Fluctuation-Correlation and Dimuon production in high energy heavy ion collision experiments

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Overview

Part 1

- Fluctuation-correlation using HRG/EVHRG model
- Beam Energy dependence of ratios of cumulants
- Chemical freeze-out parameters from fluctuation data
- Effects of finite volume on fluctuations

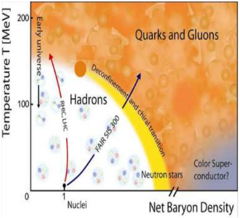
Part 2

- Thermal dimuon from QGP source at FAIR energy
- Building Event generator
- Application in MUCH simulation

Heavy-ion collision experiments

The major goals of HIC experiments at ultrarelativistic energies are

- The search and study of a new form of matter known as the Quark-Gluon Plasma (QGP).
- The mapping of the Quantum Chromodynamics (QCD) phase diagram in terms of temperature (T) and baryon chemical potential (μ_B).
- Locating the QCD critical end point (CEP). A cross-over transition is expected at high energy (LHC, top RHIC) (high T and small μ_B region) and a first order transition is expected at relatively lower energy (high μ_B region).



Part 1

Fluctuations and correlations

Motivations of Fluctuations and correlations study

- Event-by-event fluctuations of conserved charges like baryon, strangeness and electric charge are sensitive indicators of the transition between quark-gluon plasma and hadronic matter.
- The existence of the CEP can be signalled by the divergent fluctuations.
- Non-monotonic variations of observables related to the cumulants of the distributions of conserved charges with beam energy are believed to be good signatures of a phase transition and a CEP.
- Of course the signatures of phase transition or CEP are detectable if they survive during the evolution of the system.
- Correlations between conserved charges behave differently in hadronic and QGP phase.

Hadron Resonance Gas model (Non-interacting)

- Statistical thermal model.
- System consists of all the hadrons including resonances.
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- Grand canonical ensemble.
- The grand canonical partition function of a hadron resonance gas:

$$\ln Z = \sum_i \ln Z_i$$

- For particle i,

$$\ln Z_i^{id} = \pm \frac{V g_i}{2\pi^2} \int_0^\infty p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

The upper and lower sign corresponds to baryons and mesons respectively.

$$E_i = \sqrt{p^2 + m_i^2}, \quad \mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q$$

Various thermodynamical quantities of the thermal system

$$P^{id} = \sum_i P_i^{id} = \sum_i T \frac{\partial(\ln Z_i^{id})}{\partial V}$$

$$n^{id} = \sum_i n_i^{id} = \sum_i \frac{T}{V} \left(\frac{\partial \ln Z_i^{id}}{\partial \mu_i} \right)_{V,T}$$

$$\varepsilon^{id} = \sum_i \varepsilon_i^{id} = \sum_i -\frac{1}{V} \left(\frac{\partial \ln Z_i^{id}}{\partial \frac{1}{T}} \right)_{\frac{\mu}{T}}$$

$$s^{id} = \sum_i s_i^{id} = \sum_i \frac{1}{V} \left(\frac{\partial \left(T \ln Z_i^{id} \right)}{\partial T} \right)_{V,\mu}$$

Susceptibilities

$$\chi_x^n = \frac{1}{VT^3}\frac{\partial^n(lnZ)}{\partial(\frac{\mu_x}{T})^n}; \quad x = B, S, Q$$

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- χ^n is related to x^n.
- Susceptibilities are related to fluctuations via fluctuations-dissipation theorem.

Correlations

$$\chi_{xx'}^{ij} = \frac{1}{VT^3}\frac{\partial^{i+j}(lnZ)}{(\partial(\frac{\mu_x}{T})^i)(\partial(\frac{\mu_{x'}}{T})^j)}$$

Excluded volume hadron resonance gas model

In ideal HRG model, particles are point like. This simple model has only a few parameters. The interactions are also important, especially at high T or large μ_B, to catch the basic qualitative features of strong interactions where ideal gas assumption becomes inadequate. In the EVHRG model, hadronic phase is modelled by a gas of interacting hadrons, where the geometrical size of the hadrons are explicitly incorporated as the excluded volume correction to approximate the short-range van der Waals type repulsive hadron-hadron interaction.

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- Hadrons have finite hard-core radii.
- $V_{ex} = 4\frac{4}{3}\pi R^3$ is the volume (maximum) excluded for the hadron.

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- Hadrons have finite hard-core radii.
- $V_{ex} = 4\frac{4}{3}\pi R^3$ is the volume (maximum) excluded for the hadron.
- Pressure and chemical potential in EVHRG model:

$$P(T, \mu_1, \mu_2, ..) = \sum_i P_i^{id}(T, \hat{\mu}_1, \hat{\mu}_2, ..),$$

$$\hat{\mu}_i = \mu_i - V_{ev,i} P(T, \mu_1, \mu_2, ..)$$

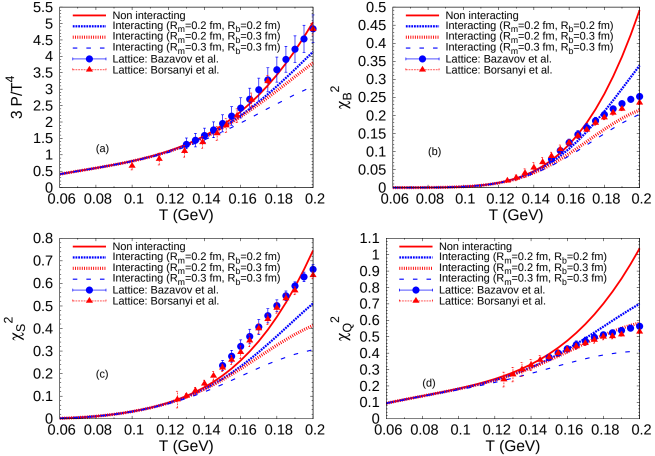
- In an iterative procedure one can get the total pressure.

