Recent Results from Dalitz Plot Analyses in D/Ds Decays.

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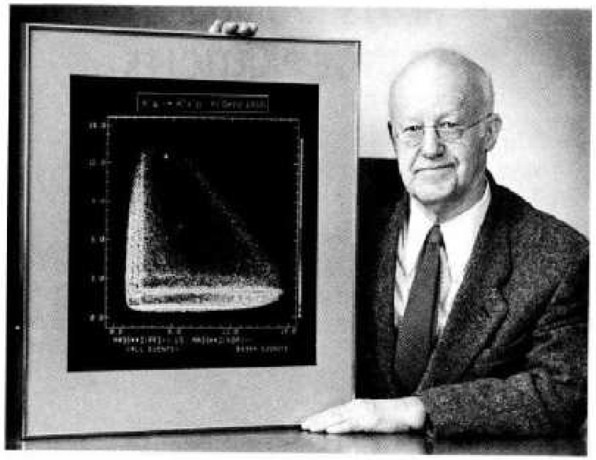
□ **Summary.**

- Dalitz plot analysis of three-body charm decays;
- Analysis methods;
- Search for CP violation;
- Conclusions.

Dalitz plot analysis.

Picture is from $K^- p \to \bar{K}^0 \pi^- p$ at 11 GeV/c, LASS.



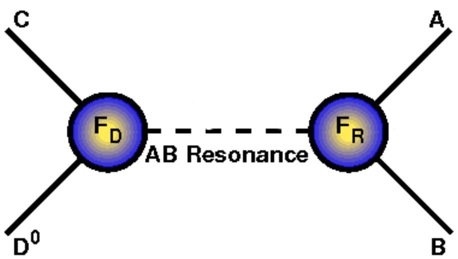
Richard Dalitz, 1925-2006

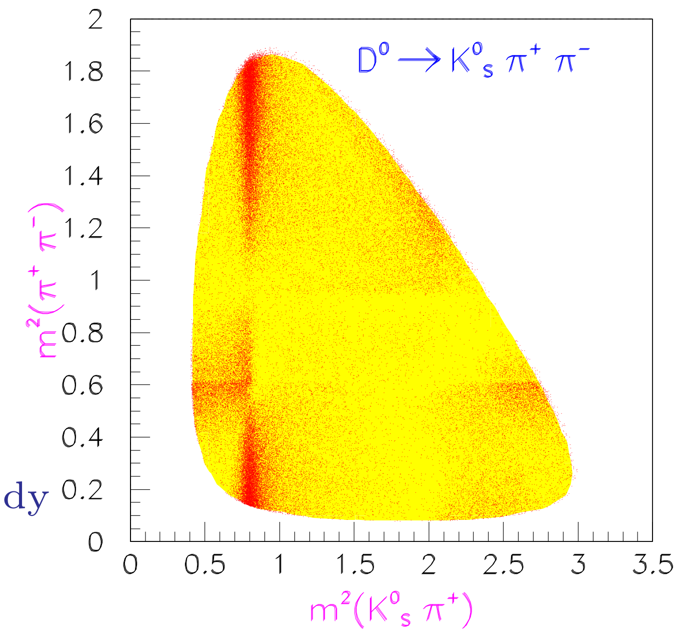
- **Lorentz invariant, and phase space flat.**
- **Allows resonance parameters and spin to be well measured.**
- **Starts from a well-defined spin 0 particle, expect intermediate resonances to have $J \leq 2$ because of limited two-body-mass range, and centrifugal barrier suppression; however, parity and isotopic spin are not conserved in the decay.**
- **Charm Dalitz plots have many uses:**
  - **New measurements in light meson spectroscopy.**
  - **Key role in CKM-γ measurement.**
  - **Fundamental information needed to understand heavy mesons decay.**
  - **Mixing and CP violation studies.**
- **Can be extended to 4-(and more) body decays.**

Dalitz plot analysis.

□ Dalitz plot analysis is an invaluable procedure exploited in many charm (and B) decay analyses. (D. Asner, hep-ex/0410014)

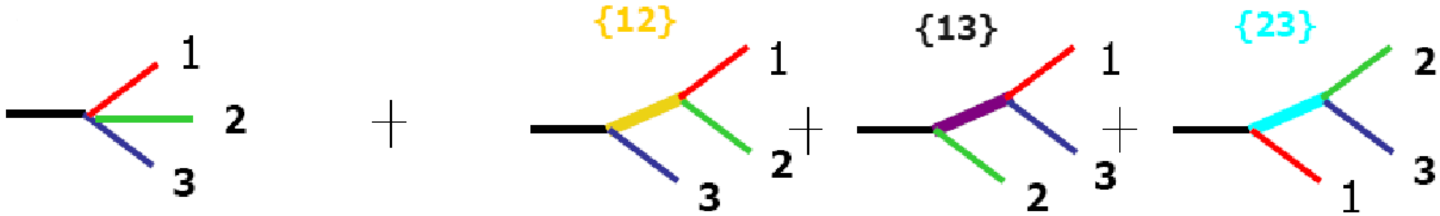
□ Kinematics of 3-body decay $D \to A, B, C$ fully described by 2 parameters.
Typical choice:
$s_{AB} = (p_A + p_B)^2$, $s_{BC} = (p_B + p_C)^2$





□ The decay is almost always mediated at least one two-body resonance.

□ Non-Resonant contribution often consistent with zero.



□ In some cases one or two contributions can be zero.

Dalitz Plot Analysis.

□ Analyses are usually performed by means of unbinned maximum likelihood fits.

□ The likelihood is parametrized as:

$$L = x \mid A_1 + c_2 A_2 e^{i\phi_2} + c_3 A_3 e^{i\phi_3} + \ldots \mid^2 + (1-x) B$$

where A_i and ϕ_i are the amplitudes and phase for contribution i, all measured with respect to a reference amplitude.

□ x is the signal fraction and B is the incoherent background.

□ Amplitudes written as:

$$A_i = BW_i \times \Omega_i$$

where BW_i is a Relativistic Breit-Wigner and Ω_i describes its decay angular dependence.

□ Standard procedure is to use the helicity formalism.

□ Zemach tensors can also be used.

□ All amplitudes normalized on the Dalitz plot.

□ **Isobar model: all intermediate resonances described by Breit-Wigner. Introduce known or unknown resonances until a good fit is obtained.**

The problem of the scalar mesons.

- ☐ The study of B decays also deals with the problem of scalar mesons.
- ☐ Too many scalar mesons below 2. GeV with uncertain parameters.

I = 1/2	I = 1	I = 0
$k(800)$		σ
	$a_0(980)$	$f_0(980)$
		$f_0(1370)$
$K_0^*(1430)$	$a_0(1490)$	$f_0(1500)$
		$f_0(1700)$
$K_0^*(1950)$		

- ☐ The interpretation of broad structures as resonances not clear.
- ☐ Several states need to be described by a coupled channel formalism.
- ☐ Broad overlapping resonances cannot be described by standard Breit-Wigner's.
- ☐ Four different approaches.

  - Isobar model;
  - K-matrix formalism;
  - Model Independent Partial Wave Analysis;
  - Direct Partial Wave Analysis.

K-matrix formalism.

☐ The K-Matrix formalism overcomes the main limitation of the BW model to parameterize large and overlapping S-wave $\pi\pi$ resonances: violation of unitarity.

