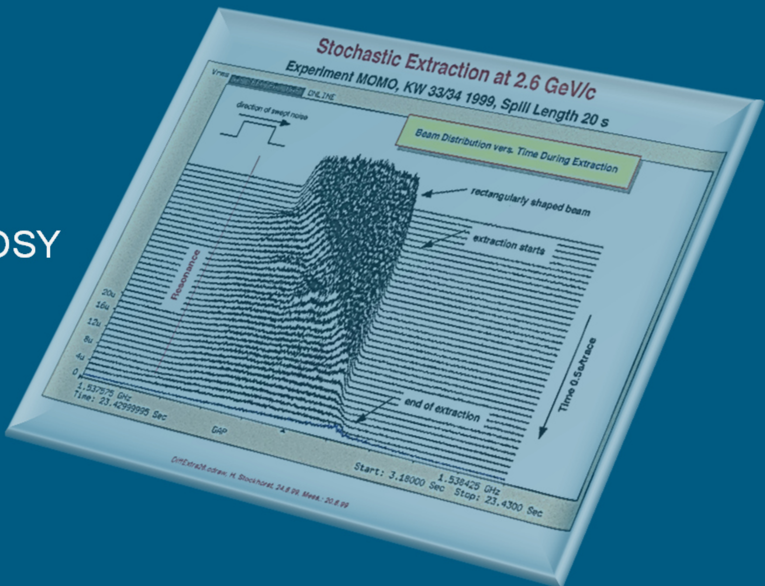
Mitglied der Helmholtz-Gemeinschaft

The procedure of beam extraction with longitudinal feeding at COSY-Juelich



H. Stockhorst for the COSY-team
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The Slow Extraction Workshop

June 01 – 04, 2016, GSI Darmstadt, Germany

Content

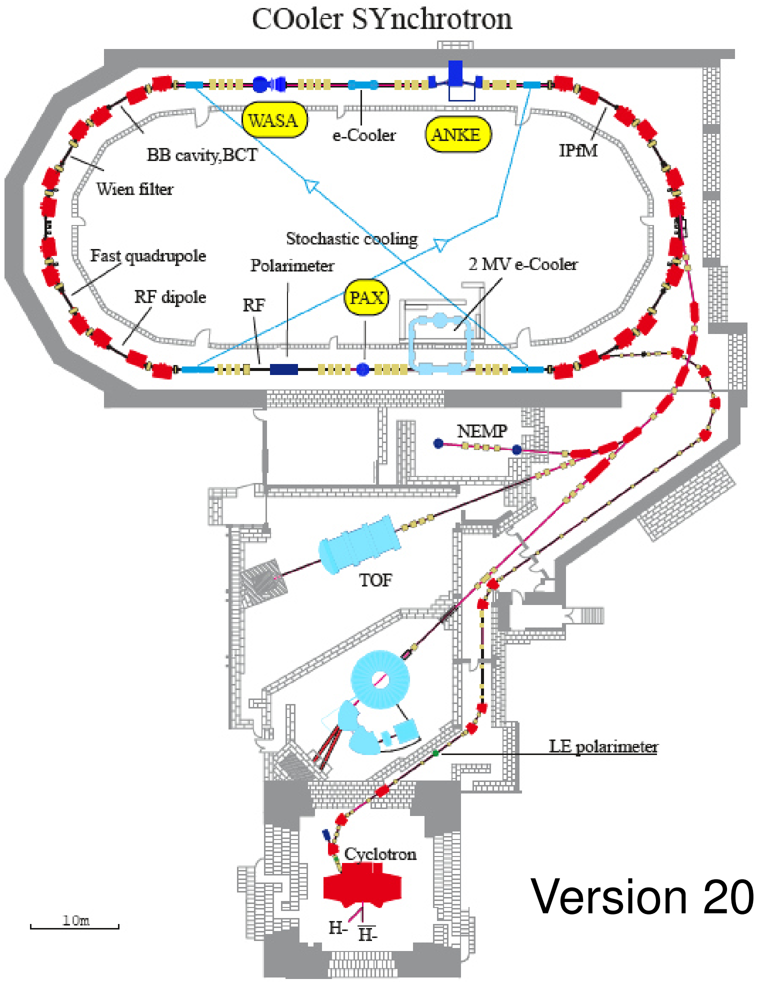
- Sketch of Extraction Development at COSY
- COSY Layout and Main Parameter
  - Extraction Layout and Components
- Extraction: Longitudinal Aspects
- Spill Time Structure (Example)
- Backup Slides Extraction: Transverse Aspects

Sketch of Extraction Development at COSY

- 1991: Theoretical Studies of Ultra Slow Extraction for COSY
- Inauguration COSY 1993
- EPAC 1996: First Resonant Extraction, Conventional Method
- EPAC 98: Stochastic Extraction at COSY
- Stochastic Extraction is now well established in a wide momentum range

Floorplan COSY and Main Parameters

- Circumference: 184
- Straight section: 40 m
  (telescopes with 2 π phase advance)
- Arc section 52 m
  (three cells, each
  QF-D-QD-D-D-QD-D-QF)
- Protons and deuterons momentum range 300 (540) MeV/c to 3300 MeV/c
- Polarized and unpolarized particles
- 100 keV electron cooler at injection and new 2 MeV cooler
- Stochastic cooling above 1.5 GeV/c
- Three extraction beam lines
- **Beam extraction in the whole momentum range**



Version 2014

Quadrupole and Sextupole Distribution

Bending Magnets

number:	24
radius:	7 m
angle:	15^0
field range:	0.23 - 1.6 T

Quadrupoles in the arcs

number:	24
no. of families:	6
eff. length:	0.3 m
aperture radius:	85 mm
max. gradient:	7.5 T/m

Quadrupoles in the telescopes

number:	32
no. of families:	8
eff. length:	0.55 m
aperture radius:	85 mm
max. gradient:	7.5 T/m

Sextupoles

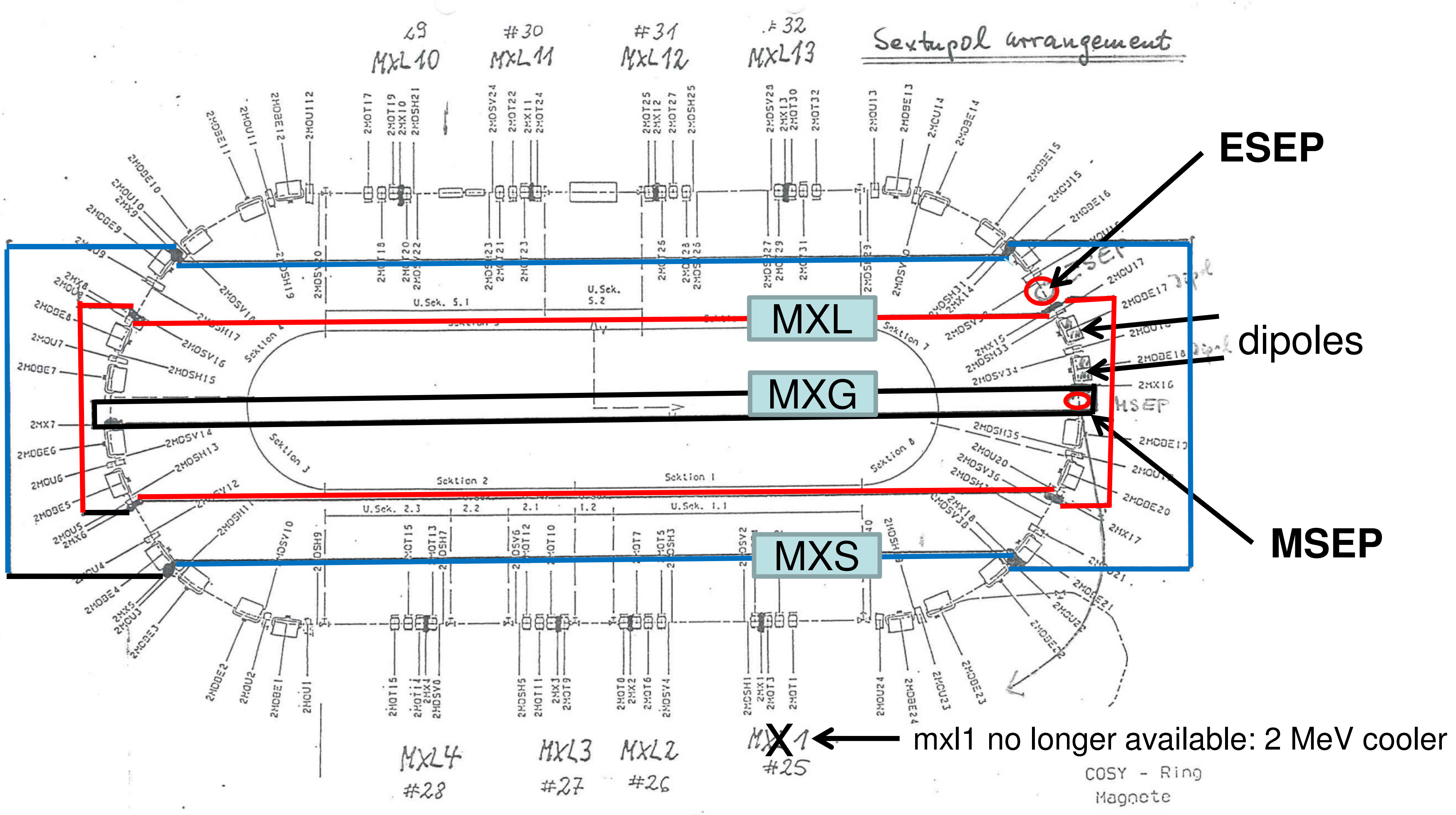
number:	~~18~~ 17
no. of families:	11
eff. length:	0.3/0.2/0.1 m
aperture radius:	85 mm
max. gradient:	30 T/m²

- **3 families of sextupoles in the arc section for chromaticity correction**
- 7 ~~8~~ **sextupoles for resonance in the straight sections**

Sextupoles:

- In the arcs to adjust chromaticity (Hardt condition)
- In the telescopes with dispersion zero to adjust resonance strength

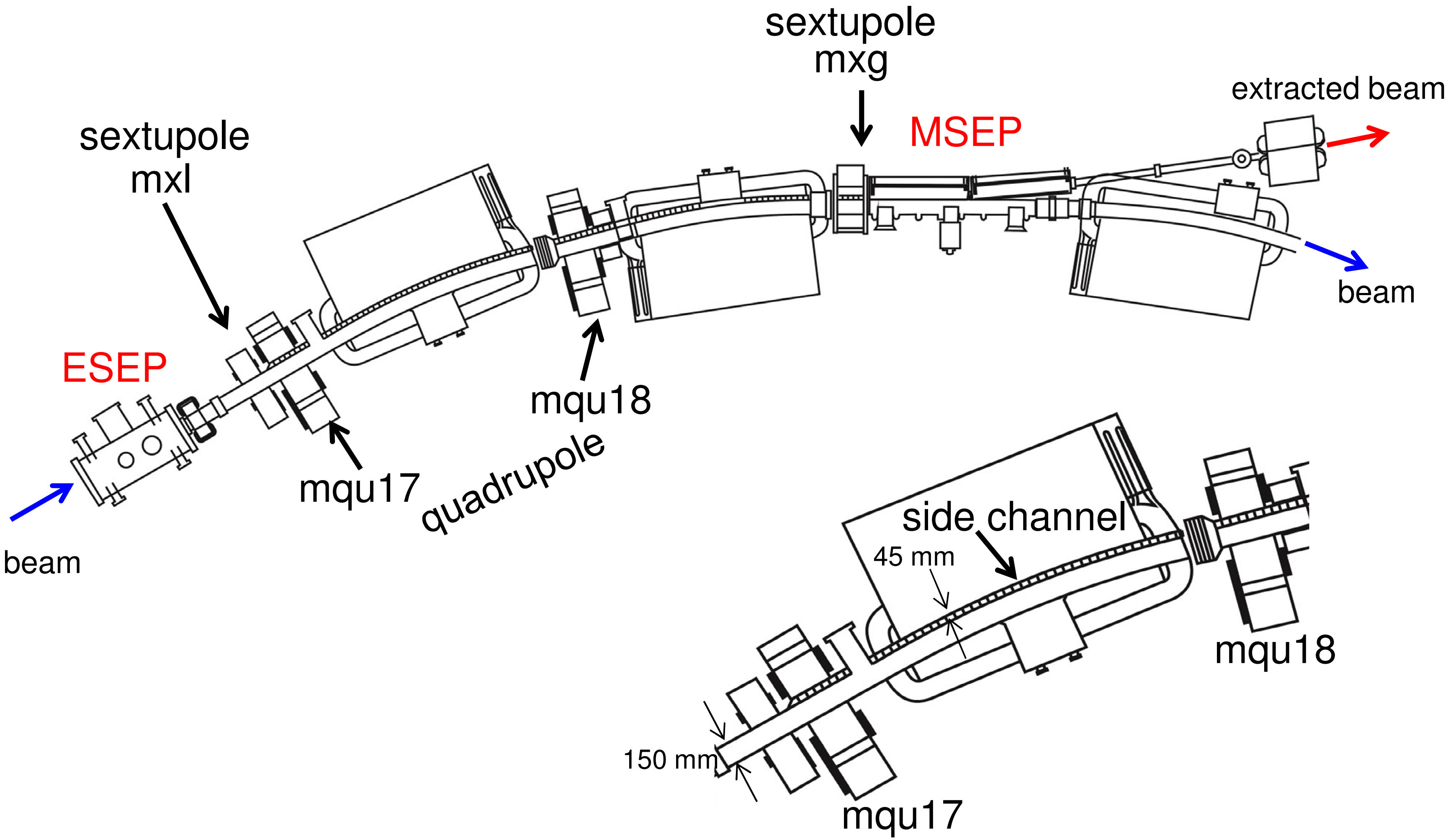
Sextupole Distribution



MXS: four sextupoles at one power supply
MXL: four sextupoles at one power supply
MXG: two sextupoles at one power supply

