Flow Power Spectrum in Relativistic Heavy-Ion Collisions

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Outline:

1. Elliptic Flow and flow fluctuations in relativistic heavy-ion collision experiments : overall view

2. Power spectrum of flow fluctuations: Acoustic peaks and superhorizon fluctuations for heavy-ion collisions, as for CMBR.

3. Hydrodynamics simulations: Non-trivial structure of flow power spectrum, information about freezeout horizon, viscosity,…..

4. Probing high baryon density phases through vorticity detection

   CFL phase, nucleon superfluidity (as in pulsar glitches): All lead to superfluid vortices. Qualitative effects on flow power spectrum. New focus for FAIR and NICA: very low energy collisions with neutron rich nuclei to get nucleon superfluidity

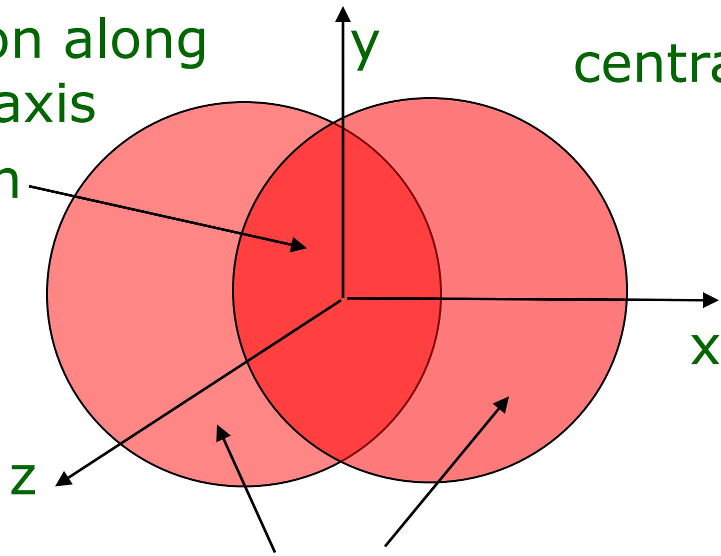
5. Effect of magnetic field on power spectrum: Enhancement of flow anisotropies, larger v_2 (larger η/s than AdS/CFT limit?)

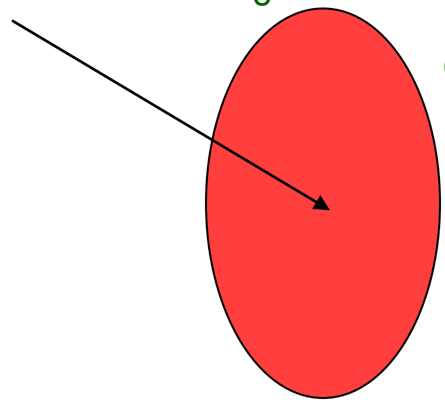
   Magnetohydrodynamics simulations: Effect of magnetic Field on power spectrum

Relativistic heavy-ion collision experiments:

Recall: Elliptic Flow

In non-central collisions: central QGP region is anisotropic



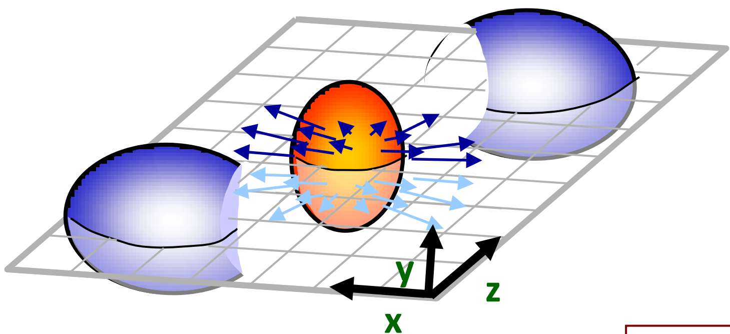


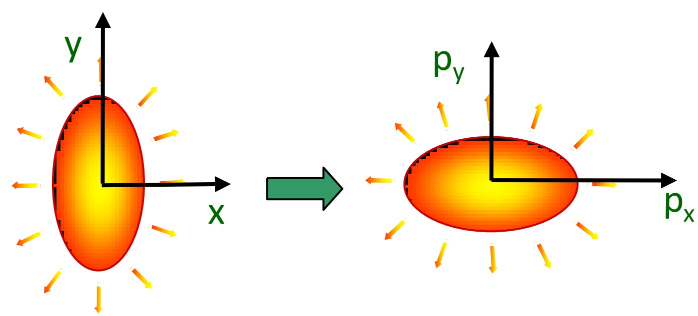
Anisotropic shape implies: $$\frac{\partial P}{\partial x} > \frac{\partial P}{\partial y}$$

Important: Initially no transverse expansion
Anisotropic pressure gradient implies:
Buildup of plasma flow larger in x direction
than in y direction

Flow is a phenomenon seen in nucleus-nucleus collisions, which correlates the momentum distributions of the produced particles with the **spatial eccentricity** of the overlap region.

azimuthal dependence of the pressure gradient.

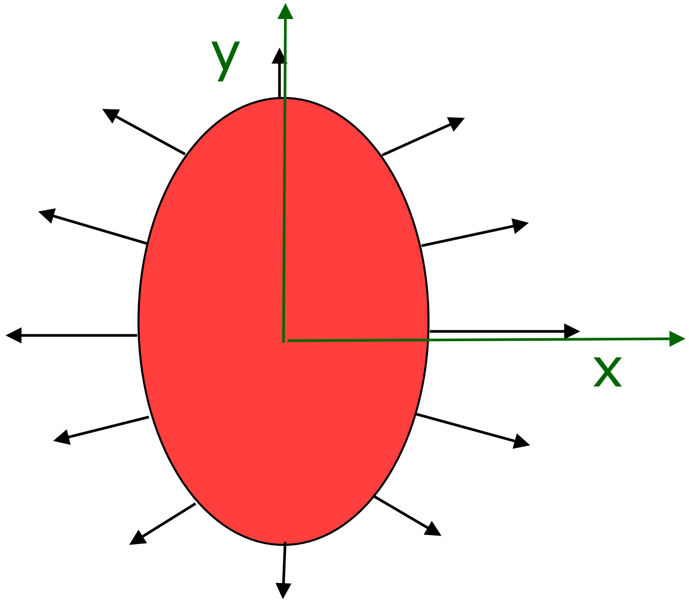




Reaction plane: z-x plane

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle} \qquad v_2 = \langle \cos 2\varphi \rangle, \quad \varphi = \tan^{-1}\left(\frac{p_y}{p_x}\right)$$

$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{dN}{p_t dp_t dy}\left(1 + \sum_{n=1} 2v_n \cos\left(n\left[\varphi - \Psi_{RP}\right]\right)\right) \qquad v_2 = \left\langle \cos\left(2\left[\varphi - \Psi_2\right]\right)\right\rangle$$



Initial particle momentum distribution isotropic : it develops anisotropy due to larger flow in x direction

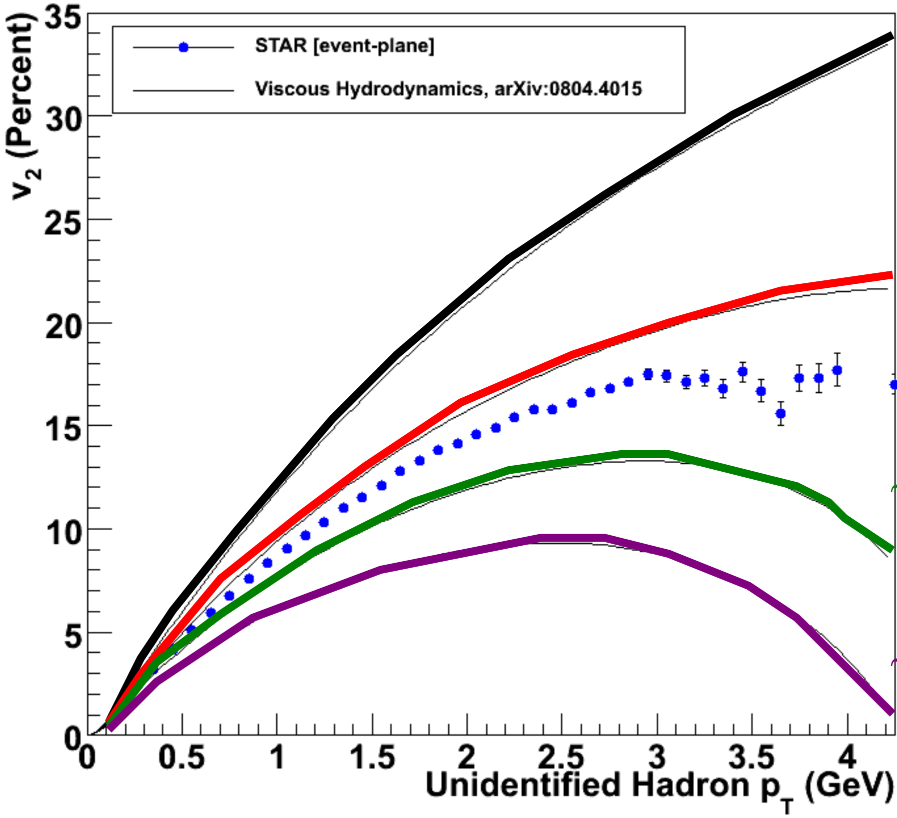
This momentum anisotropy is characterized by the 2nd Fourier coefficient V_2 (Elliptic flow)

Note: Elliptic flow strong evidence for thermalization. No other way To get anisotropic momentum distribution only from spatial Anisotropy.

Led to very important results:

Strong constraints on η/s : values determined to be in range 1 -3 times AdS/CFT bound. Lower than any known liquid.

Viscosity

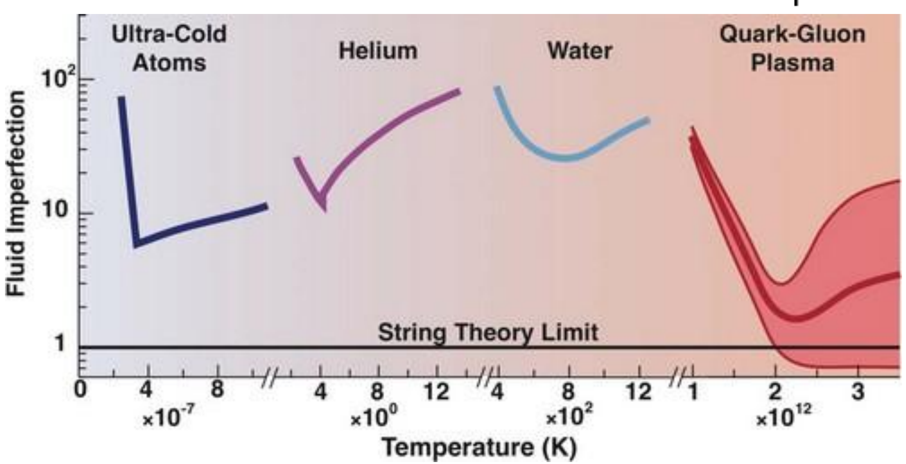


$\eta/s \sim 0$

$\eta/s = 1/4\pi$

$\eta/s = 2 \times 1/4\pi$

$\eta/s = 3 \times 1/4\pi$



Almost Perfect Fluid

Elliptic flow of thermal photons: Important probe as it contains
Information about all stages of system development.
Information about thermalization time: R. Chatterjee and D.K. Srivastava
Information about viscosity: : Shen, Heinz, et al.
(may be important for detection of superfluid phases: see later)

Elliptic flow: Important role played by initial state fluctuations:
Long realized that initial fluctuations affect determination of event
Plane and initial eccentricity.

