

# The QCD Phase diagram

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Rangpo, Sikkim*

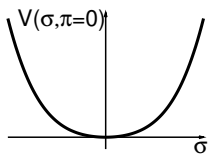
Collaborators: Zsolt Szép, Péter Kovács

# Overview

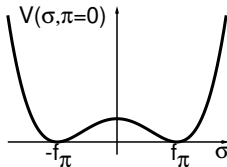
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  - Axial(vector) meson extended linear  $\sigma$ -model
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# Chiral symmetry

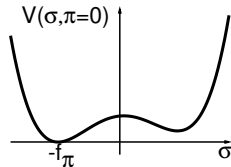
- for massless fermions: helicity  
spin has the same or opposite direction to the momentum,  
right-handed or left-handed particles
- $m_u \approx 4\text{MeV}$ ,  $m_d \approx 7\text{MeV}$ ,  $m_s \approx 150\text{MeV}$   
 $m_q \ll m_p$ , QCD is approximately chiral symmetric  
known before QCD (phenomenology of the weak interaction)
- The vacuum symmetric  $\Rightarrow$  parity doublets  
not symmetric  $\Rightarrow$  SSB  $\Rightarrow \langle \bar{q}q \rangle$  condensates in the vacuum



symmetry



spontaneous



explicit breaking

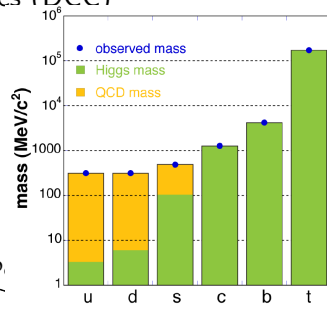
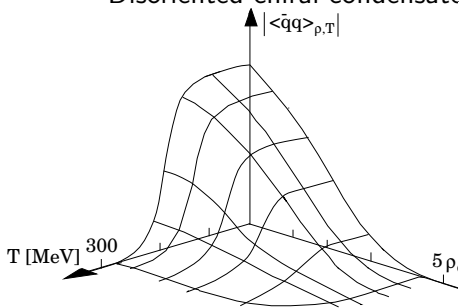
# Chiral symmetry restoration

- The order parameter of chiral symmetry is  $\langle \bar{q}q \rangle$

- Signatures:

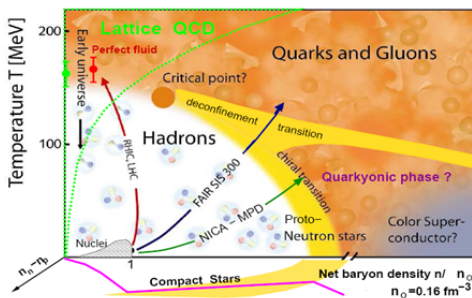
Hadrom mass modifications  $\sigma, \rho, \omega, \phi$

Disoriented chiral condensates (DCC)



# QCD phase diagram

Phase diagram in the  $T - \mu_B - \mu_I$  space



- At  $\mu_B = 0$   $T_c = 151$  MeV  
Y. Aoki, *et al.*, PLB **643**, 46 (2006)
- Is there a CP?  
( $T_{CP} = 162$  MeV,  $\mu_{CP} = 360$  MeV, Fodor-Katz)
- At  $T = 0$  in  $\mu_B$  where is the phase boundary?

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)

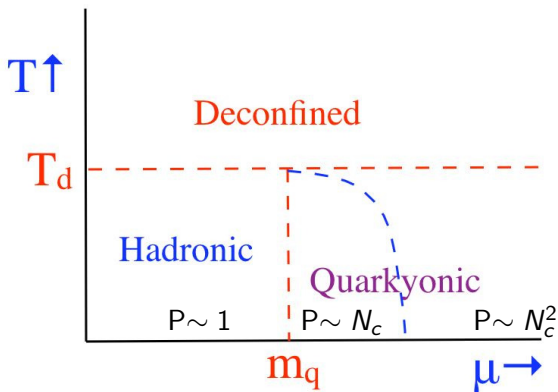
# Color Superconductivity

- The condition of color superconductivity is to have an attractive interaction between quarks on the Fermi surface  
At very large density ( $g(\mu_q) \ll 1$ ) one-gluon exchange dominates which is attractive for special quark quantum number combinations
- Condensation of Cooper pairs is the ground state
- In nature it may not be found, but this is the ground state at large enough density
- There are several possible condensate, several possible phase

- QED superconductivity in metals:  
interaction is difficult (phonon exchange)  
condensate is simple (electron Cooper-pairs)
- QCD superconductivity:  
interaction is simple (one gluon exchange)  
condensate: many possible condensate (needs numerical calculations)

# Quarkonium (McLerran-Pisarski: large $N_c$ )

$$N_c \rightarrow \infty, \quad N_f \ll N_c$$





# Addressed problems

- Which scalars are the chiral partners of the pseudoscalar nonet?
- Which parameterizations give phenomenologically good description of the phase transition?
- What is the thermodynamics of the model (compare with lattice)
- Which of them predict the existence of the CP?
- What is the order of phase transition on the  $T=0$  line?
- How the order parameters behave at finite temperature/chemical potential?
- How the masses change in medium?

# Effective models

Since QCD is very hard to solve  $\longrightarrow$  low energy effective models were set up  $\longrightarrow$  reflecting the global symmetries of QCD

- Nambu-Jona-Lasinio model (+Kobayashi-Maskawa-t'Hooft)
- Chiral perturbation theory
- Linear and nonlinear (it does not contain degrees of freedom relevant at high T) sigma model
- To study the phase diagram, we introduced the constituent quarks
- For mimicing confinement, we add the Polyakov loop variables.

extended Polyakov-Quark-Meson model

# Chiral symmetry

If the quark masses are zero (chiral limit)  $\implies$  QCD invariant under the following global transformation (**chiral symmetry**):

$$q_L = (1 - \gamma_5)/2q, \quad q_R = (1 + \gamma_5)/2q \quad \text{only the mass term mixes}$$

$$U(3)_V q = \exp(-i\alpha t)q \quad U(3)_A q = \exp(-i\beta\gamma_5 t)q$$

$$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A =$$

$$SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$$

$U(1)_V$  term  $\longrightarrow$  baryon number conservation

$U(1)_A$  term  $\longrightarrow$  broken through axial anomaly

$SU(3)_A$  term  $\longrightarrow$  broken down by any quark mass

$SU(3)_V$  term  $\longrightarrow$  broken down to  $SU(2)_V$  if  $m_u = m_d \neq m_s$   
 $\longrightarrow$  totally broken if  $m_u \neq m_d \neq m_s$  (**in nature**)

