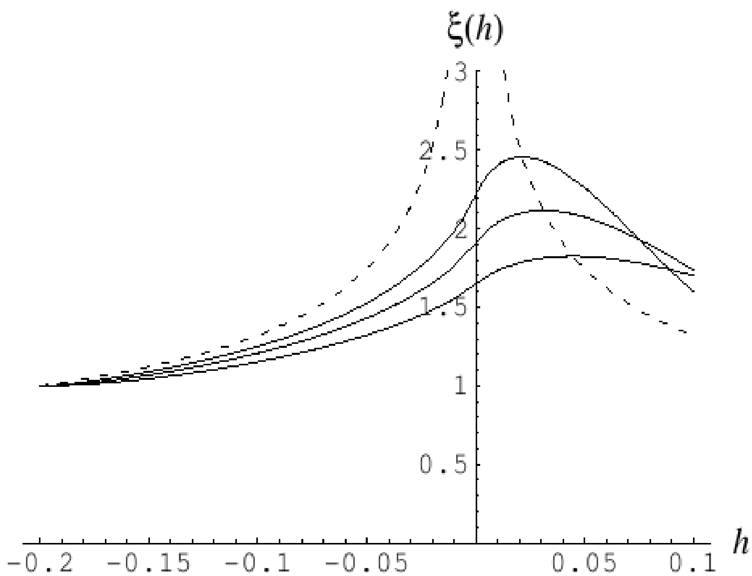
Second order



- Fluctuation of order parameter at all scales
- Diverging susceptibilities
  ~1/(”Mass”)^2
- Diverging correlation length
  ~1/(“Mass”)
- Universality
- Critical slowing down !

Second order

correlation length ~$1/m_\sigma$



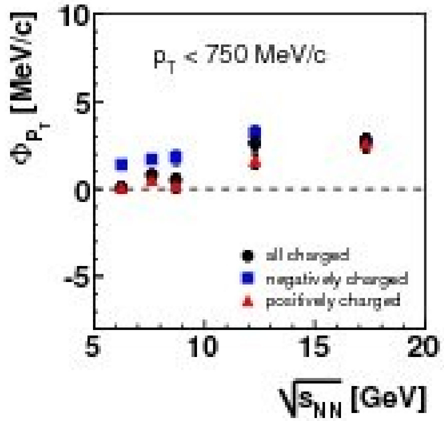
T/MeV 180 140 120

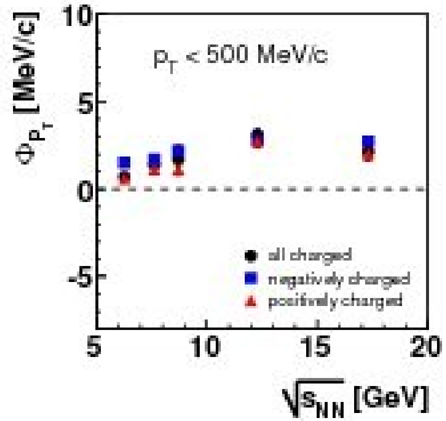
Bernikov, Rajagopal, hep-ph/9912274

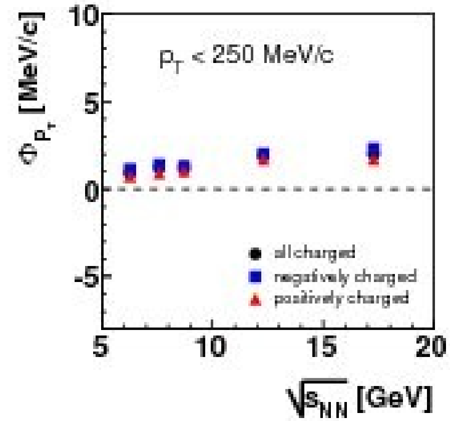
- Critical slowing down
- limited sensitivity on model parameters
- Max. correlation length 2-3 fm
- Translates in **3-5%** effect in p_t-fluctuations

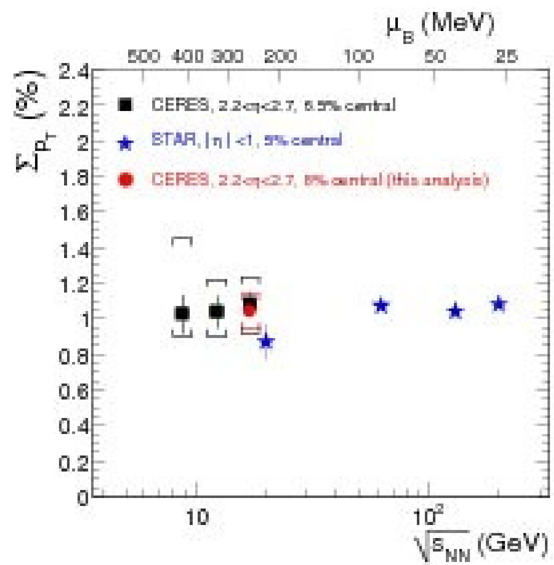
Expect:
Maximum in excitation function of p_t-fluctuations at low p_t

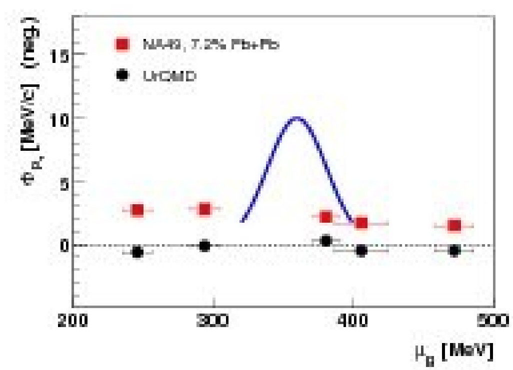
What does experiment say?











Higher cumulants?

Stephanov
arXiv:0809.3450

$$\omega_2 = \frac{\langle (\delta N)^2 \rangle}{\langle N \rangle} \sim \xi^2$$

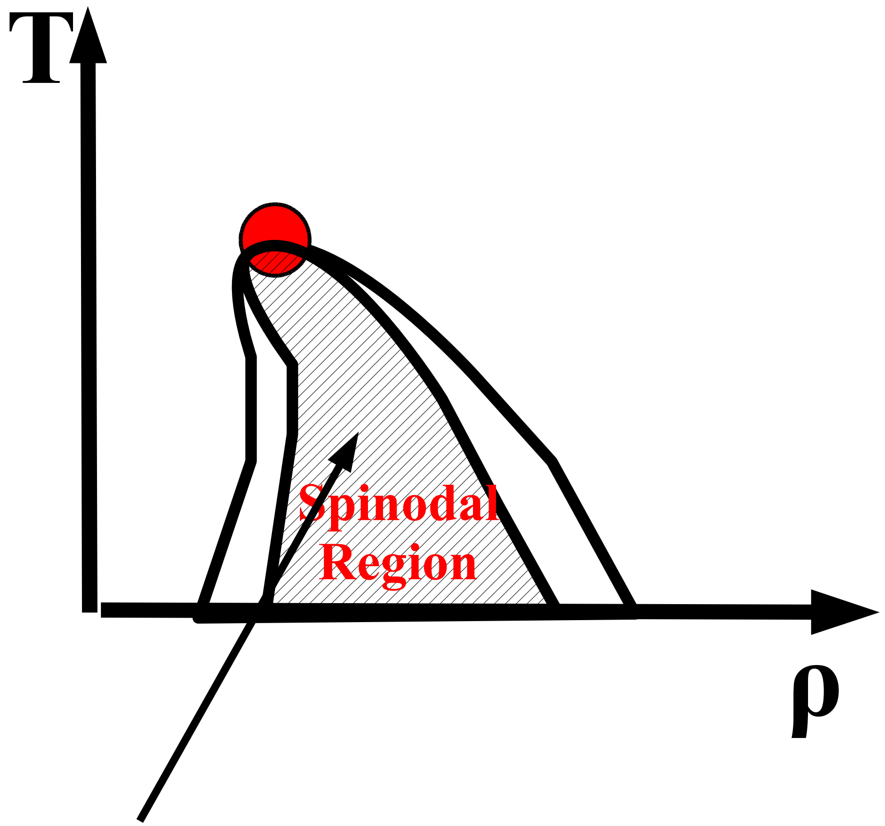
$$\omega_4 = \frac{\langle (\delta N)^4 \rangle}{\langle N \rangle} \sim \xi^7$$