Recent focus: Shape engineering of events: uses acoustic scaling
patterns of anisotropic flow for different event shapes: robust
control on initial state fluctuations

Earlier discussions mostly focused on a couple of Fourier
Coefficients, V_n with n = 2, 4, 6, 8 (Note: no odd harmonics)

Importantly: mostly discussion on average values of
(with the identification of the event plane).

Few discussions about fluctuations of V_n for n = 2,4,6

Power spectrum of flow fluctuations: New approach to flow analysis

Inhomogeneities of all scales present, even in central collisions:
Arising from initial state fluctuations

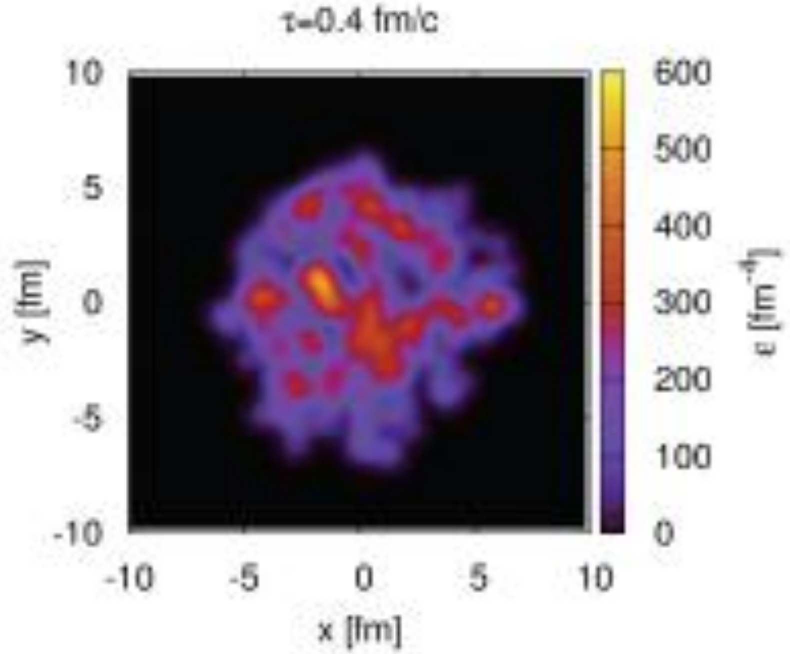
These anisotropies were known earlier, however, they were only discussed in the context of determination of the eccentricity for elliptic flow calculations.

We argued that due to these initial state fluctuations all flow coefficients will be non-zero in general:

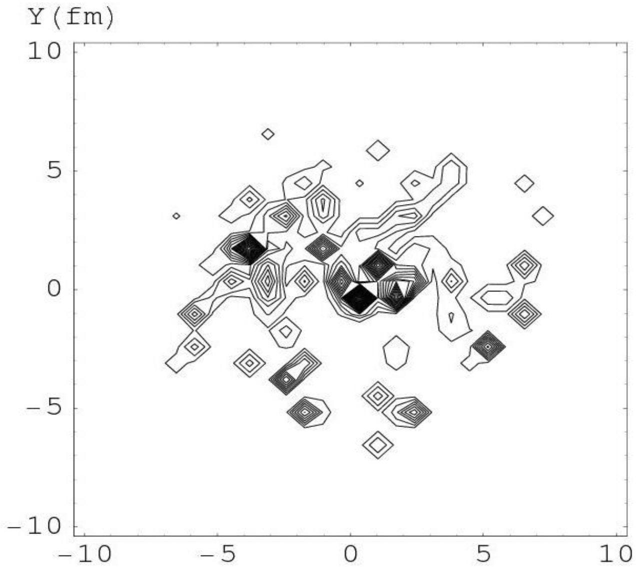
All Fourier coefficients V_n are of interest (say, n=1 to 30 -40, **including Odd harmonics,** these were never discussed earlier, For example, triangular flow is discussed now only as arising from Initial state fluctuations).

We emphasized: Learn from CMBR power spectrum analysis: Calculate root-mean-square values of V_n, and NOT their average values.

Initial state fluctuations



(Blaizot's talk, Matter at extreme conditions, 2014)



transverse energy density: Au-Au **central collision** at 200 GeV/A, HIJING

Thus: the equilibrated matter will also have azimuthal anisotropies (as well as radial fluctuations) of similar level.

We emphasized: Learn from CMBR power spectrum analysis: Calculate root-mean-square values of, and NOT their average values.

Important lesson for heavy-ion collisions from CMBR analysis

CMBR temperature anisotropies analyzed using Spherical Harmonics

$$\frac{\Delta T}{T}(\theta,\phi) = a_{lm} Y_{lm}(\theta,\varphi)$$

Now: Average values of these expansions coefficients are zero due to overall isotropy of the universe

$$< a_{lm} > = 0$$

However: their standard deviations are non-zero and contain crucial information.

$$C_l = < | a_{lm} |^2 >$$

This gives the celebrated Power Spectrum of CMBR anisotropies

Lesson : Apply same technique for RHICE also

For central events average values of flow coefficients will be zero

$$< V_n > = 0$$

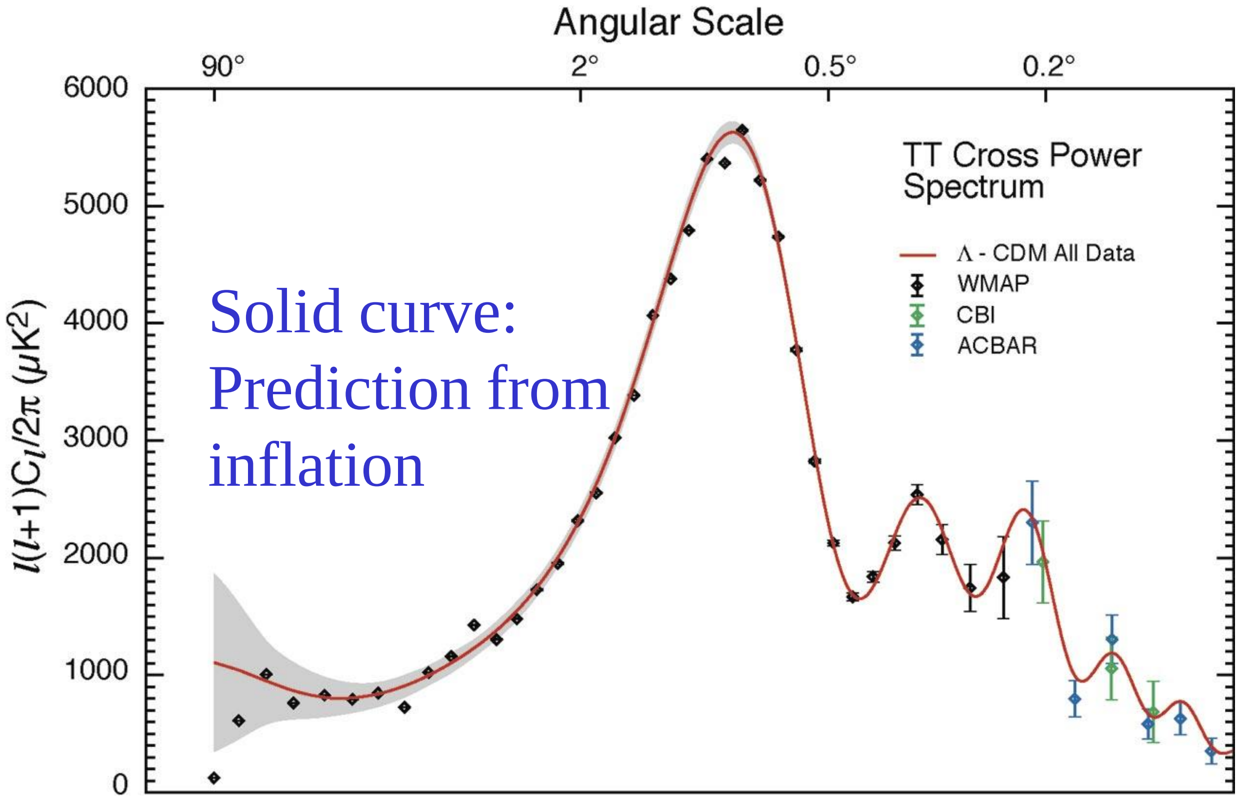
(same is true even for non-central events if a coordinate frame with fixed orientation in laboratory system is used).

Following CMBR analysis,we proposed to calculate root-mean-square values of these flow coefficients using a lab fixed coordinate system, And plot it for a large range of values of n = 1, 30-40

$$V_n^{rms} = \sqrt{< V_n^2 >}$$

These values will be generally non-zero for even very large n and will carry important information

Recall: Acoustic peaks in CMBR anisotropy power spectrum



Can such a power spectrum be plotted for heavy-ion collisions ?

So far we discussed: Plot of V_n^{rms} for large values of n will give important information about initial density fluctuations.

We now discuss: Such a plot may also reveal non-trivial structure like acoustic peaks for CMBR as above.

References :

The suggestion to plot this "power spectrum" for relativistic Heavy-ion collision experiments was first given by us (Mishra, Mohapatra, Saumia, AMS) in the following papers :

1) Super-horizon fluctuations and acoustic oscillations in relativistic heavy-ion collisions: PRC 77, 064902 (2008)

2) Using CMBR analysis tools for flow anisotropies in relativistic heavy-ion collisions: PRC 81, 034903 (2010)

It was emphasized in these papers that such a plot will have Valuable information about nature of fluctuations, especially initial fluctuations, and will be non-trivial for all n (up to about 30, **including odd n**).
This applies to all collisions, including central collisions.

An Important feature of flow power spectrum
Fluctuations with **superhorizon** wavelengths

Meaning of Horizon for the Universe:
Horizon size = speed of light c X age of the universe t
No physical effect possible for distances larger than this

In the universe, density fluctuations with wavelengths of superhorizon scale have their origin in the inflationary period.
Tiny fluctuations are stretched by superluminal expansion

Meaning of Horizon for Heavy-ion collisions:
System equilibrates in time τ_0 less than 1 fm/c. Horizon size = c τ_0
No physical effects possible for distances larger than c τ_0 = 1 fm.

Note: Fluctuations present of all wavelengths even at time τ_0
(arising from N-N collisions and fluctuations in nucleon positions).
All fluctuations larger than 1 fm are superhorizon at time τ_0.

At any later time τ, any fluctuation larger than cτ is superhorizon.

We argued that sub-horizon fluctuations in heavy-ion collisions should display oscillatory behavior just as fluctuations for CMBR

What about super-horizon fluctuations ?

Recall: For CMBR, the importance of horizon entering is for the growth of fluctuations due to gravity.

This leads to increase in the amplitude of density fluctuations, with subsequent oscillatory evolution, leaving the imprints of these important features in terms of acoustic peaks.

Superhorizon fluctuations for universe do not oscillate (are frozen, as we discussed earlier).

Importantly, they also do not grow,
That is: they are suppressed compared to the fluctuation which enters the horizon.

For heavy-ion collisions, there is a similar (though not the same, due to absence of gravity here) importance of horizon entering. One can argue that flow anisotropies for superhorizon fluctuations in heavy-ion collisions should be suppressed by a factor $\frac{H^s_{fr}}{\lambda / 2}$

where H^s_{fr} is the sound horizon at the freezeout time t_{fr} (~ 5-10 fm for heavy-ion collisions)

This is because here spatial variations of density are not directly detected, in contrast to the Universe where one directly detects the spatial density fluctuations in terms of angular variations of CMBR.

