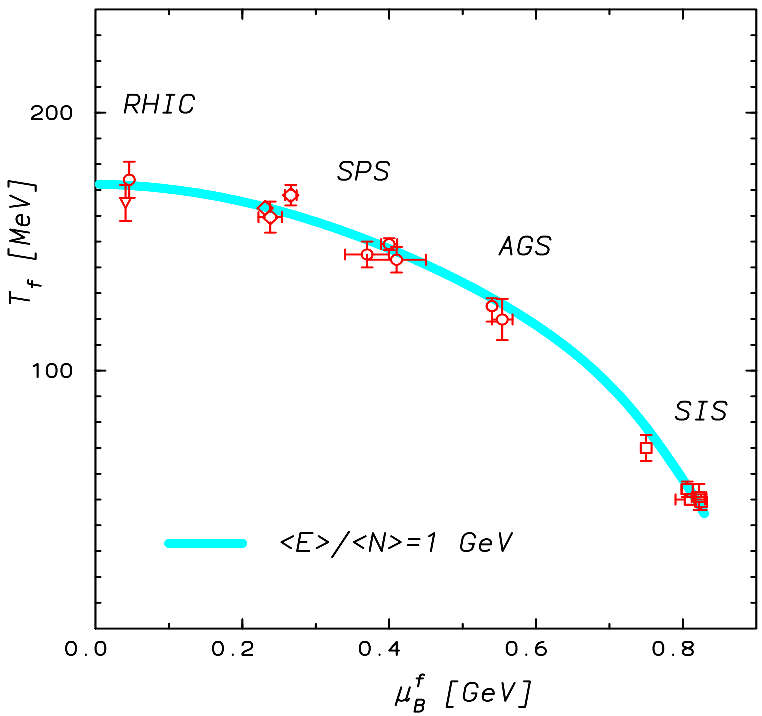
Fluctuation studies in the PNJL model

Rajarshi Ray

(Bose Institute, Kolkata)

International Conference on CBM Physics, SMIT, Sikkim

Freezeout curve



Thermal model: $Z(T, \mu_B, \mu_I, \mu_S) = \sum_i Z_i(T, \mu_B, \mu_I, \mu_S)$

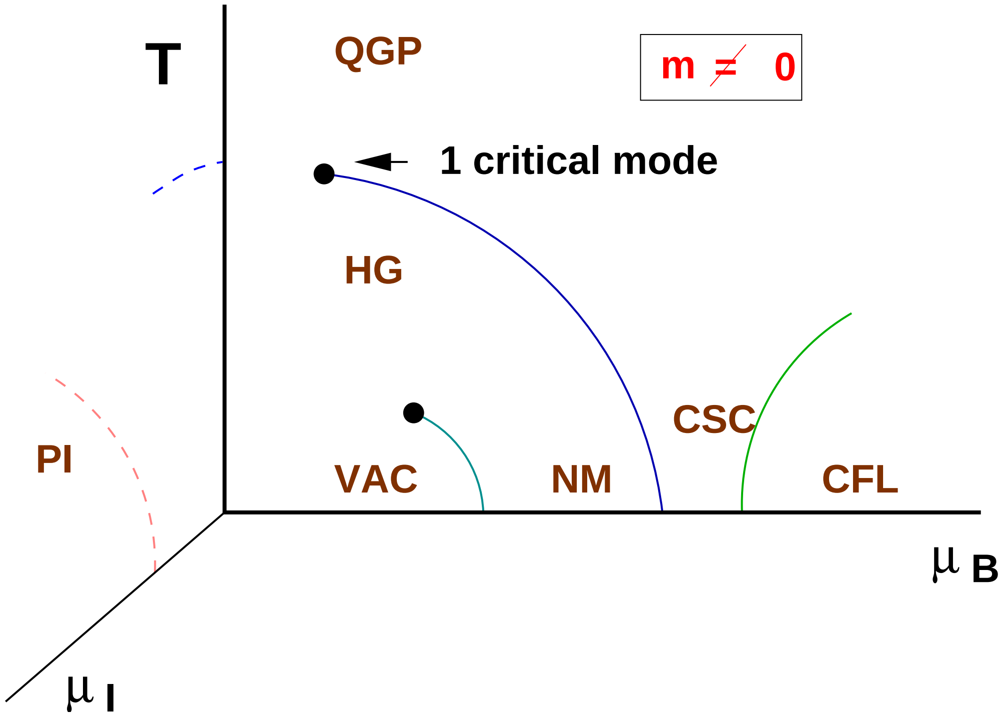
$\Rightarrow n_i(T, \mu_B, \mu_I, \mu_S)$

Cleymans and Redlich Phys. Rev. Lett 81 (1998) 5284

Braun-Munzinger, Redlich and Stachel:

'Quark Gluon Plasma 3', R.C. Hwa and X.-N. Wang, ed., (World Scientific) (nucl-th/0304013)

Phase Diagram



$$\mu_B = \frac{1}{3}\left(\frac{\mu_u + \mu_d}{2}\right) \qquad ; \qquad \mu_I = \left(\frac{\mu_u - \mu_d}{2}\right)$$

Rajagopal, Wilczek : The Condensed Matter Physics of QCD
Ch. 35, 'Handbook of QCD', M. Shifman, ed., (World Scientific) (hep-ph/0011333).

QCD

$$\mathcal{L}^E_{QCD} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_{f=1}^{N_f} \bar{q}_f \left(\gamma^E_\mu D_\mu + m_f - \mu_f \gamma_0 \right) q_f$$

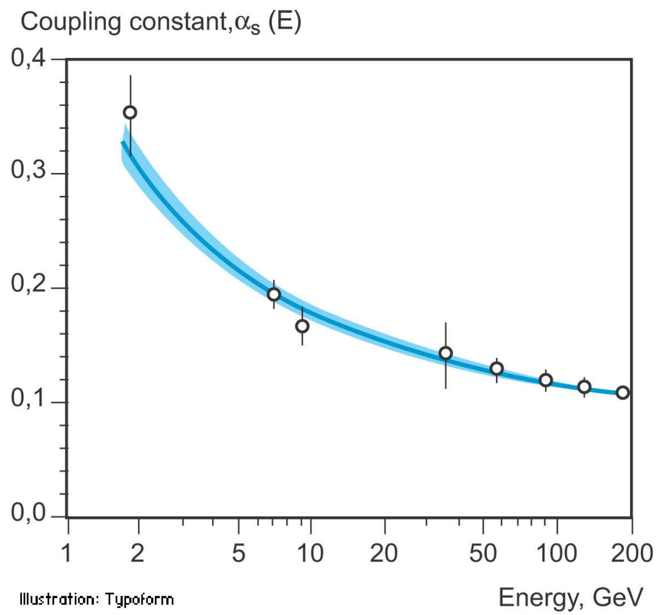
where,

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu \\ D_\mu &= \partial_\mu - i g T^a G^a_\mu \qquad\qquad a = 1, 2, ...8. \end{aligned}$$

T^a are SU(3) group generators and f^{abc} are SU(3) structure constants
The QCD partition function is,

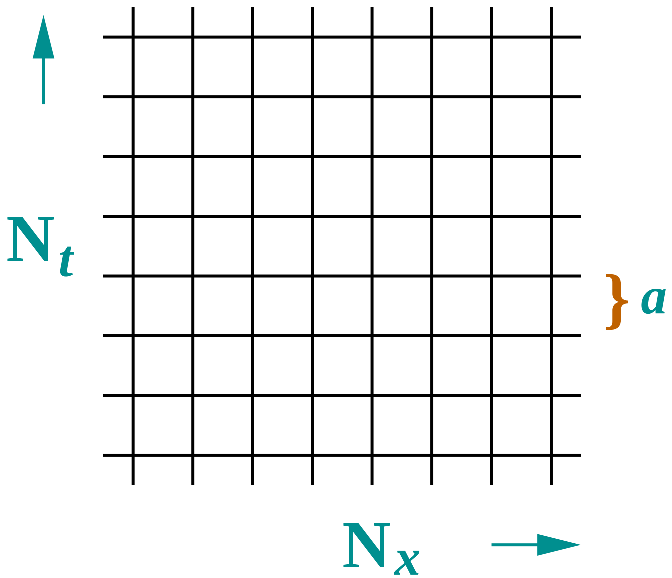
$$Z = \int DG^a_\nu Dq_f D\bar{q}_f \, \mathrm{e}^{-\int_0^\beta d\tau \int_{-\infty}^{\infty} d^3x \mathcal{L}^E_{QCD}}$$

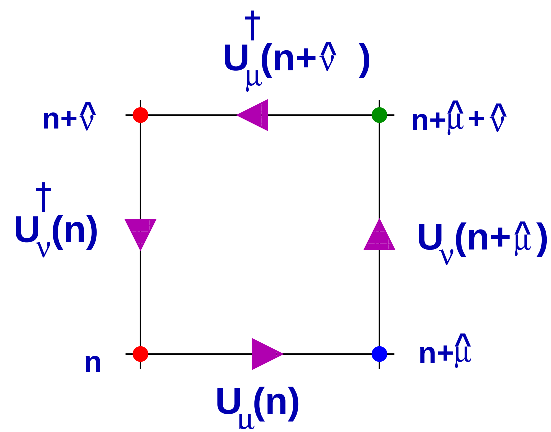
Running coupling



- Asymptotic Freedom of colour charge
- Hadrons p, n, π, ρ, ... are colour neutral composites.
  Bag models, String models ...

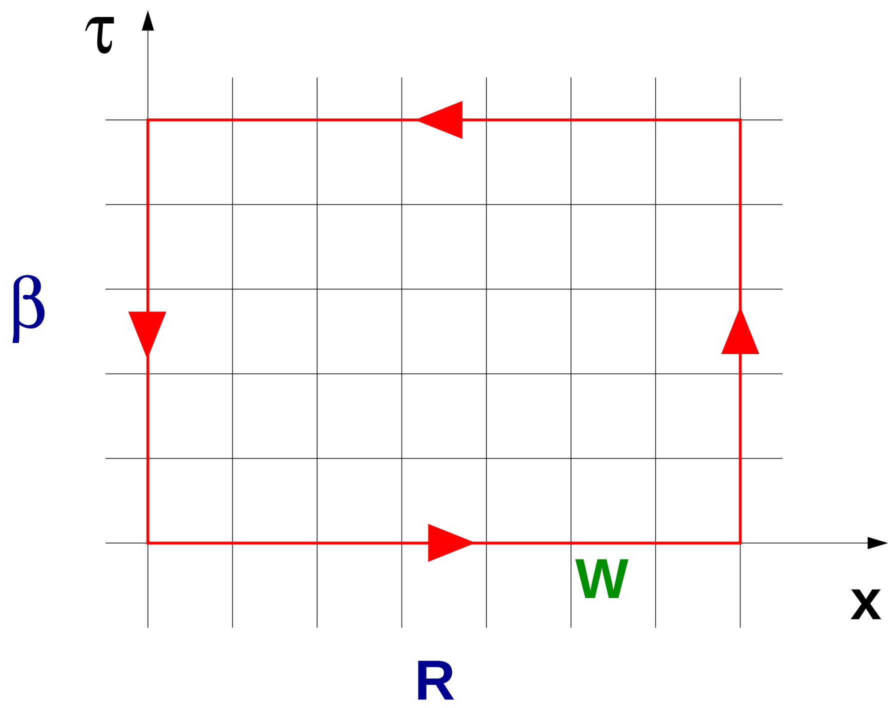
Lattice Formulation





- Quarks sit on the lattice points $q(\mathrm{n})$
- Gluons are on the links $U_\mu(\mathrm{n}) = \mathcal{P}\exp\left[ig\int_{\mathrm{n}}^{\mathrm{n}+\hat{\mu}a} \mathrm{d}y^\sigma\, G^a_\sigma(y)\, T^a\right]$
- $V = a^3(\mathrm{N}_x \times \mathrm{N}_y \times \mathrm{N}_z)$ $\qquad \beta = a\mathrm{N}_t$
- momentum cutoff $\simeq \frac{1}{a}$ $\qquad a \to 0 \Rightarrow$ Continuum physics

