Equation of State and more from lattice regularized QCD

Frithjof Karsch, BNL& Bielefeld University

- Introduction

  Phases of Nuclear Matter

- Bulk thermodynamics

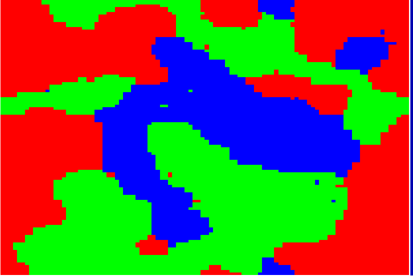
  cut-off effects in QCD thermodynamics

  the equation of state and velocity of sound at $\mu_q = 0$

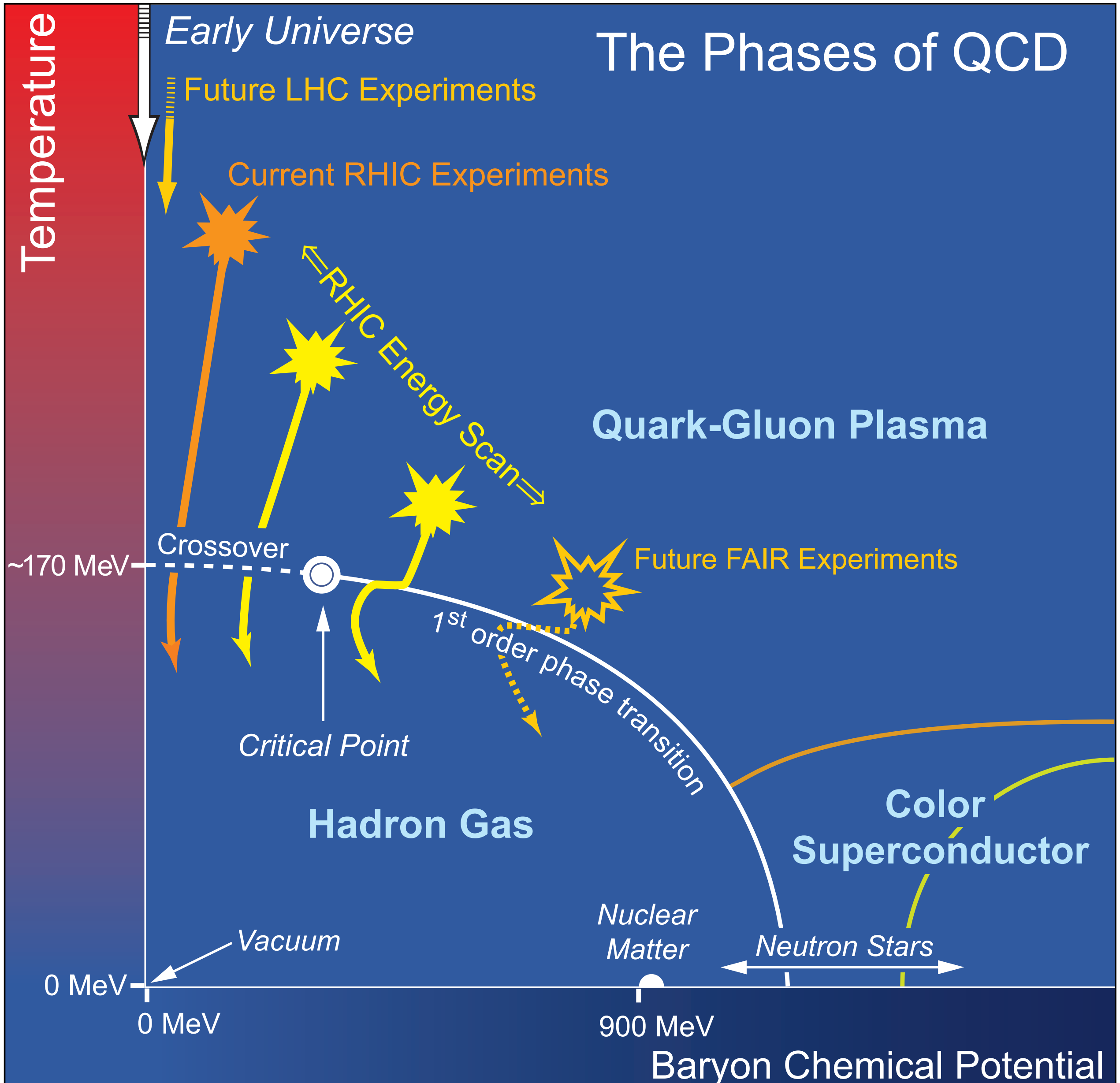
- Characterizing the QCD transition

  deconfinement and chiral symmetry restoration

- Conclusions



The Phases of Nuclear Matter



physics of the early universe

hot: $T \sim 10^{12} K$

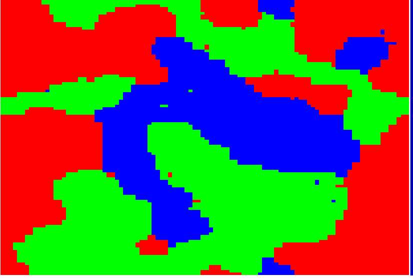
experimentally accessible

in Heavy Ion Collisions at

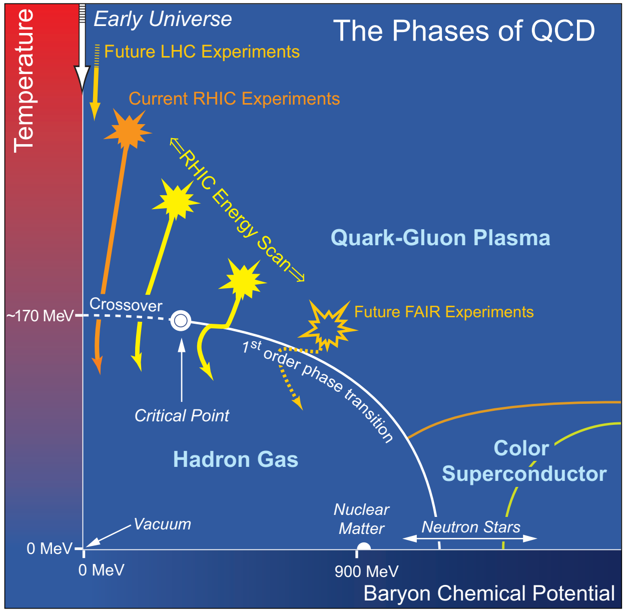
SPS, RHIC, LHC, FAIR

properties of compact stars

dense: $n_B \sim 10 n_{NM}$



The Phases of Nuclear Matter Key Questions (NP LRP 2007)



study properties of strongly interacting nuclear matter and elementary particles under extreme conditions

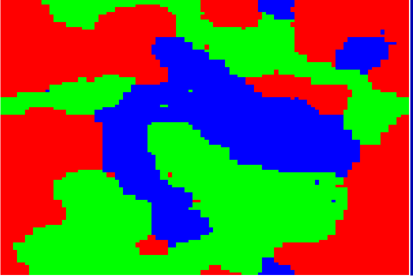
strongly interacting

$\Rightarrow$ QCD = Quantum Chromo Dynamics

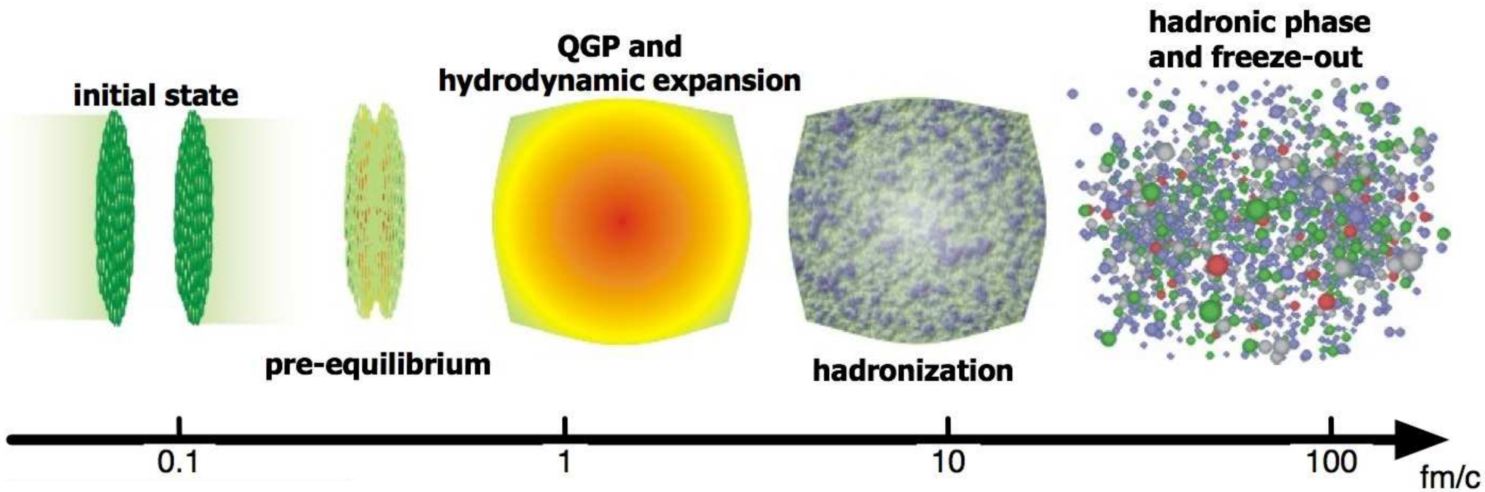
GOAL: learn about basic mechanisms that characterize QCD

chiral symmetry breaking; confinement; asymptotic freedom; axial anomaly

- What are the phases of strongly interacting matter, and what role do they play in the cosmos?
- What does QCD predict for the properties of strongly interacting matter?
- What governs the transition of quarks and gluons into pions and nucleons?



Heavy Ion collisions and the QGP

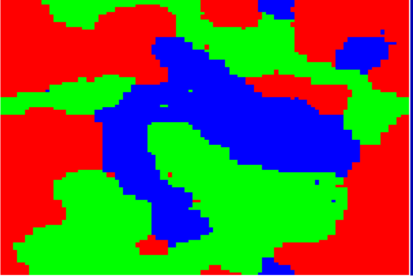


simple Bjorken model $\sim$ 1-d hydrodynamic expansion

equation of motion: $\partial_\mu T^{\mu\nu} = 0$

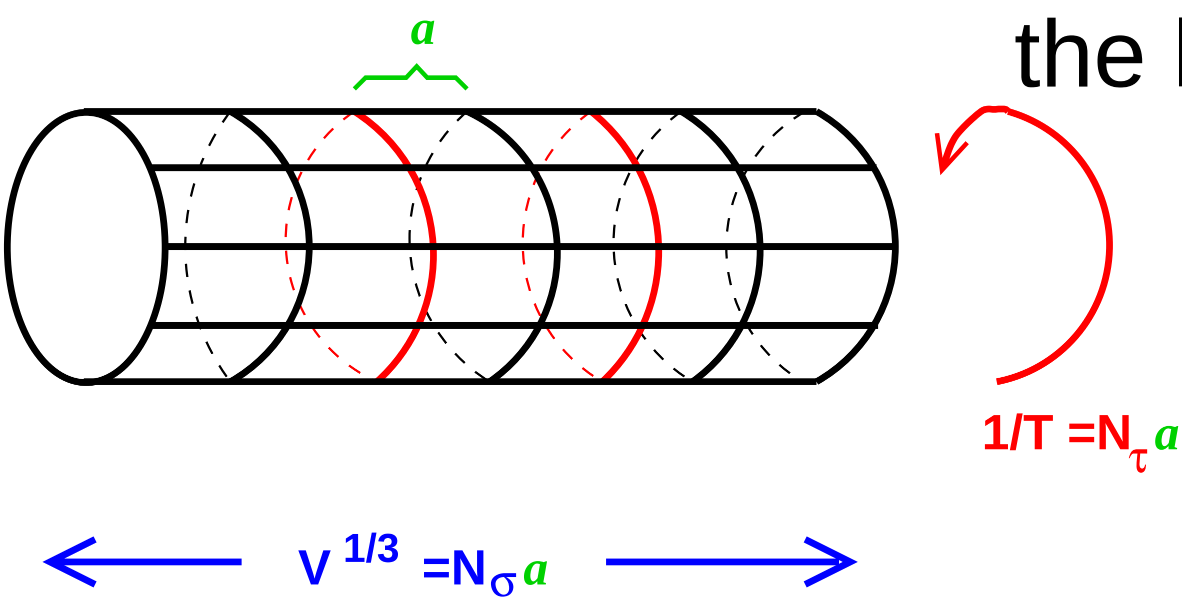
energy density: $\frac{d\epsilon}{d\tau} + \frac{1}{\tau}(\epsilon + p) - \frac{1}{\tau^2}\left(\frac{4}{3}\eta + \theta\right) = \epsilon(\tau_0)\delta(\tau - \tau_0)$