For heavy-ion collisions, spatial fluctuation of a given scale (i.e. a definite mode) has to convert to fluid momentum anisotropy of the corresponding angular scale. This will get imprinted on the final hadrons and will be experimentally measured.

This conversion of spatial anisotropy to Momentum ansitropy (via pressure gradients) is not effective for Superhorizon modes.

Thus: Superhorizon modes will be suppressed in heavy-ion collisions

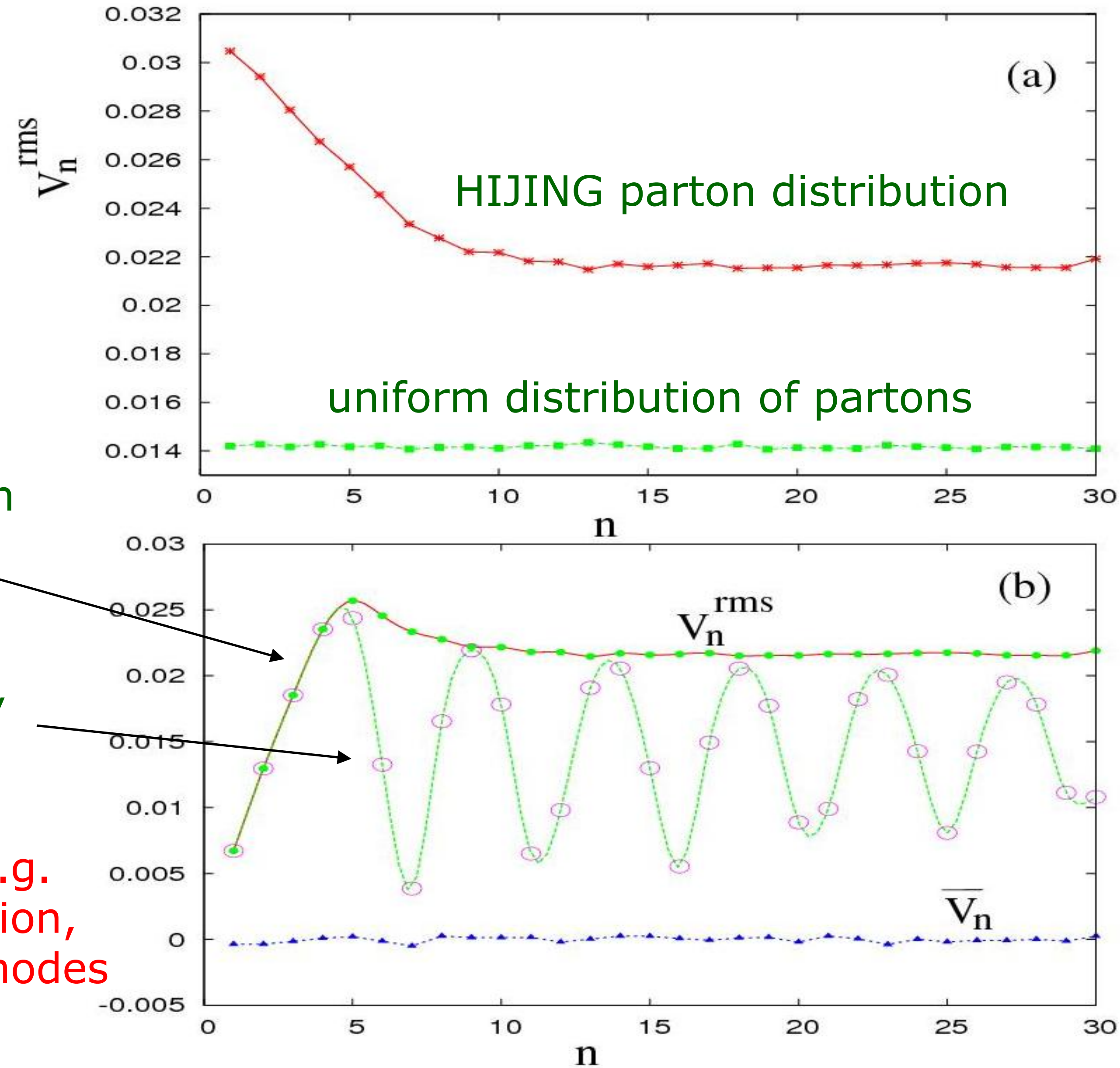
Results: (modeling only,   no hydrodynamical simulation here yet)

Errors less than ~ 2%

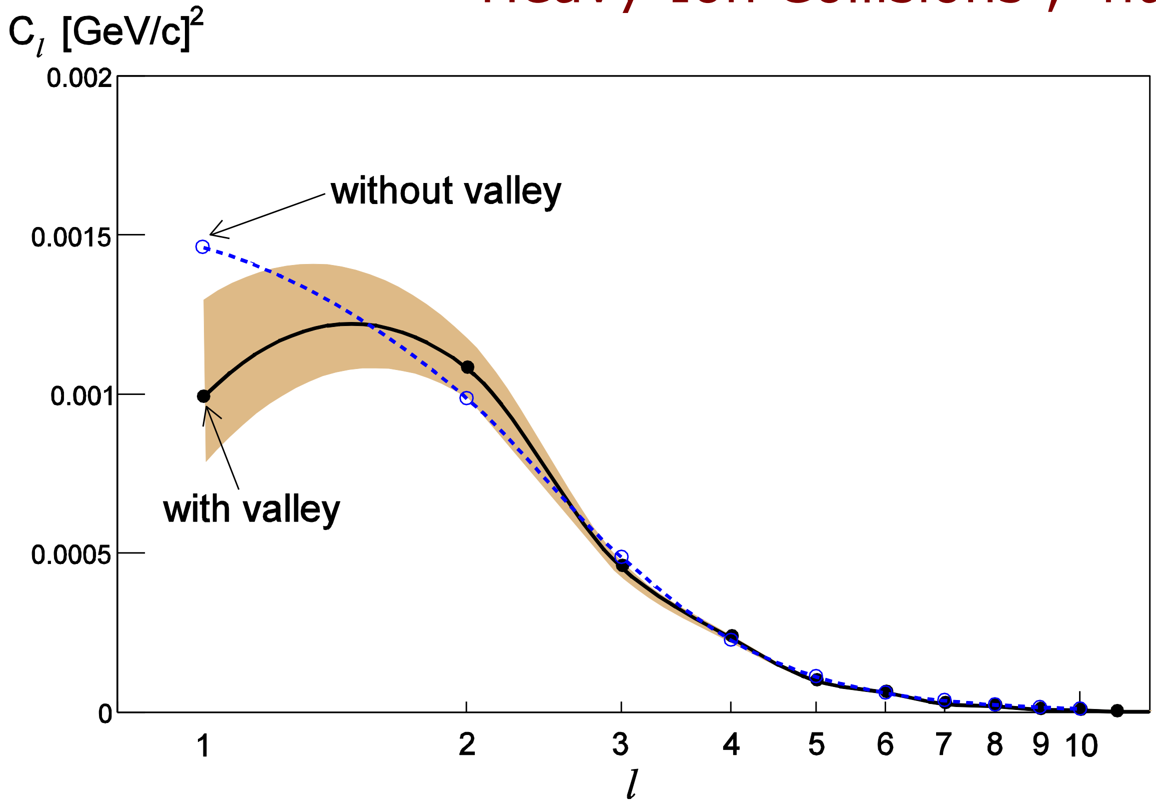
Include superhorizon suppression

Include oscillatory factor also

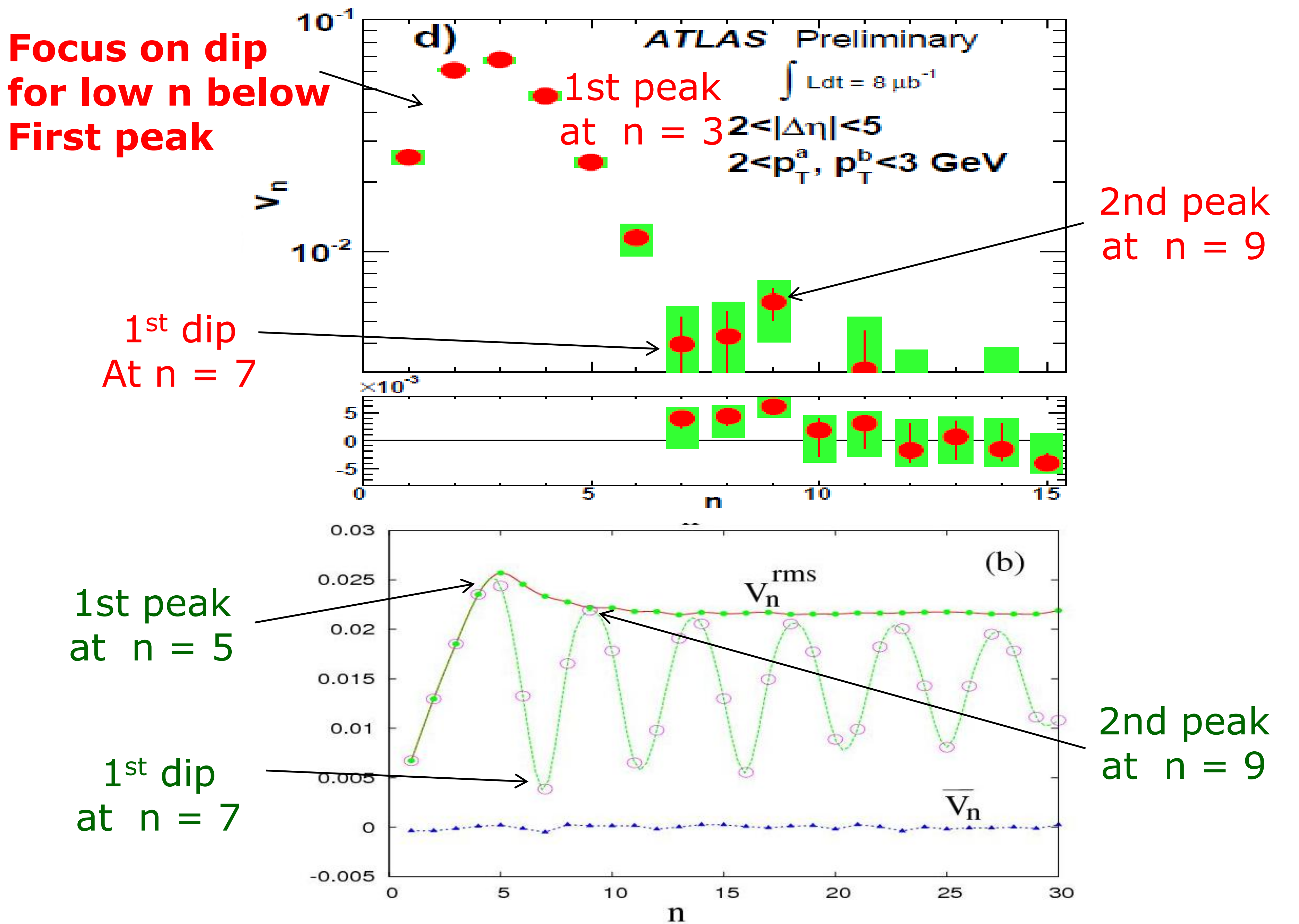
Note: Dissipation, e.g. from viscosity, diffusion, will damp higher n modes