$$n_i = n_i(T, \mu_1, \mu_2, ...) = \frac{\partial P}{\partial \mu_i} = \frac{n_i^{id}(T, \hat{\mu_i})}{1 + \sum_k V_{ev,k} n_k^{id}(T, \hat{\mu_k})}$$

$$\varepsilon = \varepsilon(T, \mu_1, \mu_2, ...) = \frac{\sum_i \varepsilon_i^{id}(T, \hat{\mu_i})}{1 + \sum_k V_{ev,k} n_k^{id}(T, \hat{\mu_k})}$$

$$\chi_q^1 = \chi_q^1(T, \mu_1, \mu_2, ...) = \frac{\sum_i \chi_{q,i}^{1,id}(T, \hat{\mu_i})}{1 + \sum_k V_{ev,k} n_k^{id}(T, \hat{\mu_k})}$$

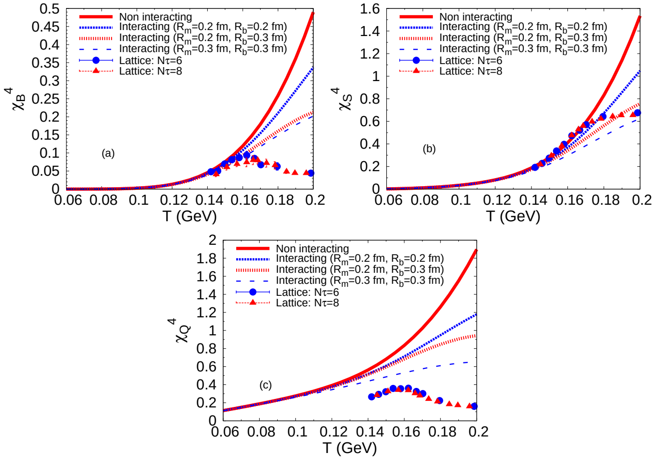
Variation of $3P/T^4$ and χ_q^2 with T at $\mu = 0$



S. Samanta *et al.*, Phys. Rev. C 90, no.3, 034909 (2014)

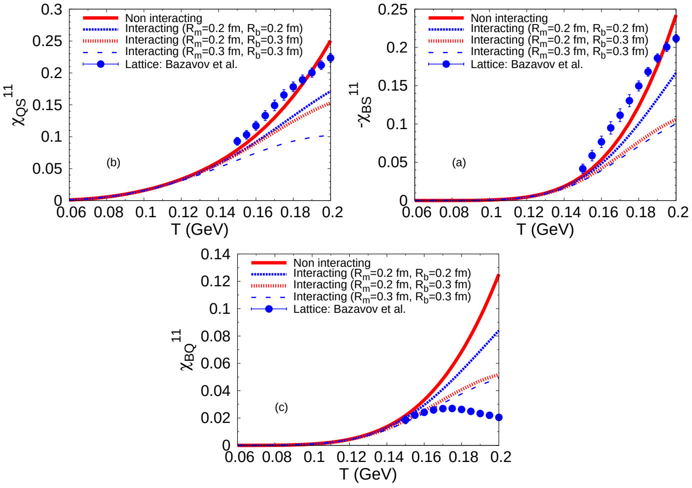
- The dominant contribution to χ_B^2, χ_S^2, χ_Q^2 at low temperatures come from nucleons, kaons and pions respectively. Since a pion is lighter compared to a nucleons and kaon, magnitude of χ_Q^2 is more than that of χ_B^2 and χ_S^2.
- With increase of T, effective d.o.f. increases (massive hadrons start contributing)
- There is almost no effect of interaction till $T = 0.13$ GeV.
- Dependence of χ_B^2 on R_m is not completely negligible.
- Odd powers of susceptibilities are zero since $\mu = 0$

Variation of χ_q^4 with temperature at $\mu = 0$



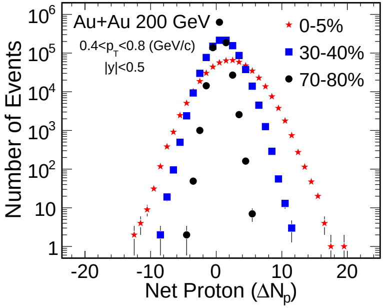
- The nature of all the fourth-order susceptibilities is similar to second-order susceptibilities.
- In the non-interacting HRG model, under Boltzmann approximation, $\chi_B^4 \approx \chi_B^2$ as only baryons with baryon number one contribute to various susceptibilities.
- Magnitudes of fourth-order susceptibilities are larger compared to that of second-order susceptibilities for strangeness and electric charge
- The LQCD data for χ_B^4 and χ_S^4 are close to HRG model up to T = 0.16 GeV. Whereas for χ_Q^4 , LQCD data is lower compared to both HRG and EVHRG.

