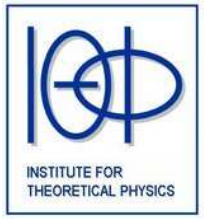

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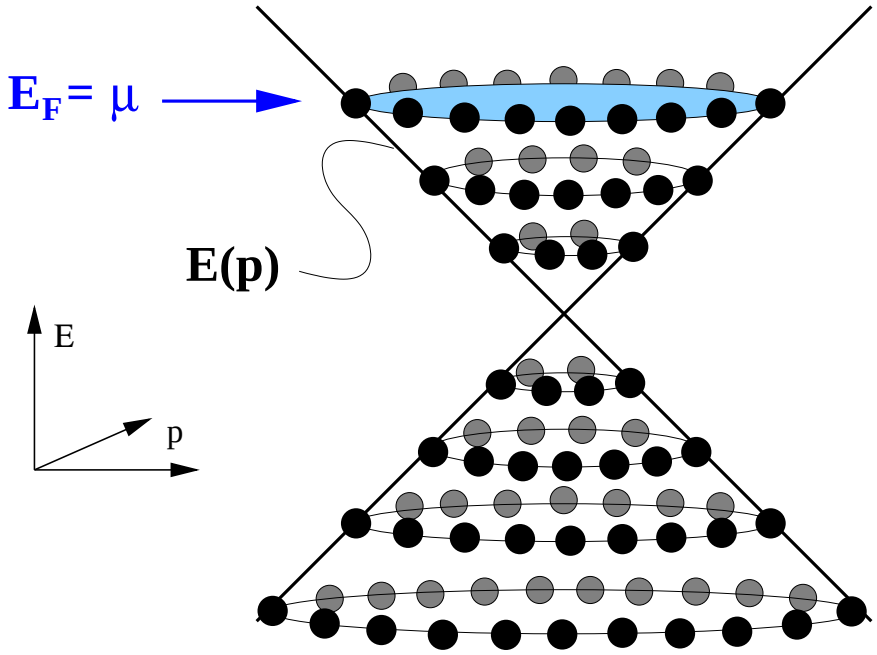
Color superconductivity in dense quark matter: properties and astrophysical applications

For a review, see

M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, Rev. Mod. Phys. 80, 1455 (2008)

- **Color-flavor locking and "stressed pairing"**
- **Color superconductivity in compact stars: viscosity & r-modes etc**

• Superconductivity: Cooper pairing of fermions

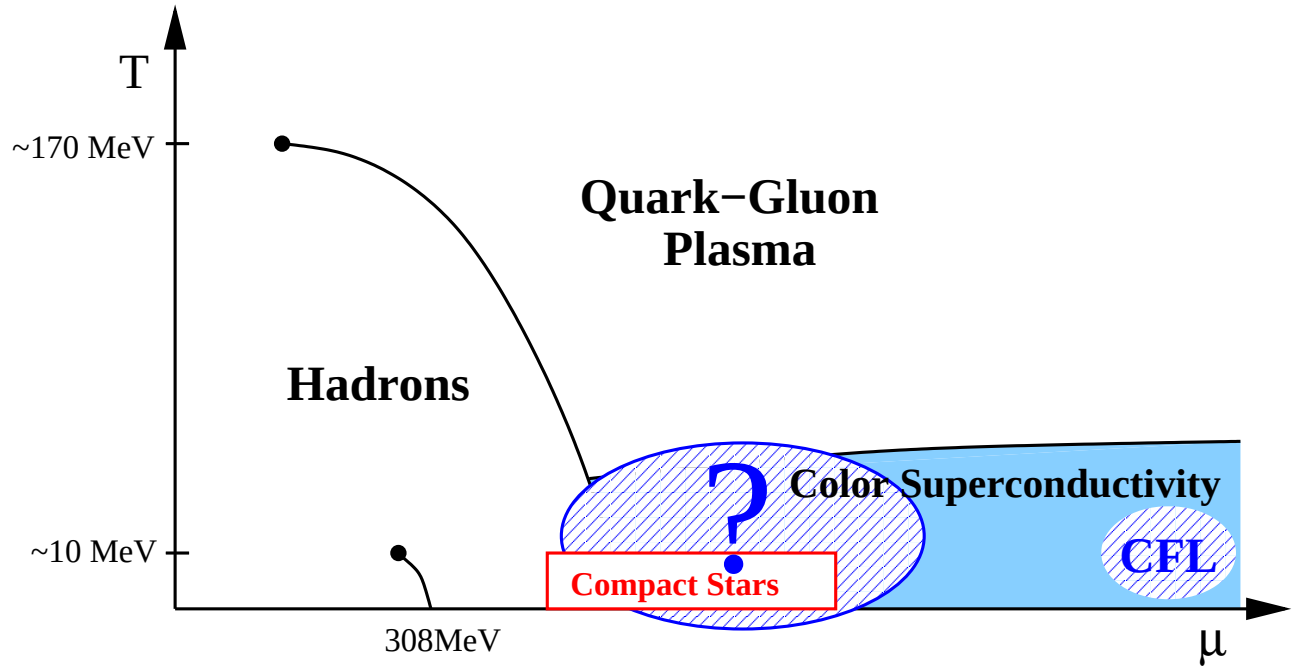


free energy $\Omega = E - \mu N$

- no interactions: add fermion at $E = \mu$ without cost
- attractive interaction: add pair with gain
- pairs condense
  $\rightarrow$ superconductivity