Chromaticity correction only

Arc Section and Extraction Elements



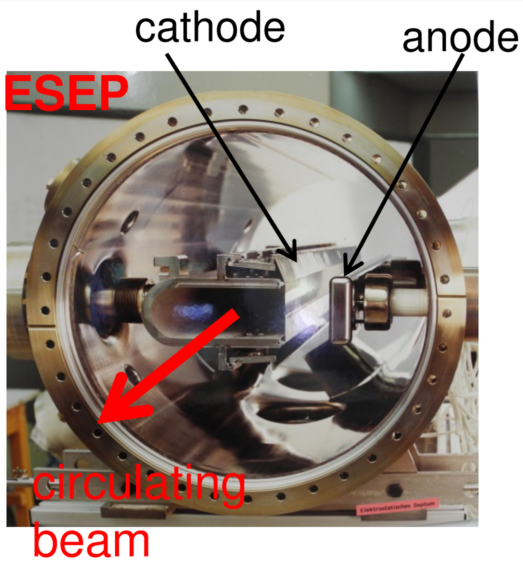
Main data ESEP and MSEP

ESEP

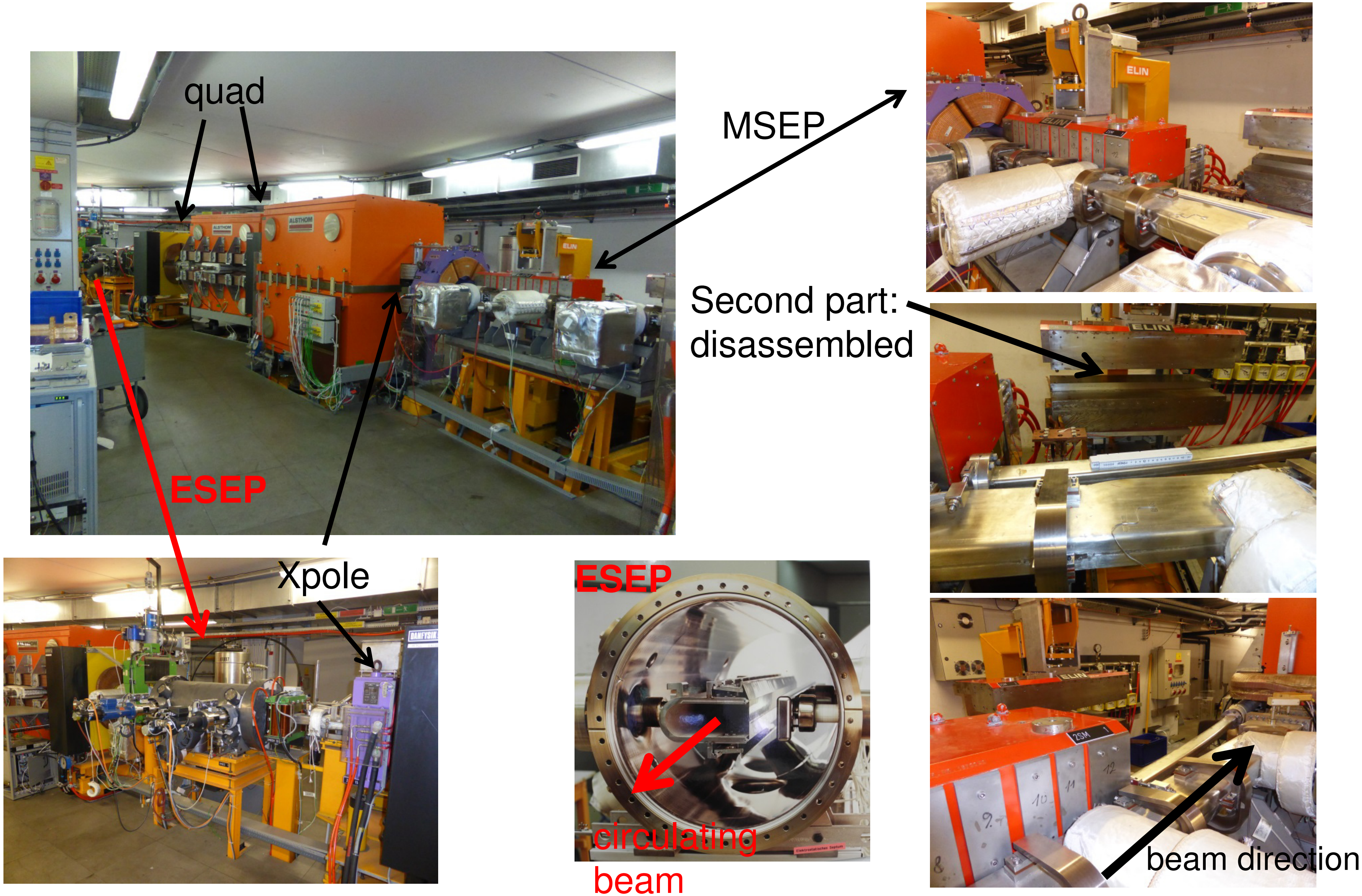
max. energy	**2.5**	**GeV**
max. momentum	3.3	GeV/c
deflection	3.5	mrad
max. voltage	200	kV
max. gradient	120	kV/cm
gap width	12 - 40	mm
radial position variation anode and cathode	± 20	mm
angle resp. to closed orbit	± 2	mrad
anode thickness	0.1	mm
length	1	m

MSEP

max. field	1.1	T
length	2 x 1	m
deflection	2 x 5	degrees
max. current	2740	A
inner side	88	mm
outer side	120	mm
gap	32	mm



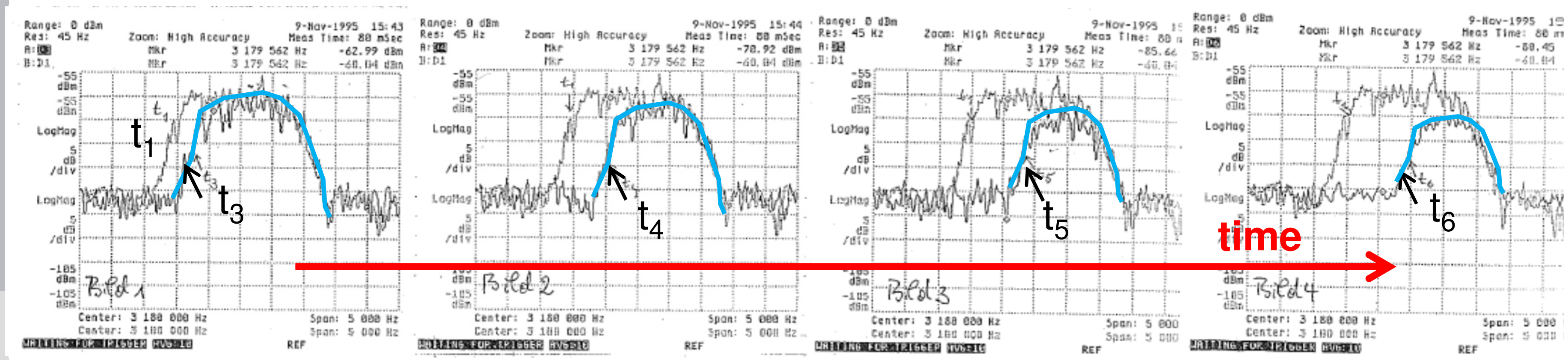
Extraction Elements at COSY



Conventional Extraction Method

- The beam is slowly swept across the resonance with constant speed by moving the tune.
- Circulating beam intensity distribution behaves like a ***rigid body***:
  Hard resonance edge for all particles!
- **For a constant spill the beam edge should never be repelled from the resonance! Otherwise ripple modulations.**

Longitudinal beam distributions during extraction of 800 MeV/c protons:



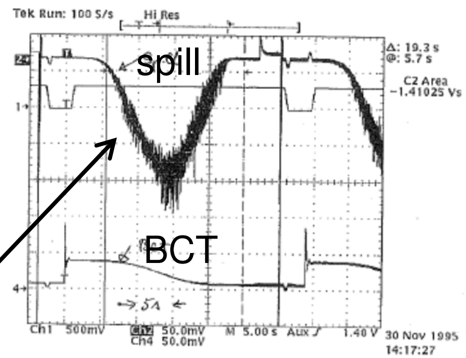
Linear tune ramp:

$$Q(t) = Q_S + \frac{dQ}{dt} \cdot (t - t_0)$$

$$\frac{dN}{dt} = \frac{dN}{dQ} \cdot \frac{dQ}{dt}$$

spill

tune distribution



Stochastic Beam Extraction

with Swept Band-Limited White Noise

- The main difference to the conventional extraction method is in the way the particles are driven to the third order resonance.
- The average tune of the beam and thus the lattice optics is **not** changed.
- Instead, the beam distribution is longitudinally heated by adding noise around a revolution harmonic.
- A diffusion in tune is created by a proper setting of the horizontal chromaticity to accelerate the particles into resonance.

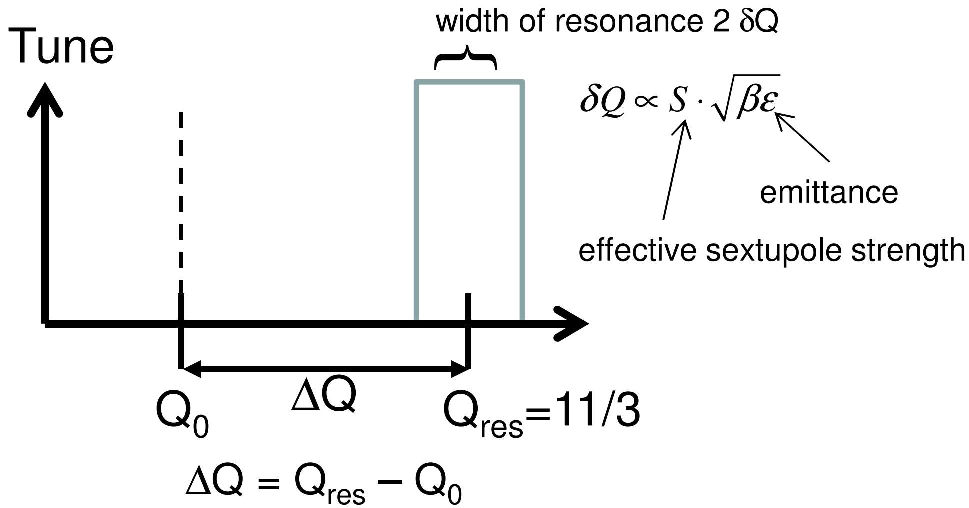
S. van der Meer, “Stochastic Extraction, a Low-Ripple Version of Resonant Extraction”, CERN/PS/AA 78-6

R. Cappi, W.E.K. Hardt and Ch.P. Steinbach, “Ultraslow Extraction with Good Duty Factor”,
11th International Conference on High-Energy Accelerators, Geneva, Switzerland, July 7–11, 1980

The LEAR team, “Performance of LEAR”, IEEE Transactions on Nuclear Science,
Vol. NS-32. No. 5, October 1985