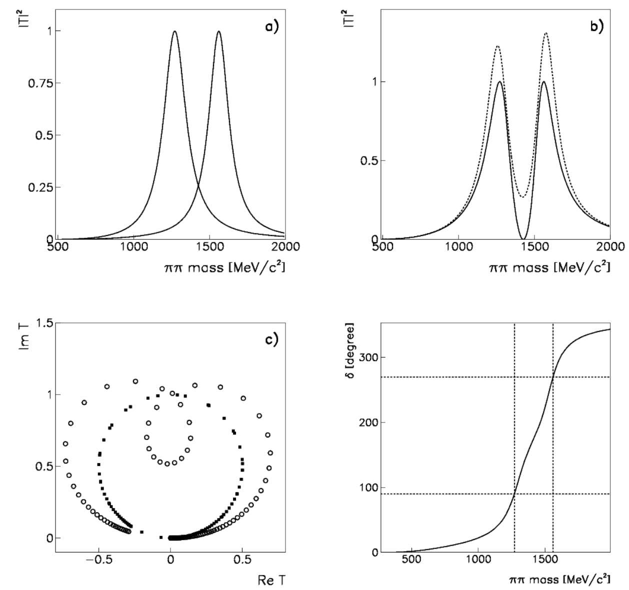
I.J.R. Aitchison, Nucl. Phys. A189, 417 (1972)

☐ The PDF is written as:

$$\mathcal{A}_D(\mathbf{m}) = F_1(s) + \sum_{r \neq (\pi\pi)_{L=0}} a_r e^{i\phi_r} \mathcal{A}_r(\mathbf{m}) + a_{\rm NR} e^{i\phi_{\rm NR}}$$

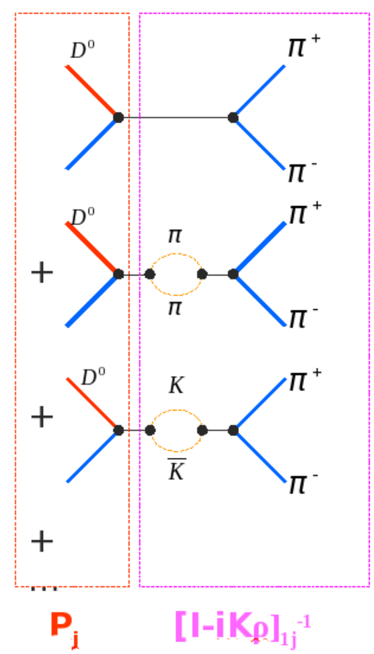
where $F_1(s)$ is the K-matrix contribution of the $\pi\pi$ S-wave

$$F_1(s) = \sum_j \left[I - iK(s)\rho(s)\right]^{-1}_{1j} P_j(s)$$



☐ P_j is the initial production vector.
5 channels: 1=$\pi\pi$, 2=$K\bar{K}$, 3=$\pi\pi\pi\pi$, 4= $\eta\eta$, 5= $\eta\eta'$

V.V. Anisovitch, A.V Sarantev Eur. Phys. Jour. A16, 229 (2003)

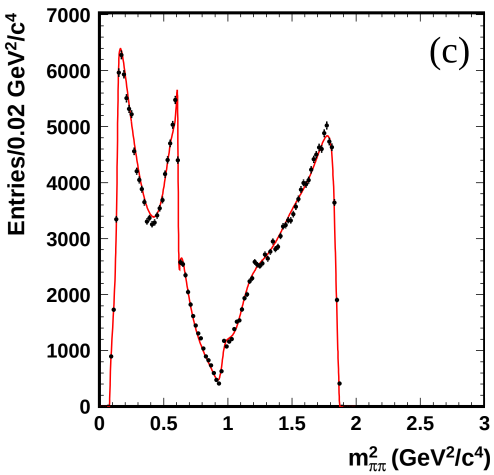


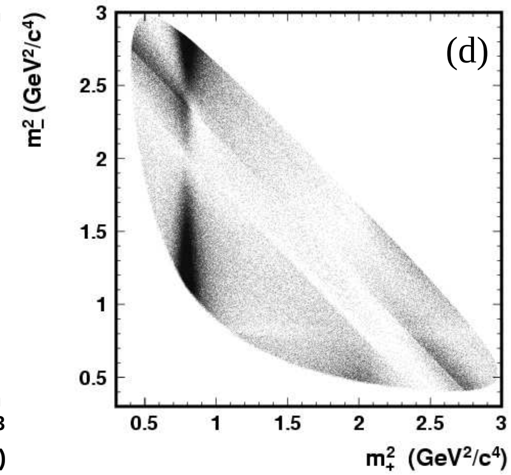
☐ Method physically correct. However it is based on results from past experiments.

Study of $D^0 \to \bar{K}^0\pi^+\pi^-$.

- Aim is to measure γ in $B \to DK$ decays.
- **Isobar model.** Dalitz plot analysis from Belle requires two more "σ" resonances. ($\chi^2/ndf = 2.72$)

arXiv:0803.3375, Phys.Rev.D73:112009,2006

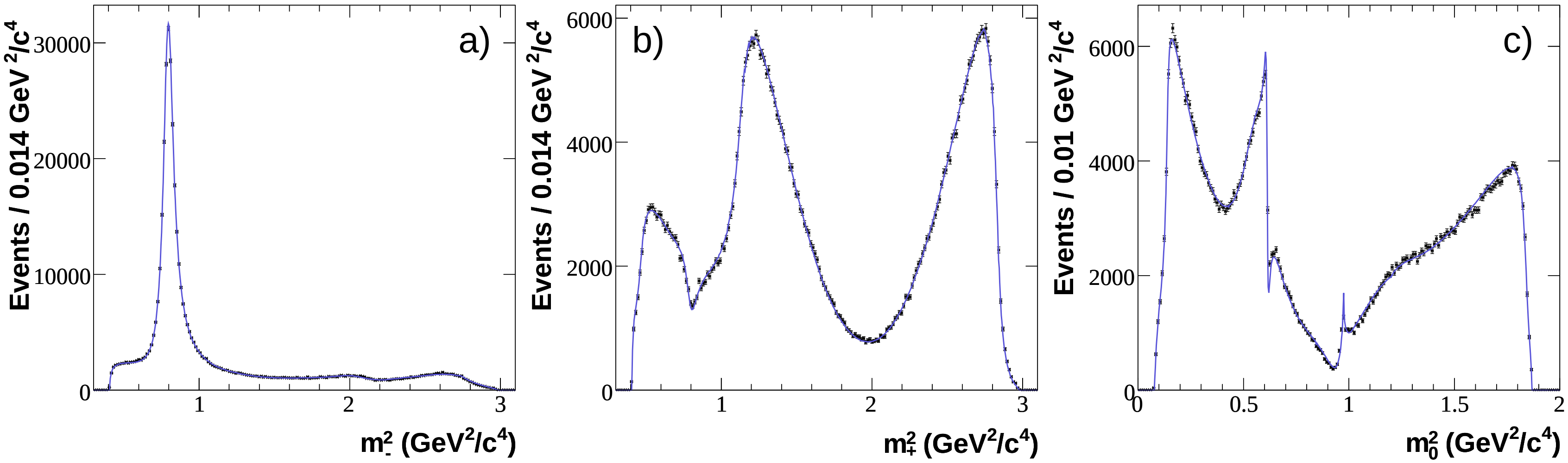




Intermediate state	Amplitude	Phase (°)	Fit fraction
$K^0_S\sigma_1$ ($M_{\sigma_1} = 519 \pm 6$ **MeV**, $\Gamma_{\sigma_1} = 454 \pm 12$ **MeV**)	1.43 ± 0.07	212 ± 3	**9.8%**
$K^0_S\rho^0$	1.0 (fixed)	0 (fixed)	21.6%
$K^0_S f_0(980)$	0.365 ± 0.006	201.9 ± 1.9	**4.9%**
$K^0_S\sigma_2$ ($M_{\sigma_2} = 1050 \pm 8$ **MeV**, $\Gamma_{\sigma_2} = 101 \pm 7$ **MeV**)	0.23 ± 0.02	237 ± 11	**0.6%**
$K^0_S f_0(1370)$	1.44 ± 0.10	82 ± 6	**1.1%**
$K^*(892)^+\pi^-$	1.644 ± 0.010	132.1 ± 0.5	61.2%
$K^*_0(1430)^+\pi^-$	2.15 ± 0.04	353.6 ± 1.2	7.4%
$K^*_2(1430)^+\pi^-$	0.88 ± 0.03	318.7 ± 1.9	2.2%
...	...	...	...
non-resonant	3.0 ± 0.3	164 ± 5	**9.7%**

Study of $D^0 \to \bar{K}^0\pi^+\pi^-$.