$\Rightarrow$ understanding the time evolution requires knowledge of the equation of state and transport coefficients (bulk (θ) and shear (η) viscosity)



QCD Thermodynamics: Simulating hot and dense matter

the lattice: $N_\sigma^3 \times N_\tau$



partition function:

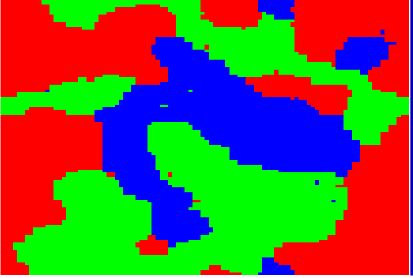
$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\; \mathrm{e}^{-S_E}$$

$$S_E = \int_0^{1/T} dx_0 \int_V d^3x\; \mathcal{L}_E(\mathcal{A},\psi,\bar{\psi},\mu)$$

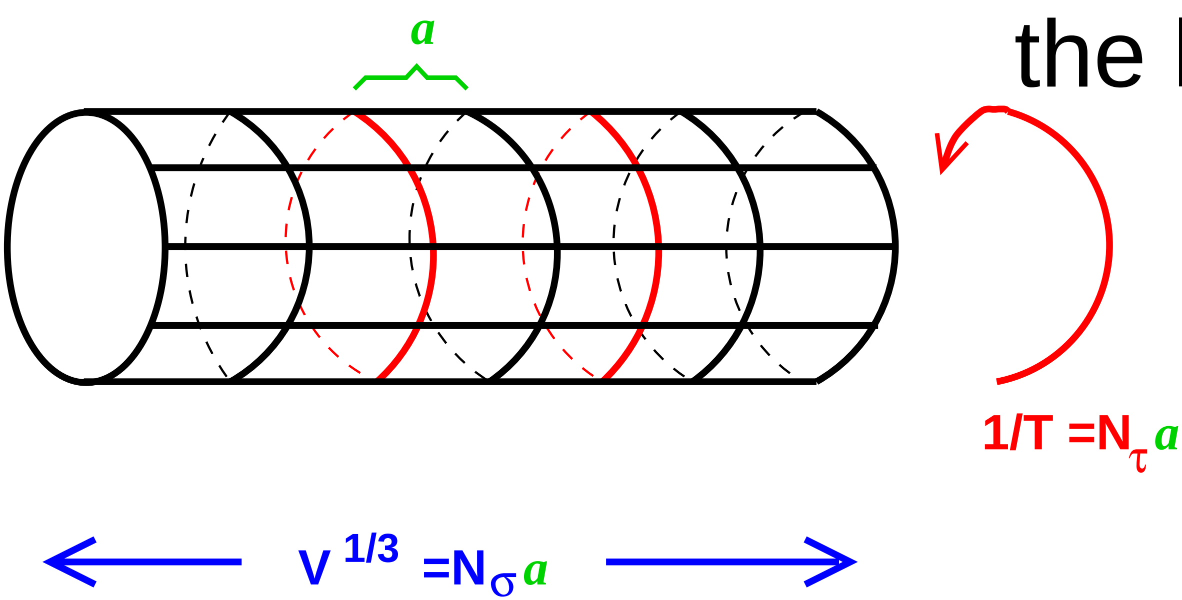
temperature volume chemical potential

$\mathcal{O}(10^6)$ grid points;
$\mathcal{O}(10^8)$ d.o.f.;

integrate eq. of motion



QCD Thermodynamics: Simulating hot and dense matter



the lattice: $N_\sigma^3 \times N_\tau$

the problem:

the fermion determinant requires

large scale computing

particularly difficult problems:

- low momentum structure of the QGP (many scales)
- finite density QCD (complex determinant)
- chiral formulation of QCD (5th dimension,..)

partition function:

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\; Det M(\mathcal{A},\mu)\; \mathrm{e}^{-S_G}$$

$$S_E = \int_0^{1/T} dx_0 \int_V d^3x\; \mathcal{L}_E(\mathcal{A},\psi,\bar{\psi},\mu)$$

temperature volume chemical potential

Bulk thermodynamics

Goal: QCD thermodynamics with realistic quark masses in (2+1)-f QCD and controlled extrapolation to the continuum limit;

$\Rightarrow$ T_c, EoS,.. for $\mu_q \geq 0$

RBC-Bielefeld collaboration

PRD77, 014511 (2008)

- $N_\tau = 4,\ 6$: bulk thermodynamics on a line of constant physics (LCP):
  (i) use $m_l = 0.1 m_s$, corresponding to $m_\pi \simeq 220$ MeV;
  (ii) tune m_s to physical strange quark mass using $m_K,\ m_{\bar{s}s}$
  at all values of the cut-off
- analyze EoS in a wide T-range: $140\ \mathrm{MeV} \lesssim T \lesssim 800\ \mathrm{MeV}$
- extend analysis to $N_\tau = 8$;
  compare p4 and asqtad results:
  joint project of RBC, Bielefeld, MILC, LANL and LLNL
  $\Rightarrow$ hotQCD collaboration

Cut-off effects and staggered fermions

the situation is more complex than in SU(3)

I) we have to deal with $\mathcal{O}(a^2)$ discretization errors;
just like in SU(3) but more severe!

$\mathcal{O}(a^2)$ improved actions for thermodynamics: Naik, p4

II) in addition we have to deal with $\mathcal{O}(a^2)$ violations of chiral symmetry

fat links in various variants, 3-link staple, 7-link staple (asqtad), stout,..

Cut-off effects with SF

the ideal gas (infinite temperature) limit (I):

- standard staggered fermions lead to $\mathcal{O}(a^2)$ errors in bulk thermodynamics
- p4-action and Naik action remove $\mathcal{O}(a^2)$ errors in bulk thermodynamics at $\mu = 0$ and $\mu > 0$

  $\Rightarrow$ $\mathcal{O}(a^2)$ improved pressure;

  $\Rightarrow$ small higher order corrections

Prasad Hegde et al.,
Eur. Phys. J. C55, 423 (2008)

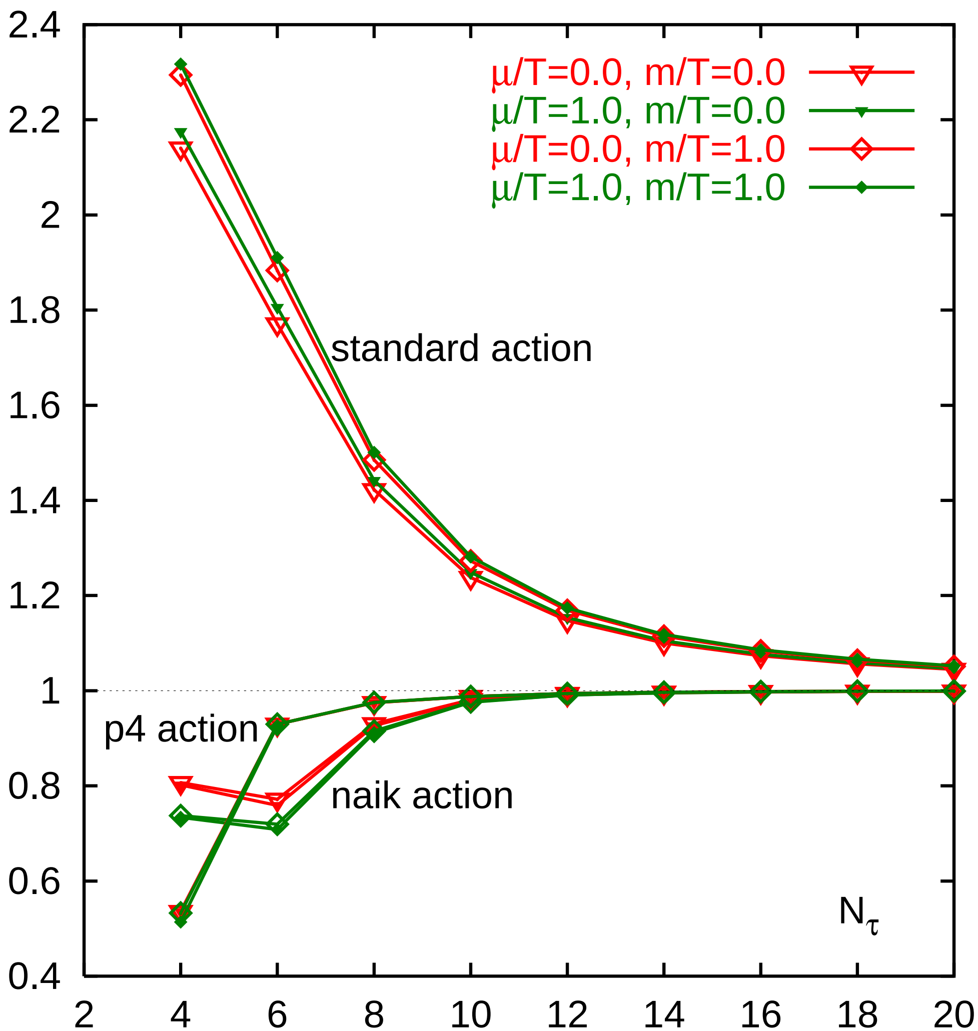
$$\frac{p}{p_{SB}} = 1 + \frac{248}{147}\left(\frac{\pi}{N_\tau}\right)^2 + \frac{635}{147}\left(\frac{\pi}{N_\tau}\right)^4 + \ldots \qquad (standard)$$

$$\frac{p}{p_{SB}} = 1 + 0 - \frac{1143}{980}\left(\frac{\pi}{N_\tau}\right)^4 + \frac{73}{2079}\left(\frac{\pi}{N_\tau}\right)^6 + \ldots \qquad (p4)$$

$$\frac{p}{p_{SB}} = 1 + 0 - \frac{1143}{980}\left(\frac{\pi}{N_\tau}\right)^4 - \frac{365}{77}\left(\frac{\pi}{N_\tau}\right)^6 + \ldots \qquad (Naik)$$

Cut-off effects with SF

the ideal gas (infinite temperature) limit (II):



cut-off effects similar for

$\mu = 0$ and $\mu > 0$

$\Rightarrow$

quark number susceptibilities

$\chi_{q,s}/T^2 \sim \partial^2 \ln Z/\partial(\mu_{q,s}/T)^2$

show similar cut-off effects as

pressure $p/T^4 \sim \ln Z$

Calculating the EoS on lines of constant physics (LCP)

- The interaction measure for $N_f = 2+1$ $\Leftrightarrow$ Trace Anomaly

$$\frac{\epsilon - 3p}{T^4} = T\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{p}{T^4}\right) = \left(a\frac{\mathrm{d}\beta}{\mathrm{d}a}\right)_{LCP} \frac{\partial p/T^4}{\partial \beta}$$

$$= \left(\frac{\epsilon - 3p}{T^4}\right)_{gluon} + \left(\frac{\epsilon - 3p}{T^4}\right)_{fermion} + \left(\frac{\epsilon - 3p}{T^4}\right)_{\hat{m}_s/\hat{m}_l}$$

