D^0-$\overline{D}{}^0$ Mixing and CP Violation: HFAG Combination of Parameters

A. J. Schwartz*
(representing the HFAG charm group)[1]
[1]Physics Department, University of Cincinnati, Cincinnati, OH 45221, USA

Abstract

We present the most recent set of world averages for D^0-$\overline{D}{}^0$ mixing and CP violation parameters, as obtained by the Heavy Flavor Averaging Group from a global fit to various measurements. The values obtained for the mixing parameters when allowing for CP violation are $x = 0.98\,^{+0.24}_{-0.26}$ and $y = 0.83 \pm 0.16$; the significance of mixing is 10.2σ. There is no evidence for CP violation at the current level of sensitivity.

Introduction

In 2006, the Heavy Flavor Averaging Group (HFAG) [1] convened a new subgroup to calculate world average (WA) values of charm mixing and CP violation (CPV) parameters [2]. Since that time, D^0-$\overline{D}{}^0$ mixing has been observed, and a wealth of mixing and CPV results have appeared. The HFAG charm group has calculated several sets of WA values, updating old averages as new results have become available. This paper presents the most recent set of averages, i.e., those based on results that appeared in preprint form by the summer of 2009.

Mixing in the B^0 and B^0_s heavy flavor systems is governed by the short-distance box diagram. In the D^0 system, however, this diagram is doubly-Cabibbo-suppressed (relative to amplitudes dominating the decay width) and also GIM-suppressed. Thus, the short-distance mixing rate is tiny, and D^0-$\overline{D}{}^0$ mixing is expected to be dominated by long-distance processes. These are difficult to calculate reliably, and theoretical estimates for D^0-$\overline{D}{}^0$ mixing range over 2-3 orders of magnitude [3, 4].

The decay rates for $D^0 \to f$ and $\overline{D}{}^0 \to \bar{f}$ are, respectively,

$$\frac{dN_{D^0}}{dt} \;\propto\; e^{-\overline{\Gamma}\,t}\,\Biggl\{R^+ + \left|\frac{q}{p}\right|\sqrt{R^+}\times (y'\cos\phi - x'\sin\phi)(\overline{\Gamma}t) + \left|\frac{q}{p}\right|^2 \frac{(x'^2+y'^2)}{4}(\overline{\Gamma}\,t)^2\Biggr\} \quad (1)$$

$$\frac{dN_{\overline{D}{}^0}}{dt} \;\propto\; e^{-\overline{\Gamma}\,t}\,\Biggl\{R^- + \left|\frac{p}{q}\right|\sqrt{R^-}\times (y'\cos\phi + x'\sin\phi)(\overline{\Gamma}t) + \left|\frac{p}{q}\right|^2 \frac{(x'^2+y'^2)}{4}(\overline{\Gamma}\,t)^2\Biggr\}. \quad (2)$$

In these expressions, $x' = x\cos\delta + y\sin\delta$ and $y' = y\cos\delta - x\sin\delta$, where $x = (M_2 - M_1)/\overline{\Gamma}$ and $y = (\Gamma_2 - \Gamma_1)/(2\overline{\Gamma})$ are mixing parameters, and δ is the strong phase difference between amplitudes $\mathcal{A}(\overline{D}{}^0 \to f)$ and $\mathcal{A}(D^0 \to f)$. Parameters M_1, M_2, Γ_1, and Γ_2 are the masses and decay widths of the mass eigenstates $|D_1\rangle \equiv p|D^0\rangle + q|\overline{D}{}^0\rangle$ and $|D_2\rangle \equiv p|D^0\rangle - q|\overline{D}{}^0\rangle$, and $\overline{\Gamma} = (\Gamma_1+\Gamma_2)/2$. Our convention is $CP|D^0\rangle = -|\overline{D}{}^0\rangle$ such that for $q = p$, D_1 is CP-odd and D_2 is CP-even. The parameters $R^+ = |\mathcal{A}(D^0 \to f)/\mathcal{A}(\overline{D}{}^0 \to f)|^2$, $R^- = |\mathcal{A}(\overline{D}{}^0 \to \bar{f})/\mathcal{A}(D^0 \to \bar{f})|^2$, and $\phi = \mathrm{Arg}(q/p)$.