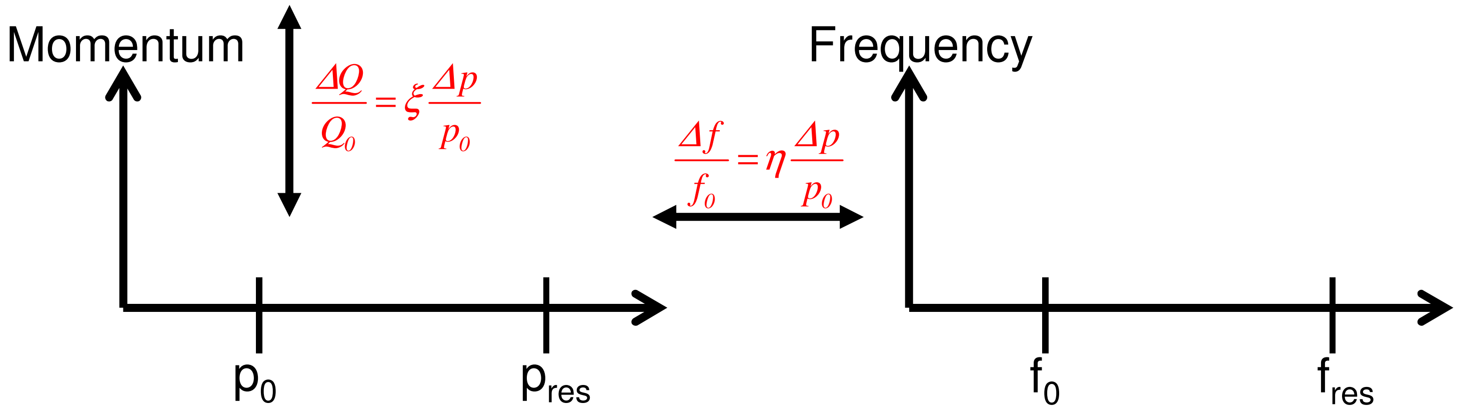
Michel Chanel, “LEAR Performances”, Proc. of the LEAR Symposium, CERN, 15th May 1999,
CERN/PS 99-040 (CA)

Feeding the Resonance



Some possibility:

- Classical method:
  Move the tune (lattice change)
- Horizontal heating the beam:
  increase the emittance
- Longitudinal heating (COSY)
- Betatron core

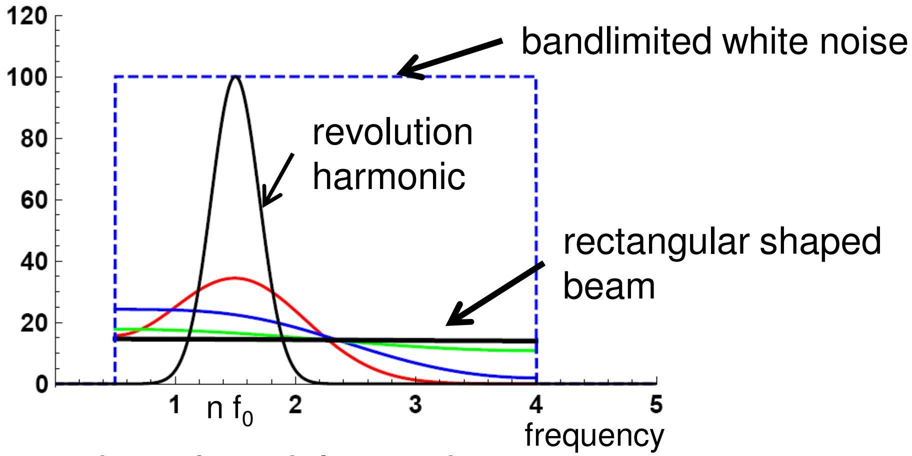


- ***Step 1:***
  - Beam Momentum Distribution Shaping
    - *Gaussian beam → Uniform beam distribution*

- ***Step 2:***
  - Uniform noise is applied **that always covers the resonance.**
  - The carrier frequency is slowly moved towards the shaped beam distribution.
    - *Particles diffuse into the resonance and are extracted.*

Beam Shaping as Preparation for Extraction

Beam shaping in longitudinal phase space



Conservation of particle number:

$$\frac{\partial}{\partial t}\Psi(x,t)=-\frac{\partial}{\partial x}\Phi(x,t)$$

Particle flux: $\Phi(x,t)=-D\cdot\frac{\partial}{\partial x}\Psi(x,t)$

is zero outside the noise bandwidth

$x=f$ or Δf or ΔQ
or $x=\Delta p/p_0$

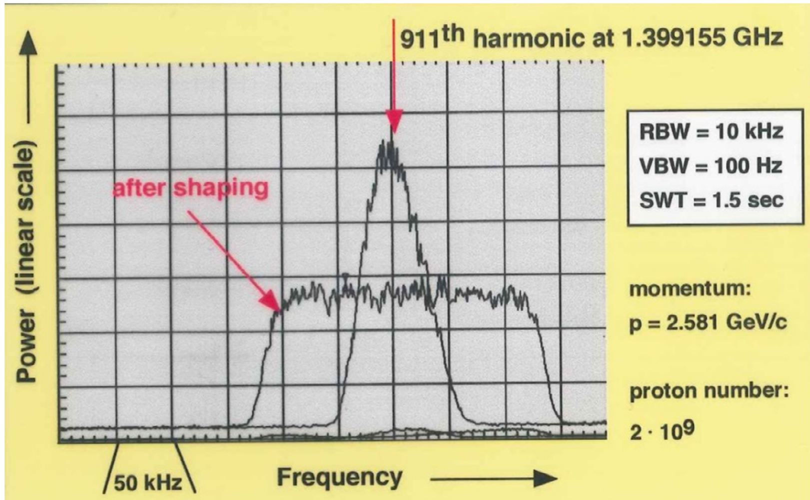
- Band-limited white noise is sent to a longitudinal kicker, BB cavity at COSY. Bandwidth *W*. Noise kicker voltage U_{rms}.
- The noise density covers a revolution harmonic $n\,f_0$ of the beam.
- Shaping is applied for approximately 1 s and the beam attains a rectangular (uniform) distribution
- Shaping with noise induces a diffusion process described with a Fokker-Planck equation:

$$\boxed{\frac{\partial}{\partial t}\Psi(x,t)=D\cdot\frac{\partial^2}{\partial x^2}\Psi(x,t)}$$

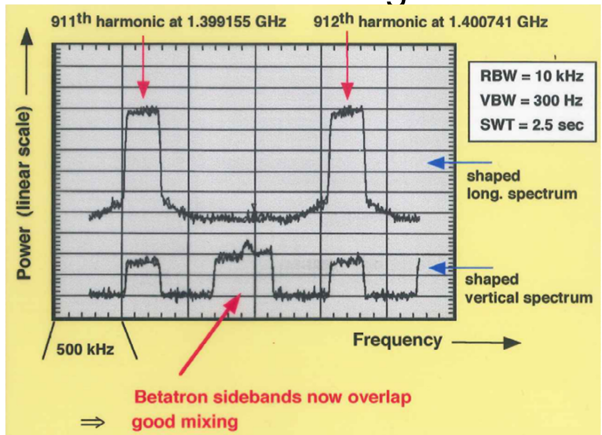
Diffusion constant D in frequency or momentum space:

$$\boxed{D_{\Delta f}=\frac{f_0^2}{2W}\frac{(\eta n f_0)^2}{\beta^2(pc)^2}(eU_{rms})^2 \qquad D_{\Delta p/p_0}=\frac{f_0^2}{2W}\frac{(eU_{rms})^2}{\beta^2(pc)^2}}$$

Longitudinal Beam Shaping Experiments



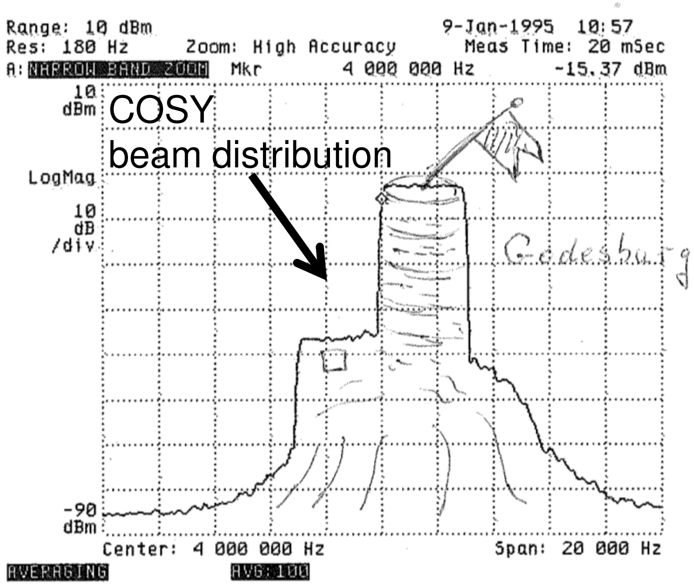
observed with PU
of stochastic cooling in band I



- Rectangular shaped noise with width W = 1 kHz applied to the 4th revolution harmonic
- Longitudinal momentum distribution rectangular ⇒ transverse sidebands rectangular
- Shaping time 1 s
- Width of 4th harmonic without noise 340 Hz
- The resulting width of the revolution harmonic 911 agrees with $W = 911 \cdot \frac{1\,kHz}{4} = 228\,Hz$
- Necessary voltage: U_{rms} = 77 V, noise power into 50 Ω: 120 W, spectral noise density: S = 120 mW/Hz

Just for Fun

Digital Noise Generator: arbitrary waveform possible



Godesburg in Bad Godesberg near Bonn 1900*)

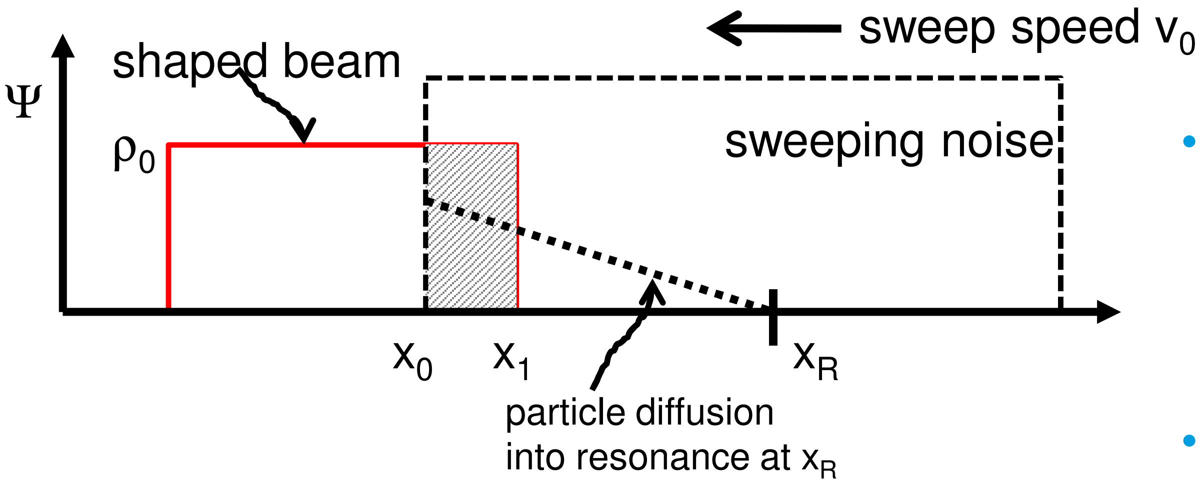
- superimposed noise on the COSY beam results in

The Godesberger spectrum

*) www.godesberger-markt.de

JÜLICH FORSCHUNGSZENTRUM

Beam Extraction with Swept Noise



- Band-limited white noise permanently covering the resonance is swept over the beam with speed v_0.
- Diffusion equation with diffusion coefficient D

In the vicinity of the resonance:

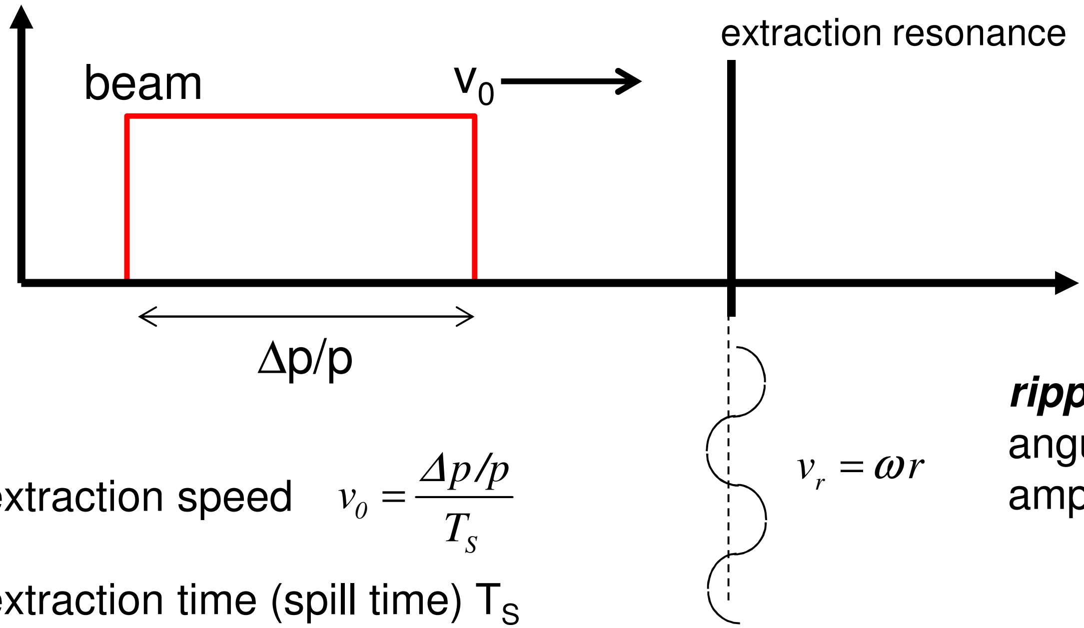
$$\Psi_0(x) \approx -\frac{v_0 \cdot \rho_0}{D}(x - x_R) \qquad \Rightarrow \qquad \Psi_0(x_R) = 0$$

