Quarkonia: a theoretical frame

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Why to study quarkonia?

Quarkonia are systems where low energy QCD may be studied in a systematic way (e.g. large order perturbation theory, non-perturbative matrix elements, QCD vacuum, exotica, confinement, deconfinement, ...).

This is because the quark mass M is the largest scale in the system:

- $M \gg p$
- $M \gg \Lambda_{\rm QCD}$

The non-relativistic expansion

• $M\gg p$ implies that quarkonia are non-relativistic and characterized by the hierarchy of scales typical of a non-relativistic bound state:

$$M \gg p \sim 1/r \sim Mv \gg E \sim Mv^2$$

Systematic expansions in the small heavy-quark velocity v may be implemented at the Lagrangian level by constructing suitable effective field theories (EFTs):

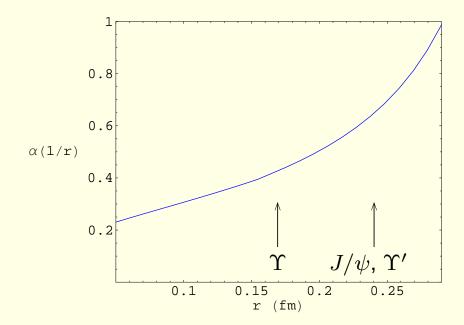
- expanding QCD in p, E/M leads to NRQCD
 - o Bodwin Braaten Lepage PRD 51(95)1125
- ullet expanding NRQCD in E/p, 1/r leads to pNRQCD
 - o Brambilla Pineda Soto Vairo RMP 77(04)1423

The hierarchy of non-relativistic scales makes the very difference of quarkonia with heavy-light mesons, which are just characterized by the two scales M and $\Lambda_{\rm QCD}$.

The perturbative expansion

• $M\gg \Lambda_{\rm QCD}$ implies $\alpha_{\rm s}(M)\ll 1$: phenomena happening at the scale M may be treated perturbatively.

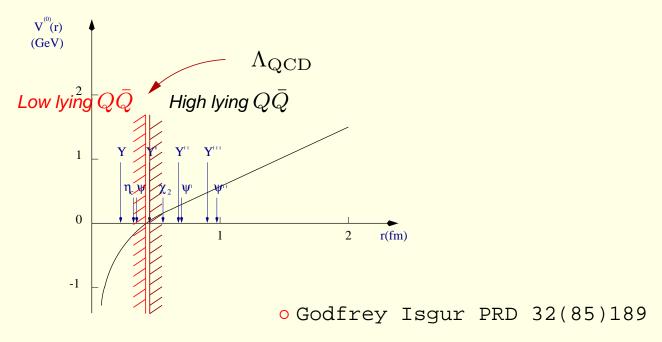
We may further have small couplings if $Mv\gg \Lambda_{\rm QCD}$ and $Mv^2\gg \Lambda_{\rm QCD}$, in which case $\alpha_{\rm s}(Mv)\ll 1$ and $\alpha_{\rm s}(Mv^2)\ll 1$ respectively. This is likely to happen only for the lowest charmonium and bottomonium states.



Quarkonium as a confinement and deconfinement probe

It is precisely the rich structure of separated energy scales that makes quarkonium an ideal probe of confinement and deconfinement.

 The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.

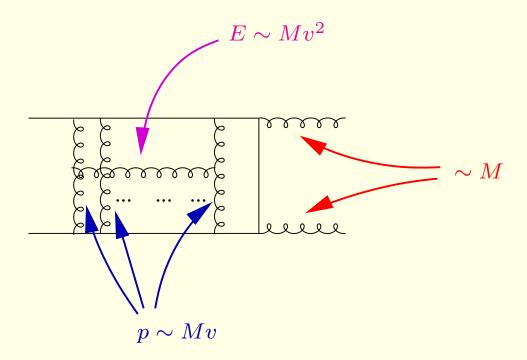


 Different quarkonia will dissociate in a medium at different temperatures, providing a thermometer for the plasma: see the talk by P. Petreczky.

o Matsui Satz PLB 178(86)416

Quarkonium scales

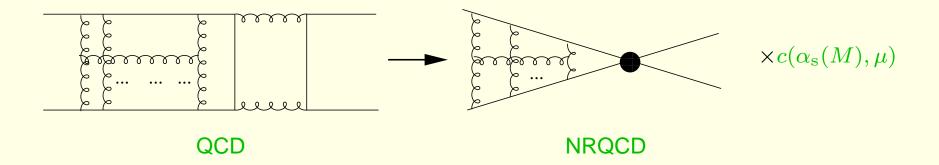
Scales get entangled:



- Quarkonium annihilation and production happen at the scale M;
- Quarkonium binding happens at a scale Mv.

Physics at the scale M: annihilation and production

Quarkonium annihilation and production happens at the scale M. The suitable EFT is NRQCD.



