Radiative capture of twisted electrons by bare ions. Going beyond the Born approximation

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in collaboration with

V.G. Serbo and V.M. Shabaev

Outline

- Introduction
- Present investigation
- Nonrelativistic results
- Relativistic results
- Summary

What is twisted electrons?

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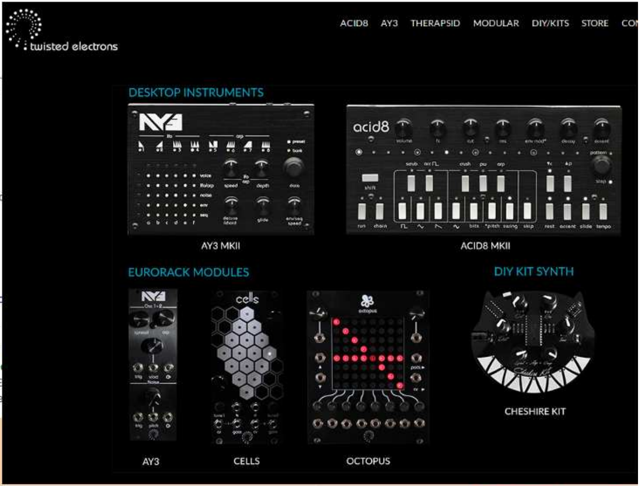

What is twisted electrons?

Short answer

Waves in systems with a cylindrical symmetry

What is twisted electrons? Details

Longer answer

Solutions of the free Dirac equation in cylindrical coordinates

Quantum numbers

E	Energy
p_z	$\mathbf{p}$ projection onto the z axis
j_z	$\mathbf{j}$ projection onto the z axis
μ	Helicity

What is twisted electrons? Details

Explicit form

$$\psi_{\varkappa m p_z \mu}(\mathbf{r}) = \int \frac{e^{im\varphi_p}}{2\pi p_\perp}\delta(p_\parallel - p_z)\delta(p_\perp - \varkappa) i^{\mu - m}\psi_{\mathbf{p}\mu}(\mathbf{r}) d\mathbf{p}$$

Notations

$\varkappa$	$\sqrt{\varepsilon^2 - 1 - p_z^2}$
m	$\mathbf{j}$ projection onto the z axis
μ	Helicity
$\psi_{\mathbf{p}\mu}$	Plane wave

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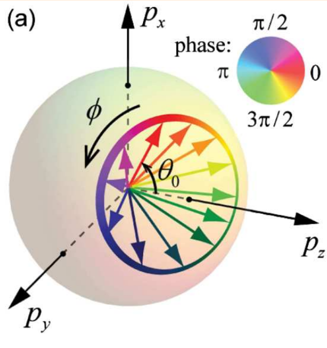

What is twisted electrons?

Predicted

K. Y. Bliokh *et al.*, PRL 2007

Realized

J. Verbeeck *et al.*, Nature 2010

M. Uchida and A. Tonomura, Nature 2010

B. J. McMorran *et al.*, Science 2011

Motivation

- additional degree of freedom m
- spin-orbit interaction increases (m up to 200)
- twistedness becomes the most pronounced for heavy ions

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- it is fashionably

Previous investigations

Nonrelativistic

- R. V. Boxem *et al.*, PRA 2014,
  "Rutherford scattering of electron vortices"
- O. Matula *et al.*, NJP 2014,
  "Radiative capture of twisted electrons by bare ions"
- R. V. Boxem *et al.*, PRA 2015,
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Relativistic

V. G. Serbo *et al.*, PRA 2015,
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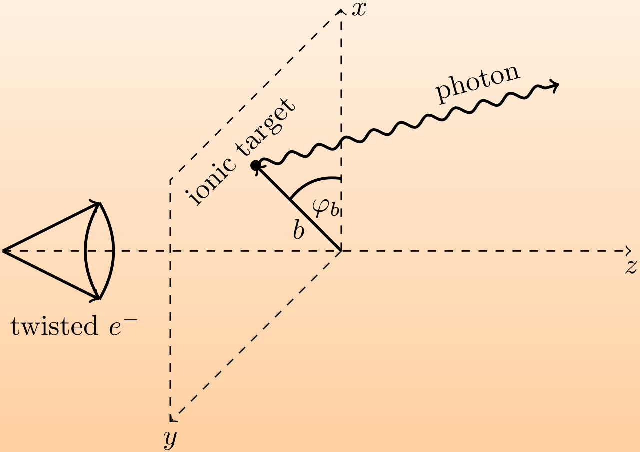
Radiative capture of twisted electrons