- BaBar makes use of the **K-matrix formalism for the $\pi\pi$ S-wave** (fraction=11.9 $\pm$ 2.6%).Phys.Rev.D78:034023,2008
- 487 000 events $\chi^2/NDF = 1.11$.
- Using the Belle isobar model: $\chi^2/NDF = 1.20$

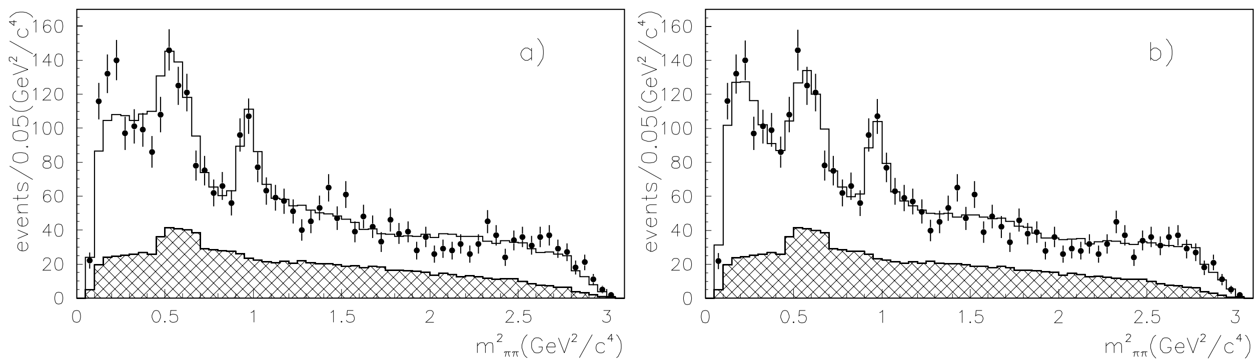


- Better fit and no need to introduce new scalar resonances.

Study of $D^+ \to \pi^+\pi^+\pi^-$.

□ Isobar model from $E791$ ($\approx$ 1200 events) requires the presence of a broad scalar $\sigma(500)$ resonance.Phys. Rev. Lett. 86, 770 (2001)

□ Fit without and with the $\sigma(500)$.



□ $\sigma(500)$ fraction: $46.3 \pm 9 \pm 2.1\%$

□ $\sigma(500)$ parameters:

$$m = 478^{+24}_{-23}, \qquad \Gamma = 324^{+42}_{-40} \qquad MeV$$

□ In some models the $\sigma(500)$ is a candidate for being a scalar glueball.

□ In others, the $\sigma(500)$ should not be interpreted as a $q\bar{q}$ state, but results from a weakly attractive potential between the bosons.(PRL 96, 132001 (2006)).

□ Recent analysis from CLEO ($\approx$ 2600 events).(Phys.Rev.D76:012001,2007)

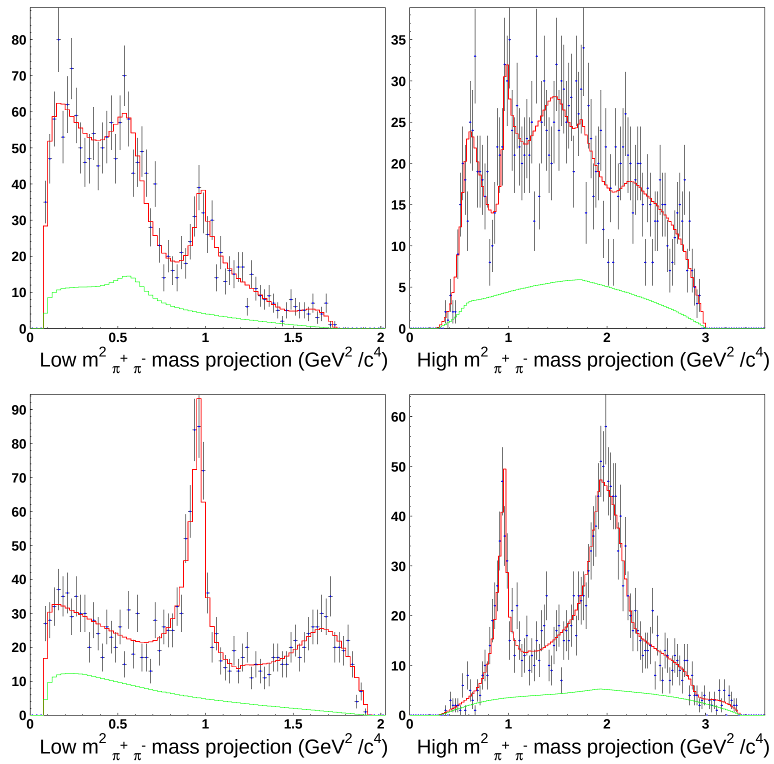
□ σ pole: (466±18, –223±28) MeV

K-matrix analysis of $D^+/D_s^+ \to \pi^+\pi^+\pi^-$.

□ K-matrix fit from FOCUS($\approx$ 1500 events) is able to fit both $D^+ \to \pi^+\pi^+\pi^-$ and $D_s^+ \to \pi^+\pi^+\pi^-$ using the same S-wave structure as in hadronic exeperiments.(Phys.Lett.B585:200-212,2004)

□ $D^+ \to \pi^+\pi^+\pi^-$

□ $D_s^+ \to \pi^+\pi^+\pi^-$



□ This does not solve the problem of the existence of the $\sigma(500)$ meson. It was present also in previous experiments.

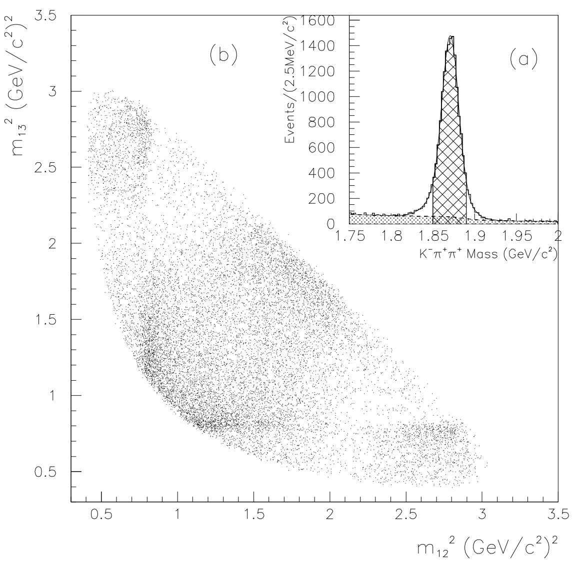
Study of $D^+ \to K^-\pi^+\pi^+$.

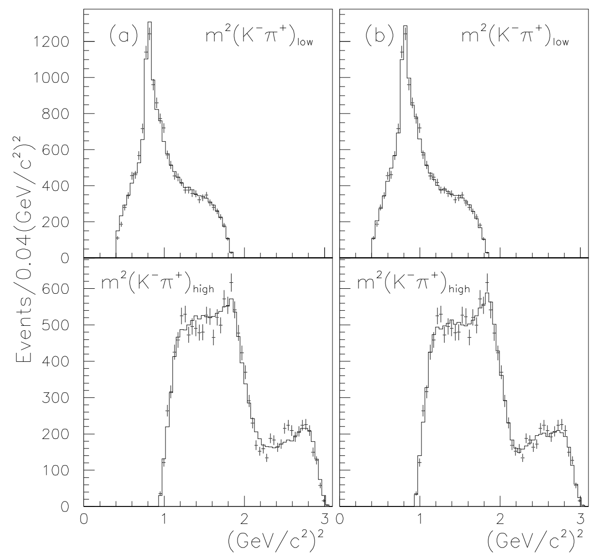
□ Isobar analysis from E791 of the decay $D^+ \to K^-\pi^+\pi^+$ ($\approx$ 15000 events). (Phys.Rev.Lett.89:121801,2002)

□ A better fit is obtained introducing a $\kappa(800)$ resonance interfering with a coherent, uniform, S-wave backgrond.

□ Resulting κ parameters.

$$m = 797 \pm 19 \pm 43 \; MeV/c^2, \qquad \Gamma = 410 \pm 43 \pm 87 \; MeV/c^2$$





□ Isobar/K-matrix analysis from FOCUS ($\approx$ 52000 events) based on LASS data.(Phys.Lett.B653:1-11,2007)

□ Both methods give good description of the data. Discrepancy at the $K\eta'$ threshold.