The effective Lagrangian is organized as an expansion in 1/M and $\alpha_s(M)$:

$$\mathcal{L}_{NRQCD} = \sum_{n} \frac{c_n(\alpha_s(M), \mu)}{M^n} \times O_n(\mu, Mv, Mv^2, ...)$$

NRQCD factorization

- Production: see the talk by P. Artoisenet.
- Annihilation: the NRQCD factorization formula reads.

$$\Gamma(H \to l.h.) = \sum_{n} \frac{2 \operatorname{Im} f^{(n)}}{M^{d_{O_n} - 4}} \langle H | O_n^{4 - \operatorname{fermion}} | H \rangle$$

Progress has been made in

- the evaluation of the factorization formula at order v^7 ;
 - O Brambilla Mereghetti Vairo JHEP 0608(06)039
 PRD 79(09)074002
- the (lattice) evaluation of the matrix elements.
 - OBodwin Lee Sinclair PRD 72(05)014009

Charmonium P-wave decays

... and in the experimental data. E.g.

Ratio	PDG09	PDG00	LO	NLO
$\frac{\Gamma(\chi_{c0} \to \gamma\gamma)}{\Gamma(\chi_{c2} \to \gamma\gamma)}$	4.9±0.8	13±10	3.75	≈ 5.43
$\frac{\Gamma(\chi_{c2} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}{\Gamma(\chi_{c0} \to \gamma\gamma)}$	440±100	270±200	≈ 347	≈ 383
$\frac{\Gamma(\chi_{c0} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}{\Gamma(\chi_{c0} \to \gamma\gamma)}$	4000±600	3500±2500	≈ 1300	≈ 2781
$ \frac{\Gamma(\chi_{c0} \to l.h.) - \Gamma(\chi_{c2} \to l.h.)}{\Gamma(\chi_{c2} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)} $	8.0±0.9	12.1±3.2	2.75	pprox 6.63
$\frac{\Gamma(\chi_{c0} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}{\Gamma(\chi_{c2} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}$	9.0±1.1	13.1±3.3	3.75	≈ 7.63

$$m_c =$$
 1.5 GeV $\alpha_{
m s}(2m_c) =$ 0.245 in NLO, v^7 terms are not included

The table clearly shows that the data are sensitive to NLO corrections in the Wilson coefficients $f^{(n)}$ (and perhaps also to relativistic corrections).

$\alpha_{\rm s}$ extraction

The achieved sensitivity may allow

- for determinations of the decay matrix elements (also related to the production matrix elements);
- for the determination of α_s at the quarkonium scale. As an example, an analysis of $\Gamma(\Upsilon(1S) \to \gamma \ l.h.)/\Gamma(\Upsilon(1S) \to l.h.)$ along this line has led to

$$\alpha_{\rm s}(M_Z) = 0.119^{+0.006}_{-0.005}$$

o Brambilla Garcia Soto Vairo PRD 75(07)074014

Physics at the scale Mv: bound state formation

Quarkonium formation happens at the scale Mv. The suitable EFT is pNRQCD.

$$\frac{1}{E-p^2/m-V(r,\mu',\mu)}$$
 NRQCD

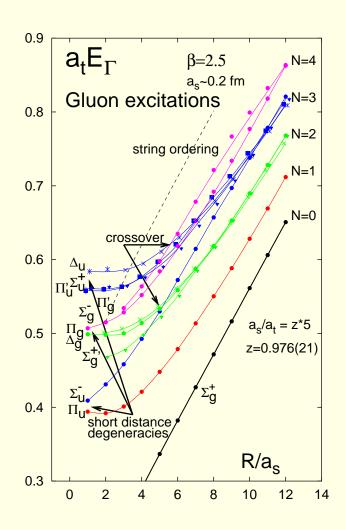
The effective Lagrangian is organized as an expansion in 1/M , $\alpha_{\rm s}(M)$ and r:

$$\mathcal{L}_{\text{pNRQCD}} = \int d^3r \sum_{n} \sum_{k} \frac{c_n(\alpha_s(M), \mu)}{M^n} \times V_{n,k}(r, \mu', \mu) \ r^k \times O_k(\mu', Mv^2, ...)$$

- $V_{n,0}$ are the potentials in the Schrödinger equation.
- $V_{n,k\neq 0}$ are the couplings with the low-energy degrees of freedom, which provide corrections to the potential picture.

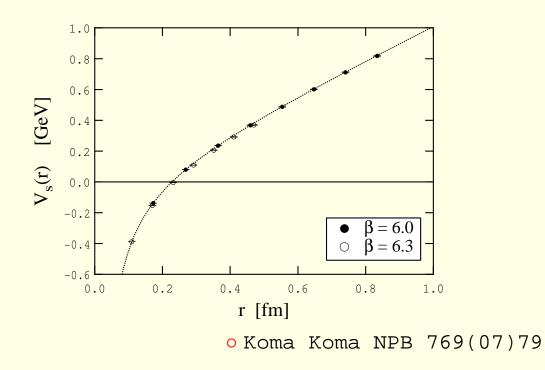
The static QCD spectrum without light quarks

- At short distances, it is well described by the Coulomb potentials: $V_s=-4\alpha_{\rm s}/3r$ and $V_o=\alpha_{\rm s}/6r$.
- At large distances, the energies rise linearly with r.
- Higher excitations develop a mass gap $\sim \Lambda_{\rm QCD}$ with respect to the lowest one.
- Reintroducing the heavy quark mass M: the spectrum of the Mv^2 fluctuations around the lowest state is the quarkonium spectrum; the spectrum of the Mv^2 fluctuations around the higher excitations is the hybrid spectrum.



