D and D_s decay constants from lattice QCD

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Abstract: I describe the issues around simulating charm quarks in lattice QCD calculations and how the development of an improved method for doing this has revolutionised the accuracy possible from lattice QCD results for D and D_s masses and decay constants, which can be compared to experiment. It has also allowed the accurate determination of other quantities such as the charm and light quark masses. I discuss the current status of results and prospects for the future.

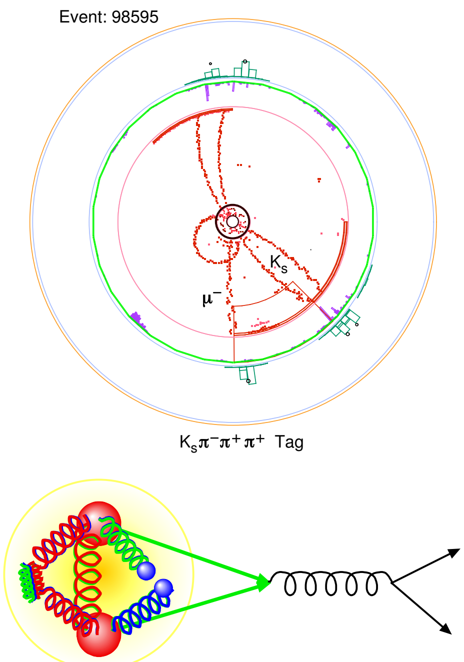

Figure 1: The top figure shows a CLEO-c event containing a $D_s^- \to \mu\nu$ decay (the accompanying D_s^+ decayed to K_s and 3 π). The lower figure sketches the process in which the valence quark and antiquark inside a meson annihilate to a W boson that is seen as a lepton and antineutrino.

Introduction

Quantum Chromodynamics (QCD), which describes the strong force, is a key part of the Standard Model of particle physics. One reason it is so important is because of the confinement property of the strong force that means that the fundamental particles described by QCD, quarks and gluons, are never seen in the real world. The only particles accessible to experiment are their bound states called hadrons. To connect experiment to the quark and gluon world then requires calculations in QCD. A lot of our confidence in QCD comes from high energy experiments where behaviour such as the appearance of 'jets' can be described by perturbation theory in α_s, the coupling constant of QCD. These tests are compelling evidence for QCD but not precision tests of the theory because it is generally not possible to do the perturbation theory to high enough order and/or there are systematic errors affecting the interpretation of the experiment, for example from the identification of jets with quark or gluon production at the fundamental vertex.

The techniques of lattice QCD, on the other hand, allow us to calculate simple properties of hadrons fully nonperturbatively. Masses for 'gold-plated' hadrons, i.e. those which are stable in QCD, are often known extremely accurately from experiment [1]. They can now be calculated accurately in lattice QCD because the full effect of 'sea quarks', missing for many years, can be included using current supercomputers [2].

The simplest quantities to calculate in a lattice QCD calculation besides hadron masses are hadron decay constants. These are defined from the amplitude for that hadron to annihilate to the QCD vacuum via a W boson (for a pseudoscalar) or a photon (for a vector) given the appropriate valence quark content (see Fig. 1). In the pseudoscalar case:

$$< 0|\overline{\psi}\gamma_0\gamma_5\psi|H >= f_H m_H \quad (1)$$

where m_H is the mass of the hadron and f_H is its decay constant. This decay constant is then directly related to the branching fraction to leptons that can be measured in experiment via

$$\Gamma(H \to \mu\nu) = \frac{G_F^2|V_{ab}|^2}{8\pi} f_H^2 m_\mu^2 m_H \left(1 - \frac{m_\mu^2}{m_H^2}\right)^2 . \quad (2)$$

Here V_{ab} is the Cabibbo-Kobayashi-Maskawa matrix element, a parameter in the Standard Model, that links quark of flavor a and antiquark of flavor b to the W boson. f_H is a property of the meson in QCD. It can loosely be thought of as the amplitude for the quark and antiquark to be in the same place to annihilate, given all the QCD interactions going on inside the meson. Comparison of the lattice QCD calculation to the experimental hadronic decay rate given above can be used to determine the CKM element V_{ab}, or, if this is known from other processes, it can be used as a stringent test of lattice QCD and QCD itself.

Lattice QCD calculations also enable the parameters of QCD such as the quark masses and coupling constant to be accurately determined, because we have direct access to these parameters in the lattice QCD Lagrangian. QCD

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has very few parameters and consequently lattice QCD has enormous predictive power if we can do the calculations accurately enough.

Lattice QCD calculations

Lattice QCD calculations proceed by a Monte Carlo evaluation of the Feynman Path Integral (FPI) representation of the vacuum expectation value of some operator, $\mathcal{O}$. To calculate a hadron correlation function, for example, i.e. the amplitude to create a hadron at some initial time and then destroy it at some later time, $\mathcal{O}$ becomes the product of a hadron creation operator and a hadron annihilation operator. The correlation function is then represented by $< 0|H^{\dagger}(0)H(t)|0 >$. The expectation value is given by the ensemble average of the hadron correlation function calculated on sets of gluon field configurations that are chosen using appropriate importance sampling for the FPI. This requires that the probability distribution of the gluon fields follows that of $\exp(-S_{QCD})$ where S_{QCD} is the QCD action (integral over the Lagrangian). In this sense the gluon field configurations are said to be 'typical snapshots of the vacuum'.

Quarks cause technical problems because, having anticommuting fields, they cannot be readily represented on a computer. Instead the quark fields are integrated out of the FPI (this is straightforward because of the form of the QCD action), leaving behind a function of the gluon fields. This function depends on the matrix $M \equiv \gamma \cdot D + m$ through which gluon fields interact with quarks in QCD (the gluon field appears in the covariant derivative D). The function has two pieces because quark fields appear in two places in the FPI; in the quark piece of the QCD action and in the hadron creation and annihilation operators. The first piece represents the presence of the sea quarks in the vacuum produced by energy fluctuations that give rise to a quark-antiquark pair. This piece gives a factor of $\det(M)$. The second piece represents the propagation through the gluon fields of the valence quarks that make up the hadron H and give it its quantum numbers. This piece gives factors of M^{-1}. The FPI is then a well-defined integral which can be calculated numerically if space-time is split up into a 4-d lattice of points so that there are only a finite number of fields.