# Meson fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^8 \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & K^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_S = \sum_{i=0}^8 \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & K_S^0 & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j)$$

## Particle content:

Pseudoscalars:  $\pi(138)$ ,  $K(495)$ ,  $\eta(548)$ ,  $\eta'(958)$

Scalars:  $a_0(980 \text{ or } 1450)$ ,  $K_0^*(800 \text{ or } 1430)$ ,

$(\sigma_N, \sigma_S)$  : 2 of  $f_0(500, 980, 1370, 1500, 1710)$

# Structure of scalar mesons

	Mass (MeV)	width (MeV)	decays
$A_0(980)$	$980 \pm 20$	$50 - 100$	$\pi\pi$ dominant
$A_0(1450)$	$1474 \pm 19$	$265 \pm 13$	$\pi\eta, \pi\eta', K\bar{K}$
$K_s(800) = \kappa$	$682 \pm 29$	$547 \pm 24$	$K\pi$
$K_s(1430)$	$1425 \pm 50$	$270 \pm 80$	$K\pi$ dominant
$f_0(500) = \sigma$	400–550	400 – 700	$\pi\pi$ dominant
$f_0(980)$	$980 \pm 20$	40 – 100	$\pi\pi$ dominant
$f_0(1370)$	1200–1500	200 – 500	$\pi\pi \approx 250, K\bar{K} \approx 150$
$f_0(1500)$	$1505 \pm 6$	$109 \pm 7$	$\pi\pi \approx 38, K\bar{K} \approx 9.4$
$f_0(1710)$	$1722 \pm 6$	$135 \pm 7$	$\pi\pi \approx 30, K\bar{K} \approx 71$

Possible scalar states:  $\bar{q}q, \bar{q}\bar{q}qq$ , meson-meson molecules, glueballs

pseudoscalar nonet:  $\pi, K, \eta, \eta'$ , scalar nonet:  $A_0, K_0, 2 f_0$

multiquark states:  $f_0(980), A_0(980) f_0(600), K_0(800) ???$

meson-meson bound state ( $K\bar{K}$ ):  $f_0(980) ???$

glueballs:  $f_0(1500)$  (weak coupling to  $\gamma\gamma$ ),  $f_0(1710) ???$

## Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^8 \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & K^{*0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^8 b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N} + a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & K_1^0 & f_{1S} \end{pmatrix}^\mu$$

## Particle content:

Vector mesons:  $\rho(770)$ ,  $K^*(894)$ ,  $\omega_N = \omega(782)$ ,  $\omega_S = \phi(1020)$

Axial vectors:  $a_1(1230)$ ,  $K_1(1270)$ ,  $f_{1N}(1280)$ ,  $f_{1S}(1426)$

## Lagrangian (2/1)

$$\begin{aligned}
\mathcal{L}_{\text{Tot}} = & \text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\
& - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \text{Tr} \left[ \left( \frac{m_1^2}{2} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + \text{Tr}[H(\Phi + \Phi^\dagger)] \\
& + c_1 (\det \Phi + \det \Phi^\dagger) + i \frac{g_2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \\
& + \bar{\Psi} i \not{\partial} \Psi - g_F \bar{\Psi} (\Phi_S + i\gamma_5 \Phi_{PS}) \Psi + g_V \bar{\Psi} \gamma^\mu \left( V_\mu + \frac{g_A}{g_V} \gamma_5 A_\mu \right) \Psi
\end{aligned}$$

+Polyakov loops

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke,  
 Phys. Rev. D87 (2013) 014011

## Lagrangian (2/2)

where

$$D^\mu \Phi = \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ieA_e^\mu [T_3, \Phi]$$

$$\Phi = \sum_{i=0}^8 (\sigma_i + i\pi_i) T_i, \quad H = \sum_{i=0}^8 h_i T_i \quad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^8 (\rho_i^\mu - b_i^\mu) T_i, \quad L^\mu = \sum_{i=0}^8 (\rho_i^\mu + b_i^\mu) T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu [T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu [T_3, R^\mu]\}$$

$$\bar{\Psi} = (\bar{u}, \bar{d}, \bar{s})$$

non strange – strange base:

$$\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8,$$

$$\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \quad \varphi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates  $\langle \sigma_N \rangle, \langle \sigma_S \rangle \longleftrightarrow \phi_N, \phi_S$



# Symmetry properties of the model

Global  $U(3)_L \times U(3)_R$  transformation:

$$\begin{aligned}\Phi &\rightarrow U_L \Phi U_R^\dagger \\ L^\mu &\rightarrow U_L L^\mu U_L^\dagger & R^\mu &\rightarrow U_R R^\mu U_R^\dagger\end{aligned}$$

Consequences (using the unitarity of U's):

$$\begin{aligned}D^\mu \Phi &\rightarrow U_L D^\mu \Phi U_R^\dagger \\ L^{\mu\nu} &\rightarrow U_L L^{\mu\nu} U_L^\dagger & R^{\mu\nu} &\rightarrow U_R R^{\mu\nu} U_R^\dagger\end{aligned}$$

All terms are invariant except

the determinant: breaks  $U_A(1)$

the explicit symmetry breaking term H: breaks  $SU_A(3)$  and remains  $U_V(1) \times SU_V(3)$  if all 3 eigenvalues of H are equal  
remains  $U_V(1) \times SU_V(2)$  if 2 eigenvalues of H are equal  
remains  $U_V(1)$  if all 3 eigenvalues of H are different.

# Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \rightarrow \sigma_{N/S} + \phi_{N/S} \quad \phi_{N/S} \equiv \langle \sigma_{N/S} \rangle$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like  $\text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)]$ :

$$\pi_N - a_{1N}^\mu : -g_1 \phi_N a_{1N}^\mu \partial_\mu \pi_N,$$

$$\pi - a_1^\mu : -g_1 \phi_N (a_1^{\mu+} \partial_\mu \pi^- + a_1^{\mu 0} \partial_\mu \pi^0) + \text{h.c.},$$

$$\pi_S - a_{1S}^\mu : -\sqrt{2} g_1 \phi_S a_{1S}^\mu \partial_\mu \pi_S,$$

$$K_S - K_\mu^* : \frac{ig_1}{2} (\sqrt{2} \phi_S - \phi_N) (\bar{K}_\mu^{*0} \partial^\mu K_S^0 + K_\mu^{*-} \partial^\mu K_S^+) + \text{h.c.},$$

$$K - K_1^\mu : -\frac{g_1}{2} (\phi_N + \sqrt{2} \phi_S) (K_1^{\mu 0} \partial_\mu \bar{K}^0 + K_1^{\mu+} \partial_\mu K^-) + \text{h.c.}$$