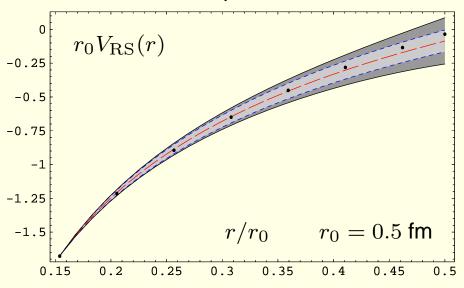
The quark-antiquark static energy

The energy of the lowest state is the quark-antiquark static energy.



Low-lying quarkonia: spectrum at $\mathcal{O}(M\alpha_{\rm s}^5)$

At short distances the potential is well described by PT up to NNNLL accuracy.



- O Necco Sommer PLB 523(01)135
- o Pineda JPG 29(03)371

Physical observables of the $\Upsilon(1S)$, η_b , B_c , J/ψ , η_c , ... may be understood in terms of PT. E.g. the spectrum up to $\mathcal{O}(M\alpha_s^5)$

$$E_n = \langle n | \frac{\mathbf{p}^2}{M} + V_s + ... | n \rangle - i \frac{g^2}{3N_c} \int_0^\infty dt \, \langle n | \mathbf{r} e^{it(E_n^{(0)} - H_o)} \mathbf{r} | n \rangle \, \langle \mathbf{E}(t) \, \mathbf{E}(0) \rangle$$

- oBrambilla Pineda Soto Vairo PLB 470(99)215 Kniehl Penin NPB 563(99)200
- o Kniehl Penin Smirnov Steinhauser NPB 635(02)357

Non-perturbative corrections are small and encoded in (local or non-local) condensates.

c and b masses

reference	order	$\overline{M}_b(\overline{M}_b)$ (GeV)	
Brambilla et al 01	NNLO +charm ($\Upsilon(1S)$)	$4.190 \pm 0.020 \pm 0.025$	
Penin Steinhauser 02	NNNLO* ($\Upsilon(1S)$)	4.346 ± 0.070	
Lee 03	NNNLO* ($\Upsilon(1S)$)	4.20 ± 0.04	
Contreras et al 03	NNNLO* ($\Upsilon(1S)$)	4.241 ± 0.070	
Pineda Signer 06	NNLL* high moments SR	4.19 ± 0.06	
reference	order	$\overline{M}_c(\overline{M}_c)$ (GeV)	
Brambilla et al 01	NNLO (J/ψ)	1.24 ± 0.02	
Eidemüller 02	NNLO high moments SR	1.19 ± 0.11	

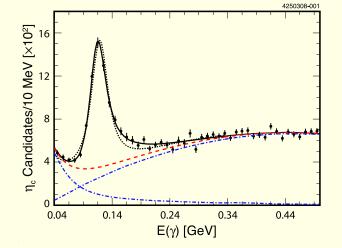
$J/\psi \to \eta_c \gamma$ at NNLO

$$\Gamma(J/\psi \to \eta_c \gamma) = \frac{16}{3} \alpha e_c^2 \frac{k_{\gamma}^3}{M_{J/\Psi}^2} \left(1 + \frac{4}{3} \frac{\alpha_s(M_{J/\Psi}/2)}{\pi} - \frac{2}{3} \frac{\langle 1|rV_s'|1\rangle}{M_{J/\Psi}} + 2 \frac{\langle 1|V_s|1\rangle}{M_{J/\Psi}} \right)$$

$$= \frac{16}{3} \alpha e_c^2 \frac{k_{\gamma}^3}{M_{J/\psi}^2} \left[1 + C_F \frac{\alpha_s(M_{J/\psi}/2)}{\pi} - \frac{2}{3} (C_F \alpha_s(p_{J/\psi}))^2 \right]$$

o Brambilla Jia Vairo PRD 73(06)054005

If we assume the J/ψ to be a weakly coupled bound state, then, up to order v^2 , the transition $J/\psi \to \eta_c \gamma$ is completely accessible to perturbation theory, leading to $\Gamma(J/\psi \to \eta_c \gamma) = (1.5 \pm 1.0)$ keV. To be compared with



$$\Gamma(J/\psi \to \eta_c \gamma) = (1.85 \pm 0.08 \pm 0.28)\,\mathrm{keV}$$

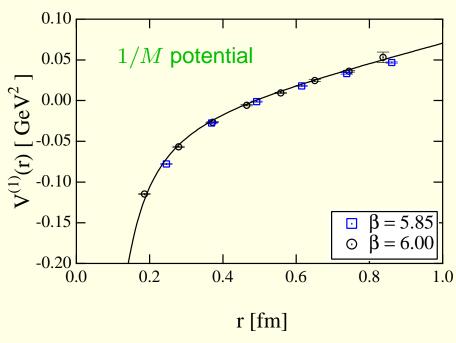
OCLEO PRL 102(09)011801

High-lying quarkonia: the 1/M potentials

The long range tail of the potential describes high-lying quarkonium resonances. 1/M and $1/M^2$ terms of the potential may be systematically included.