- The number of particles that are extracted per sec is given by

$$\Phi_{ex}(t) = \Phi(x_R, t) = -D \cdot \frac{\partial}{\partial x}\Psi(x_R, t)$$ and yields the **constant flux** $\Phi_{ex}(t) = \Phi_0$

W. Hardt, “Remarks on Stochastic Extraction”, PS/DL/Note 78-5

Classical extraction:



ripple:
angular frequency ω
amplitude r

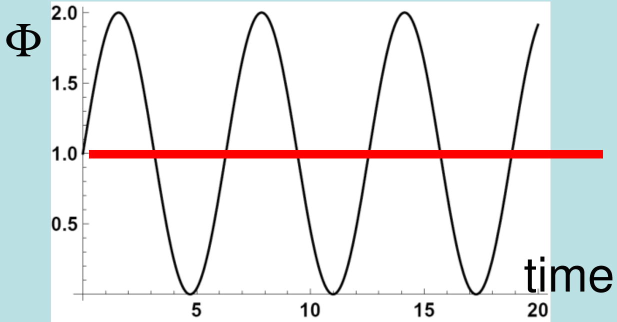
extraction speed $v_0 = \dfrac{\Delta p/p}{T_S}$

extraction time (spill time) T_S

$v_0 << v_r$	$\Rightarrow$	high ripple modulation: BAD
$v_0 >> v_r$	$\Rightarrow$	Low ripple modulation: but spill time T_S small

Stochastic extraction:

- Particles diffuse into the resonance with speed $\langle v \rangle = \dfrac{v_0 \cdot \rho_0}{\Psi_0(x)} >> v_0$
- In the vicinity of the resonance the average speed <v> becomes large
- Spill time can be made large
- Less sensitivity to power supply ripple



$$\omega r = v_0 \quad \Rightarrow \quad F = 2/3$$

$$\omega = 2\pi\, 50 Hz \quad r = 10^{-4}$$
$$\Delta p/p = 1.5 \cdot 10^{-3} \quad \Rightarrow T_S \leq 420\, ms$$

Spill quality (Duty factor)):*

conventional:

$$F = \frac{\langle \Phi \rangle^2}{\langle \Phi^2 \rangle} = \frac{1}{1 + \frac{1}{2}\left(\frac{\omega r}{v_0}\right)^2} \qquad 0 < F < 1$$

stochastic:

$$F = \frac{\langle \Phi \rangle^2}{\langle \Phi^2 \rangle} = \frac{1}{1 + \frac{1}{2}\frac{\omega r^2}{D}}$$

adjust: $D > \omega r^2$

*) L. Badano et al., "PROTON-ION MEDICAL MACHINE STUDY (PIMMS) PART I", CERN/PS 99-010 (DI), 1999

Marco Pullia, PhD Thesis, 1999: "Dynamics of slow extraction and its influence on transfer lines design",
http://spazioweb.inwind.it/mgp_homepage/

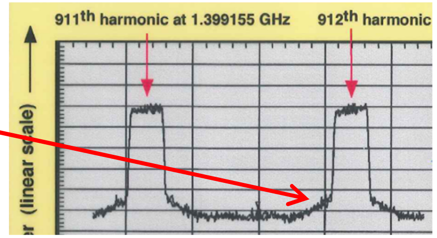
the mean diffusion velocity <v> is given by:

$$\langle v \rangle = \frac{v_0 \cdot \rho_0}{\Psi_0(x)} >> v_0$$

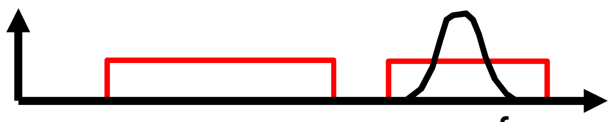
- In the vicinity of the resonance ($x \approx x_R$) the diffusion velocity becomes large.
- **Less sensitivity against ripple as compared to conventional extraction method where the sweep is carried out with the speed v_0 to move the tune.**
- The mean velocity depends upon the diffusion coefficient D.
- Therefore D should be large.
- **But:** If D is too large some particles will eventually not be extracted.

Limitations

- Too much power (too large diffusion D) should be avoided:
  - more unwanted amplifier noise
  - non-linearities of power amplifiers introduce intermodulation products with the result:
    - *undesired diffusion outside the selected bandwidth*



- The selected bandwidth should be not too small: correlation time $\tau \sim 1/W$ of noise becomes small
  $\Rightarrow$ spill modulated with carrier frequency of the (shaping/sweeping)noise
- If t_{ex} is the time an unstable particle needs to move along the separatrix branch to the ESEP, then, to avoid ripple: $t_{ex} >> \tau$
  - Example: a particle needs roughly 1000 revolutions in the ring to reach the ESEP. One revolution 1 μs. Thus $t_{EX} = 1\ ms$ and the bandwidth should be $W > 1\ kHz$.
- **Remedy:**
  - Use a high harmonic number n for the center frequency of the noise to make the bandwidth W large.
  - Apply **additional** noise between the revolution harmonics. The additional noise does not affect the extraction but reduces the coherence time.
  - Apply “chimney” noise covering the resonance.



Digital Noise Generator

Three independent systems available:

USE1: beam shaping

USE2: swept noise

USE3: chimney noise

Noise generator based completely on digital signal processing

Complex random sequence:

$$x(n) = \frac{1}{N}\sum_{s=0}^{N-1} \hat{x}(s) e^{-i2\pi sn/N}$$ random sequence $\hat{x}(s)$ *n, s* integers

Band-limited white noise:

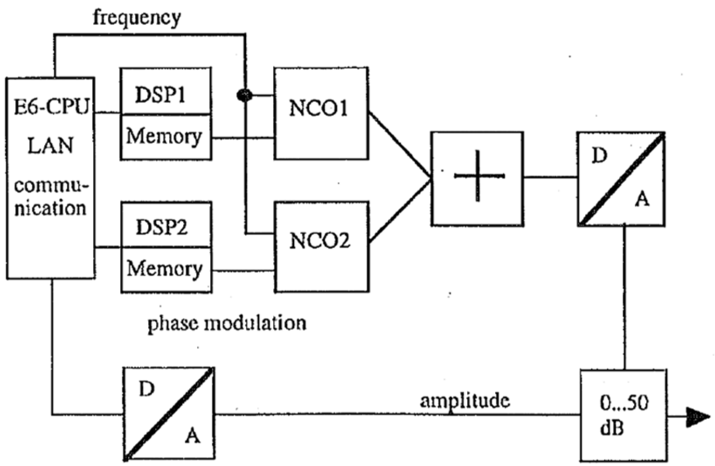
- The amplitudes $\hat{x}(s) = Ae^{-i\varphi(s)}$ are non-zero for $s_{-} \le s \le s_{+}$
- Phases $\varphi(s)$ uniformly distributed in $[0, 2\pi[$
- The discrete power density is $S(f) = A^2/(N^2 \Delta f)$ at frequency $f = n\,\Delta f$ with frequency resolution $\Delta f = N\,\Delta t$.
- N large and Δf small: real and imaginary part nearly Gaussian distributed with zero mean.

The real part $X_R(n)$ of the complex band-limited signal x(n) centered at frequency f_0 can be written as **sum of two phase modulated cosine sequences**

$$x_R(n) = A\{\cos(2\pi f_0 n\Delta t + \Theta_{-}(n)) + \cos(2\pi f_0 n\Delta t + \Theta_{+}(n))\}$$

where the phase sequences $\Theta_{-/+}(n)$ are determined by the desired noise signal.

General layout*):



- Two digital signal processors (DSP) are used to calculate the desired phase sequences $\Theta_{-/+}(n)$ for a wanted noise signal off-line.
- The outputs of the numerically controlled oscillators (NCO) are added and fed into the DAC to produce the analog noise signal.
- No filters are necessary.

- Completely predictable design (investigate quantization/truncation impact etc. on spectrum)
- Arbitrarily shaped spectra possible (see Godesberger spectrum)
- Signal-to-noise ratio 60 dB
- Typical roll-off 180 dB/octave
- Spectrum is calculated only once. The center frequency up to 20 MHz is chosen online.
- Bandwidth 0.05 kHz up to 500 kHz in different steps
- Resolution Δf = 6 Hz, now down to 0.1 Hz
- Clock frequency 50 MHz, phase modulation 12.5 MHz
- Available noise power up to 500 W

*) G. Heinrichs, H. Meuth, H. Stockhorst, A. Schnase, *A Narrow-Band Digital RF-Noise Generator*, Proc. of EPAC94, London, June 27 –July 1, 1994
- patents in Europe, Japan and the US have been released by the FZJ and the patent owner in 2001.

Beam Extraction

by Noise Feeding the Resonance

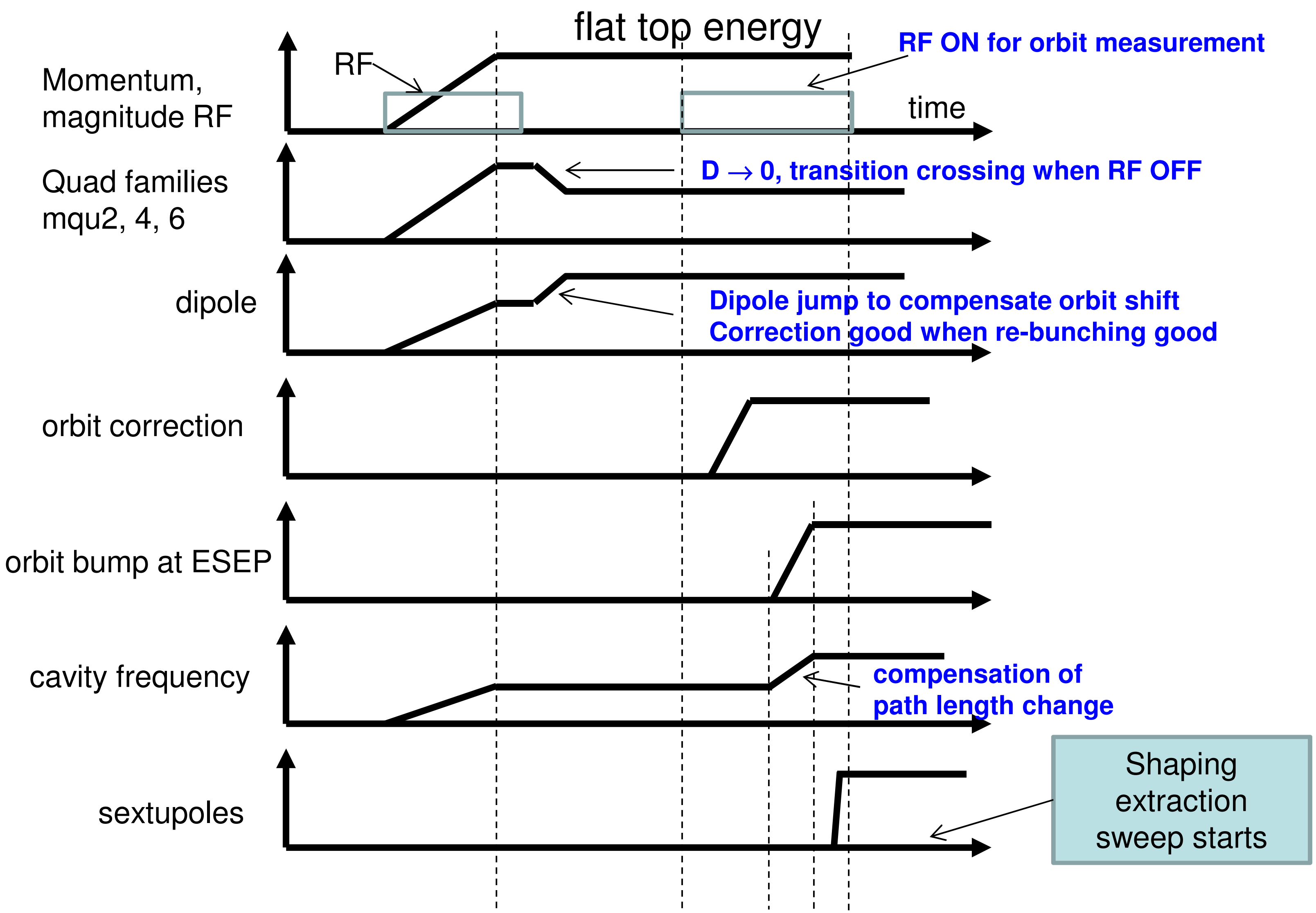
Extraction of an e-cooled Proton Beam

Momentum	**1.57**	**GeV/c**
Revolution frequency	1.402381	MHz
β	0.859	
γ	1.950	
Tune Q_x	3.655	
Tune Q_y	3.570	
Slip factor η	0.1	
Chromaticity ξ_x	+1.2	
ξ_y	-2.8	