To obtain WA values of $x, y, \delta, |q/p|$, and ϕ, we perform a global fit to 28 measured observables. These observables are from measurements of $D^0 \to K^+\ell^-\nu$, $D^0 \to K^+K^-/\pi^+\pi^-$, $D^0 \to K^+\pi^-$, $D^0 \to K^+\pi^-\pi^0$, $D^0 \to K^0_S\,\pi^+\pi^-$, and $D^0 \to K^+K^-K^0_S$ decays [5], and from double-tagged branching fractions measured in $e^+e^- \to \psi(3770) \to DD$ reactions. To fit these observables, we must include an additional strong phase $\delta_{K\pi\pi}$ (see below). For $D^0 \to K^+\pi^-$ decays, we combine R^+ and R^- into parameters $R_D \equiv (R^+ + R^-)/2$ and $A_D \equiv (R^+ - R^-)/(R^+ + R^-)$. Correlations among observables are accounted for by using covariance matrices provided by the experimental collaborations.

With the exception of the $\psi(3770) \to DD$ measurements, all methods identify the flavor of the D^0 or $\overline{D}{}^0$ when produced by reconstructing the decay $D^{*+} \to D^0\pi^+$ or $D^{*-} \to \overline{D}{}^0\pi^-$; the charge of the accompanying pion identifies the D flavor. For signal decays, $M_{D^*} - M_{D^0} - M_{\pi^+} \equiv Q \approx 6$ MeV, which is relatively close to the threshold. Thus, analyses typically require that the reconstructed Q be small to suppress backgrounds. For time-dependent measurements, the D^0 decay time is calculated as $(\ell/p) \times M_{D^0}$, where ℓ is the distance between the D^* and D^0 decay vertices and p is the D^0 momentum. The D^* vertex position is taken to be either the primary vertex position ($\bar{p}p$ experiments) or else is calculated from the intersection of the D^0 momentum vector with the beam-spot profile (e^+e^- experiments).

Input Observables

The global fit determines central values and errors for x, y, δ, R_D, A_D, $|q/p|$, ϕ, and $\delta_{K\pi\pi}$ using a χ^2 statistic. Parameters x and y govern mixing, and parameters A_D, $|q/p|$, and ϕ govern CPV. The parameter $\delta_{K\pi\pi}$ is the strong phase difference between amplitudes $\mathcal{A}(\overline{D}{}^0 \to K^+\pi^-\pi^0)$ and $\mathcal{A}(D^0 \to K^+\pi^-\pi^0)$ evaluated at $M_{K^+\pi^-} = M_{K^*(890)}$.

All input values are listed in Table 1. The values for observables $R_M = (x^2+y^2)/2$ [6], y_{CP} [7], and A_Γ [7] are HFAG WA values [8]. They are calculated as weighted averages of measurements, taking into account correlations

* alan.j.schwartz@uc.edu

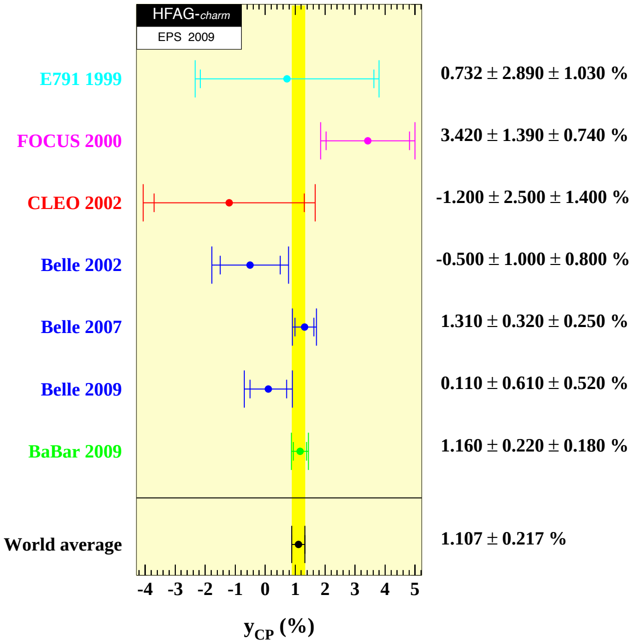

Figure 1: WA value of y_{CP} as calculated from $D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$, and $D^0 \to K^+K^-K^0_S$ measurements [7].