- The pressure

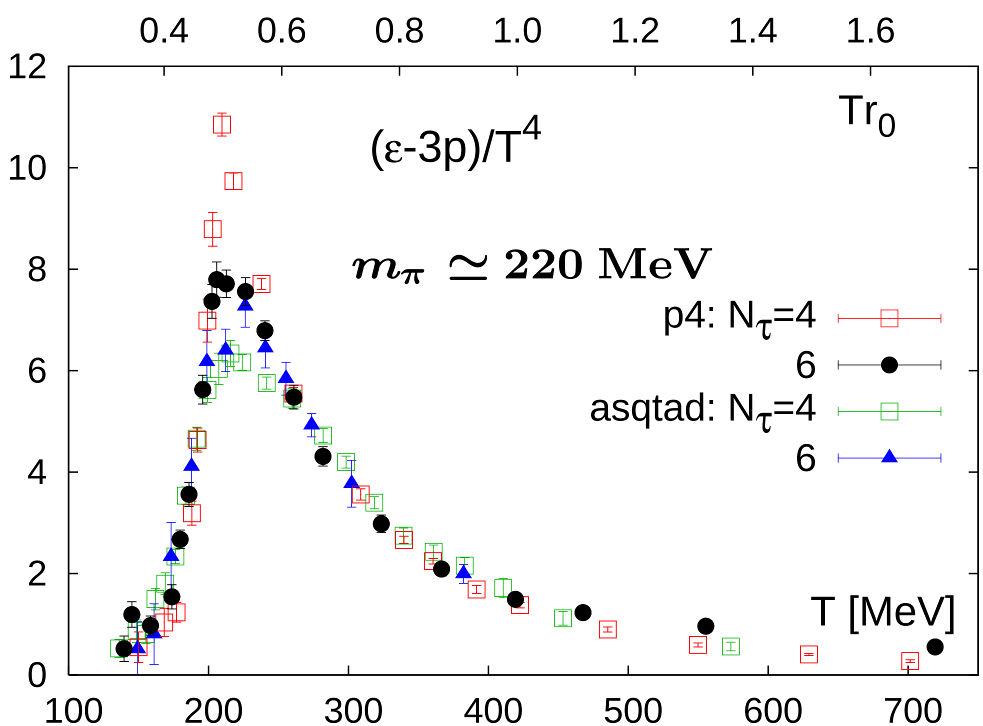
$$\left.\frac{p}{T^4}\right|_{\beta_0}^{\beta} = \int_{T_0}^{T} \mathrm{d}T\, \frac{1}{T}\left(\frac{\epsilon - 3p}{T^4}\right)$$

- need T-scale, $aT = 1/N_\tau$ and its relation to the gauge coupling $a \equiv a(\beta)$

N.B.: $a(\beta)$ is only defined through physical observables
$\Rightarrow$ choose a simple one

$(\epsilon - 3p)/T^4$ on LCP

- requires good control over $T > 0$ observables (action differences, chiral condensates); difficult: CPU requirement $\sim a^{-(10-12)}$
- requires accurate determination of $T = 0$ scales



p4 vs. asqtad: overall good agreement

However: We still need to...

$T < T_c$	make contact to hadron gas phenomenology
$T < 2T_c$	analyze large deviation from conformal limit ($\epsilon = 3p$)
$T > 2T_c$	make contact to (resummed) perturbation theory

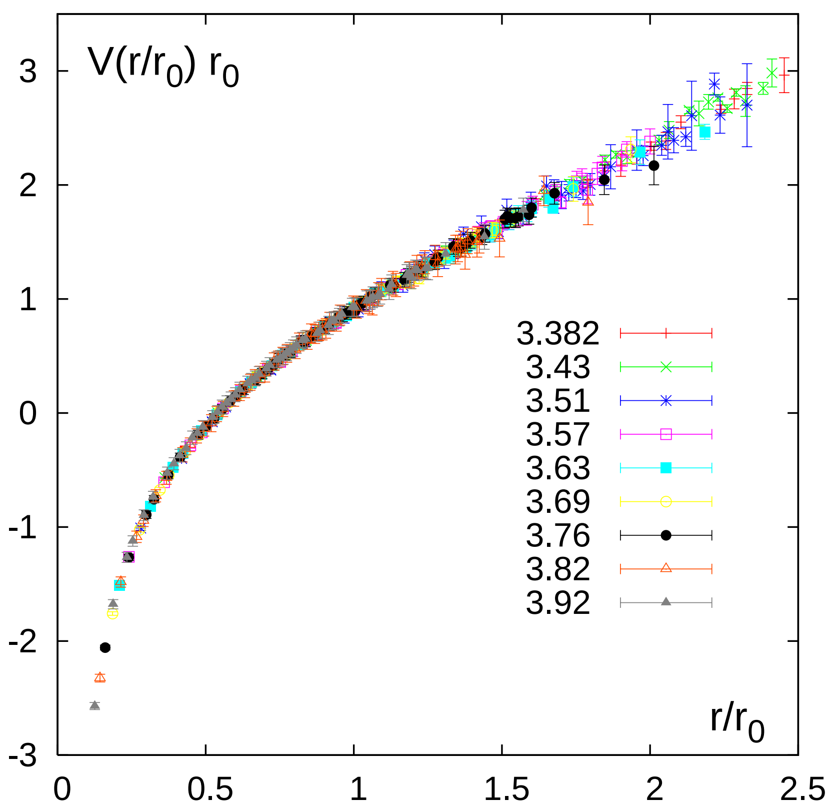
p4-data: RBC-Bielefeld
M. Cheng et al, PRD77, 014511 (2008)

asqtad data: MILC
C. Bernard et al., PRD 75, 094505 (2007)

$T = 0$ scale setting using the heavy quark potential

use r_0 or string tension to set the scale for $T = 1/N_\tau a(\beta)$

$$V(r) = -\frac{\alpha}{r} + \sigma r \quad , \quad r^2 \frac{\mathrm{d}V(r)}{\mathrm{d}r}\Big|_{r=r_0} = 1.65$$

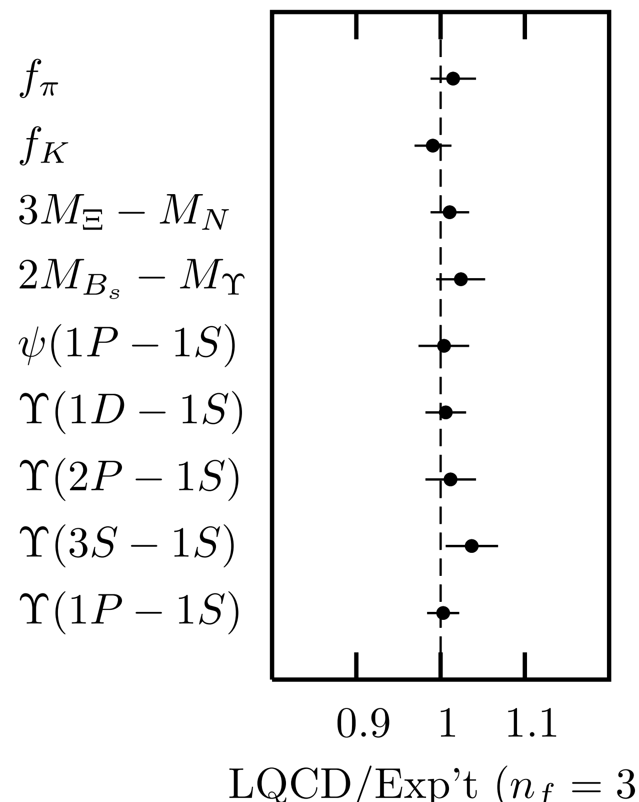


no significant cut-off dependence when cut-off varies by a factor 5

i.e. from the transition region on $N_\tau = 4$ lattices ($a \simeq 0.25$ fm) to that on $N_\tau = 20$ lattices ($a \simeq 0.05$ fm) !!

scales extracted from 'gold plated observables'

- high precision studies of several experimentally well known observables in lattice calculations with staggered (asqtad) fermions led to convincing agreement $\Rightarrow$ gold plated observables
- simultaneous determination of r_0/a in these calculations determines the scale r_0 in MeV



knowing any of these experimentally accessible quantities accurately from a lattice calculation is equivalent to knowing r_0, which is a fundamental parameter of QCD

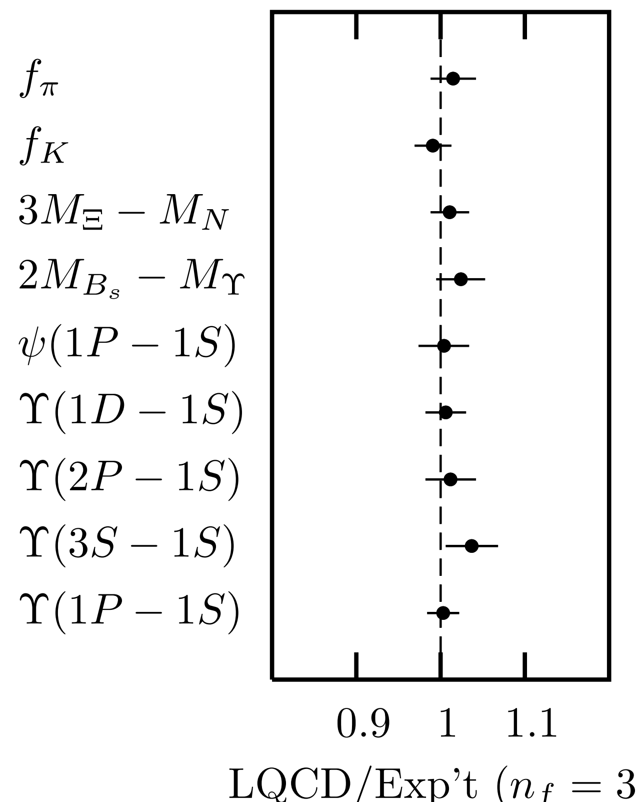
C.T.H. Davies et al., PRL 92 (2004) 022001
A. Gray et al., PRD72 (2005) 094507

scales extracted from 'gold plated observables'

- high precision studies of several experimentally well known observables in lattice calculations with staggered (asqtad) fermions led to convincing agreement $\Rightarrow$ gold plated observables
- simultaneous determination of r_0/a in these calculations determines the scale r_0 in MeV

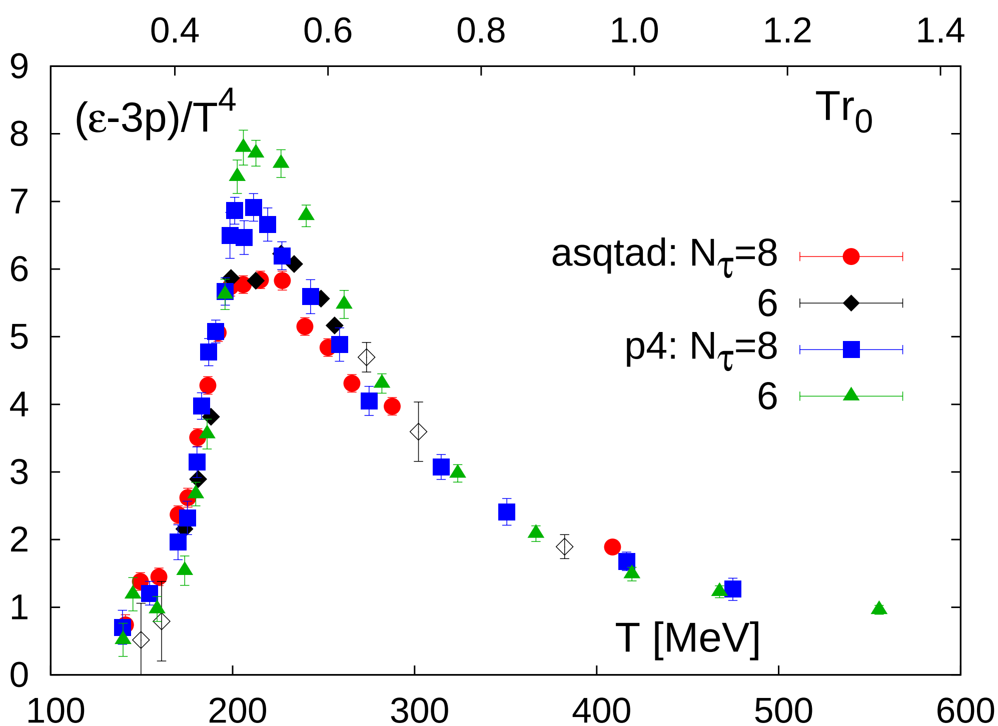
we use $r_0 = 0.469(7)$ fm determined from quarkonium spectroscopy
A. Gray et al, Phys. Rev. D72 (2005) 094507

