Heavy Quark Production, Propagation and Energy-Loss in Hot and Dense QCD Medium

MUNSHI G. MUSTAFA

Saha Institute of Nuclear Physics, Kolkata

Content:

- Why Heavy Quark?
- Heavy Quark Production and Initial Distribution
- Heavy Quark Propagation
  - Collisional Energy Loss
  - Radiative Energy Loss
  - Jet Quenching
- Conclusion

Why Heavy Quarks ?

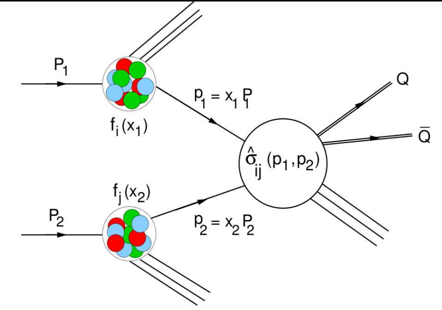
- Heavy Quarks ➤ **Charm (c) & Bottom (b)**
  - **Charm:** $M_c = 1.27^{+0.07}_{-0.09}$ **GeV**
  - **Bottom:** $M_b = 4.19^{+0.18}_{-0.06}$ **GeV**
- Production time: $\tau_Q = 1/2M_Q \leq 0.1$ fm$/c$
- Thermalisation time (τ_{th}):
  - Successive equlibrium: $\tau_{th}^{g} < \tau_{th}^{u,d} < \tau_{th}^{c} < \tau_{th}^{b} < \tau_{th}^{life}$

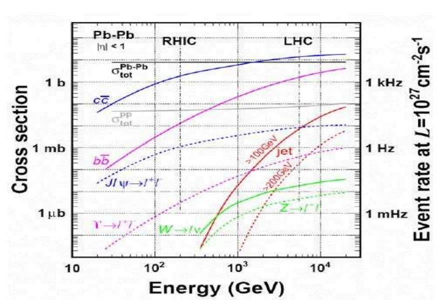
    [J. Alam, S. Raha & B. Sinha, PRL 73 (1994) 1895]
  - Therma. time for HQs $\tau_{th}^{Q} \sim \frac{M_Q}{T} \times \tau_{th}^{u,d}$ [G. Moore & D. Teaney, PRC 2005]
  - $\tau_{th}^{u,d} < \tau_{th}^{Q}$: no production at QGP and hadronic phase
  - Produced at very early time interactions in Hard Scattering of partons in Nucleons. Initial distribution of HQs ➤ frozen
  - g, u, d thermalize early and provide an expanding thermal background

Why Heavy Quarks ?

- Heavy Quarks propagation:
  - in foreground with gluons & light quarks as an expanding thermal background for the non-equilibrated HQs ➤ expansion dynamics
  - interacts with equilibrated degrees of freedom ➤ energy-loss of HQ
  - requires dynamics ➤ final distr^n of HQs (Transport Eq.)
- Uniqueness of Heavy Quarks:
  - distinguishable down to lowest momenta ➤ medium/quark coupling (Hydro)
  - cleaner energy-loss probe ➤ reflected in leading particle $p_\perp$ spectra, flow
  - tests understanding of mass dependence
  - Quarkonia: QGP thermometer
- True probe: Not glue or light quarks

Heavy Quark Production in Heavy-Ion Collisions:



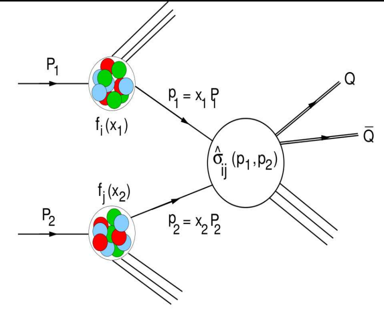


- Heavy-Ion Coll.: **ALICE: 0809.1062[nucl-ex]**
  - ☛ **Charm:** **LHC** $\sim 10\times$ **RHIC** ☛**Bottom:** **LHC** $\sim 100\times$ **RHIC**
- Produced in pairs $(Q\bar{Q})$ at early times
- QCD describes the structure and dynamics of hadrons interms of their constituents: quarks and gluons
- QCD provides a framework to compute production x-section and distribution

HQs Production and Initial Distribution

- Factorisations allows that an observable can be expressed as a convolution of short distance (hard scattering of partons) and the long distance contribution describing the initial hadrons

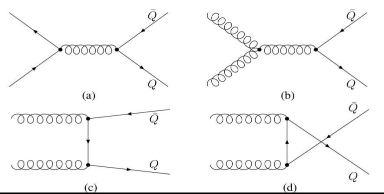
$$\frac{d\sigma}{dy_1\, dy_2\, dp_\perp} = x_1 x_2 p_\perp \sum_{ij} f_i^{(1)}(x_1,\mu_F^2) f_j^{(2)}(x_2,\mu_F^2)\hat{\sigma}_{ij}(x_i,x_j,\alpha_s(\mu_F^2),\mu_F^2)$$