among systematic errors and sometimes also statistical errors. As an example, the weighted average for y_{CP} is shown in Fig. 1. The values of observables from $D^0 \to K^0_S\,\pi^+\pi^-$ decays [9] for no-CPV are HFAG WA values [8], but for the CPV-allowed case only Belle measurements are available. The $D^0 \to K^+\pi^-$ results used [10] are from Belle, Babar, and CDF, as these results have much greater precision than earlier ones. The $D^0 \to K^+\pi^-\pi^0$ results are from Babar [11], and the $\psi(3770) \to DD$ results are from CLEOc [12].

The relationships between the observables and the fitted parameters are listed in Table 2. For each set of correlated observables, we construct a difference vector $\vec{V}$. For example, $\vec{V} = (\Delta x, \Delta y, \Delta|q/p|, \Delta\phi)$ for $D^0 \to K^0_S\,\pi^+\pi^-$ decays, where Δ represents the difference between the measured value and the fitted parameter value. The contribution of a set of observables to the χ^2 is calculated as $\vec{V} \cdot (M^{-1}) \cdot \vec{V}^T$, where M^{-1} is the inverse of the covariance matrix for the measurement. All covariance matrices used are listed in Table 1.

Fit results

The global fit uses MINUIT with the MIGRAD minimizer, and all errors are obtained from MINOS. Three separate fits are performed: *(a)* assuming CP conservation (A_D and ϕ are fixed to zero, $|q/p|$ is fixed to one); *(b)* assuming no direct CPV (A_D is fixed to zero); and *(c)* allowing full CPV (all parameters floated). Results from the first and last fits are listed in Table 3. For the CPV-allowed fit, individual contributions to the χ^2 are listed in

Table 3: Results of the global fit for the cases of no CPV and all-CPV-allowed.

Parameter	No CPV	CPV-allowed	CPV 95% CL
x (%)	$0.99\,^{+0.24}_{-0.25}$	$0.98\,^{+0.24}_{-0.26}$	[0.46, 1.44]
y (%)	0.81 ± 0.16	0.83 ± 0.16	[0.51, 1.14]
δ (°)	$25.2\,^{+9.6}_{-9.9}$	$26.4\,^{+9.6}_{-9.9}$	[5.9, 45.8]
$\delta_{K\pi\pi}$ (°)	$13.5\,^{+20.2}_{-22.1}$	$14.8\,^{+20.2}_{-22.1}$	[-30.3, 53.8]
R_D (%)	0.336 ± 0.008	0.337 ± 0.009	[0.320, 0.353]
A_D (%)	–	-2.2 ± 2.4	[-6.9, 2.6]
$\|q/p\|$	–	$0.87\,^{+0.17}_{-0.15}$	[0.60, 1.22]
ϕ (°)	–	$-8.5\,^{+7.4}_{-7.0}$	[-22.1, 6.3]

Table 4: Contributions to the χ^2 (CPV-allowed fit).

Observable	χ^2	$\sum\chi^2$
y_{CP}	1.85	1.85
A_Γ	0.15	2.00
$x_{K^0\pi^+\pi^-}$	0.23	2.23
$y_{K^0\pi^+\pi^-}$	2.49	4.73
$\|q/p\|_{K^0\pi^+\pi^-}$	0.00	4.73
$\phi_{K^0\pi^+\pi^-}$	0.67	5.39
$R_M(K^+\ell^-\nu)$	0.03	5.42
$x_{K^+\pi^-\pi^0}$	2.94	8.36
$y_{K^+\pi^-\pi^0}$	1.67	10.04
$R_M/y/R_D/\sqrt{R_D}\cos\delta$ (CLEOc)	5.72	15.76
$R^+/x'^{2+}/y'^+$ (Babar)	2.74	18.50
$R^-/x'^{2-}/y'^-$ (Babar)	2.01	20.51
$R^+/x'^{2+}/y'^+$ (Belle)	3.72	24.23
$R^-/x'^{2-}/y'^-$ (Belle)	1.28	25.51
$R_D/x'^2/y'$ (CDF)	0.75	26.26

Table 4. The total χ^2 is 26.3 for $28 - 8 = 20$ degrees of freedom; this corresponds to a confidence level of 0.16.