Variation of correlations with temperature at $\mu = 0$



- At very low T the system is dominated by pions ($B = 0$, $S = 0$), all the correlations remain zero.
- The next particles to be excited are kaons ($K^{\pm}$) and as a result χ_{QS}^{11} becomes nonzero around $T = 0.075$ GeV. However other correlations are zero at lower T.
- Leading contribution to χ_{BS}^{11} is Λ ($S = -1, B = 1$) other contributing particles are Σ, Ξ. All the particles have relative negative sign between B and S. As a result χ_{BS}^{11} is negative.
- The dominant contribution in χ_{BQ}^{11} at low temperature is due to protons.

Fluctuations of conserved charges in relativistic heavy-ion collisions



- If we measure an observable in some system, we will not get the same result in each time of the measurement even if the measurement is performed with an ideal detector with an infinitesimal resolution. Rather, we would get a distribution of measured values around some mean value. How much the measured values fluctuate about the mean value is referred as fluctuation.
- If this measurement is performed in a thermal system then this fluctuation is called thermal fluctuation.
- Experimentally fluctuations are measured in event-by-event basis within certain acceptances.
- $\Delta y_{total} >> \Delta y_{accept}$. This condition ensures that the rest of the system acts as a reservoir.
- The main reason of variation of number of conserved charge in event-by-event is the thermal fluctuation.

Ref: M. M Aggarwal et al., Phys.Rev. Lett. 105, 022302 (2010))

Cumulants

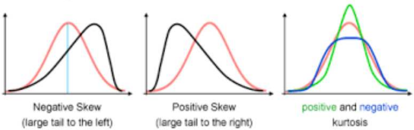
$$C_1 = \langle N_q \rangle$$
$$C_2 = \langle (\delta N_q)^2 \rangle$$
$$C_3 = \langle (\delta N_q)^3 \rangle$$
$$C_4 = \langle (\delta N_q)^4 \rangle - \langle (\delta N_q)^2 \rangle^2$$

N_q $(= N_+ - N_-)$ is the net-number and

$\delta N_q = N_q - \langle N_q \rangle$

Mean, variance, skewness, kurtosis

$$M = C_1$$
$$\sigma^2 = C_2$$
$$S = \frac{C_3}{\sigma^3}$$
$$\kappa = \frac{C_4}{\sigma^4}$$



- Both S and κ of a Gaussian distribution function are zero.
- The skewness is a measure of degree of asymmetry whereas, kurtosis is a measure of degree of peakness of a distribution function.

Relations of χ_q with the various order cumulants (C_n) of distributions of conserved charges

$$C_1 = M_q = \langle N_q \rangle = VT^3 \chi_q^1$$

N_q ($= N_+ - N_-$) is the net-number

$$C_2 = \sigma_q^2 = \left\langle (\delta N_q)^2 \right\rangle = VT^3 \chi_q^2$$

where $\delta N_q = N_q - \langle N_q \rangle$

$$C_3 = s\sigma_q^3 = \left\langle (\delta N_q)^3 \right\rangle = VT^3 \chi_q^3$$

$$C_4 = \kappa\sigma_q^4 = \left\langle (\delta N_q)^4 \right\rangle - \left\langle (\delta N_q)^2 \right\rangle^2 = VT^3 \chi_q^4$$

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$$C_4 = \kappa\sigma_q^4 = \left\langle (\delta N_q)^4 \right\rangle - \left\langle (\delta N_q)^2 \right\rangle^2 = VT^3 \chi_q^4$$

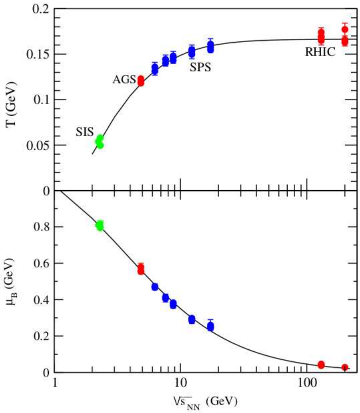
$$\frac{C_2}{C_1} = \frac{\sigma^2}{M} = \frac{\chi^2}{\chi^1}$$

$$\frac{C_3}{C_2} = s\sigma = \frac{\chi^3}{\chi^2}$$

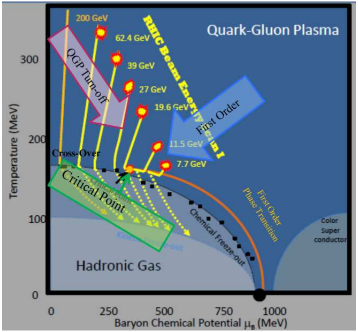
$$\frac{C_4}{C_2} = \kappa\sigma^2 = \frac{\chi^4}{\chi^2}$$

Energy dependence of the chemical freezeout parameters

Chemical freeze-out is the situation when inelastic interactions among the particles stop and hence particle yields get fixed.