Diagonalization  $\rightarrow$  Wave function renormalization

# Determination of the parameters of the Lagrangian

16 unknown parameters ( $m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, \mathbf{g}_F, \mathbf{g}_V, \mathbf{g}_A$ )  $\longrightarrow$  Determined by the **min. of  $\chi^2$** :

$$\chi^2(x_1, \dots, x_N) = \sum_{i=1}^M \left[ \frac{Q_i(x_1, \dots, x_N) - Q_i^{\text{exp}}}{\delta Q_i} \right]^2,$$

where  $(x_1, \dots, x_N) = (m_0, \lambda_1, \lambda_2, \dots)$ ,  $Q_i(x_1, \dots, x_N)$  calculated from the model, while  $Q_i^{\text{exp}}$  taken from the PDG

multiparametric minimalization  $\longrightarrow$  **MINUIT**

- PCAC  $\rightarrow$  2 physical quantities:  $f_\pi, f_K$
- Tree-level masses  $\rightarrow$  15 physical quantities:

$$m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^*}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_S}, m_{f_0^L}, m_{f_0^H}$$

- Decay widths  $\rightarrow$  12 physical quantities:

$$\Gamma_{\rho \rightarrow \pi\pi}, \Gamma_{\Phi \rightarrow KK}, \Gamma_{K^* \rightarrow K\pi}, \Gamma_{a_1 \rightarrow \pi\gamma}, \Gamma_{a_1 \rightarrow \rho\pi}, \Gamma_{f_1 \rightarrow KK^*}, \Gamma_{a_0}, \Gamma_{K_S \rightarrow K\pi},$$

$$\Gamma_{f_0^L \rightarrow \pi\pi}, \Gamma_{f_0^L \rightarrow KK}, \Gamma_{f_0^H \rightarrow \pi\pi}, \Gamma_{f_0^H \rightarrow KK}$$

- $T_c = 155$  MeV from lattice

Parametrization at  $T = 0$ 

## Results

	Cal(GeV)	Mass		Cal(GeV)	Width
$m_\pi$	0.1405	0.1380	$f_\pi$	0.0955	0.0922
$m_K$	0.4995	0.4956	$f_K$	0.1094	0.1100
$m_\eta$	0.5421	0.5479	$\Gamma_{f_0L \rightarrow KK}$	0.0	0.0
$m_{\eta'}$	0.9643	0.9578	$\Gamma_{f_0H \rightarrow KK}$	0.0	0.0
$m_\rho$	0.8064	0.7755	$\Gamma_\rho$	0.1515	0.149
$m_\phi$	0.9901	1.0195	$\Gamma_\phi$	0.003534	0.003545
$m_{K^*}$	0.9152	0.8938	$\Gamma_{K^*}$	0.04777	0.048
$m_{f_1H}$	1.4160	1.4264	$\Gamma_{f_1 \rightarrow KK}$	0.04451	0.0445
$m_{a_1}$	1.0766	1.2300	$\Gamma_{A_1 \rightarrow \rho\pi}$	0.1994	0.425
$m_{K_1}$	1.2999	1.2720	$\Gamma_{A_1 \rightarrow \gamma\pi}$	0.0003670	0.000640
$m_{a_0}$	0.7208	0.980	$\Gamma_{A_0}$	0.06834	0.075
$m_{K_0^*}$	0.7529	0.682	$\Gamma_{K_0^*}$	0.006001	0.00547
$m_{f_0L}$	0.2823	0.475	$\Gamma_{f_0L \rightarrow \pi\pi}$	0.5542	0.550
$m_{f_0H}$	0.7376	0.990	$\Gamma_{f_0H \rightarrow \pi\pi}$	0.08166	0.07
$m_{ud}$	0.3224	0.308	$m_s$	0.4577	0.483

## Parameters

Parameter	Value
$\phi_N$ [GeV]	0.1411
$\phi_S$ [GeV]	0.1416
$m_0^2$ [GeV <sup>2</sup> ]	$2.3925E - 4$
$m_1^2$ [GeV <sup>2</sup> ]	$6.3298E - 8$
$\lambda_1$	-1.6738
$\lambda_2$	23.5078
$\delta_S$ [GeV <sup>2</sup> ]	0.1133
$c_1$ [GeV]	1.3086
$g_1$	5.6156
$g_2$	3.0467
$h_1$	27.4617
$h_2$	4.2281
$h_3$	5.9839
$g_F$	4.5708
$M_0$ [GeV]	0.3511

- with this set  
 $f_0^I = 0.2837$  GeV

# Polyakov loops in Polyakov gauge

**Polyakov loop variables:**  $\Phi(\vec{x}) = \frac{\text{Tr}_c L(\vec{x})}{N_c}$  and  $\bar{\Phi}(\vec{x}) = \frac{\text{Tr}_c \bar{L}(\vec{x})}{N_c}$  with

$$L(x) = \mathcal{P} \exp \left[ i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right]$$

→ signals center symmetry ( $\mathbb{Z}_3$ ) breaking at the deconfinement transition

low  $T$ : confined phase,  $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$

high  $T$ : deconfined phase,  $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

**Polyakov gauge:** the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space

$$G_{4,d}(\vec{x}) = \phi_3(\vec{x})\lambda_3 + \phi_8(\vec{x})\lambda_8; \quad \lambda_3, \lambda_8 : \text{Gell-Mann matrices.}$$

In this gauge the Polyakov loop operator is

$$L(\vec{x}) = \text{diag}(e^{i\beta\phi_+(\vec{x})}, e^{i\beta\phi_-(\vec{x})}, e^{-i\beta(\phi_+(\vec{x})+\phi_-(\vec{x}))})$$

where  $\phi_{\pm}(\vec{x}) = \pm\phi_3(\vec{x}) + \phi_8(\vec{x})/\sqrt{3}$

# Improved Polyakov loops potential

Logarithmic potential K. Fukushima, Phys. Lett. **B591**, 277 (2004)

$$\frac{\mathcal{U}_{\log}(\Phi, \bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\Phi\bar{\Phi} + b(T)\ln\left[1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2\right]$$

with  $a(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2}$ ,  $b(T) = b_3\frac{T_0^3}{T^3}$

$\mathcal{U}(\Phi, \bar{\Phi})$  models the free energy of a pure gauge theory

Within FRG, the glue potential  $U^{glue}(\Phi, \bar{\Phi})$  coming from the gauge dof propagating in the presence of dynamical quarks can be matched to the potential  $U^{YM}(\Phi, \bar{\Phi})$  of the SU(3) YM theory by relating the reduced temperatures:

$$\frac{U^{glue}}{T^4}(\Phi, \bar{\Phi}, t_{glue}) = \frac{U^{YM}}{(T^{YM})^4}(\Phi, \bar{\Phi}, t_{YM}(t_{glue})), \quad t_{YM}(t_{glue}) \approx 0.57t_{glue}$$