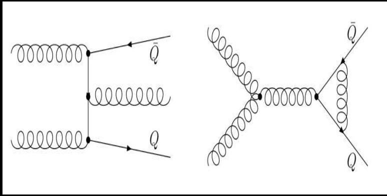


- i, j are partons in hadrons
- σ_{ij} are partonic x-sections ➤ pQCD
- f_i are parton distr^n fn. (PDFs) in hadron which are nonperturbative
- μ_F is the factorization scale between hard processes and the non-perturbative PDFs
- μ_F dependence should cancel in order by order in pQCD calculation

HQs Production and Initial Distribution

- Factorisations:
  - The short distance contribution describing the matrix elements of hard processes are free from mass singularities and calculable in pQCD
  - $gg \to Q\bar{Q}$; $q\bar{q} \to Q\bar{Q}$; $gg \to Q\bar{Q}g$; $q\bar{q} \to Q\bar{Q}g$ [Leading Order (LO), NLO & FONLL]





  - PDFs are process independent (DIS, DY etc) ➤ it can be extracted from one expt, and used in another expt.
  - PDFs obey DGLAP evolution equation ➤ knowing PDFS at one scale can be calculated at any other scale
  - In elimentary [e^+e^-, pp and $p\bar{p}$] collisions an observable is well controlled through factorization

Heavy-Ion Collisions

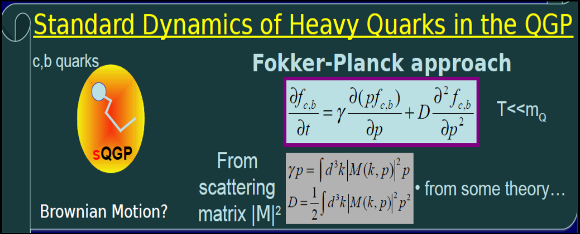
- Does factorisation work in A+A ? May or may not! But assumed!
- PDFs in nuclei (nPDFs) are different from those in hadrons: geometry, wavefunction are different, nPDFs require appropriate modifications
- Several nPDFS that prametrise the nuclear dependence of incoming parton distribution functions
- ☛ "uncertainity in intial state interactions"
- The energy degradation observed in $A+A$ collisions in $p_\perp$ spectra of hadrons ➤ the dynamical effects occuring after initial hard processes ➤ " final state effects"
- "Final state effects" for $p_\perp$ spectra of hadrons are partonic in nature before hadronisation
- The frgmentation function, used to obtain $p_\perp$ spectra, should also be afftected by the presence of hot and dense QCD medium!
- ☛ "uncertainity in final state interactions"

Heavy Quark Propagation and Final distribution

- Produced at early time ($\tau_Q < \tau_{th}$); No production at later time

  - **Total no. of HQ gets frozen very early in the history of collisions**
  - **Immediately upon their production they will propagate through QGP**
  - **One is left with the task of determining the HQ distribution**
  - **Details of the distribution may reflect the characteristics and development of QGP**

Fokker-Planck Equation



Extensive Body of Literatures

- Chakraborty & Syam, Lett. Nuvo Cim.'84
- Svetitsky, PRD'88
- Alam, Raha & Sinha, PRL'94
- Mustafa, Pal & Srivastava, PRC'97
- Mustafa PRC'05
- Moore & Teany PRC'05; Rapp &Hees'05

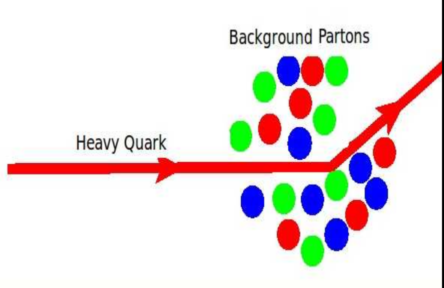
Fokker-Planck Eq. ➤ Simplified Boltzmann Eq.

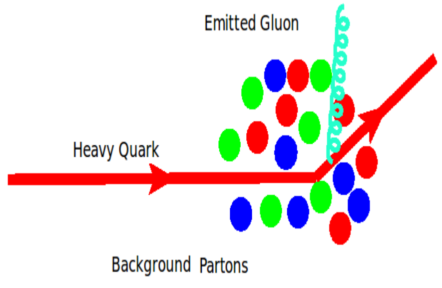
- Boltzmann Eq ➤ No external force and isotropic in space
- Soft Scatt. (Landau approx.) ➤ Taylor expan. ➤ Landau Eq.
- Landau Eq. ➤ an integro-diff^l Eq. involving transport coeffs. ➤ describes collision processes of two particles
- Landau Eq. ➤ FP Eq. when distribution of the background particles (QGP) is thermal whereas foreground particle (HQs) is non-thermal