Confidence contours in the two dimensions (x, y) and $(|q/p|, \phi)$ are obtained by letting, for any point in the two-dimensional plane, all other fitted parameters take their preferred values. The resulting 1σ-5σ contours are shown in Fig. 2 for the CP-conserving case, and in Fig. 3 for the CPV-allowed case. The contours are determined from the increase of the χ^2 above the minimum value ($\chi^2_{\rm min}$). One observes that the (x, y) contours for no-CPV and for CPV-allowed are almost identical. In the latter case, the χ^2 at the no-mixing point $(x, y) = (0, 0)$ is 110 units above the minimum value; this difference corresponds to a confidence level of 10.2σ. Thus, no mixing is excluded at this high level. In the $(|q/p|, \phi)$ plot, the no-CPV point $(1, 0)$ is within the 1σ contour; thus the data is consistent with CP conservation.

One-dimensional confidence curves for individual parameters are obtained by letting, for any value of the pa-

Table 1: Input values used for the global fit, from Refs. [6, 7, 9, 10, 11, 12].

Observable	**Value**	**Comment**
y_{CP}	$(1.107 \pm 0.217)\%$	WA $D^0 \to K^+K^-/\pi^+\pi^-$ and $D^0 \to K^+K^-K^0_S$ results [8]
A_Γ	$(0.123 \pm 0.248)\%$	
x (no CPV)	$(0.811 \pm 0.334)\%$	No CPV:
y (no CPV)	$(0.309 \pm 0.281)\%$	WA $D^0 \to K^0_S\,\pi^+\pi^-$ results [8]
$\|q/p\|$ (no direct CPV)	$0.95 \pm 0.22^{+0.10}_{-0.09}$	
ϕ (no direct CPV)	$(-0.035 \pm 0.19 \pm 0.09)$ rad	
x	$(0.81 \pm 0.30^{+0.13}_{-0.17})\%$	CPV-allowed: Belle $D^0 \to K^0_S\,\pi^+\pi^-$ results. Correlation coefficients: $\left\{\begin{matrix} 1 & -0.007 & -0.255\alpha & 0.216 \\ -0.007 & 1 & -0.019\alpha & -0.280 \\ -0.255\alpha & -0.019\alpha & 1 & -0.128\alpha \\ 0.216 & -0.280 & -0.128\alpha & 1 \end{matrix}\right\}$
y	$(0.37 \pm 0.25^{+0.10}_{-0.15})\%$	
$\|q/p\|$	$0.86 \pm 0.30^{+0.10}_{-0.09}$	
ϕ	$(-0.244 \pm 0.31 \pm 0.09)$ rad	Note: $\alpha = (\|q/p\|+1)^2/2$ is a variable transformation factor
R_M	$(0.0130 \pm 0.0269)\%$	WA $D^0 \to K^+\ell^-\nu$ results [8]
x''	$(2.61\,^{+0.57}_{-0.68} \pm 0.39)\%$	Babar $D^0 \to K^+\pi^-\pi^0$ result. Correlation coefficient $= -0.75$.
y''	$(-0.06\,^{+0.55}_{-0.64} \pm 0.34)\%$	Note: $x'' \equiv x\cos\delta_{K\pi\pi} + y\sin\delta_{K\pi\pi}$, $y'' \equiv y\cos\delta_{K\pi\pi} - x\sin\delta_{K\pi\pi}$.
R_M	$(0.199 \pm 0.173 \pm 0.0)\%$	CLEOc results from “double-tagged” branching fractions measured in $\psi(3770) \to DD$ decays. Correlation coefficients: $\left\{\begin{matrix} 1 & -0.0644 & 0.0072 & 0.0607 \\ -0.0644 & 1 & -0.3172 & -0.8331 \\ 0.0072 & -0.3172 & 1 & 0.3893 \\ 0.0607 & -0.8331 & 0.3893 & 1 \end{matrix}\right\}$
y	$(-5.207 \pm 5.571 \pm 2.737)\%$	
R_D	$(-2.395 \pm 1.739 \pm 0.938)\%$	
$\sqrt{R_D}\cos\delta$	$(8.878 \pm 3.369 \pm 1.579)\%$	Note: the only external input to these fit results are branching fractions.
R_D	$(0.303 \pm 0.0189)\%$	Babar $D^0 \to K^+\pi^-$ results. Correlation coefficients: $\left\{\begin{matrix} 1 & 0.77 & -0.87 \\ 0.77 & 1 & -0.94 \\ -0.87 & -0.94 & 1 \end{matrix}\right\}$
x'^{2+}	$(-0.024 \pm 0.052)\%$	
y'^{+}	$(0.98 \pm 0.78)\%$	
A_D	$(-2.1 \pm 5.4)\%$	Babar $D^0 \to K^+\pi^-$ results; correlation coefficients same as above.
x'^{2-}	$(-0.020 \pm 0.050)\%$	
y'^{-}	$(0.96 \pm 0.75)\%$	
R_D	$(0.364 \pm 0.018)\%$	Belle $D^0 \to K^+\pi^-$ results. Correlation coefficients: $\left\{\begin{matrix} 1 & 0.655 & -0.834 \\ 0.655 & 1 & -0.909 \\ -0.834 & -0.909 & 1 \end{matrix}\right\}$
x'^{2+}	$(0.032 \pm 0.037)\%$	
y'^{+}	$(-0.12 \pm 0.58)\%$	
A_D	$(2.3 \pm 4.7)\%$	Belle $D^0 \to K^+\pi^-$ results; correlation coefficients same as above.
x'^{2-}	$(0.006 \pm 0.034)\%$	
y'^{-}	$(0.20 \pm 0.54)\%$	
R_D	$(0.304 \pm 0.055)\%$	CDF $D^0 \to K^+\pi^-$ results. Correlation coefficients: $\left\{\begin{matrix} 1 & 0.923 & -0.971 \\ 0.923 & 1 & -0.984 \\ -0.971 & -0.984 & 1 \end{matrix}\right\}$
x'^{2}	$(-0.012 \pm 0.035)\%$	
y'	$(0.85 \pm 0.76)\%$	