Confinement



$$\mathcal{C}\, \exp[-\beta \cdot \mathrm{V(R)}] = \langle \mathrm{W(R}, \beta) \rangle$$

Wilson Loop

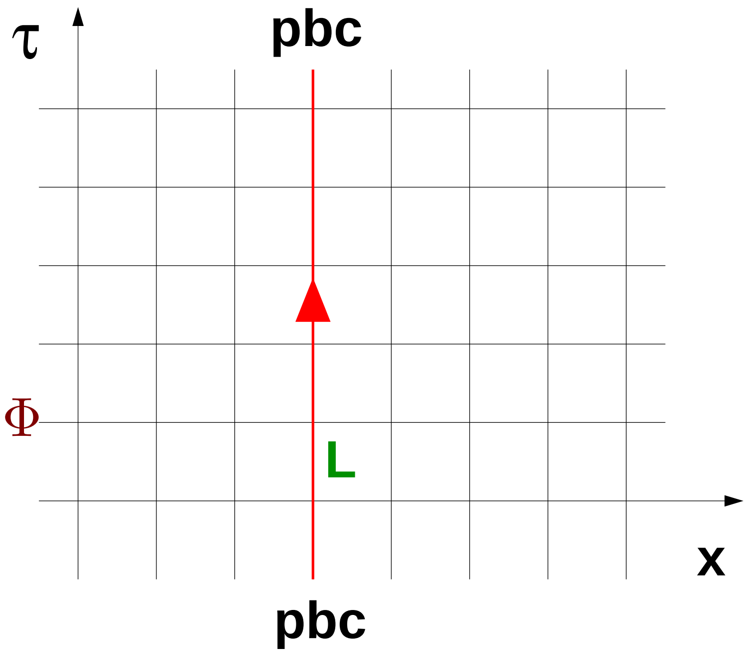
- Pair of q, $\bar{q}$ sitting a distance R apart, parallel transported in time τ.
- $\mathrm{V(R)} = \begin{cases} -\frac{g^2}{3\pi \mathrm{R}} & \text{for weak coupling} \\ \frac{\mathrm{R}}{a^2} \ln(3g^2) & \text{for strong coupling} \end{cases}$
- "Millenium Prize Problem" - Analytically for $a \to 0$.

(De)Confinement

- Finite Temperature
  $\beta = 1/T$
- Free energy of vacuum
  $F_0 = -T \log Z$
- Free energy of a single quark
  $F_q - F_0 = -T \log \langle \mathrm{TrL}(\vec{x})/3 \rangle = -T \log \Phi$

  $\mathrm{L}(\vec{x}) = \mathcal{P}\exp\left[- \int_0^{1/T} \mathrm{d}\tau G_0(\vec{x}, \tau) \right]$

  Wilson Line/Polyakov Loop



- $\Phi(\vec{x}, \tau) = \begin{cases} \neq 0 \Rightarrow F_q \text{ finite} \Rightarrow \text{deconfined} \\ = 0 \Rightarrow F_q \text{ infinite} \Rightarrow \text{confined} \end{cases}$
- Use Φ as OP for finite temperature phase transition.
  Mc Lerran and Svetitsky '81

Chiral Symmetry

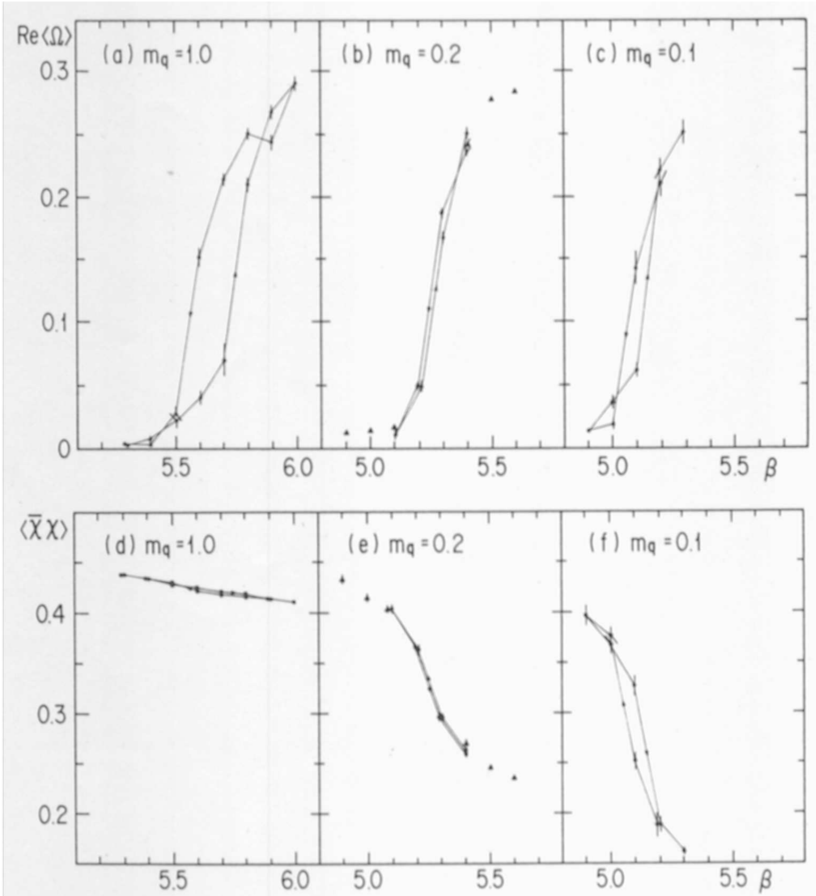
$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \sum_{f=u,d}\left[i\,\bar{q}_f\gamma^\mu D_\mu q_f - m_f\bar{q}_f q_f\right]$$

Symmetries: $SU(3)_c \otimes \underbrace{SU(2)_V \otimes SU(2)_A \otimes U(1)_B \otimes U(1)_A}_{Fermionic}$

- $U(1)_A$ broken by quantum anomalies.
- $SU(2)_V$ broken explicitly when flavour degeneracy is lifted
  e.g. proton and neutron mass splitting.
- $SU(2)_A$ broken explicitly for non-zero quark mass ....
  where are the chiral partners !!
- $SU(2)_A$ broken spontaneously; pions are the Goldstone Bosons
  $\rightarrow$ Measure is the chiral condensate $\langle\bar{q}q\rangle$.

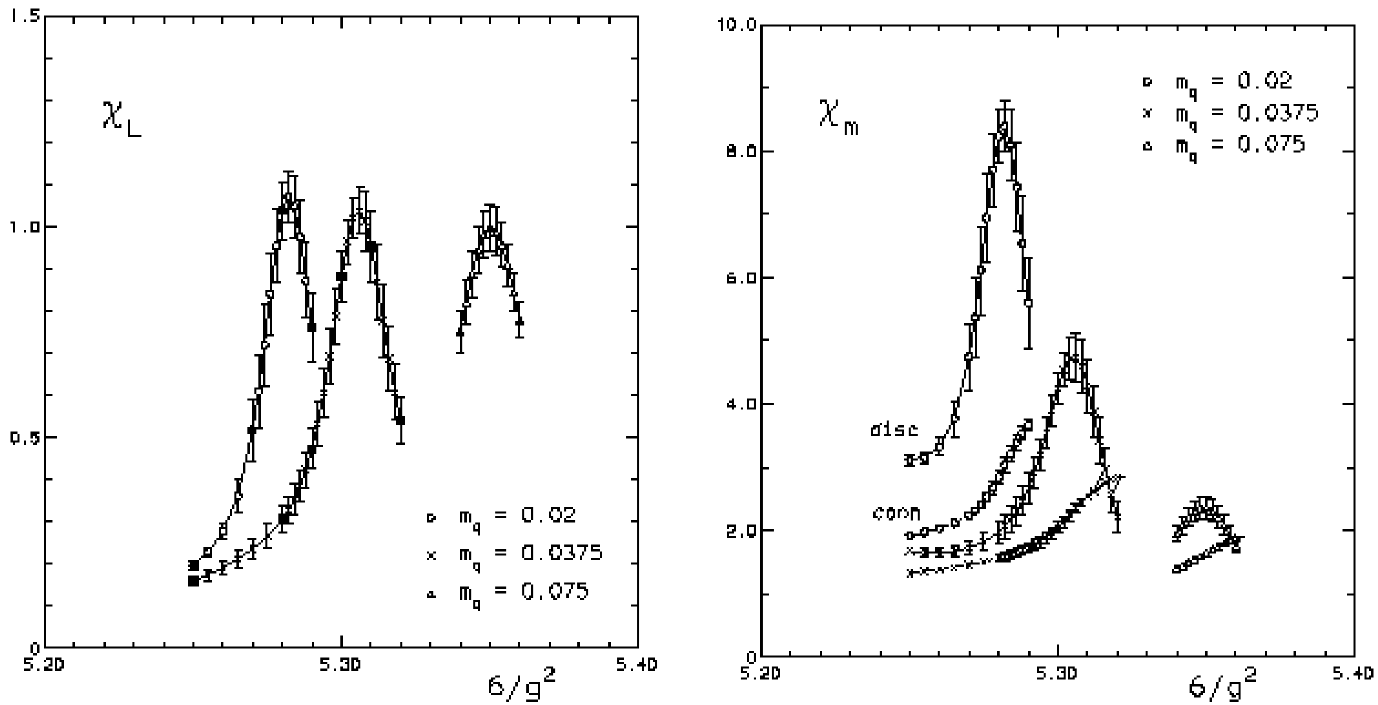
$$\langle\bar{q}q\rangle = \begin{cases} \neq 0 \Rightarrow \text{symmetry broken} \\ = 0 \Rightarrow \text{symmetry restored} \end{cases}$$

Order parameters



Variation of OPs Fukugita et.al. PRL 57 503 '86

Coincidence



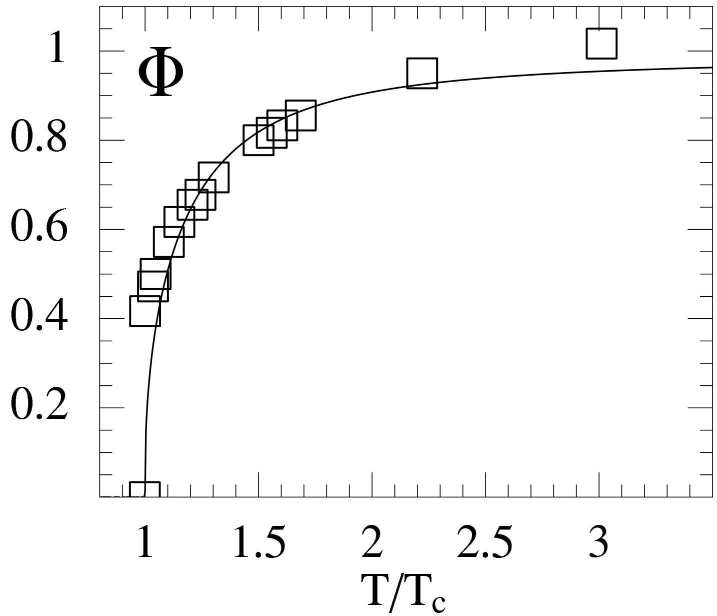
Variation of susceptibility Karsch et.al. PRD 50 6954 ’94

Recent estimates of transition temperature with 2 quark flavours:

$T_c = (171 \pm 4)$ MeV for Lattice Wilson fermions CP-PACS, PRD 63 034502 ’01

$T_c = (173 \pm 8)$ MeV for Lattice staggered fermions Bielefeld, NPB 605 579 ’02

Polyakov Loop Model



- Choose some $U(\Phi)$ as a polynomial, parametrized using Lattice:

  Lattice EOS: Scavenius et.al. PRC 66 034903 '02.

$$\frac{U\left(\bar{\Phi},\Phi,T\right)}{T^4} = -\frac{b_2\left(T\right)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2$$

where, $\Phi = \langle \mathrm{TrL} \rangle$; $\bar{\Phi} = \langle \mathrm{TrL}^\dagger \rangle$

