

Slow extraction notes

Re: Observations and their rough comparison

Take the slow spill record $\Phi(t)$.

Apply to the identity

$$\frac{1}{1 - i \omega / \Delta \omega} + \frac{-i \omega / \Delta \omega}{1 - i \omega / \Delta \omega} = 1.$$

It is a sum of 1st order low-pass and high-pass filters. These are plain integrating and differentiating circuits which are:

1. complementary (no loss of acquired data in parallel processing), and
2. realizable (either analogue circuit or time-domain DSP, as required).

Transfer of a parent signal $\Phi(t)$ through the LPF produces a DC (intensity) spill signal $\Phi_{DC}(t)$.

Transfer of a parent signal $\Phi(t)$ through HPF produces an AC (ripple) spill signal $\Phi_{AC}(t)$. By definition, time average $\langle \Phi_{AC}(t) \rangle = 0$.

The AC content may also be retained, as expected, through subtraction (see the above identity)

$$\Phi_{AC} = \Phi - \Phi_{DC}.$$

Still, use of LPF and HPF offers a tool for online stream measurement of $\Phi_{DC}(t)$ and $\Phi_{AC}(t)$.

The “-3 dB” separation roll-off frequency $\Delta \omega = 2\pi/\Delta t$ (here Δt is time boundary between DC and AC contents of the spill) is a subject to convention that complies with the demand of a particular beam user (experiment).

Signal $\Phi_{DC}(t)$ is a subject to discussion in the context of intensity control, feedback circuits, feedforward schemes, etc. It is not necessarily the square-shaped one and may be stepwise, natural (without any intensity control) etc.

Signal $\Phi_{AC}(t)$ represents ripple behavior of spills. Further discussion aims at this signal and its interpretation.

It makes sense to apply to reduced (and centered) spill ripple signal $\varphi(t)$, $\langle \varphi(t) \rangle \approx 0$

$$\varphi(t) = \frac{\Phi_{AC}(t)}{\Phi_{DC}(t)}.$$

Only few numerical parameters are adequate for the rough analysis, 2 extreme peak-to-average bursts and 2 lowest moments of distribution:

$$\varphi_m = \min(\varphi(t)) \geq -1 \quad (\text{"shut-off"} = -1),$$

$$\varphi_M = \max(\varphi(t)),$$

$$\sigma^2 = \langle \varphi^2(t) \rangle > 0, \quad (\text{"power"})$$

$$\sigma^3 = \langle \varphi^3(t) \rangle. \quad (\text{"skewness"})$$

The ultimate goal (perfect spill) is to obtain vanishing values everywhere. Observation over entire spill duration resembles ensemble averaging of a random process under ergodic hypothesis. It improves statistical quality of data-processing outcome proceeding from a particular long spill sample.

For a stochastic extraction, one observes $|\varphi_m| \neq |\varphi_M|$ and $\sigma^3 > 0$ (excess in positive bursts).

The conventional mean-square duty factor is

$$\frac{\langle \Phi \rangle^2}{\langle \Phi^2 \rangle} = \frac{1}{1 + \sigma^2}.$$

Out- and in-bursts $\varphi_{M,m}$ bear a surprisingly representative insight into extraction. Indeed, the 2-brach plots of $1 + \varphi_{M,m}$ are plotted in Figure to follow.

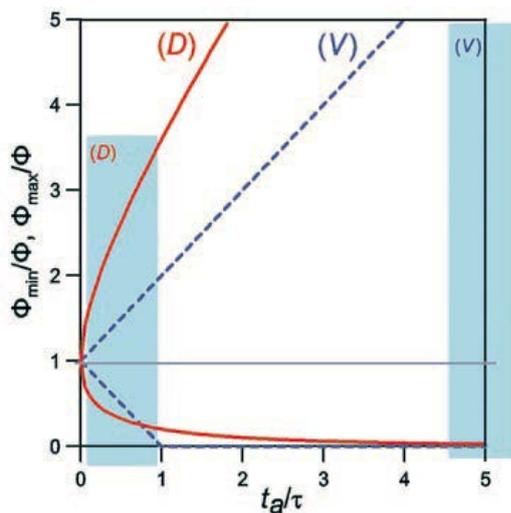


Figure 1.

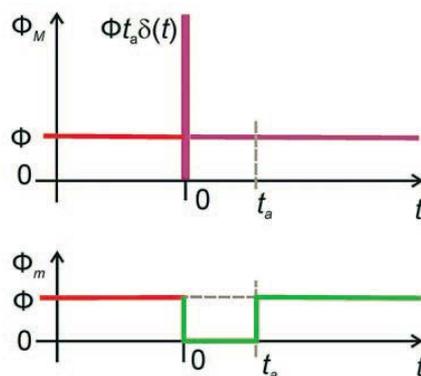
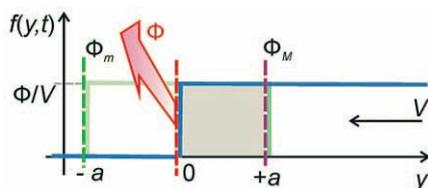
These are substantially different for conventional (translational, with a velocity V) and stochastic (with a diffusion coefficient D) feeding the extraction resonance. The underlying formulae and their derivation are given in and before Eqs. (35) and (36) of Ref.[1].