Propagation of Heavy Quark in QGP

- Energy-Loss

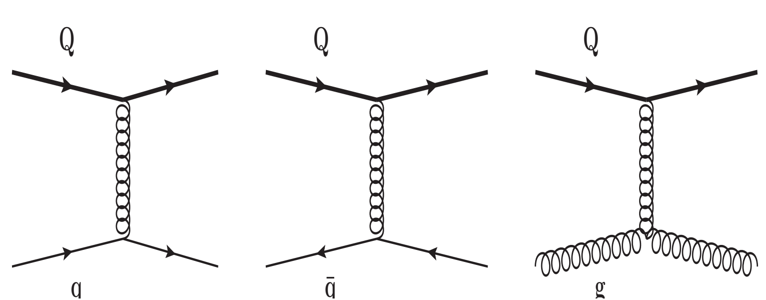
  - **Collisional Loss**
  - **Radiative Loss**





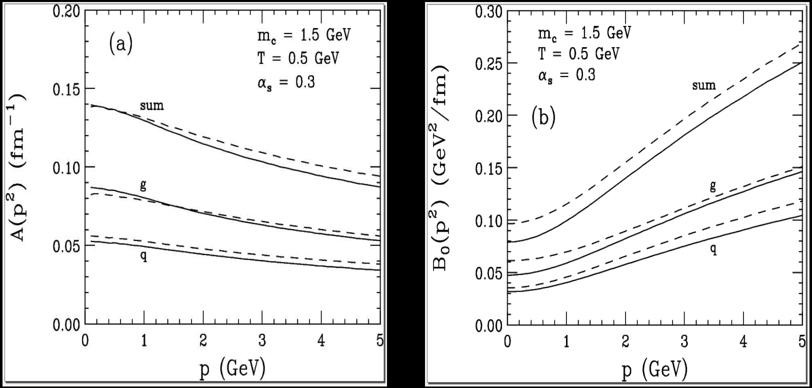
Collisional Processes

- Feynman Diagram



Heavy Quark Transport Coefficients

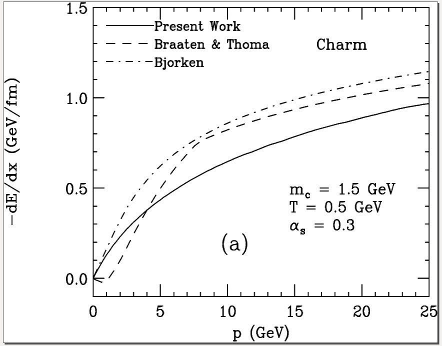
- Drag $\mathcal{A}(p)$ and Diffusions $\mathcal{B}_0(p)$

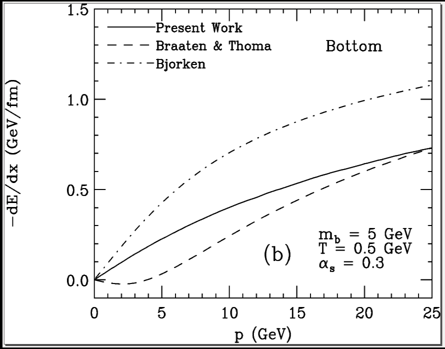


- **Mustafa, Pal & Srivastava, PRC'97**
- **Mustafa PRC'05**

Differential Collisional Energy Loss

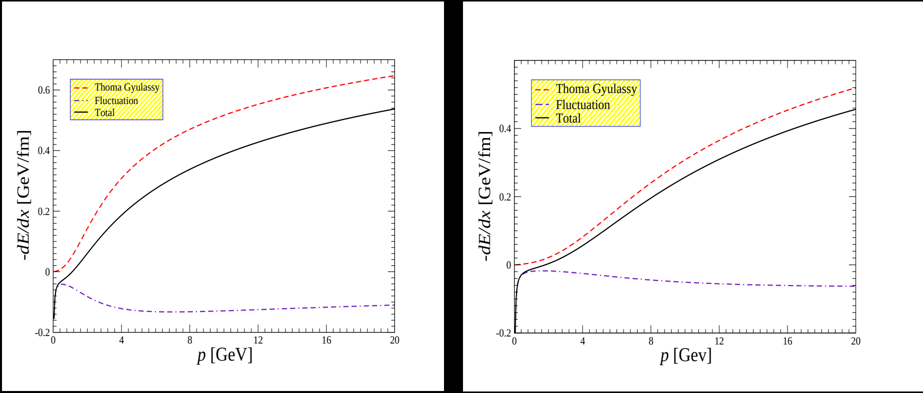
- Drag Coeff.: $\mathcal{A}(p) = -\frac{1}{p}\frac{\mathrm{dE}}{\mathrm{dx}}$
  - Differnetial E-Loss: $\frac{\mathrm{dE}}{\mathrm{dx}}$ [Mustafa, Pal, Srivastava, PRC57 (1998) 889]