Table 2: Left: decay modes used to determine the parameters x, y, δ, R_D, A_D, $|q/p|$, ϕ, and $\delta_{K\pi\pi}$. Middle: observables measured for each decay mode. Right: the relationships between the observables and the fitted parameters.

Decay Mode	**Observables**	**Relationship**
$D^0 \to K^+K^-/\pi^+\pi^-/K^+K^-K^0_S$	y_{CP} A_Γ	$2y_{CP} = (\|q/p\| + \|p/q\|)\, y\cos\phi \;-\; (\|q/p\| - \|p/q\|)\, x\sin\phi$ $2A_\Gamma = (\|q/p\| - \|p/q\|)\, y\cos\phi \;-\; (\|q/p\| + \|p/q\|)\, x\sin\phi$
$D^0 \to K^0_S\,\pi^+\pi^-$	x y $\|q/p\|$ ϕ	
$D^0 \to K^+\ell^-\nu$	R_M	$R_M = (x^2+y^2)/2$
$D^0 \to K^+\pi^-\pi^0$ (Dalitz plot analysis)	x'' y''	$x'' = x\cos\delta_{K\pi\pi} + y\sin\delta_{K\pi\pi}$ $y'' = y\cos\delta_{K\pi\pi} - x\sin\delta_{K\pi\pi}$
"Double-tagged" branching fractions measured in $\psi(3770) \to DD$ decays	R_M y R_D $\sqrt{R_D}\cos\delta$	$R_M = (x^2+y^2)/2$
$D^0 \to K^+\pi^-$	$R^+,\ R^-$ $x'^{2+},\ x'^{2-}$ $y'^+,\ y'^-$	$R_D = (R^+ + R^-)/2$ $A_D = (R^+ - R^-)/(R^+ + R^-)$ $x' = x\cos\delta + y\sin\delta$ $y' = y\cos\delta - x\sin\delta$ $A_M \equiv (\|q/p\|^4 - 1)/(\|q/p\|^4 + 1)$ $x'^{\pm} = [(1 \pm A_M)/(1 \mp A_M)]^{1/4}(x'\cos\phi \pm y'\sin\phi)$ $y'^{\pm} = [(1 \pm A_M)/(1 \mp A_M)]^{1/4}(y'\cos\phi \mp x'\sin\phi)$