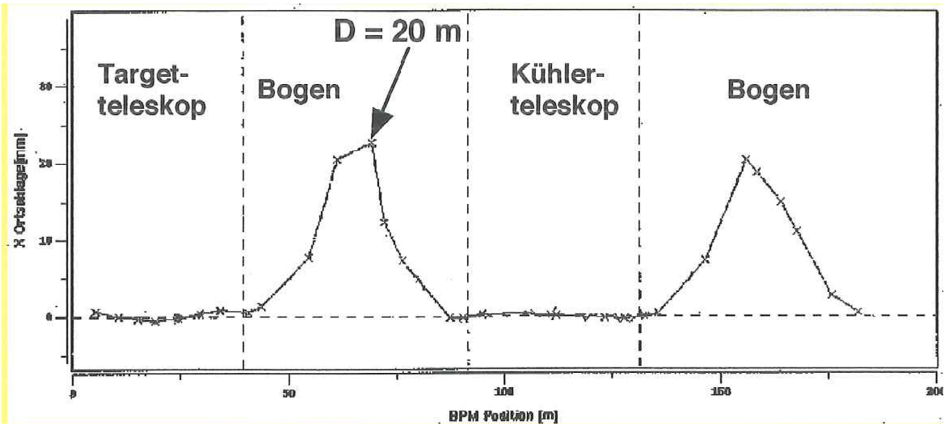
Choice and Control of Extraction Parameters

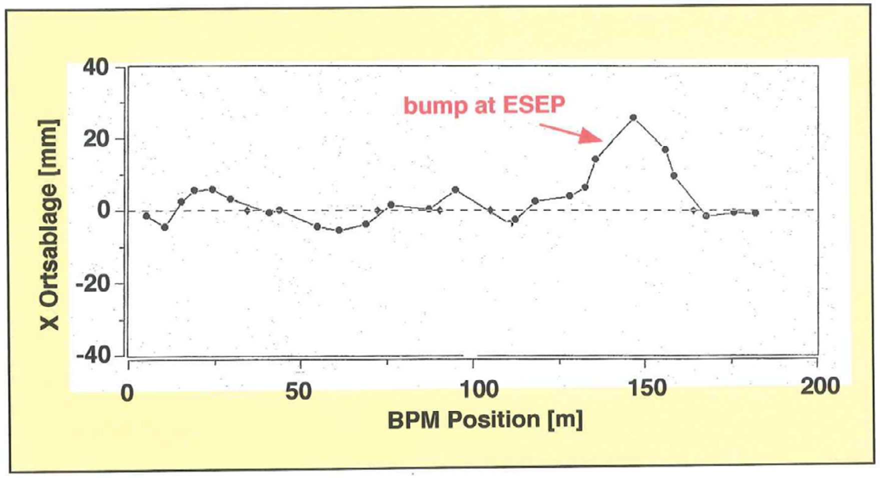
- **Many parameters must be controlled and adjusted:**
  - Sextupole strength (emittance should be known)
  - Angle of separatrix at ESEP (little program)
  - Chromaticity (can be measured)
  - Hardt condition (can be checked at wire chamber after MSEP, optimum if beam position does not vary during extraction)
  - Momentum spread (can be measured)
  - Orbit bump (can be measured, choice of amplitude depends on sextupole strength and thus spiral step)
  - Angle kick in ESEP and deflection in MSEP
  - Tune and beam shaping prior to extraction (shaping can be tuned so that spill becomes flat, no “initial” peak)
  - Extraction noise (width and center frequency, no “initial” peak)
  - Power adjustment
- Parameters are not independent from each other
  - Parameters are tuned to optimum extraction efficiency (> 80 %)

Ramp Procedure for Beam Extraction

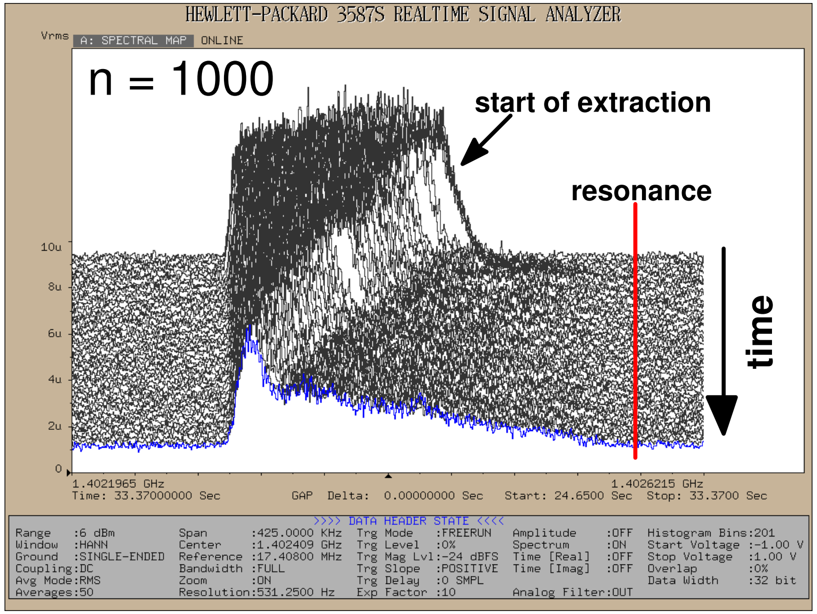


Dispersion and Orbit Bump at ESEP

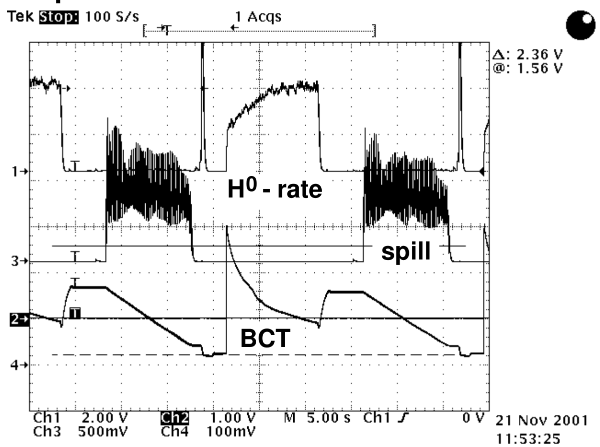




Longitudinal spectra of an e-cooled beam at 1.57 GeV/c

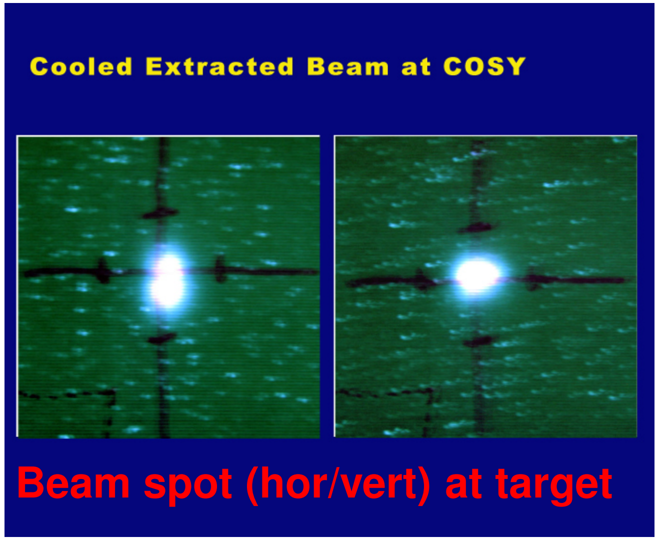


Experiment at BIG KARL



No losses when extraction starts!

- The resulting beam spot on a viewer placed in the scattering chamber of the experiment had a radius of 2 mm.
- The halo ratio for a 3 mm veto hole could be reduced from $2.5 \cdot 10^{-2}$ to $4.0 \cdot 10^{-4}$ for a cooled beam.
- Thus the primary beam intensity can be increased by a factor of 60 for the same veto counting rate.
- Spill rate 10^9 protons/s
- Extraction efficiency > 80%