NJL Model

Lagrangian:

$$\mathcal{L}_{NJL} \quad = \quad \bar{q}\left(i\gamma^{\mu}\partial_{\mu} - m_0 + \mu\gamma^0\right) q + \frac{\mathcal{G}}{2}\left[(\bar{q}q)^2 + \left(\bar{q}i\gamma^5\vec{\tau}q\right)^2\right]$$

- Symmetries: $SU(2)_V \otimes SU(2)_A \otimes U(1)_B$.
- Introducing auxillary field variables σ and $\vec{\pi}$ an $\mathcal{L}_{eff}$ is obtained.
- The mean fields $\langle\sigma\rangle = \mathcal{G}\langle\bar{q}q\rangle$ and $\langle\vec{\pi}\rangle = 0\,\mathrm{for}\,\mu_I < m_\pi$.
- Fit emperical values of m_π, f_π and $g_{\pi NN}$ (RMP 64 649 '92).
  Obtain $m_0 = 5.5\,\mathrm{MeV}$, $\mathcal{G} = 10.08\,\mathrm{GeV}^{-2}$, cutoff $\Lambda = 0.651\,\mathrm{GeV}$.
- Thermodynamic properties studied with the thermodynamic potential $\Omega[\sigma, T, \mu_0, \mu_I]$,
  where $\mu_0 = \frac{\mu_u+\mu_d}{2}$; $\mu_I = \frac{\mu_u-\mu_d}{2}$

PNJL Model

Lagrangian:

$$\mathcal{L}_{PNJL} = \bar{q}\left(i\gamma^{\mu}D_{\mu} - m_0 + \mu\gamma^0\right)q + \frac{\mathcal{G}}{2}\left[\left(\bar{q}q\right)^2 + \left(\bar{q}i\gamma^5\vec{\tau}q\right)^2\right] - U(\bar{\Phi}, \Phi)$$

where $D_{\mu} = \partial_{\mu} - igG_{\mu}$,
and $G_{\mu} = \delta_{\mu 0}G_0$

- Introducing auxillary field variables σ and $\vec{\pi}$ an $\mathcal{L}_{eff}$ is obtained, with the replacement $\exp\left[-G_0/T\right] \rightarrow \Phi$
- The mean fields $\langle\sigma\rangle = \mathcal{G}\langle\bar{q}q\rangle$ and $\langle\vec{\pi}\rangle = 0$ for $\mu_I < m_{\pi}$.
- Thermodynamic properties studied with $\Phi(T)$ and σ from the thermodynamic potential $\Omega[\bar{\Phi}, \Phi, \sigma, T, \mu_0, \mu_I]$, where $\mu_0 = \frac{\mu_u + \mu_d}{2}$; $\mu_I = \frac{\mu_u - \mu_d}{2}$

PNJL Model

$$
\begin{aligned}
\Omega &= \mathcal{U}\left(\Phi,\bar{\Phi},T\right) + 2G_1(\sigma_u^2+\sigma_d^2) + 4G_2\sigma_u\sigma_d \\
&- \sum_{f=u,d} 2\,T \int \frac{\mathrm{d}^3p}{(2\pi)^3} \left\{ \ln\left[1+3\left(\Phi+\bar{\Phi}\mathrm{e}^{-(E_f-\mu_f)/T}\right)\mathrm{e}^{-(E_f-\mu_f)/T} + \mathrm{e}^{-3(E_f-\mu_f)/T}\right]\right. \\
&+ \left.\ln\left[1+3\left(\bar{\Phi}+\Phi\mathrm{e}^{-(E_f+\mu_f)/T}\right)\mathrm{e}^{-(E_f+\mu_f)/T} + \mathrm{e}^{-3(E_f+\mu_f)/T}\right]\right\} \\
&- \sum_{f=u,d} 6 \int \frac{\mathrm{d}^3p}{(2\pi)^3} E_f \theta\left(\Lambda^2 - \vec{p}^{\,2}\right)
\end{aligned}
$$

where,

$$
\frac{\mathcal{U}\left(\Phi,\bar{\Phi},T\right)}{T^4} = -\frac{b_2\left(T\right)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2
$$

PNJL Model

- Attempt to understand the coincidence of chiral and deconfinement transition.

  Meisinger, Ogilvie '96: ($U(\Phi) = 0$)

- Dynamical understanding of the coincidence

  Fukushima '04: ($U(\Phi)$ from Lattice strong coupling)

- Pressure difference at finite μ, number density, interaction measure compared with the Lattice data.

  Ratti, Thaler, Wiese '05: ($U(\Phi)$ from Lattice EOS)

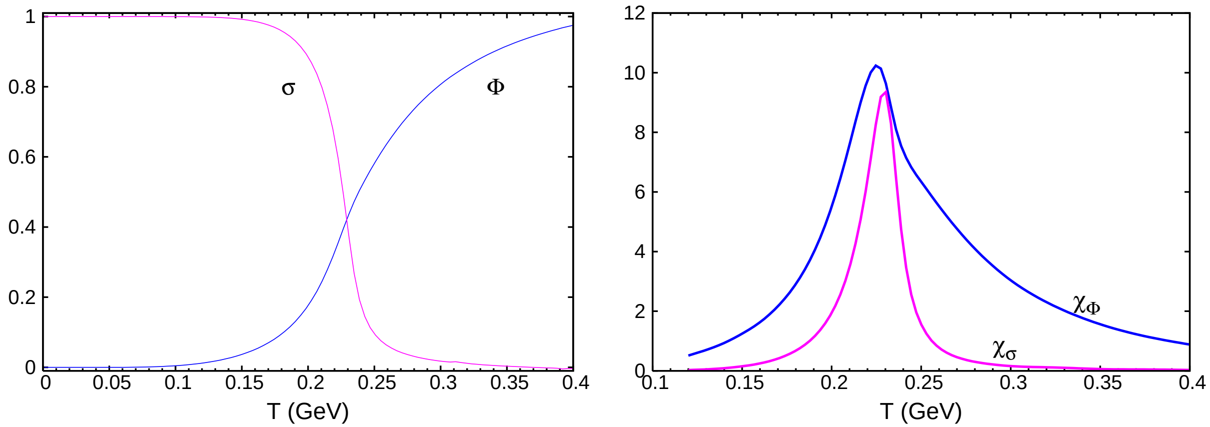
- Susceptibilities, specific heat, speed of sound, conformal measure compared with the Lattice data.

  Ghosh, Mukherjee, Mustafa, Ray '06: (Same model as Ratti et.al.)

- PNJL model improved with Van der Monde term

  Ghosh, Mukherjee, Mustafa, Ray '08

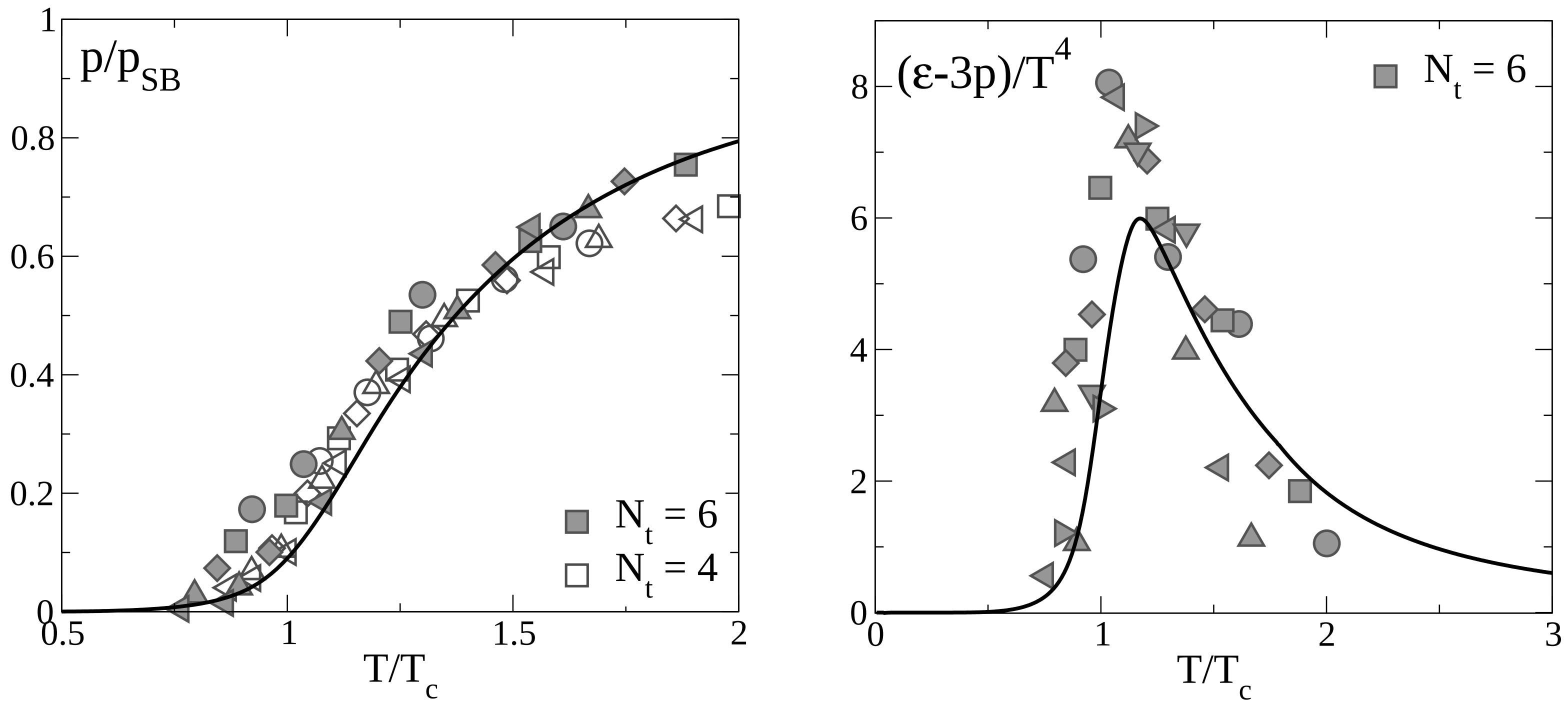
Order parameters



Variation of the OPs and coincidence of transition

Ratti et.al. PRD 73 014019 '06, RR et.al. PRD 73 114007 '06.

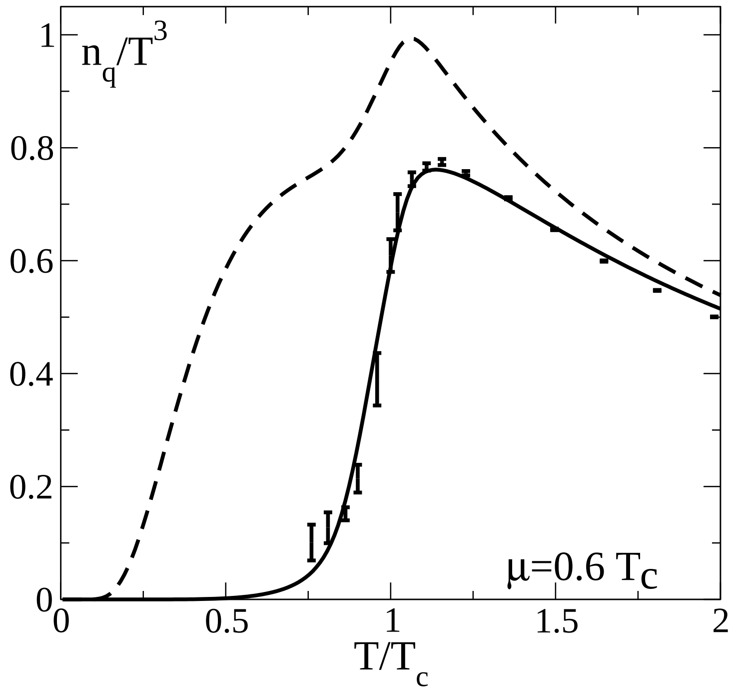
EOS



$$P = -\Omega/V = \left(\frac{T}{V}\right) \ln Z$$

PNJL model Ratti et.al.; Lattice CP-PACS PRD 64 074510 '01.