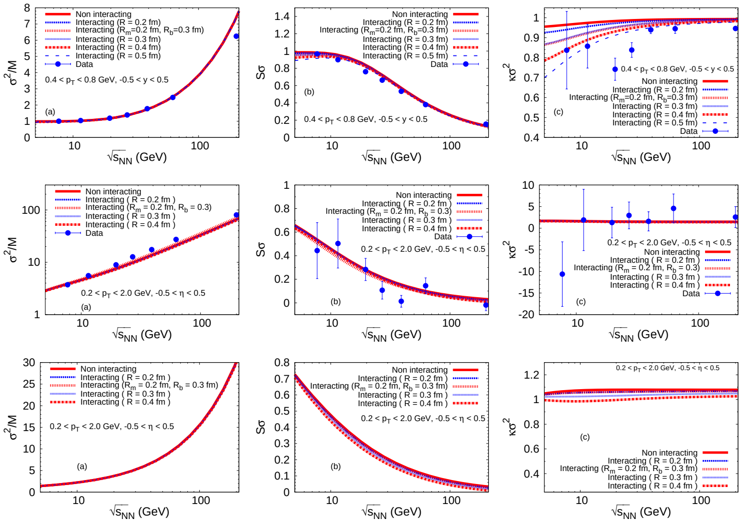


Ref: J. Cleymans *et al.*, PRC 73, 034905 (2006)



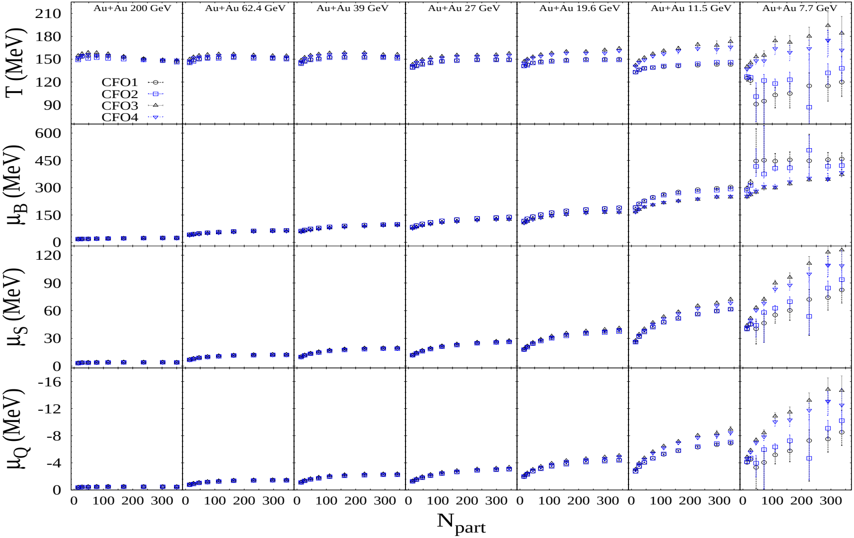
- Small variation of T_f(140-166 MeV)
- $\mu_{B,f}$ varies from 23-421 MeV
- Non-monotonic variations of observables related to the cumulants of the distributions of conserved charges with beam energy.

Energy dependence of σ^2/M, $s\sigma$ and $\kappa\sigma^2$



Net-proton data: X. Luo (for the STAR collaboration), Nucl. Phys. A 904905,911c (2013)

Net-charge data:L. Adamczyk et al. (STAR Collaboration), Phys. Rev. Lett. 113, 092301 (2014)

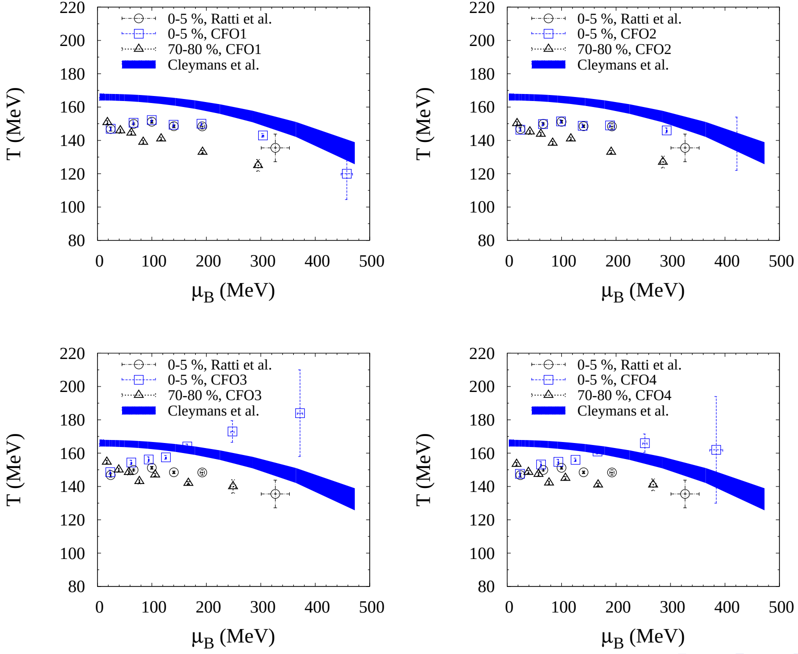


Sets of parameters	Experimental data used	Model used
CFO1	$(\sigma^2/M)_{np}$, $(\sigma^2/M)_{nc}$	HRG
CFO2	$(\sigma^2/M)_{np}$, $(\sigma^2/M)_{nc}$	EVHRG
CFO3	$(\sigma^2/M)_{nc}$, $(S\sigma)_{np}$, $(S\sigma)_{nc}$	HRG
CFO4	$(\sigma^2/M)_{nc}$, $(S\sigma)_{np}$, $(S\sigma)_{nc}$	EVHRG