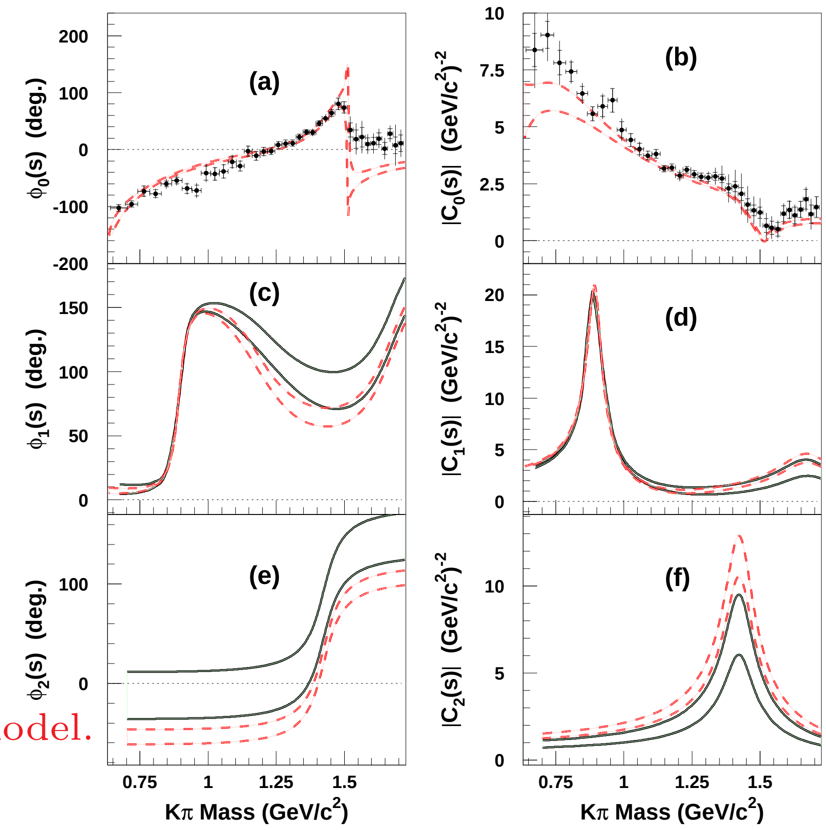
Model Independent Partial Wave Analysis ($D^+ \to K^- \pi^+ \pi^+$).

- □ E791. Study of $D^+ \to K^- \pi^+ \pi^+$ (Phys.Rev.D73:032004,2006).
- □ $K\pi$ P-wave described with $K^*(892), K_1^*(1410), K_1^*(1680)$ Breit-Wigner's.
- □ D-wave described by $K_2^*(1430)$ Breit-Wigner.
- □ S-wave extracted by spline-interpolation over 40 points.

At each point amplitude and phase are free parameters.

$$C_0(s_k)e^{i\phi_0(s_k)}$$

- □ $K\pi$ S-wave: Broad structure with dip at the $K_0^*(1430)$ resonance.

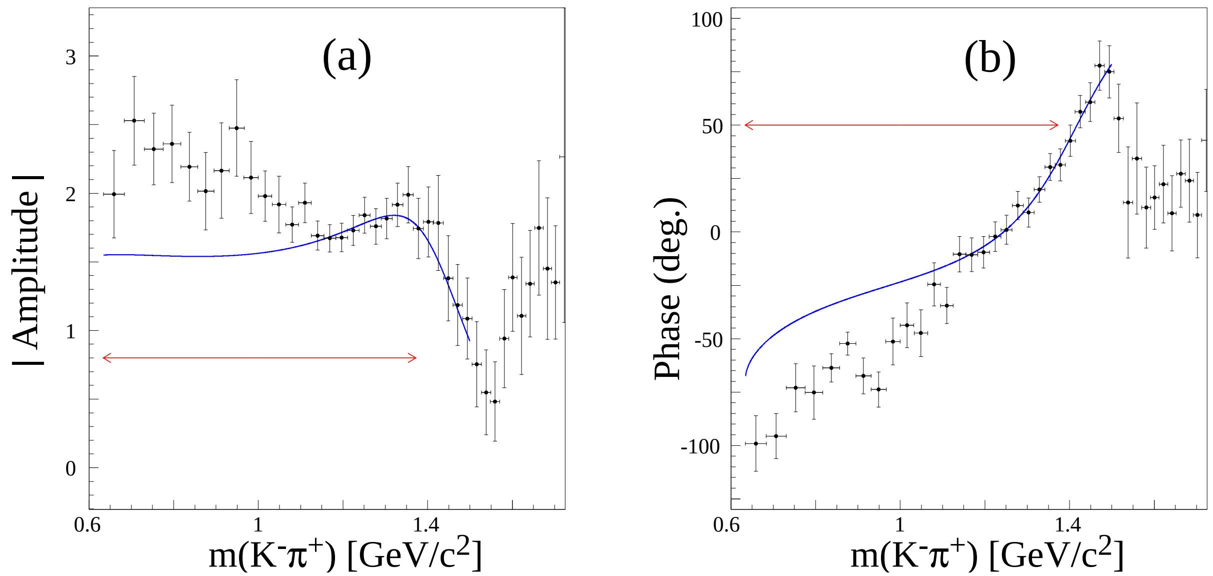


- □ Solution compared with the fit from the isobar model.
- □ Tests have been made for multiple solutions.

Model Independent Partial Wave Analysis ($D^+ \to K^-\pi^+\pi^+$).

- Comparison with results from I=1/2 $K^-\pi^+$ S-wave amplitude measurements (blue curves).

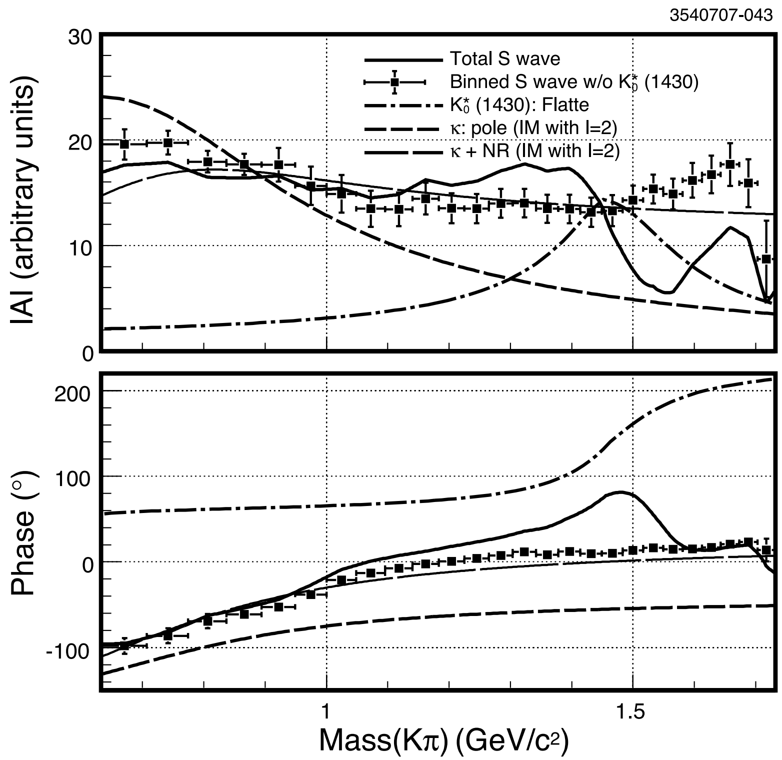
$$I = 1/2\ S - wave$$



- LASS curve normalized to the E791 data at 1.3 GeV.
- Phase normalized to the same mass shifting down by 70^0.
- Watson theorem requires elastic phase to be the same.

Model Independent Partial Wave Analysis ($D^+ \to K^-\pi^+\pi^+$).