Number Density



$$n = \frac{\partial P}{\partial \mu}$$

NJL and PNJL model Ratti et.al.; Lattice Bielefeld PRD 68 014507 ’03.

Taylor Expansion

$$\frac{1}{T^4} P(T, \mu_0, \mu_I) = \frac{1}{T^4} P(T, 0, 0) + \sum_{n_0, n_I} c^{n_0 n_I} \left(\frac{\mu_0}{T}\right)^{n_0} \left(\frac{\mu_I}{T}\right)^{n_I} + \cdots$$

- Number Density:

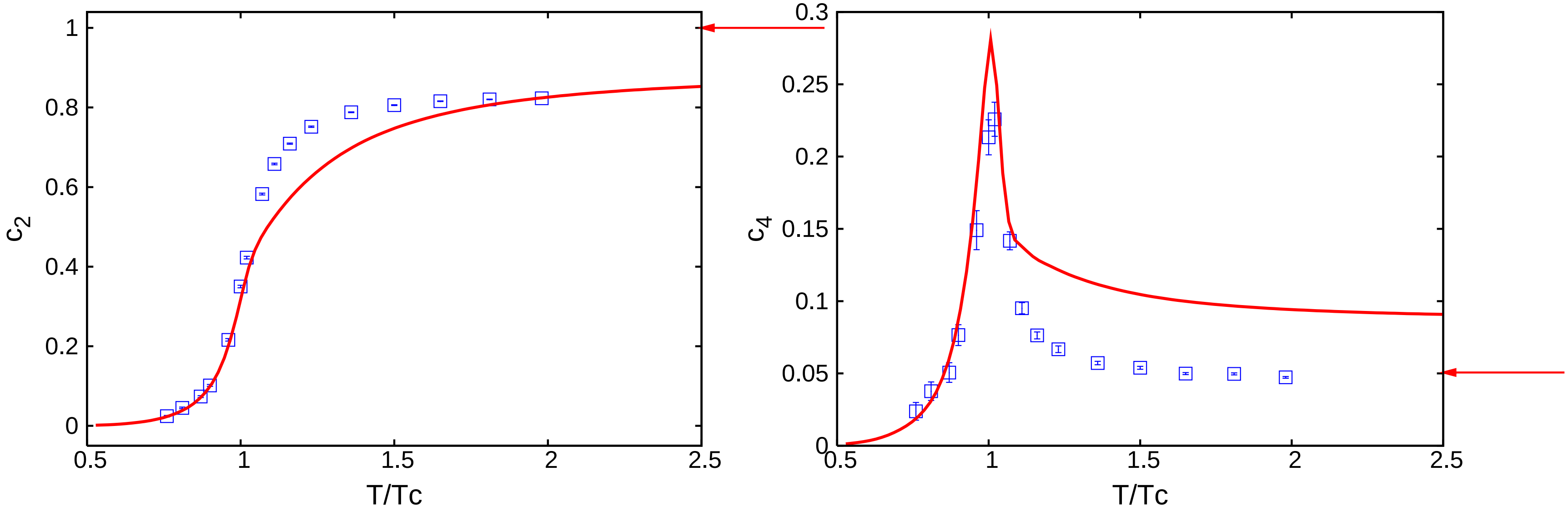
$$c^{10} = \left.\frac{\partial P}{\partial \mu_0}\right|_{\mu_0=\mu_I=0} = n_q \qquad c^{01} = \left.\frac{\partial P}{\partial \mu_I}\right|_{\mu_0=\mu_I=0} = n_I$$

- Fluctuations:

$$2T^2 c^{20} = \left.\frac{\partial^2 P}{\partial \mu_0^2}\right|_{\mu_0=\mu_I=0} = \chi_q \qquad 2T^2 c^{02} = \left.\frac{\partial^2 P}{\partial \mu_I^2}\right|_{\mu_0=\mu_I=0} = \chi_I$$

- Higher order terms give μ dependence of Fluctuations

Fluctuations

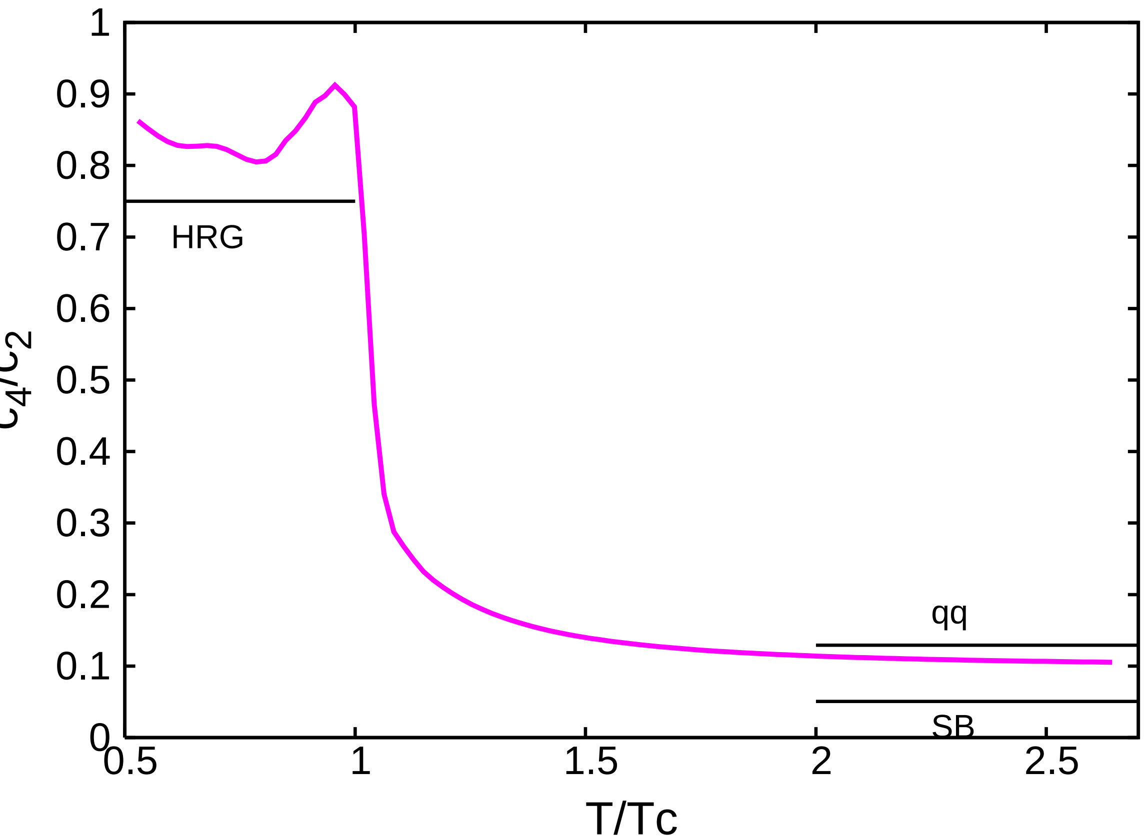


- Fluctuations are different in different phases.

  Asakawa, Heinz and Müller, '02; Jeon and Koch, '02.

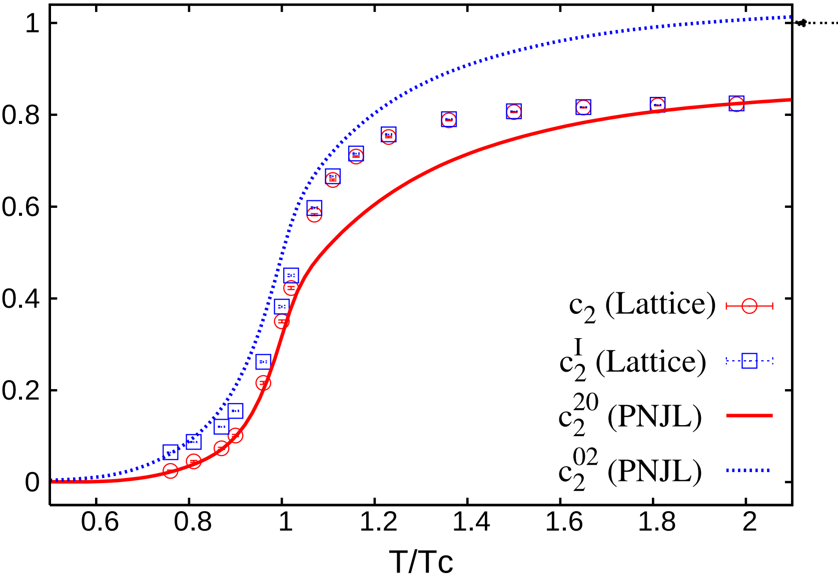
PNJL model RR et.al.; Lattice Bielefeld PRD 71 054508 '05

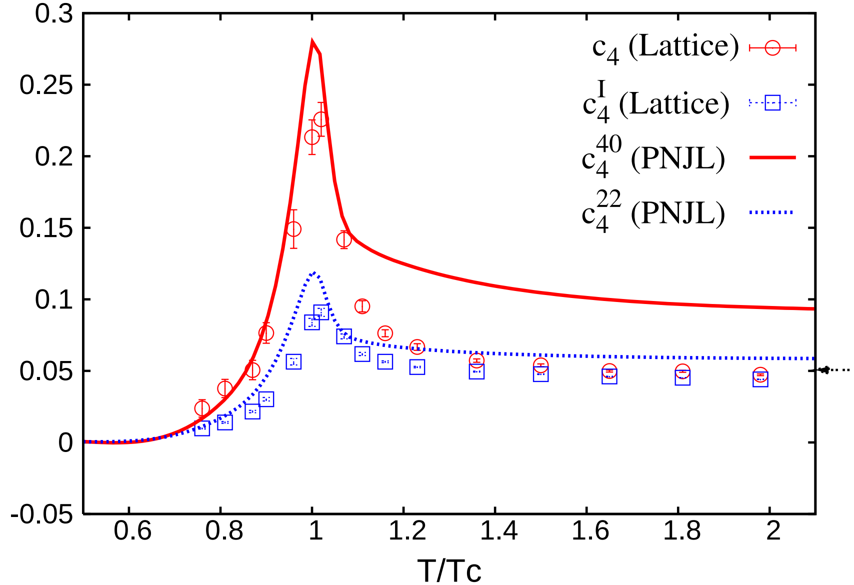
Fluctuations



- PNJL model works in a large range of temperature.

sospin Fluctuation - I





- Lattice: $c_2 \sim c_2^I \sim 80\%$ SB limit ; $c_4 \sim c_4^I \rightarrow$ SB limit
- PNJL: $c_2 \neq c_2^I$, $c_4 \neq c_4^I$