Collisional Energy Gain

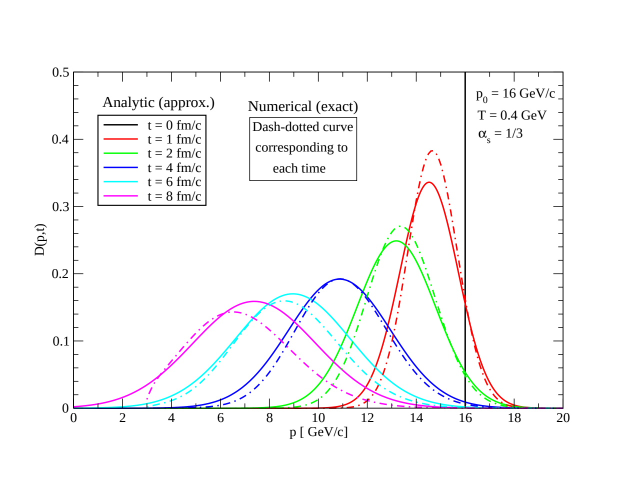
- Fluctuation of chromo-electromagnetic fields leads to energy gain



☛ **Chakraborty, Mustafa, Thoma, PRC75 (2007) 064908**

HQ Energy-Loss Distribution **[Mustafa, PRC72 (2005) 014905]**

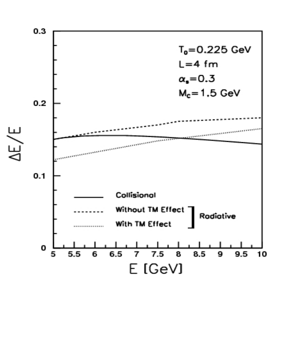
- Collisional Energy-Loss ➤ Transport-coeffs. ➤ Fokker-Planck Eq. ➤ Energy-Loss Distribution

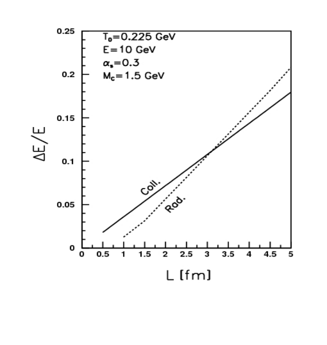


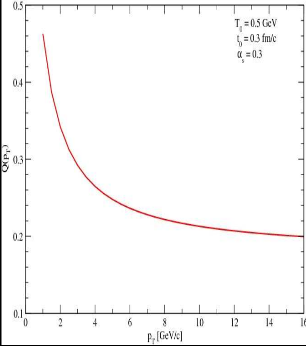
- Peak Shifts to lower p
  - **Drag Force on mean p**
- Peak broadens
  - **Diffusion in p**
  - **Momentum Dispersion**

Substantial Collisional E-Loss

- **Mustafa, PRC72 (2005) 014905**







- Coll. E-Loss is important
- Predicted R_{AA} but no data then
- For light quarks R_{AA} :

☛ **Mustafa et al., PRL100 (2008) 072301; Acta Phys.Hung. A22 (2005)**

Boltzmann vs Langevin Dynamics

[Das, Scardina, Plumari and Greco, PRC90 (2014) 044901]

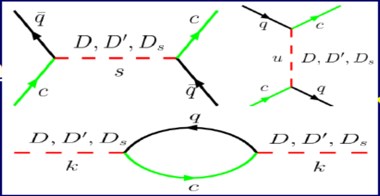
- Dynamics of a heavy quark propagation as a foreground particle in a gluonic plasma between the Langevin and the Boltzmann approach when the background bulk medium (gluons) is in thermal equilibrium
- The calculation has common origin which is the Boltzmann collisional integral involving scattering matrix element in collision process
- This Boltzmann collision term, under Landau approximation (soft approximation), leads to Fokker-Planck equation involving transport coefficients, viz., drag and diffusion. In one hand, the Fokker-Planck equation has been solved using Ito-Langevin approach, in which the matrix element in the collision process is related to those transport coefficients
- On the other hand, in the Boltzmann approach the matrix element in the full collision integral is related to the cross-section. The Boltzmann equation is solved by simulating an ensemble of particle in a box which evolve dynamically

Boltzmann vs Langevin Dynamics

[Das, Scardina, Plumari and Greco, PRC90 (2014) 044901]

Common Origin is the Matrix Element

M scattering matrix of the collisions process



Langevin approa

M -> A_i, B_{ij}

Boltzmann appro

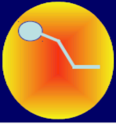
M -> $d\sigma/d\Omega$

Drag Coefficient -> <p>

$$A_i = \frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3q'}{(2\pi)^3 2E_{q'}}\int\frac{d^3p'}{(2\pi)^3 2E_{p'}}\frac{1}{\gamma_c}\sum|M|^2 (2\pi)^4\delta^4(p+q-p'-q')f(q)[(p-p')_i] = \langle\langle(p-p')_i\rangle\rangle$$