C.T.H. Davies et al., PRL 92 (2004) 022001
A. Gray et al., PRD72 (2005) 094507

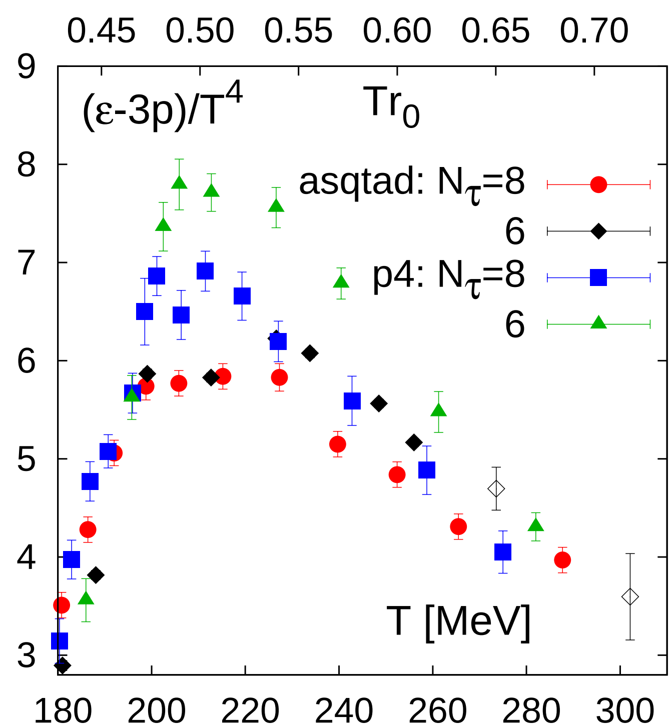


..towards the cont. limit: $N_\tau = 8$

hotQCD-collaboration PRELIMINARY



trace anomaly



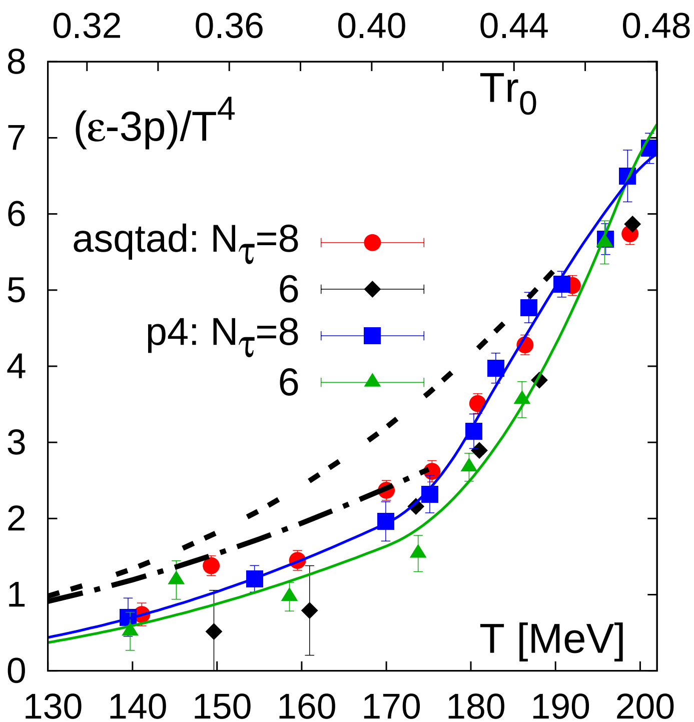
peak region

- $[(\epsilon - 3p)/T^4]_{max} \gtrsim 200$ MeV
  $\sim$ softest point of EoS
  $\sim$ minimum of velocity of sound
- cut-off effects persist in the peak region: discretization errors for $T \in [200\mathrm{MeV}, 220\mathrm{MeV}]$: $\sim 15\%$

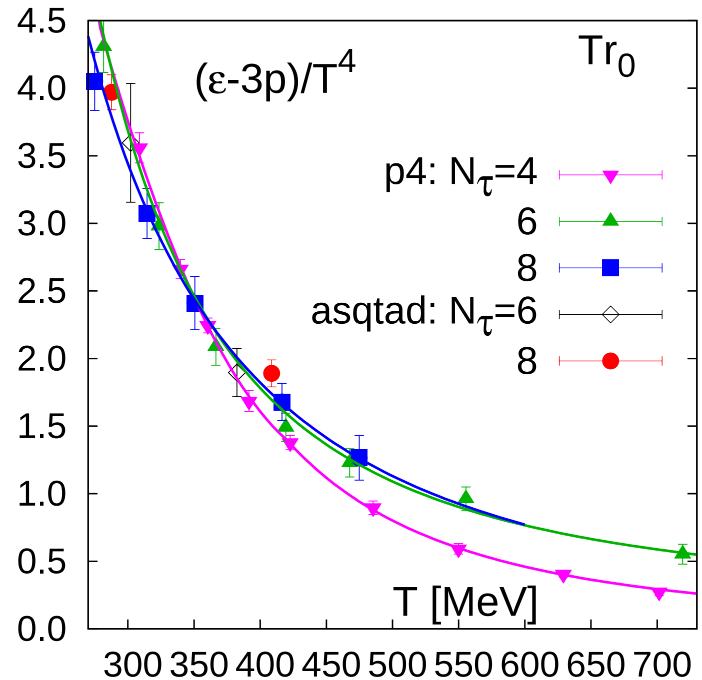
- high-T: good agreement between $N_\tau = 6$ and 8 results for $T \gtrsim 300$ MeV;

  significant deviations from perturbative behavior

EoS: low and high T regime



LGT vs resonance gas

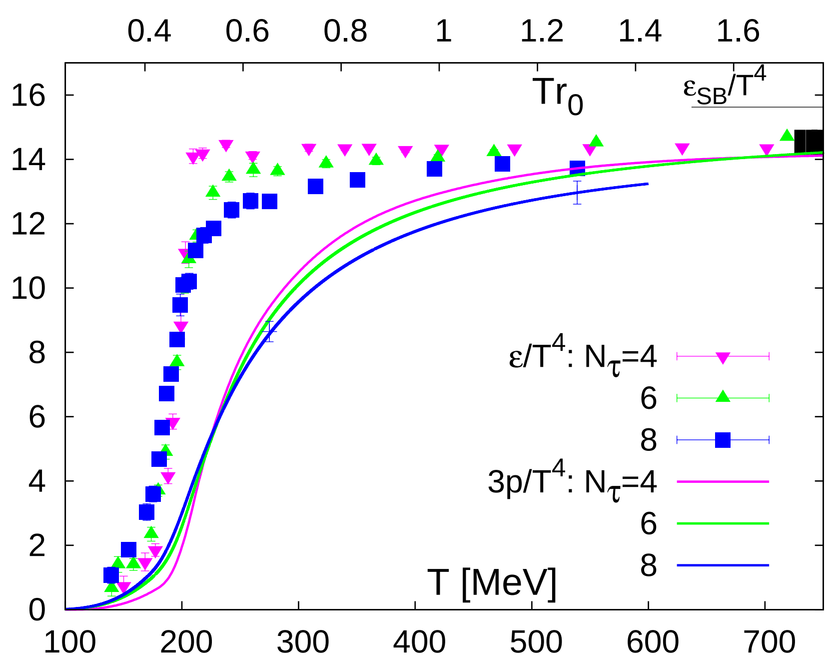


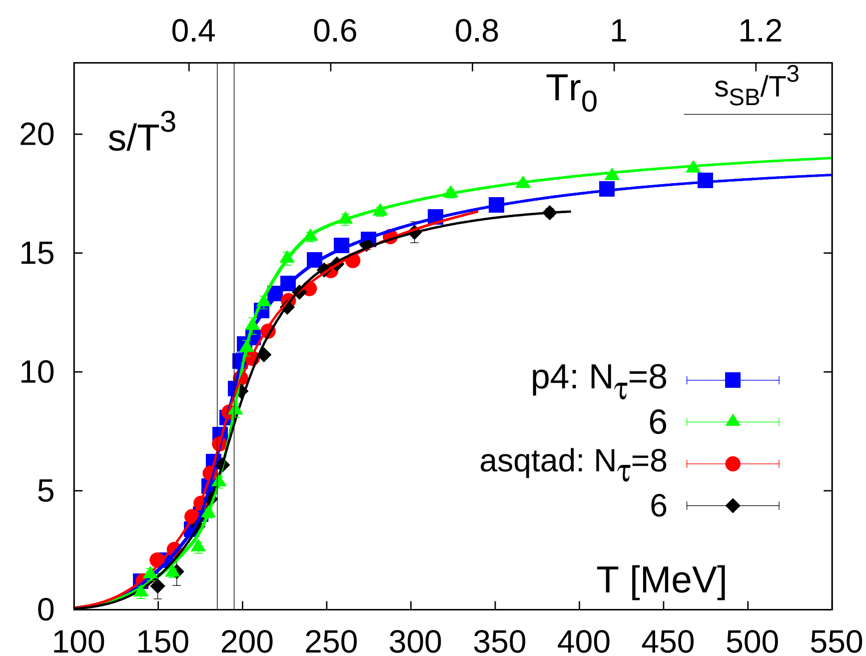
LGT vs. pert. theory

- approach to continuum limit
  $\rightarrow$ $N_\tau = 6,\ 8$
  $\rightarrow$ $\mathcal{O}(5MeV)$ shift of T-scale
- LGT below HRG for $T \lesssim 180$ MeV
  coarser lattice, larger cut-off effects
  but: Which HRG?
  M_{max} =1.5 GeV, 2.5 GeV,..
- approach to continuum limit
  $\rightarrow$ $N_\tau = 6,\ 8$: no significant cut-off dependence for $T \gtrsim 300$ MeV
- strong deviations from conformal limit:
  find $(\epsilon - 3p)/T^4 \sim a/T^2 + b/T^4$ for $300\text{MeV} \lesssim T \lesssim 700\text{MeV}$

Pressure, Energy and Entropy

- p/T^4 from integration over $(\epsilon - 3p)/T^5$;
starting integration at $T = 0$ MeV with $p(0) = 0$;
use hadron resonance gas at $T_0 = 100$ MeV to estimate systematic error:
$[p(T_0)/T_0^4]_{HRG} \simeq 0.265$
- high-T region is well under control; significant deviations from conformal limit



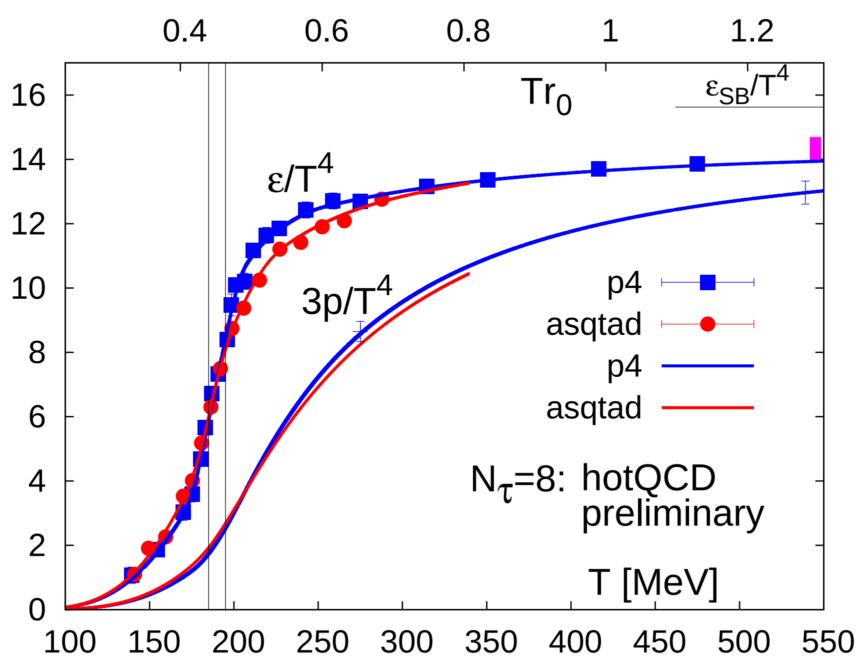


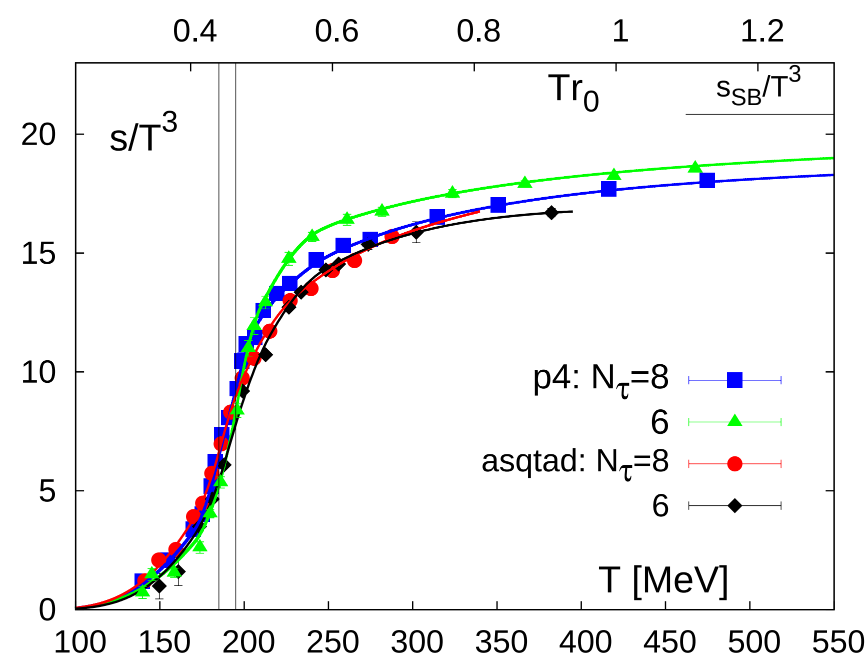
band:
$185 \text{ MeV} \leq T \leq 195 \text{ MeV}$

hotQCD, preliminary: p4 vs. asqtad

Pressure, Energy and Entropy

- p/T^4 from integration over $(\epsilon - 3p)/T^5$;
  starting integration at $T = 0$ MeV with $p(0) = 0$;
  use hadron resonance gas at $T_0 = 100$ MeV to estimate systematic error:
  $[p(T_0)/T_0^4]_{HRG} \simeq 0.265$
- high-T region is well under control; significant deviations from conformal limit