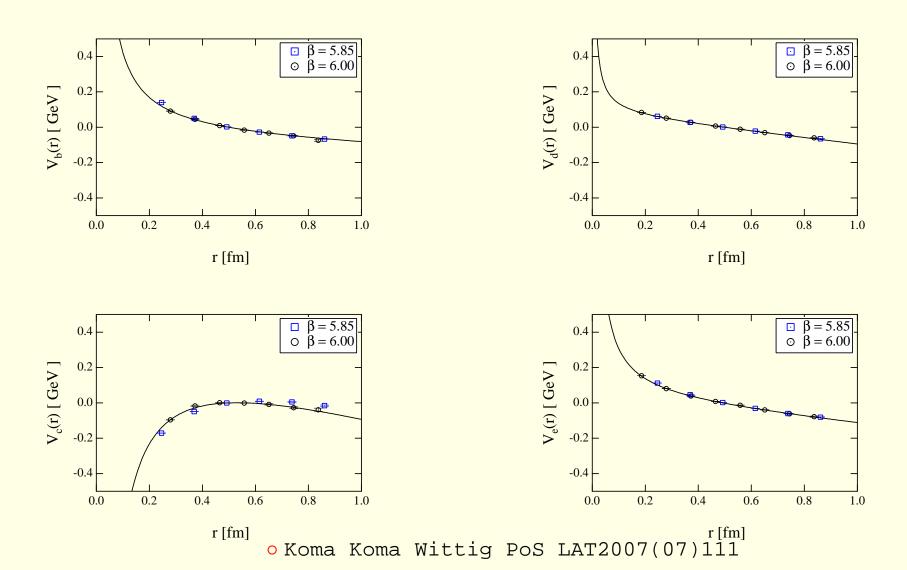
- o Brambilla Pineda Soto Vairo PRD 63(01)014023
- o Pineda Vairo PRD 63(01)054007

Lattice provides a non-perturbative determination of the potentials.

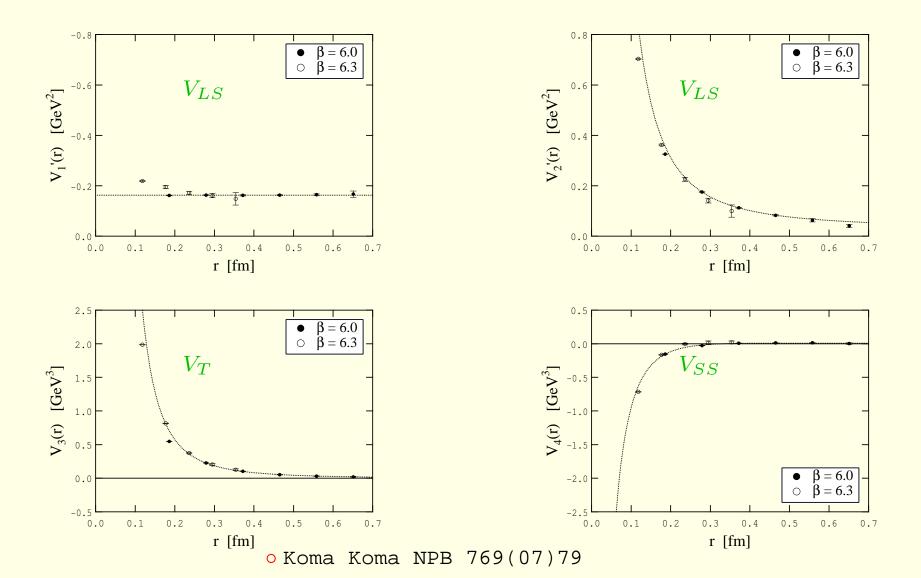


o Koma Koma Wittig PoS LAT2007(07)111

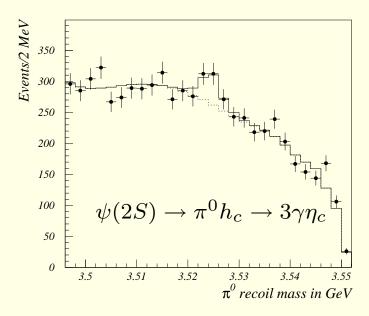
Spin-independent p^2/M^2 potentials



Spin-dependent $1/M^2$ potentials



h_c



$$M_{h_c} = 3524.4 \pm 0.6 \pm 0.4 \text{ MeV}$$

o CLEO PRL 95(05)102003

Also

$$M_{h_c} = 3525.8 \pm 0.2 \pm 0.2 \; \mathrm{MeV}, \qquad \Gamma < 1 \; \mathrm{MeV}$$