Higher cumulants diverge with higher power:

5% in second order translates 20% in fourth order

Question: How does critical slowing down affect higher cumulants

Co-existence region



System should spent long time
in spinodal region

Relativistic fluid dynamics

Energy-momentum tensor: $T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}$ $p(\varepsilon)$

Equation of motion: $\partial_\mu T^{\mu\nu} = 0$ $u^\mu = (\gamma, \gamma\mathbf{v})$

Small disturbance in a uniform stationary fluid

$$\varepsilon(x,t) = \varepsilon_0 + \delta\varepsilon(x,t)\ ,\ \delta\varepsilon \ll \varepsilon_0$$

First order in $\delta\varepsilon$:

$$\partial_t \delta\varepsilon(x,t) \approx (\varepsilon_0 + p_0)\partial_x v_x(x,t) \qquad p_0 \equiv p(\varepsilon_0)$$

$$(\varepsilon_0 + p_0)\partial_t v_x(x,t) \approx \partial_x p(x,t) \approx \frac{\partial p_0}{\partial \varepsilon_0}\partial_x \delta\varepsilon(x,t)$$

Sound waves:

$$\partial_t^2 \delta\varepsilon(x,t) = \frac{\partial p_0}{\partial \varepsilon_0}\partial_x^2 \delta\varepsilon(x,t) \qquad v_s^2 = \frac{\partial p}{\partial \epsilon}$$

J. Randrup

Growth rates γ_k

Small disturbance: $\varepsilon(x,t) = \varepsilon_0 + \delta\varepsilon(x,t)$, $\delta\varepsilon \ll \varepsilon_0$

Evolution: $\partial_t^2 \delta\varepsilon(x,t) = \frac{\partial p_0}{\partial \varepsilon_0} \partial_x^2 \delta\varepsilon(x,t)$ => $\delta\varepsilon_k(x,t) \sim \mathrm{e}^{ikx - i\omega_k t}$

Dispersion relation: $\omega_k^2 = \frac{\partial p_0}{\partial \varepsilon_0} k^2 = -\gamma_k^2 k^2$ => $\gamma_k = |v_s| k$

Local average: $p(\boldsymbol{r}) = \langle p(\varepsilon(\boldsymbol{r}))\rangle$ $\omega_k^2 = \frac{\partial p_0}{\partial \varepsilon_0} g_k k^2$, $g_k = \mathrm{e}^{-a^2 k^2/2}$

$\gamma \sim$ k OK for small *k*
But what about $k \to \infty$?
Ideal hydro has no scale!

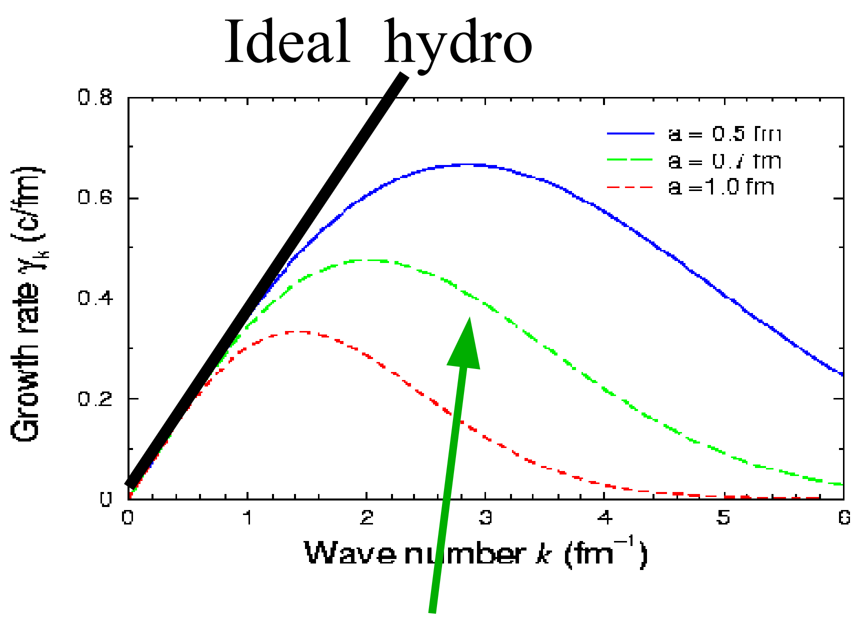
a: smearing range
suppresses large *k*

⇩

γ_k has a maximum

⇩

Spinodal pattern
may develop



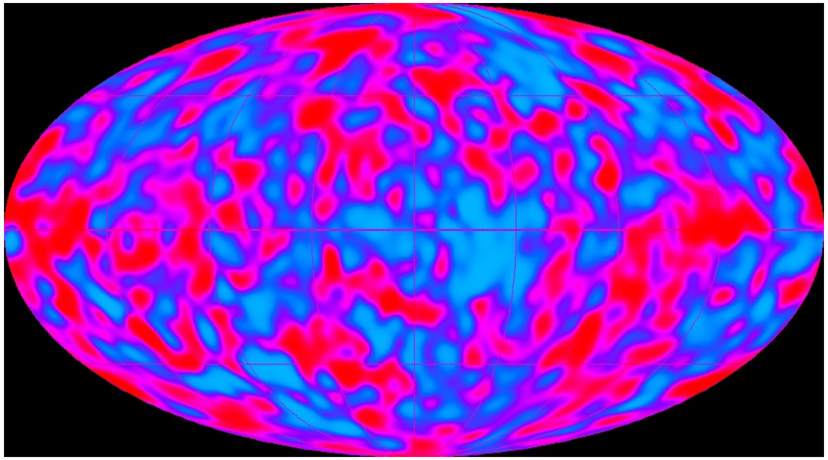
- *if* there is enough time!

Need a length scale!!
Interface tension from lattice?!