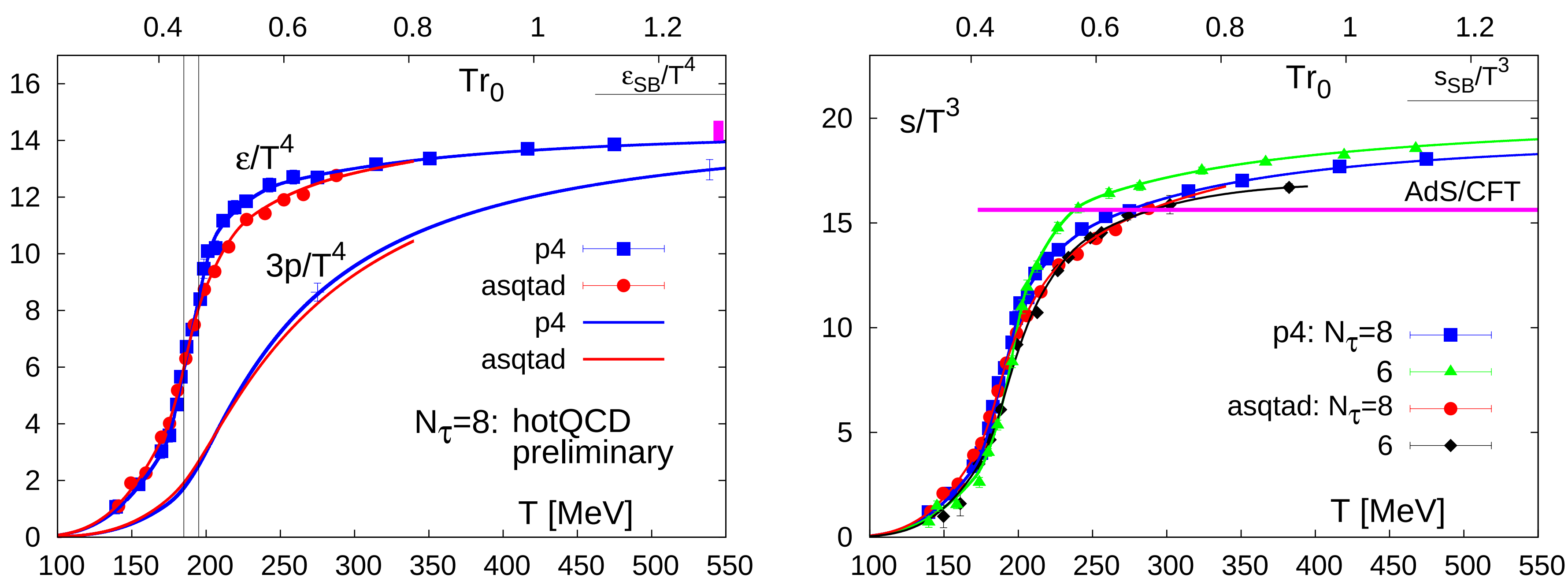


band:
$185 \text{ MeV} \leq T \leq 195 \text{ MeV}$

hotQCD, preliminary: p4 vs. asqtad

Pressure, Energy and Entropy

- p/T^4 from integration over $(\epsilon - 3p)/T^5$;
  starting integration at $T = 0$ MeV with $p(0) = 0$;
  use hadron resonance gas at $T_0 = 100$ MeV to estimate systematic error:
  $[p(T_0)/T_0^4]_{HRG} \simeq 0.265$
- high-T region is well under control; significant deviations from conformal limit
  AND AdS/CFT



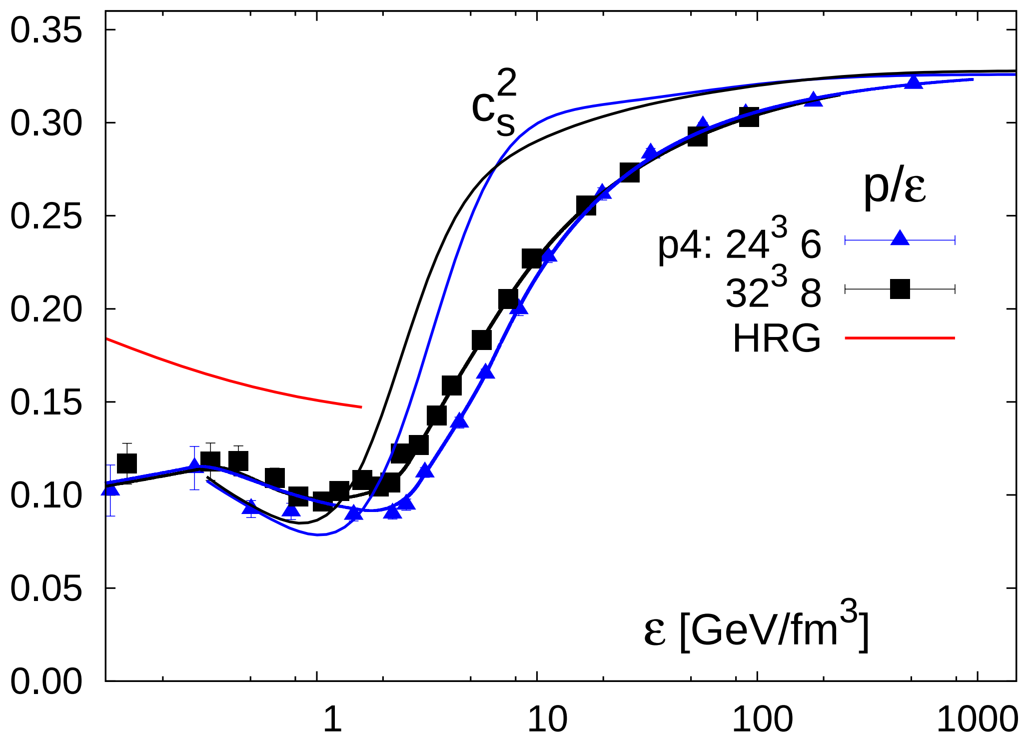
band:
$185\ \mathrm{MeV} \leq T \leq 195\ \mathrm{MeV}$

hotQCD, preliminary: p4 vs. asqtad

EoS and velocity of sound

- $p/\epsilon \Rightarrow$ velocity of sound:

$$c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}\epsilon} = \epsilon \frac{\mathrm{d}(p/\epsilon)}{\mathrm{d}\epsilon} + \frac{p}{\epsilon} \equiv \frac{s}{c_V}$$



hydro-expansion:

$p/\epsilon < 1/3$

$\Rightarrow$ slows down expansion;

$\Rightarrow$ increases plasma lifetime

e.g.

$1 \leq \epsilon$ [GeV/fm^3]≤ 10

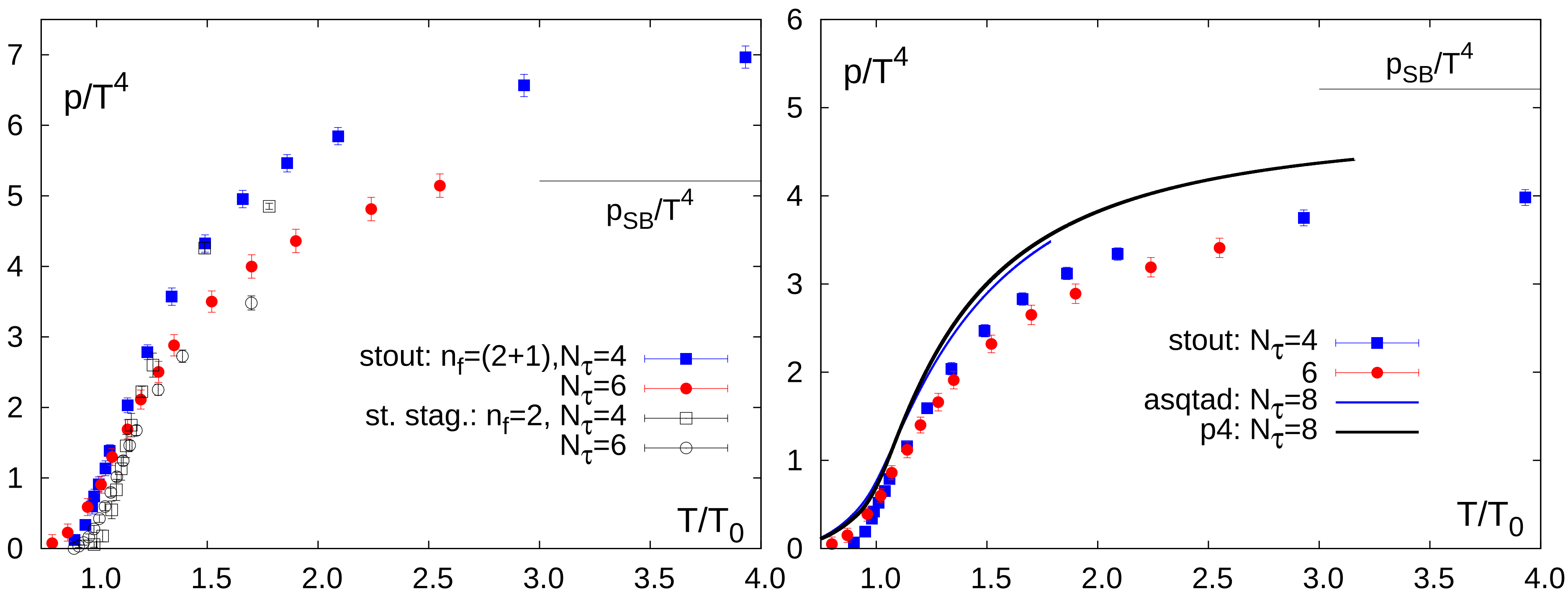
$\Rightarrow \Delta\tau \simeq$ 5.5 fm (ideal gas)

$\Rightarrow \Delta\tau \simeq$ 7 fm (LGT EoS)

hotQCD, preliminary

Cut-off effects with SF ($T < \infty$)

- $\mathcal{O}(a^2)$ improvement crucial for controling the high-T structure of the EoS
- same holds true for quark number susceptibilities
  R. V. Gavai, S. Gupta and P. Majumdar, PRD65, 054506 (2002)
- dividing out the lattice-SB value over-compensates cut-off effects



st.stag.: C.W. Bernard et al. (MILC), PRD55, 6861 (1997)
stout: Y. Aoki et al., JHEP 0601, 089 (2006)

hotQCD, preliminary

Deconfinement and χ-symmetry

- The chiral phase transition (i.e. at $m_q = 0$) is deconfining
  - true in QCD, i.e. SU(3) + fermions in the fundamental representation
  - SU(3) + fermions in the adjoint representation: $T_{deconf} < T_\chi$
- The transition in QCD with physical quark masses is a crossover

  In which sense is the transition

  deconfining and chiral symmetry restoring?