 • E835 PRD 72(05)032001

• To be compared with $M_{\rm c.o.g.}(1P) = 3525.36 \pm 0.2 \pm 0.2 \; {\rm MeV}.$

Gluonic excitations

A plethora of states built on each of the hybrid potentials is expected. These states typically develop a width also without including light quarks, since they may decay into lower states, e.g. like hybrid \rightarrow glueball + quark-antiquark.

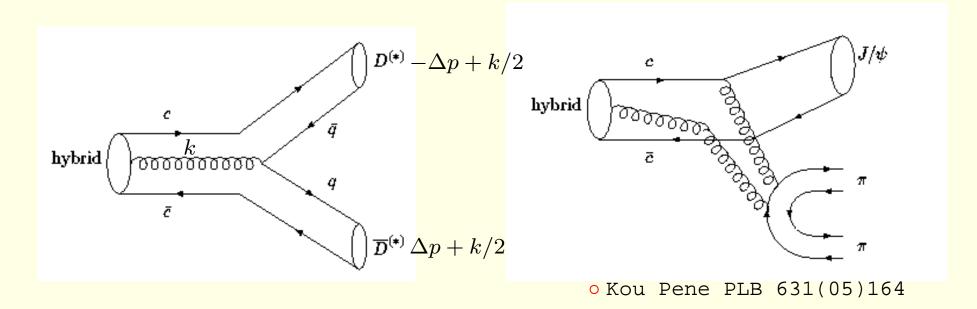
Y(4260): some properties

•
$$J^{PC} = 1^{--}$$

•
$$\frac{\mathcal{B}(Y \to D\bar{D})}{\mathcal{B}(Y \to J/\psi \pi^+ \pi^-)} < 1.0$$
 (~ 500 for $\psi(3770)$)

•
$$\frac{\mathcal{B}(Y \to D^* \bar{D})}{\mathcal{B}(Y \to J/\psi \pi^+ \pi^-)} < 34, \quad \frac{\mathcal{B}(Y \to D^* \bar{D}^*)}{\mathcal{B}(Y \to J/\psi \pi^+ \pi^-)} < 40$$

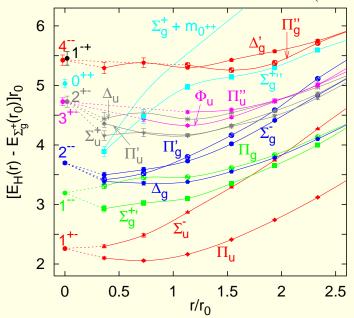
Y(4260): a $c\bar{c}$ hybrid candidate



$$|Y\rangle = |\Pi_u\rangle \otimes |\phi\rangle$$

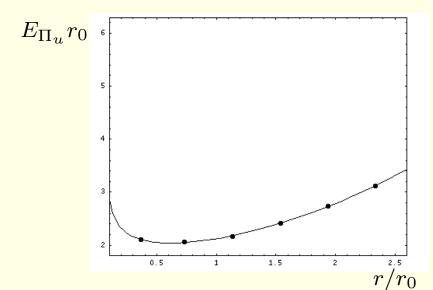
- $|\Pi_u\rangle$ is a 1⁺⁻ static hybrid state that encodes the glue content.
- $|\phi\rangle$ is a 0^{-+} solution of the Schrödinger equation whose potential is the static energy of $|\Pi_u\rangle$.
- At leading order, decays into $D^{(*)}\bar{D}^{(*)}$ are forbidden.
- $Y \to \pi^+\pi^-J/\psi$ decay is induced by the emission of an additional magnetic gluon. It is suppressed by $1/M_c$, but with a large available phase space.

Y(4260): a $c\bar{c}$ hybrid candidate



H	$\Lambda_H^{ m RS} r_0$	$\Lambda_H^{ m RS}/{ m GeV}$
B_i	2.25(39)	0.87(15)
$ E_i $	3.18(41)	1.25(16)
$D_{\{i}B_{j\}}$	3.69(42)	1.45(17)
$D_{\{i}E_{j\}}$	4.72(48)	1.86(19)
$D_{\{i}D_{j}B_{k\}}$	4.72(45)	1.86(18)
\mathbf{B}^2	5.02(46)	1.98(18)
$D_{\{i}D_{j}D_{k}B_{l\}}$	5.41(46)	2.13(18)
$(\mathbf{B} \wedge \mathbf{E})_i$	5.45(51)	2.15(20)
	$egin{array}{c c} B_i & & & & & & & & & & & & & & & & & & &$	$egin{array}{ c c c c c c c c c c c c c c c c c c c$