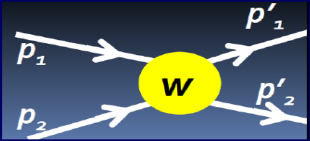
Diffusion coefficient -> <Δp²>

$$B_{ij} = \frac{1}{2}\langle\langle(p-p')_i(p'-p)_j\rangle\rangle$$



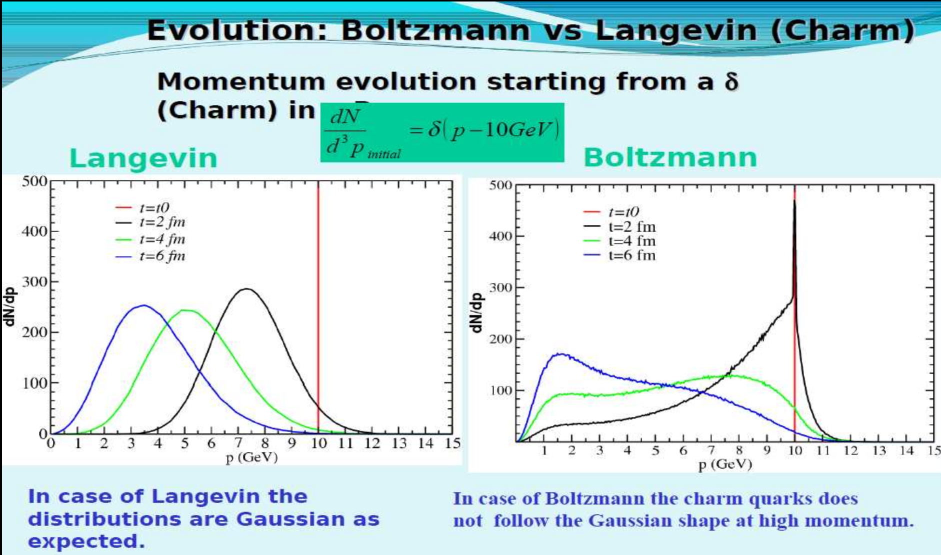
Total and differential cross sect

$$\sigma_{gc\to gc} = \frac{1}{16\pi\left(s-M_c^2\right)^2}\int_{-(s-M^2)^2/s}^{0} dt\sum|M|^2$$



Boltzmann vs Langevin Dynamics

[Das, Scardina, Plumari and Greco, PRC90 (2014) 044901]

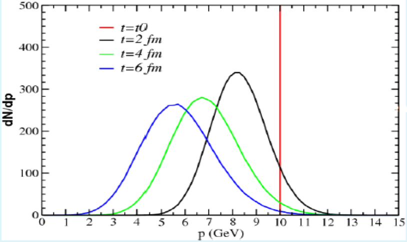


Boltzmann vs Langevin Dynamics

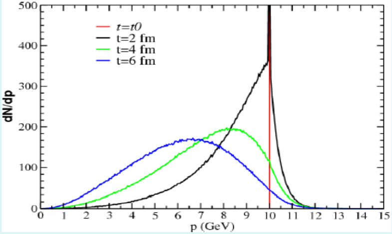
[Das, Scardina, Plumari and Greco, PRC90 (2014) 044901]

Momentum evolution starting from a δ (Bottom)

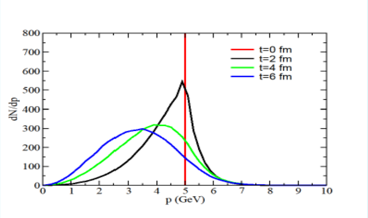
Langevin



Boltzmann

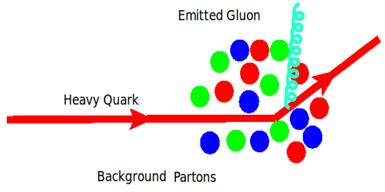


For bottom quarks it works better.



Radiative Energy Loss

- HQ encounters inelastic scattering as it traveses a medium
  - Emits a gluon



- Differential Loss: dE/dx
- Very simple but first estimation:
  - **Mustafa, Pal, Srivastava, Thoma, PLB 428 (1998) 234**
- Improvements in formalism

Radiative Energy Loss

- Singnificant Points:
  - The nature of the medium through which the energetic parton propagates ➤ thermally equilibrated perturbative medium with a collection of static scatt. centres with specified density
  - Kinematic approximations for interation between medium partons (thermal background) and the projectile parton that is propagating in the medium
  - The virtuality and branching/splitting of hard parton to reduce its off-shellness ➤ multiple splitting may occure in the medium ➤ multigluon final state should include interference of emitted gluons

Radiative Energy Loss

- Kinematic Approximations:
  - Eikonal-I: leading parton energy $(E = p_z,\ p_\perp = 0)$ $E >> q_\perp$, the transverse mom. of the exchaged gluons ➤ doesn't give sufficient transverse kick to deflect the parton from straightline trajectory
  - Eikonal-II: leading parton energy $(E = p_z,\ p_\perp = 0)$ $E >> k_\perp$, the transverse mom. of the emitted gluons ➤ doesn't get enough transverse kick from emitted gluons too
  - Soft gluon emission: the energy of the emitted gluon, $\omega << E$ ➤ $x = \frac{\omega}{E} \sim 0$
  - Small angle or colinear emission: the energy of the emitted gluons, $\omega >> k_\perp$, its transeverse momentum. In broader sense $E >> \omega >> k_\perp$, which includes Eikonal-II