This Bardeen-Cooper-Schrieffer (BCS) argument holds for electrons in metal, ^{3}He atoms, ...,
and quarks in quark matter

- **QCD phase diagram (page 1/2): Known and unknown territories**



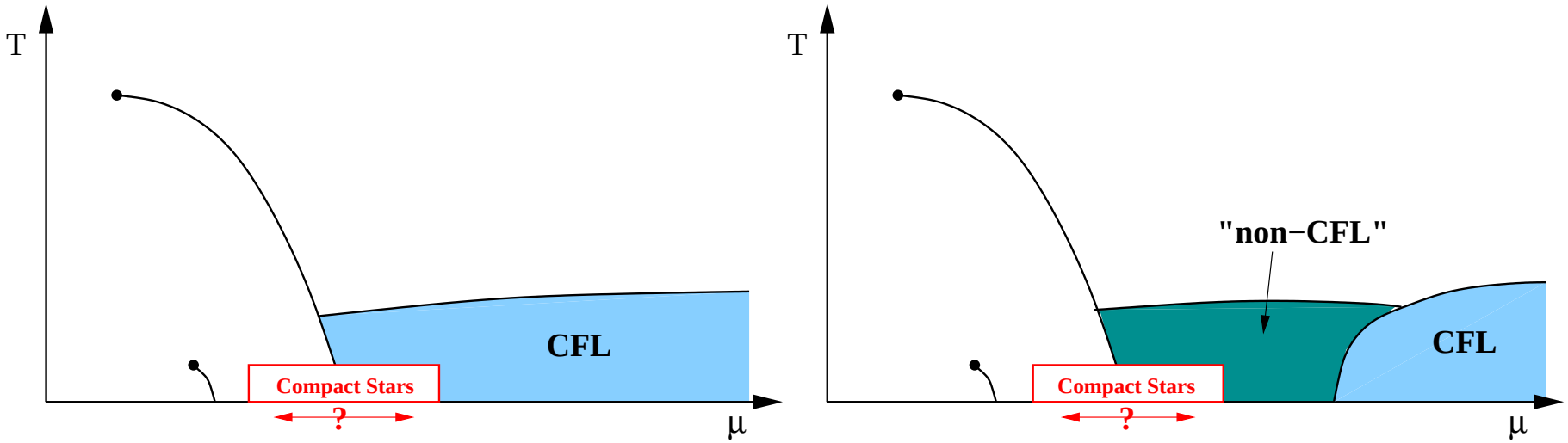
High densities:

- rigorous theoretical control
- no nonperturbative gaps in our understanding

Moderate densities:

- perturbative QCD not valid
- strange mass & neutrality: stress on Cooper pairing

• QCD phase diagram (page 2/2): 2 Possible scenarios



CFL superseded by nuclear matter:

- effective theory for CFL in strongly-coupled regime
- CFL matter in the core of compact stars?

CFL superseded by "non-CFL" matter:

- complicated phase structure?
- rely on Nambu-Jona-Lasinio-type models

Question:

What is the ground state of deconfined quark matter at moderate densities (in the interior of compact stars)?

1. **Theoretical approach: start from CFL and ask "what is next phase down in density?" (if not hadronic matter)**
2. **Phenomenological approach: "guess" possible phase, compute its properties and compare with astrophysical observations**

3. (Tabletop approach: learn from parallels to cold fermionic atoms in magnetic trap)
   E. Gubankova, A. Schmitt, F. Wilczek, PRB 74, 064505 (2006)

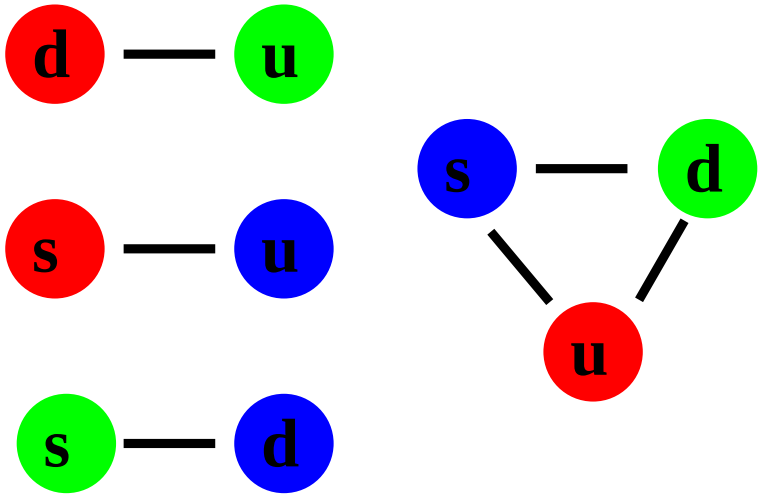
- **On safe grounds: Asymptotically large density**

$$0 \simeq m_s \simeq m_u \simeq m_d \ll \mu \qquad \text{all quark masses negligible}$$