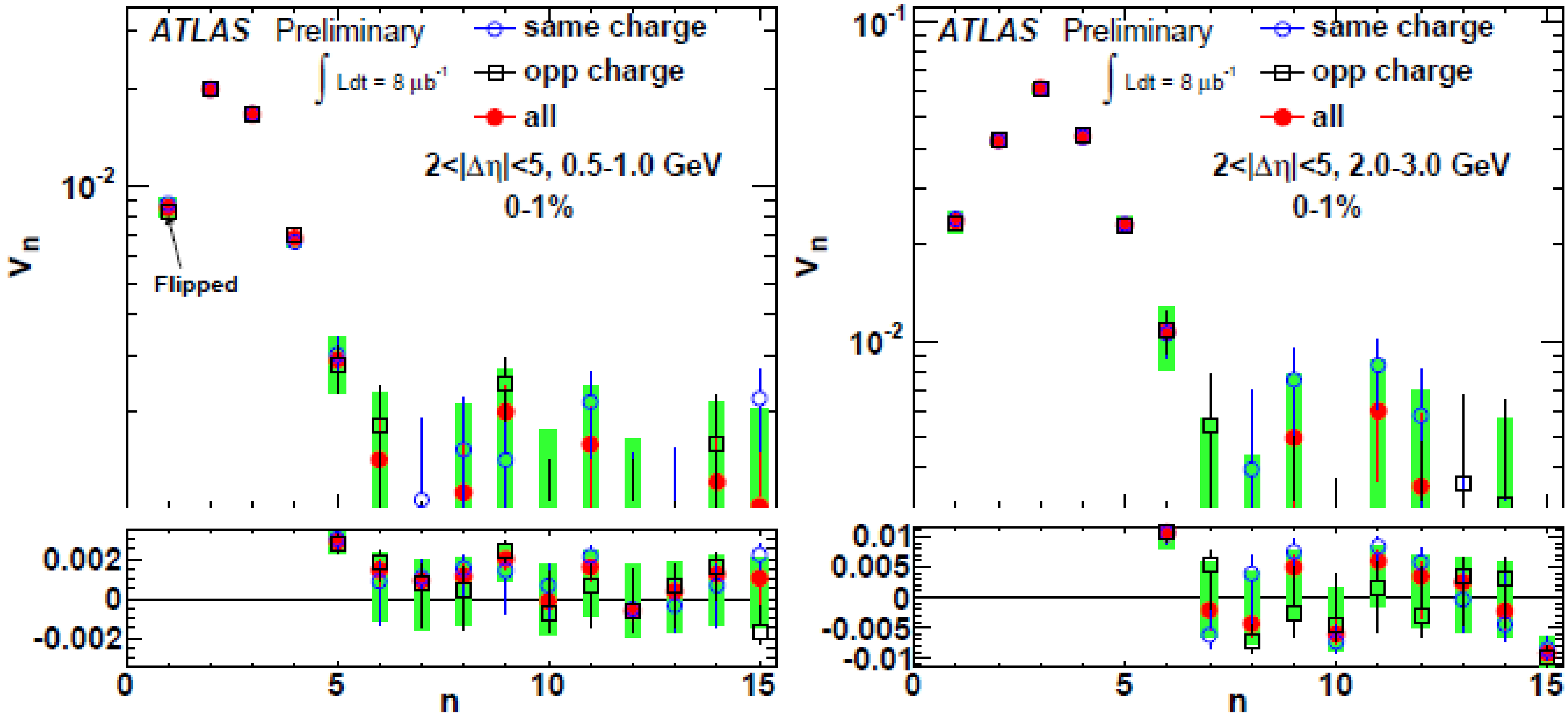
Paul Sorensen “Searching for Superhorizon Fluctuations in Heavy-Ion Collisions”, nucl-ex/0808.0503



See, also, youtube video by Sorensen from STAR:
http://www.youtube.com/watch?v=jF8QO3Cou-Q


Focus on dip for low n below First peak
d)
ATLAS Preliminary
∫ Ldt = 8 μb^-1
2<|Δη|<5
2<p_T^a, p_T^b<3 GeV
1st peak at n = 3
2nd peak at n = 9
1st dip At n = 7
v_n
n
(b)
V_n^rms
V̄_n
1st peak at n = 5
2nd peak at n = 9
1st dip at n = 7
n

arXiv: 1107.1468, LHC Pb-Pb at 2.76 TeV



Note: Very important to understand suppression of low n harmonics
It contains the information about freezeout horizon size

Relativistic Hydrodynamics Simulations:

(Saumia P.S., AMS, arXiv: 1512.02136)

We have developed two independent codes for hydrodynamic simulation for heavy-ion collisions. We compare the results of both Simulations and cross-check for consistency.

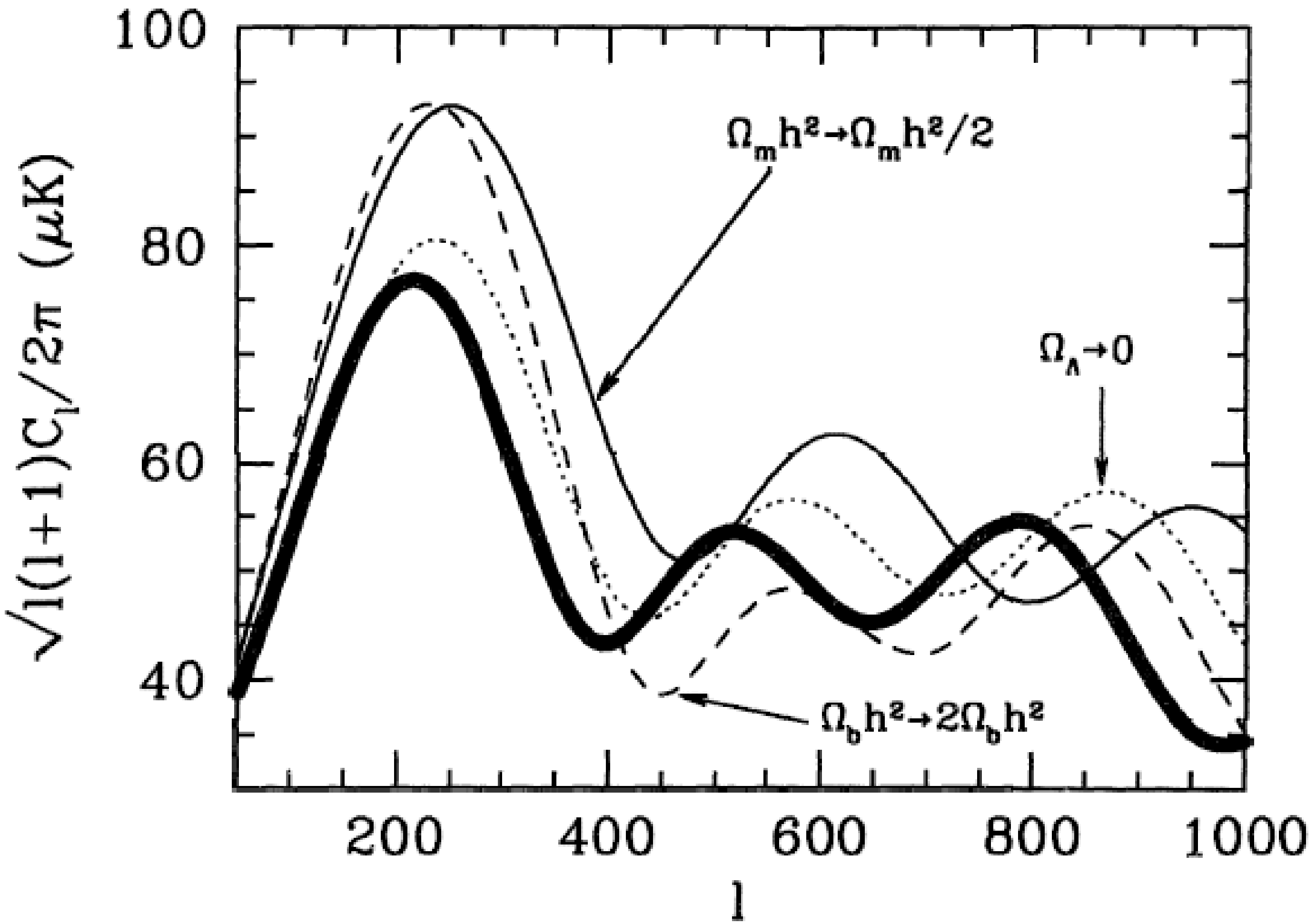
One simulation is 3-D simulation with random Gaussian fluctuations of specific width on a Woods-Saxon background corresponding to QGP with initial temperature of 500 MeV.
Power spectrum are obtained for the central rapidity region.

Second one is a 2-D simulation with Bjorken scaling model. Initial density profile is obtained from Glauber initial conditions for Pb-Pb collision at 200 GeV energy.

We study power spectrum of spatial anisotropies, momentum anisotropies, time evolution of power spectrum and the peak structure in the power spectrum.

Our results show startling correspondence with evolution of fluctuations in the universe and CMBR power spectrum, in complete agreement with our earlier predictions.

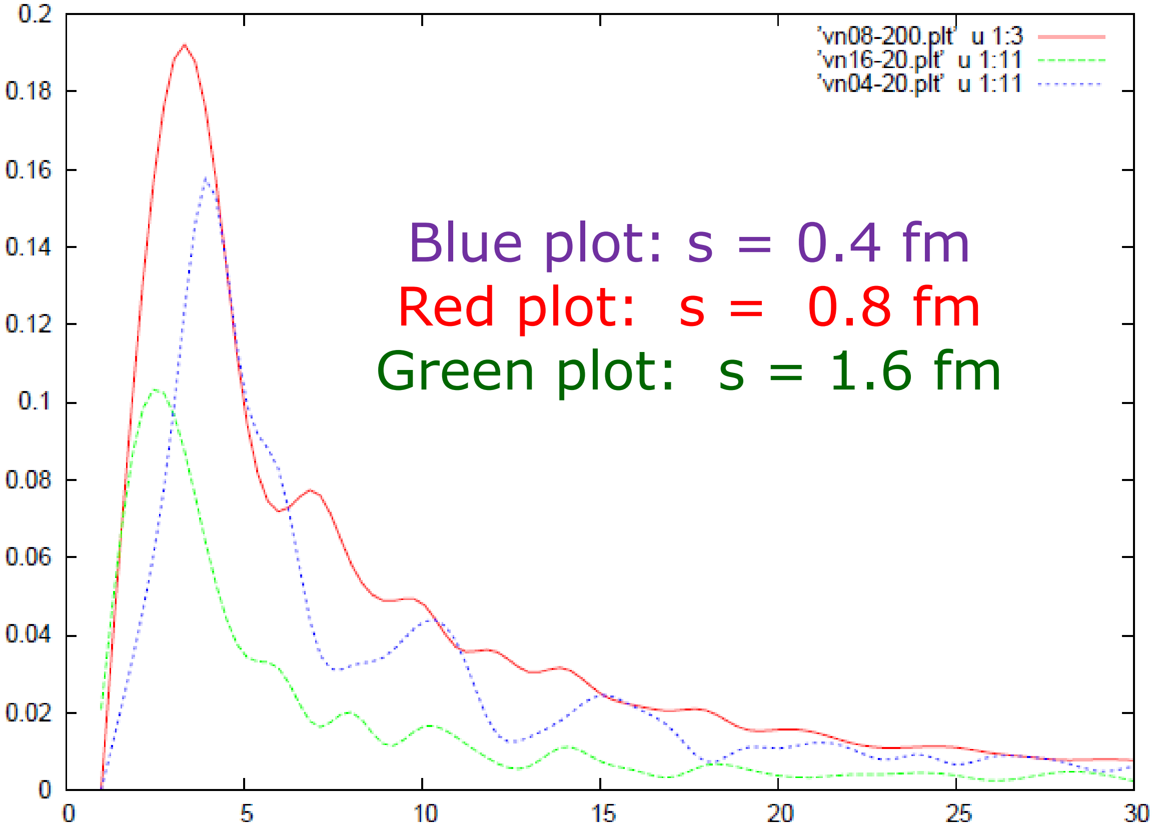
Changes in the location of peaks with energy-matter density of the Universe, (apparent horizon size changes)