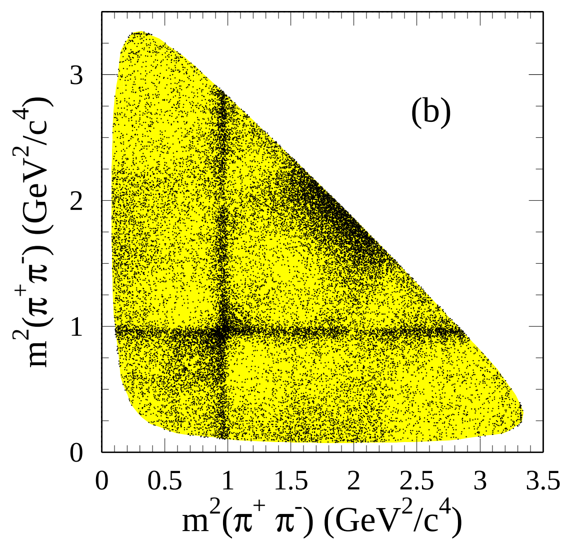
- $D^+ \to K^-\pi^+\pi^+$ from CLEO (141 000 events).(Phys.Rev.D78:052001,2008)
- In this case the $K_0^*(1430)$ resonance is described by a Breit-Wigner.
- Introduced a I=2 $\pi^+\pi^+$ amplitude.N.N. Achasov and G.N. Shestakov, Phys. Rev. D**67**, 114018 (2003).
- Mass dependent phase from scattering experiments.

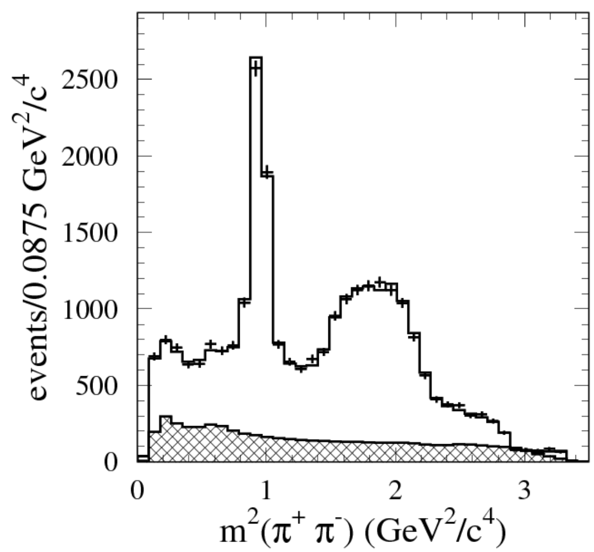


- S-wave almost flat.

Model Independent Partial Wave Analysis ($D_s^+ \to \pi^-\pi^+\pi^+$).

- ◻ BaBar ($\approx$ 13 000 events).(Phys. Rev. D 79, 032003 (2009)
- ◻ Dalitz plot and $\pi^+\pi^-$ projection: strong $f_0(980)$ signal and broad structure at $\approx$ 1.3 GeV.





- ◻ Extraction of the $\pi^+\pi^-$ amplitude and phase coupled to $s\bar{s}$.

$$A_{\mathcal{S}-\text{wave}}(m_{\pi\pi}) = \text{Interp}(c_k(m_{\pi\pi})e^{i\phi_k(m_{\pi\pi})})_{k=1,..,30}$$

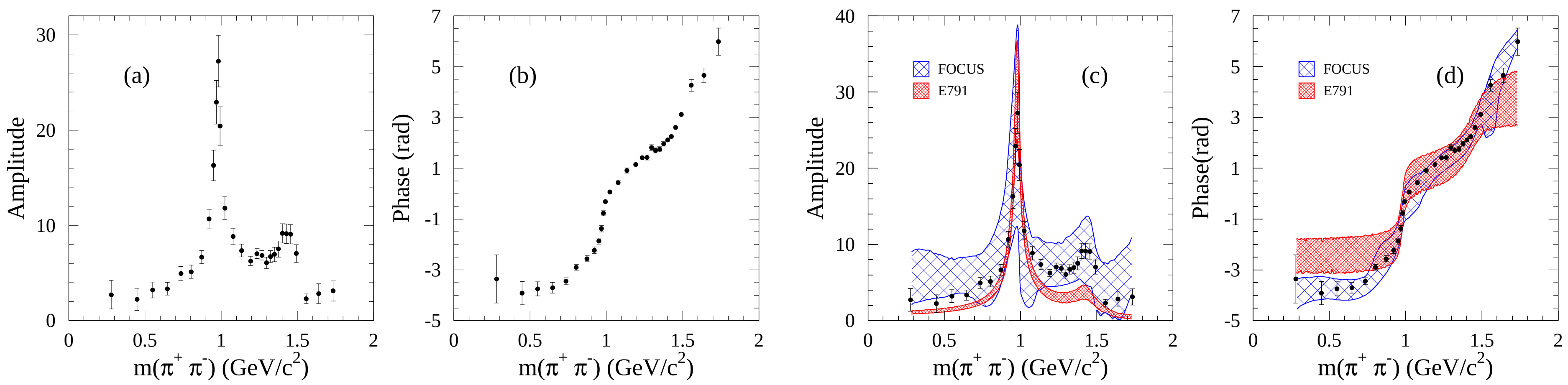
Model Independent Partial Wave Analysis ($D_s^+ \to \pi^-\pi^+\pi^+$).

- Large $\mathcal{S}$-wave contribution.

Measured with respect to the $f_2(1270)$ resonance.

Decay Mode	Decay fraction(%)	Amplitude	Phase(rad)
$f_2(1270)\pi^+$	$10.1 \pm 1.5 \pm 1.0$	1.(Fixed)	0.(Fixed)
$\rho(770)\pi^+$	$1.8 \pm 0.5 \pm 1.0$	$0.19 \pm 0.02 \pm 0.12$	$1.1 \pm 0.1 \pm 0.2$
$\rho(1450)\pi^+$	$2.3 \pm 0.8 \pm 1.7$	$1.2 \pm 0.3 \pm 1.0$	$4.1 \pm 0.2 \pm 0.5$
$\mathcal{S}$-wave	$83.0 \pm 0.9 \pm 1.9$		
Total	$97.2 \pm 3.7 \pm 3.8$		
χ^2/NDF	$\frac{437}{422-64} = 1.2$		

- Measurement of the $\mathcal{S}$-wave amplitude and phase.
- Comparison with the isobar and K-matrix analyses from $E791$ and FOCUS.

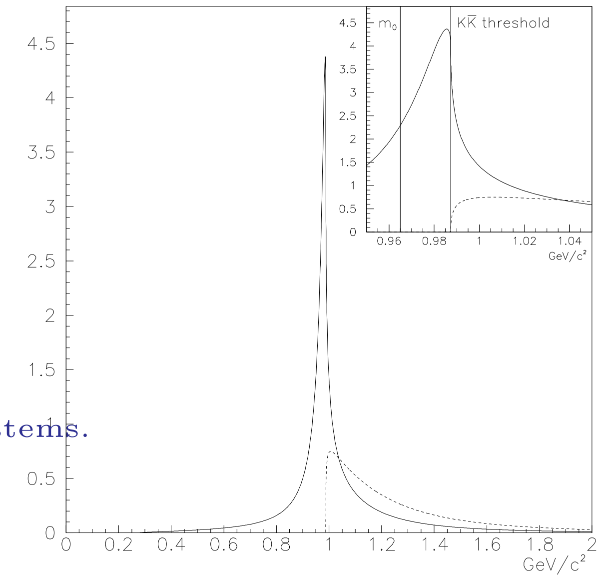


Coupled channel resonances.