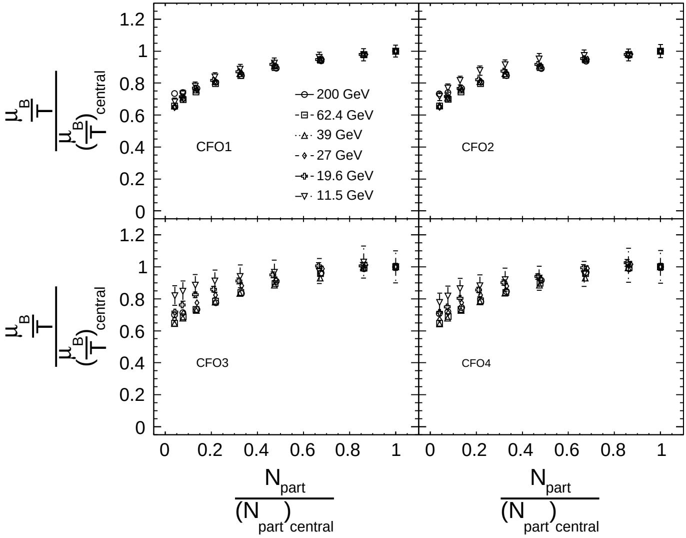
Refs. of Expt. data measured by STAR collaboration:

Phys. Rev. Lett. 112, 032302 (2014)

Phys. Rev. Lett. 113, 092301 (2014)


0-5 %, Ratti et al.
0-5 %, CFO1
70-80 %, CFO1
Cleymans et al.
0-5 %, CFO2
70-80 %, CFO2
0-5 %, CFO3
70-80 %, CFO3
0-5 %, CFO4
70-80 %, CFO4
T (MeV)
μB (MeV)

Scaling


$\frac{\mu_B/T}{(\mu_B/T)_{central}}$
1.2
1
0.8
0.6
0.4
0.2
0
200 GeV
62.4 GeV
39 GeV
27 GeV
19.6 GeV
11.5 GeV
CFO1
CFO2
CFO3
CFO4
0
0.2
0.4
0.6
0.8
1
$\frac{N_{part}}{(N_{part})_{central}}$

HRG model in a finite volume

- Usually any thermodynamic study assumes the system volume to be infinite.
- However, the fireball created in the relativistic heavy-ion collision experiments has a finite spatial volume.
- The size of such spatial volume critically depends on three parameters: the size of the colliding nuclei, $\sqrt{s_{NN}}$, and the centrality of collisions

Modification of HRG model in a finite volume

- We incorporate this by considering a lower momentum cutoff $p_{min} = \pi/R = \lambda$, say, where R is the size of a cubic volume

HRG

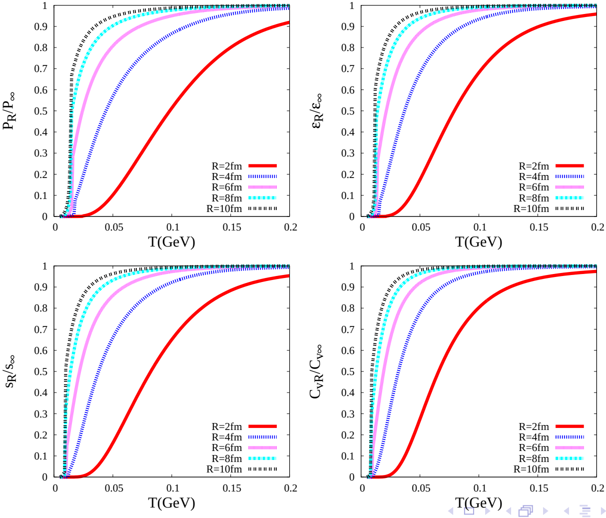
$$\ln Z_i^{id} = \pm \frac{Vg_i}{2\pi^2} \int_0^\infty p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