Radiative Energy Loss

- Radiative E-Loss Models:

Radiative Energy-Loss Model

Model	Virtuality	Medium	Eikonal	Soft Emission	Collinear / Small Angle
BDMPS-Z (PA)	Multiple gluon emis. + no inter	Static Scatt. centers	Yes	Yes	Yes
ASW (PA)	Mult. Gluons + Inter.	Do	Yes	Yes	Yes
GLV (Opacity Exp.; Poission anstaz)	Do	Do	Yes	Yes	Yes
AMY (FP Eq.)	Mult. Gluons + no inter	Do	Yes	No	Yes (also large angle)

Suppression due to Mass and Dead Cone

- General notion: **heavy quark radiates less than light quark**

qq'→qq' $$|\mathcal{M}_{qq'\to qq'}|^2 = \frac{8}{9} g^4 \frac{s^2}{t^2}$$

Qq→Qq $$|\mathcal{M}_{Qq\to Qq}|^2 = \frac{8}{9} g^4 \frac{s^2}{t^2} \left(1 - \frac{M^2}{s}\right)^2$$

qq'→qq'g $$|\mathcal{M}_{qq'\to qq'g}|^2 = 12 g^2 \, \frac{8}{9} g^4 \frac{s^2}{t^2} \, \frac{1}{k_\perp^2}$$

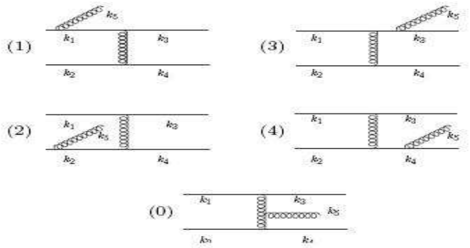
Qq→Qqg $$|\mathcal{M}_{Qq\to Qqg}|^2 = 12 g^2 \, \frac{8}{9} g^4 \frac{s^2}{t^2} \, \frac{1}{k_\perp^2}$$?

2001 Dokshitzer and Kharzeev proposed (Phys. Lett. B 519, 199 (2001)) "dead cone" effect => heavy quark small energy loss.

$$\left(1 + \frac{\theta_0^2}{\theta^2}\right)^{-2} \qquad \theta_0 = M/E$$

Generalised Dead Cone

- RHIC data (PHENIX) ➤ $R_{AA}^{HQ} \sim R_{AA}^{LQ}$ ➤ **(Heavy quark puzzle)**



Hierarchy employed in this study

$$\sqrt{s}, E \gg \sqrt{|t|} \sim q_\perp \gg \omega > k_\perp \gg m_D$$

Mass Range $0 < M/E < 1$

Emission angle Range $-\pi < \theta < +\pi$

$$|\mathcal{M}_{Qq\to Qqg}|^2 = 12g^2\,|\mathcal{M}_{Qq\to Qq}|^2\,\frac{1}{k_\perp^2}\left(1+\frac{M^2}{s\tan^2(\frac{\theta}{2})}\right)^{-2}$$

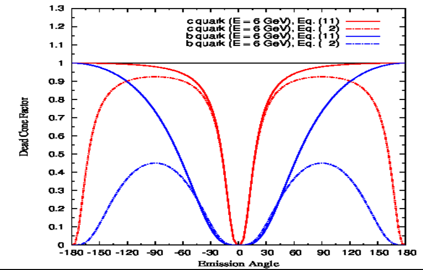
$$= 12g^2\,|\mathcal{M}_{Qq\to Qq}|^2\,\frac{1}{k_\perp^2}\left(1+\frac{M^2}{s}e^{2\eta}\right)^{-2}$$

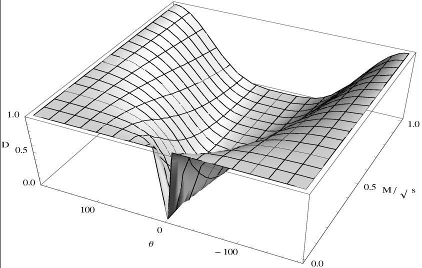
$$\mathcal{D} = \left(1+\frac{M^2}{s\tan^2(\frac{\theta}{2})}\right)^{-2}$$

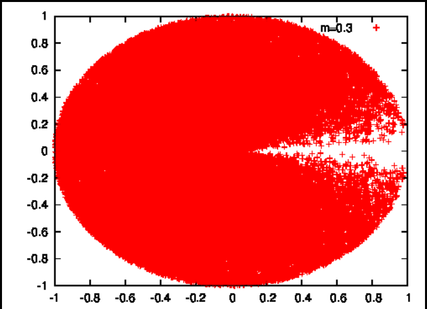
..... *Dead Cone Factor*

☛ **Abir, Greiner, Martinez, Mustafa, Uphoff, PRD85 (2012) 054012**

Generalised Dead Cone [Abir, Greiner, Martinez, Mustafa, Uphoff, PRD85 (2012) 054012]







$$\mathcal{D} = \left(1 + \frac{M^2}{s\tan^2(\frac{\theta}{2})}\right)^{-2}$$

Radiation Spectrum [Abir,Greiner,Martinez,Mustafa,Uphoff, PRD85 (2012)054012]