- o Foster Michael PRD 59(99)094509
- o Bali Pineda PRD 69(04)094001



Fitting the Π_u curve, $E_{\Pi_u}=(0.87+0.11/r+0.24\,r^2)$ GeV and solving the Schrödinger equation, one gets

$$M(Y) = 2 \times 1.48 + 0.87 + 0.53 = 4.36$$
 GeV

• Vairo IJMP A22(07)5481

The QCD spectrum with light quarks

• We still have states just made of heavy quarks and gluons. They may develop a width because of the decay through pion emission. If new states made with heavy and light quarks develop a mass gap of order $\Lambda_{\rm QCD}$ with respect to the former ones, then these new states may be asborbed into the definition of the potentials or of the (local or non-local) condensates.

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o Brambilla et al. PRD 67(03)034018
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In addition new states built using the light quark quantum numbers may form.

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o Soto NP PS 185(08)107
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• Pairs of heavy-light mesons: $D\bar{D}, B\bar{B}, ...$

Molecular states, i.e. states built on the pair of heavy-light mesons.

o Tornqvist PRL 67(91)556

• The usual quarkonium states, built on the static potential, may also give rise to molecular states through the interaction with light hadrons (hadro-quarkonium).

o Dubynskiy Voloshin PLB 666(08)344

Pairs of heavy-light baryons.

o Qiao PLB 639(06)263

Tetraquark states.

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o Jaffe PRD 15(77)267
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o Ebert Faustov Galkin PLB 634(06)214

Having the spectrum of tetraquark potentials, like we have for the gluonic excitations, would allow us to build a plethora of states on each of the tetraquark potentials, many of them developing a width due to decays through pion (or other light hadrons) emission. Diquarks have been recently investigated on the lattice.

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o Alexandrou et al. PRL 97(06)222002
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o Fodor et al. PoS LAT2005(06)310

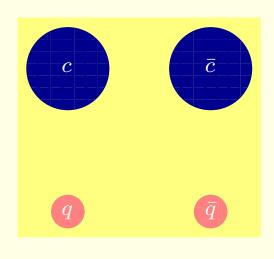
And likely many other states ...

Experimental evidences of new states

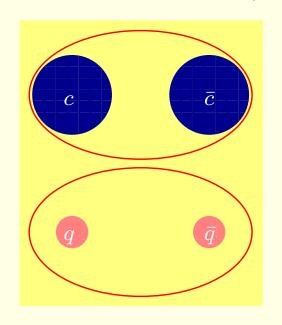
- Clear evidence for four-quark states may be provided by a charged resonance, like the $Z^+(4430)$, $Z_1^+(4050)$ and $Z_2^+(4250)$ signals, detected by BELLE, possibly are. See the talks by R. Chistov, C. Patrignani and the panel discussion.
- There is accumulating evidence, although not yet conclusive, that the X(3872) may be a four quark state.

X(3872): some properties

- $\Gamma_X < 2.3 \text{ MeV}$
- The dominant decay mode is: $\frac{\mathcal{B}(X\to D^0D^0\pi^0)}{\mathcal{B}(X\to \pi^+\pi^-J/\psi)} = 9.4^{+3.6}_{-4.3},$ but the threshold enhancement peaks at $3875.4\pm0.7^{+1.2}_{-2.0}$ MeV: is it X(3872)?
- $X \to \gamma J/\psi \Rightarrow C = +.$
- Angular distribution analyses favour the spin parity: $J=1^+$
- $\frac{\mathcal{B}(X \to \pi^+ \pi^- \pi^0 J/\psi)}{\mathcal{B}(X \to \pi^+ \pi^- J/\psi)} = 1.0 \pm 0.4 \pm 0.3$ $\Rightarrow X \text{ is a mixture of } I = 1 \text{ and } I = 0 \text{ states.}$
- The substantial I=1 component requires that X contains $u\bar{u}/d\bar{d}$ pairs in addition to hidden charm, which thus qualifies it as a four-quark state.



4-quark state with $J^{PC} = 1^{++}$



o Høgaasen Richard Sorba PRD 73(06)054013

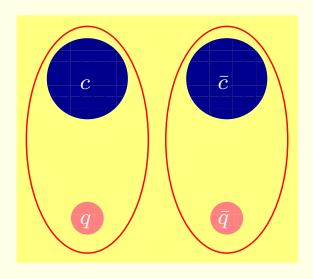
$$X \sim (c\bar{c})_{S=1}^{8} \otimes (q\bar{q})_{S=1}^{8}$$

$$\sim (c\bar{q})_{S=0}^{1} \otimes (q\bar{c})_{S=1}^{1} + (c\bar{q})_{S=1}^{1} \otimes (q\bar{c})_{S=0}^{1}$$

Predictions based on the phenomenological Hamiltonian:

$$H = -\sum_{ij} C_{ij} T^a \otimes T^a \boldsymbol{\sigma} \otimes \boldsymbol{\sigma};$$

- decays into charmonium plus light vector mesons are suppressed with respect to those into heavy-light mesons like $D\bar{D}^*$;
- decays into charmonium plus light pseudoscalar mesons are not allowed by simple quark rearrangements.
- Two neutral states made of $cu\bar{c}\bar{u}$ and $cd\bar{c}\bar{d}$ and two charged ones made of $cu\bar{c}\bar{d}$ and $cd\bar{c}\bar{u}$ are predicted.