HRG in a finite volume

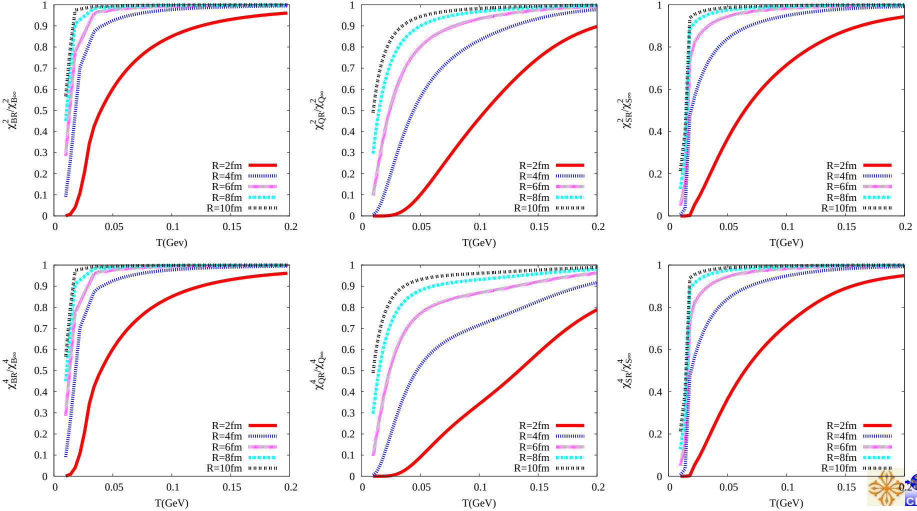
$$\ln Z_i^{id} = \pm \frac{Vg_i}{2\pi^2} \int_\lambda^\infty p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

S. Samanta *et al.*, Phys. Rev. C 91, no. 4, 041901 (2015) (Rapid Communication)

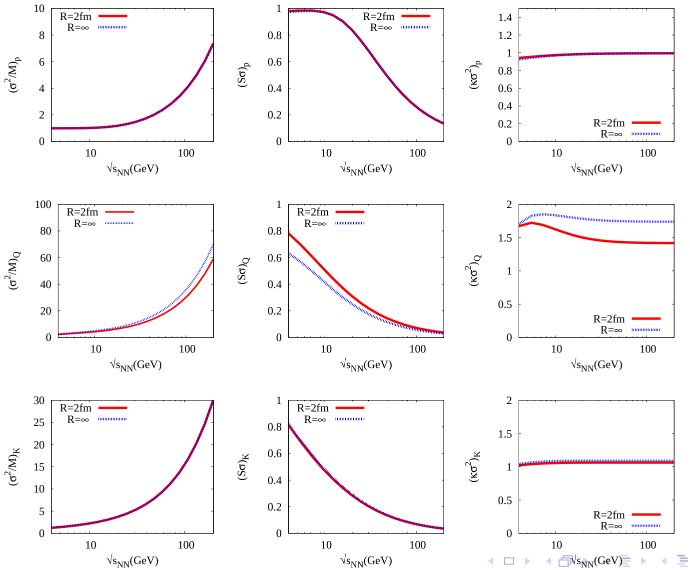
Variation of scaled pressure, scaled energy, scaled entropy, and scaled specific heat with T at $\mu = 0$



Variation of scaled susceptibilities with T at $\mu = 0$



Energy dependence of $\sigma^2/M, S\sigma$ & $\kappa\sigma^2$



Conclusion

- Interacting HRG model is important at high temperature.
- At high temperatures and $\mu_B = 0$, compared to HRG, χ_B^2 and χ_Q^2 from EVHRG fits better to LQCD continuum data for radius of hadrons between $0.2 - 0.3$ fm .
- The LQCD data ($N_\tau = 6, 8$) for χ_B^4 and χ_S^4 are in good agreement with HRG/EVHRG model up to $T = 0.16$ GeV. Whereas, both HRG and EVHRG model overestimates LQCD data for χ_Q^4.
- Lattice continuum data for both χ_{BS}^{11} and χ_{QS}^{11} are closer to HRG results at lower T. On the other hand, χ_{BQ}^{11} calculated in lattice is close to EVHRG results at lower T.
- The effect of repulsive interaction, as present in EVHRG, is distinguishable for lower energies. Moreover this difference is more pronounced for higher order susceptibilities.
- Although the variations of σ^2/M and $S\sigma$ with $\sqrt{s_{NN}}$ seem to describe the experimental data well, higher moment $\kappa\sigma^2$ shows large deviations.
- Centrality dependence of chemical freezeout have heen extracted using fluctuation data
- Maximum system volume dependence is observed for charge fluctuation.

Part 2

Thermal Dimuon production at FAIR energy

Detection of thermal dimuon: Motivation

- The leptons are electromagnetically interacting particles.
- The dilepton yields from thermal radiation is expected to serve as a probe to the QGP.
- Dimuon from thermal source (QGP) produce continuum in invariant mass spectra. Our main motivation is to disentangle thermal continuum from low mass vector mesons in dimuon invariant mass spectra.

Thermal dimuon production from QGP source

- In QGP phase, dileptons are produced dominantly from quark-antiquark annihilation process.
- A quark interacts with an antiquark to form a virtual photon which subsequently decays into a lepton pair.
- In case of massless quark and antiquark

$$\frac{dN_{\mu^+\mu^-}}{dM^2 d^4x} = N_c N_s^2 \sum_{f=1}^{N_f} (\frac{e_f}{e})^2 \frac{\sigma(M)}{2(2\pi)^4} M^2 f_1(\epsilon) F_2(\frac{M^2}{4\epsilon})(\frac{2\pi}{w(\epsilon)})^{1/2}. \quad (3)$$

N_c, N_s are color and spin degeneracy factors respectively
N_f is the number of flavour
e_f is the electric charge of a quark with flavour f
f_1 (f_2) is the quark (antiquark) distribution function.