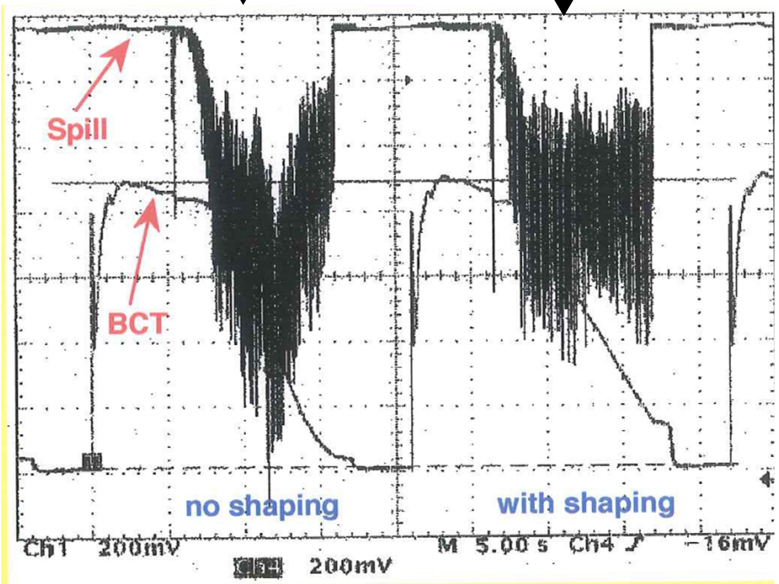


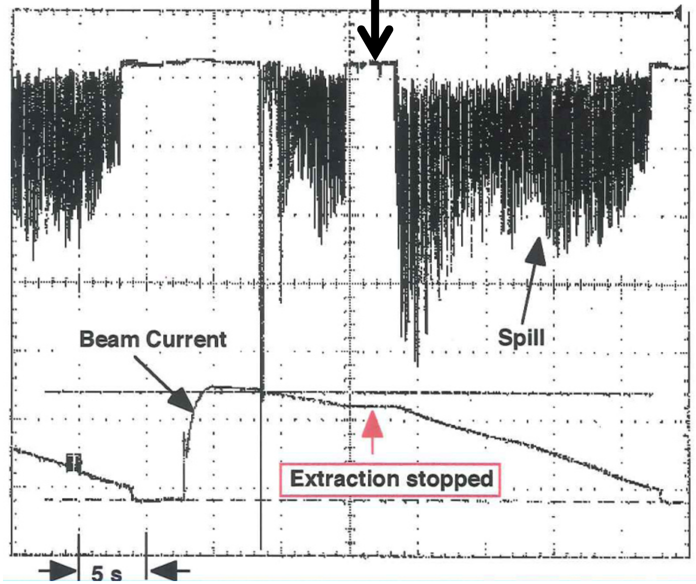
Spill Shape During Extraction

No shaping, spill Gaussian

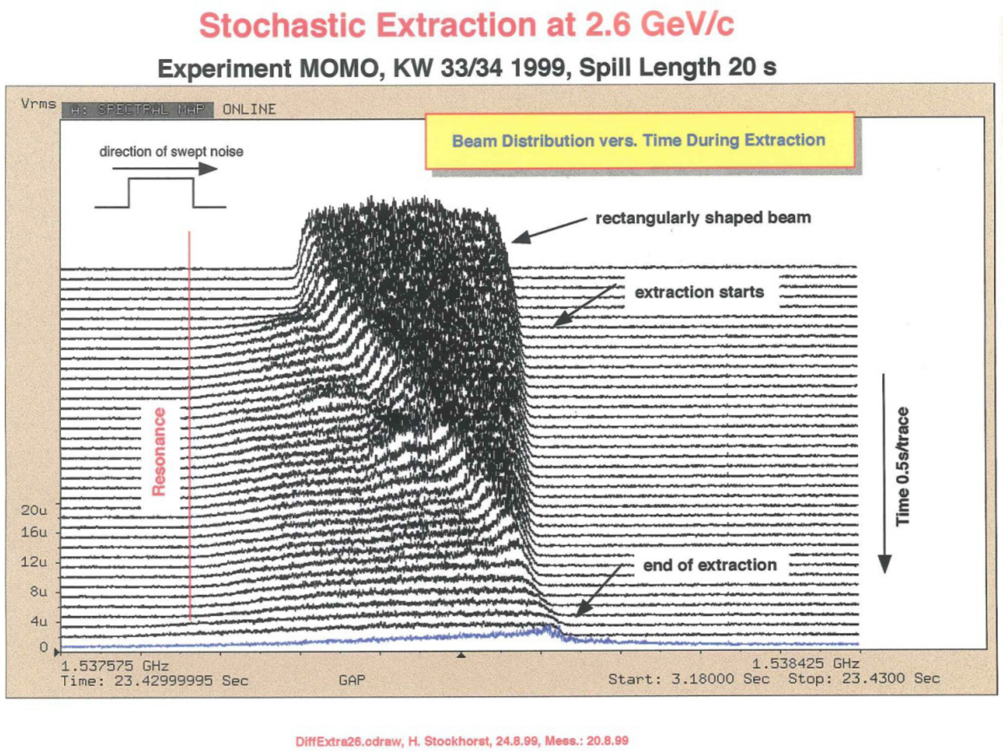
Rectangular shaping, spill flat

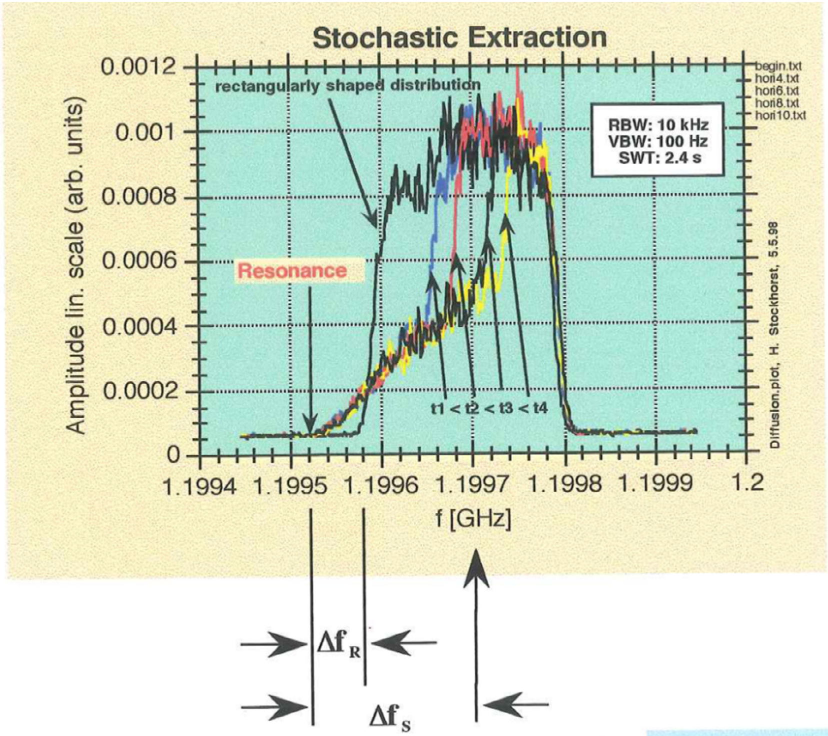
Noise sweep stopped:
Extraction stops





Extraction above Transition Energy

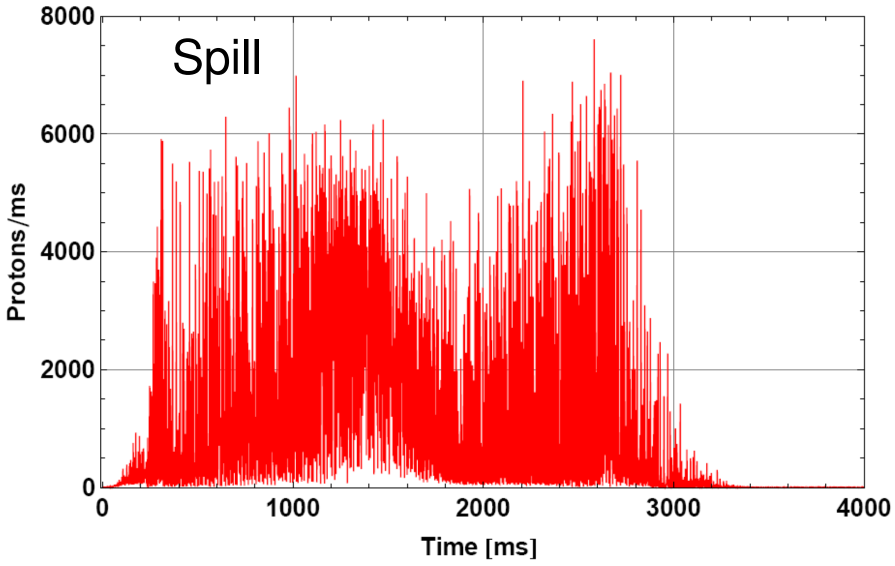


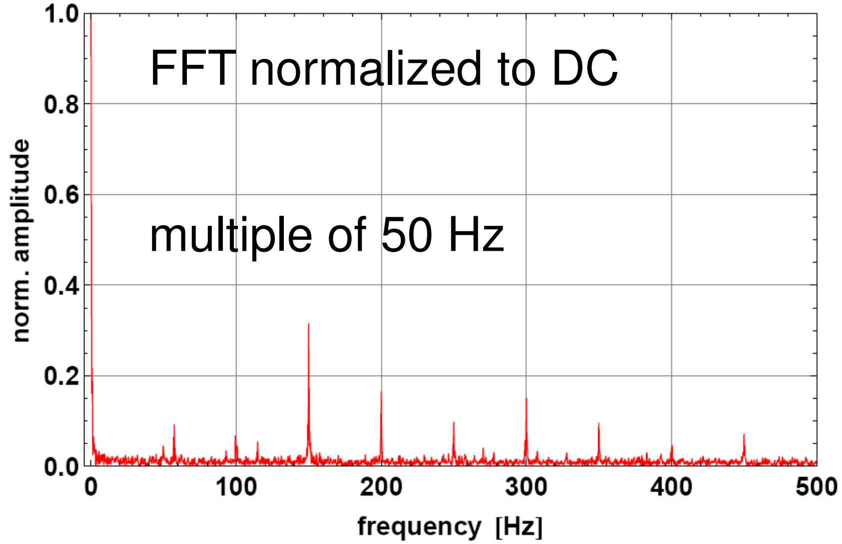


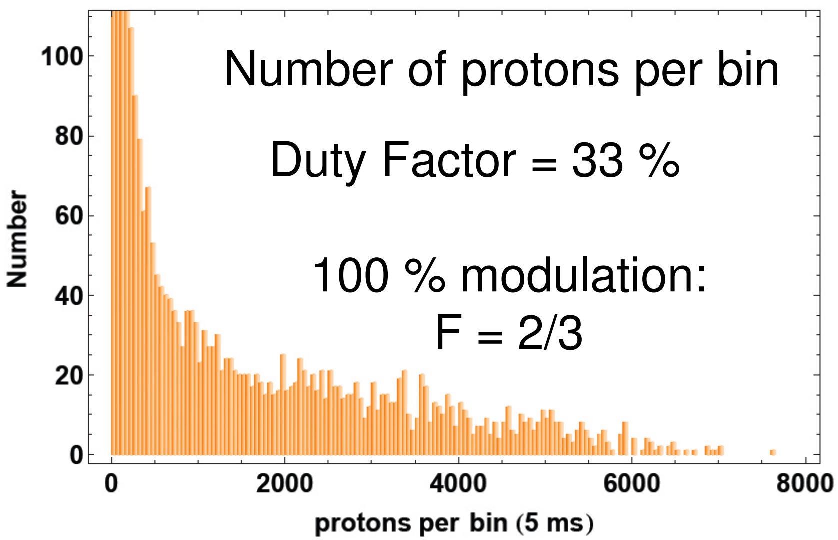
Spill Structure (Example)

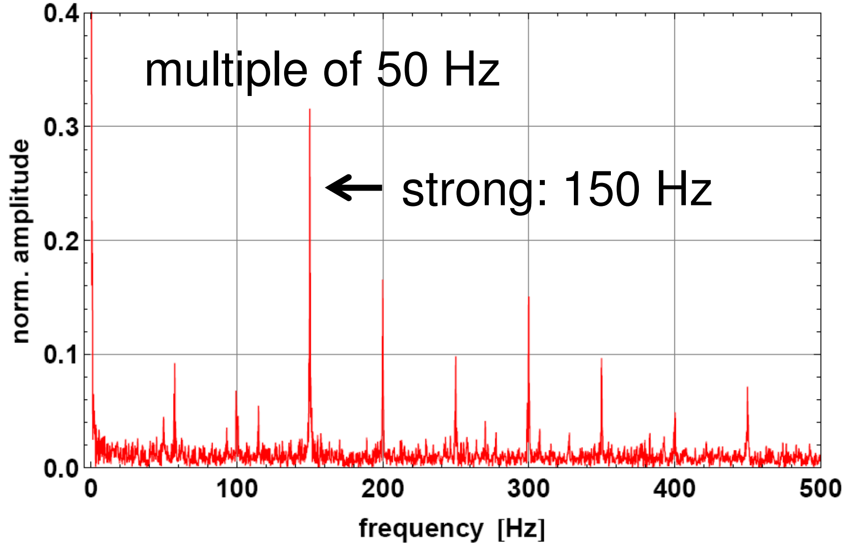
- Proton momentum 796 MeV/c
- Spill duration app. 4 s
- Hodoscope signals measured with time interval analyzer HP 5372 A
- Bin width 1 ms, 4096 bins