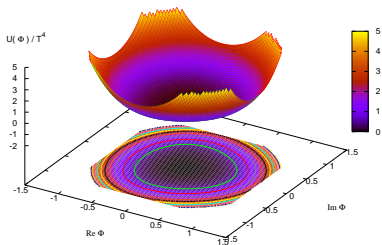
$$t_{glue} \equiv \frac{T - T_c^{glue}}{T_c^{glue}}, \quad t_{YM} \equiv \frac{T^{YM} - T_0^{YM}}{T_0^{YM}}, \quad T_c^{glue} \in (180, 270) \text{ MeV}$$

L.M.Haas et al., PRD 87, 076004 (2013)

# Polyakov loop potential

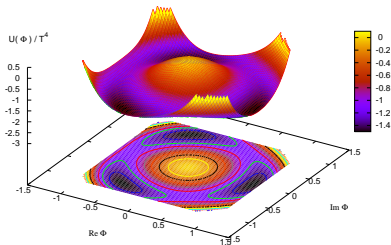
“Color confinement”

$\langle \Phi \rangle = 0 \rightarrow$  no breaking of  $\mathbb{Z}_3$   
one minimum



“Color deconfinement”

$\langle \Phi \rangle \neq 0 \rightarrow$  spontaneous breaking of  $\mathbb{Z}_3$   
minima at  $0, 2\pi/3, -2\pi/3$   
one of them spontaneously selected



from H. Hansen et al., PRD75, 065004 (2007)



# Effects of Polyakov loops on FD statistics

Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

$$f(E_p - \mu_q) \longrightarrow f_{\bar{\Phi}}^+(E_p) = \frac{(\bar{\Phi} + 2\Phi e^{-\beta(E_p - \mu_q)}) e^{-\beta(E_p - \mu_q)} + e^{-3\beta(E_p - \mu_q)}}{1 + 3(\bar{\Phi} + \Phi e^{-\beta(E_p - \mu_q)}) e^{-\beta(E_p - \mu_q)} + e^{-3\beta(E_p - \mu_q)}}$$

$$f(E_p + \mu_q) \longrightarrow f_{\Phi}^-(E_p) = \frac{(\Phi + 2\bar{\Phi} e^{-\beta(E_p + \mu_q)}) e^{-\beta(E_p + \mu_q)} + e^{-3\beta(E_p + \mu_q)}}{1 + 3(\Phi + \bar{\Phi} e^{-\beta(E_p + \mu_q)}) e^{-\beta(E_p + \mu_q)} + e^{-3\beta(E_p + \mu_q)}}$$

$$\Phi, \bar{\Phi} \rightarrow 0 \implies f_{\Phi}^{\pm}(E_p) \rightarrow f(3(E_p \pm \mu_q)) \quad \Phi, \bar{\Phi} \rightarrow 1 \implies f_{\Phi}^{\pm}(E_p) \rightarrow f(E_p \pm \mu_q)$$

**three-particle state appears:** mimics confinement of quarks within baryons

at  $T = 0$  there is no difference between models with and without Polyakov loop

Extremum equations for  $\phi_{N/S}$  and  $\Phi, \bar{\Phi}$

## $T/\mu_B$ dependence of the condensates

$\Omega$ : grand canonical potential

$$\frac{\partial \Omega}{\partial \Phi} = \left. \frac{\partial \Omega}{\partial \bar{\Phi}} \right|_{\varphi_N = \phi_N, \varphi_S = \phi_S} = 0$$

$$\frac{\partial \Omega}{\partial \phi_N} = \left. \frac{\partial \Omega}{\partial \phi_S} \right|_{\Phi, \bar{\Phi}} = 0, \quad (\text{after the SSB})$$

**Hybrid approach:** fermions at one-loop, mesons at tree-level (their effects are much smaller)

Extremum equations for  $\phi_{N/S}$  and  $\Phi, \bar{\Phi}$ 

## Masses

$$M_{i,ab}^2 = \left. \frac{\partial^2 \Omega(T, \mu_f)}{\partial \varphi_{i,a} \partial \varphi_{i,b}} \right|_{\min} = m_{i,ab}^2 + \Delta_0 m_{i,ab}^2 + \Delta_T m_{i,ab}^2,$$

$m_{i,ab}^2 \longrightarrow$  tree-level mass matrix,

$\Delta_0/T m_{i,ab}^2 \longrightarrow$  fermion vacuum/thermal fluctuation,

$$\Delta_0 m_{i,ab}^2 = \left. \frac{\partial^2 \Omega_{q\bar{q}}^{\text{vac}}}{\partial \varphi_{i,a} \partial \varphi_{i,b}} \right|_{\min} = -\frac{3}{8\pi^2} \sum_{f=u,d,s} \left[ \left( \frac{3}{2} + \log \frac{m_f^2}{M^2} \right) m_{f,a}^{2(i)} m_{f,b}^{2(i)} + m_f^2 \left( \frac{1}{2} + \log \frac{m_f^2}{M^2} \right) m_{f,ab}^{2(i)} \right],$$

$$\begin{aligned} \Delta_T m_{i,ab}^2 = \left. \frac{\partial^2 \Omega_{q\bar{q}}^{\text{th}}}{\partial \varphi_{i,a} \partial \varphi_{i,b}} \right|_{\min} &= 6 \sum_{f=u,d,s} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_f(p)} \left[ (f_f^+(p) + f_f^-(p)) \left( m_{f,ab}^{2(i)} - \frac{m_{f,a}^{2(i)} m_{f,b}^{2(i)}}{2E_f^2(p)} \right) \right. \\ &\quad \left. + (B_f^+(p) + B_f^-(p)) \frac{m_{f,a}^{2(i)} m_{f,b}^{2(i)}}{2TE_f(p)} \right], \end{aligned}$$

where  $m_{f,a}^{2(i)} \equiv \partial m_f^2 / \partial \varphi_{i,a}$ ,  $m_{f,ab}^{2(i)} \equiv \partial^2 m_f^2 / \partial \varphi_{i,a} \partial \varphi_{i,b}$

# Features of our approach

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables,  $\Phi, \bar{\Phi}$  with  $U^{YM}$  or  $U^{glue}$
- u,d,s constituent quarks, ( $m_u = m_d$ )
- no mesonic fluctuations included in the grand canonical potential:

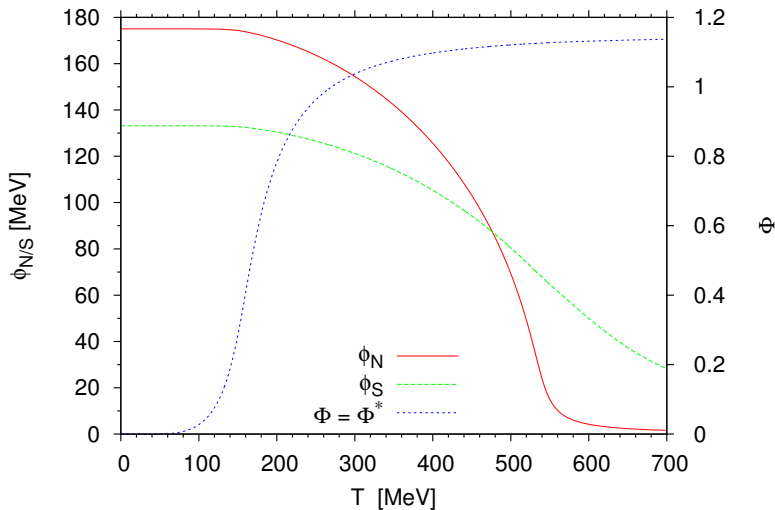
$$\Omega(T, \mu_q) = -\frac{1}{\beta V} \ln(Z)$$

- Fermion **vacuum** and **thermal** fluctuations
- quarks do not couple to (axial) vector meson yet
- Four order parameters ( $\phi_N, \phi_S, \Phi, \bar{\Phi}$ )  $\rightarrow$  four  $T/\mu$ -dependent equations
- thermal contribution of  $\pi, K, f_0^L$  included in the pressure

$T$  dependence of the order parameters

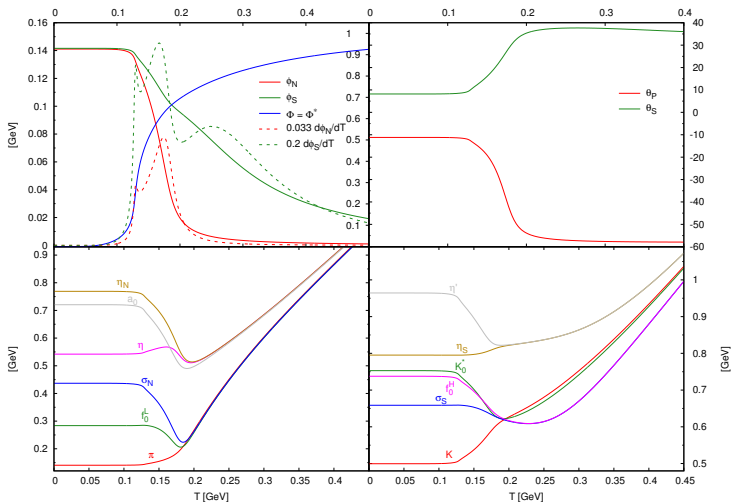
$\phi_{N/S}(T), \Phi/\bar{\Phi}(T)$  *with* Polyakov loop  $m_{f_0^L} = 1326$  MeV

Condensates and Polyakov loop variables with vacuum fluctuations



$T$  dependence of the order parameters

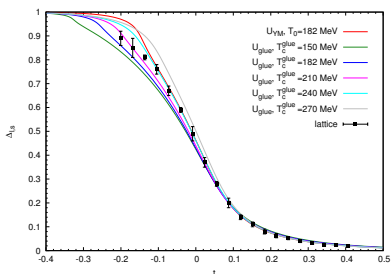
With low mass scalars,  $m_{f_0^L} = 300$  MeV



Our pattern:  $m_\eta \leq m_{\eta_N} < m_{\eta_S} \leq m_{\eta'}$  in contrast to others

Schaefer, PRD79 014018, Tiwari PRD88, 074017

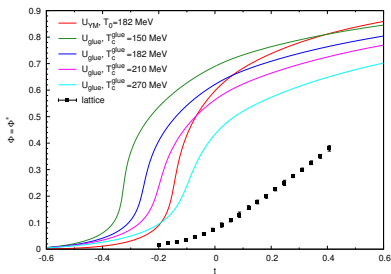
# $T$ -dependence of condensates compared to lattice results



subtracted chiral condensate

$$\Delta = \frac{(\Phi_N - h_N/h_S \Phi_S)_T}{(\Phi_N - h_N/h_S \Phi_S)_{T=0}}$$

$U^{glue}$  with  $T_C^{glue} \in (210 - 240)$  MeV gives good agreement with the lattice result of [Borsanyi et al., JHEP 1009, 073 \(2010\)](#)



- lattice shows smooth transition
- our result is completely off
- renormalization of the Polyakov loop may explain part of the discrepancy [Andersen et al., PRD92, 114504](#)

# Thermodynamical Observables

We include mesonic thermal contribution to  $p$  for  $(\pi, K, f_0^I)$

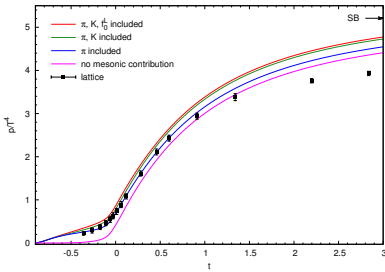
$$\Delta p(T) = -nT \int \frac{d^3q}{(2\pi)^3} \ln(1 - e^{-\beta E(q)}), \quad E(q) = \sqrt{q^2 + m^2}$$

- pressure:  $p(T, \mu_q) = \Omega_H(T = 0, \mu_q) - \Omega_H(T, \mu_q)$
- entropy density:  $s = \frac{\partial p}{\partial T}$
- quark number density:  $\rho_q = \frac{\partial p}{\partial \mu_q}$
- energy density:  $\epsilon = -p + Ts + \mu_q \rho_q$
- scaled interaction measure:  $\frac{\Delta}{T^4} = \frac{\epsilon - 3p}{T^4}$
- speed of sound at  $\mu_q = 0$ :  $c_s^2 = \frac{\partial p}{\partial \epsilon}$

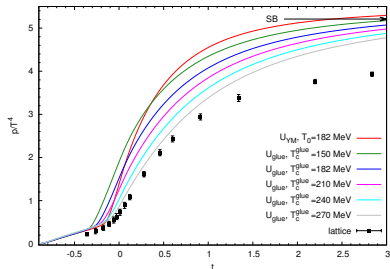


$T$  dependence of the order parameters

# Normalized pressure



we use  $U^{glue}$  with  $T_c^{glue} = 270$  MeV  
 pion dominates at low  $T$   
 at high  $T$  pressure overshoots the lattice data