The factor of $\det(M)$ is part of the probability distribution of the gluon fields in the FPI and must be included in the importance sampling. This is numerically expensive and was ignored in the early years of lattice QCD when we used the 'quenched approximation'. This approximation does not destroy QCD but clearly misses out an important piece of it in the form of the sea quarks. The most important sea quarks are the lightest ones, since they are the most readily produced by vacuum fluctuations. So we need to include u, d and s quarks in the sea and now 'unquenched' or 'dynamical' calculations are able to do this. u and d quarks are taken to have the same mass and this is generally larger than their physical mass, because this is so light. Inclusion of $\det(M)$ becomes numerically very expensive as the quark mass becomes smaller. How important it is to have a very light u/d mass does depend on the calculation of interest. For the calculations I shall describe here extrapolations to the physical u/d mass point from results at a range of accessible values is mild, but for other calculations it can be more of an issue.

The calculation of a hadron correlation function then proceeds by inverting the matrix M for each of the valence quarks needed for the hadron in question on each of an ensemble of unquenched gluon configurations made with a particular set of QCD parameters (about which more below). These quark propagators are tied together to make the hadron correlation function, inserting appropriate Dirac γ matrices to give the right spin-parity quantum numbers. We often sum over spatial sites at the annihilation time, t, to project onto zero spatial momentum. The result for the hadron correlation function is then averaged over the entire ensemble. To determine the hadron mass m_H we have to fit the averaged hadron correlation function as a function of t to the form:

$$\begin{aligned} < 0|H^{\dagger}(0)H(t)|0 > &= A_0(e^{-m_H t} + e^{-m_H(T-t)}) + \\ \sum_i & A_i(e^{-m_i t} + e^{-m_i(T-t)}) \quad (3) \end{aligned}$$

where T is the time extent of the lattice. All hadrons of the spin, parity and flavour created by $H^{\dagger}$ appear in the correlation function in principle and this is why there are multiple exponentials. The ground state hadron in this channel, denoted by H, is the one of lightest mass which therefore dominates at large t. Here I will be focussing on ground states only. They are relatively easy to pick out of correlation functions but multi-exponential fits must nevertheless be used if this is to be done accurately [10]. On the lattice we only have values for the hadron correlation function for t an integral number of time slices from the source (creation) time, say $t = \tilde{t}a$, where a is the lattice spacing. Then we actually fit the hadron correlation function for the integers $\tilde{t} = 1, 2, 3 \ldots$ to the form above and extract $\tilde{m}_H = m_H a$, the hadron mass *in lattice units*. In order to extract m_H to physical units we need to know the value of a.

It is important to realise that we do not in principle know a until after we have done a lattice QCD calculation. a does not appear explicitly anywhere in the calculation. The parameters in the QCD action are the coupling g and the quark masses, m_q. In the lattice QCD action the quark masses appear as dimensionless parameters $m_q a$. Before the calculation we need to set these parameters to some values (of course by now we have a lot of experience in knowing roughly what values to take) but then after the calculation we must determine the lattice spacing that has resulted from these parameter choices. We determine the lattice spacing by fixing it from a particular hadron mass (which is then no longer an output from the calculation). For example, using the hadron H, above, we would have $a^{-1} = m_H/\tilde{m}_H$ in GeV. Once a is determined then all

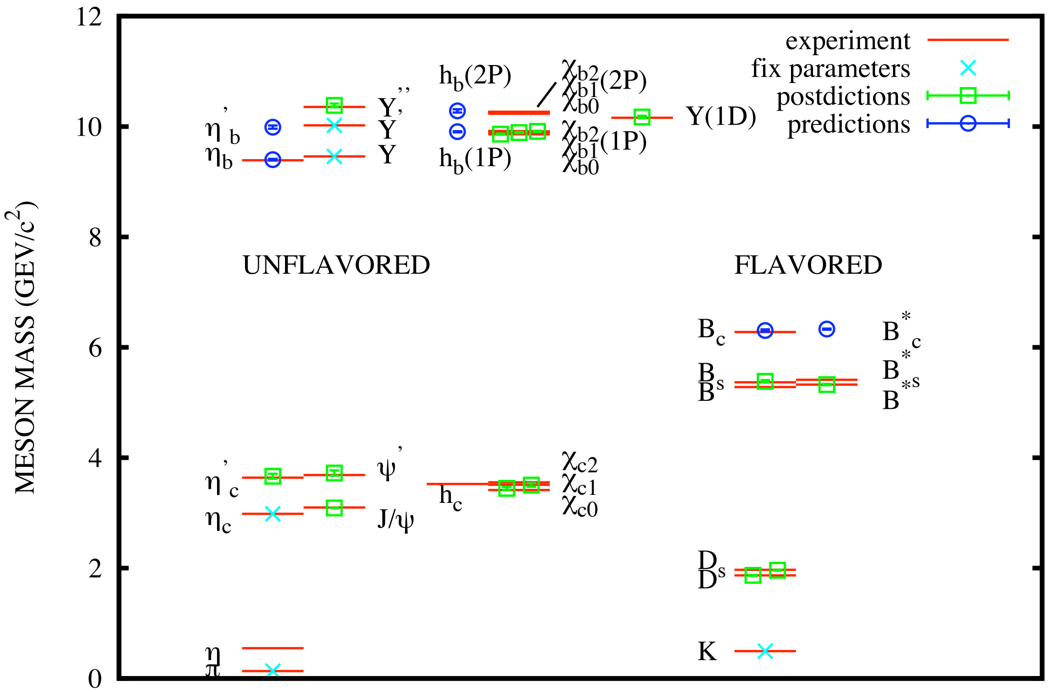

Figure 2: The spectrum of 'gold-plated' mesons from lattice QCD calculations by the HPQCD collaboration. Results are divided into those used to fix the parameters of QCD (4 quark masses and a coupling constant); those which are postdictions [3, 4, 5] and those which are predictions [3, 6, 7].

other hadron masses can be converted into GeV and compared to experiment. It is then necessary to tune the quark masses by adjusting these until other hadron masses (one for each quark mass) are correct. From this it is clear that it is a good idea to choose a hadron mass to fix the lattice spacing that is not strongly dependent on quark masses. For obvious reasons it should be gold-plated. A good example of such a quantity is the radial excitation energy in the Υ system [3, 11]. Clearly the hadron mass used to fix a particular quark mass should also be gold-plated and preferably strongly dependent on that quark mass and not strongly dependent on others.