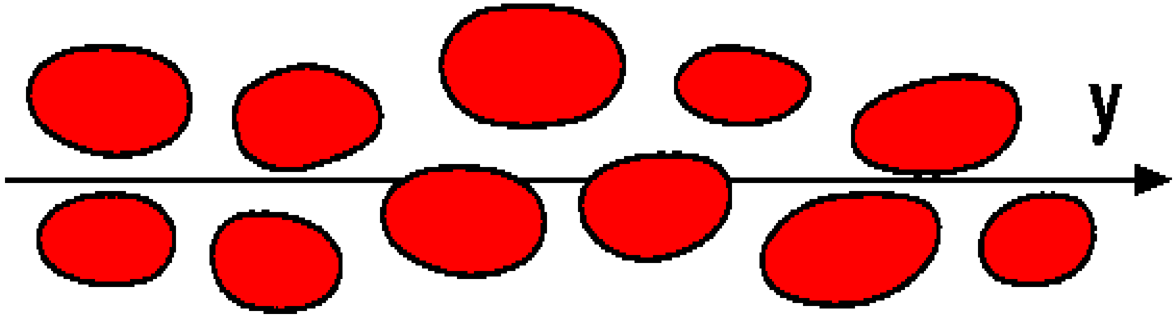
J. Randrup

How to detect clumping?

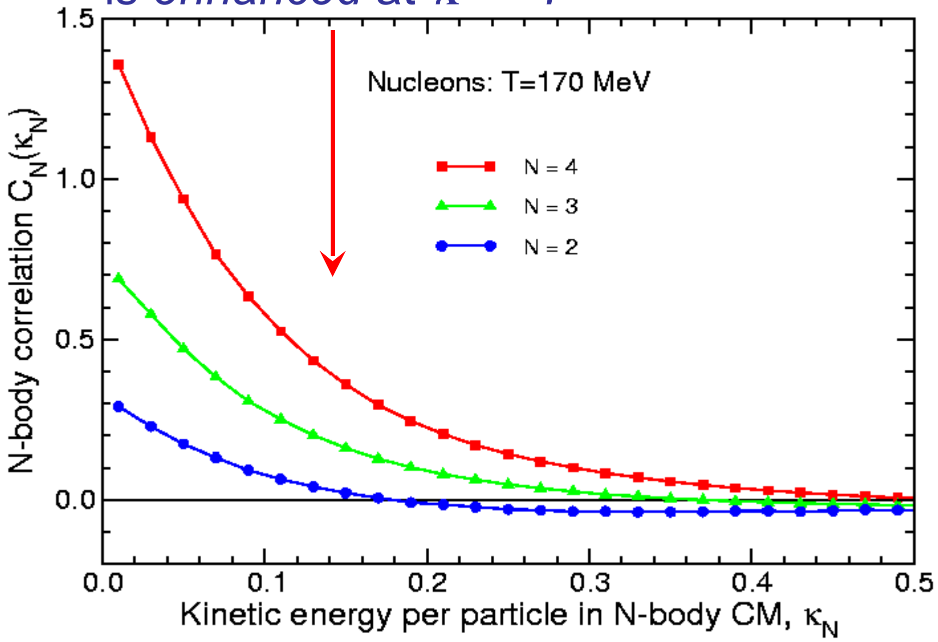
- No obvious candidates for clumps contrary to nuclear liquid gas
  - Kinematic correlations
  - Flavor correlations
- Fluctuations due to clumping

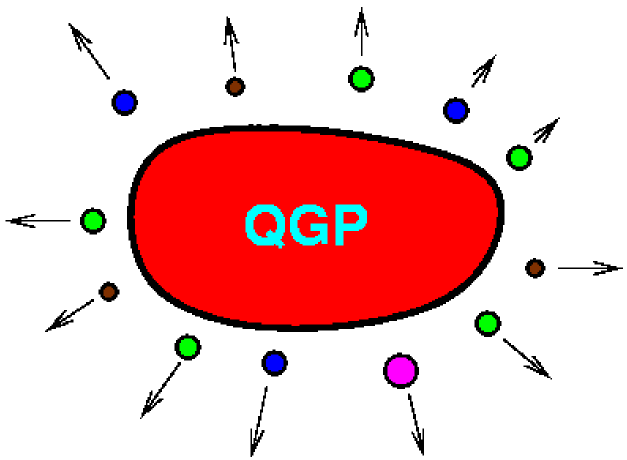


N-particle correlations



- is *enhanced* at $\kappa \approx T$

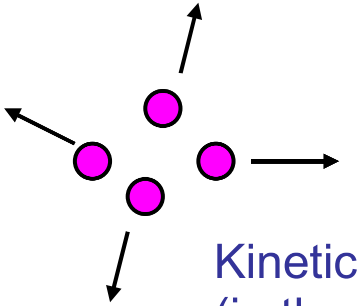




[J. Randrup, J. Heavy Ion Physics 22 (2005) 69]

Kinematic clumping =>

Invariant-mass correlations



Total four-momentum:

$$P\{\boldsymbol{p}_n\} = \sum_n (E_n, \boldsymbol{p}_n)$$

Kinetic energy per particle (in the *N*-body CM frame):

$$\kappa_N\{\boldsymbol{p}_n\} = \frac{1}{N}\left[[P\{\boldsymbol{p}_n\} \cdot P\{\boldsymbol{p}_n\}]^{\frac{1}{2}} - \sum_n m_n \right]$$

Distribution of κ :

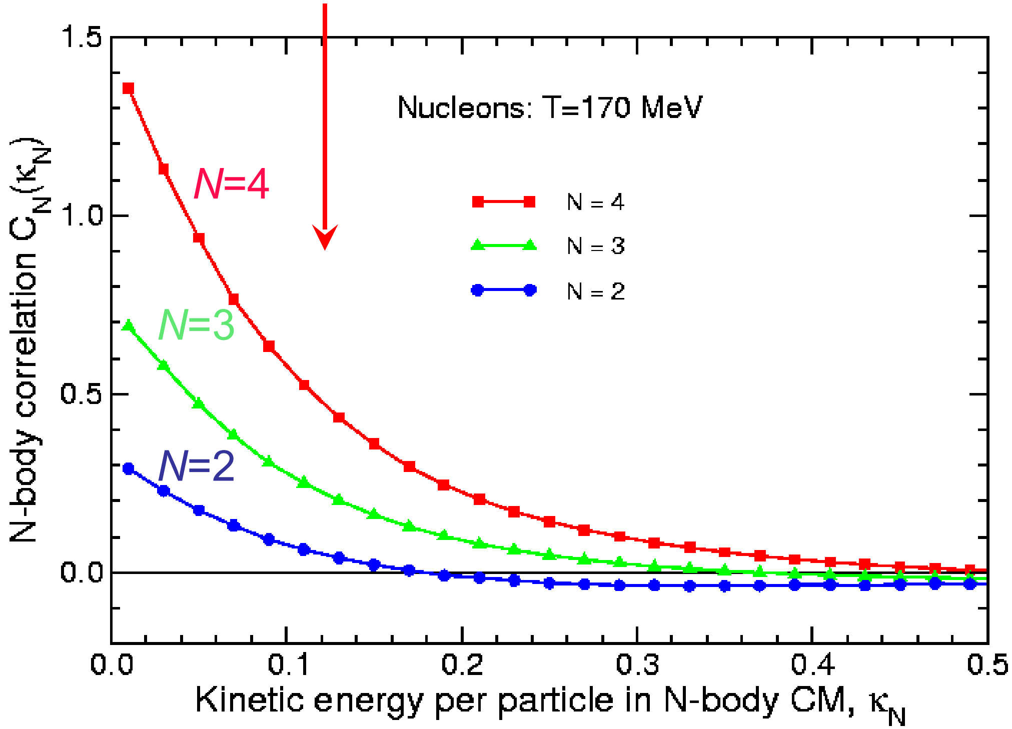
$$P_N(\kappa) \equiv \prec \delta(\kappa - \kappa_N\{\boldsymbol{p}_n\}) \succ$$