Spill during conventional extraction



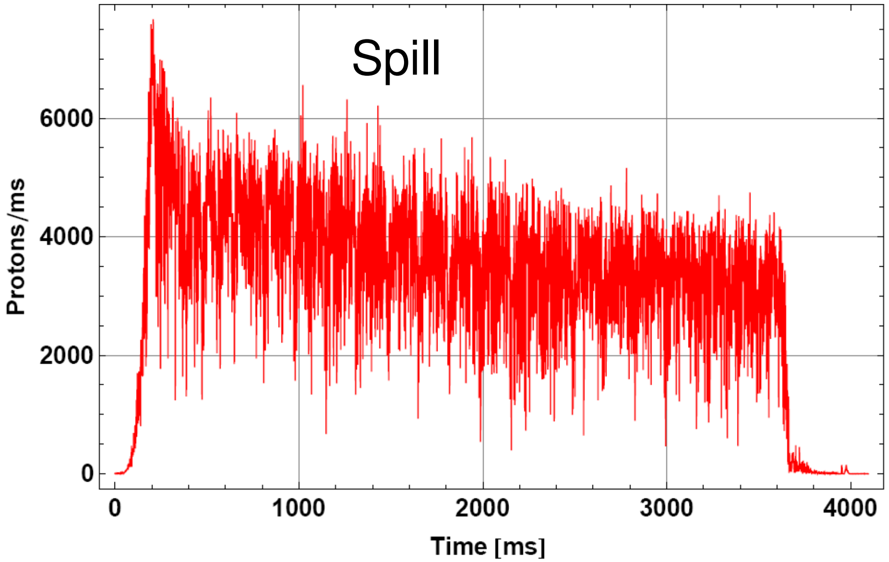


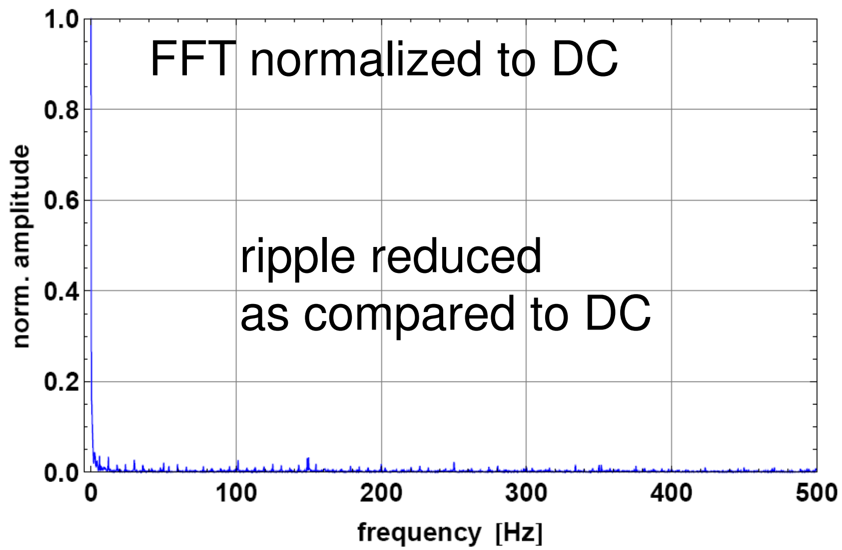


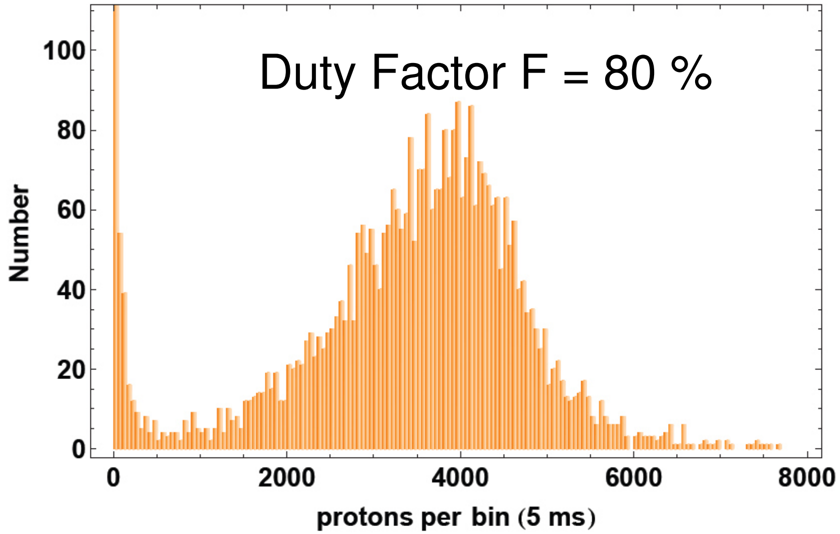


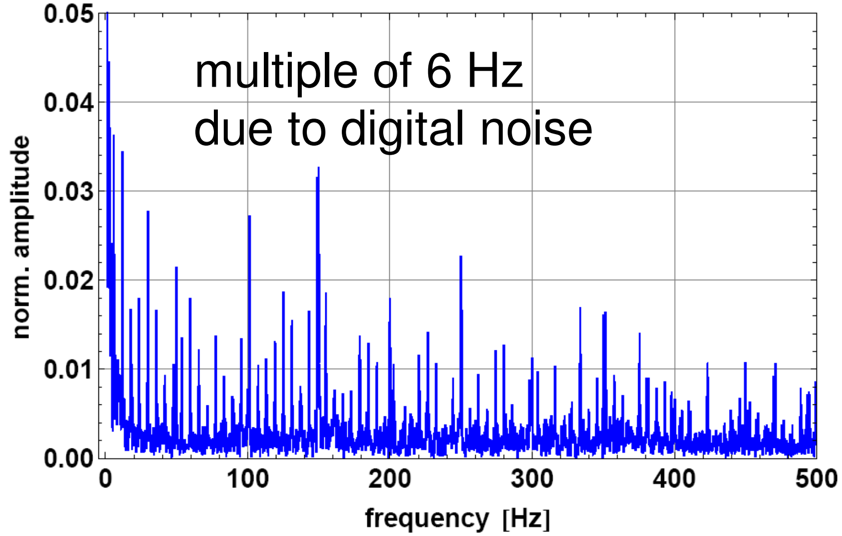
Stochastic Extraction

Same conditions as before, but tune fixed









Summary

- Stochastic extraction at COSY in the whole momentum range
- Spill duration several minutes (1 h spill duration has been done)
- Extraction efficiency larger than 80 %
- Extracted beam width at target $\sigma_x \approx \sigma_y = 0.3$ mm
- Extraction of polarized beams possible

Continuation: next talk

D. Prasuhn, Observations with slow extraction at COSY and cures

Thank you for your attention

Special thanks to

Michel Chanel

Backup Slides

Power Consumption (1)

diffusion in tune space: $$D_{\Delta Q} = \frac{1}{2}\frac{d\left\langle (\Delta Q)^2 \right\rangle}{dt}$$

with $\Delta Q = \frac{\xi Q}{\beta pc}\Delta E$ $$D_{\Delta Q} = \frac{1}{2}\frac{(\xi Q)^2}{\beta^2 (pc)^2}\frac{d\left\langle (\Delta E)^2 \right\rangle}{dt}$$

Band-limited white noise at longitudinal kicker (BB cavity at COSY) of width *W*

$$\frac{d\left\langle (\Delta E)^2 \right\rangle}{dt} = \frac{f_0^2}{W}\left(eU_{rms}\right)^2$$

U_{rms}: rms kicker voltage

ΔQ-space:

$$D_{\Delta Q} = \frac{f_0^2}{2W}\frac{(\xi Q)^2}{\beta^2 (pc)^2}\left(eU_{rms}\right)^2$$

Δf-space:

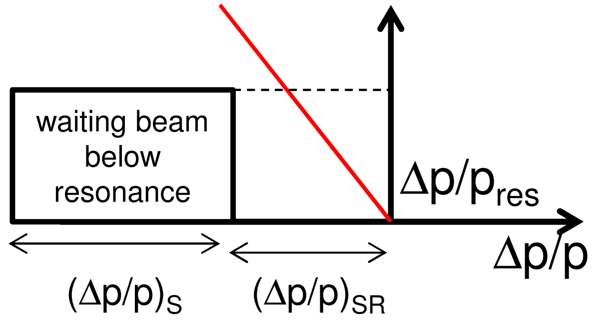
$$D_{\Delta f} = \frac{f_0^2}{2W}\frac{(\eta n f_0)^2}{\beta^2 (pc)^2}\left(eU_{rms}\right)^2$$

Δp/p-space:

$$D_{\Delta p/p_0} = \frac{f_0^2}{2W}\frac{\left(eU_{rms}\right)^2}{\beta^2 (pc)^2}$$

Power Consumption (2)

Extraction duration (spill time): T_S



wanted $\Delta t_{diffusion} \ll T_S$

$$\langle v \rangle \approx \frac{D_{\Delta p/p}}{(\Delta p/p)_{SR} + (\Delta p/p)_S}$$

extraction speed: $v_0 = \frac{(\Delta p/p)_S}{T_S}$

$$D_{\Delta p/p} = \frac{d(\Delta p/p)^2}{2\Delta t_{diffusion}} = \frac{\{(\Delta p/p)_{SR} + (\Delta p/p)_S\}^2}{2\Delta t_{diffusion}}$$

allows to determine
the required voltage or power

Example: COSY at 2.6 GeV/c (f_0 = 1.538 MHz β = 0.94)

$(\Delta p/p)_{SR} = 5 \cdot 10^{-4}$

$(\Delta p/p)_S = 1.5 \cdot 10^{-3}$ after shaping

T_S = 10 s
$\Delta t_{diffusion}$ = 1 s
W = 1 kHz

$D_{\Delta p/p} = 2 \cdot 10^{-6} s^{-1}$

yields

$U_{rms} = 100V$

$P = 200W \text{ at } 50\Omega$

$S = 0.2W/Hz$

$\langle v \rangle = 10^{-3} s^{-1}$

$v_0 = 1.5 \cdot 10^{-4} s^{-1}$

Principle of Third Order Resonance Extraction

- transverse aspects -

Extraction Principle

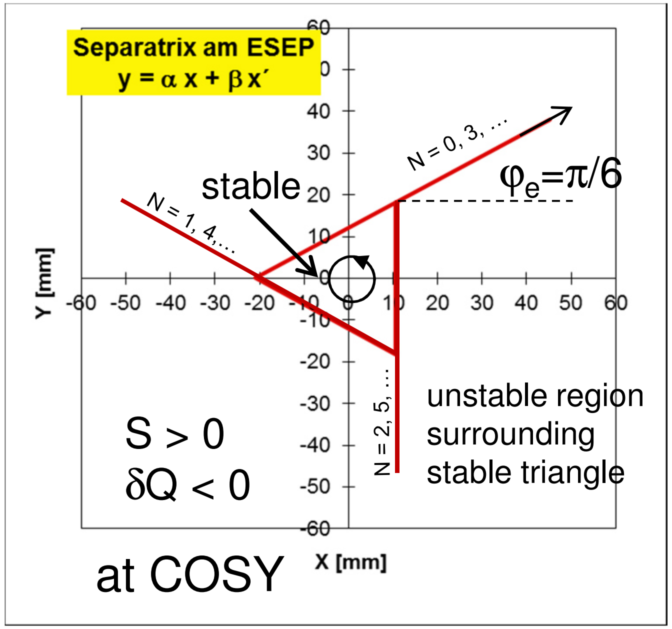
- ***11/3 – order resonance extraction excited by sextupoles***

Sextupolar perturbation in the plane of extraction (horizontal) by single sextupole, thin lens

Normalized phase space $(\tilde{x}, \tilde{y})$
$$\begin{cases} \tilde{x} = \sqrt{\dfrac{\beta_{ES}}{\beta}}\, x \\ \tilde{y} = \sqrt{\dfrac{\beta_{ES}}{\beta}}\,(\alpha x + \beta x') \end{cases}$$

Normalized phase space for single sextupole:

$$x^2 + (\alpha x + \beta x')^2 = \beta \varepsilon$$



Beam emittance ε Beam phase space area $A = \pi \cdot \varepsilon$

Distance to the resonance Q_{res} $\delta Q = Q - Q_{res}$

Normalized sextupole strength*)
$$\boxed{S = \frac{k'\ell}{2}\frac{\beta(s)^{3/2}}{\sqrt{\beta_{ES}}}\left[\frac{1}{m}\right]}$$

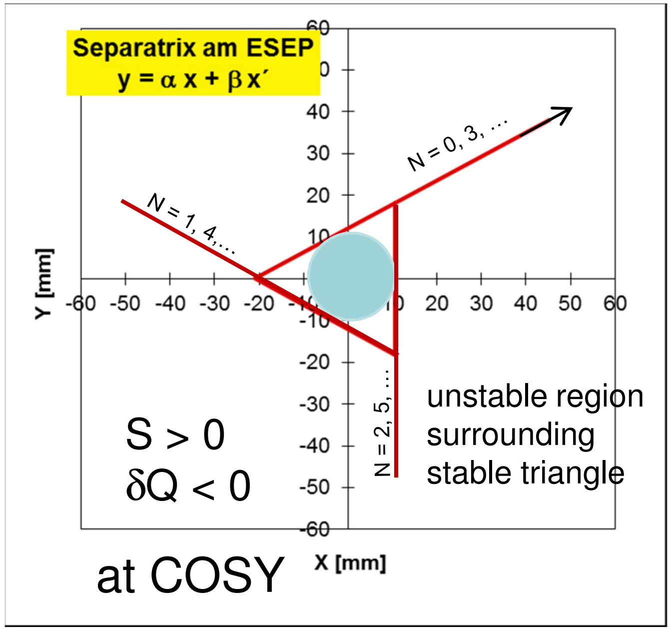
$$k' = -\frac{B''}{B\rho}\left[\frac{1}{m^3}\right]$$

Area of stable triangle:
$$\boxed{F = 3\sqrt{3}\left(\frac{4\pi\,\delta Q}{S}\right)^2 \quad [m^2]}$$

betratron function at ESEP β_{ES}
betratron function along ring $\beta(s)$

*) R. Giannini, PS/CD/Note 81-4

Extraction Principle (2)



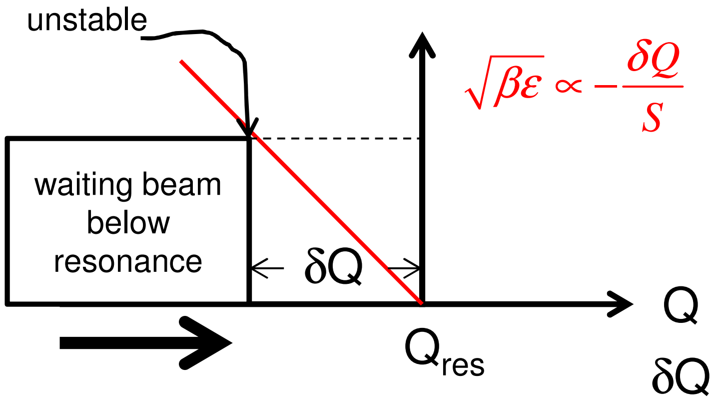
Fix points:

$$x = -\frac{4\pi\,\delta Q}{S} \qquad y = \pm\sqrt{3}\,\frac{4\pi\,\delta Q}{S}$$

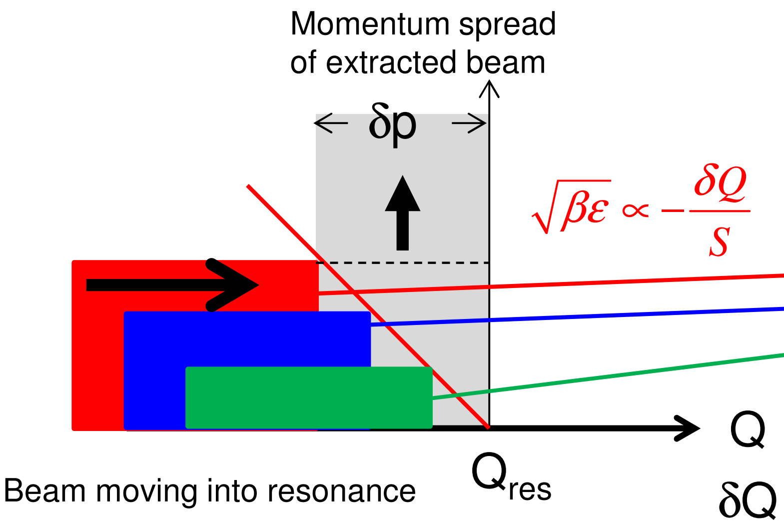
$$x = \frac{8\pi\,\delta Q}{S} \qquad y = 0$$