Relativistic Hydrodynamics Simulations:

(Saumia P.S., AMS, arXiv: 1512.02136)

Plots of Vn(rms) vs. n for Gaussian fluctuations of width s

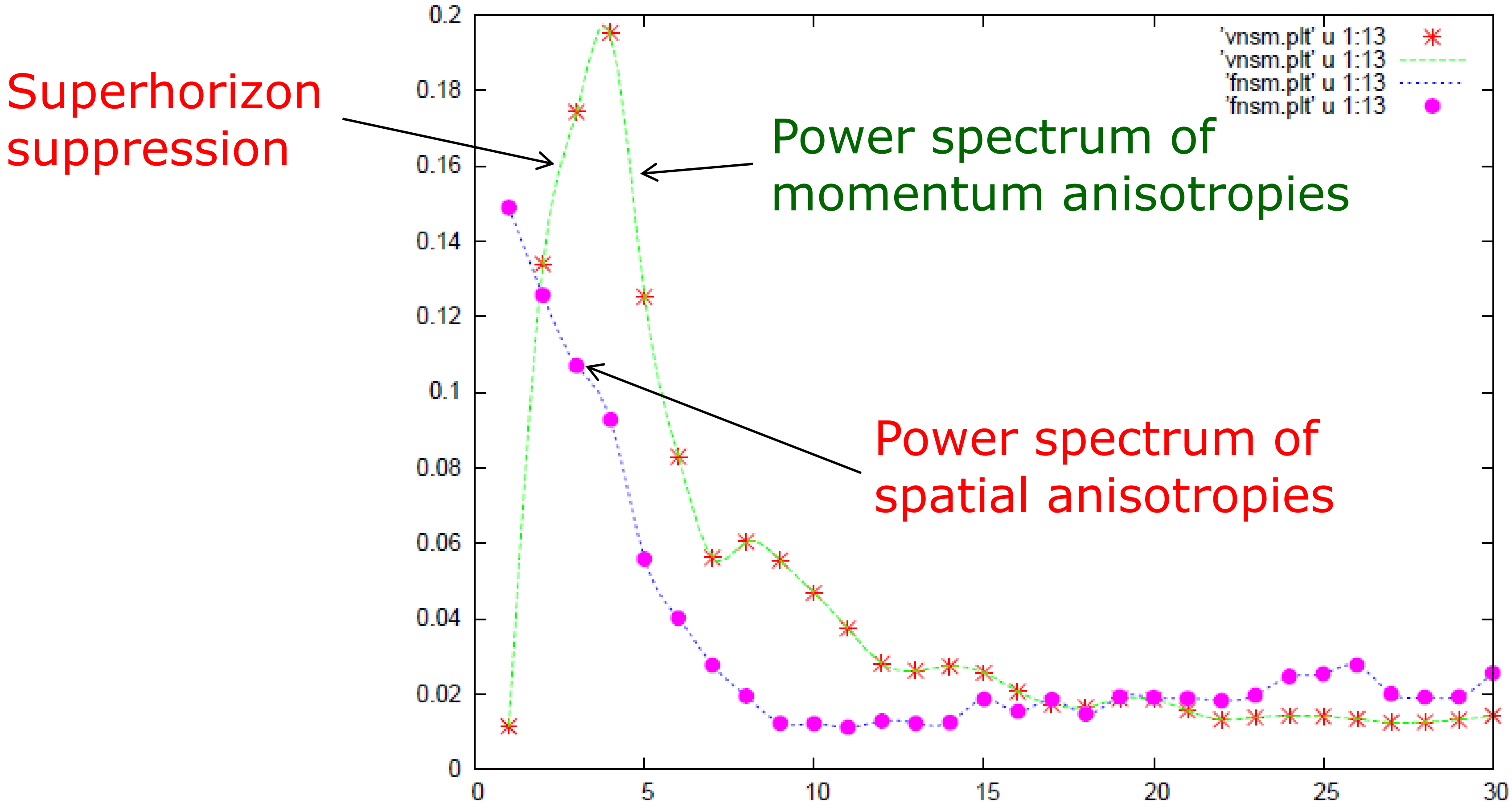


Note: Here peak position gives information about length scale of fluctuations

Just as for CMBR, where first peak location directly gives size of largest fluctuation at last scattering surface

Woods-Saxon density profile, 2 fm radius with 10 Gaussian fluctuations, To=500 MeV

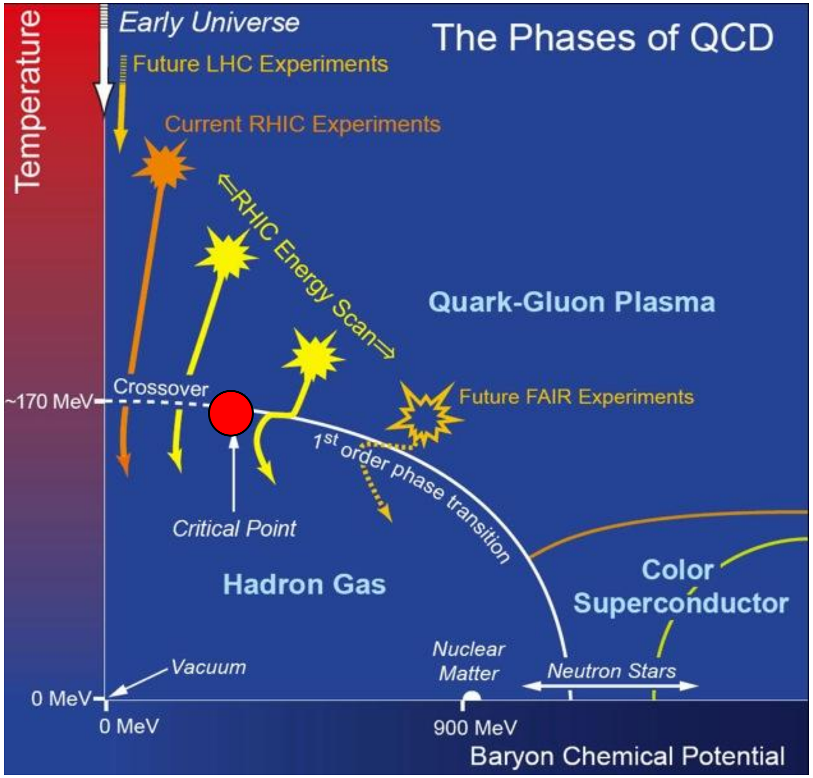
Evidence for Superhorizon suppression



Probing high baryon density phases through vorticity detection

(Arpan Das, Shreyansh Dave, Somnath De, AMS)

Recall: QCD
Phase diagram



High density transition to exotic phases: superfluidity: Vortices ?

Note: Possibility of Nucleon superfluidity for low energy collisions ?

Qualitative effects on the power spectrum, clean signal of vorticities.

Different phases at high baryon density:

At ultra-high baryon density – very large chemical potential.
asymptotic freedom makes Perturbative calculations reliable.

One-gluon exchange: quark-quark interaction attractive
in 3* channel

Cooper problem: Any attractive interaction destabilizes Fermi surface

BCS pairing of quarks in 3* channel: **Color Superconductivity**

Quark Cooper pair: $< q^{\alpha}_{ia} q^{\beta}_{jb} >$, α,β color, i,j, flavor, a,b spin

Consider most symmetric case: all three flavors masssless
(not unreasonable for very high chemical potential)

Color-Flavor Locked (CFL) phase:
color antisymmetric, spin antisymmetric, flavor antisymmetric:

pairing pattern:

$$< q_i^\alpha q_j^\beta > \sim \Delta_{CFL} (\delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta) = \Delta_{CFL} \varepsilon^{\alpha\beta n} \varepsilon_{ijn}$$

Note: This is invariant under equal and opposite rotations of color and (vector) flavor:

Leads to following spontaneous symmetry breaking

$$SU(3)_{color} \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{C+L+R} \times Z_2$$

Color SU(3) spontaneously broken

Other possible pairings: 2 light flavors: 2SC pairing
Breaks Color SU(3) to color SU(2) symmetry.

Topological properties of the CFL phase:

CFL spontaneous symmetry breaking pattern:

$$G \equiv SU(3)_{color} \times SU(3)_L \times SU(3)_R \times U(1)_B$$
$$\rightarrow SU(3)_{C+L+R} \times Z_2 \equiv H$$

Check: Homotopy groups of the vacuum manifold

$$\pi_0\left(\frac{G}{H}\right) = 1 \qquad \pi_2\left(\frac{G}{H}\right) = 1$$

So: No domain wall, or monopole defects

$$\pi_1\left(\frac{G}{H}\right) = Z$$

String defects: superfluid vortices

$$\pi_3\left(\frac{G}{H}\right) = Z \otimes Z$$

Doubling of colored Skyrmions (baryons)

Implications of the Topological Analysis:

Universality implies that:
Transition to CFL phase will invariably lead to formation of suerfluid vortices:

These will lead to strong fluid circulation right from the initial stage of transition:

It is obvious that resulting flow pattern will be dramatically different from the case without any vortex.

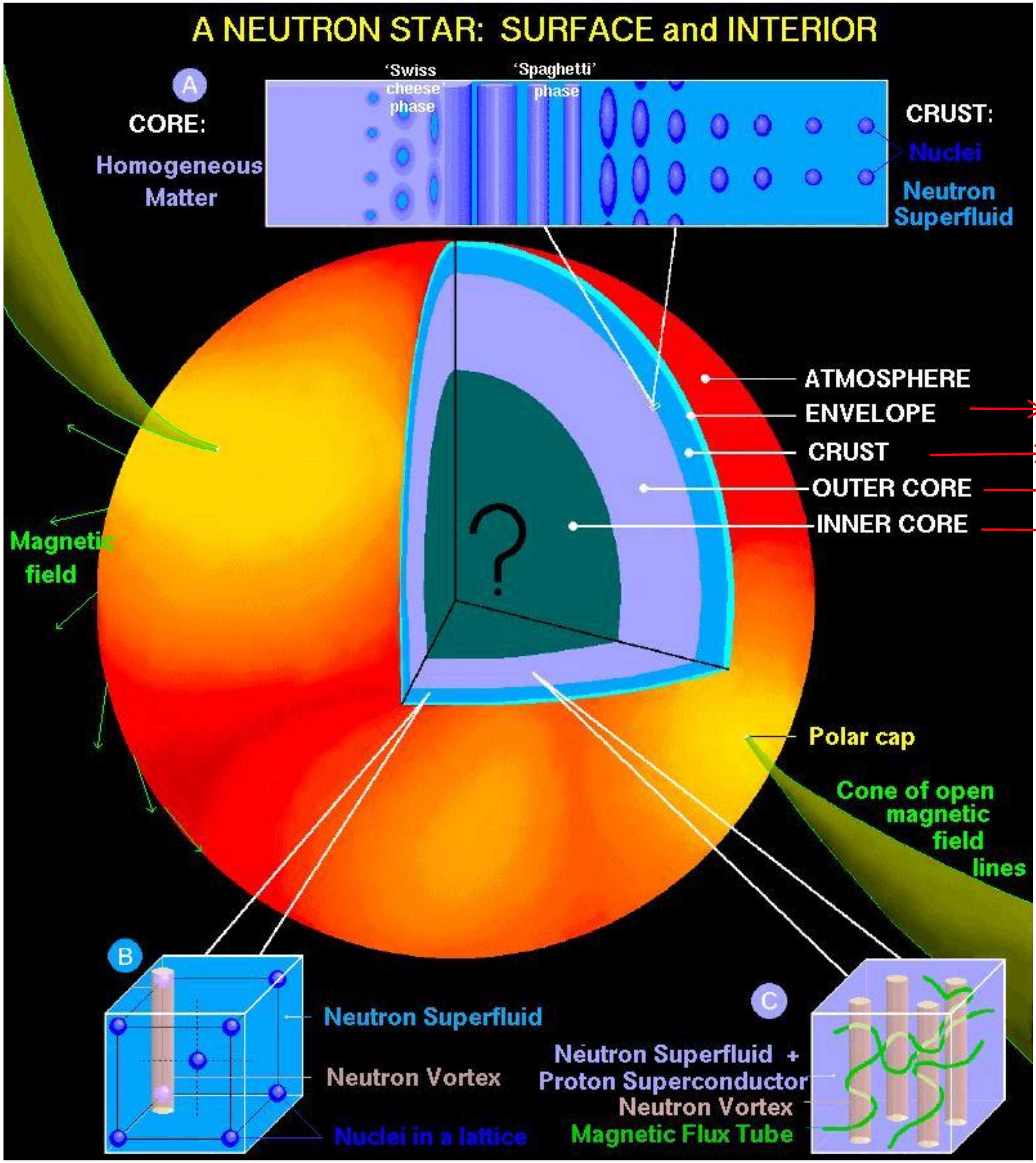
Thus: Qualitative changes in the flow pattern, specifically the flow power spectrum can give clean signal of transition to these Exotic phases of QCD.