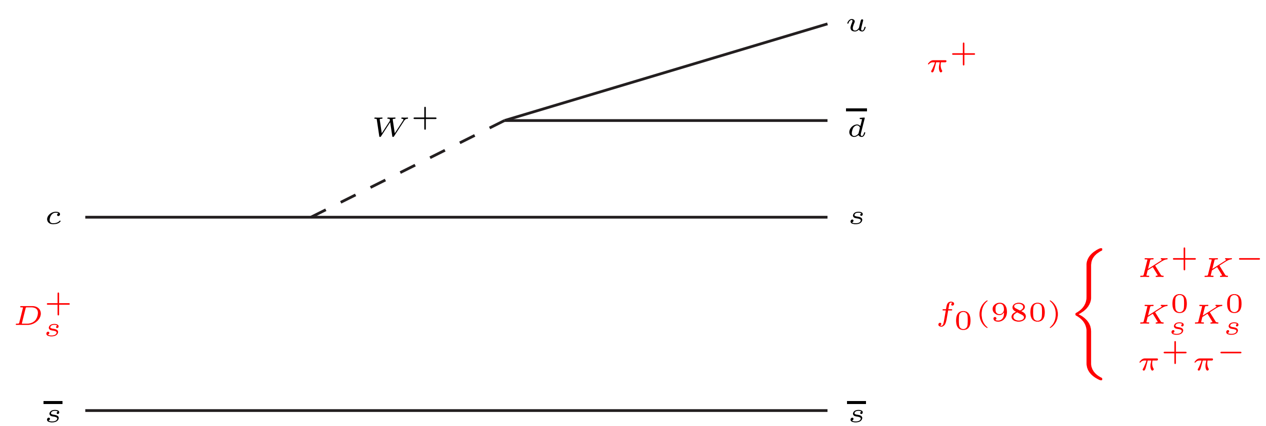
□ The $f_0(980)$ resonance is usually described by a coupled channel Breit Wigner, *the Flatté Formula*:

$$A = \frac{1}{m_0^2 - s - i m_0 (g_\pi \rho_\pi + g_K \rho_K)} \qquad \rho_i = \sqrt{1 - \frac{m_i^2}{s}}$$

- ρ_π and ρ_K describe the couplings to the two channels. ρ_k becomes imaginary below the $K\bar{K}$ threshold.
- Not symmetric shape.
- Narrow apparent width.
- Maximum not at the pole mass.



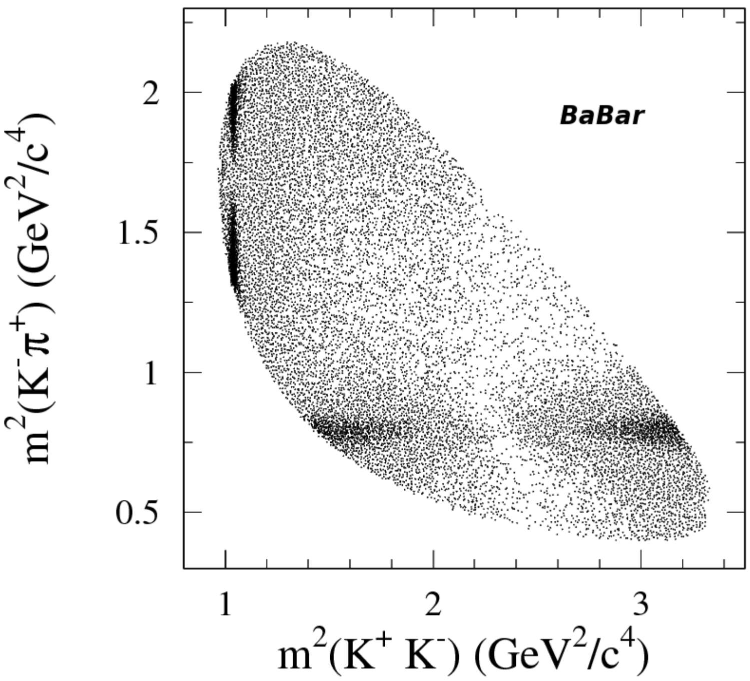
□ $D_s^+ \to K^+K^-\pi^+$, $D_s^+ \to K_s^0 K_s^0 \pi^+$, and $D_s^+ \to \pi^+\pi^-\pi^+$ decays provide a way to study the coupling of $f_0(980)$ to $K\bar{K}$ and $\pi^+\pi^-$ systems.

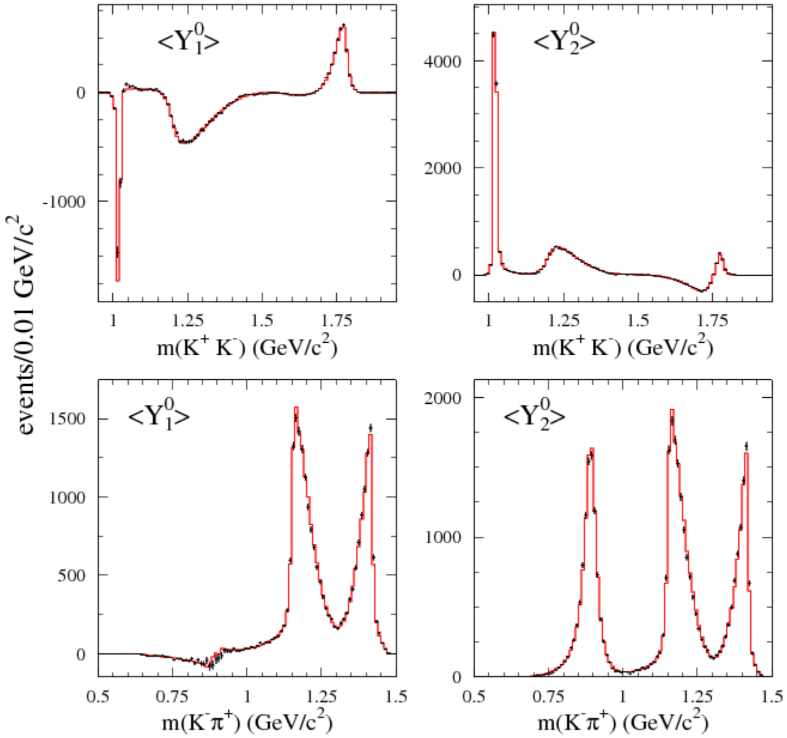


Dalitz plot Analysis of $D_s^+ \to \pi^+ K^+ K^-$

□ Preliminary isobar BaBar anaysis (≈ 101 000 events).

□ The spherical harmonic moment Y_1^0 show large S-P interference in the K^+K^- channel and very small in the $K^-\pi^+$ channel.



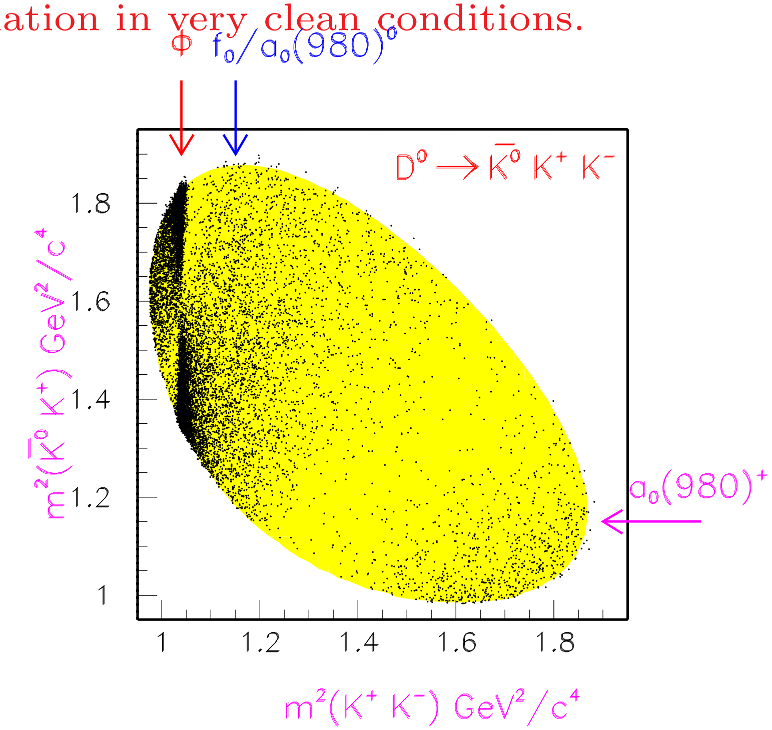


□ See also CLEO analysis (≈ 14 000 events.)(Phys. Rev. D 79, 072008 (2009))

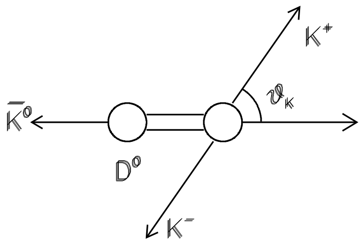
Direct Partial Wave Analysis.

- Charm decays in some cases allow to extract new information in very clean conditions.
- BaBar: Dalitz plot Analysis of $D^0 \to \bar{K}^0 K^+ K^-$.