- $Qq \to Qqg$ and $Q\bar{q} \to Q\bar{q}g$; Replace $Q \to \bar{Q}$

$$\left.\frac{dn_g}{d\eta\ dk_\perp^2}\right|_{Qq\to Qqg} = \frac{C_A\alpha_s}{\pi}\frac{1}{k_\perp^2}\mathcal{D}$$

- $Qg \to Qgg$ and Replace $Q \to \bar{Q}$

$$\left.\frac{dn_g}{d\eta\ dk_\perp^2}\right|_{Qg\to Qgg} = \frac{C_A}{C_F}\left.\frac{dn_g}{d\eta\ dk_\perp^2}\right|_{Qq\to Qqg};\ \frac{C_A}{C_F} = \frac{9}{4}$$

Radiative E-Loss [Abir,Jamil,Mustafa, Srivastava, PLB715 (2012) 183]

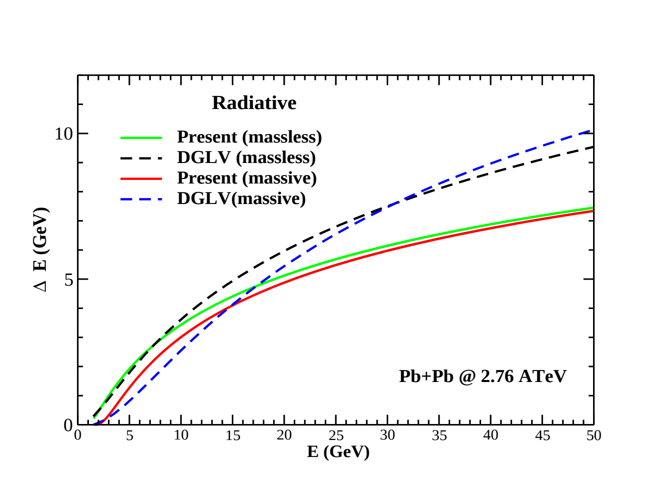
- Various Models (Armesto et al.: 1106.1106; Renk: 1112.2503)
  - **Kinematical cuts**
  - **Large angle radiation**
- Differential Radiative E-Loss

$$-\frac{dE}{dx} = \frac{\langle\omega\rangle}{\langle\lambda\rangle}$$

$\langle\omega\rangle$ = **Mean energy of the emitted gluon**

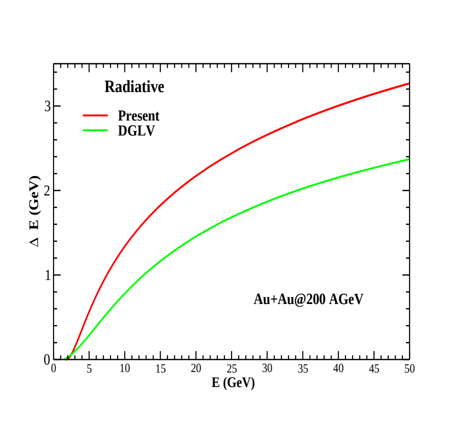
$\langle\lambda\rangle$ = **Mean free path of the heavy quark**

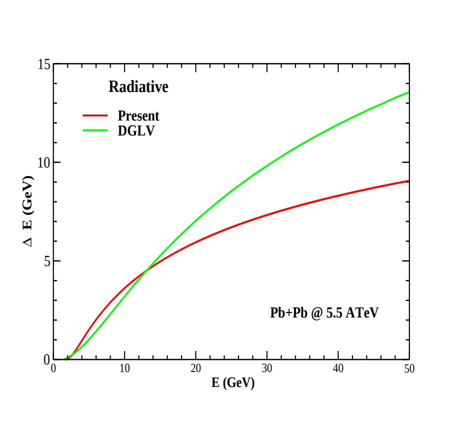
Radiative E-Loss [Abir,Jamil,Mustafa,Sivastava, PLB715 (2012) 183]



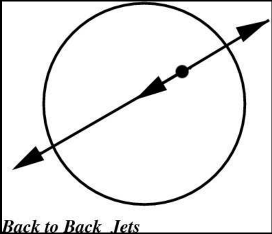
- DGLV: **PRL85 (2000); NPA784 (2007); NPA783 (2000); NPA733 (2004)**

Radiative E-Loss [Abir,Jamil,Mustafa,Sivastava, PLB715 (2012) 183]