As explained earlier, u and d quark masses are taken to have larger values than their physical ones, because they are so light. We also take $m_u = m_d = m_l$. m_l can then only be determined by extrapolation of the mass of the π meson to the physical point. There is no problem in having a physical value for the s quark mass in the calculation, but its value in principle is tuned from that of the K meson which also contains a u/d quark. In practice, we can use the mass of the fictitious η_s particle (an $s\overline{s}$ pseudoscalar) to tune the s mass. The η_s mass can be determined in terms of the K and π masses from lattice QCD calculations [11] and it is not sensitive to the u/d mass. The reason it does not appear in the real world is because of mixing with other flavour-singlets, which can be prevented in the lattice calculation. There is no difficulty in tuning the values of the c and b masses to their physical values. The obvious mesons to use for these are the η_c or J/ψ and the Υ. Having fixed the quark masses and the lattice spacing there are no more parameters to fix, and all other hadron masses are an output of the calculation that can be compared to experiment. In this way QCD as a theory covers a huge range of hadron physics (from π meson physics to Υ physics) with only 5 parameters.

Many gluon field configurations are now available that include the effect of u/d and s sea quarks (although some collaborations include just u/d). The figures of 'merit' for these configurations are the lattice spacing (as small as possible) and $m_{u/d}$ (also as small as possible) and the spatial volume (as large as possible). Most of the results I will describe here come from gluon configurations generated by the MILC collaboration [12]. They include u, d and s sea quarks using the improved staggered quark formalism. They have many sets of configurations, all of size $>$ (2.4fm)^3 with a range of lattice spacing values from 0.15fm to 0.045 fm and values of $m_{u/d}$ down to $m_s/10$. Their configurations are the most extensive set available because the quark formalism is a numerically fast one.

A compendium of first results from the HPQCD/Fermilab Lattice and MILC collaborations using these configurations was given in [2] showing accurate agreement with experiment across the range of hadron physics once sea quarks were included. Fig. 2 gives an update from the HPQCD collaboration of the gold-plated hadron spectrum since then showing the *predictions* of hadron masses that have been made. The masses of the B_c and η_b were subsequently measured by

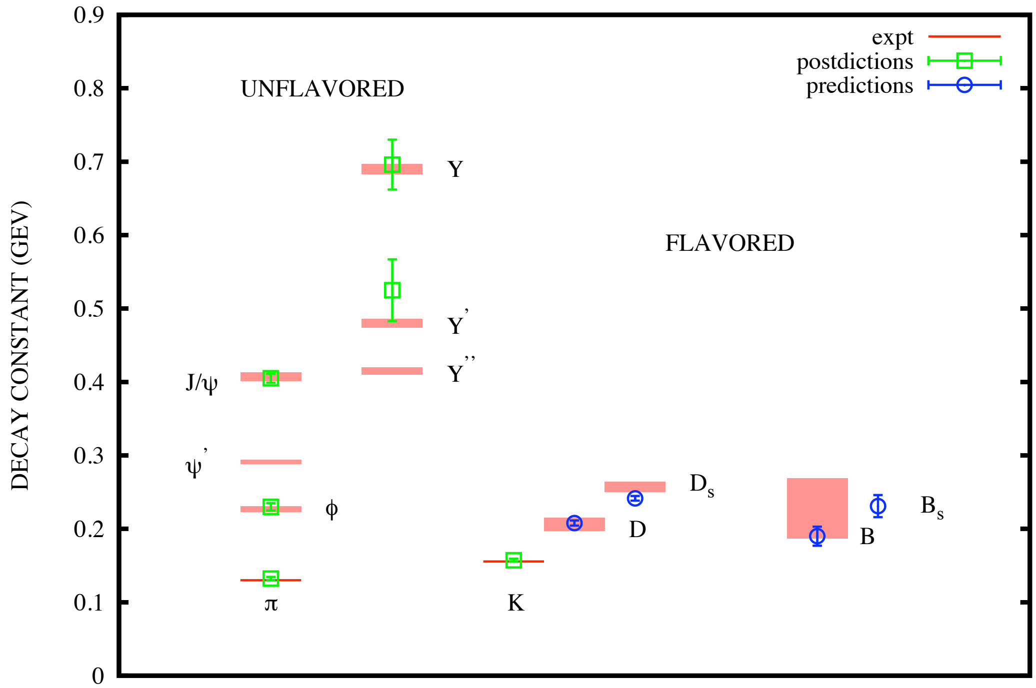

Figure 3: A summary of decay constant values for gold-plated vector and pseudoscalar mesons from lattice QCD calculations by the HPQCD collaboration [5, 8]. Results for vectors are preliminary [9].

experiment [13, 14].

Decay constants are also obtained from the fit to hadron correlation functions in eq. 3 if we use the operator of eq. 1 that couples to the W or γ as the hadron creation/annihilation operator. Then A_0 is simply related to $f_H^2 a^2$ [5]and f_H can easily be extracted. Figure 3 gives a summary of the current status of decay constants from lattice QCD calculations by the HPQCD collaboration for pseudoscalar and vector mesons.