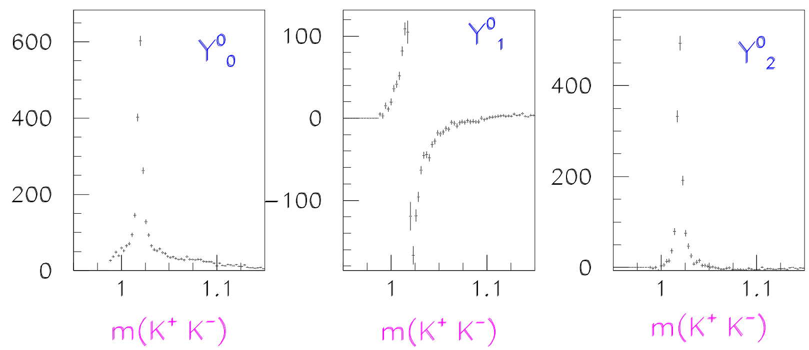
(Phys.Rev.D72:052008,2005)



- Presence of $\phi(1020)$ interfering with a threshold scalar $f_0/a_0(980)^0$.
- Presence of $a_0(980)^+$.
- Assume, in the K^+K^- threshold region, a diagram:



- Unnormalized Y_l^m moments:

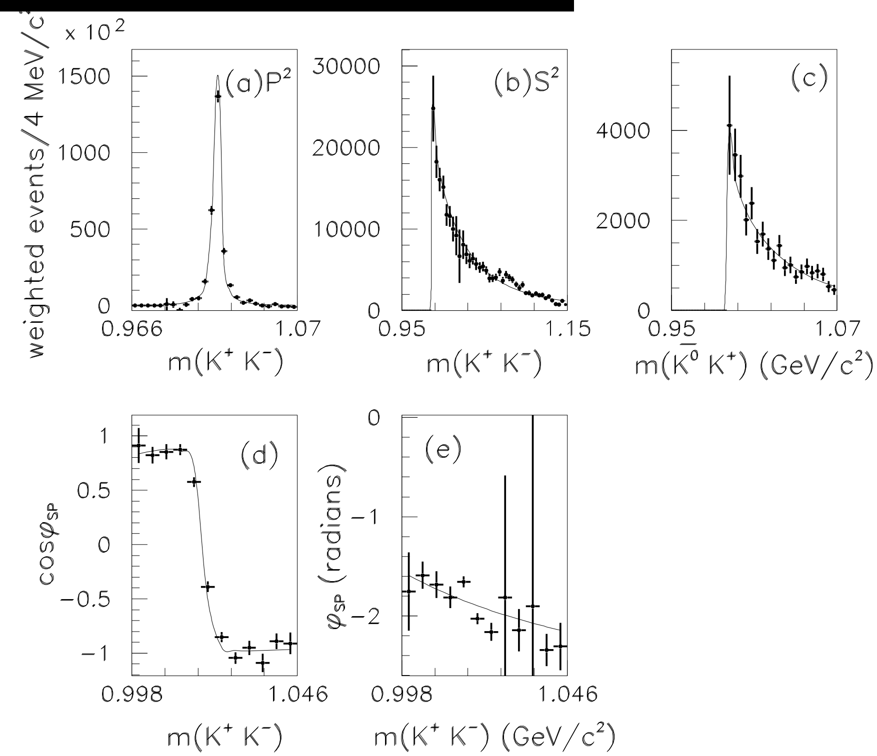


- Y_1^0 shows a strong S-P interference. Y_2^0 shows all the P-wave.

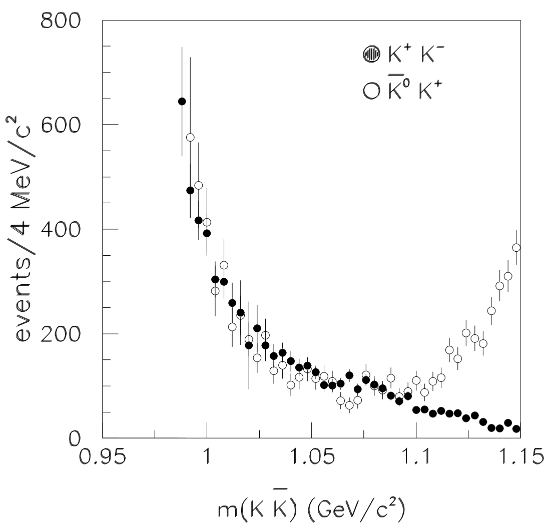
Partial Wave Analysis of the K^+K^- system.

☐ S, P waves and relative phase can be extracted using:

$$\sqrt{4\pi}Y_0^0 = S^2 + P^2$$
$$\sqrt{4\pi}Y_1^0 = 2SPcos\phi$$
$$\sqrt{4\pi}Y_2^0 = 0.894P^2$$



☐ Phase space corrected. Superposition of the $\bar{K}^0K^+$ and K^+K^- normalized projections



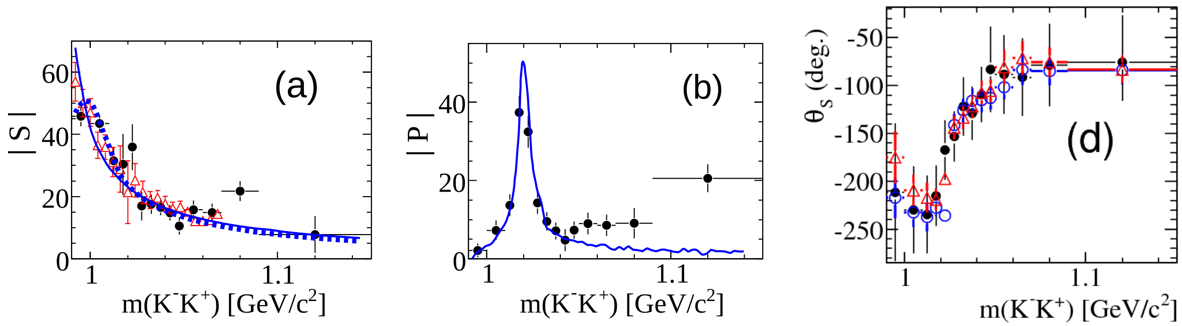
☐ Clear $\phi(1020)$ in the P-wave.

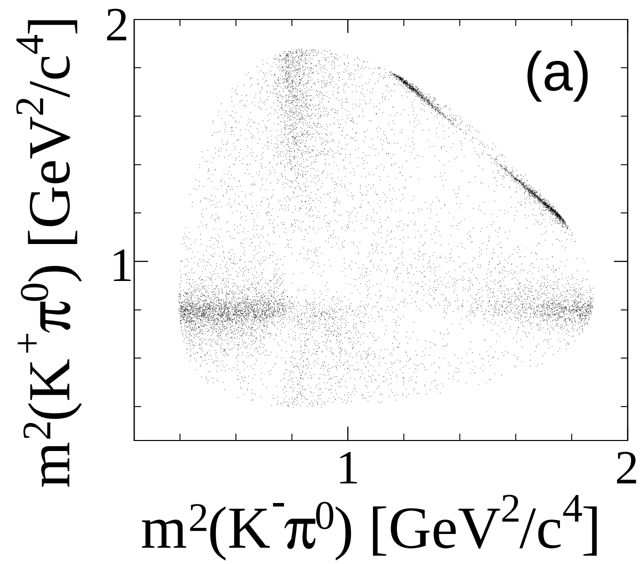
☐ The scalar contribution in the K^+K^- mass projection and $\overline{K}^0K^+$ are entirely due to the $a_0(980)$.

☐ Extracted phase motions of $\phi(1020)$ and $a_0(980)^0$.

Dalitz plot analysis of $D^0 \to K^+K^-\pi^0$.

- BaBar $\approx$ 11 300 events.(Phys. Rev. D 76, 011102(R) (2007))
- Direct Partial Wave Analysis in the K^+K^- threshold region
- $S-wave$, $P-wave$ and $S-wave$ phase.