$$e^-_{\rm tw} + A^{Z+} \to A^{(Z-1)+}(n\kappa) + \gamma_{\rm ph}$$



Going beyond the Born approximation

Wave function construction

as a solution of the Dirac equation *in the nucleus field* with asymptotics

$$\Psi^{(+)}_{\varkappa m p_z \mu}(\mathbf{r}) \xrightarrow[r\to\infty]{} \psi_{\varkappa m p_z \mu}(\mathbf{r}) + G^{(\mathrm{tw})}\frac{e^{ipr}}{r}$$

the explicit form

$$\Psi^{(+)}_{\varkappa m p_z \mu}(\mathbf{r}) = \int \frac{e^{im\varphi_p}}{2\pi p_\perp}\delta(p_\parallel - p_z)\delta(p_\perp - \varkappa) i^{\mu - m}\Psi^{(+)}_{\mathbf{p}\mu}(\mathbf{r}) d\mathbf{p}$$

where

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Macroscopic target

Target

- Infinite size
- Ions are distributed randomly and uniformly

Integrated over **b** cross section

$$\frac{d\sigma^{(\mathrm{tw})}}{d\Omega_k} = \frac{1}{\cos\theta_p} \int_0^{2\pi} \frac{d\varphi_p}{2\pi} \frac{d\sigma^{(\mathrm{PW})}}{d\Omega_k}$$

Only the capture into $1s$ state is studied

Nonrelativistic formalism. Cross section

$$R_{\mathrm{NR}}(\nu) = \frac{d\sigma^{(\mathrm{NR})}/d\Omega_k}{d\sigma^{(\mathrm{NR,\ B})}/d\Omega_k} = \frac{2\pi\nu}{(1+\nu^2)^2}\frac{e^{-4\nu\cot^{-1}\nu}}{1-e^{-2\pi\nu}}$$

$\nu = \alpha Z/p$
$\nu \to 0$ corresponds to the Born approxiamtion

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O. Matula *et al.*, NJP 2014

For $E_{\rm kin} = 2$ keV and $Z = 1$

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For $E_{\rm kin} = 2$ keV and $Z = 1$ one gets $R_{\rm NR} = 0.77$

Nonrelativistic formalism. Relative observables

Angular distribution

$$\frac{d\overline{W}^{(\mathrm{tw,\ NR})}}{d\Omega_k} = \frac{3}{4}\left[\left(2 - 3\sin^2\theta_p\right)\sin^2\theta_k + 2\sin^2\theta_p\right]$$

Polarization

$$P_l^{(\mathrm{tw,\ NR})} = \frac{\left(2 - 3\sin^2\theta_p\right)\ \sin^2\theta_k}{\cos^2\theta_k\sin^2\theta_p + 2\sin^2\theta_k\cos\theta_p^2 + \sin^2\theta_p}$$

No dependence on ν parameter!

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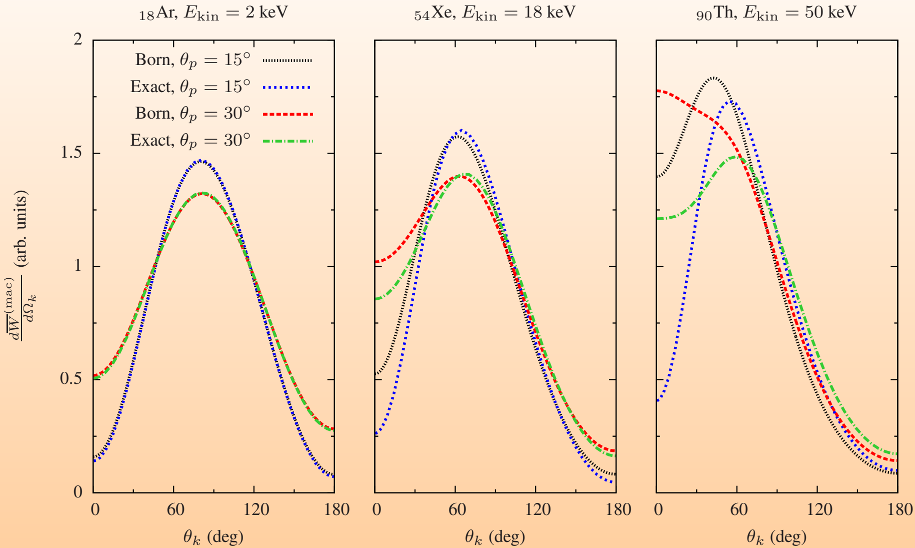
Relativistic formalism. Angular distribution