Since [2] results have significantly improved in charm physics and the reason for this is a much improved handling of charm quarks in lattice QCD that has better control of a number of systematic errors. This has allowed accurate masses to be determined for the D and D_s mesons as well as their decay constants.

Heavy quarks in lattice QCD

There are a number of different discretisations of the quark action possible in lattice QCD and each formalism has advantages and disadvantages. An important issue is that of discretisation errors. When a hadron mass m_H (in GeV) is determined in a lattice QCD calculation the result will still depend on the lattice spacing because of these errors. The dependence will typically be of the form:

$$m_H = m_{H,a=0}(1 + A(\Lambda a)^2 + B(\Lambda a)^4 + \ldots) \qquad (4)$$

for a good discretisation of the quark action (early versions had errors at $\mathcal{O}(a)$ as well and some discretisations do have errors with odd powers of a). These errors must be extrapolated away by fitting results as a function of a, but clearly the smaller the errors are, the more accurate the extrapolated result. To reduce the errors we can 'improve' the discretisation of the Dirac action so that the first powers of a that appear above are as as high as possible. Then, if $\Lambda a < 1$ the errors will be reduced. The quantity Λ that sets the scale for discretisation errors is a typical QCD scale for light hadrons, i.e. of order a few hundred MeV. For charm quarks, however, Λ can be set by m_c, which is much larger than a few hundred MeV. Then, if we have a lattice spacing such that $m_c a > 1$, the discretisation errors will be large and no amount of improvement will necessarily help. However, $m_c a \approx 0.5$ for current values of a and so then improving the discretisation of the Dirac action to remove errors at $(m_c a)^2$ and the leading errors at $(m_c a)^4$ leaves discretisation errors at the few percent level (A and B are power series in α_s, so removing the tree level pieces of A and B significantly improves the size of the error even if it does not completely remove the error at that order in a). This is what we have been able to do with the Highly Improved Staggered quark (HISQ) action [4]. This action can also be used for u, d and s quarks where it is also an improvement over the standard improved staggered action. Using the same action for all 4 lightest quarks has significant advantages as we will see below.

The alternative to using a highly improved action of this kind is to take advantage of the nonrelativistic nature of the bound states of heavy quarks to effectively replace m_c with Λ again in the forms above. However, the price paid is that we are then using a nonrelativistic effective theory, and we no longer have some of the symmetries of the continuum quark action. For example it is no longer true that the energy of the hadron at zero momentum is its mass, since we have an energy offset. To determine the hadron mass (and therefore tune the quark mass) is more complicated and statistically less accurate as a result. We have to account for relativistic corrections to our nonrelativistic action. We no longer have a partially conserved axial current and this means that the decay constant that we calculate in our lattice QCD calculation must be renormalised to match the continuum current, before we can compare to experiment. This brings in a large systematic uncertainty unless this matching can be done accurately.

For b quarks, because $m_b a$ is so large on existing lattices, we have no choice but to use nonrelativistic methods, such as NRQCD [3] (but see Future Prospects below). Nonrelativistic methods also work well for b because it is so nonrelativistic. For c quarks the issue is more finely balanced. The early work on c physics on the MILC configurations was done using the 'Fermilab' heavy quark formalism [15] developed over ten years ago. This is a hybrid nonrelativistic/relativistic method which for small values of ma becomes the standard 'clover' formalism (which in principle has $\mathcal{O}(a)$ errors). For large ma, for c and b, a nonrelativistic approach to determining the meson mass allows the discretisation errors to be reduced at the cost of other systematic errors discussed above. The decay constant of the D and D_s mesons was determined with 6% errors using this method before experimental results were available [16]. Improvements can be made to this calculation [17] but it seems clear that a fully relativistic and highly improved formalism has advantages for c quarks, as we move to finer and finer lattices and $m_c a$ becomes smaller. Another promising formalism of this kind is the twisted mass formalism espoused by the European Twisted Mass Collaboration [18]. This formalism comes from the same generic set as the clover formalism but is more highly improved so that discretisation errors start at $\mathcal{O}(a^2)$.

Errors in lattice masses and decay constants

The method of extracting a hadron mass and decay constant from the lattice calculation of a hadron correlation function is described above. How accurately the calculation can be done depends on several factors. It is important to understand what these factors are, how they vary between different calculations and what checks of the errors can and have been done so that it is clear whether they are reasonable. It is obvious, for example, that one might compare the error in calculating masses and decay constants of D and D_s mesons to that for calculating these quantities for K and π mesons, and expect to be able to achieve similar accuracy. Early calculations of D and D_s meson decay constants [16], however, were much poorer than those for π and K for a number of reasons, and this led to unnecessarily low expectations for D and D_s lattice calculations despite the fact that this was a key physics goal of the CLEO-c experiment.

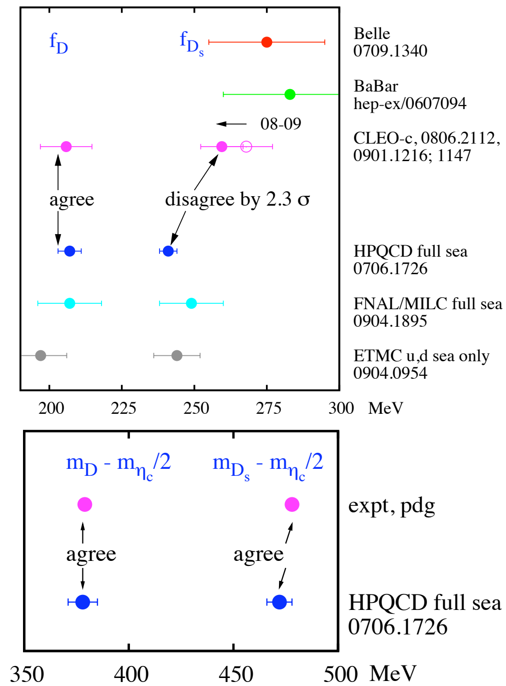

Figure 4: The upper plot shows the status of D and D_s decay constants from lattice QCD and experiment as of Spring 2009. The shift in the CLEO-c average from summer 2008 to January 2009 is shown. The BaBar result has since been superseded by that in Fig. 5. The lower figure shows the status of lattice calculations of the mass differences between D and D_s and one half the η_c mass that acts as an important check of the decay constant calculation.