- The $f_0(980)$ and $a_0(980)$ lineshapes are very similar.
- Neglecting CP violation, the strong phase difference, δ_D, between the $\overline{D}^0$ and D^0 decays to $K^*(892)^+K^-$ state and their amplitude ratio, r_D, are given by:

$$r_D e^{i\delta_D} = \frac{a_{D^0\to K^{*-}K^+}}{a_{D^0\to K^{*+}K^-}} e^{i(\delta_{K^{*-}K^+} - \delta_{K^{*+}K^-})}$$

- BaBar finds $\delta_D = -35.5^\circ \pm 1.9^\circ$ (stat) $\pm 2.2^\circ$ (syst) and $r_D = 0.599 \pm 0.013$ (stat) $\pm$ 0.011 (syst).
- These results are consistent with CLEO measurements, $\delta_D = -28^\circ \pm 8^\circ$ (stat) $\pm 11^\circ$ (syst) and $r_D = 0.52 \pm 0.05$ (stat) $\pm$ 0.04 (syst).

Dalitz plot analysis and search for CP violation.

□ CLEO. In the limit of CP conservation, charge conjugate decays will have the same Dalitz plot distribution.

□ CP violation expected in Cabibbo-Suppressed charm decays.

□ The integrated CP violation across the Dalitz plot is determined from:

$$\mathcal{A}_{CP} = \int \frac{|\mathcal{M}|^2 - \left|\overline{\mathcal{M}}\right|^2}{|\mathcal{M}|^2 + \left|\overline{\mathcal{M}}\right|^2} \, dm^2_{ab} \, dm^2_{bc} \Bigg/ \int dm^2_{ab} \, dm^2_{bc} \; ,$$

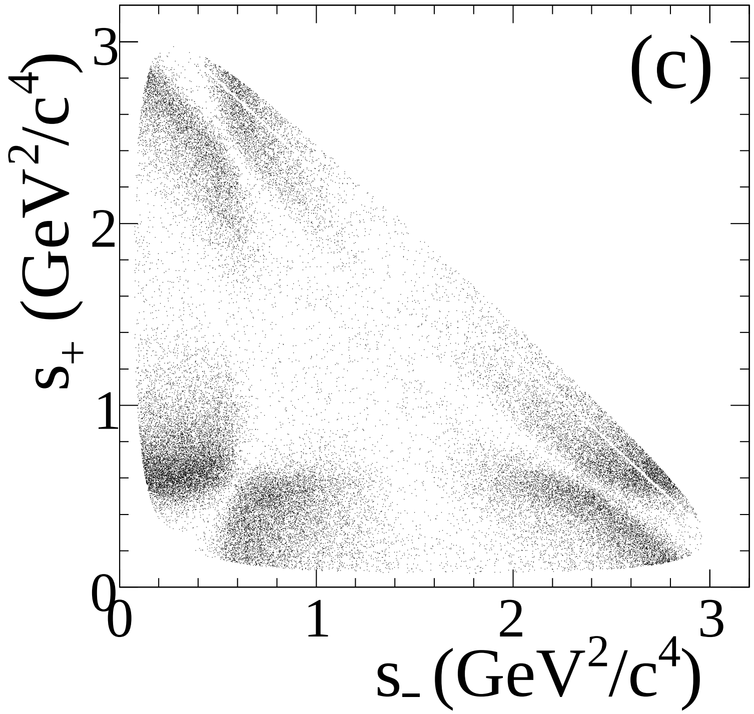
where $\mathcal{M}$ and $\overline{\mathcal{M}}$ are the D^0 and $\overline{D}^0$ Dalitz plot amplitudes.

□ No evidence of CP violation has been observed.

Decay mode	$\mathcal{A}_{CP}$(%)
$D^0 \to K^-\pi^+\pi^0$	-3.1 ± 8.6
$D^0 \to K^+\pi^-\pi^0$	$+9^{+22}_{-25}$
$D^0 \to K^0_S\pi^+\pi^-$	$-0.9 \pm 2.1^{+1.0+1.3}_{-4.3-3.7}$
$D^0 \to \pi^+\pi^-\pi^0$	$+1^{+9}_{-7} \pm 9$
$D^+ \to K^+K^-\pi^+$	$-0.4 \pm 2.0^{+0.2}_{-0.5}$

Dalitz plot analysis of $D^0 \to \pi^+\pi^-\pi^0$.

- CLEO: Dalitz analysis of $D^0 \to \pi^+\pi^-\pi^0$ ($\approx$ 1 900 events) Phys.Rev.D72:031102,2005
- BaBar: Dalitz analysis of $D^0 \to \pi^+\pi^-\pi^0$ ($\approx$ 45 000 events) Phys.Rev.D78:052001,2008



State	R_r (%)	$\Delta\phi_r$ (°)	f_r(%)
$\rho^+(770)$	100	0	67.8±0.0±0.6
$\rho^0(770)$	58.8±0.6±0.2	16.2±0.6±0.4	26.2±0.5±1.1
$\rho^-(770)$	71.4±0.8±0.3	−2.0±0.6±0.6	34.6±0.8±0.3
...	...	...	
Non-Res	57±7±8	−11±4±2	0.84±0.21±0.12

- Decay dominated by ρ resonances (including $\rho(1700)$).
- $\pi\pi$ $S-wave$ consistent with zero.

Search for CP violation.

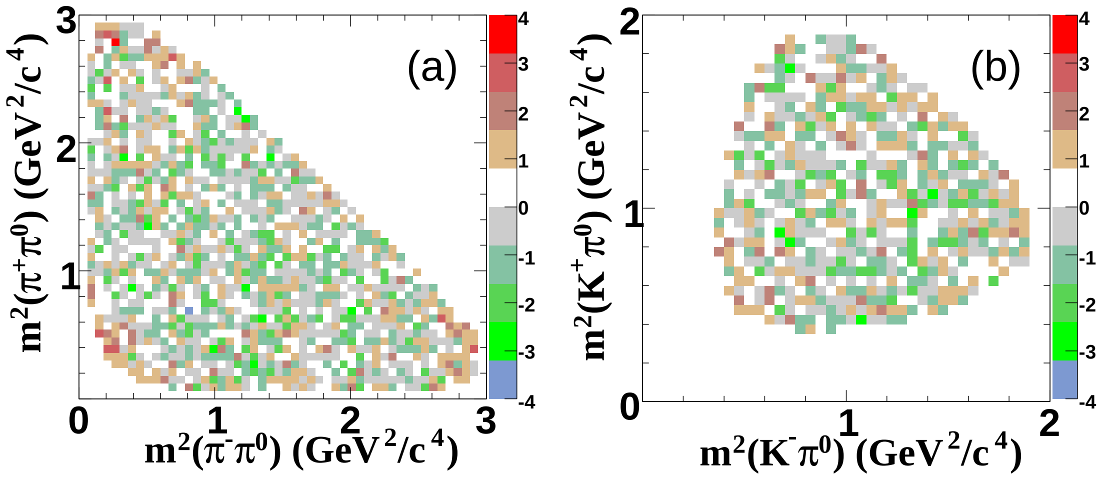
□ BaBar. Study of $D^0 \to \pi^+\pi^-\pi^0$ and $D^0 \to K^+K^-\pi^0$.

□ Compute the asymmetry on the Dalitz plot. (Phys.Rev.D78:051102,2008)

$$\Delta = \left(n_{\overline{D}^0} - R \cdot n_{D^0}\right) / \sqrt{\sigma^2_{n_{\overline{D}^0}} + R^2 \cdot \sigma^2_{n_{D^0}}}$$

n denotes the number of events in a DP element and σ its uncertainty.

□ The factor R, equal to 0.983 ± 0.006 for $D^0 \to \pi^+\pi^-\pi^0$ and 1.020 ± 0.016 for $D^0 \to K^+K^-\pi^0$, is the ratio of the number of efficiency-corrected $\bar{D}^0$ to D^0 events.



□ Computed asymmetries on fractions, phases and spherical harmonics moments. No evidence for CP violation.

Conclusions.

□ Dalitz plot analysis of three-body charm decays is still a fundamental tool for the study of heavy mesons decays.
□ High statistics studies of charm decays provide information needed for the study of B decays.
□ *Examples:*

- Hot topic is the amount of S-wave below the ϕ in $B_s \to J/\psi\phi$.
- 3-body or 4-body Dalitz analyses needed for measuring γ.

□ Many analyses are still in progress.
□ Isobar model often requires the introduction of "new resonances".
□ K-matrix fits give good description of the data. However does not provide new information on critical issues.

□ Partial Wave Analyses are providing new information and new data which are waiting to be understood.