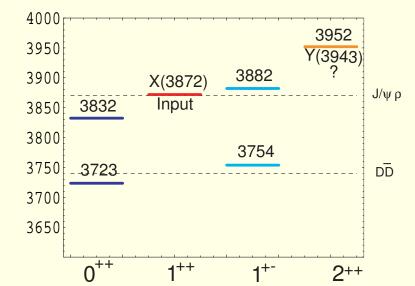
• Maiani et al. PRD 71(05)014028

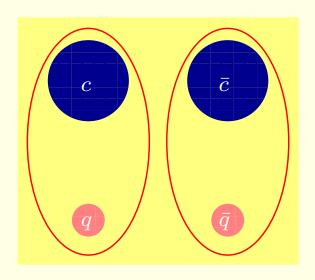
$$X \sim (cq)_{S=1}^{\bar{3}} \otimes (\bar{c}\bar{q})_{S=0}^3 + (cq)_{S=0}^{\bar{3}} \otimes (\bar{c}\bar{q})_{S=1}^3$$

the dynamical assumption is that quark pair cluster in tightly bound color triplet diquarks (see 1-gluon exchange); the difficulty in breaking the system explains the narrow width.

• Predictions based on the phenomenological Hamiltonian: $H = \sum_{ij} \kappa_{ij} \ \sigma \otimes \sigma$; the framework has been applied to a large variety of systems $(D_s, X, Y, ...)$ and observables.







o Maiani et al. PRD 71(05)014028

$$X \sim (cq)_{S=1}^{\bar{3}} \otimes (\bar{c}\bar{q})_{S=0}^3 + (cq)_{S=0}^{\bar{3}} \otimes (\bar{c}\bar{q})_{S=1}^3$$

the dynamical assumption is that quark pair cluster in tightly bound color triplet diquarks (see 1-gluon exchange); the difficulty in breaking the system explains the narrow width.

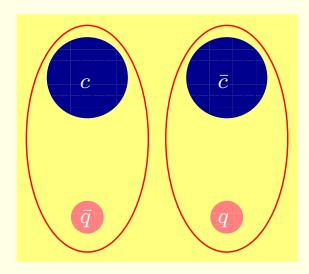
• Two neutral states made of $X_u \sim cu\bar{c}\bar{u}$ and $X_d \sim cd\bar{c}\bar{d}$ (and two charged ones) are predicted, with mass difference

$$\Delta M = \frac{2(m_d - m_u)}{\cos(2\theta)} \approx (8 \pm 3) \text{ MeV}$$

if the mixing angle is fixed on $\Gamma(X \to \pi^+\pi^-\pi^0 J/\psi)/\Gamma(X \to \pi^+\pi^- J/\psi)$.

The two resonances discovered by BELLE and BABAR, the first decaying in $J/\psi\rho$ and $J/\psi\omega$, and the second preferably in $D^0\bar{D}^0\pi^0$ have been suggested as possible candidates for the X_d and X_u .

o Maiani Polosa Riquer PRL 99(07)182003

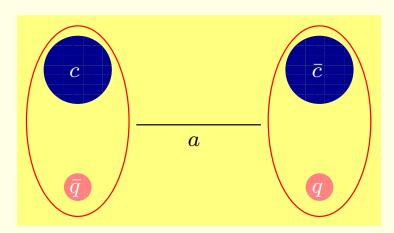


- o Törnqvist ZPC 61(94)525
- o Swanson, PLB 588(04)189 (2004)

$$X \sim (c\bar{q})_{S=0}^{1} \otimes (q\bar{c})_{S=1}^{1} + (c\bar{q})_{S=1}^{1} \otimes (q\bar{c})_{S=0}^{1}$$
$$\sim D\bar{D}^{*} + D^{*}\bar{D}$$

This is assumed to be the dominant long-range Fock component; short-range components of the type $(c\bar{c})_{S=1}^1 \otimes (q\bar{q})_{S=1}^1 \sim J/\psi \ \rho, \omega$ are assumed as well.

- Predictions are strongly based on the assumed phenomenological Hamiltonians: short range ($\sim \Lambda_{\rm QCD}$): potential model interaction at the quark level; long range ($\sim m_{\pi}$): one pion exchange.
- The prediction $\Gamma(X \to \pi^+\pi^- J/\psi) \approx \Gamma(X \to \pi^+\pi^-\pi^0 J/\psi)$ turned out to be consistent with BELLE.
- However, $\Gamma(X \to \pi^+\pi^-J/\psi) \approx 20 \, \Gamma(X \to D^0 \bar{D}^0\pi^0)$ is two orders of magnitude far from the BELLE measurement. This may point to a smaller $J/\psi \, \rho$ component in the Fock space, which may conflict with the charmonium-like production mechanism.