One issue in doing lattice calculations is the statistical accuracy of the ensemble average. High statistical accuracy is obviously good, but it also helps in being able to determine systematic errors, for example from discretisation effects. For good statistical accuracy you need a large ensemble with many configurations (which are suitably decorrelated with respect to each other despite having been generated in sequence). Reasonable ensemble sizes in current calculations are of order 500 configurations. You can improve statistical accuracy by calculating hadron correlators from several different points within the configuration. Using random sources also works well for mesons - by putting random numbers as a source for the inversion of M it is equivalent to having several different sources for a meson when the propagators are combined together. This was standard practice for π and K mesons [19] but is only now being used for D and D_s calculations [5].

In addition there is the issue of intrinsic signal/noise in the meson correlator. This depends on the meson you are studying. Using the methods above it has been possible for some time to make statistically very accurate hadron correlation functions for π and K mesons, because they have very good signal/noise properties. The noise in a meson correlation function is related to the square of that corre-

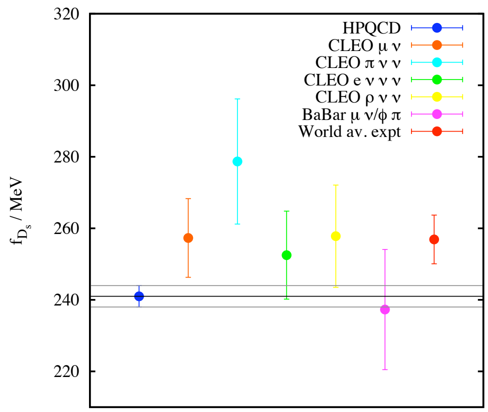

Figure 5: A comparison of the HPQCD lattice calculation of f_{D_s} [5] with results from individual channels for the CLEO-c experiment [27, 30]. The updated result using BaBar data by HFAG is also given, as well as the HFAG average of September 2009 [31].

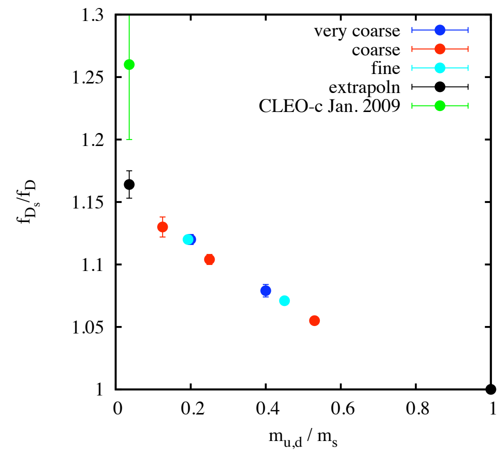

Figure 7: The ratio f_{D_s}/f_D as a function of the light quark mass in units of the s quark mass at 3 different values of the lattice spacing [5].

lation function. The square consists of 2 quark and 2 antiquark propagators but they can rearrange themselves inside the noise to correspond to two different mesons. For example the noise in a K meson can correspond to a π and an η_s (pseudoscalar $s\overline{s}$) meson. If $m_\pi + m_{\eta_s} < 2m_K$ the noise will decay in time more slowly than the signal (from eq. 3) leading to poor signal/noise. This is not a considerable problem for the K, but becomes worse for heavier mesons (because the corresponding heavy-heavy mesons are more deeply bound). Thus it is starting to become noticeable for the D and D_s mesons and it becomes worse for the B and B_s mesons [20]. The statistical error on D and D_s is then slightly worse than that possible for K and π but not by a large amount.

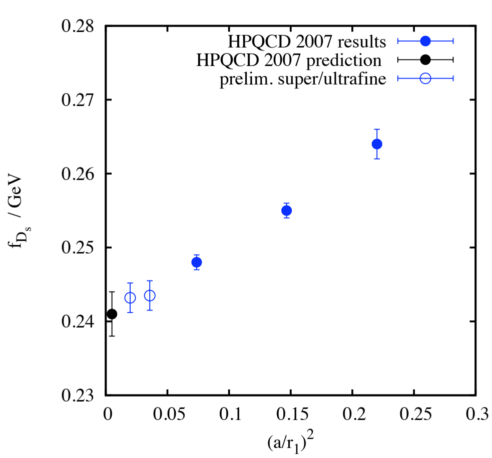

Figure 6: HPQCD results for f_{D_s} plotted against the square of the lattice spacing (in units of the parameter r_1 [11]) comparing results from [5] with new results on two sets of MILC configurations with finer lattice spacing.

The other issues for accuracy are the various sources of systematic error. As discussed above, it is generally not possible to include u/d quarks with their physical masses and so one must do the calculations at unphysical masses and extrapolate to the physical point guided by chiral perturbation theory. Obviously π and K meson masses and decay constants depend on the value of the u/d mass since they contain valence light quarks and so this extrapolation will be important. (It is worth pointing out here that lattice QCD calculations can have different valence u/d quark masses from the sea u/d quark masses and in principle these two mass dependences can be separated. In the real world, of course, all u/d quarks are the same). At the same time one has to extrapolate the results from several values of the lattice spacing to $a = 0$. These two extrapolations are generally done together using various versions of lattice chiral perturbation theory. The size of discretisation errors for a given quantity depends on how sensitive that quantity is to the lattice spacing. For light hadrons, as discussed above, the scale that sets the size of discretisation errors is typically Λ_{QCD}, i.e. a few hundred MeV. Thus for π and K mesons the chiral extrapolation is a much bigger issue than the discretisation errors. For mesons containing charm quarks, the size of discretisation errors will be set by the charm quark mass and so will be typically much larger. This is why it was so important to develop the HISQ action, improved to high order, to reduce the size of these errors. However, for the D_s at least, the chiral extrapolation is only a very small effect because the only u/d quarks appear in the sea for this meson. π and K mesons have some sensitivity to the finite volume of the lattice, which also gives a systematic error. For D and D_s mesons, because they are much smaller, this is much less of an issue.