"color-flavor locked phase (CFL)"

M. Alford, K. Rajagopal, F. Wilczek, NPB 537, 443 (1999)

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}}$$



- all quarks pair
- color and electrically neutral
- chiral symmetry broken
- superfluid
- Meissner mass for all gluons
- massless "rotated" photon

- **Low-energy degrees of freedom in CFL**

$\longrightarrow$ Goldstone octet (K^0, $K^\pm$, ...) + "superfluid mode" ϕ

- **effective theory for mesons and superfluid mode**

P. F. Bedaque, T. Schäfer, NPA 697, 802 (2002)

D. T. Son, M. A. Stephanov, PRD 61, 074012 (2000)

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[(\partial_0\Sigma + i[A_0,\Sigma])(\partial_0\Sigma^\dagger - i[A_0,\Sigma]^\dagger) - v_\pi^2\partial_i\Sigma\partial_i\Sigma^\dagger\right] + \frac{a f_\pi^2}{2}\mathrm{det}\hat{M}\,\mathrm{Tr}[\hat{M}^{-1}(\Sigma+\Sigma^\dagger)]$$

$$\Sigma = e^{i\theta/f_\pi}\,, \qquad \theta \in SU(3)\,, \qquad A_0 \equiv \mu_Q Q - \frac{\hat{M}^2}{2\mu}$$

$$\mathcal{L} = \frac{1}{2}(\partial_0\phi)^2 - \frac{v_H^2}{2}(\nabla\phi)^2 - \frac{\pi}{9\mu^2}\partial_0\phi\partial_\mu\phi\partial^\mu\phi + \frac{\pi^2}{108\mu^4}(\partial_\mu\phi\partial^\mu\phi)^2$$

$\longrightarrow$ nonzero strange mass $\longrightarrow$ **Kaon condensation ("CFL-K^0")**

- condensate (if present at $T=0$) very robust, $T_c \sim T_c^{\mathrm{CFL}}$

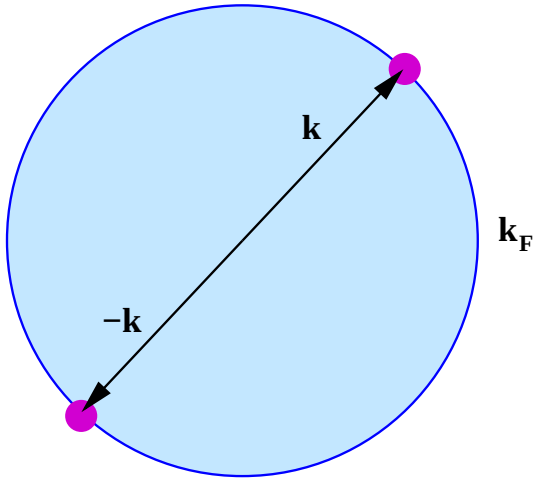
M. G. Alford, M. Braby, A. Schmitt, J. Phys. G 35, 025002 (2008)

- **Large, but not asymptotically large densities**

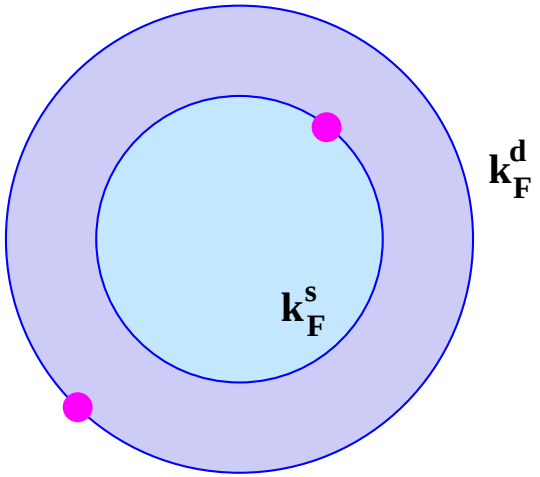
going down in density $\Leftrightarrow$ "switching on" m_s and maintaining neutrality

$\rightarrow$ **mismatch in Fermi momenta of pairing quarks ("stressed" pairing)**

BCS-pairing:



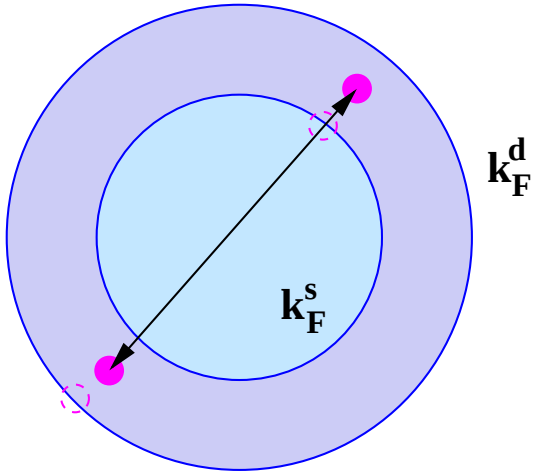
Neutral quark matter:



$\rightarrow$ stressed pairing is **unavoidable**

K. Rajagopal, A. Schmitt, PRD 73, 045003 (2006)

• Less (and less symmetric) pairing (page 1/4)



- create common Fermi surface:
  cost in free energy $\sim \delta\mu^2\mu^2$
- form pairs:
  gain in free energy $\sim \Delta^2\mu^2$

$\rightarrow$ "gapless CFL" for $\delta\mu > \Delta$

I. Shovkovy, M. Huang, PLB 564, 205 (2003) (2SC)
M. Alford, C. Kouvaris, K. Rajagopal, PRL 92, 222001 (2004)

$\rightarrow$ *however:* chromomagnetic instability

M. Huang, I. A. Shovkovy, PRD 70, 051501 (2004)
R. Casalbuoni, R. Gatto, M. Mannarelli, G. Nardulli, M. Ruggieri, PLB 605, 362 (2005)
K. Fukushima, PRD 72, 074002 (2005)

• Less (and less symmetric) pairing (page 2/4)

- Kaon-condensed phases:
  CFL-K^0, curCFL-K^0

  P. Bedaque, T. Schäfer, NPA 697, 802 (2002)

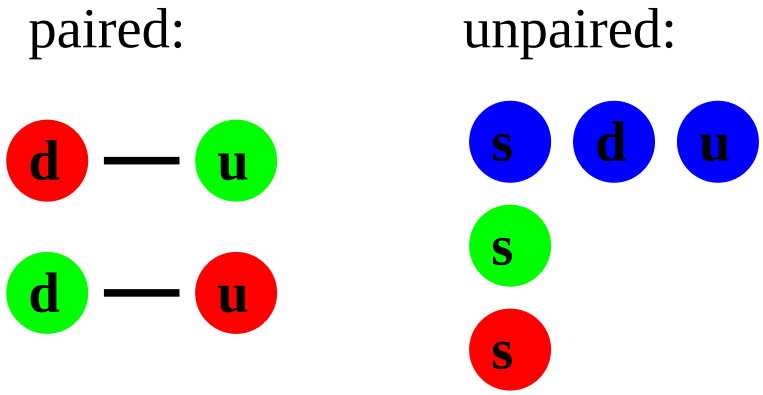
  T. Schäfer, PRL 96, 012305 (2006)

curCFL-K^0

counterpropagating currents:
K^0-condensate
+ gapless fermions

- 2SC phase

  D. Bailin, A. Love,
  Phys. Rept. 107, 325 (1984)

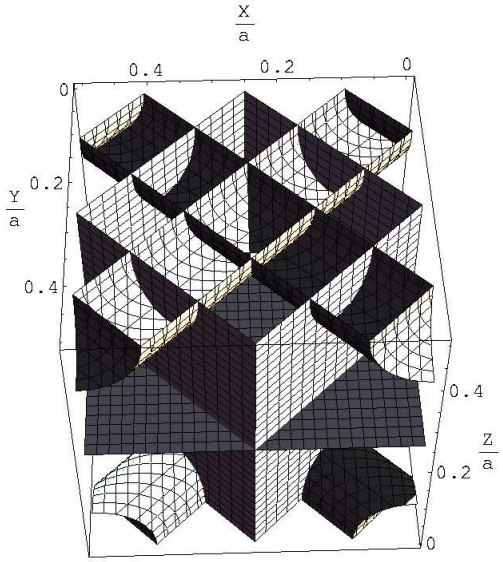


• Less (and less symmetric) pairing (page 3/4)

- Crystalline phases: LOFF

  M. Alford, J. Bowers, K. Rajagopal, PRD 63, 074016 (2001)

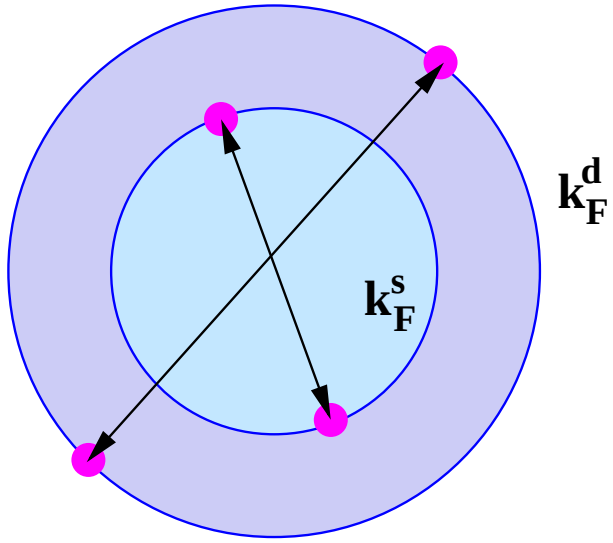
  M. Mannarelli, K. Rajagopal and R. Sharma, PRD 73, 114012 (2006)