Relativistic formalism. Polarization

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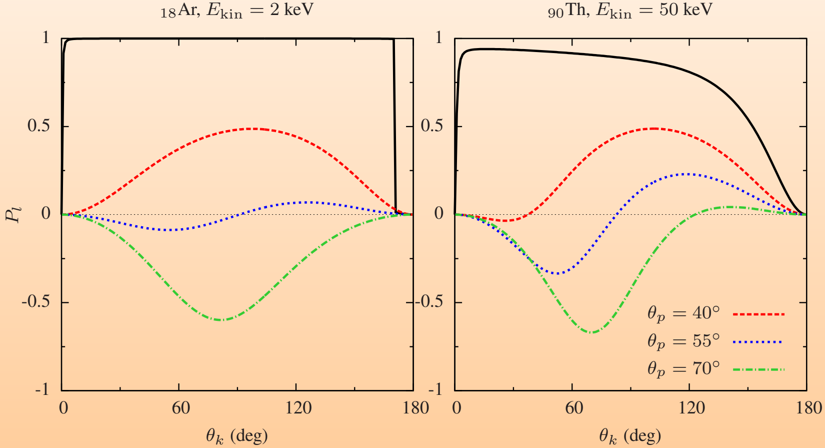


Summary

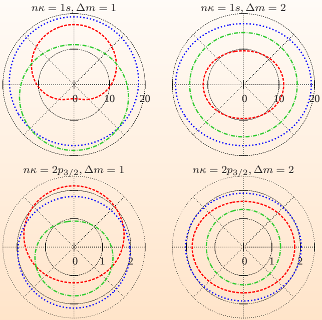
- Wave function

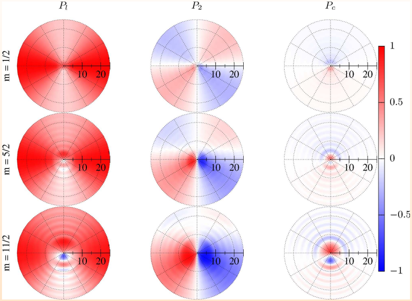
$$\Psi^{(+)}_{\varkappa m p_z \mu}(\mathbf{r}) = \int \frac{e^{im\varphi_p}}{2\pi p_\perp} \delta(p_\parallel - p_z)\delta(p_\perp - \varkappa) i^{\mu - m} \Psi^{(+)}_{\mathbf{p}\mu}(\mathbf{r}) d\mathbf{p}$$

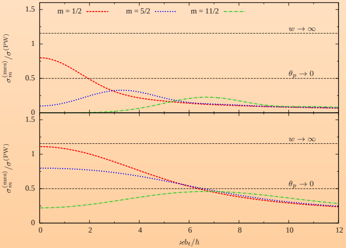
- Importance of the calculations beyond the Born approximation

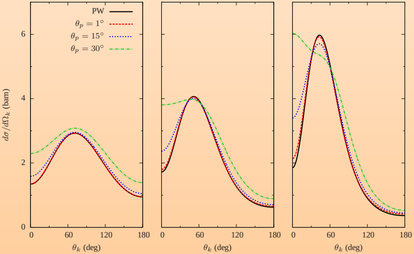
$$R_{\mathrm{NR}}(\nu) = \frac{d\sigma^{(\mathrm{NR})}/d\Omega_k}{d\sigma^{(\mathrm{NR,\ B})}/d\Omega_k} = 0.77\ (e^{(\mathrm{tw})}_{2\mathrm{keV}} + H^+)$$

- Relativistic description was performed


$n\kappa = 1s, \Delta m = 1$
$n\kappa = 1s, \Delta m = 2$
$n\kappa = 2p_{3/2}, \Delta m = 1$
$n\kappa = 2p_{3/2}, \Delta m = 2$
0
10
20
1
2


P_l
P_2
P_c
m = 1/2
m = 5/2
m = 11/2
10
20
1
0.5
0
−0.5
−1


m = 1/2
m = 5/2
m = 11/2
$w \to \infty$
$\theta_p \to 0$
$\sigma_m^{(\mathrm{mes})}/\sigma^{(\mathrm{PW})}$
$\varkappa b_t/\hbar$
0
2
4
6
8
10
12
0.5
1
1.5


PW
$\theta_p = 1°$
$\theta_p = 15°$
$\theta_p = 30°$
$d\sigma/d\Omega_k$ (barn)
θ_k (deg)
0
60
120
180
2
4
6