Courtesy: C. Y. Wong, Phys. Rev. C 48, 902 (1993)

$$\frac{dN_{\mu^+\mu^-}}{dM^2d^4x} = N\frac{\sigma(M)}{2(2\pi)^4}M^2f_1(\epsilon)F_2(\frac{M^2}{4\epsilon})(\frac{2\pi}{w(\epsilon)})^{1/2}$$

- $N = N_c N_s^2 \sum(e_f/e)^2$ (= 20/3 for 2 flavour)
- $\sigma(M) = \frac{4\pi}{3}\frac{\alpha^2}{M^2}\sqrt{(1 - \frac{4m_\mu^2}{M^2})}(1 + \frac{2m_\mu^2}{M^2})$.
  α is the fine structure constant
- $F_2(E) = -\int_\infty^E f_2(E')dE'$.
- $\epsilon(M)$ is the root of the equation: $\frac{d}{dE}[lnf_1(E) + lnF_2(\frac{M^2}{4E})]_{E=\epsilon} = 0$.
- $w(\epsilon) = -[\frac{d^2}{dE^2}(lnf_1(E) + lnF_2(\frac{M^2}{4E}))]_{E=\epsilon}$
- Carry the information of the T, μ of the system
- The relation is for static system.

$d^4x = ?$

- Let us choose 1D Bjorken expansion
- $d^4x = \pi R^2 dz d\tau = \pi R^2 \tau d\tau$ dy
- τ is the proper time
- y is the fluid rapidity
- R is the radius of colliding ions (For Au, $R \simeq 7fm$)

$$\frac{dN}{(dMdy)} = \int_{\tau_i}^{\tau_f} \frac{dN_{\mu^+\mu^-}}{dM^2 d^4x} (2M\pi R^2 \tau d\tau)$$

- Initial and final time of QGP phase ?
- T and μ depend on τ

Initial condition at $\mu \neq 0$

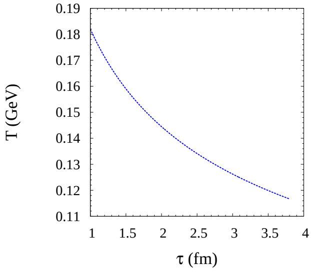
- Three unknown τ_i, T_i and μ_i
- $\tau_i \sim 1 - 2fm$ $(\tau_i \sim \frac{2R}{\gamma\beta})$
- We need two equations to estimate T_i and μ_i
- Conservations of energy density and net-baryon density
  $s\tau = constant$
  $n_b\tau = constant$
- $\frac{dS}{dy} = \pi R^2 s\tau = constant$
- $\frac{dN_b}{dy} = \pi R^2 n_b\tau = constant$

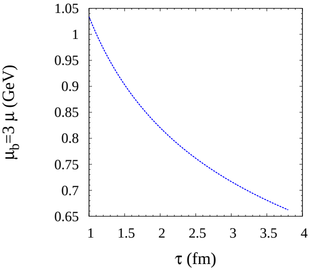
Initial condition at $\mu \neq 0$ continued

- Entropy is conserved during the evolution in QGP phase.
- Entropy is generated during phase transition and also during freezeout (assuming them negligible).
- Assuming that the entire evolution of the fireball from the initial time τ_i till the freeze-out time τ_f is strictly adiabatic.
- $s_i \simeq \frac{1}{\pi R^2 \tau_i}(\frac{dS}{dN}\frac{dN}{dy})_f$
  $s = 1/3(37/45)\pi^2 T^3 + 2\mu^2 T$
- $n_{b_i} \simeq \frac{1}{\pi R^2 \tau_i}(\frac{dn_b}{dy})_f$
  $n_b = 2/3(\mu T^2 + \mu^3/\pi^2)$
- $\frac{dn_{ch}}{dy} \simeq 270$ at $\sqrt{s_{NN}} = 7$ A GeV (PLB 571 (2003) 3644)
- $\frac{dn_p}{dy} \simeq 30$ for Pb+Pb collisions at 40 A GeV (arXiv:1009.1747v2)
- $T_i = 181$ MeV $\mu_{B_i} = 1034 MeV$

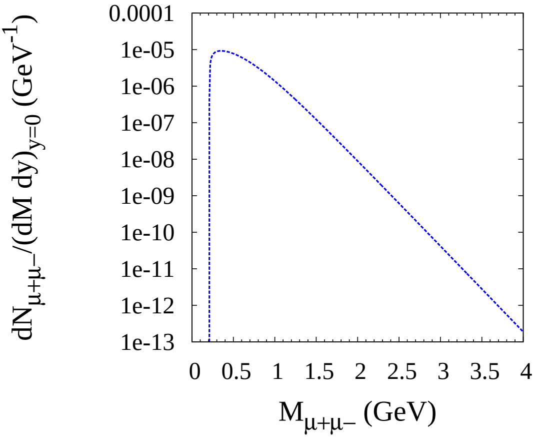
Cooling law in QGP phase

- $n_b \tau = constant$
- $\epsilon \tau^{1+c_s^2} = constant$ assuming $P = c_s^2 \epsilon$ ($c_s^2 = constant$)
- Here we have assumed that the system is in QGP phase till $\epsilon = 0.6$ GeV /fm^3 which corresponds to four times the nuclear density (150 MeV/fm^3)