Jet Quenching

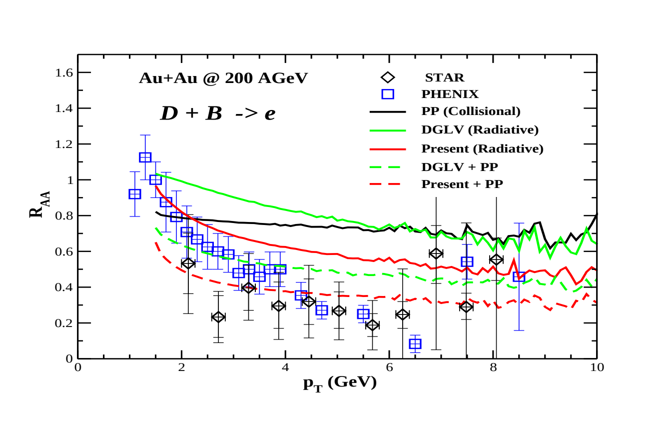


- Jets are produced back to back
  - **Awayside will pass through medium**
  - **Interact with the medium**
  - **Lose energy and quenched**
  - **Results in suppression of hadronic yields in AA than NN**

- Nuclear Suppression Factor: $R_{AA} = \frac{(\text{Yield})^{AA}}{N_{\text{coll}} \ (\text{Yield})^{NN}}$

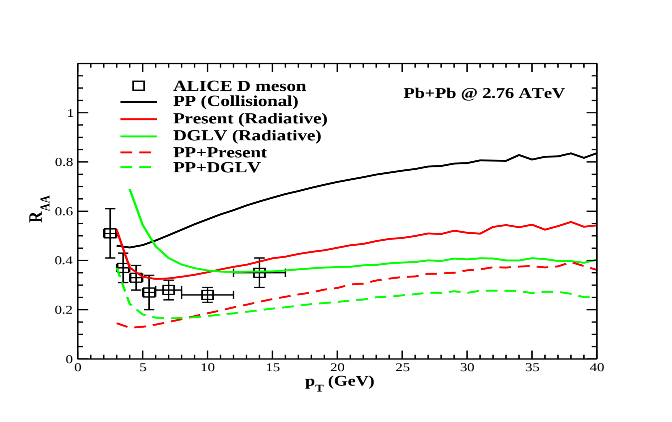
- Quenching depends on amout of energy-loss suffered in the medium

Single Electron @ RHIC



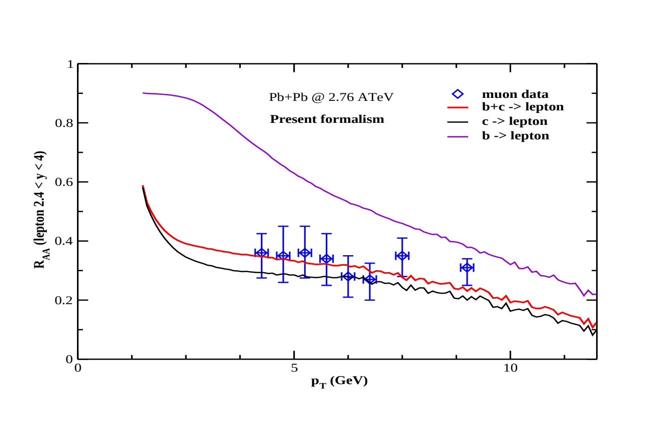
- STAR Data: **Abelev et al., PRL98 (2007) 192301; PRL106 (2011) (E)**
- PHENIX Data: **Adler et al., PRL98 (2007) 172301**
- Theory: **Abir, Jamil, Mustafa, Srivastava, PLB715 (2012) 183**

D-Meson @ LHC 2.76ATeV



- ALICE Data: **Abelev et al., ⟨arXiv:1203.2160⟩[nucl-ex]**
- Theory: **Abir, Jamil, Mustafa, Srivastava, PLB715 (2012) 183**

Single muon @ LHC 2.76ATeV



- ALICE Data: **Abelev et al., PRL109 (2012) 11231**
- Theory: **Abir, Jamil, Mustafa, Srivastava, under preparation**

Conclusion:

- **Discussed why are heavy quarks important**
- **Heavy quark production in high energy HIC: X-section in pQCD**
- **Collisional E-Loss & E-Gain**
  - Relation to Transport Coeffs: Drag and Diffusion
  - Coll. E-Loss Distr. & **importance of Coll. E-Loss**
- **Radiative E-Loss**
  - Discussed in general various Rad. E-Loss Model
  - Generalised Dead Cone and radiative E-Loss
  - Light and heavy quark lose energy in a similar fashion
- **Heavy quark nuclear suppression R_{AA}; Jet quenching**
  - Single lepton @ RHIC for 200 AGeV in Au+Au
  - D-Meson @LHC for 2.76ATeV in Pb+Pb
  - Single lepton @ LHC for 2.76ATeV in Pb+Pb

Collaborators:

- Raktim Abir (AMU, Aligarh)
- Purnendu Chakraborty (Basirhat College, N-24 Parganas)
- Umme Jamil (D.R. College, Golaghata, Assam)
- Dipali Pal (USA)

- D. K. Srivastava (VECC, Kolkata)

- Markus H. Thoma (Giessen Univ., Germany)
- Carsten Greiner (Goethe Univ. Frankfurt, Germany)
- Jan Uphoff (Goethe Univ., Frankfurt, Germany)
- M. Martinez (FIAS, Frankfurt, Germany)

THANK YOU