  - deconfinement: heavy hadrons $\Rightarrow$ light quarks and gluons; liberation of many new light degrees of freedom
    $\Rightarrow$ rapid change in ϵ/T^4, s/T^3, ...
  - chiral symmetry restoration: vanishing mass splittings, no new degrees of freedom
    $\Rightarrow$ minor effect on bulk thermodynamics, but
    rapid change of chiral condensate

Deconfinement and χ-symmetry

- The chiral phase transition (i.e. at $m_q = 0$) is deconfining
  - true in QCD, i.e. SU(3) + fermions in the fundamental representation
  - SU(3) + fermions in the adjoint representation: $T_{deconf} < T_\chi$
- The transition in QCD with physical quark masses is a crossover

In which sense is the transition

deconfining and chiral symmetry restoring?

- deconfinement: heavy hadrons $\Rightarrow$ light quarks and gluons; liberation of many new light degrees of freedom
  $\Rightarrow$ rapid change in ϵ/T^4, s/T^3, ...
- chiral symmetry restoration: vanishing mass splittings, no new degrees of freedom
  $\Rightarrow$ minor effect on bulk thermodynamics, but
  rapid change of chiral condensate

Critical behavior & chiral limit of QCD

- Universal critical behavior (thermal): $f(T,\mu_q) = f_s + f_r$

$$
\begin{aligned}
f_s(T,\mu_q) &= b^{-1} f_s(tb^{1/(2-\alpha)}) \sim t^{2-\alpha} \\
t &= \left|\frac{T-T_c}{T_c}\right| + A\left(\frac{\mu_q}{T_c}\right)^2 \quad , \qquad \alpha < 0 \text{ for } O(N)
\end{aligned}
$$

- fluctuations of Goldstone modes influence behavior in the chiral limit also away from (thermal) criticality

$$
\langle\bar\psi\psi\rangle \sim \begin{cases} c(T)\sqrt{m_q} + d(T)m_q + \text{regular} & T < T_c \\ c_\delta m_q^{1/\delta} + d(T_c)m_q + \text{regular} & T = T_c \\ d(T)m_q + \text{regular} & T > T_c \end{cases}
$$

$$
\Rightarrow \quad \chi_m \sim \left.\frac{\partial\langle\bar\psi\psi\rangle}{\partial m_q}\right|_{m_q=0} \sim \begin{cases} \infty & T \le T_c \\ t^{-\gamma} & T > T_c \end{cases}
$$

Quark number susceptibility...
...and its susceptibility

- rapid change in quark/baryon/strangeness number susceptibility reflects change in mass of the carrier of these quantum numbers ⇔ DECONFINEMENT
- quark number susceptibility feels nearby singular point just like the energy density

scaling field: $t = \left|\frac{T - T_c}{T_c}\right| + A\left(\frac{\mu_q}{T_c}\right)^2 \quad , \quad \mu_{crit} = 0$

singular part: $f_s(T, \mu_q) = b^{-1} f_s(t b^{1/(2-\alpha)}) \sim t^{2-\alpha}$

Y. Hatta, T. Ikeda, PRD67 (2003) 014028

$$c_2 \equiv \chi_q \sim \frac{\partial^2 \ln Z}{\partial \mu_q^2} \sim t^{1-\alpha} \quad , \quad c_4 \sim \frac{\partial^4 \ln Z}{\partial \mu_q^4} \sim t^{-\alpha} \qquad (\mu = 0)$$

$$\epsilon \sim \frac{\partial \ln Z}{\partial T} \sim t^{1-\alpha} \quad , \quad C_V \sim \frac{\partial^2 \ln Z}{\partial T^2} \sim t^{-\alpha} \qquad (\mu = 0)$$

$\Rightarrow 2^{nd}$ derivative w.r.t μ_q "looks like energy density"

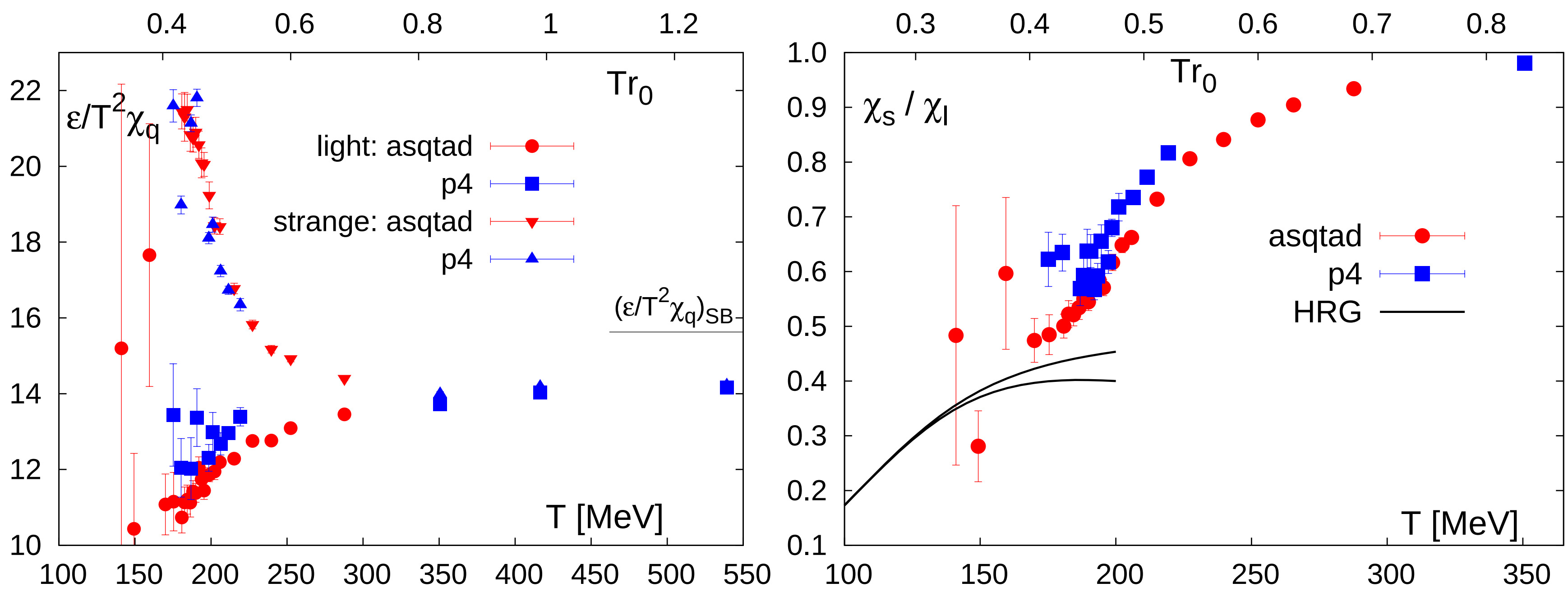
$\Rightarrow 4^{th}$ derivative w.r.t μ_q "looks like specific heat"

Energy Density and Light Quark Susceptibility

- singular parts of ϵ/T^4 and χ_l/T^2 have identical T-dependence
- χ_s and ϵ couple to different excitations at low T:

$$\chi_s \sim \exp(-m_K/T)\ ,\ \epsilon \sim \exp(-m_\pi/T)$$

- χ_s/T^2 not sensitive to critical behavior in the chiral limit

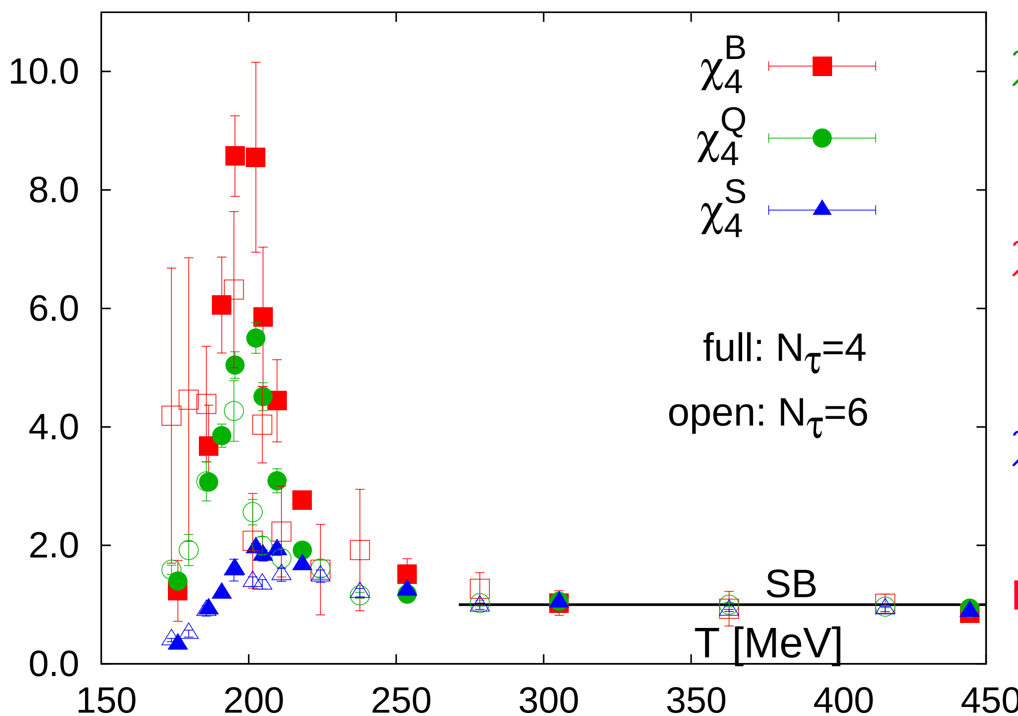


$N_\tau = 8$: hotQCD preliminary

Quartic fluctuations of baryon number charge & strangeness in (2+1)-flavor QCD

RBC-Bielefeld, arXiv:0811.1006

vanishing chemical potentials:



$$\chi_4^Q = \frac{1}{VT^3}\left(\langle Q^4\rangle - 3\langle Q^2\rangle^2\right)$$

$$\chi_4^B = \frac{1}{VT^3}\left(\langle N_B^4\rangle - 3\langle N_B^2\rangle^2\right)$$

$$\chi_4^S = \frac{1}{VT^3}\left(\langle N_S^4\rangle - 3\langle N_S^2\rangle^2\right)$$

rapid approach to SB limit

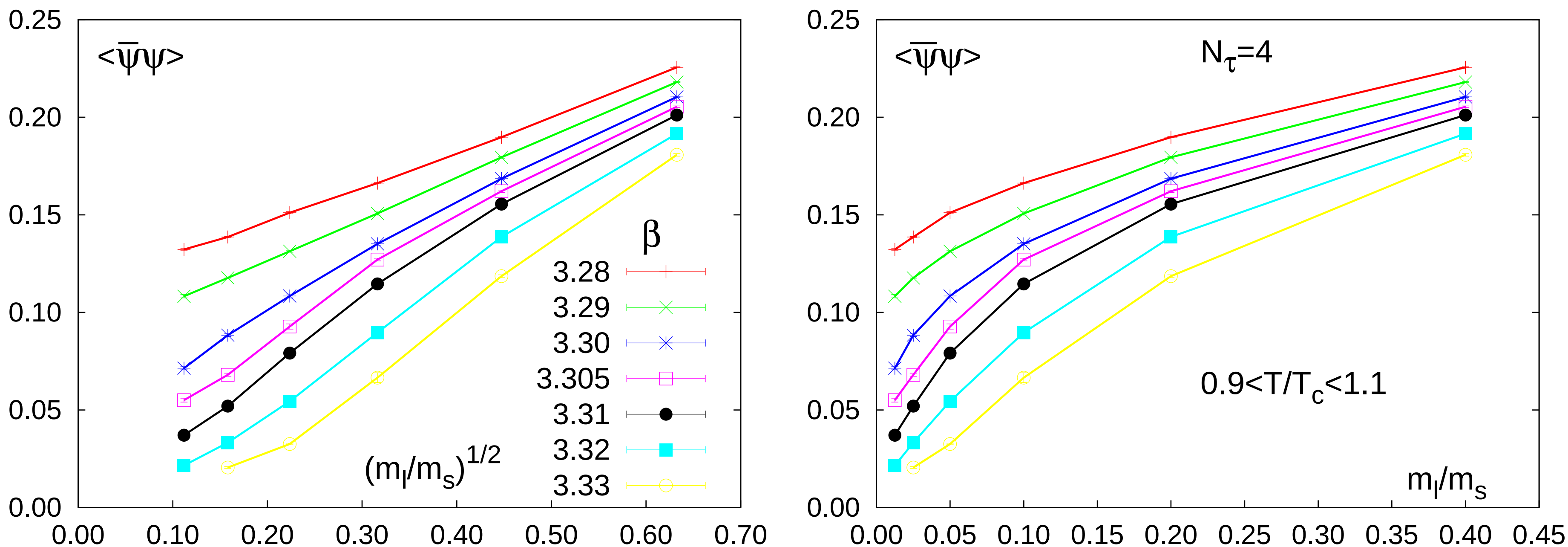
$\Rightarrow$ large light quark number & charge fluctuations across transition region

chiral limit: $\chi_4^B,\ \chi_4^Q \sim |T - T_c|^{-\alpha} + \mathbf{regular}$

$N_\tau = 4$: chiral condensate

(RBC-Bielefeld collaboration, in preparation)

$$\langle \bar\psi\psi \rangle \quad = \quad \frac{1}{N_\sigma^3 N_\tau} \frac{n_f}{4} \frac{\partial \ln Z}{\partial m_l a}$$