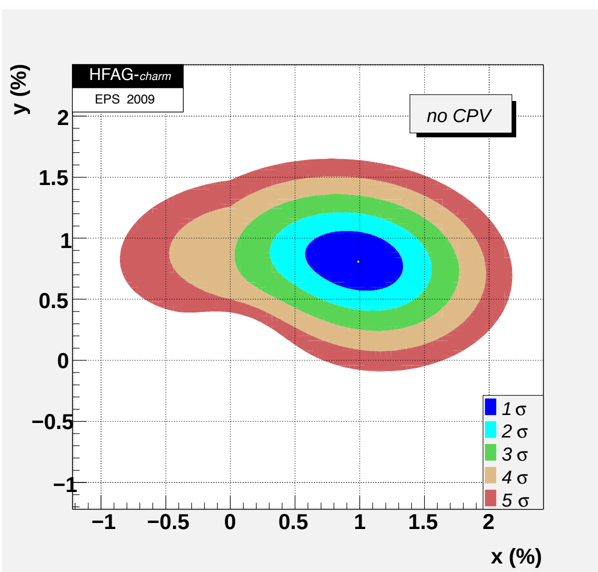

Figure 2: Two-dimensional contours for mixing parameters (x, y), for no CPV.

rameter, all other fitted parameters take their preferred values. The resulting functions $\Delta\chi^2 = \chi^2 - \chi^2_{\rm min}$ are shown in Fig. 4. The points where $\Delta\chi^2 = 3.84$ determine 95% C.L. intervals for the parameters, as shown in the figure. These intervals are listed in Table 3.

Summary

We summarize the fit results listed in Table 3 and shown in Figs. 3 and 4 as follows:

- the experimental data consistently indicate that D^0 mesons undergo mixing. The no-mixing point $(x, y) = (0, 0)$ is excluded at 10.2σ. The parameter x differs from zero by 3.2σ, and the parameter y differs from zero by 4.8σ. The effect is presumably dominated by long-distance processes, which are difficult to calculate.

- Since $y_{\rm CP}$ is positive, the CP-even state is shorter-lived, as in the K^0-$\overline{K}^0$ system. However, since x is also positive, the CP-even state is heavier, unlike in the K^0-$\overline{K}^0$ system.

- The strong phase difference δ is probably not small: the fitted value is $(26.4\,^{+9.6}_{-9.9})^\circ$.

- There is no evidence yet for CPV in the D^0-$\overline{D}^0$ system. Observing CPV at the current level of sensitivity would indicate new physics.

Acknowledgments

We thank the organizers of the CHARM 2009 workshop for excellent hospitality and for a stimulating scientific program.