Important to note: Nucleon superfluidity is believed to exist in the interiors of neutron stars.

Strong support from pulsar glitches

Inner Core:
Size ~ 1 Km
QCD phases:
QGP, CFL, etc.

(from Internet)



0.2 km
1 km
9 km
1 km

Outer Core:
Size ~ 9 Km
Superfluid
Phases of
Nucleons

Superfluid
Vortices play
Important role
in glitches

Inner Crust region:

Superfluid $^1S_0$ phase appears in the inner crust region:

Extending from $\rho \sim 4$ x 10^{11} g/cm^3 to 2 x 10^{14} g/cm^3

For the $^3P_2$ *channel,* the interaction becomes strongly attractive only at higher density, this occurs in the core of the neutron star.

This phase is quite like the superlfuid phase of ^{3}He

Allows for non-trivial order parameter, and hence associated topological defects.

Even though nucleon superfluid vortices play crucial role in understanding of pulsar glitches,

Only theoretically investigated so far

Thee have never been seen in any experiment.

Proposal: Very low energy collisions of neutron rich nuclei (possibly superheavy, near neutron dripline) may create bulk phase of nucleon superfluidity.

Possible at FAIR and NICA

Then our signals can establish nucleon superfluid vortices and investigate their properties under direct experimental control. Very important for understanding of neutron star properties.

UrQMD simulations: It may be possible to get temperatures of few MeV for nucleonic system for low energy collision (U-U) (less than 1 GeV lab energy). (Some fraction of particles freezeout at low temperatures, most freezeout at high temperature).

(thus a small part of fluid will be superfluid)

Smaller temperature will be better

Nucleon superfluid transition temperautre: 0.2 MeV to 5 Mev

Collisions of Au on Au with the FOPI-facility at GSI Darmstadt.
E/A 150 250 400 MeV/A (incident)
T 17.2 ± 3.4 26.2 ± 5.1 36.7 ± 7.5 MeV
(results from blast model fit)

Various multiplicities

	150 A MeV			250 A MeV			400 A MeV		
proton	26.1	±	1.4	31.9	±	1.6	38.7	±	2.0
deuteron	18.6	±	1.0	23.0	±	1.2	27.9	±	1.4
triton	17.2	±	0.9	21.0	±	1.1	25.5	±	1.3
^{3}He	5.7	±	0.3	9.1	±	0.5	10.6	±	0.6
^{4}He	21.0	±	1.1	18.2	±	1.0	13.6	±	0.7
neutron	92.6	±	10.8	97.9	±	8.2	101.7	±	6.2
charged p.	99.0	±	0.7	110.7	±	0.8	121.4	±	0.7
IMF	10.4	±	0.1	7.6	±	0.1	5.2	±	0.1

Freezeout density =
0.4 x nuclear saturation density
nucl-ex/9610009

Intermediate mass fragments z>2

Au-Au collisions at E/A = 50, 100, 150, and 200 MeV, at heavy-ion synchrotron SIS, T ~ 4-5 MeV is reached (nucl-ex/9801006)

Lower energy collisions or, a more dilute system quite likely to give lower T and correct ρ for superfluid transition

Properties of CFL vortices:

Correlation length:

$$\xi = 0.26\left(\frac{100MeV}{T_c}\right)\left(1-\frac{T}{T_c}\right)^{-1/2} fm$$

Velocity profile:

$$v(r) = v_0 \frac{r}{\xi} \quad \text{For } r < \xi$$

$$v(r) = v_0 \frac{\xi}{r} \quad \text{For } r > \xi$$

$$v_0 = \frac{1}{2\mu_q \xi}$$

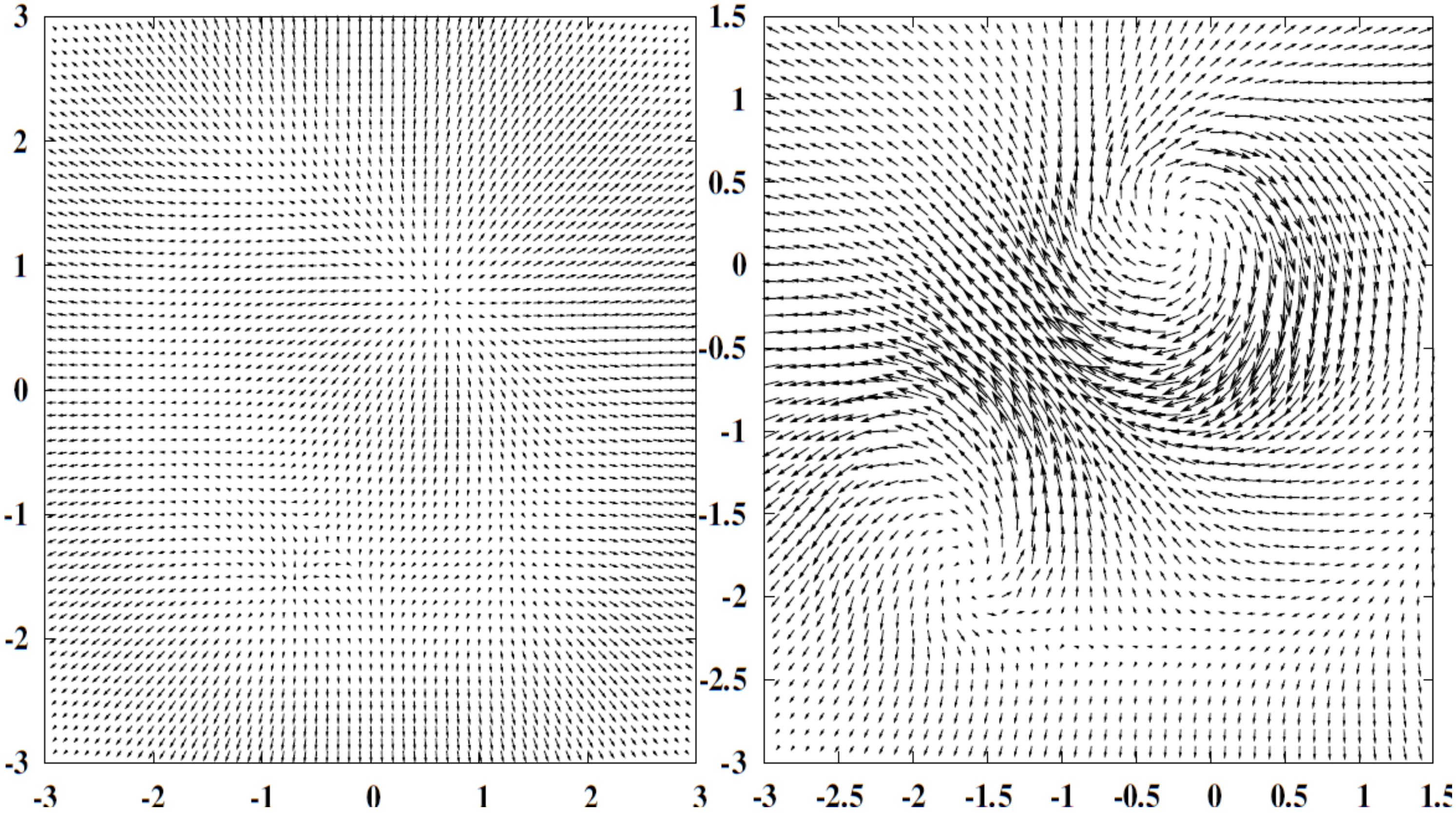
Results shown for T_c = 50 MeV, μ_q = 500 MeV, T = 25 MeV

Resulting $\xi = 0.7$ fm, $v_0 = 0.3$

Same velocity profile holds for nucleon superfluid vortices,
For that case, T_c = 0.3-5 MeV, $\xi \sim 1$ fm

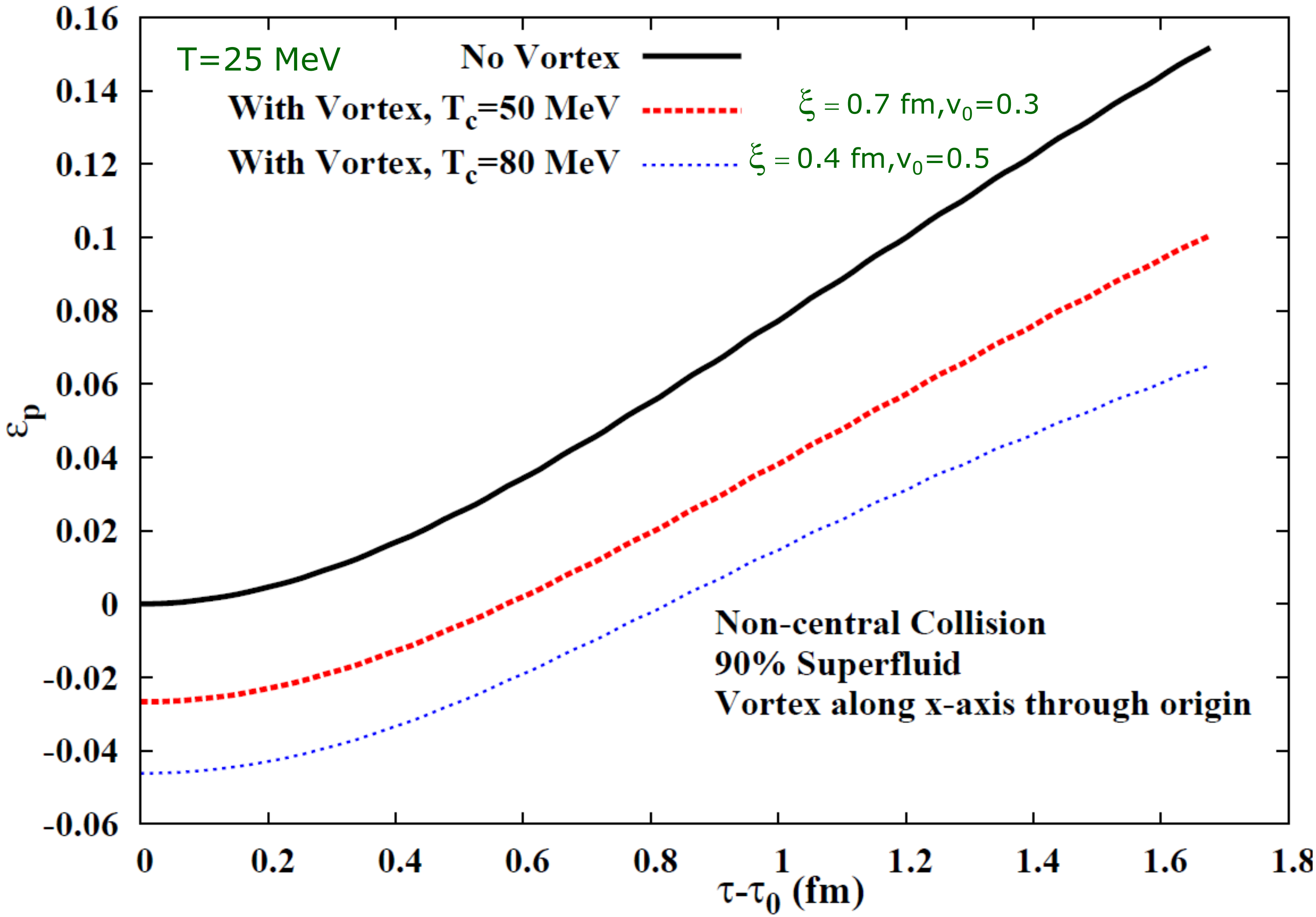
Results of relativistic hydrodynamics simulations with two-fluid picture of superfluid (normal component and superfluid component)