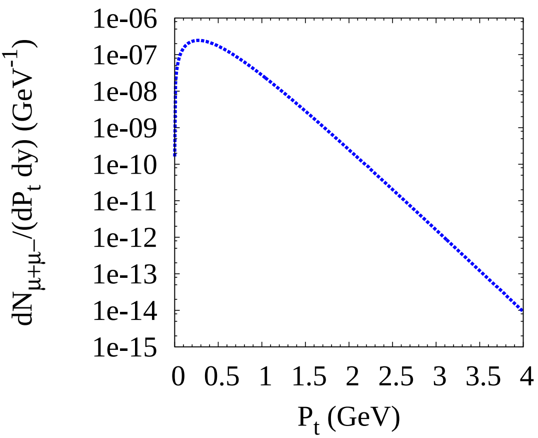




Invariant mass distribution



P_t distribution



Event generator

Input

Theoretical distributions of M and P_t

Output

Four momentum of μ^+ and μ^- (4 unknown quantities, $P_{t1}, P_{t2}, P_{l1}, P_{l2}$)

Approximation

$|P_{t1}| = |P_{t2}|$ and $P_{l1} = -P_{l2}$ (in CM frame)

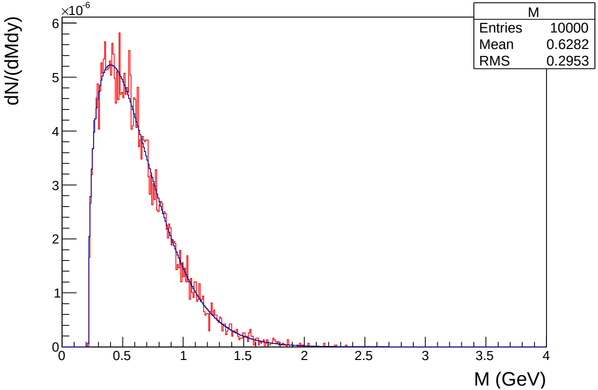
- Proper Lorentz boost is applied

output format

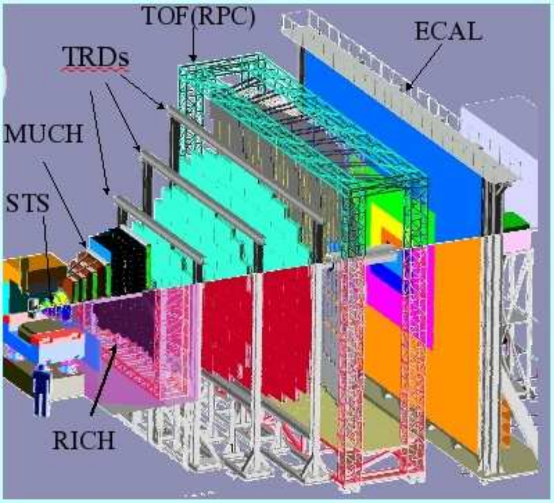
ntracks eventID vx vy vz
pdgID px py pz

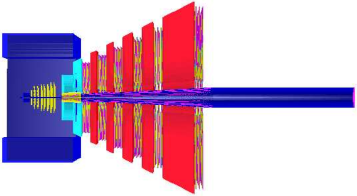
Reconstruction of events

- Blue one is theoretically calculated invariant mass.
- Red one is reconstructed invariant mass from generated μ^+ and μ^- (without detector).



CBM experiment: Detectors





- STS consists of 8 tracking layers of silicon detectors.
- MUCH: Total absorber is divided into thinner segments.
- First one is made of 60 cm carbon and rest are made of irons (20 + 20 + 30 + 35 + 100 cm).
- Tracking chamber triplets placed in between the absorber segments.
- This system allows us to identify muons over a wide range of momenta depending on the number of absorber segments that the particles have passed through.

Simulation

Geometry

SIS300

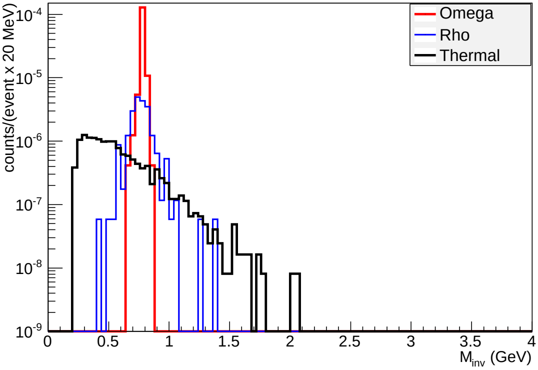
CBMROOT and FAIRSOFT version

CBMROOT: JUN13
FAIRSOFT: sep12

Class for input primary event generator

FairAsciiGenerator

Invariant mass reconstruction and comparison with rho and omega meson



Cuts used

hits in
MUCH layers ≥ 14
STS layers ≥ 4
$\chi^2 \leq 3$

- Thermal dimuon is important in intermediate invariant mass region.

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Thank you
The End.