In short, it is assumed that there occurs the worst-case event – a sudden jump (without subsequent correction) in beam-edge-to-resonance distance by $\pm a$. Waiting beam stack size is A in the same units as a . The prescribed duration of spill is t_A . There is a typical relaxation time t_a required to cure the near-to-edge perturbation of magnitude a in beam profile:

Conventional feeding with V

$$t_a = \frac{a}{V} = \frac{A}{V} \left(\frac{a}{A} \right) = t_A \left(\frac{a}{A} \right).$$

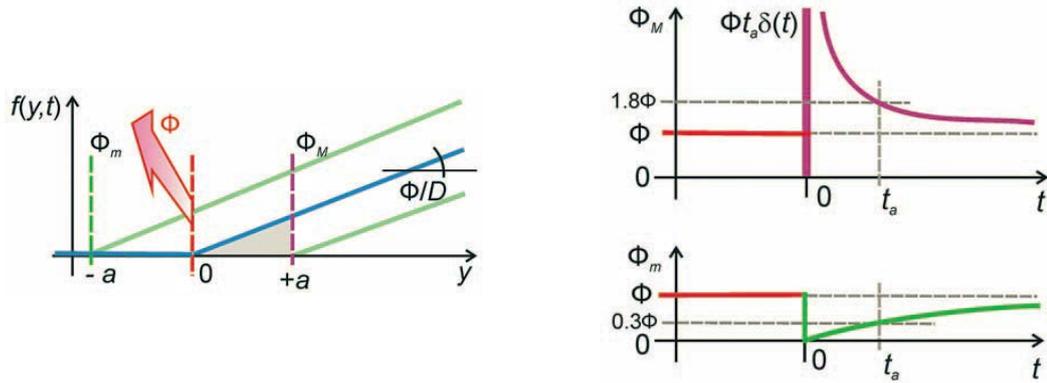
The out-burst (delta-function) is caused by the scraping effect.



Diffusive feeding with D

$$t_a = \frac{a^2}{2D} = \frac{A^2}{2D} \left(\frac{a}{A}\right)^2 \cong t_A \left(\frac{a}{A}\right)^2.$$

The out-burst is caused by the scraping effect and a wake of enhanced diffusion from a newly acquired abrupt beam edge. The in-burst is smoothly smeared by diffusion into an emptied layer.



On averaging the right-hand-side patterns containing delta-spikes with a time constant τ (running integrating window width), one gets the plots of Figure 1.

There, the return point at zero abscissa represents the ultimate perfect spill. By the way, it can be approached by (cheating) over-averaging the observations ($\tau \rightarrow \infty$) with measuring tools. Stochastic spills are inherently more immune to flux cut-offs.

Other conditions being equal, relaxation times for diffusion are by a (normally small) factor a/A shorter than those for a translational feeding.

Preference for shorter ratios t_a/τ is obvious from Figure 1 and quantifies all the common cures (mentioned at the WS by many reporters) to lower ripples:

1. to suppress pulsations in optics a (normalized to units A of waiting beam stack width),
2. to enlarge this width A with RF gymnastics or noise,
3. to relax on beam-user time-resolution demands τ ,
4. to shorten spill duration t_a ,
5. and, to the final end, to go from conventional to stochastic feeding.

Figure 1 also emphasizes the need to better quantify the unavoidable contribution to τ due to nonlinear motion of extracted fraction in the outskirts of

resonance separatrix triangle. It is the inverse of “inherent low-pass bandwidth” of beam transfer function through the resonance.

Table to follow shows numerical example for slow extraction from the U-70, protons.

Table. Slow extractions in the U-70

Feeding	t_A, sec	$a= \delta Q $	A	τ, sec	t_a/τ	φ_m	φ_M
V	2	$6 \cdot 10^{-4}$	$3 \cdot 10^{-2}$	$4 \cdot 10^{-3}$	10	-1	10
D	2	$6 \cdot 10^{-4}$	$3 \cdot 10^{-2}$	$4 \cdot 10^{-3}$	0.2	-0.5	0.9

As the first step for sound decision making concerning FAIR slow extractions it makes sense to indicate the other machines' experience on the “ φ versus t_a/τ ” plot.

Moments σ^2 and σ^3 depend on more subtle details of slow extraction process involved.

References

- [1]
 Stochastic Slow Extraction from the U-70. News and Problems of Fundamental Physics (in Russian). Vol. 2(6), 2009, p. 5- 25. # ISSN: 1999-2858.
<http://exwww.ihep.su/ihep/journal/IHEP-2-2009.pdf>