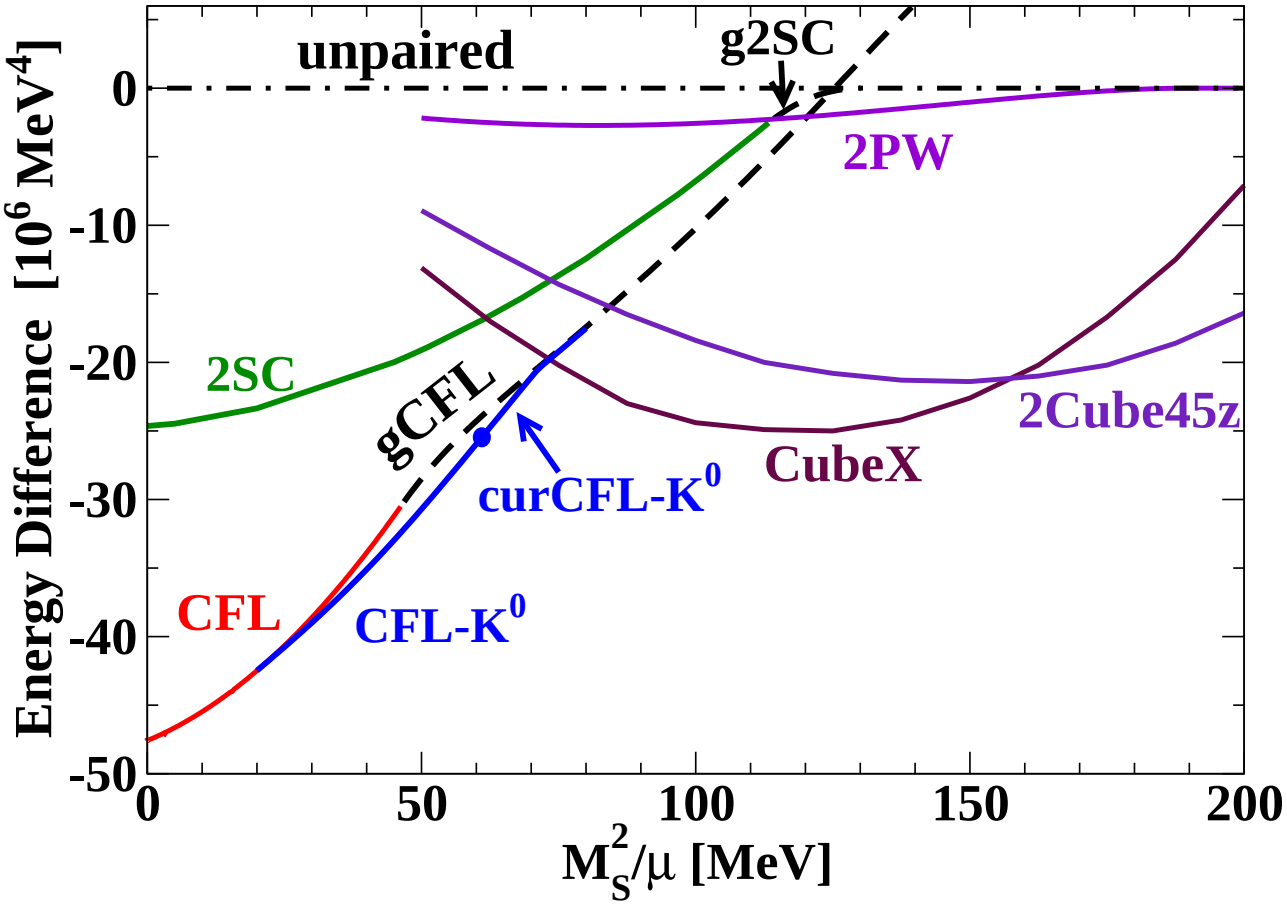
- Single-flavor pairing: CSL, A-phase, polar phase ...

  T. Schäfer, PRD 62, 094007 (2000)

  A. Schmitt, PRD 71, 054016 (2005)



- **Less (and less symmetric) pairing (page 4/4)**



Free energy comparison of 3-flavor quark phases for $\Delta_{\mathrm{CFL}} = 25$ MeV.