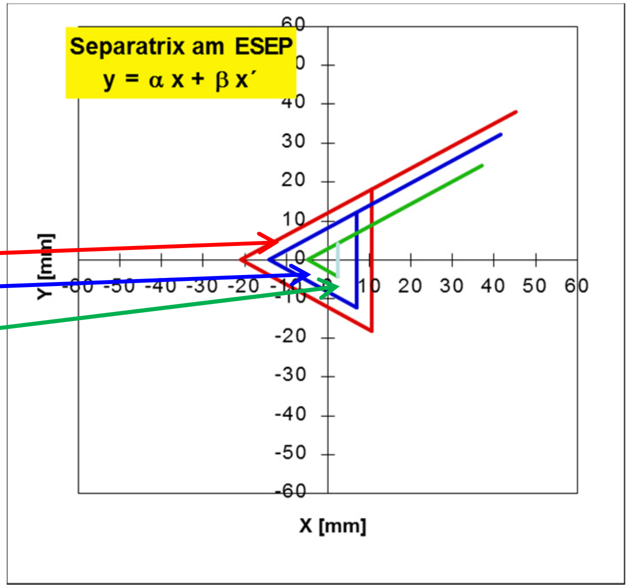
Distance to the resonance Q_{res} $\boxed{\delta Q = -\frac{S}{4\sqrt{3\pi\sqrt{3}}}\cdot\sqrt{\beta\varepsilon}}$

Emittance that touches the separatrix: ε



- Increase the tune of the waiting beam to move the beam into the resonance
- Exactly on resonance δQ = 0



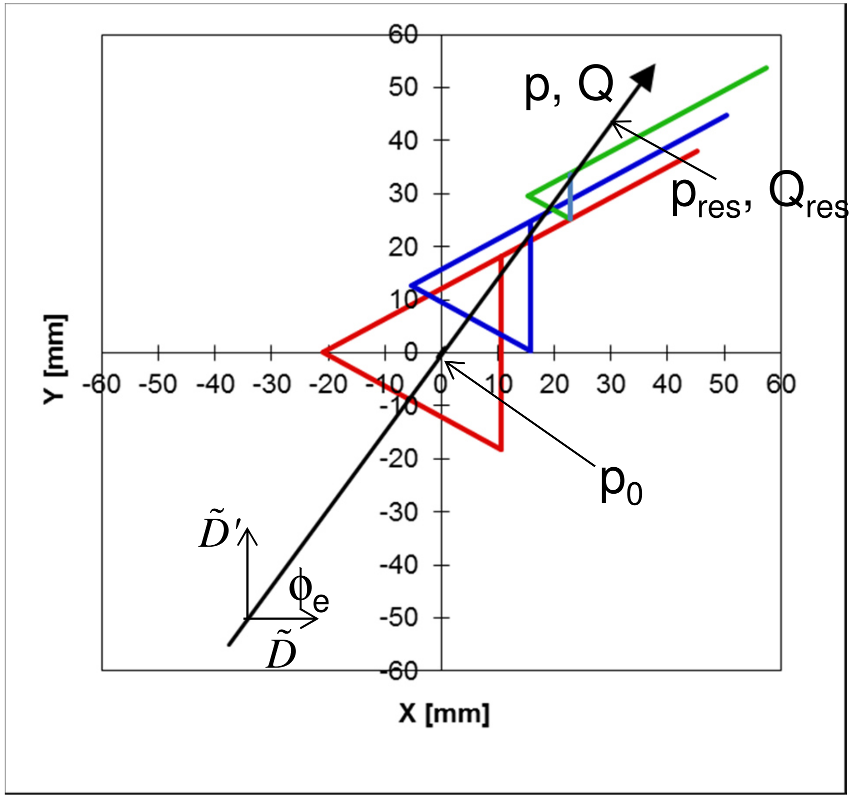


Position D and angle dispersion D' zero

Chromaticity ξ: $\boxed{\delta p = p_0\, \xi\, \frac{\delta Q}{Q_0}}$

- Increase the tune of the waiting stack to move the beam into the resonance
- Particles with different momenta are extracted on different separatrices
  $\Rightarrow$ angle variation! $\Rightarrow$ position variation at MSEP
- Position of separatrix depends on momentum spread if there is dispersion D and D' at the ESEP

At COSY: Position D and angle dispersion D' **non-zero** at ESEP and MSEP:



Positive chromaticity ξ

$$\tilde{D} = D$$

$$\tilde{D}' = (\alpha D + \beta D')$$

$$p_{res} = p_0\left(1 + \frac{1}{\xi}\frac{Q_{res} - Q_0}{Q_0}\right) > p_0$$ at COSY

- Hardt condition to align separatrices of different momenta:

$$\boxed{\xi_h = \frac{S}{4\pi Q_h}\left|\tilde{D}\right| \sin(\phi_e - \varphi_e)}$$

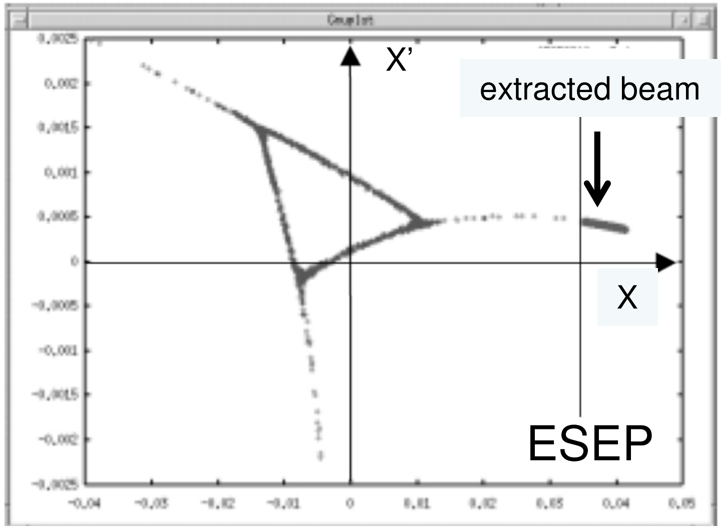
COSY:
at ESEP D > 0 and D' > 0

Better for Hardt condition*):
D < 0 and D' > 0

*) W. Hardt, "Ultraslow Extraction out of LEAR", PS/DL/LEAR Note 81-6

Emittance of Extracted Beam

Particle tracking*)



*) Figure taken from **Marco Pullia, PhD Thesis, 1999**: "Dynamics of slow extraction and its influence on transfer lines design", http://spazioweb.inwind.it/mgp_homepage/

- The **horizontal emittance** of the extracted beam becomes very small.
- A first order estimate follows with *Liouville's* theorem:

$$\varepsilon_{ex} = \varepsilon_h \frac{(\Delta p/p)_{ring}}{(\Delta p/p)_{ex}} \cdot \frac{T_0}{T_S}$$

T_0: revolution period
T_S: extraction duration (spill time)
ring: ring
ex: quantities of extracted beam

- The **vertical emittance** of the extracted and circulating beam are the same.

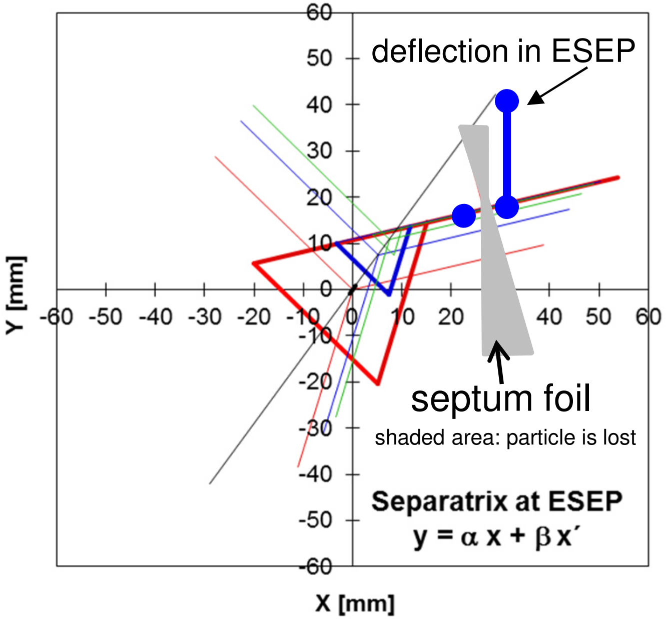
Example Beam Optics for Extraction at 1.9 GeV/c

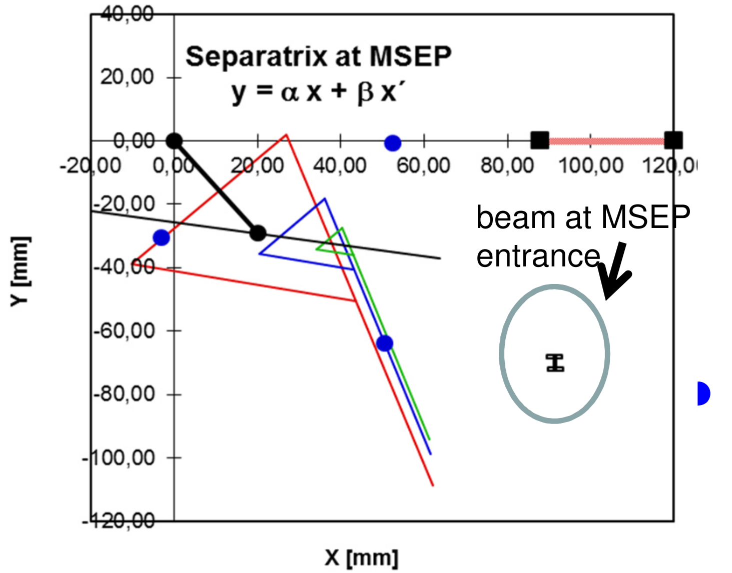
ESEP		
D	5.8	m
D’	1.1	
beta	4.5	m
alfa	0.6	

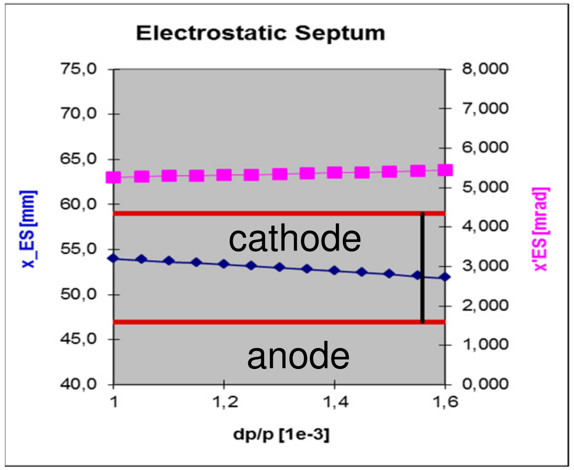
MSEP		
D	14.6	m
D’	0.03	
beta	10.4	m
alfa	-0.2	

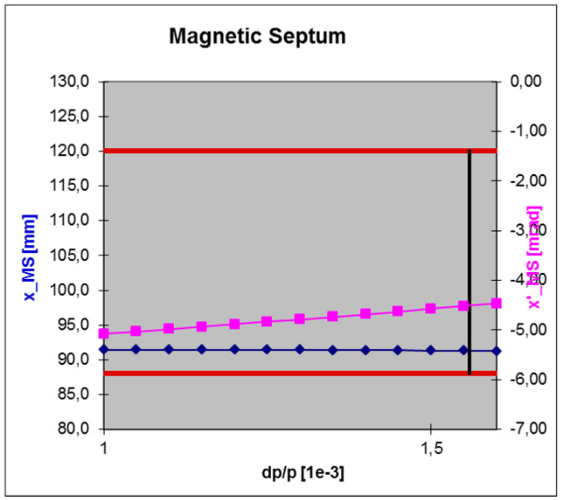
Phase advance	86	degrees
Orbit bump	20	mm
Deflection	5.5	mrad
Position ESEP	47	mm
Gap	12	mm
Separatrix angle	14	degrees
Sextupole strength S	20	m^{-1}
Chromaticity ξ	2.9	

- Hardt condition: all particles leave on the same separatrix branch and jump into the septum without losses.









Effective Sextupole

N sextupoles with strength S_n located at phase μ_n add up to an ***effective sextupole*** with magnitude S and phase μ_S according to*)

$$Se^{i3\mu_S} = \sum_{n=1}^{N} S_n e^{i3\mu_n}$$

where for a single sextupole $$S_n = \frac{k_n' \ell_n}{2} \frac{\beta_n^{3/2}}{\sqrt{\beta_{ES}}} \qquad k_n' = -\frac{B_n''}{B\rho}$$

Example **MXG**: Two sextupoles with **one** power supply

$$\mu_2 = \mu_1 + \frac{Q}{2} \cdot 2\pi$$

$$\beta_1 = \beta_2 \qquad \ell_1 = \ell_2$$

on resonance results in ***S = 0***

contribution ***only*** to chromaticity

*) W. Hardt, "Ultraslow Extraction out of LEAR", PS/DL/LEAR Note 81-6

Calculation of Effective Sextupole

Effective sextupole:

- magnitude S
- angle of outgoing separatrix φ_e

Wanted at the location of the ESEP

$$\varphi_e = \frac{\pi}{6} + \langle \mu_S - \mu_e \rangle \qquad \langle \ldots \rangle = \text{ smallest value mod } \frac{2\pi}{6}$$

μ_e: betatron phase at ESEP $\qquad$ μ_S: betatron phase effective sextupole

- With eight sextupoles in the telescopic straight sections of COSY:

$$Se^{i3\mu_S} = \sum_{n=1}^{8} S_n e^{i3\mu_n}$$

- Equation can be approximately solved: **S**ingle **V**alue **D**ecomposition