c_2 and c_4 away from SB limit ; c_2^I and $c_4^I \rightarrow$ SB limit

PNJL model RR et.al. PRD '07

PNJL + Van der Monde

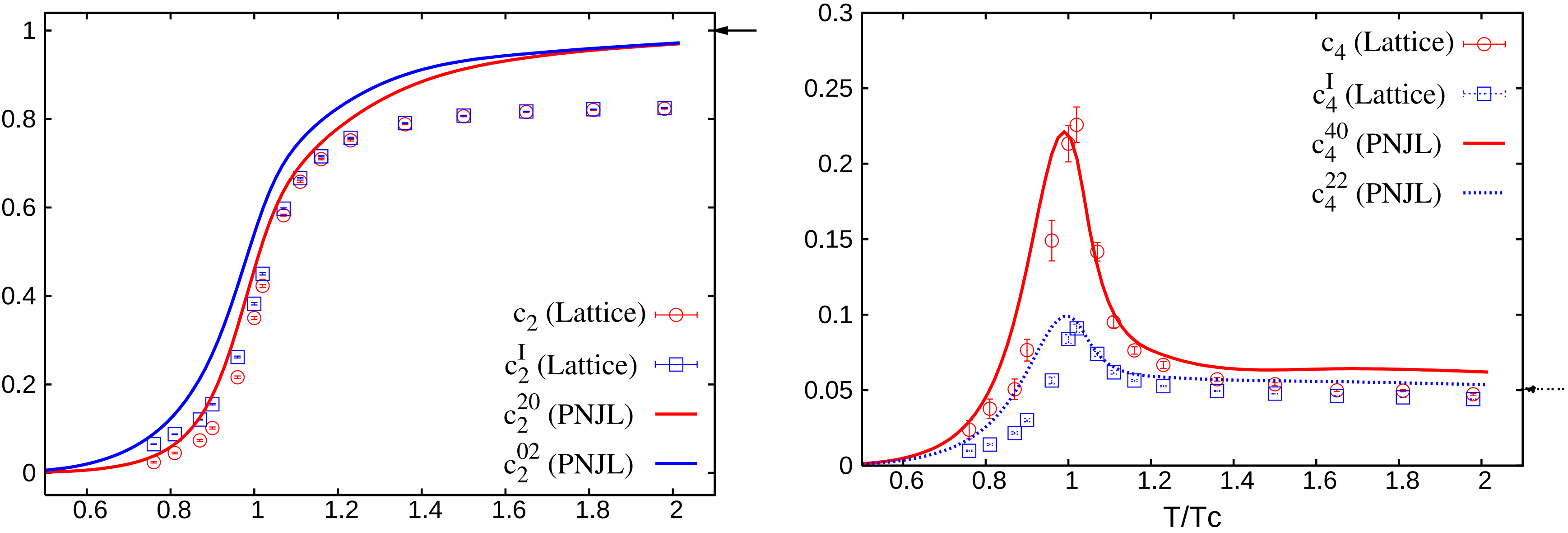
$$
\begin{aligned}
\Omega &= \mathcal{U}\left(\Phi,\bar{\Phi},T\right) + 2G_1(\sigma_u^2+\sigma_d^2) + 4G_2\sigma_u\sigma_d \\
&- \sum_{f=u,d} 2\,T \int \frac{\mathrm{d}^3p}{(2\pi)^3} \left\{ \ln\left[1 + 3\left(\Phi + \bar{\Phi}\mathrm{e}^{-(E_f-\mu_f)/T}\right)\mathrm{e}^{-(E_f-\mu_f)/T} + \mathrm{e}^{-3(E_f-\mu_f)/T}\right] \right. \\
&+ \left. \ln\left[1 + 3\left(\bar{\Phi} + \Phi\mathrm{e}^{-(E_f+\mu_f)/T}\right)\mathrm{e}^{-(E_f+\mu_f)/T} + \mathrm{e}^{-3(E_f+\mu_f)/T}\right]\right\} \\
&- \sum_{f=u,d} 6 \int \frac{\mathrm{d}^3p}{(2\pi)^3} E_f\,\theta\left(\Lambda^2 - \vec{p}^{\,2}\right)
\end{aligned}
$$

where,

$$
\frac{\mathcal{U}\left(\Phi,\bar{\Phi},T\right)}{T^4} = -\frac{b_2\left(T\right)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2
$$

$$
+\kappa\ln[1 - 6\,\bar{\Phi}\Phi + 4\,\left(\Phi^3+\bar{\Phi}^3\right) - 3\,\left(\bar{\Phi}\Phi\right)^2]
$$

Isospin Fluctuation - II



- Lattice: $c_2 \sim c_2^I \sim 80\%$ SB limit ; $c_4 \sim c_4^I \rightarrow$ SB limit
- PNJL: $c_2 \sim c_2^I \rightarrow$ SB limit ; $c_4 \sim c_4^I \rightarrow$ SB limit
- At low temperatures PNJL coeff. greater than lattice.

PNJL model RR et.al. PRD '08

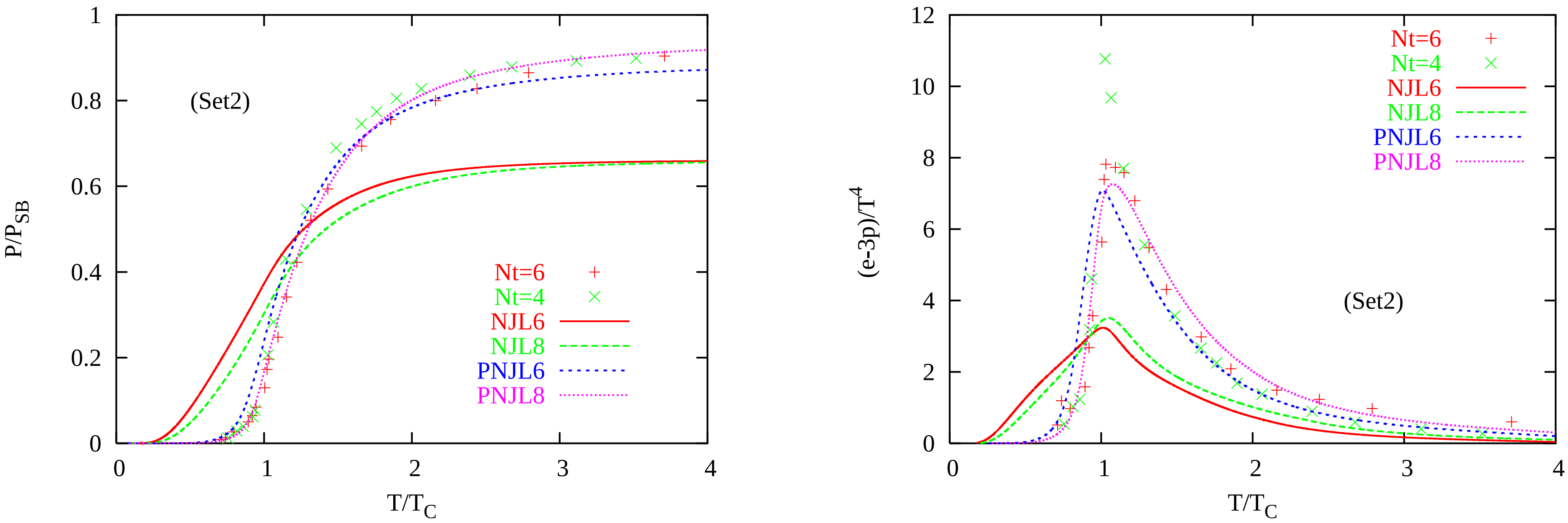
2+1 flavors

$$\begin{aligned}\Omega \quad &= \mathcal{U}'[\Phi,\bar{\Phi},T] + 2g_S\sum_{f=u,d,s}\sigma_f^2 - \frac{g_D}{2}\sigma_u\sigma_d\sigma_s + 3\frac{g_1}{2}(\sum_{f=u,d,s}{\sigma_f}^2)^2 \\
&+3g_2\sum_{f=u,d,s}\sigma_f^4 - 6\sum_{f=u,d,s}\int_0^\Lambda \frac{d^3p}{(2\pi)^3}E_f\Theta(\Lambda-|\vec{p}|) \\
&-2T\sum_{f=u,d,s}\int_0^\infty \frac{d^3p}{(2\pi)^3}\ln\left[1+3(\Phi+\bar{\Phi}e^{-\frac{(E_f-\mu)}{T}})e^{-\frac{(E_f-\mu)}{T}}+e^{-3\frac{(E_f-\mu)}{T}}\right] \\
&-2T\sum_{f=u,d,s}\int_0^\infty \frac{d^3p}{(2\pi)^3}\ln\left[1+3(\bar{\Phi}+\Phi e^{-\frac{(E_f+\mu)}{T}})e^{-\frac{(E_f+\mu)}{T}}+e^{-3\frac{(E_f+\mu)}{T}}\right]\end{aligned}$$

where, g_S is the usual four-fermi interaction, g_D is the coupling for the 't Hooft determinant and g_1 and g_2 are the 8q coupling constants needed to remove an infinite potential well close to the classical vacuum.

Abhijit Bhattacharya, Paramita Deb, Sanjay Ghosh, Anirban Lahiri, Sarbani Majumder, RR