Calculations of D and D_s masses and decay constants that are almost as accurate as those of K and π should therefore be possible provided that the discretisation errors are well controlled. Indeed for the D_s meson this is really the only issue. With the HISQ action this control of discretisation errors is possible - and statistically accurate results enable the fit to the lattice spacing dependence to be well determined and extrapolated to $a = 0$ [5].

Lattice QCD results

D and D_s mesons – masses and decay constants

Fig. 4 shows the status from Spring 2009 of lattice and experimental results for the masses and decay constants of the D and D_s mesons. The experimental results for the decay constants are obtained from the leptonic decay rate using $V_{cd} = V_{us}$ and $V_{cs} = V_{ud}$. The conclusion from the figure is that f_D agrees well between lattice QCD and experiment but that f_{D_s} does not. The lattice QCD results are dominated in error by those from the HPQCD collaboration [5] and the experimental results are dominated by those from CLEO-c [27].

This figure has developed a lot over the last two years and it is worth discussing its history briefy. The original Fermilab Lattice/MILC results including sea u/d and s quarks for the first time [16] were :

$$\begin{aligned} f_D &= 201(3)(17) \quad \text{MeV} \\ f_{D_s} &= 249(3)(17) \quad \text{MeV} \end{aligned} \tag{5}$$

where the error budget was dominated by uncertainties in determining a and m_c, relativistic corrections and chiral extrapolations. Experimental results at that stage had uncertainties of $> 10\%$. Subsequent experimental results, appearing shortly afterwards, were:

$$\begin{aligned} f_D &= 223(16)(9)\text{MeV} \quad \text{CLEO}-\text{c}\,[21] \\ f_{D_s} &= 283(17)(7)(14)\text{MeV} \quad \text{BaBar}\,[22]. \end{aligned} \tag{6}$$

Although the differences with lattice QCD at this stage were not significant, these early experimental results already set the tone of being larger, particularly for f_{D_s}. Subsequent results from CLEO-c for f_{D_s} were also high.

During 2007 the HPQCD collaboration produced 2% accurate results for f_D and f_{D_s} as a result of developing the HISQ action described above and using the MILC gluon configurations at 3 values of the lattice spacing down to 0.09fm. We spent a significant amount of time testing the action in the charmonium sector [4] and more work there is ongoing. Charmonium mesons, being smaller, are more susceptible to discretisation errors which means that they are an excellent test of actions for charm physics, that gives confidence in our understanding of systematic errors. Charmonium calculations are statistically very precise which is also good because we can use the η_c meson to fix m_c very accurately. Our paper gave [5] :

$$\begin{aligned} f_D &= 207(4) \quad \text{MeV} \\ f_{D_s} &= 241(3) \quad \text{MeV} \end{aligned} \tag{7}$$

with a complete error budget. The uncertainties are dominated by uncertainty in the value of the lattice spacing and errors arising from the $a \to 0$ extrapolation. This is not surprising given the discussion about the importance of these in charm physics above. There are no errors coming from relativistic effects or uncertainties in normalising the decay constant because we are using a relativistic discretisation with an absolutely normalised current.

We also gave a number of other quantities that allowed tests of the systematic errors quoted. These included f_K and f_π using the HISQ formalism for the u/d and s quarks in agreement with results from experiment with 1-2% errors. In addition we calculated the mass of the D and D_s mesons. This last test is a particularly important one because the masses come from exactly the same correlators as those used for the decay constants and the chiral extrapolations to the physical u/d mass limit are linked together in chiral perturbation theory. In fact what is calculated is the mass difference between the D and D_s masses and one half the η_c mass. This is a much smaller number than the D/D_s mass itself and so, because we can obtain the mass difference directly in lattice units from our calculation, we have a smaller absolute error coming from the uncertainty in the lattice spacing than if we directly determine the D/D_s mass. The lower figure in Fig. 4 gives these results. This test has not been done in any other formalism.

Subsequent experimental results in the summer of 2008 caused quite a stir. The CLEO-c measurement of f_D [21] :

$$f_D = 205.8(8.5)(2.5) \quad \text{MeV} \tag{8}$$

agreed very well with the HPQCD result. The results for f_{D_s}, along with a result from Belle [23] did not. The 2008 particle data tables [24] quoted an average of $f_{D_s} = 273(10)$ MeV excluding the BaBar result. The discrepancy with HPQCD exceeded 3σ where the size of σ was dominated by the experimental uncertainty because the HPQCD result was so accurate. This was a shocking result because it was rather difficult to think of new physics processes that could affect D_s leptonic decay and not the D [25]. At least the fact that the experimental result was not below the lattice QCD result could be used to rule out parameter space for a charged Higgs [26].

Meanwhile the Fermilab Lattice and MILC collaborations concentrated on improving their statistical precision and extrapolation uncertainties continuing to use the Fermilab method, with a Lattice 2008 conference result of $f_{D_s} = 249(11)$ MeV [28], plotted in Fig. 4. At the same conference the European Twisted Mass Collaboration produced new results using the twisted mass formalism. This is another fully relativistic formalism, like HISQ. It has discretisation errors at $\mathcal{O}(a^2)$ (so in principle somewhat worse than HISQ) and it has an absolutely normalised decay constant. Currently the results from this formalism are obtained on configurations which include only u/d quarks in the sea and not the full complement of light quarks. It is unclear what error to take from the omission of sea s quarks, and none is quoted. The ETMC results, plotted in Fig. 4 are [29]:

$$f_D = 197(9) \quad \text{MeV}$$

$$f_{D_s} \quad = \quad 244(8) \quad \text{MeV} \tag{9}$$

The error budget for the calculations gives discretisation errors as the main source of uncertainty.