Qualitatively different features of flow pattern and power spectrum Signalling superfluid transition

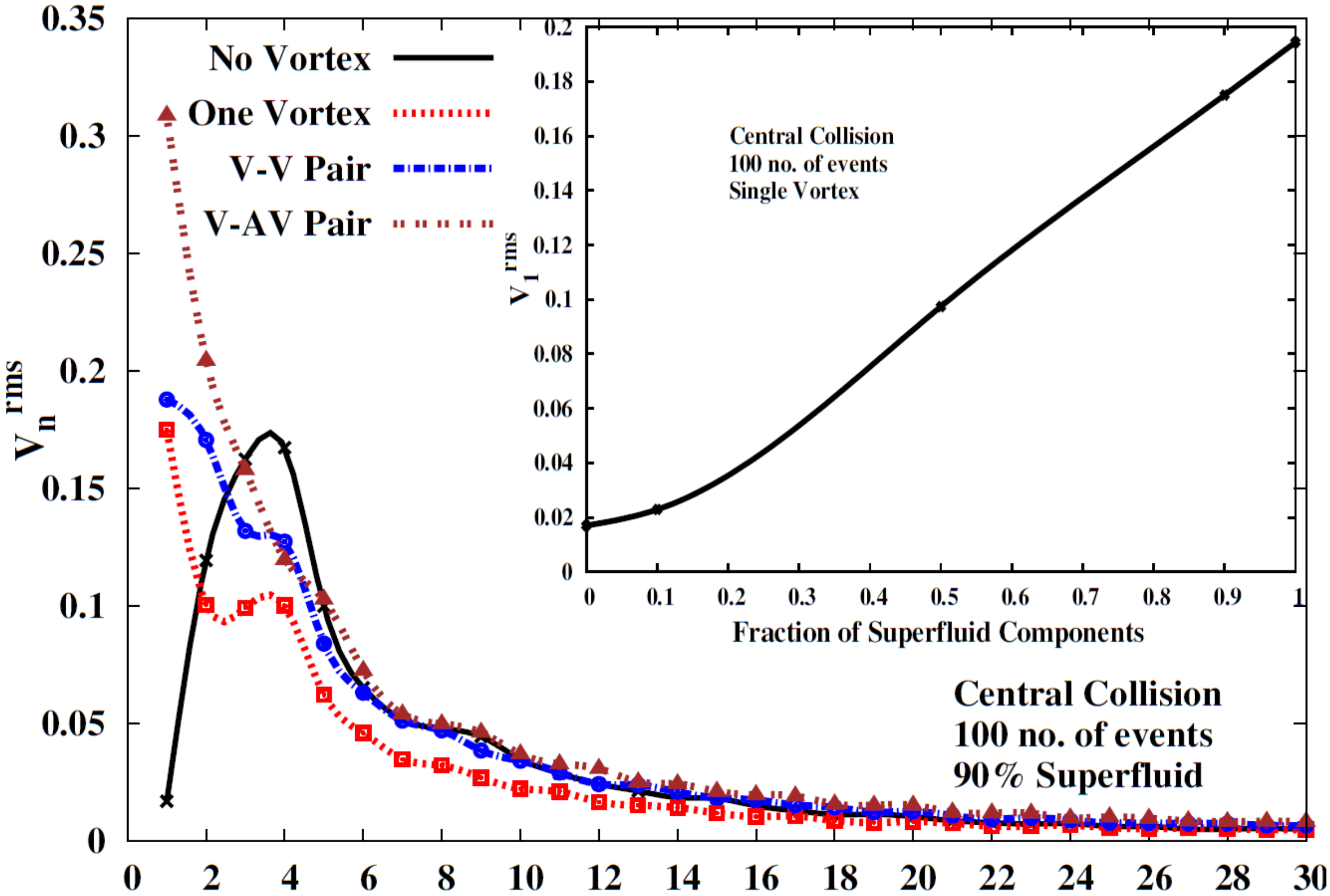


Velocity profile at central rapidity without (left) and With (right) vortices: $\tau-\tau_0$ = 0.84 fm/c

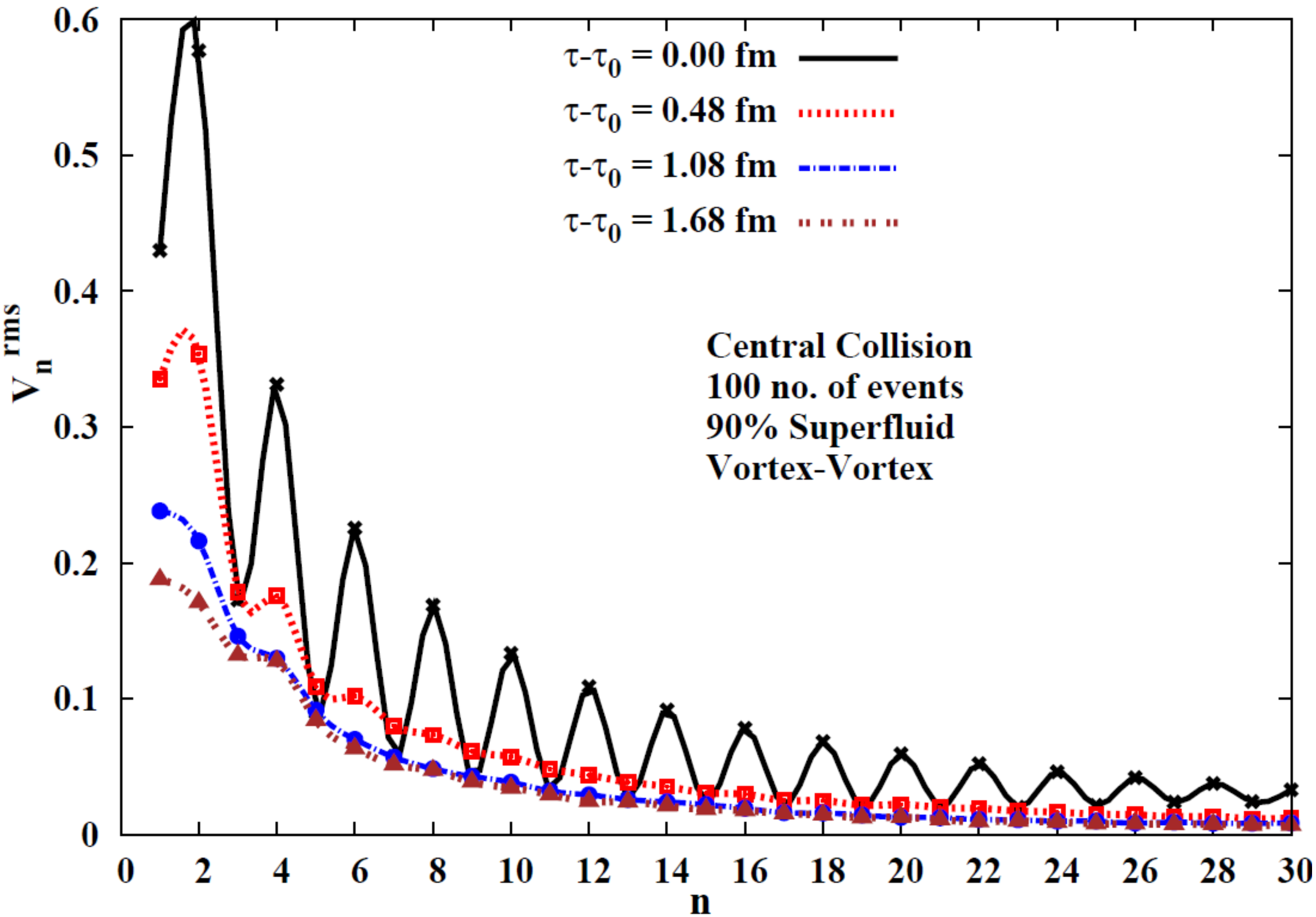
Strong negative elliptic flow due to vortices



Strong directed flow in the presence of vortices



In certain cases: different powers in even-odd harmonics in
The presence of vortices during initial stages: Could be probed by
Thermal photon elliptic flow, or peripheral collisions



Relativistic Magneto-Hydrodynamics: present work

(Arpan das, Shreyansh Dave, Saumia P.S. AMS)

Presence of **magnetic fields** in QGP strongly affects its evolution
Due to Complex dependence of magnetosonic waves on magnetic field and on density gradients

We study such effects in relativistic heavy-ion collisions in view of the fact that in non-central collisions there is a large magnetic field of the order of $B = 10^{15}$ Tesla present

