Next-to-leading-order EFT($\not\pi$)

for three *single* nucleons.

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in collaboration with

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The Hebrew University of Jerusalem

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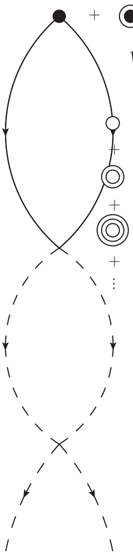
NLO EFT($\not\pi$) WITH SINGLE NUCLEONS.



$V = C_0 + C_{0,0} + C_2 q^2 + \ldots$

$\leftrightarrow \frac{\vec{\nabla}^2}{2m} + \frac{\vec{\nabla}^4}{8m^3} + \ldots$

- Leading order:

$$V = \mathring{\mathring{\mathring{C}}}_{0,s}\,\hat{P}^{(^1S_0)} + \mathring{\mathring{\mathring{C}}}_{0,t}\,\hat{P}^{(^3S_1)} + \mathring{\mathring{\mathring{D}}}_{(*)}\,\hat{P}^{(S)}$$

- Next-to-leading order:

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$$+\mathring{C}_{pp}\hat{P}^{(^1S_0)}_{pp} + \frac{e^2}{4|r|}$$

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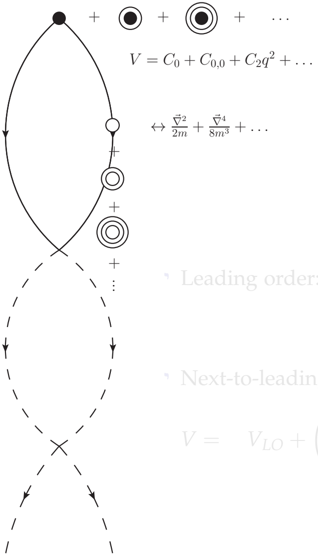

"Natural", renormalized LECs:

$$C_{2n} = \frac{4\pi\mathcal{O}(1)}{m\aleph(M\aleph)^n} \qquad C'_{2n} = \frac{4\pi\mathcal{O}(1)}{mM^{2n+1}}$$

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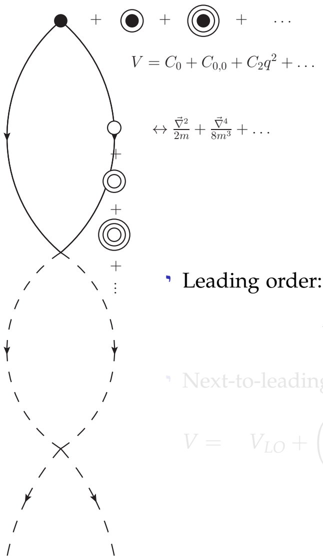

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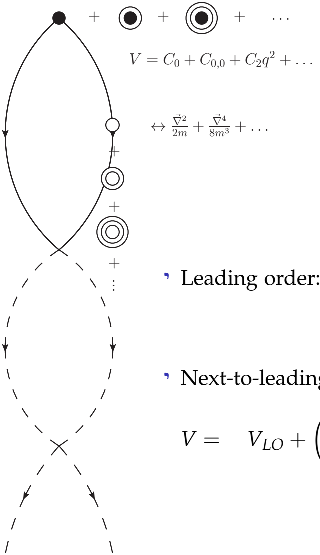

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for $T(E_p \pm i\epsilon)|\vec{p}\,\rangle_0 = V|\vec{p}\pm\rangle_V$:

$$\begin{aligned}
{}_0\langle\vec{p}\,'|T|\vec{p}\,\rangle_0 &= {}_0\langle\vec{p}\,'|V|\vec{p}\,\rangle_0 + {}_0\langle\vec{p}\,'|VG_0T|\vec{p}\,\rangle_0 \\
&= {}_0\langle\vec{p}\,'|V|\vec{p}\,\rangle_0 + {}_0\langle\vec{p}\,'|VG_0V + VG_0VG_0V|\vec{p}\,\rangle_0 \\
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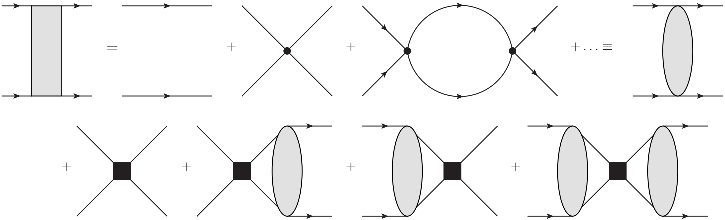
$\langle\vec{p}'|\vec{p}\rangle \;+\; \langle\vec{p}'|V_{LO}|\vec{p}\rangle \;+\; \langle\vec{p}'|V_{LO}G_0V_{LO}|\vec{p}\rangle \;+\; \ldots \equiv \langle\vec{p}'|V_{LO}G_0V_{LO}|\vec{p}\pm\rangle$