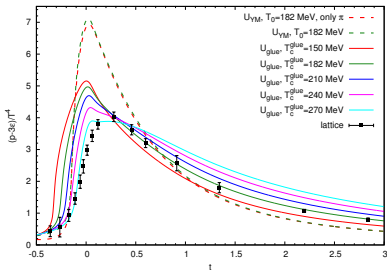


overshooting increases with decreasing  $T_c^{glue}$   
 lattice: [Borsányi et al., JHEP 1011, 077 \(2010\)](#)

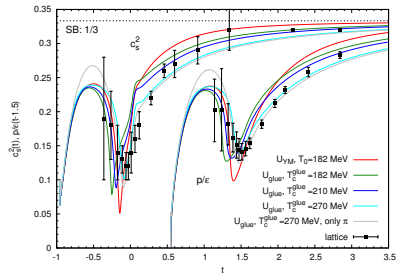
$T$  dependence of the order parameters

# Observables

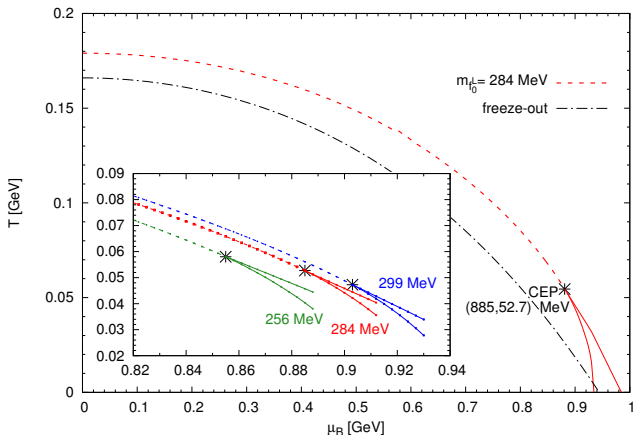
## interaction measure



## speed of sound



# $T - \mu_B$ phase diagram



- we use  $U^{glue}$  with  $T_c^{glue} = 210$  MeV
- freeze-out curve from Cleymans et al., J.Phys.G32, S165

## Summary and Conclusions

- The thermodynamics of the ePQM was studied after parametrizing of the model with a modification of the method used in [Parganlija et al., PRD87, 014011](#)
- 40 possible assignments of the scalars to the nonet states were investigated. Lowest  $\chi^2$  for  $a_0^{\bar{q}q} \rightarrow a_0(980)$ ,  $K_0^{*,\bar{q}q} \rightarrow K_0^*(980)$ ,  $f_0^{l,\bar{q}q} \rightarrow f_0(500)$ ,  $f_0^{h,\bar{q}q} \rightarrow f_0(980)$
- The phase transition temperature requires low mass ( $\leq 400$  MeV)  $f_0$
- For the best set of parameters CEP was found in the  $T - \mu_B$  plane
- The  $T$ -dependence of various thermodynamical observables measured on the lattice is reasonable well reproduced with an improved Polyakov loop potential. [L.M. Hass et al., PRD87, 076004](#)
- The model can be used to give predictions at finite  $\mu_B$

- To do...
- Improve the vacuum phenomenology by tetraquarks (and glueballs)
- coupling the quarks to the (axial)vectors
- including mesonic fluctuations

# Chiral symmetry

If the quark masses are zero (chiral limit)  $\implies$  QCD invariant under the following global transformation (**chiral symmetry**):

$$q_L = (1 - \gamma_5)/2q, \quad q_R = (1 + \gamma_5)/2q \quad \text{only the mass term mixes}$$

$$U(3)_V q = \exp(-i\alpha t)q \quad U(3)_A q = \exp(-i\beta\gamma_5 t)q$$

$$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A =$$

$$SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$$

$U(1)_V$  term  $\longrightarrow$  baryon number conservation

$U(1)_A$  term  $\longrightarrow$  broken through axial anomaly

$SU(3)_A$  term  $\longrightarrow$  broken down by any quark mass

$SU(3)_V$  term  $\longrightarrow$  broken down to  $SU(2)_V$  if  $m_u = m_d \neq m_s$   
 $\longrightarrow$  totally broken if  $m_u \neq m_d \neq m_s$  (**in nature**)

# Meson fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^8 \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & K^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_S = \sum_{i=0}^8 \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & K_S^0 & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j)$$

## Particle content:

Pseudoscalars:  $\pi(138)$ ,  $K(495)$ ,  $\eta(548)$ ,  $\eta'(958)$

Scalars:  $a_0(980 \text{ or } 1450)$ ,  $K_0^*(800 \text{ or } 1430)$ ,

$(\sigma_N, \sigma_S)$  : 2 of  $f_0(500, 980, 1370, 1500, 1710)$

# Structure of scalar mesons

	Mass (MeV)	width (MeV)	decays
$A_0(980)$	$980 \pm 20$	$50 - 100$	$\pi\pi$ dominant
$A_0(1450)$	$1474 \pm 19$	$265 \pm 13$	$\pi\eta, \pi\eta', K\bar{K}$
$K_s(800) = \kappa$	$682 \pm 29$	$547 \pm 24$	$K\pi$
$K_s(1430)$	$1425 \pm 50$	$270 \pm 80$	$K\pi$ dominant
$f_0(500) = \sigma$	400–550	400 – 700	$\pi\pi$ dominant
$f_0(980)$	$980 \pm 20$	40 – 100	$\pi\pi$ dominant
$f_0(1370)$	1200–1500	200 – 500	$\pi\pi \approx 250, K\bar{K} \approx 150$
$f_0(1500)$	$1505 \pm 6$	$109 \pm 7$	$\pi\pi \approx 38, K\bar{K} \approx 9.4$
$f_0(1710)$	$1722 \pm 6$	$135 \pm 7$	$\pi\pi \approx 30, K\bar{K} \approx 71$

Possible scalar states:  $\bar{q}q, \bar{q}\bar{q}qq$ , meson-meson molecules, glueballs

pseudoscalar nonet:  $\pi, K, \eta, \eta'$ , scalar nonet:  $A_0, K_0, 2 f_0$

multiquark states:  $f_0(980), A_0(980) f_0(600), K_0(800) ???$

meson-meson bound state ( $K\bar{K}$ ):  $f_0(980) ???$

glueballs:  $f_0(1500)$  (weak coupling to  $\gamma\gamma$ ),  $f_0(1710) ???$



## Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^8 \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & K^{*0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^8 b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N} + a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & K_1^0 & f_{1S} \end{pmatrix}^\mu$$

## Particle content:

Vector mesons:  $\rho(770)$ ,  $K^*(894)$ ,  $\omega_N = \omega(782)$ ,  $\omega_S = \phi(1020)$