Correlation function:

$$C_N(\kappa) \equiv P_N(\kappa) \,/\, P_N^0(\kappa) - 1$$

Same event / *Mixed events*

- is *enhanced* at $\kappa \approx T$



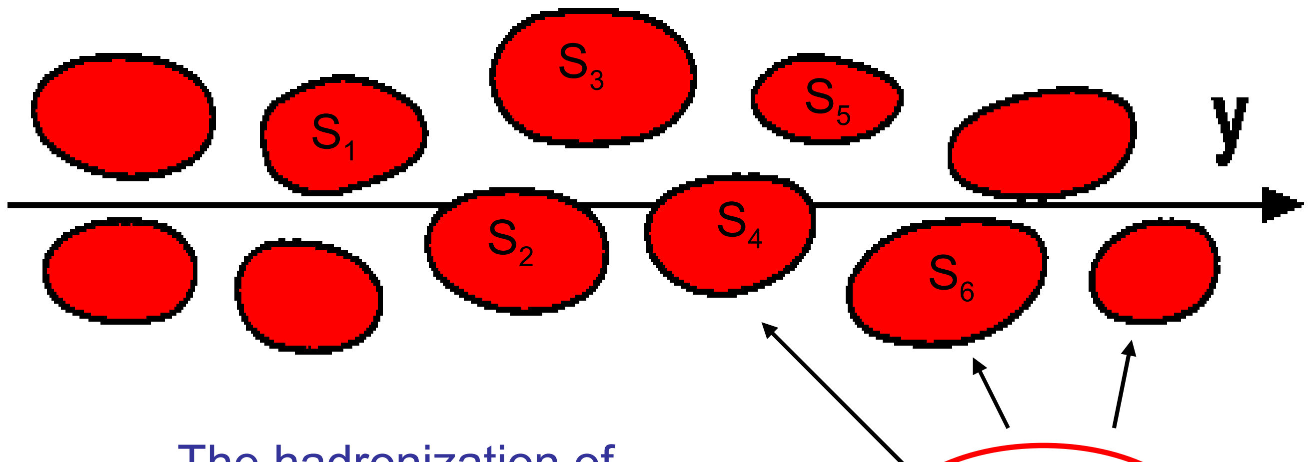
Higher-order correlations stand out more clearly!

(but require larger samples)

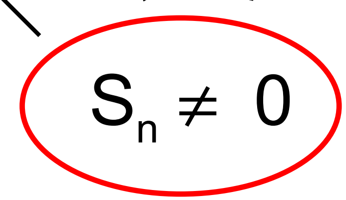
[J. Randrup, J. Heavy Ion Physics 22 (2005) 69]

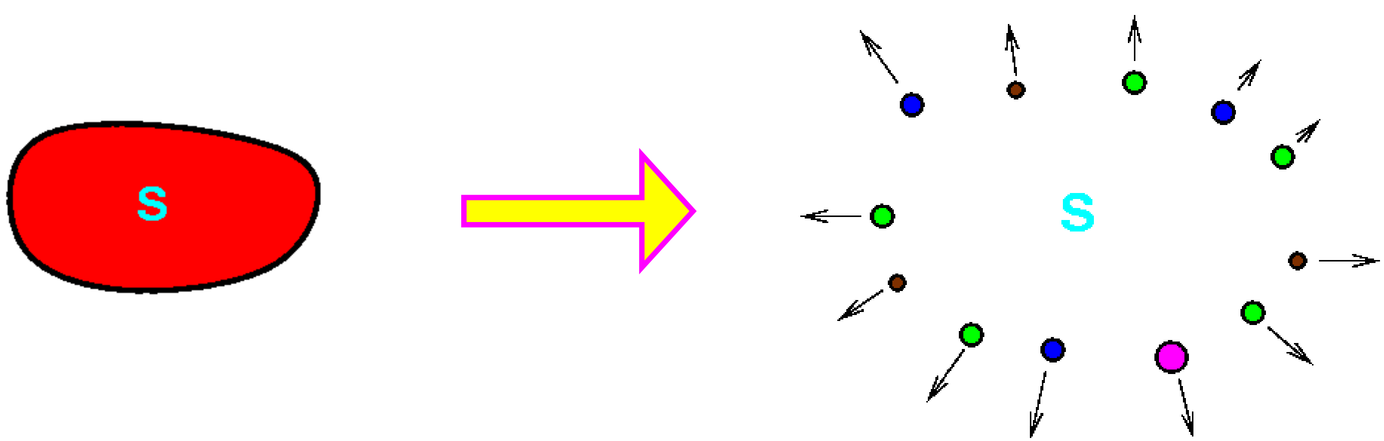
Strangeness correlations

The expanding system decomposes into plasma blobs which each contain a certain amount of strangeness:



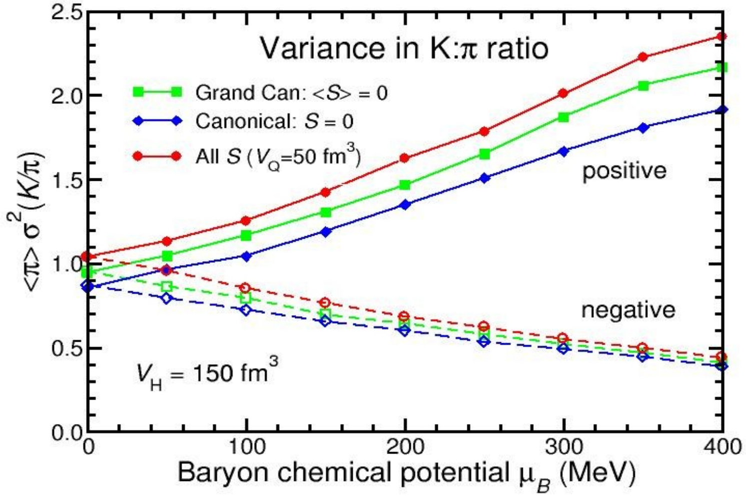
The hadronization of each isolated blob *conserves* strangeness:

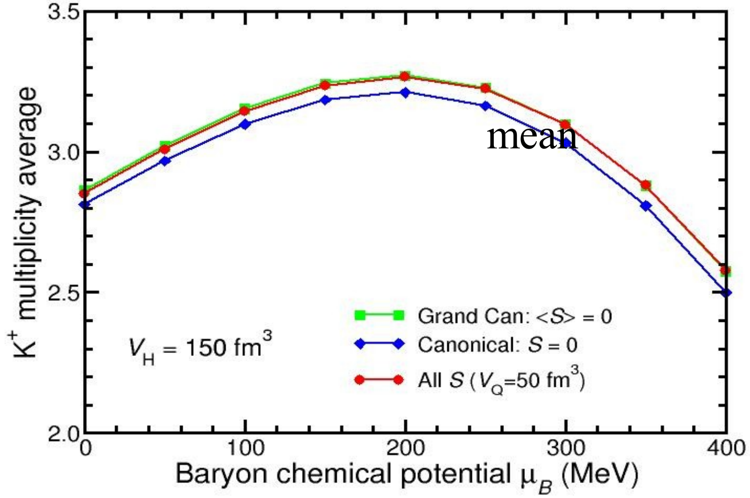




[V. Koch, A. Majumder, J. Randrup, Phys. Rev. C 72:064903,2005]