• Color superconductivity in compact stars (page 1/3)

Cooling of the star

- **neutrino emissivity**

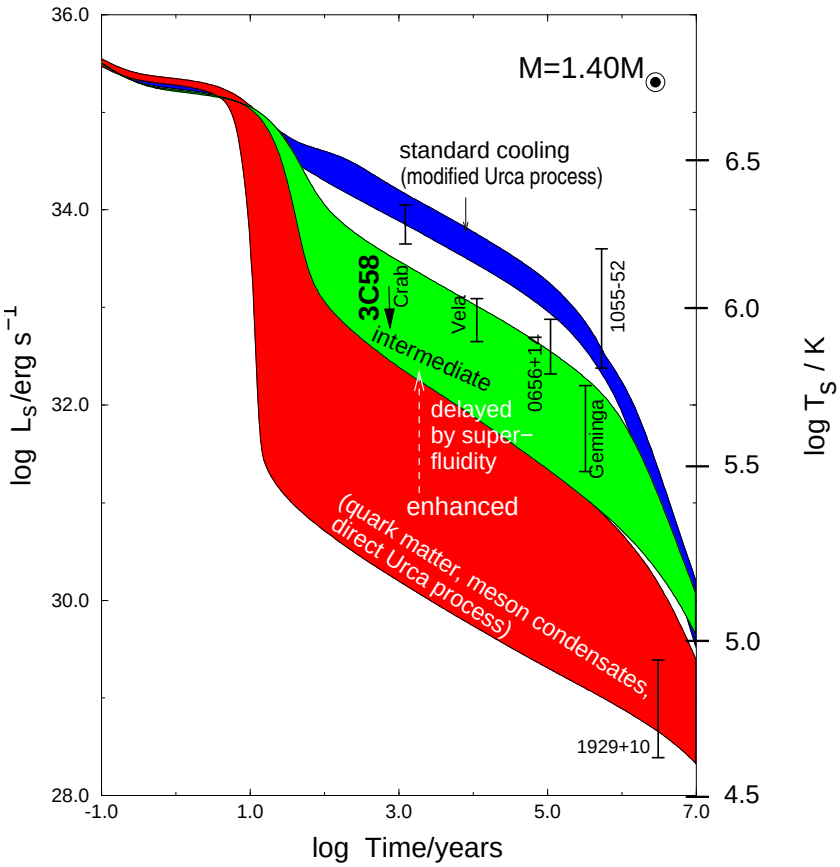
  $T \sim 10\,\mathrm{MeV} \rightarrow 0.1\,\mathrm{keV}$

  $(t \sim 0 \rightarrow 10^6\,\mathrm{yr})$

- **direct Urca processes**

  $u + e \rightarrow d + \nu\,, \quad d \rightarrow u + e + \bar{\nu}$

F. Weber, Prog. Part. Nucl. Phys. 54, 193 (2005)



unpaired quark matter Iwamoto, PRL 44, 1637 (1980) **CFL** Jaikumar, Prakash, Schäfer, PRD 66, 063003 (2002) **2SC** Jaikumar, Roberts, Sedrakian, PRC 73, 042801 (2006) **spin-1** Schmitt, Shovkovy, Wang, PRD 73, 034012 (2006) **LOFF** Anglani, Nardulli, Ruggieri, Mannarelli, PRD 74, 074005 (2006)

• Color superconductivity in compact stars (page 2/3)

magnetic fields

- spin-0 **no Meissner effect**

  M.G. Alford, J. Berges, K. Rajagopal,

  NPB 571, 269 (2000)

- spin-1 **Meissner effect**

  A. Schmitt, Q. Wang, D.H. Rischke,
  PRL 91, 242301 (2003)

- "magnetic" CFL

  E.J. Ferrer, V. de la Incera, C. Manuel,

  PRL 95, 152002 (2005)

- de Haas-van Alphen oscillations

  K. Fukushima and H. J. Warringa,
  PRL 100, 032007 (2008)
  J. L. Noronha and I. A. Shovkovy,
  PRD 76, 105030 (2007)

glitches

- **vortex pinning at lattice**

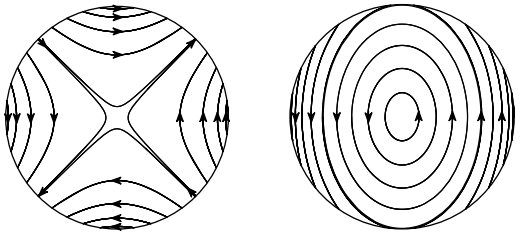
  $\longrightarrow$ neutron superfluidity + ion lattice

  $\longrightarrow$ or better (?): crystalline color superconductivity

  M. Mannarelli, K. Rajagopal, R. Sharma, PRD 76, 074026 (2007)

• Color superconductivity in compact stars (page 3/3)