Axial vectors:  $a_1(1230)$ ,  $K_1(1270)$ ,  $f_{1N}(1280)$ ,  $f_{1S}(1426)$

## Lagrangian (2/1)

$$\begin{aligned}
\mathcal{L}_{\text{Tot}} = & \text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\
& - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \text{Tr} \left[ \left( \frac{m_1^2}{2} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + \text{Tr}[H(\Phi + \Phi^\dagger)] \\
& + c_1 (\det \Phi + \det \Phi^\dagger) + i \frac{g_2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \\
& + \bar{\Psi} i \not{\partial} \Psi - g_F \bar{\Psi} (\Phi_S + i\gamma_5 \Phi_{PS}) \Psi + g_V \bar{\Psi} \gamma^\mu \left( V_\mu + \frac{g_A}{g_V} \gamma_5 A_\mu \right) \Psi
\end{aligned}$$

+Polyakov loops

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke,  
 Phys. Rev. D87 (2013) 014011

## Lagrangian (2/2)

where

$$D^\mu \Phi = \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ieA_e^\mu [T_3, \Phi]$$

$$\Phi = \sum_{i=0}^8 (\sigma_i + i\pi_i) T_i, \quad H = \sum_{i=0}^8 h_i T_i \quad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^8 (\rho_i^\mu - b_i^\mu) T_i, \quad L^\mu = \sum_{i=0}^8 (\rho_i^\mu + b_i^\mu) T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu [T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu [T_3, R^\mu]\}$$

$$\bar{\Psi} = (\bar{u}, \bar{d}, \bar{s})$$

non strange – strange base:

$$\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8,$$

$$\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \quad \varphi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates  $\langle \sigma_N \rangle, \langle \sigma_S \rangle \longleftrightarrow \phi_N, \phi_S$

# Symmetry properties of the model

Global  $U(3)_L \times U(3)_R$  transformation:

$$\begin{aligned}\Phi &\rightarrow U_L \Phi U_R^\dagger \\ L^\mu &\rightarrow U_L L^\mu U_L^\dagger & R^\mu &\rightarrow U_R R^\mu U_R^\dagger\end{aligned}$$

Consequences (using the unitarity of U's):

$$\begin{aligned}D^\mu \Phi &\rightarrow U_L D^\mu \Phi U_R^\dagger \\ L^{\mu\nu} &\rightarrow U_L L^{\mu\nu} U_L^\dagger & R^{\mu\nu} &\rightarrow U_R R^{\mu\nu} U_R^\dagger\end{aligned}$$

All terms are invariant except

the determinant: breaks  $U_A(1)$

the explicit symmetry breaking term H: breaks  $SU_A(3)$  and  
remains  $U_V(1) \times SU_V(3)$  if all 3 eigenvalues of H are equal  
remains  $U_V(1) \times SU_V(2)$  if 2 eigenvalues of H are equal  
remains  $U_V(1)$  if all 3 eigenvalues of H are different.

# Determinant term

$$U_L = e^{-i\omega_L^a T^a} \quad U_R = e^{-i\omega_R^a T^a}$$

$$\omega_V^a = 0.5(\omega_L^a + \omega_R^a) \quad \omega_A^a = 0.5(\omega_L^a - \omega_R^a)$$

By  $SU(3)_L \times SU(3)_R$  transformation (if  $\omega_L^0 = \omega_R^0 = 0 = \omega_V^0 = \omega_A^0$ )

$$(\det \Phi)' = \det(U_L \Phi U_R^\dagger) = \det U_L \det \Phi \det U_R^\dagger = \det \Phi$$

Similarly  $\det \Phi^\dagger$  is also invariant.

If  $\omega_V^0 \neq 0$  and all the other  $\omega$ 's are 0 ( $[T^a, T^0] = 0$ )

$$(\det \Phi)' = \det(e^{-i\omega_V^0 T^0} \Phi e^{i\omega_V^0 T^0}) = \det(e^{-i\omega_V^0 T^0} e^{i\omega_V^0 T^0} \Phi) = \det \Phi$$

On the other hand, if  $\omega_A^0 \neq 0$  and all the other  $\omega$ 's are 0

$$(\det \Phi)' = \det(e^{-i\omega_A^0 T^0} \Phi e^{-i\omega_A^0 T^0}) = \det(e^{-i\omega_A^0 T^0} e^{-i\omega_A^0 T^0} \Phi) = e^{-i2\omega_A^0} \det \Phi \text{Tr } T^0$$

So the determinant term is invariant under  $U(3)_V \times SU(3)_A$  transformation and breaks explicitly the  $U(1)_A$  symmetry.

Explicit breaking term:  $\text{Tr}[\hat{\epsilon}(\Phi + \Phi^\dagger)]$ 

$$\hat{\epsilon} = \sum_{i=0}^8 \epsilon_i T_i = \begin{pmatrix} \frac{\epsilon_N}{2} & 0 & 0 \\ 0 & \frac{\epsilon_N}{2} & 0 \\ 0 & 0 & \frac{\epsilon_S}{\sqrt{2}} \end{pmatrix} \quad \text{only } \epsilon^0, \epsilon^8 \neq 0$$

- axial transformation: if at least  $\epsilon^0 \neq 0$   $U(3)_A$  is broken:

$$(\text{Tr}[\hat{\epsilon}(\Phi)])' = \text{Tr}(e^{-i2\omega_A^a T^a} \hat{\epsilon} \Phi)$$

- vector transformation

$$(\text{Tr}[\hat{\epsilon}(\Phi)])' = \text{Tr}(e^{-i\omega_V^a T^a} \hat{\epsilon} e^{i\omega_V^a T^a} \Phi)$$

Since  $[\hat{\epsilon}, T^0] = 0$ ,  $U(1)_V$  symmetry is preserved.

If all  $\epsilon^a = 0$  except  $\epsilon^0$ ,  $U(3)_V$  is preserved.

If  $\epsilon^8$  also non zero, then since  $[T^K, T^8] = 0$  if  $k = 1, 2, 3$ ,  $U(1)_V \times SU(2)_V$  survives (isospin symmetry)

(If  $\epsilon^3 \neq 0$  too, then the isospin symmetry is broken, only  $U(1)_V$ .)

# Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \rightarrow \sigma_{N/S} + \phi_{N/S} \quad \phi_{N/S} \equiv \langle \sigma_{N/S} \rangle$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like  $\text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)]$ :

$$\pi_N - a_{1N}^\mu : -g_1 \phi_N a_{1N}^\mu \partial_\mu \pi_N,$$

$$\pi - a_1^\mu : -g_1 \phi_N (a_1^{\mu+} \partial_\mu \pi^- + a_1^{\mu 0} \partial_\mu \pi^0) + \text{h.c.},$$

$$\pi_S - a_{1S}^\mu : -\sqrt{2} g_1 \phi_S a_{1S}^\mu \partial_\mu \pi_S,$$

$$K_S - K_\mu^* : \frac{ig_1}{2} (\sqrt{2} \phi_S - \phi_N) (\bar{K}_\mu^{*0} \partial^\mu K_S^0 + K_\mu^{*-} \partial^\mu K_S^+) + \text{h.c.},$$

$$K - K_1^\mu : -\frac{g_1}{2} (\phi_N + \sqrt{2} \phi_S) (K_1^{\mu 0} \partial_\mu \bar{K}^0 + K_1^{\mu+} \partial_\mu K^-) + \text{h.c.}$$

Diagonalization  $\rightarrow$  Wave function renormalization

# Determination of the parameters of the Lagrangian

16 unknown parameters ( $m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, \mathbf{g}_F, \mathbf{g}_V, \mathbf{g}_A$ )  $\longrightarrow$  Determined by the **min. of  $\chi^2$** :

$$\chi^2(x_1, \dots, x_N) = \sum_{i=1}^M \left[ \frac{Q_i(x_1, \dots, x_N) - Q_i^{\text{exp}}}{\delta Q_i} \right]^2,$$

where  $(x_1, \dots, x_N) = (m_0, \lambda_1, \lambda_2, \dots)$ ,  $Q_i(x_1, \dots, x_N)$  calculated from the model, while  $Q_i^{\text{exp}}$  taken from the PDG

multiparametric minimalization  $\longrightarrow$  **MINUIT**

- PCAC  $\rightarrow$  2 physical quantities:  $f_\pi, f_K$
- Tree-level masses  $\rightarrow$  15 physical quantities:  
 $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^*}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_S}, m_{f_0^L}, m_{f_0^H}$
- Decay widths  $\rightarrow$  12 physical quantities:  
 $\Gamma_{\rho \rightarrow \pi\pi}, \Gamma_{\Phi \rightarrow KK}, \Gamma_{K^* \rightarrow K\pi}, \Gamma_{a_1 \rightarrow \pi\gamma}, \Gamma_{a_1 \rightarrow \rho\pi}, \Gamma_{f_1 \rightarrow KK^*}, \Gamma_{a_0}, \Gamma_{K_S \rightarrow K\pi},$   
 $\Gamma_{f_0^L \rightarrow \pi\pi}, \Gamma_{f_0^L \rightarrow KK}, \Gamma_{f_0^H \rightarrow \pi\pi}, \Gamma_{f_0^H \rightarrow KK}$
- $T_c = 155$  MeV from lattice



Parametrization at  $T = 0$ 

## Results

	Cal(GeV)	Mass		Cal(GeV)	Width
$m_\pi$	0.1405	0.1380	$f_\pi$	0.0955	0.0922
$m_K$	0.4995	0.4956	$f_K$	0.1094	0.1100
$m_\eta$	0.5421	0.5479	$\Gamma_{f_0L \rightarrow KK}$	0.0	0.0
$m_{\eta'}$	0.9643	0.9578	$\Gamma_{f_0H \rightarrow KK}$	0.0	0.0
$m_\rho$	0.8064	0.7755	$\Gamma_\rho$	0.1515	0.149
$m_\phi$	0.9901	1.0195	$\Gamma_\phi$	0.003534	0.003545
$m_{K^*}$	0.9152	0.8938	$\Gamma_{K^*}$	0.04777	0.048
$m_{f_1H}$	1.4160	1.4264	$\Gamma_{f_1 \rightarrow KK}$	0.04451	0.0445
$m_{a_1}$	1.0766	1.2300	$\Gamma_{A_1 \rightarrow \rho\pi}$	0.1994	0.425
$m_{K_1}$	1.2999	1.2720	$\Gamma_{A_1 \rightarrow \gamma\pi}$	0.0003670	0.000640
$m_{a_0}$	0.7208	0.980	$\Gamma_{A_0}$	0.06834	0.075
$m_{K_0^*}$	0.7529	0.682	$\Gamma_{K_0^*}$	0.006001	0.00547
$m_{f_0L}$	0.2823	0.475	$\Gamma_{f_0L \rightarrow \pi\pi}$	0.5542	0.550
$m_{f_0H}$	0.7376	0.990	$\Gamma_{f_0H \rightarrow \pi\pi}$	0.08166	0.07
$m_{ud}$	0.3224	0.308	$m_s$	0.4577	0.483

Parametrization at  $T = 0$ 

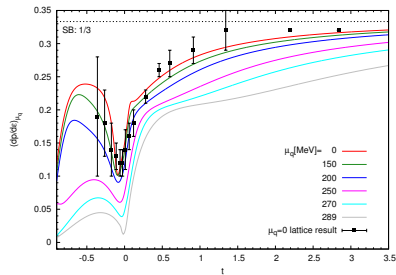
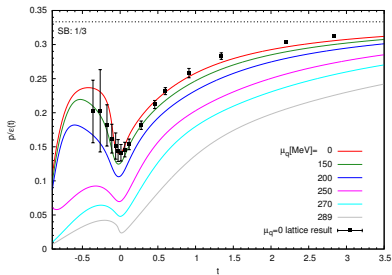
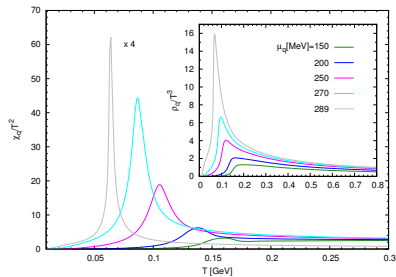
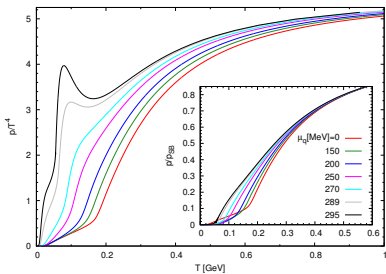
## Parameters

Parameter	Value
$\phi_N$ [GeV]	0.1411
$\phi_S$ [GeV]	0.1416
$m_0^2$ [GeV <sup>2</sup> ]	$2.3925E - 4$
$m_1^2$ [GeV <sup>2</sup> ]	$6.3298E - 8$
$\lambda_1$	-1.6738
$\lambda_2$	23.5078
$\delta_S$ [GeV <sup>2</sup> ]	0.1133
$c_1$ [GeV]	1.3086
$g_1$	5.6156
$g_2$	3.0467
$h_1$	27.4617
$h_2$	4.2281
$h_3$	5.9839
$g_F$	4.5708
$M_0$ [GeV]	0.3511

- with this set  
 $f_0^I = 0.2837$  GeV

Parametrization at  $T = 0$

# Observables at $\mu_B \neq 0$



Thank you for your attention!