Both the Fermilab Lattice/MILC and ETMC results agree well with the HPQCD result. However, their errors are such that they do not disagree radically with the experimental average either. Both methods need to improve their accuracy but also provide more test of their systematics. The Fermilab Lattice/MILC results use the D_s itself to fix m_c (along with a less accurate method for determining masses) because of worries about large systematic errors for that formalism for charmonium. This means that the simple test of calculating $m_{D_s} - m_{\eta_c}/2$ has never been done for that formalism. f_K and f_π have also not been accurately calculated in the clover formalism because it does not have an absolutely normalised decay constant. The twisted mass formalism also uses the D_s to fix m_c (although with a direct method) and has done no tests in charmonium. Light hadron decay constants have been calculated in this formalism [29] (f_π is used to fix a).

In January 2009 CLEO-c produced their final results for f_{D_s}, from both $\tau\nu$ and $\mu\nu$ channels, superseding their earlier results. The τ channel results are seen in 3 decay modes; $\pi\nu$, $e\nu\overline{\nu}$ and, now, $\rho\nu$ [30]. These results have moved down towards the lattice QCD number, as shown in Figure 4 which includes the CLEO average from January 2009 of $f_{D_s} = 259.5(7.3)$ MeV . The discrepancy with HPQCD is then 2.3σ, with σ still dominated by the experimental uncertainty.

Fig. 5 compares the HPQCD result for f_{D_s} with the results from individual channels for CLEO-c. It is clear that, although all the results are on the same side of the lattice QCD result, none of the individual channels show a serious discrepancy. Also included is an updated of the BaBar result by the Heavy Flavor Averaging Group (HFAG) [31]. The BaBar result includes a normalisation of the leptonic decay to the channel ($D_s \rightarrow \phi\pi$), but this was being done inconsistently. Correcting this, HFAG obtain $f_{D_s} = 237.3(16.7)(1.7)$ MeV from BaBar which agrees very well with HPQCD. The HFAG world average of September 2009:

$$f_{D_s} = 256.9(6.8) \quad \text{MeV} \tag{10}$$

is also plotted, 2σ from the HPQCD result. It begins to appear less likely that there is new physics in the D_s decay constant.

HPQCD is currently updating its D_s decay constant to include results on even finer lattices provided by the MILC collaboration, down to lattice spacing values of 0.045 fm. Fig. 6 show that these new results agree with the previous ones and simply lie closer to $a = 0$ on the same trajectory. Sea quark mass effects are not shown in this plot since, as discussed earlier, these are a very small effect for f_{D_s}, whereas the continuum extrapolation is not, and must be well-controlled. The x-axis plots the lattice spacing in units of a parameter r_1 which is used for calibration. We now believe that we need to adjust our calibration slightly [11] and this will move our answer for f_{D_s} (and f_D) up slightly. This was allowed for in our original errors, so it will not be a substantial move. However, it will go in the same direction of reducing the discrepancy between theory and experiment for f_{D_s}. An updated result for f_{D_s} will appear shortly. Fermilab/MILC results have also increased at least partly because of this calibration [32].

For f_D the chiral extrapolation to the physical u/d mass is important because the D meson contains valence u/d quarks. In fact the ratio of the D_s to the D decay constant is almost independent of the lattice spacing, as can be seen in Fig. 7. This ratio can then be determined more accurately from lattice QCD than the individual quantities. HPQCD obtain a value 1.164(11) for this ratio, and this can be compared between lattice calculations. The result from Fermilab Lattice/MILC is 1.200(27) [28] and from the European Twisted Mass Collaboration 1.24(3) [29]. To improve the lattice value for f_D clearly requires going to lighter $m_{u/d}$ values. Finer lattices are not obviously necessary.

The charm quark mass and quark mass ratios

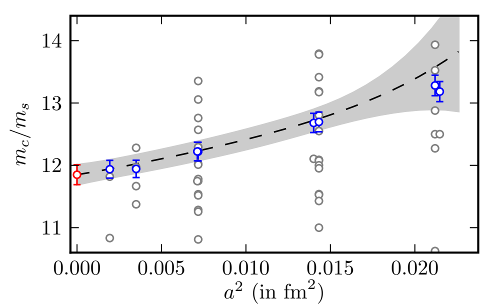

Figure 8: The ratio of m_c to m_s calculated in lattice QCD using HISQ quarks as a function of lattice spacing. The dashed line is a fit as a function of a that allows us to extrapolate to the continuum value, given by the value, with error bars, at $a = 0$.

As described above, lattice QCD calculations give direct access to the quark mass parameters in the lattice QCD action. To be useful to others, however, these quark masses need to be converted to renormalisation schemes that are used in continuum QCD, the standard one being $\overline{MS}$. Since they are running masses they also need to be quoted at a standard scale. In fact it is the conversion between the lattice mass and the $\overline{MS}$ scheme that causes most of the error from lattice QCD quark mass determinations. Because this is simply a conversion from one renormalisation scheme to another it can be done perturbatively but it is hard to do high order perturbative calculations in lattice QCD perturbation theory [33].

For charm quarks, we can make use of the high order continuum QCD perturbation theory that has been done for the heavy quark vacuum polarisation [34]. This leads to $\mathcal{O}(\alpha_s)^3$ accurate moments of heavy quark-antiquark cor-

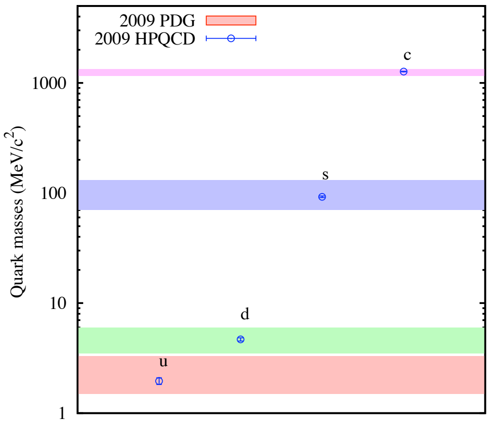

Figure 9: HPQCD results for the 4 lightest quark masses compared to the current PDG evaluations (shaded bands) [1]. Each mass is quoted in the $\overline{MS}$ scheme at its conventional scale: 2 GeV for u, d, s and m_c for m_c [37].