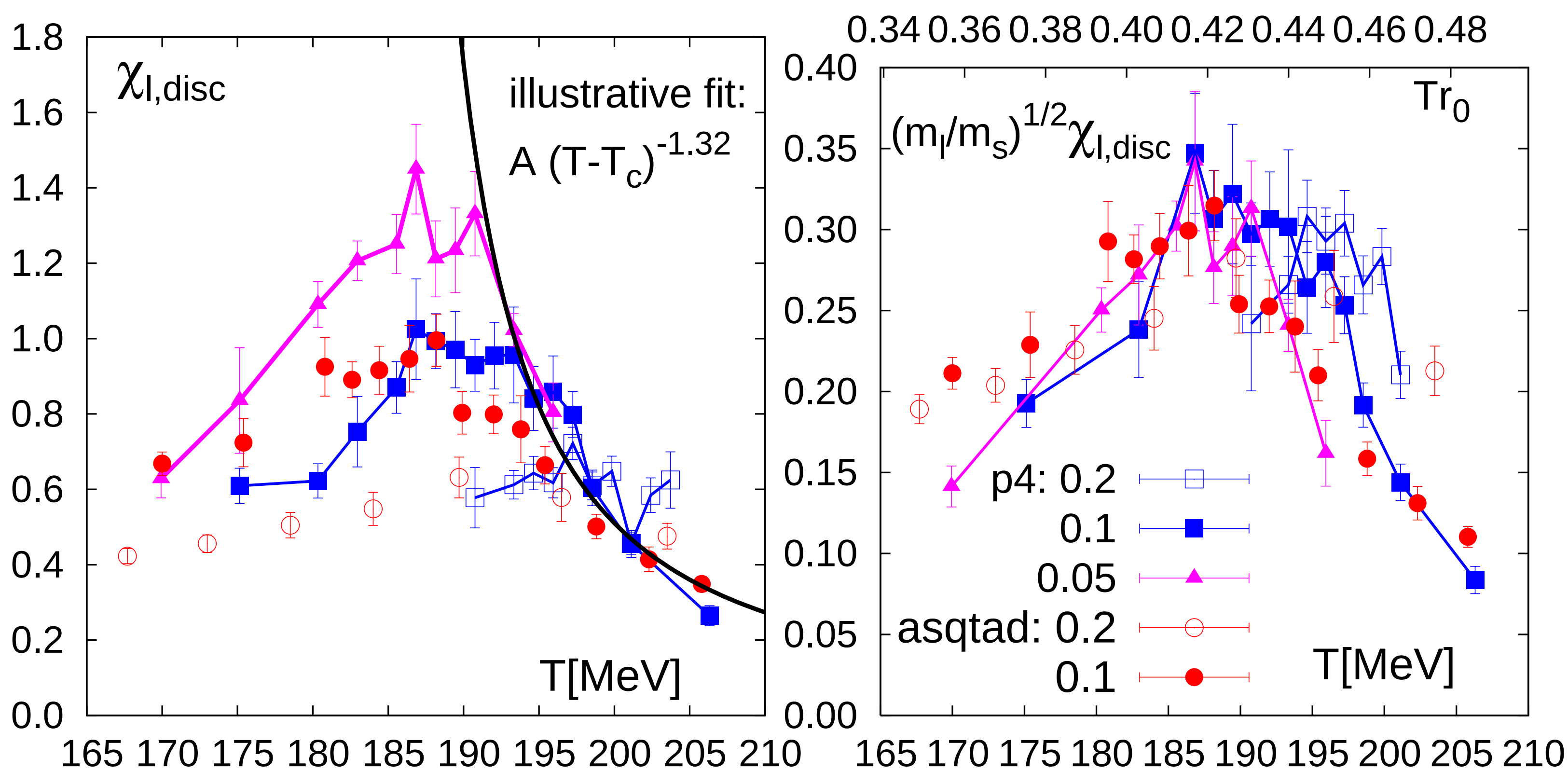


- evidence for $\sqrt{m_l}$ term in $\langle \bar\psi\psi \rangle$

for orientation: $\beta = 3.28$ $T \simeq 188$ MeV, $\beta = 3.30$ $T \simeq 196$ MeV

$N_\tau = 8$: p4 and asqtad

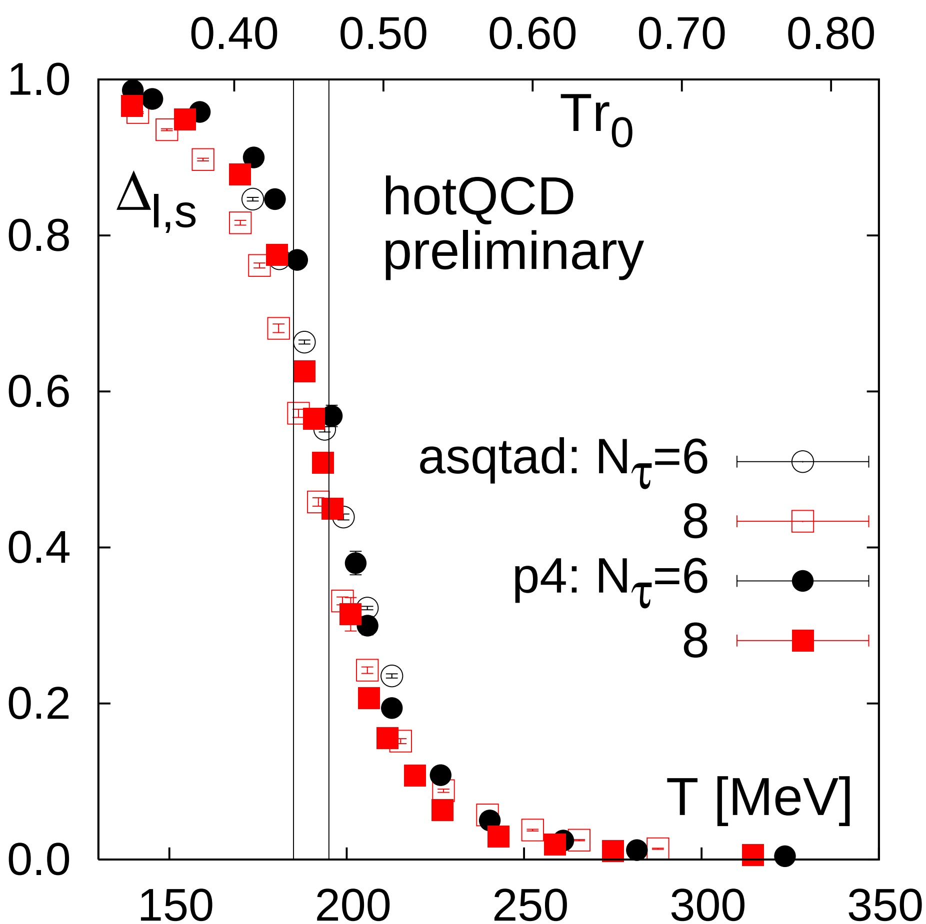
hotQCD and RBC-Bielefeld collaborations, preliminary



- p4 and asqtad calculation lead to similar quark mass dependence
- the rapid drop at large temperature is consistent with the expected O(2) [O(4)] scaling; however no 'critical behavior' of peak heights
- thermal critical behavior competes with fluctuations of Goldstone modes in the symmetry broken phase

Chiral condensates

- sudden change in ratios of finite and zero temperature condensates reflects chiral symmetry restoration



$$\Delta_{l,s}(T) = \frac{\langle\bar{\psi}\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T}}{\langle\bar{\psi}\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,0}}$$

subtracted a fraction of the strange quark condensate to eliminate additative renormalization terms

- 'normal' cut-off dependence of the subtracted and normalized chiral condensate; no 'unusually' large effects for $N_\tau = 8$
- consistent with confinement observables
- good agreement between p4 and asqtad results for $N_\tau = 8$

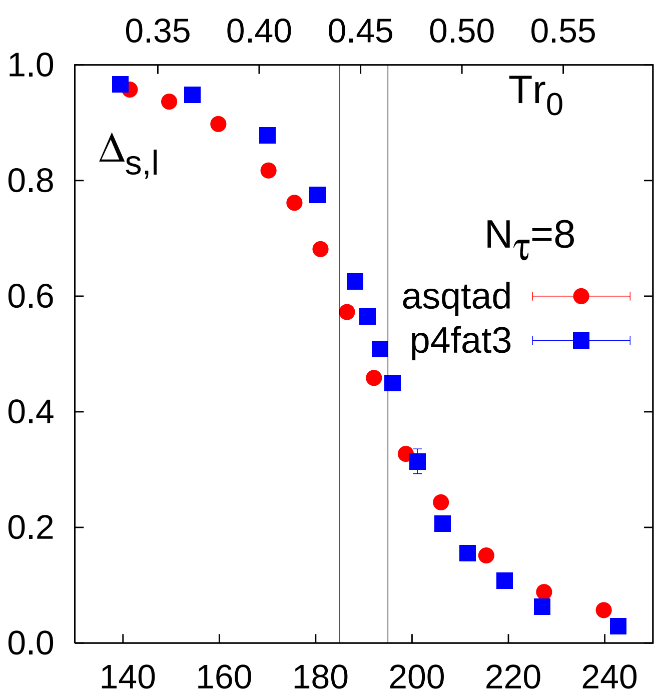
$N_\tau = 6$ (p4): RBC-Bielefeld, PRD77, 014511 (2008)
$N_\tau = 8$, and $N_\tau = 6$ (asqtad): hotQCD, preliminary

χ-condensate and susceptibility

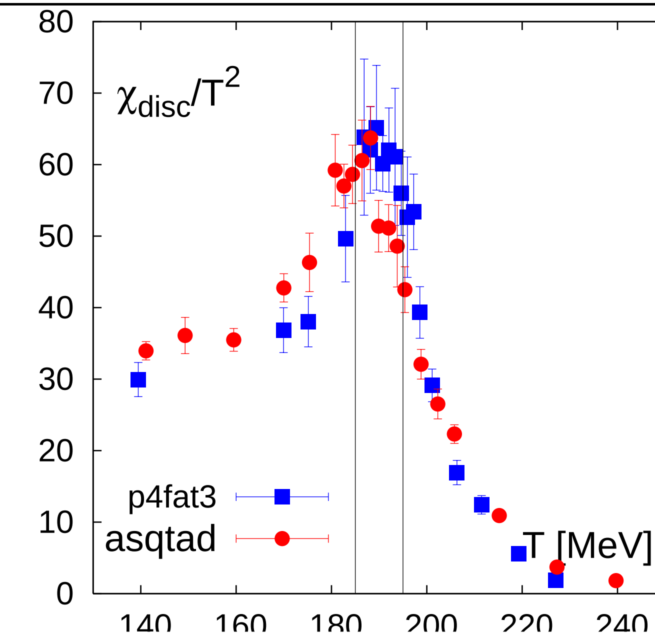
- sudden change in chiral condensate is, of course, related to peaks in the (singlet) chiral susceptibility