Some numbers





Variance: enhanced by ~10 %

V_{QGP}= 50 fm^3

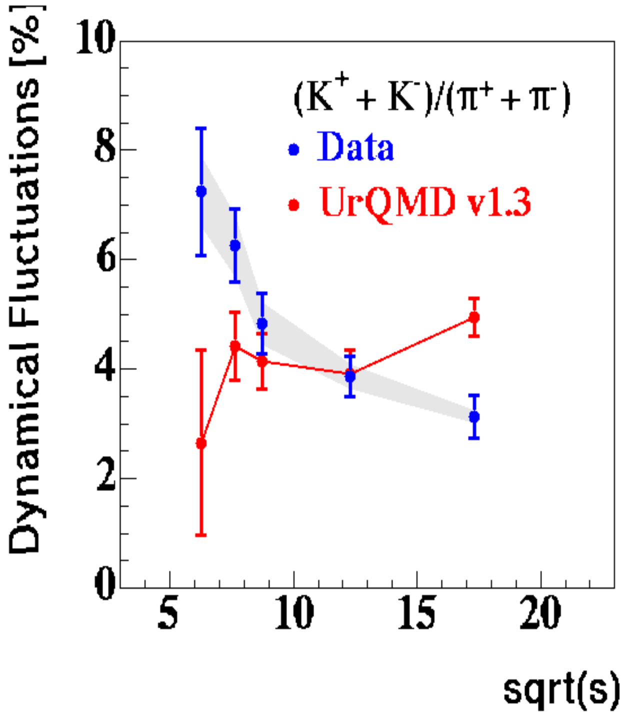
V_{hadron}= 150 fm^3

T = 170 MeV

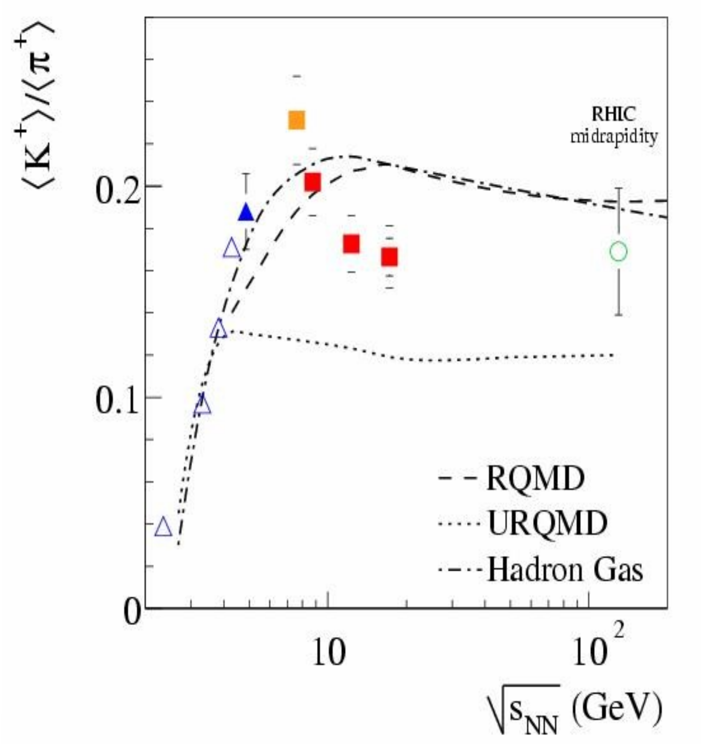
Generally: variance is more enhanced than mean

Strange things...

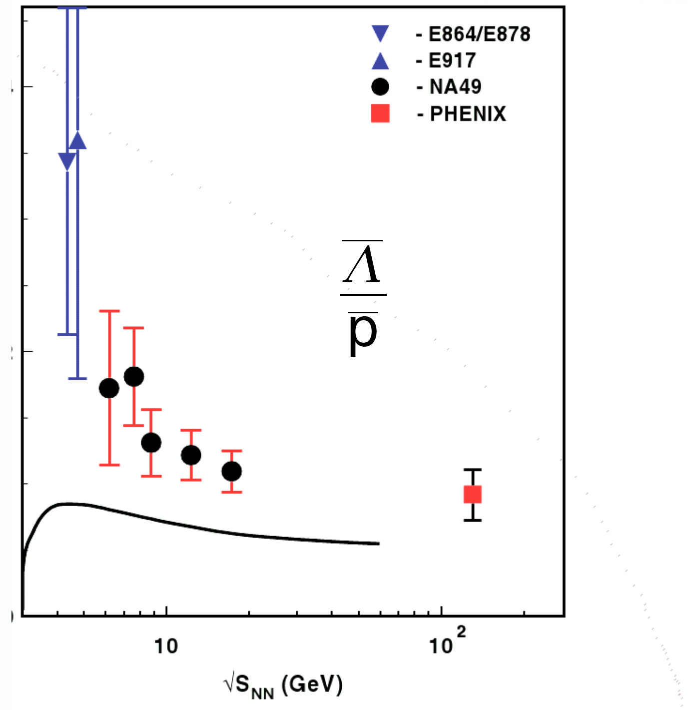
C. Roland, QM04, NA49



NA49



NA49, PRC73, 044910 (2006)