relation functions and these have been used to determine the charm quark mass very accurately using experimental results for $R(e^+e^- \rightarrow \text{hadrons})$, after isolating the charm quark contribution [35]. The perturbative series can be evaluated accurately because it is calculated to high order and because the natural scale for α_s is a few times the charm quark mass. Since the current-current correlators for accurately tuned charm quarks can be extracted from lattice QCD calculations, extrapolating to the continuum limit, we can also use the same continuum perturbation theory to determine the charm quark mass in the $\overline{MS}$ scheme. In fact we can determine it most accurately from pseudoscalar correlators that we do not have direct access to from experiment. Experiment enters these calculations through the tuning of the charm quark mass using the η_c meson mass. A 1% determination of the charm quark mass is possible : $m_c(m_c) = 1.268(9)$ GeV [36].

Lighter quark masses cannot be determined this way. However, the ratio of m_c to, say, m_s at a fixed scale is the same on the lattice as in the $\overline{MS}$ scheme since the renormalisation from the lattice to $\overline{MS}$ cancels. This is *only* true provided that the same formalism is used for both quarks. Thus the HISQ formalism is very well suited to this because we can handle light, strange and charm quarks in the same formalism. Fig. 8 shows the ratio of m_c/m_s obtained from lattice QCD using HISQ quarks at 5 values of the lattice spacing, and a fit as a function of the lattice spacing that enables us to extrapolate the ratio to the continuum $a = 0$ limit [37]. The value we obtain, 11.85(16), can then be used with the value of m_c above, to yield an accurate value for the s quark mass (92.4(1.5) MeV). Ratios of $m_s/m_{u/d}$ can then be used to determine the light quark masses accurately. In this way, the accuracy we are able to obtain in charm physics is cascaded down to the light quark masses. Figure 9 summarises the current accuracy on quark masses compared to that available from non-lattice methods as quoted in the Particle Data Tables.

Prospects for the future

Further work on D and D_s decay constants will be done by HPQCD and Fermilab Lattice/MILC on finer lattices. In addition new gluon configurations that include charm quarks in the sea (i.e. $n_f = 2+1+1$) are being made using the HISQ formalism by the MILC collaboration [38] and using the twisted mass formalism by the ETMC collaboration. This will allow the twisted formalism to include the full effect of sea quarks and allow both formalisms to test the effect of sea c quarks (which should be small). Work on charm quarks with other formalisms such as the overlap formalism, is also starting. The BES experiment plans in future to determine f_{D_s} to 1% so there is plenty of room yet for improvements to the lattice QCD result.

Semileptonic form factors for D decay are also required for comparison to experiment. Here also we expect significant improvements from using the HISQ formalism [39], and this work is underway.

As finer and finer lattices become available the prospects open up of having lattices for which $m_b a$ is small enough to use a relativistic formalism. Fig. 10 shows current results for the decay constant of the η_h meson made of 2 heavy quarks, as a function of the mass of the η_h, on MILC lattices ranging in lattice spacing from 0.12fm to 0.045fm [40]. A fit as a function of m_{η_h} can be made to the data, including discretisation errors. It is clear from the figure that discretisation errors are small at the c quark but rapidly become larger as the quark mass increases. On the coarser lattices it is not possible to reach very high meson masses with quark masses $m_h a < 1$. However, on the finest lattices, we have a much greater reach. The fit curve then gives, in the $a \rightarrow 0$, not only m_{η_b} but also the physical curve for the dependence of f_{η_h} on m_{η_h} which is of interest to heavy quark modellers. Lattices with spacing of 0.03fm are now being made and these will enable more accurate results for a range of quantities at the b quark mass, and it will be possible to test the heavy quark mass dependence accurately against ideas from nonrelativistic methods such as Heavy Quark Effective Theory.

Conclusions

The calculation of the D and D_s decay constants have been an interesting lesson in the power of a modern lattice QCD calculation to produce a result that has real consequences. At the same time we have to accept that a 3σ discrepancy between experiment and theory is not that compelling as a signal of new physics. That should not stop us, however, looking everywhere we can for discrepancies, and we may yet discover something in f_{D_S} when errors are improved further. Quantities like decay constants and gold-plated masses that are very 'clean', both theoretically and experimentally, provide at the very least stringent tests of QCD. On the lattice QCD side we can see the importance of using a modern highly improved discretisation of QCD coupled with ensembles of gluon field configurations that include the full effect of u/d and s quarks, and that cover

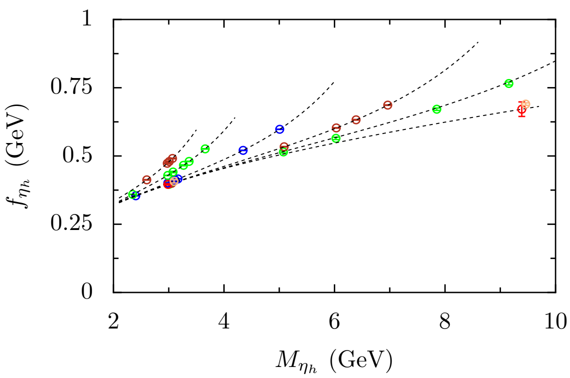

Figure 10: Decay constant of the η_h meson as a function of the mass of the η_h meson for the mass region between c and b. The heavy quarks are handled in the HISQ formalism and the lattice spacings vary (from left to right) from 0.12fm to 0.045fm.

a range of different lattice spacing values. Now that lattice QCD is doing 'real' QCD it is possible to get the full range of QCD physics results from fixing only 5 parameters, as in QCD itself and it is very important that calculational results that are linked together and provide crosschecks of each other e.g. masses and decay constants, are done simultaneously. This will allow lattice QCD to make further compelling predictions for experiment in future.

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