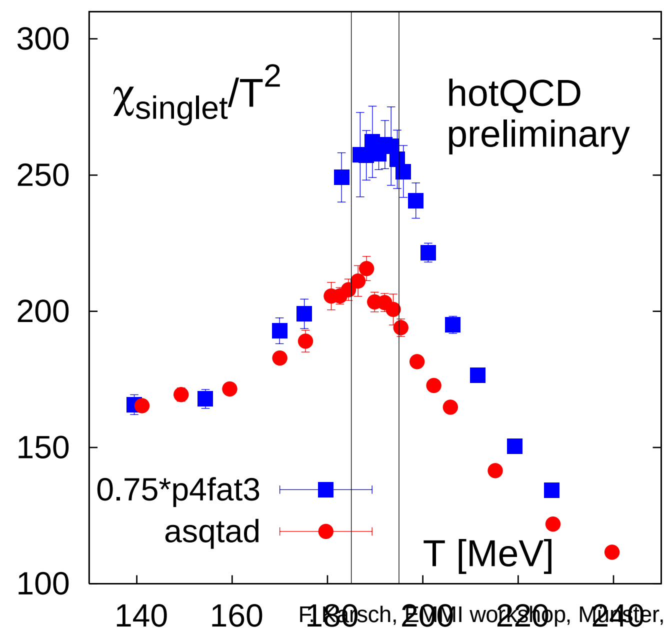
$$\chi_{tot}/T^2 = 2\chi_{dis}/T^2 + \chi_{con}/T^2$$

band: 185 MeV $< T < 195$ MeV



p4 and asqtad: hotQCD, preliminary



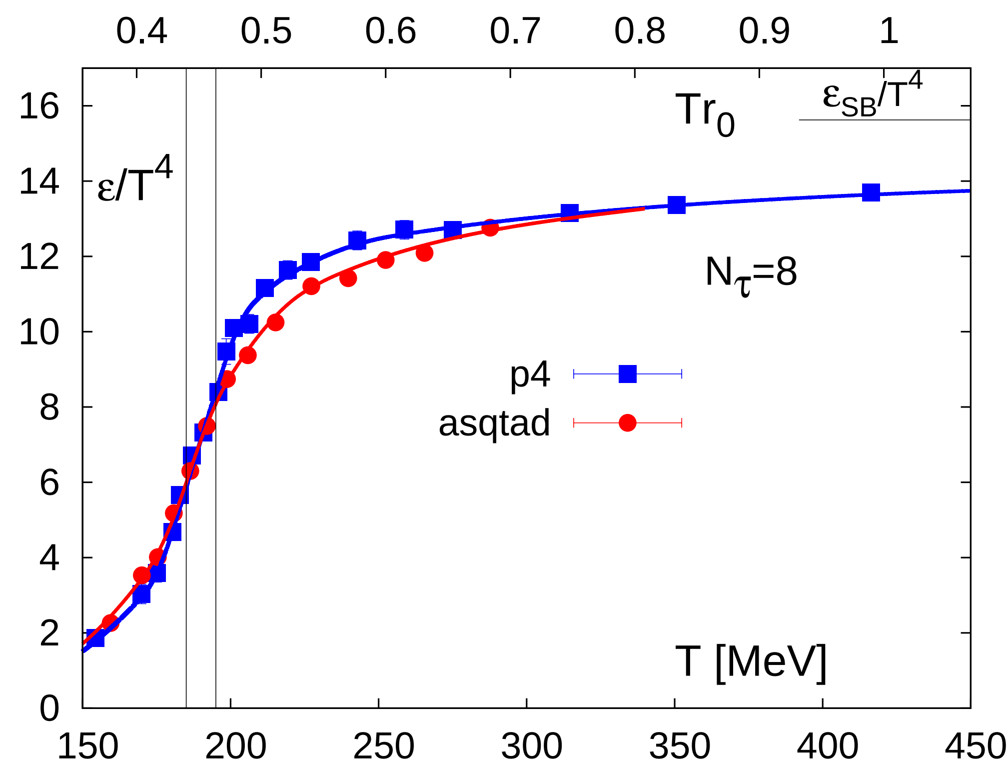


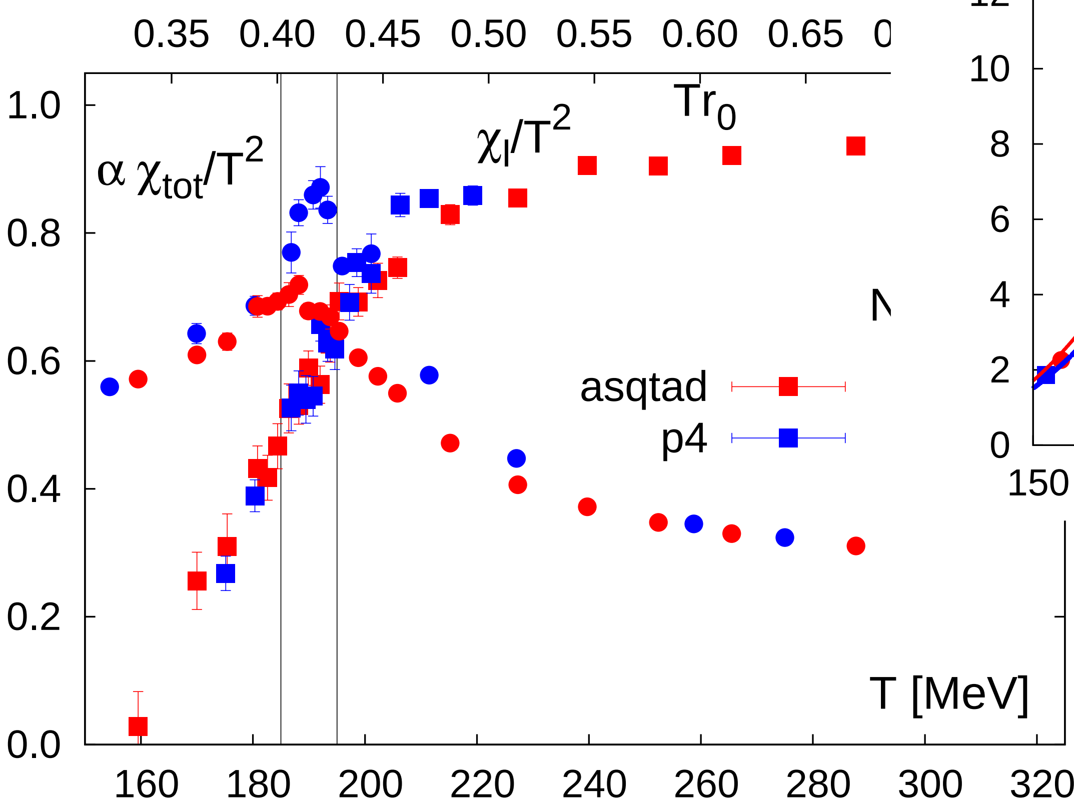
C. DeTar, Lat'08

Deconfinement and χ-symmetry and bulk thermodynamics

- most prominent features of bulk thermodynamics are related to deconfinement:

  e.g. rapid rise in energy density and quark number susceptibility





band: $185\mathrm{MeV} \leq T \leq 195\mathrm{MeV}$

- no indication for 'low' chiral transition temperature

Conclusions

- $\mathcal{O}(a^2)$ improved actions drastically reduce cut-off effects

  p4 and asqtad actions lead to consistent thermodynamics on lattices of temporal extent $N_\tau = 6$ and 8, although the handling of flavor symmetry breaking (fat-links) and $\mathcal{O}(a^2 g^2)$ corrections as well as cut-off effects in the free limit are quite different

- deconfinement and chiral symmetry restoration happen at roughly the same temperature that also characterizes the crossover region seen in bulk thermodynamics

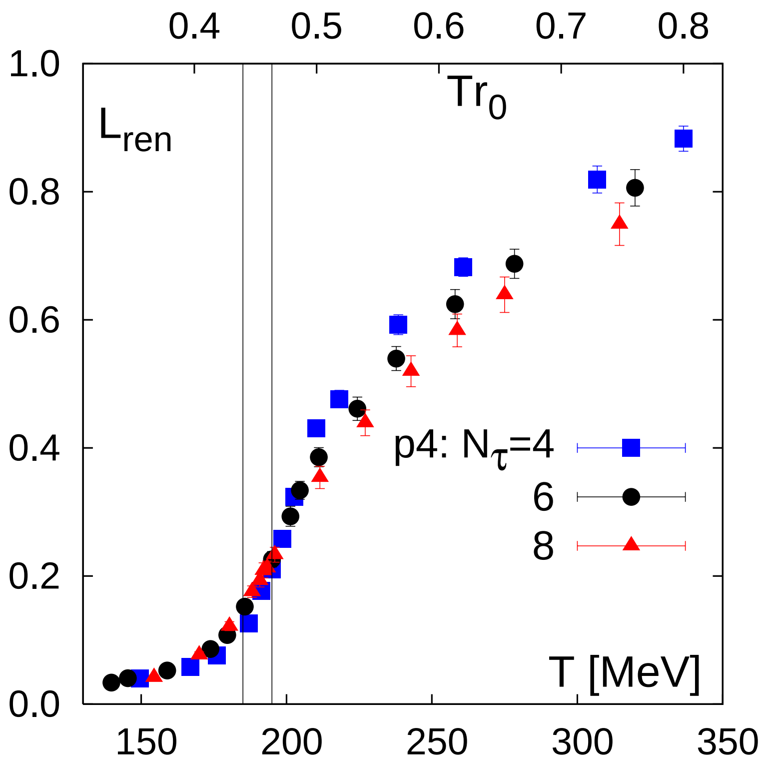
- T_c should be extracted from observables that are linked to critical behaviour in the chiral limit;
  analysis on $N_\tau = 8$ lattices, including updated $N_\tau = 6$ results, is in progress

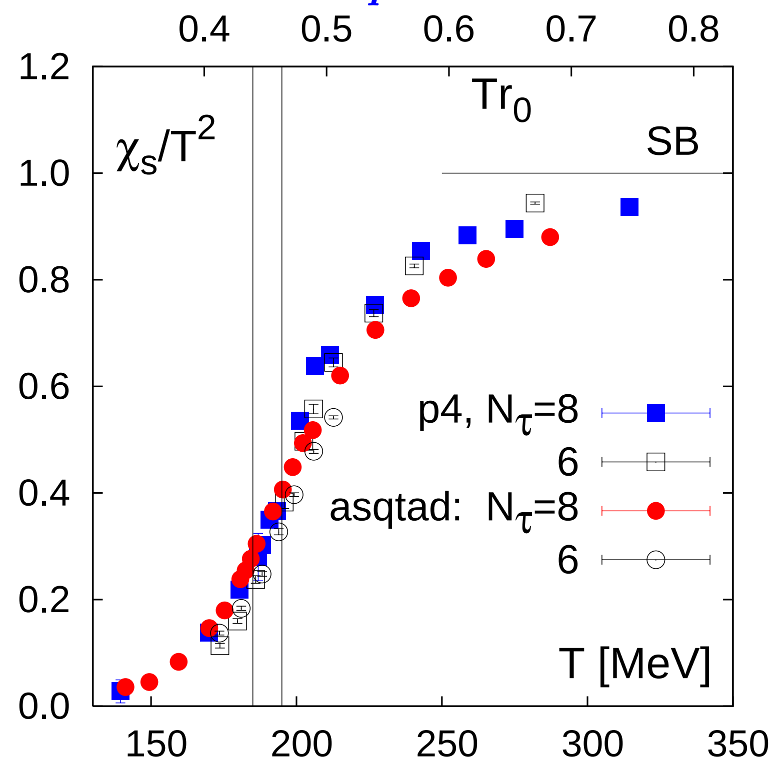
Deconfinement

- renormalized Polyakov loop and strange quark number susceptibility

$$L_{ren} \sim \mathrm{e}^{-F_Q(T)/T}, \qquad \chi_s/T^2 \sim \langle N_s^2 \rangle$$

rapid change, however 'non-singular' even for $m_q \to 0$





$N_\tau = 4,\ 6$ (p4): RBC-Bielefeld, PRD77, 014511 (2008)

$N_\tau = 8$, and $N_\tau = 6$ (asqtad): hotQCD, preliminary