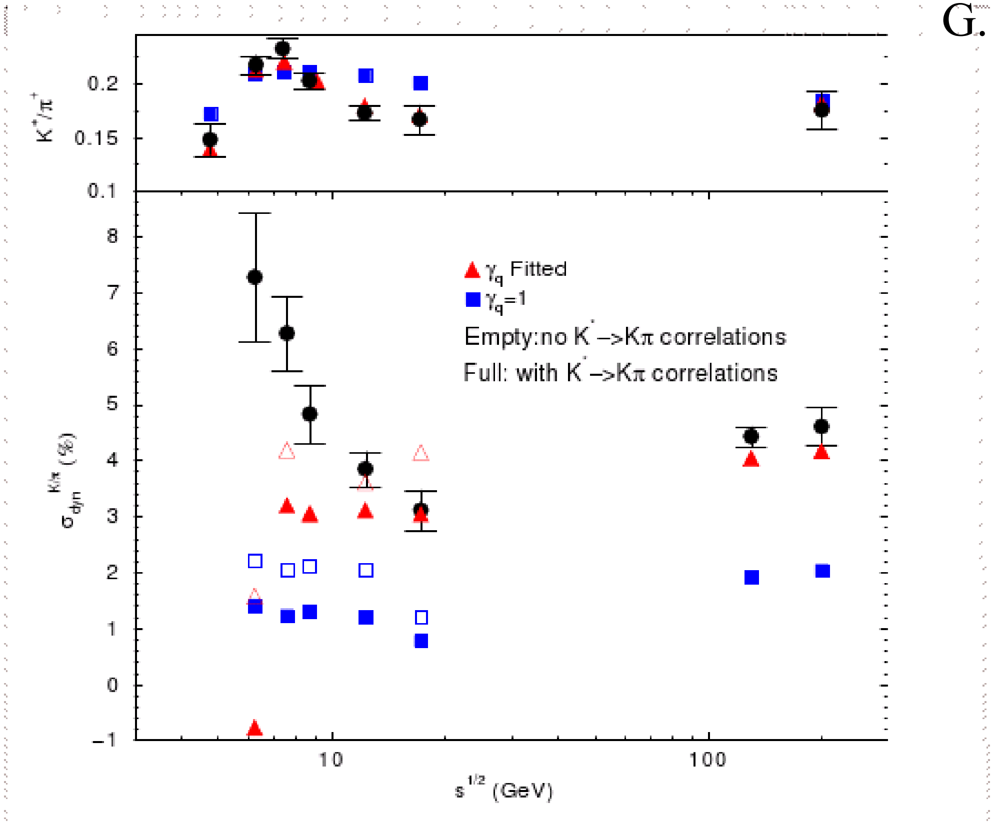


Hadron gas predictions

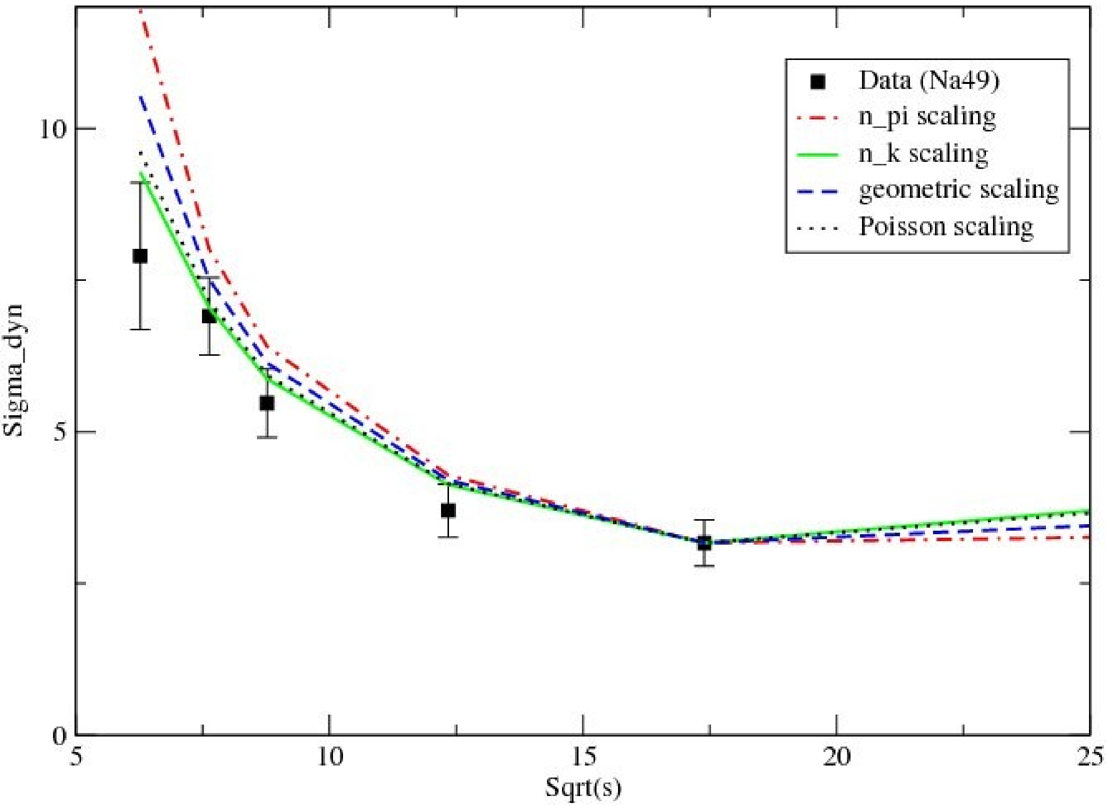
G. Torrieri, QM2006

$$\langle \frac{K}{\pi} \rangle$$

$$\langle \delta \left(\frac{K}{\pi}\right)^2 \rangle$$



Some trivial effects...



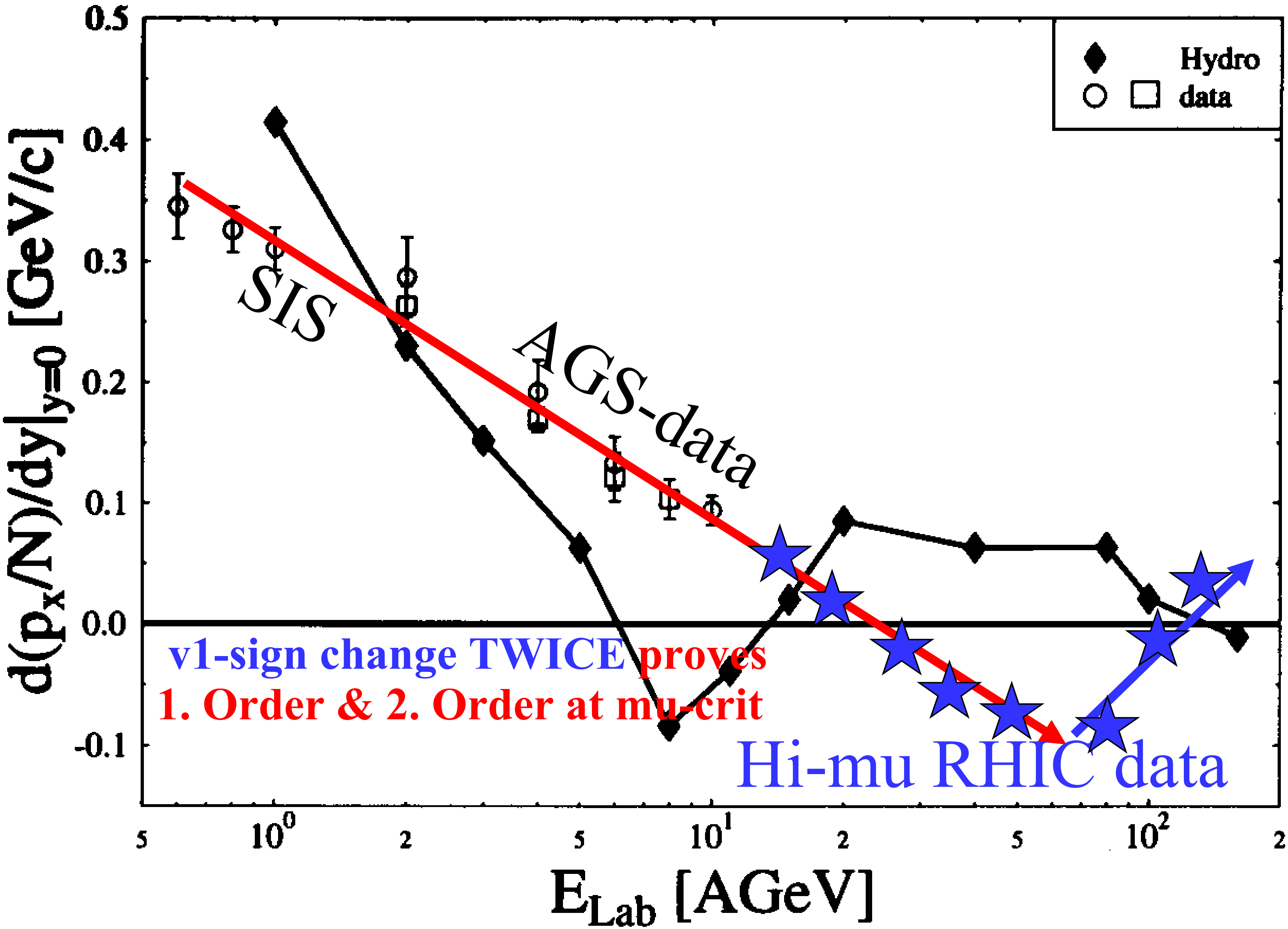
$$\sigma^2_{\mathrm{dyn}} = \frac{\langle(\delta K)^2 - K\rangle}{\langle K\rangle^2} + \frac{\langle(\delta\pi)^2 - \pi\rangle}{\langle\pi\rangle^2} - 2\frac{\langle\delta K\,\delta\pi\rangle}{\langle K\rangle\langle\pi\rangle}$$

$$= \frac{(\omega_K - 1)}{\langle K\rangle} + \frac{(\omega_\pi - 1)}{\langle\pi\rangle} - 2\frac{(\omega_{K\pi} - 1)}{\sqrt{\langle K\rangle\langle\pi\rangle}}$$

$$\sim 1/(\text{accepted Mulitplicity})$$

Other (“indirect”) observables

- Flow measurements (EOS, viscosity?)
- Lepton pairs? Only in conjunction with something else, such as baryon number fluctuations
  - Correlate baryon number with lepton yield in order to get after density fluctuations