- o Voloshin PLB 579(04)316
- o Braaten Kusunoki PRD 69(04)074005
- AlFiky Gabbiani Petrov PLB 640(06)238

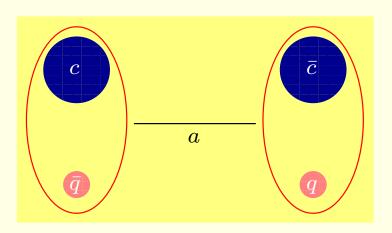
$$\Lambda_{\rm QCD} \gg m_\pi \gg m_\pi^2/(2m_{\rm red}) \approx 10 \; {\rm MeV} \gg E_{\rm binding}$$

 $E_{\rm binding} \approx M_X - (M_{D^0*} + M_{D^0}) = (0.4 \pm 0.8) \; {\rm MeV}$

- BELLE measure of M_X in $D^0 \bar{D}^0 \pi^0$ is 3-4 MeV larger.
- Systems with a short-range interaction and a large scattering length $a\gg 1/m_\pi$ have universal properties. If $\gamma_{\rm re}={\rm Re}(1/a)>0$ there is a shallow bound state of binding energy $E_X=\gamma_{\rm re}^2/(2m_{\rm red})$, width $\Gamma_X=2\gamma_{\rm re}\gamma_{\rm im}/m_{\rm red}$ and wave function

$$\phi(r) = \sqrt{\frac{1}{2\pi a}} \frac{e^{-r/a}}{r}$$

• Mass and width of the X constrain $\gamma_{\rm re}$ and $\gamma_{\rm im}$ and imply $\langle r \rangle_X > 3$ fm.



- o Voloshin PLB 579(04)316
- o Braaten Kusunoki PRD 69(04)074005
- AlFiky Gabbiani Petrov PLB 640(06)238

$$\Lambda_{\rm QCD} \gg m_{\pi} \gg m_{\pi}^2/(2m_{\rm red}) \approx 10 \text{ MeV} \gg E_{\rm binding}$$
 $E_{\rm binding} \approx M_X - (M_{D^0*} + M_{D^0}) = (0.4 \pm 0.8) \text{ MeV}$

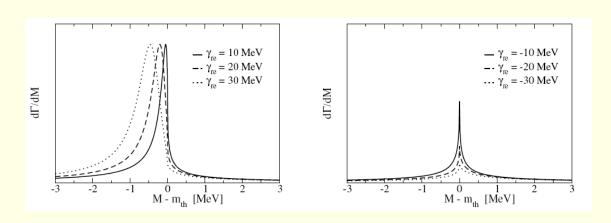
• BELLE measure of M_X in $D^0 \bar{D}^0 \pi^0$ is 3-4 MeV larger.

Universal properties:

$$\Gamma(X \to D^0 \bar{D^0} \pi^0) = \Gamma(D^0 {}^* \to \bar{D^0} \pi^0), \dots$$

 $A = A_{\mathrm{short}} \times A_{\mathrm{long}}(E_X, \Gamma_X)$ in decay and production amplitudes.

Shape of the invariant mass distribution in a short distance decay channel:



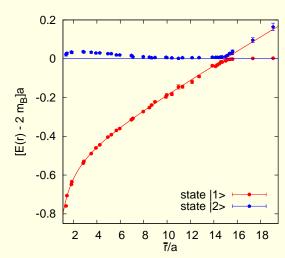
Coupled channels

An important (and unsolved) issue is how all the different kind of states (with and without light quarks) interact with each other.

A systematic treatment does not exist so far. For the coupling with two-meson states, most of the existing analyses rely on two models, which are now more than 30 years old:

- the Cornell coupled-channel model;
 - o Eichten et al. PRD 17(78)3090, 21(80)313
 - o Eichten et al. PRD 69(04)094019, 73(06)014014, 73(06)079903
- and the 3P_0 model.
 - o Le Yaouanc et al. PRD 8(73)2223
 - o Kalashnikova PRD 72(05)034010

Steps towards a lattice based approach have been undertaken: see the talk by G. Bali.



Conclusions

Our understanding of how a (effective field) theory of quarkonium should look like has dramatically increased over the last decade.

For states below threshold such a theory exists and allows a systematic study of the quarkonium lowest resonances. Even precision physics is possible. Higher resonances may need to be supplemented by lattice data. High quality lattice data have become available in the last years for some crucial quantities (e.g. potentials, decay matrix elements, ...).

For states above threshold the picture appears much more uncertain. Many degrees of freedom seem to show up, and the absence of a clear systematics is an obstacle to an universal picture. Most likely descriptions will be found that suite specific families of states, the near threshold molecular states providing an example.