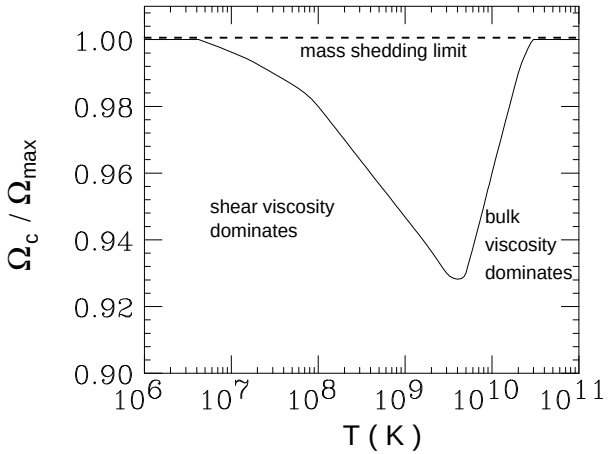
r-mode instability



L. Lindblom, arXiv:astro-ph/0101136

- **r-modes:**
  non-radial pulsation modes
- **grow unstable**
  in a perfect-fluid rotating star
  → emission of gravitational waves
- **spin down**
  the star drastically and quickly (within days)

- fast rotating stars are observed!
  $\omega \simeq 1\text{ms}^{-1}$
- must be some damping mechanism → bulk/shear viscosity



• What is bulk viscosity (in CFL-K^0)?

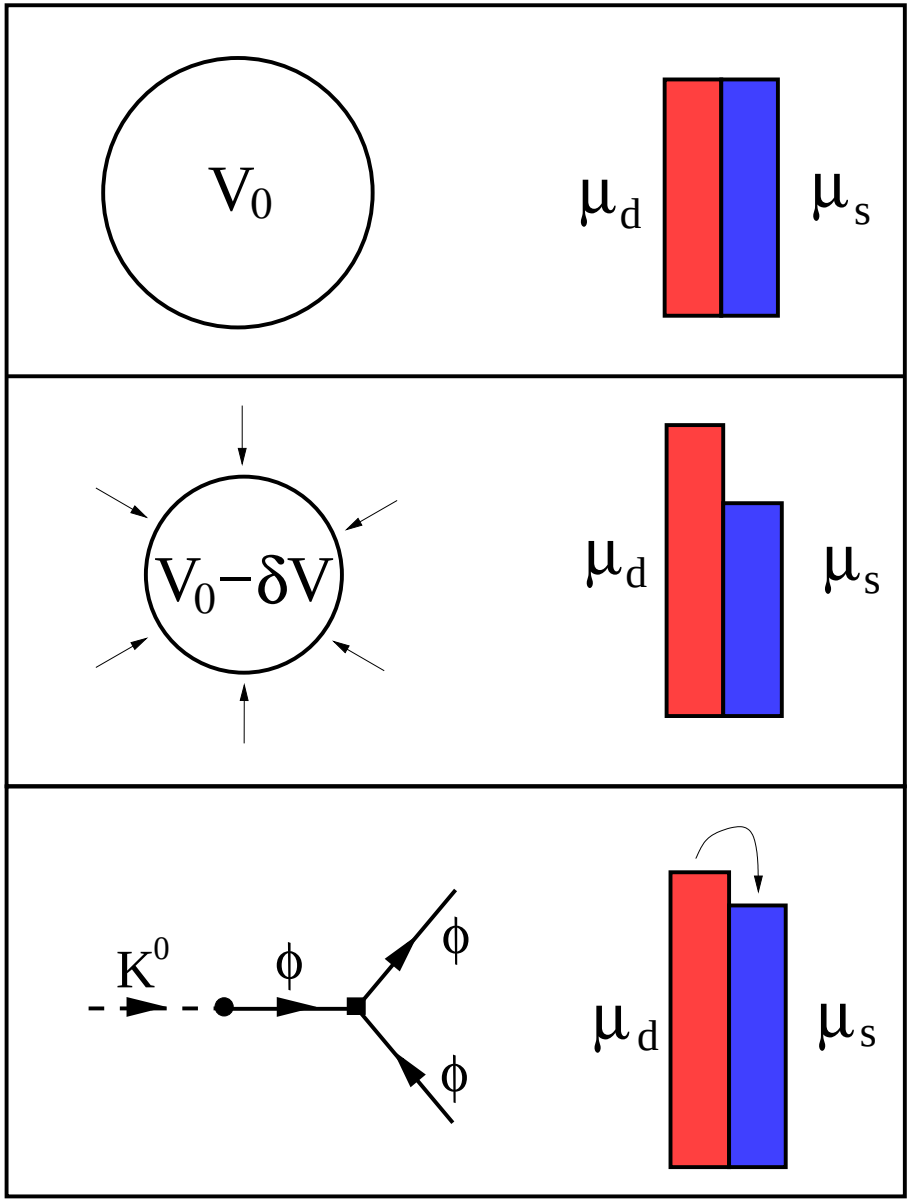
- volume oscillation
  $\longrightarrow$ chemical
  non-equilibrium