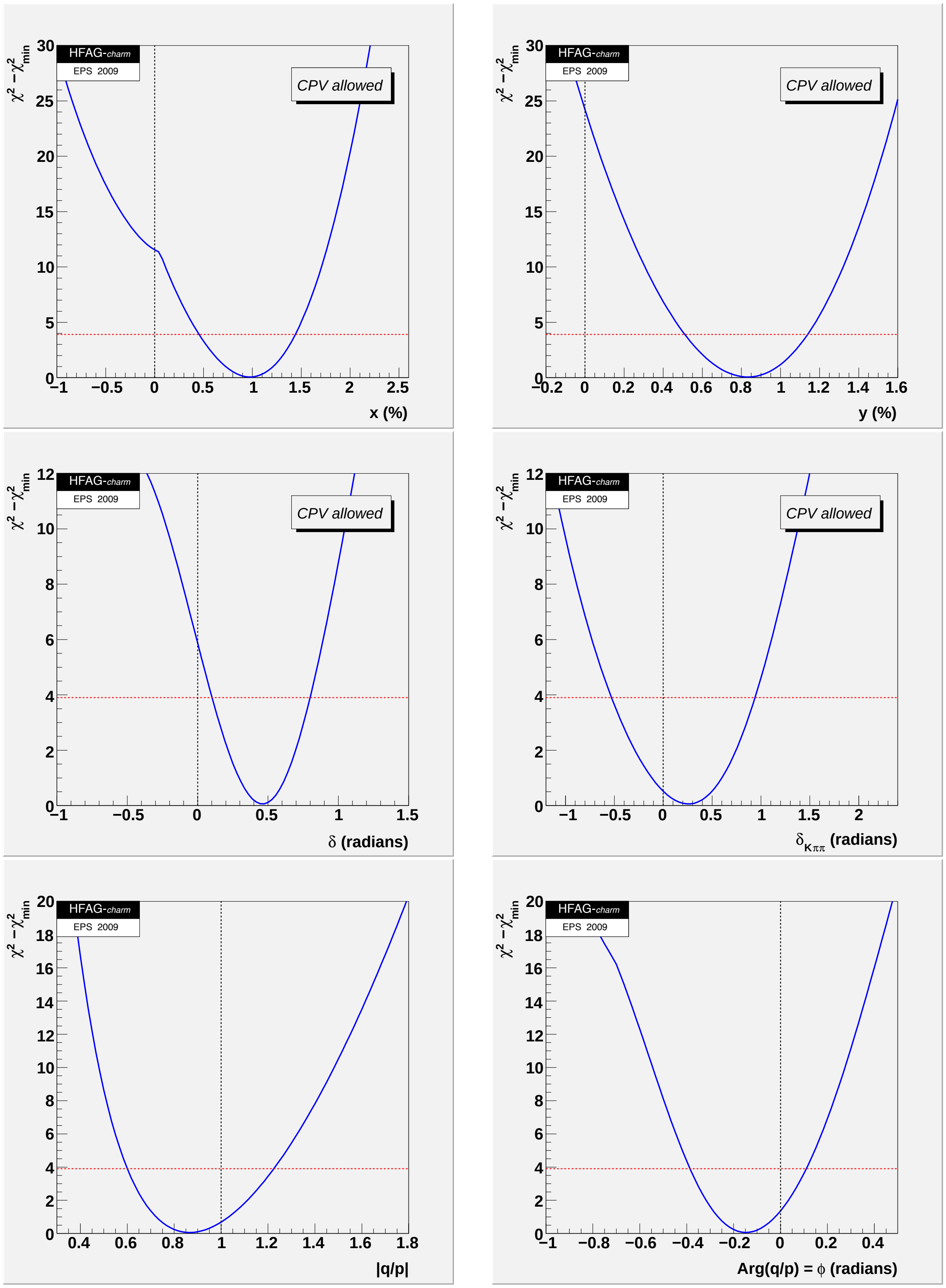

Figure 4: The function $\Delta\chi^2 = \chi^2 - \chi^2_{\rm min}$ for parameters x, y, δ, $\delta_{K\pi\pi}$, $|q/p|$, and ϕ. The points where $\Delta\chi^2 = 3.84$ (denoted by the dashed horizontal line) determine a 95% C.L. interval.