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Critical Point and viscosities

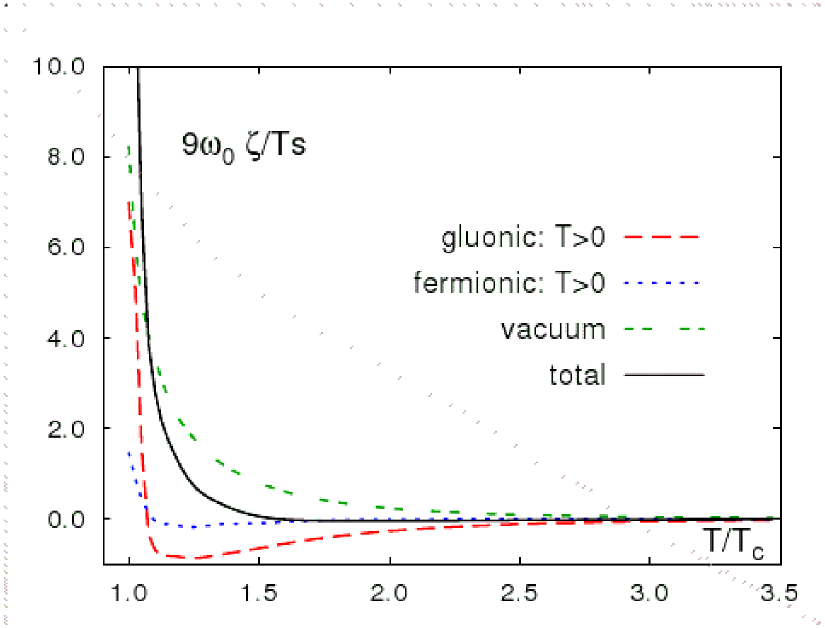
CP is in universality class of liquid gas (Son, Stephanov)

Hohenberg - Halperin Model H (Rev. Mod. Phys 49 (1977)):

$$\eta \sim \xi^{0.065,} \qquad \xi = \text{Correlation Length}$$

Shear viscosity diverges at CP

Bulk viscosity also diverges:
(Kharzeev, Turchin, Karsch arXiv:0711.0914)



Note: even large increase without PT
due to vacuum contribution

QCD critical point

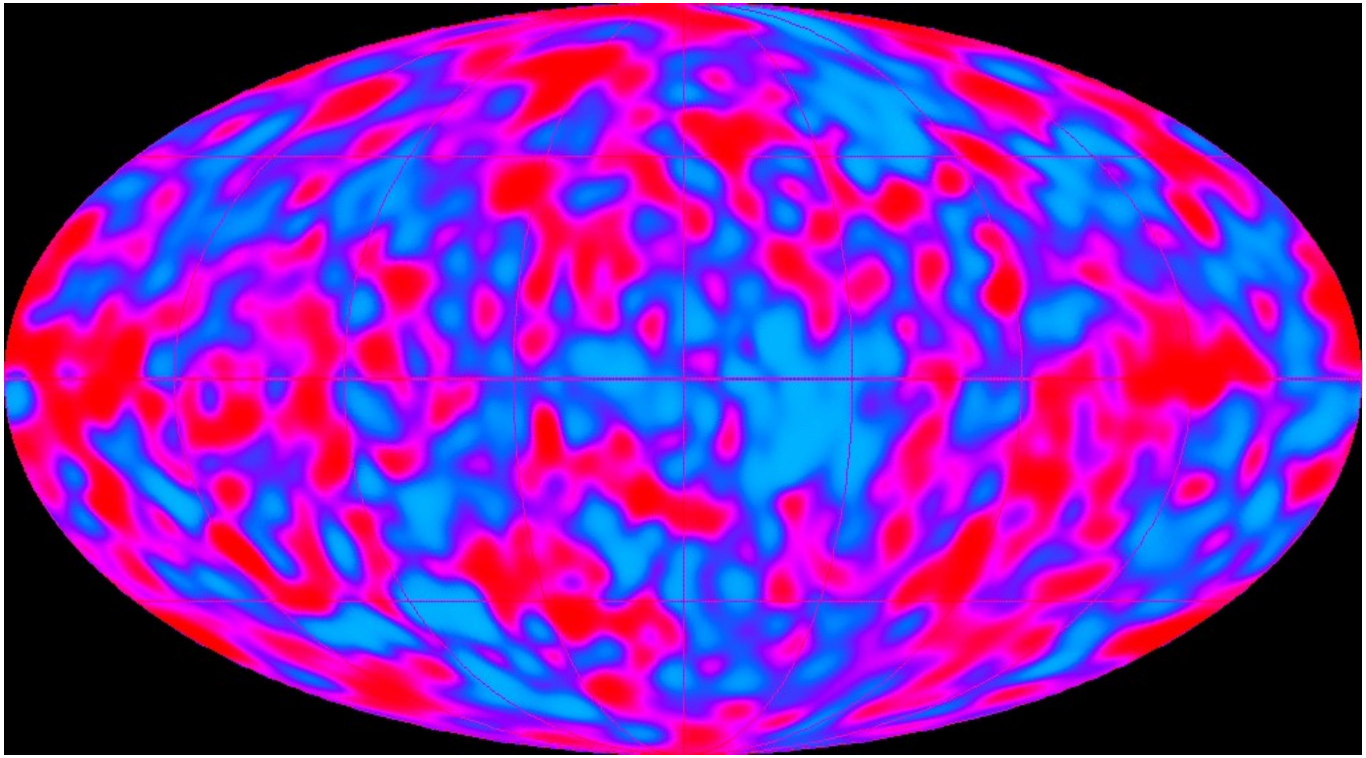
- Order parameter: baryon density or scalar density
  - Actually it is a superposition
- Both scalar (chiral) an quark number susceptibilities diverge
- **Screening** (“space like”) masses vanish (“omega”, “sigma”)
  - not accessible by (time-like) dileptons
- Is it related to chiral transition at m_q=0 ?
- The transition is in same universality class as liquid gas! (Son, Stephanov)
  - Fluctuations are driven by density fluctuations; chiral field is just tagging
- CP “just” the end of of 1^{st} Order transiition
  - Spinodal instabilities