$$\mu_d - \mu_s \neq 0$$

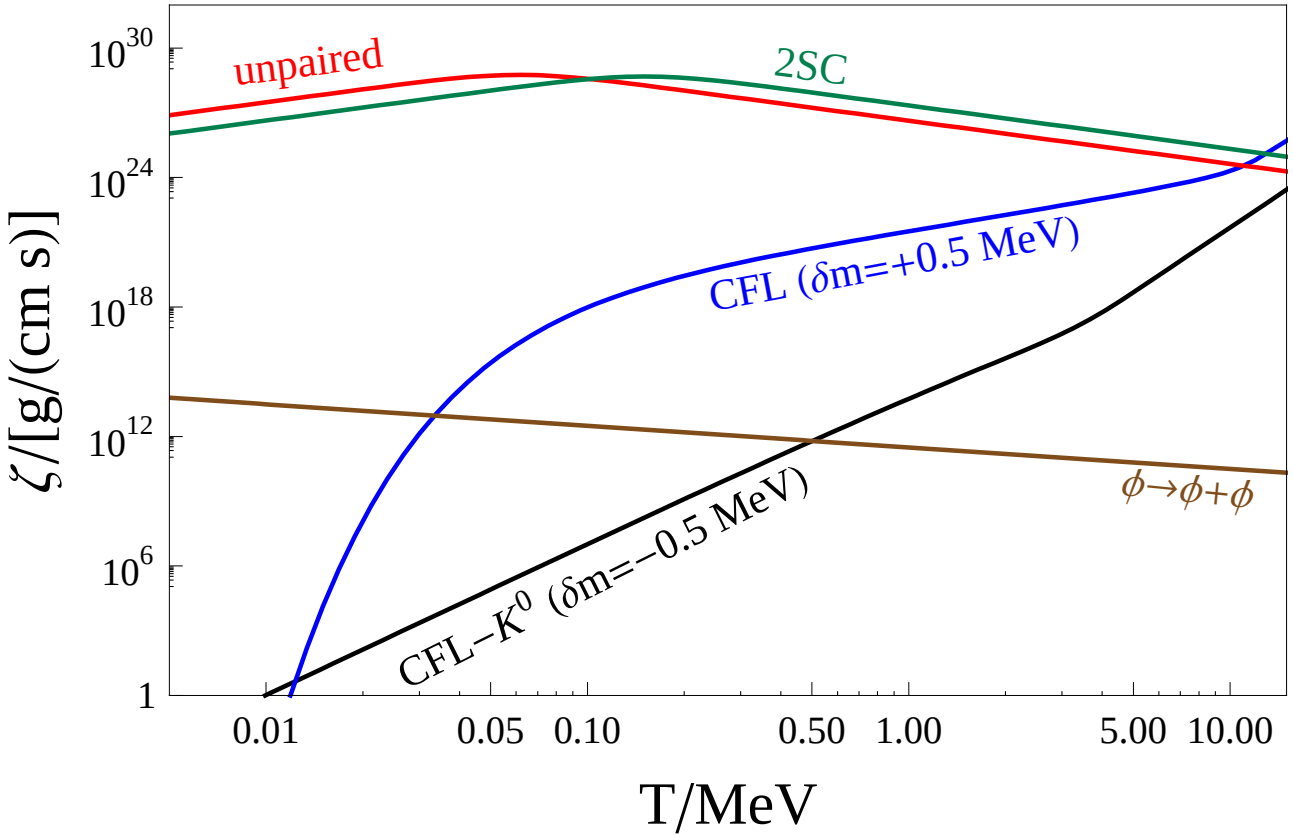
- re-equilibration via

$$K^0 \leftrightarrow \phi + \phi$$

- **resonance phenomenon**:
  external oscillation
  vs. microscopic rate



• Quark matter bulk viscosity: recent results



$\omega/(2\pi) = 1\,\mathrm{ms}^{-1}$

$\mu = 400\,\mathrm{MeV}$

$\delta m \equiv m_{K^0} - \mu_{K^0}$

unpaired J. Madsen, PRD 46, 3290 (1992)
2SC M.G. Alford, A. Schmitt, JPG 34, 67-101 (2007)
CFL from $K^0 \leftrightarrow \phi + \phi$ M.G. Alford, M. Braby, S. Reddy, T. Schafer, PRC 75, 055209 (2007)
CFL-K^0 from $K^0 \leftrightarrow \phi + \phi$ M.G. Alford, M. Braby, A. Schmitt, JPG 35, 115007 (2008)
CFL from $\phi \leftrightarrow \phi + \phi$ C. Manuel, F. Llanes-Estrada, JCAP 0708, 001 (2007)
See also: Spin-1 B.A. Sa'd, I.A. Shovkovy, D.H. Rischke, PRD 75, 065016 (2007)
Vortices in CFL M. Mannarelli, C. Manuel, B. A. Sa'd, PRL 101, 241101 (2008)

- **Conclusions**

  - **QCD phase diagram**
    - 3-flavor quark matter at asymptotically large densities is in the **color-flavor locked (CFL)** state
    - "switching on" a strange mass $m_s \rightarrow$ **kaon condensation**
    - phase(s) between **CFL** and **hadronic matter** are unknown

  - **Compact stars**
    - use **astrophysical observations** to rule out or confirm phases
    - e.g., **viscosity** – r-mode instability;
      neutrino emissivity – cooling curve