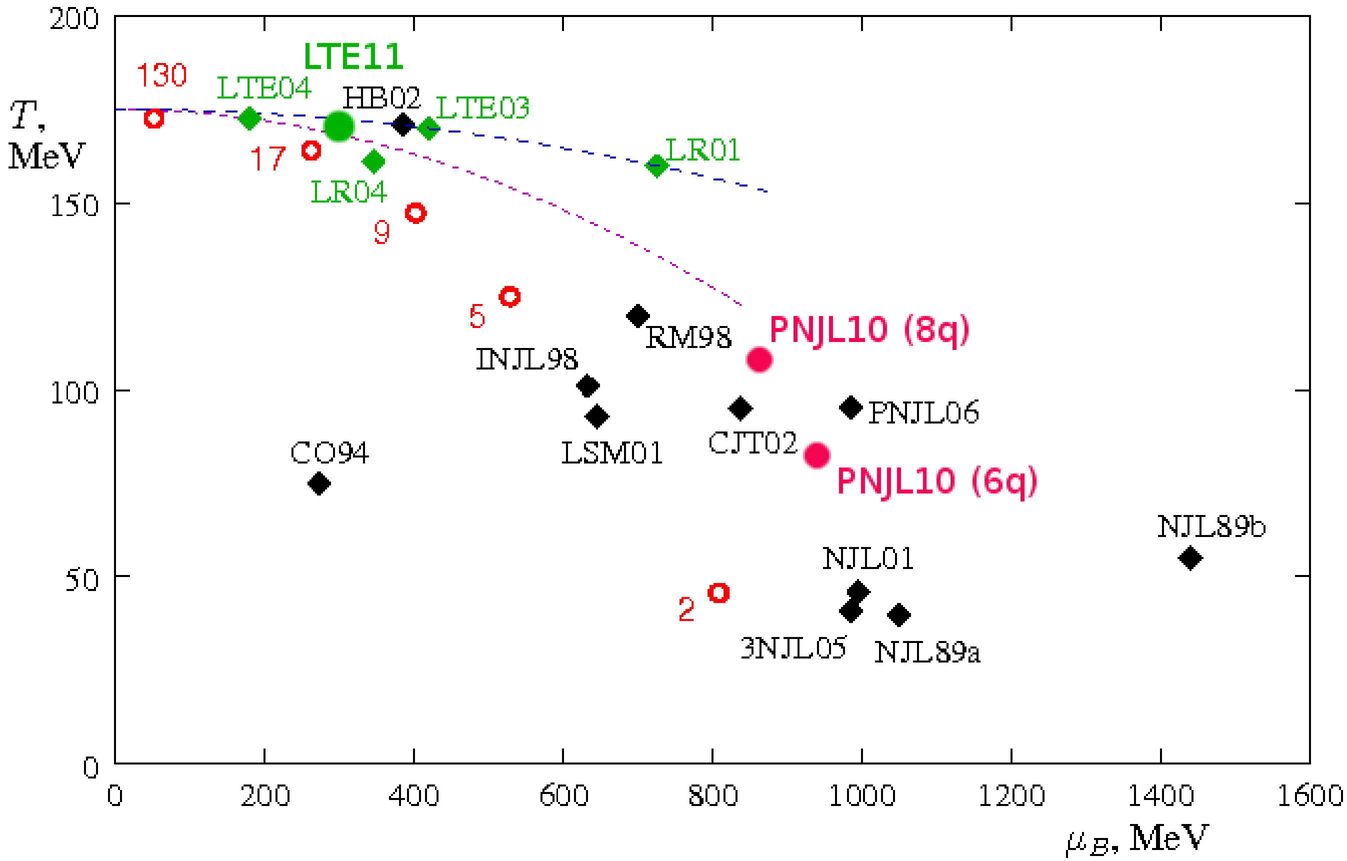
EOS - 2+1 flavor



$$P = -\Omega/V = \left(\frac{T}{V}\right) \ln Z$$

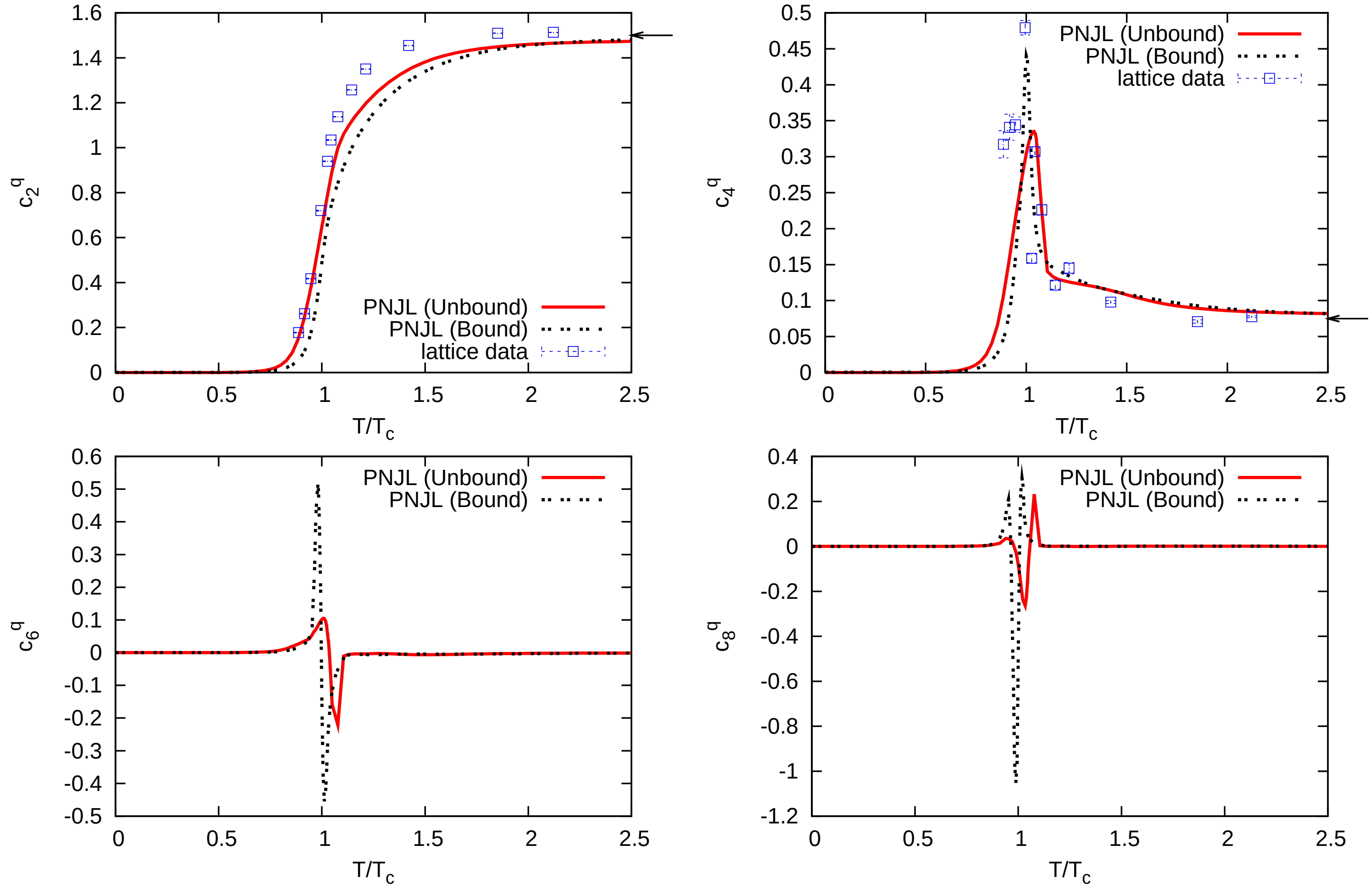
PNJL model RR et.al.; Lattice Cheng et. al. PRD 77 014511 ’08.

Phase Diagram....



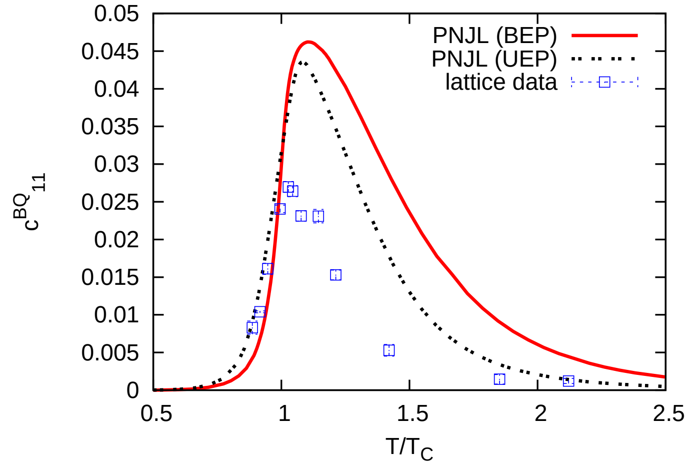
M. Stephanov (hep-lat:0701002) (except PNJL10(6q), PNJL10(8q))

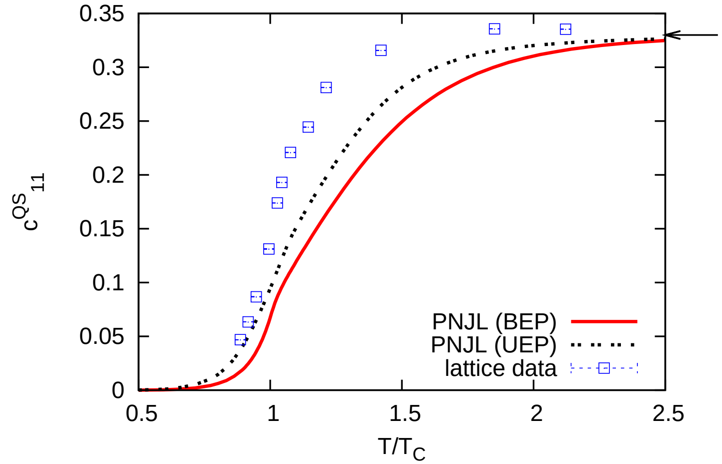
Fluctuations - 2+1 flavors

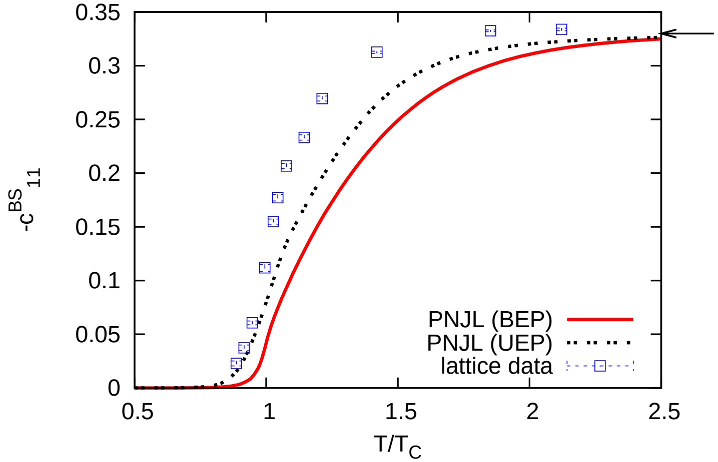


PNJL model RR et.al. ’10   LQCD Cheng et.al. ’08

Correlations - 2+1 flavors







PNJL model RR et.al. ’10 LQCD Cheng et.al. ’08

Baryon-Isospin correlations

Going back to 2 flavors

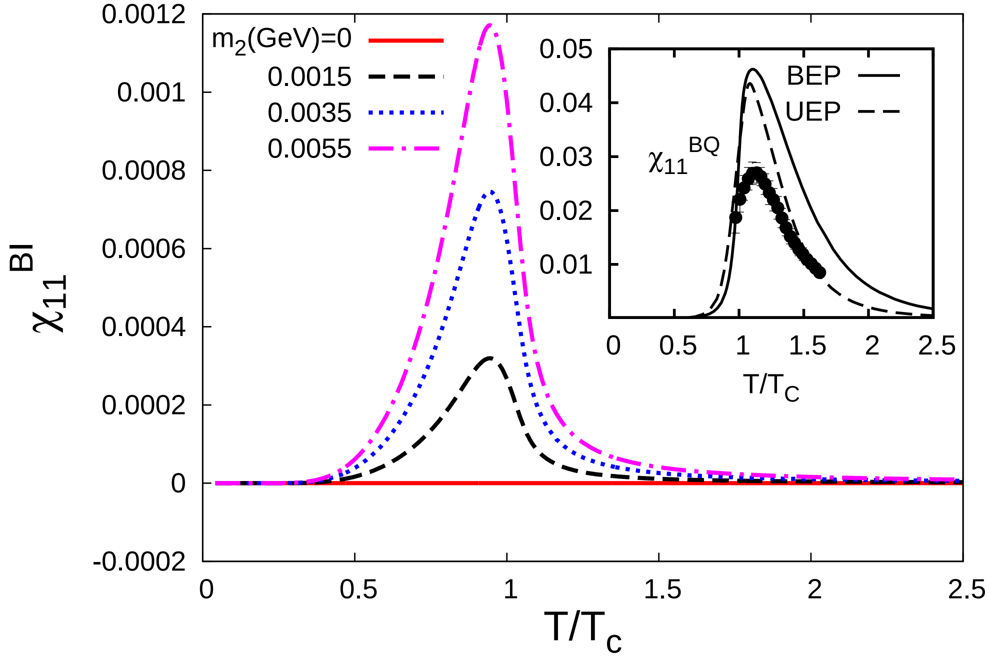
- *Non-degenerate* masses for different flavors $\rightarrow SU_V(2)$ broken explicitly.
- Mass matrix is given as;

$$\hat{m} \equiv m_1 \mathbb{1}_{2\times 2} - m_2 \tau_3 = \begin{pmatrix} m_1 - m_2 & 0 \\ 0 & m_1 + m_2 \end{pmatrix} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}.$$

- three different values of m_2 are taken keeping m_1 fixed at 5.5 MeV.

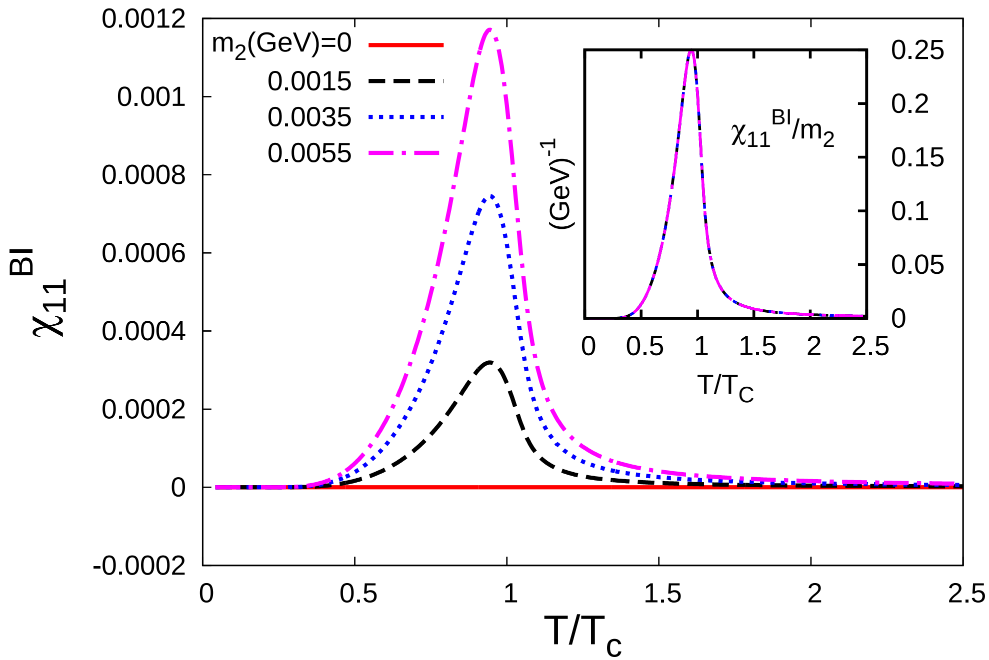
Abhijit Bhattacharya, Sanjay Ghosh, Anirban Lahiri, Sarbani Majumder, Sibaji Raha, RR