Observables for CP and co-existence

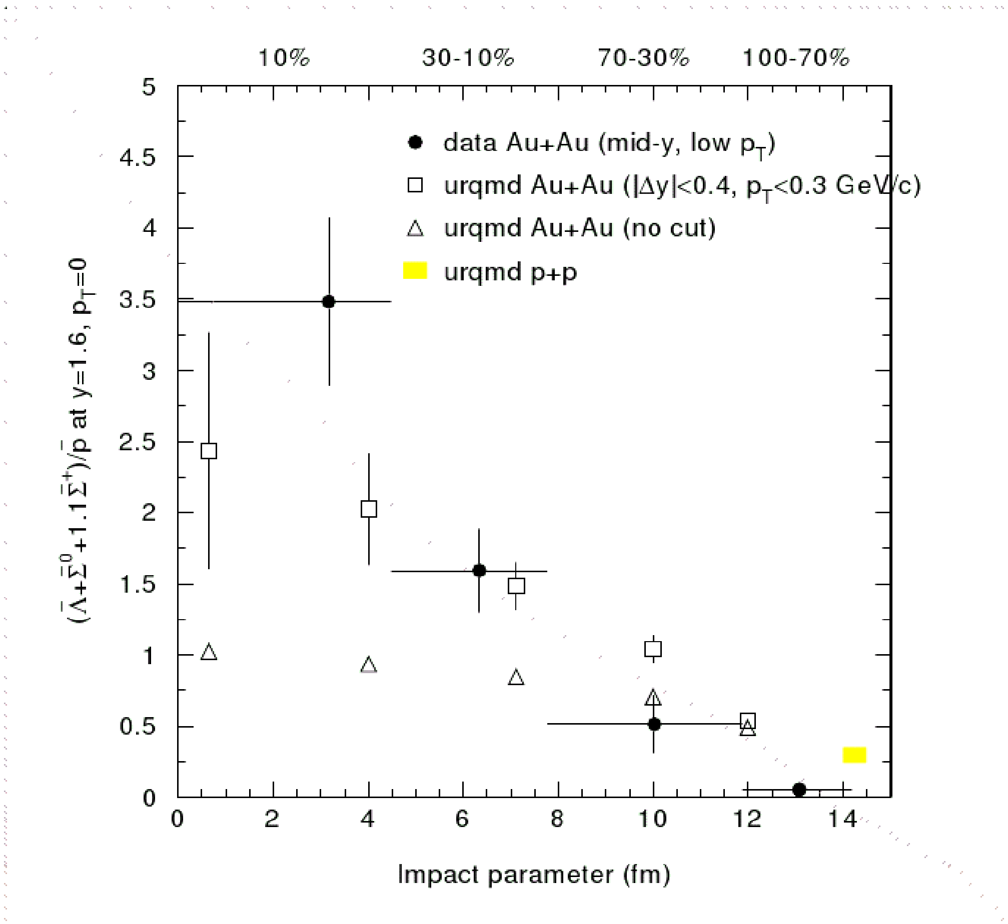
- Fluctuations (probably not of conserved charges)
- Correlations (spiondal blobs)
- Energy scan
- System size dependence (finite volume scaling)
  - centrality may not do
- Be prepared to measure everything
  - not clear (yet?) which observable couples strongest to baryon density
  - Would like to see finite volume scaling in more than one observable
- So far NOTHING seen

Summary

- Sign of phase co-existence CAN be seen in these type of experiments (Liquid Gas)
- Situation for QCD PT rather unsatisfactory
  - No firm theoretical guidance (Not even qualitative!)
  - Not clear how the phases present themselves (What are the “droplet”?
  - So far no evidence for or against PT of whatever kind
-



UrQMD and Lambda-bar / p-bar



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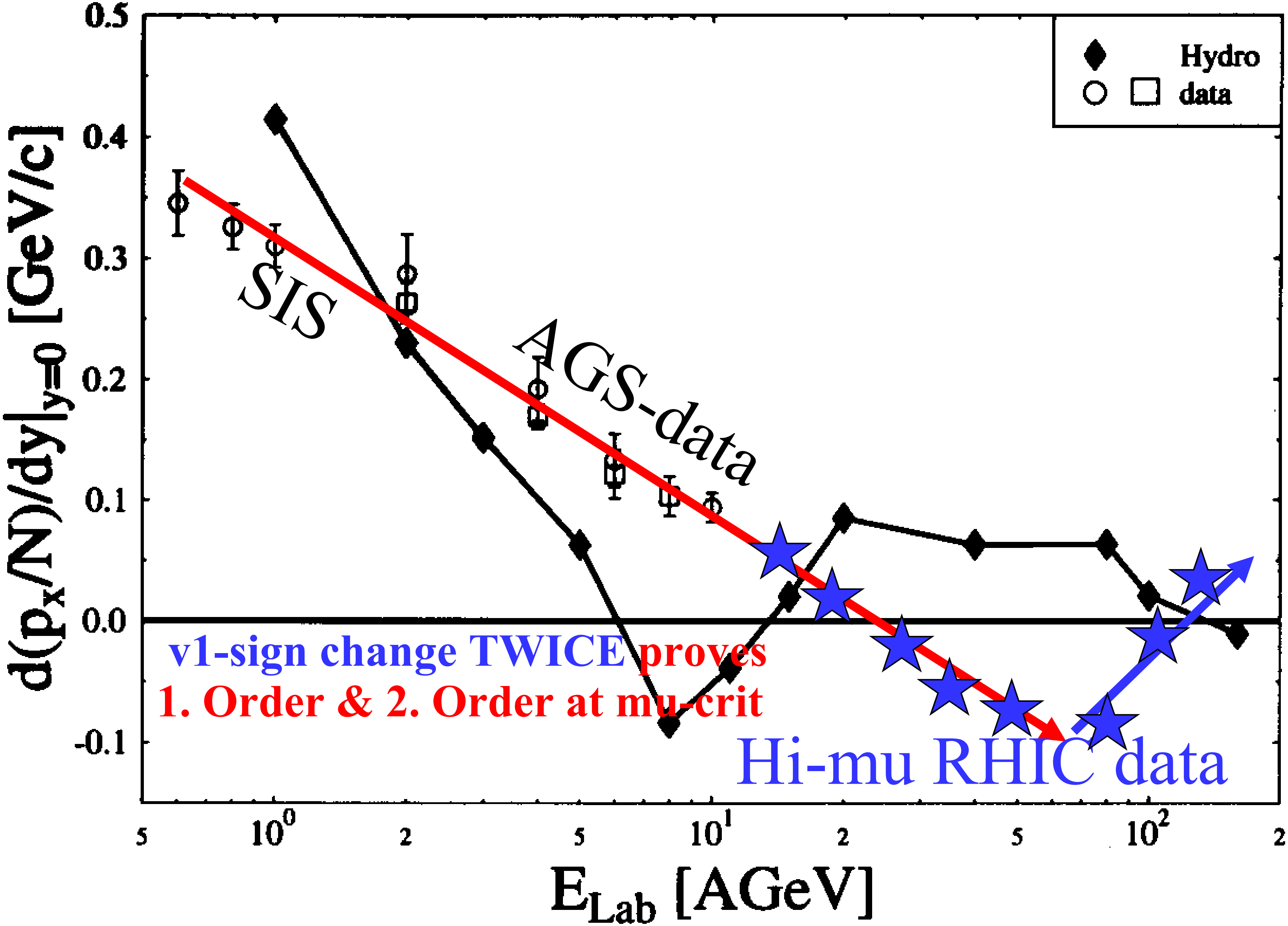
nucl-ex/0010002

Strong enhancement mostly an effect of acceptance cut !?



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