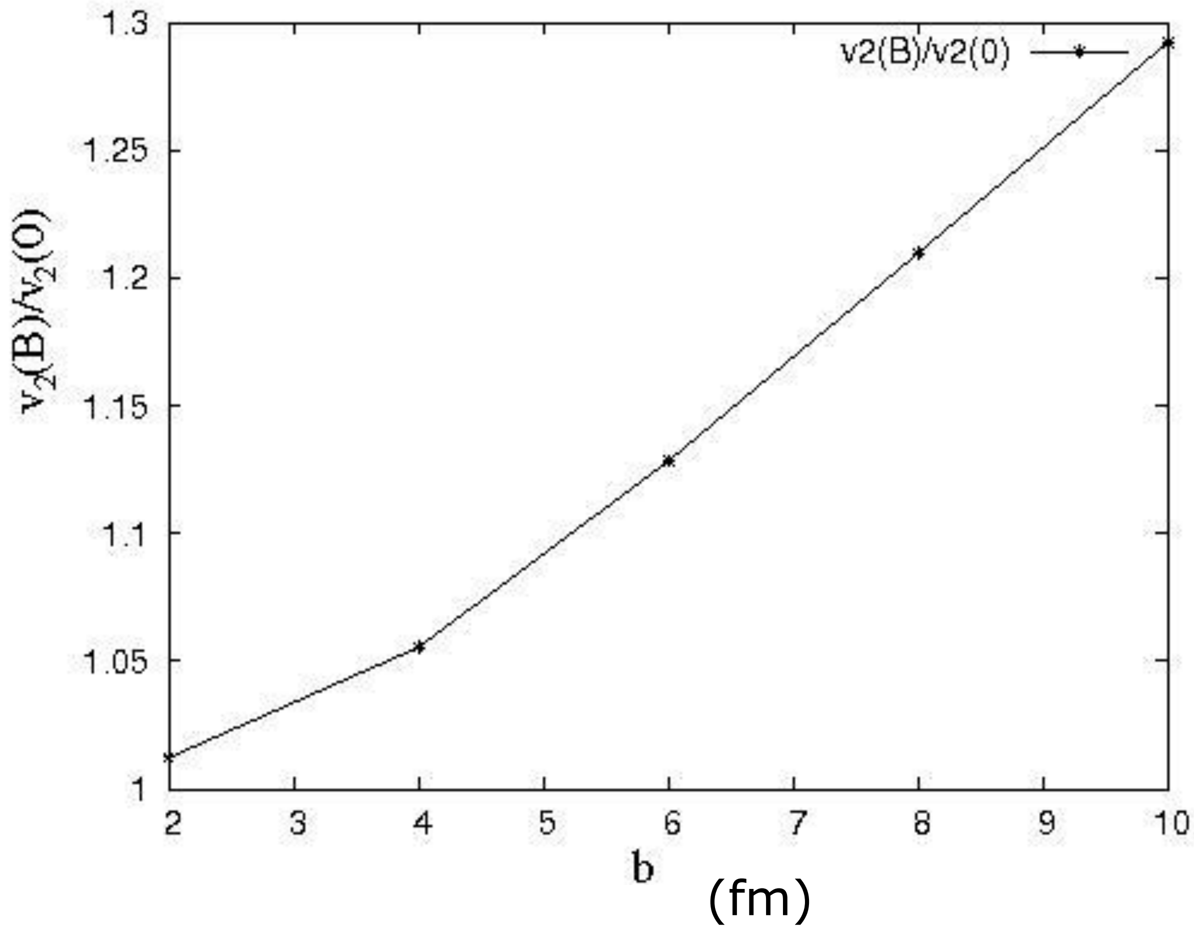
Note: due to induced currents the time scale for this can be several fm. (Tuchin)

We show that magnetic field can enhance flow anisotropies, e.g. elliptic flow can be increased by about 30 %

Can this accommodate larger values of viscosity ?

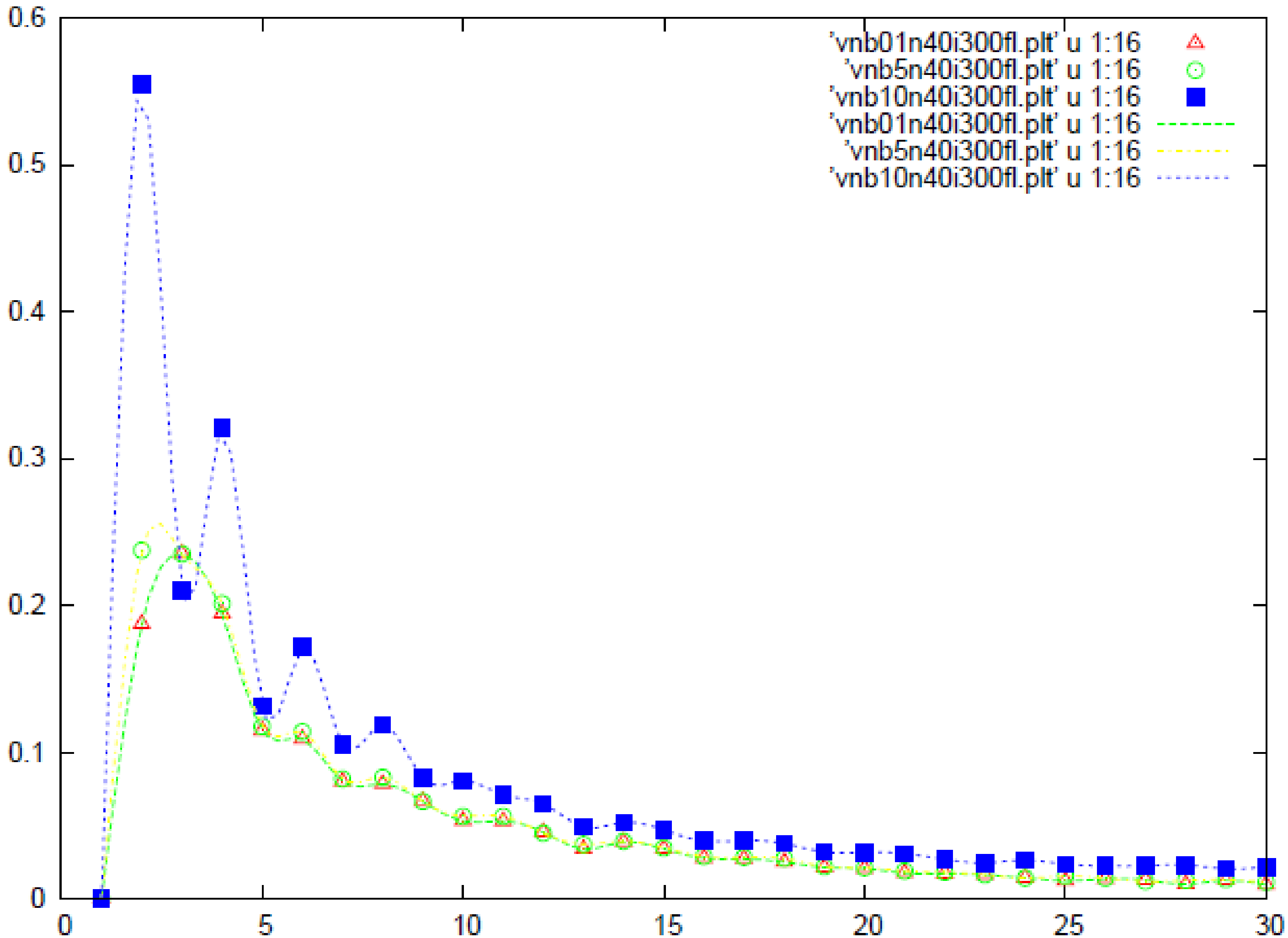
Enhancement of elliptic flow v_2 with magnetic field (up to 30 %)

R.K.Mohapatra, P.S. Saumia, AMS, MPLA26, 2477 (2011).

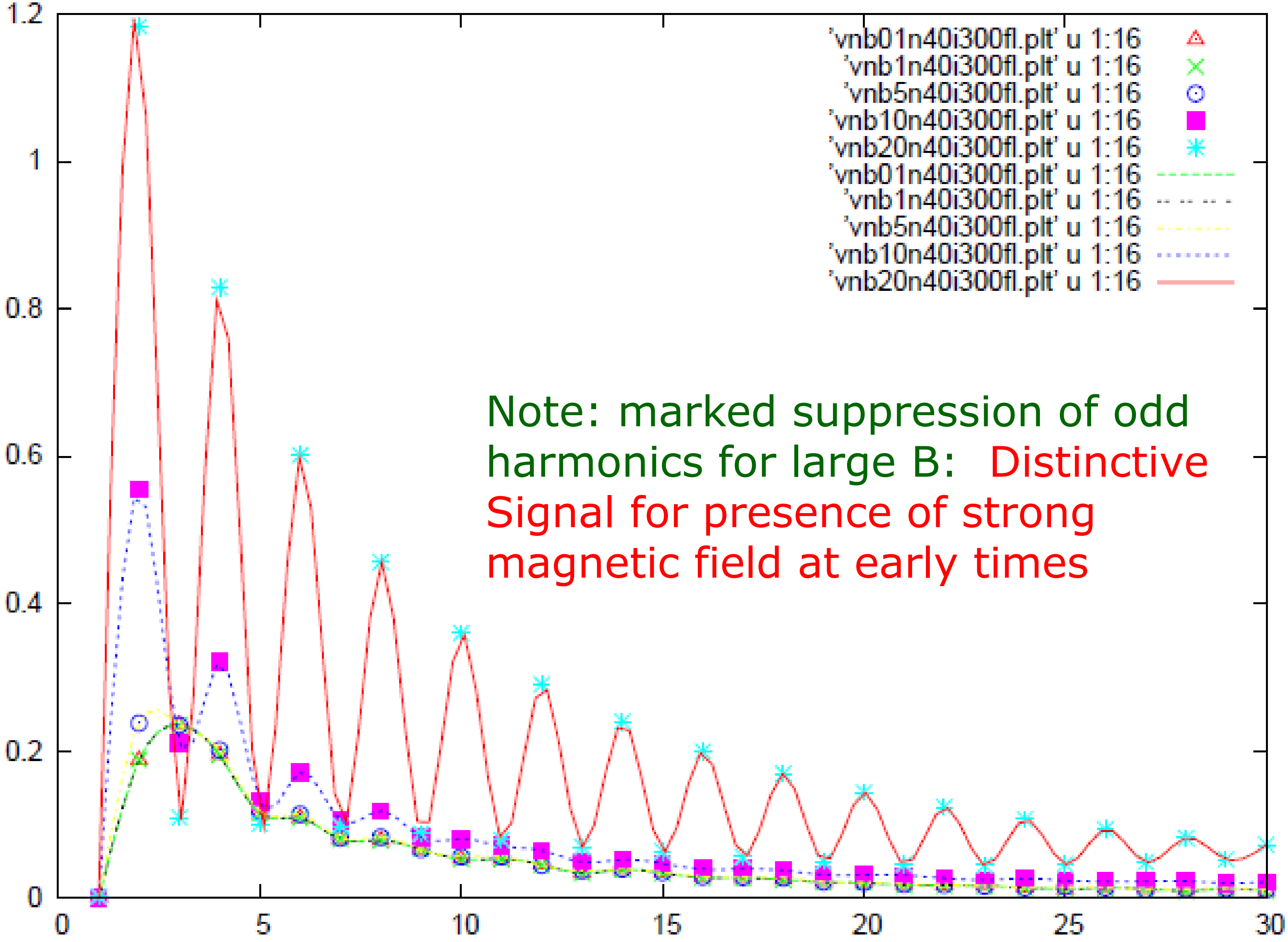


Subsequent analysis by Tuchin: Results in agreement with this
K. Tuchin, J.Phys. G 39, 025010 (2012)

By = 0.01,5,10 $m\pi^2$, t = 1.3 fm, with fluctuations
Plots of V_n(rms) vs. n - Full Power spectrum



By = 0.01,5,10,20 mπ^2, t = 1.3 fm, with fluctuations
Plots of V_n(rms) vs. n - Full Power spectrum



Concluding remarks: Flow pattern and the power spectrum of flow fluctuations an important tool for heavy-ion collisions

1) Most important feature in the power spectrum: Bending down of curve for low n values: Signal of superhorizon suppression

2) Detailed analysis of the peak structure, time evolution of power spectrum and development of acoustic oscillations shows very close correspondence with CMBR power spectrum. Just like CMBR, can provide crucial importance about initial state fluctuations and medium properties

3) Qualitative changes in power spectrum in presence of superfluid vortices: clean signal for CFL, nucleon superfuid phases Important for understanding pulsar dynamics

4) Magnetic field can strongly affect plasma evolution. (Qualitative signals like suppression of odd harmonics, may signal initial strong magnetic field).

Magnetohydrodynamics simulations needed for proper understanding of flow and its fluctuations

Thank You