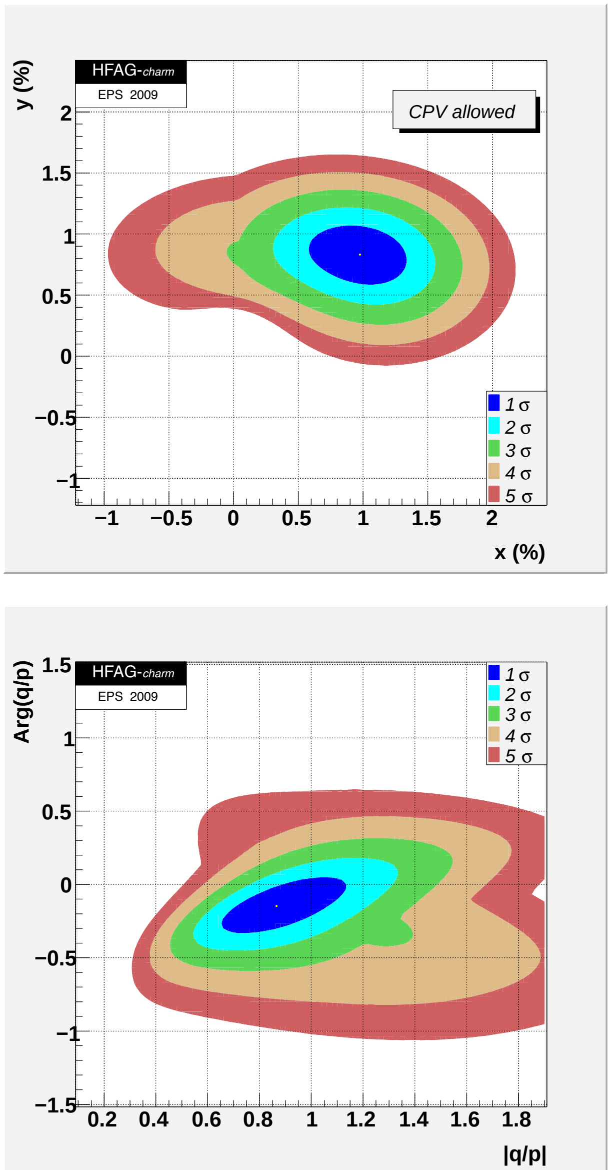

Figure 3: Two-dimensional contours for parameters (x, y) (top) and $(|q/p|, \phi)$ (bottom), allowing for CPV.

References

[1] See http://www.slac.stanford.edu/xorg/hfag.

[2] The group has since expanded its activities to include calculating WA values of hadronic branching fractions, semileptonic form factors, properties of excited D_s^{**} states, and the D_s^+ decay constant f_{D_s}.

[3] I. Bigi and N. Uraltsev, Nucl. Phys. B592 (2001) 92.

[4] A. A. Petrov, "Charm Physics: Theoretical Review", eConf C030603 (2003), arXiv:hep-ph/0311371; Nucl. Phys. Proc. Suppl. 142 (2005) 333.

[5] Charge-conjugate modes are implicitly included unless noted otherwise.

[6] E. M. Aitala *et al.* (E791 Collab.), Phys. Rev. Lett. 77 (1996) 2384; C. Cawlfield *et al.* (CLEO Collab.), Phys. Rev. D 71 (2005) 071101; B. Aubert *et al.* (Babar Collab.), Phys. Rev. D 76 (2007) 014018; U. Bitenc *et al.* (Belle Collab.), Phys. Rev. D 77 (2008) 112003.

[7] E. M. Aitala *et al.* (E791 Collab.), Phys. Rev. Lett. 83 (1999) 32; J. M. Link *et al.* (FOCUS Collab.), Phys. Lett. B485 (2000) 62; S. E. Csorna *et al.* (CLEO Collab.), Phys. Rev. D 65 (2002) 092001; K. Abe *et al.* (Belle Collab.), Phys. Rev. Lett. 88 (2002) 162001; M. Staric *et al.* (Belle Collab.), Phys. Rev. Lett. 98 (2007) 211803; B. Aubert *et al.* (Babar Collab.), Phys. Rev. D 78 (2008) 011105; A. Zupanc *et al.* (Belle Collab.), Phys. Rev. D 80 (2009) 052006; B. Aubert *et al.* (Babar Collab.), arXiv:0908.0761.

[8] http://www.slac.stanford.edu/xorg/hfag/charm/EPS09/results_mixing.html.

[9] D. M. Asner *et al.* (CLEO Collab.), Phys. Rev. D 72 (2005) 012001; arXiv:hep-ex/0503045 (revised April 2007); L. M. Zhang *et al.* (Belle Collab.), Phys. Rev. Lett. 99 (2007) 131803.

[10] L. M. Zhang *et al.* (Belle Collab.), Phys. Rev. Lett. 96 (2006) 151801; B. Aubert *et al.* (Babar Collab.), Phys. Rev. Lett. 98 (2007) 211802; T. Aaltonen *et al.* (CDF Collab.), Phys. Rev. Lett. 100 (2008) 121802.

[11] B. Aubert *et al.* (Babar Collab.), arXiv:0807.4544. See also: X. C. Tian *et al.* (Belle Collab.), Phys. Rev. Lett. 95 (2005) 231801.

[12] D. M. Asner *et al.* (CLEOc Collab.), Phys. Rev. D 78 (2008) 012001.