Baryon-Isospin correlations



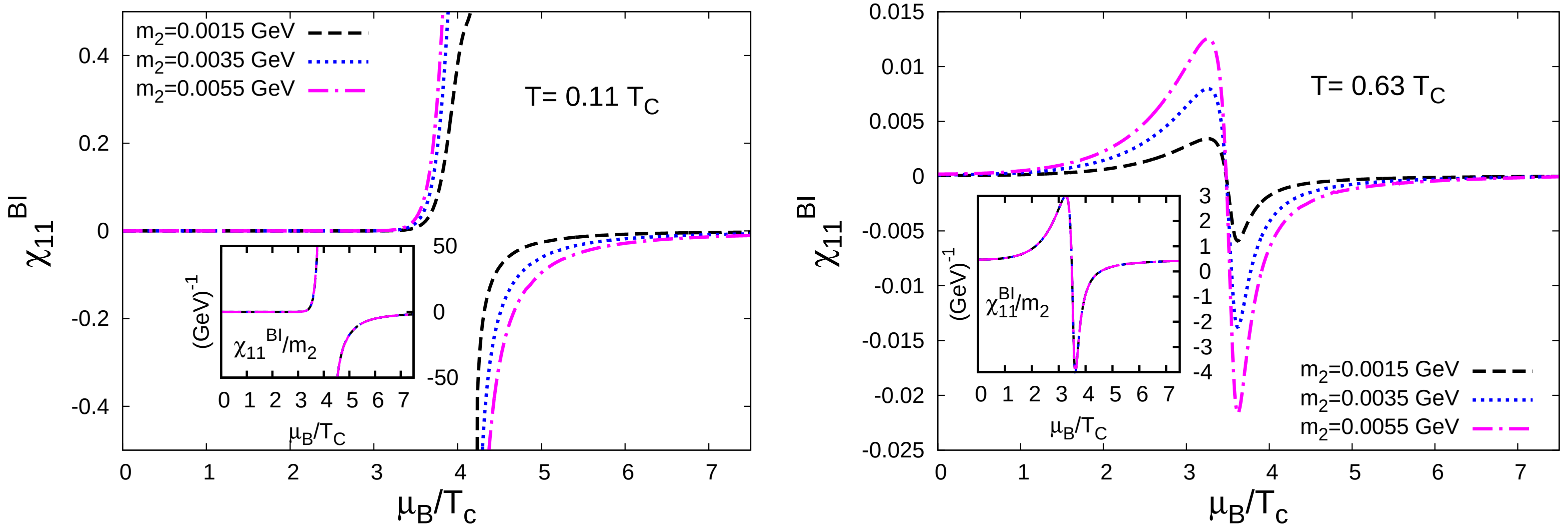
- $\chi_{11}^{BI} = \frac{1}{6}(\chi_2^u - \chi_2^d)$
- $\chi_{11}^{BQ} = \frac{1}{9}(2\chi_2^u - \chi_2^d - \chi_2^s + \chi_{11}^{ud} + \chi_{11}^{us} - 2\chi_{11}^{ds})$

Baryon-Isospin correlations



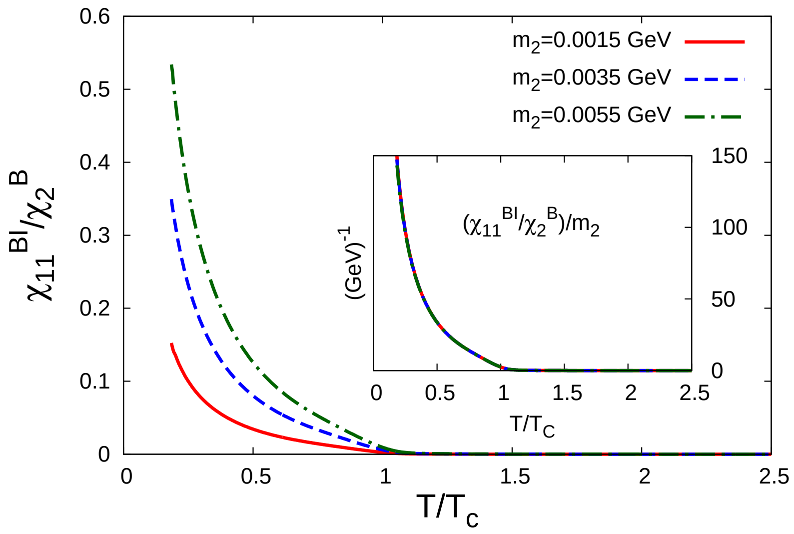
- $\chi_{11}^{BI} = \frac{1}{6}(\chi_2^u - \chi_2^d)$
- Almost linear scaling with m_2.

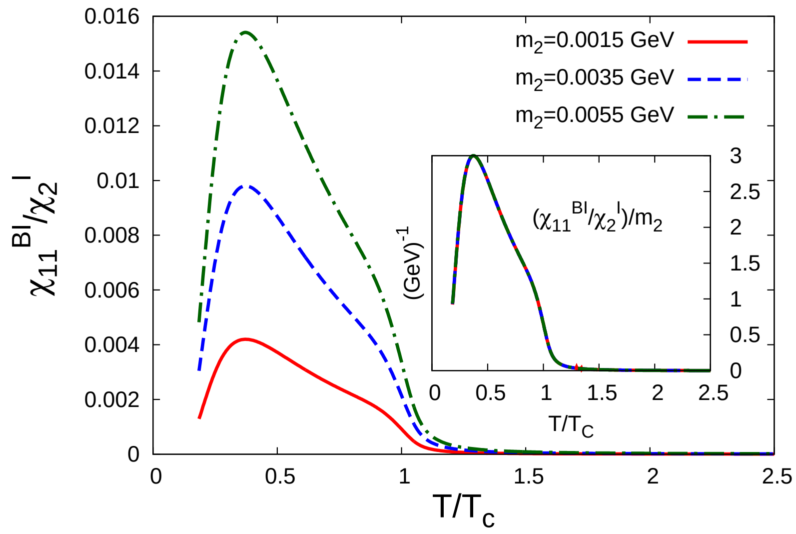
Baryon-Isospin correlations



- Expected scaling in finite chemical potential

Baryon-Isospin correlations





- Setting a baseline for experimental exploration

Finite Volumes

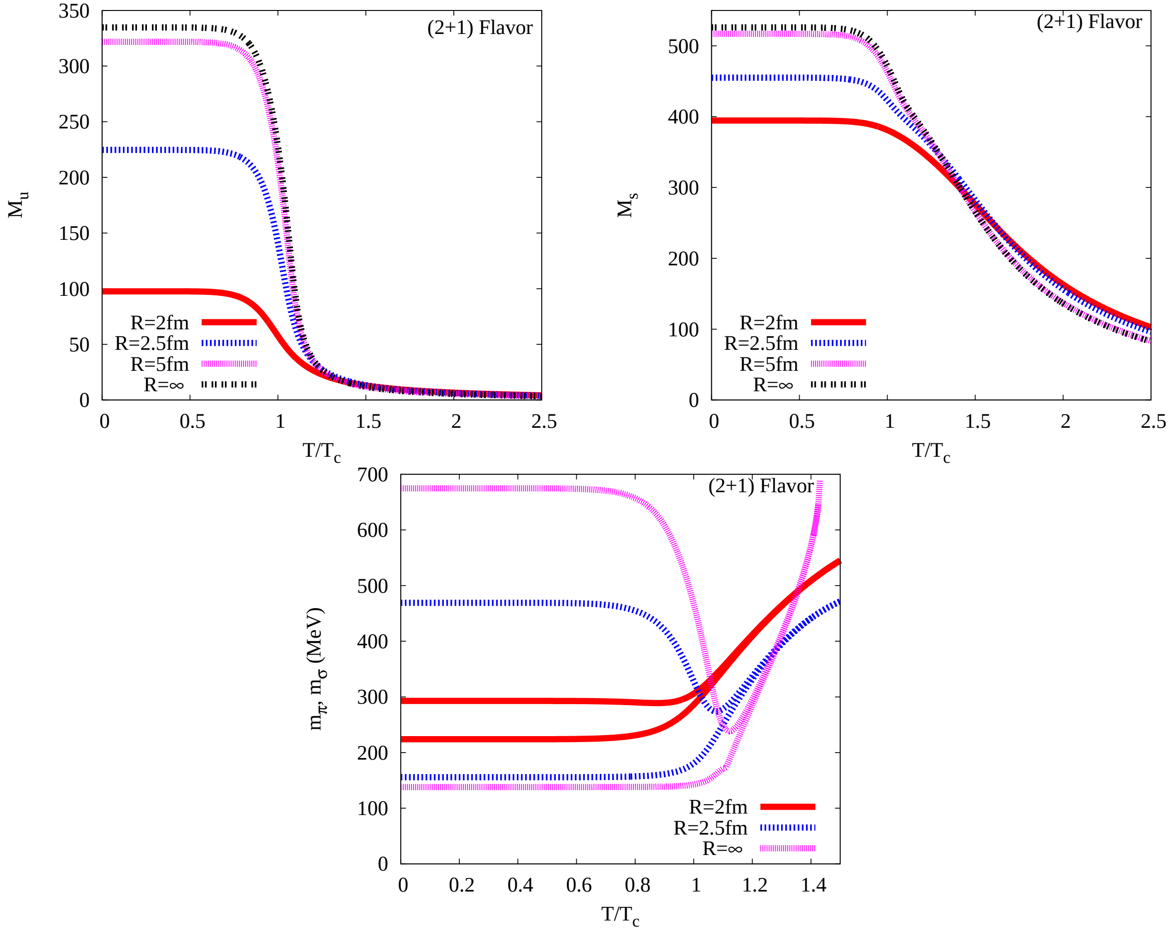
Oversimplifications:

- Neglecting volume effects on $U(\Phi, \bar{\Phi})$
- Neglecting surface curvature effects
- Replacing discrete momentum sum by integral with IR cut-off

$P = -\partial\Omega/\partial V$

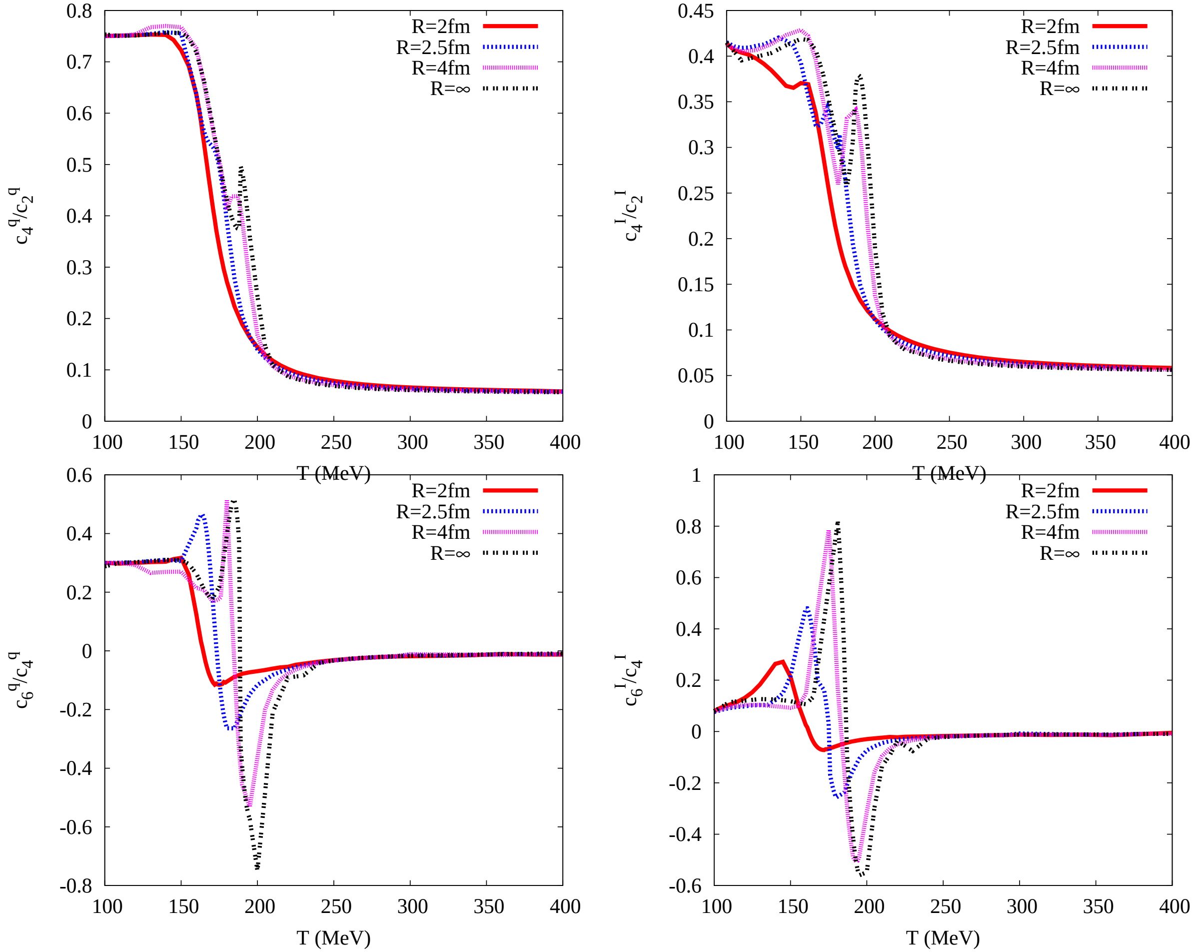
Abhijit Bhattacharya, Paramita Deb, Sanjay Ghosh, Kinkar Saha, Subrata Sur, Sudipa Upadhaya, RR

Finite Volumes



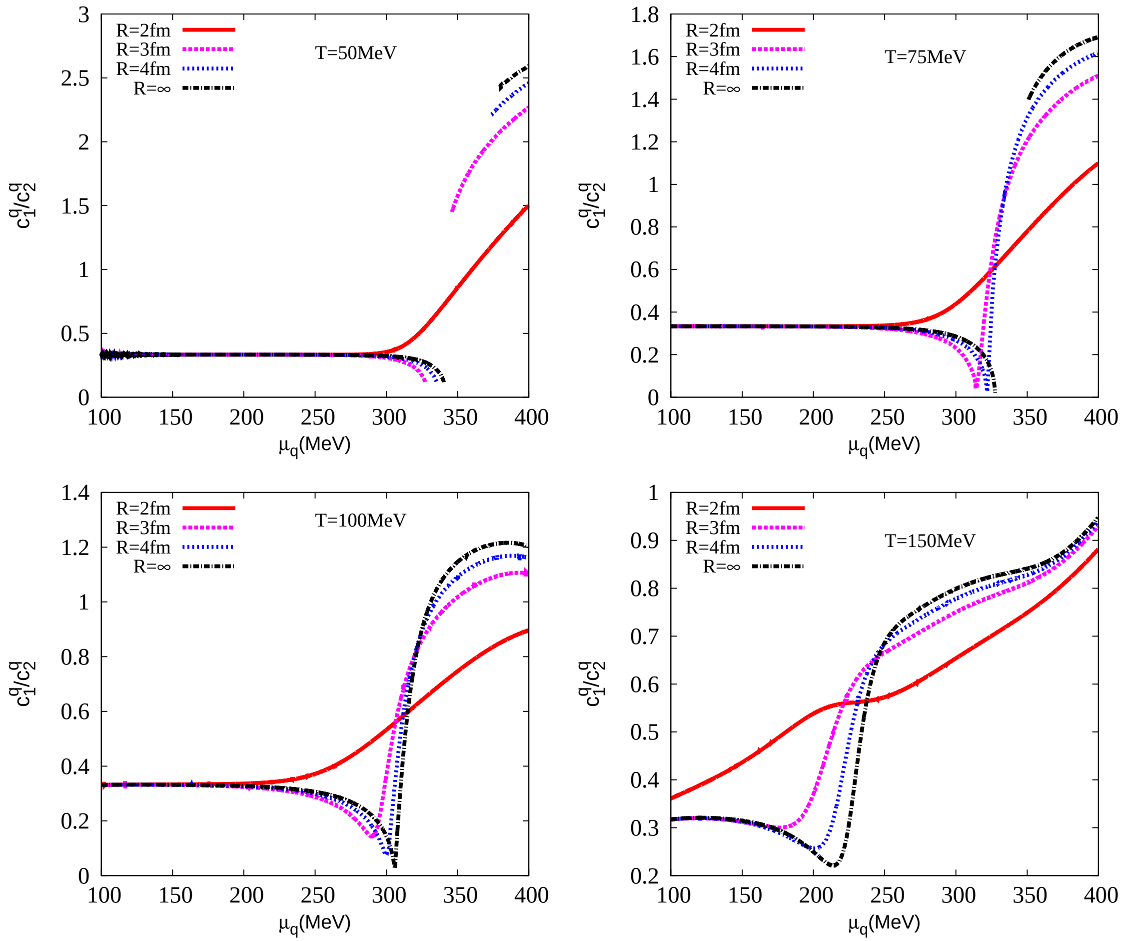
PNJL model RR et.al. ’12

Finite Volume Fluctuations



- Volume scaling shaky only close to T_c (PNJL model 2-flavor RR et.al. '14)

Finite Volume + Finite Density

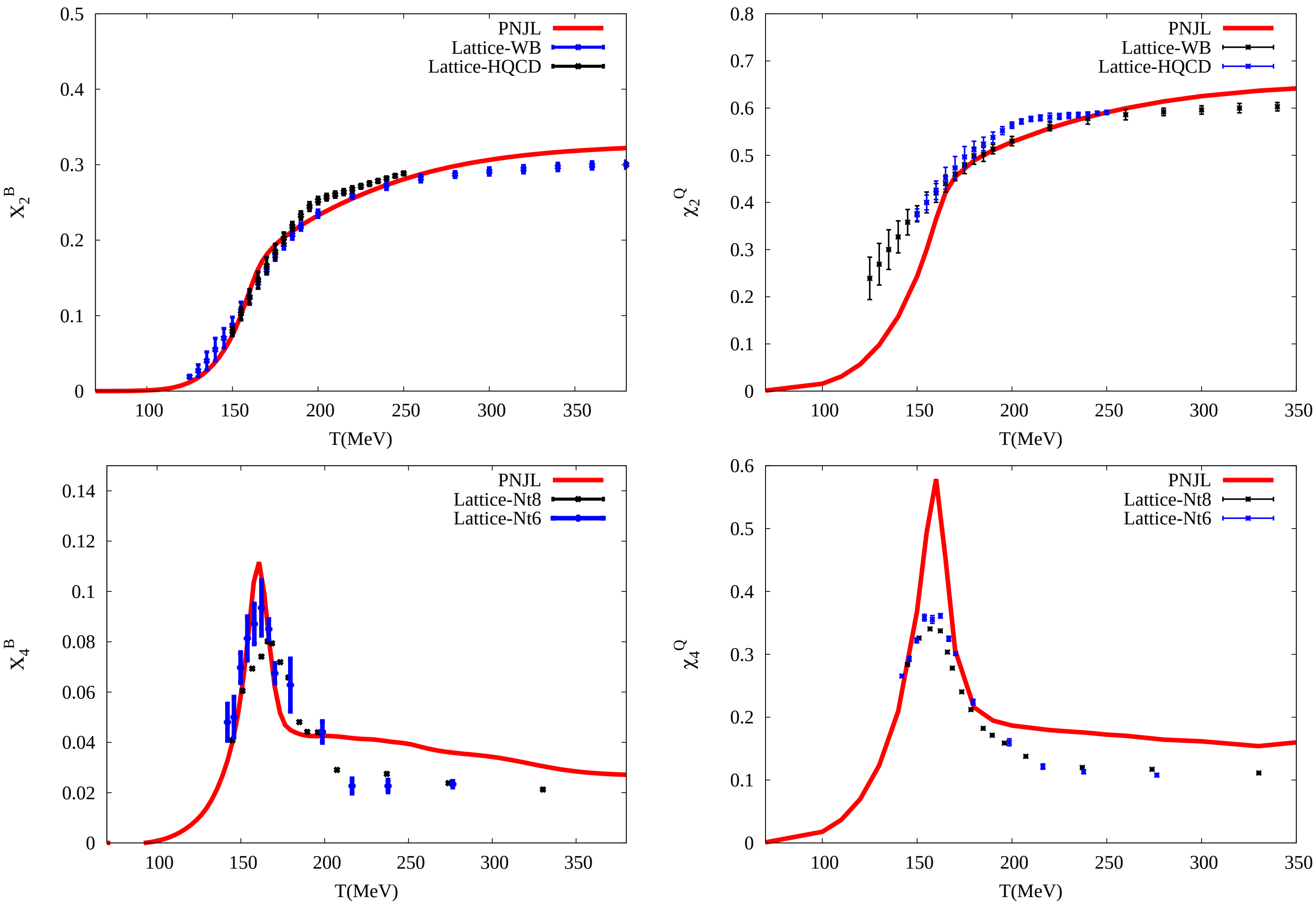


- Volume scaling shaky only close to T_c (PNJL model 2+1 flavor RR et.al. '15)

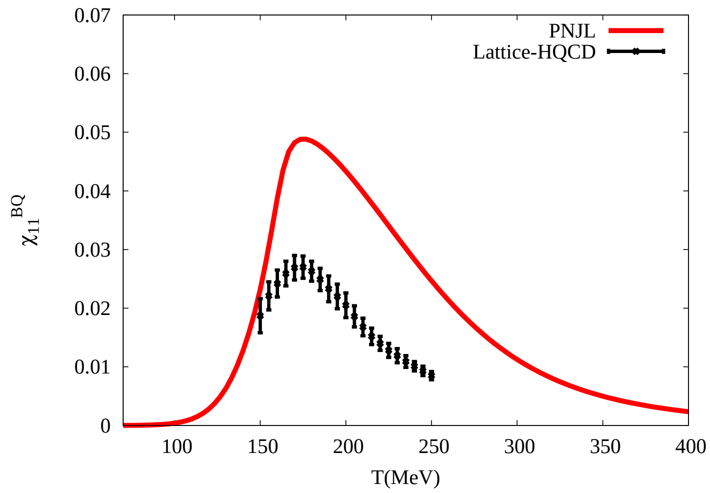
Setting the scale

- Till now we found qualitative physics information to be in good agreement with LQCD data where available. We are now working for quantitative contrast as well as setting up the transition temperature.
- Changing the scale in Polyakov loop model has limited control over the chiral transition.
- Time to remodel NJL
  - Four quark coupling $G = G(\Phi, \bar{\Phi})$ (entangled PNJL)
    Aminul Islam, Munshi Golam Mustafa....
  - Four quark coupling $G = G(T) = G\exp^{T^2/\alpha}$
    Soumitra Maity.....

Setting the scale



Setting the scale

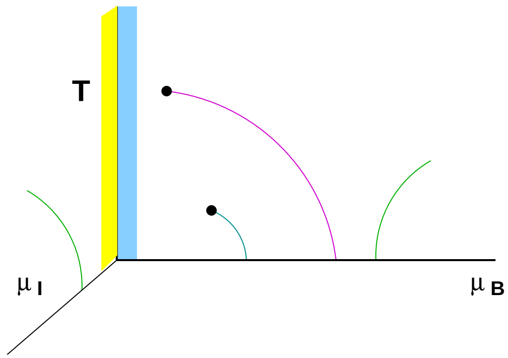


- Temperature dependence of G helps in softening the condensate behaviour: the scale may now be set
- Quantitative mismatch in the charge sector is mainly due to absence of light hadrons in our current model.

Lattice data: A. Bazavov et al. [HotQCD Collaboration], Phys. Rev. D 86;
S. Borsanyi et al., JHEP 1201, 138 (2012)

Summary

- Exploratory study of PNJL model to mimic physics of QCD
  - All the qualitative features of LQCD results are captured
- Baryon-isospin correlation offers a unique signature for finite temperature as well as finite density scenario
  - χ_{11}^{BI} changes sign at high density and low temperature.
- Finite volume fluctuation ratios follow system size scaling except very close to T_c at $\mu_B = 0$
- The higher order fluctuations break the scaling in a wider range of temperature.
- For $\mu_B \neq 0$ the region of scaling violation even for lower order fluctuations is considerable.



Thank You