$\langle\vec{p}'|V_{NLO}|\vec{p}\rangle + \langle\vec{p}'|V_{NLO}G_0T_{LO}|\vec{p}\rangle + \langle\vec{p}'|T_{LO}G_0V_{NLO}|\vec{p}\rangle + \langle\vec{p}'|T_{LO}G_0V_{NLO}G_0T_{LO}|\vec{p}\rangle$

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and the full solution reads:

$$_0\langle\vec{p}\,'|T|\vec{p}\,\rangle_0 = {}_{LO}\langle\vec{p}\,'\text{-}|V_{LO}|\vec{p}\,\rangle_0 + {}_{LO}\langle\vec{p}\,'\text{-}|V_{NLO}|\vec{p}+\rangle_{NLO} \quad .$$

then approximate

$$|\vec{p}+\rangle_{NLO} \approx |\vec{p}+\rangle_{LO} = |\vec{p}\,\rangle_0 + G_0T_{LO}|\vec{p}\,\rangle_0$$

to get

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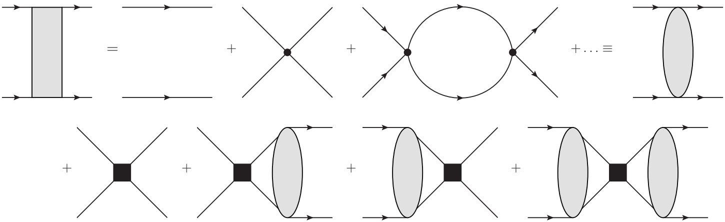
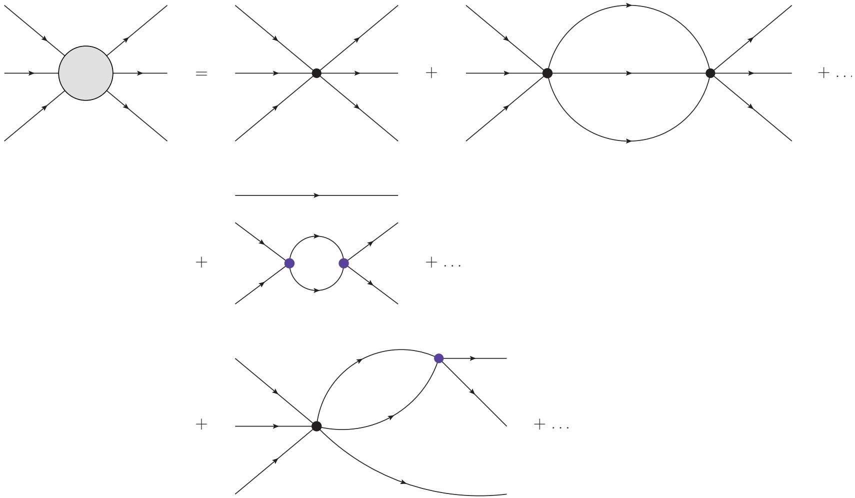
$$\begin{aligned} {}_{LO}\langle\vec{p}\,'\text{-}|V_{NLO}|\vec{p}+\rangle_{NLO} &\approx {}_0\langle\vec{p}\,'|V_{NLO}|\vec{p}\,\rangle_0 + {}_0\langle\vec{p}\,'|T_{LO}G_0V_{NLO}|\vec{p}\,\rangle_0 \\ &\quad + {}_0\langle\vec{p}\,'|V_{NLO}G_0T_{LO}|\vec{p}\,\rangle_0 + {}_0\langle\vec{p}\,'|T_{LO}G_0V_{NLO}G_0T_{LO}|\vec{p}\,\rangle_0 \\ &= {}_{LO}\langle\vec{p}\,'\text{-}|V_{NLO}|\vec{p}+\rangle_{LO} \end{aligned}$$

NLO EFT($\not{\pi}$) $\hat{=}$ DWBA.



$$
\begin{aligned}
{}_{LO}\langle \vec{p}\,'\text{-} | V_{NLO} | \vec{p}+ \rangle_{NLO} &\approx {}_0\langle \vec{p}\,' | V_{NLO} | \vec{p}\,\rangle_0 + {}_0\langle \vec{p}\,' | T_{LO} G_0 V_{NLO} | \vec{p}\,\rangle_0 \\
&\quad + {}_0\langle \vec{p}\,' | V_{NLO} G_0 T_{LO} | \vec{p}\,\rangle_0 + {}_0\langle \vec{p}\,' | T_{LO} G_0 V_{NLO} G_0 T_{LO} | \vec{p}\,\rangle_0 \\
&= {}_{LO}\langle \vec{p}\,'\text{-} | V_{NLO} | \vec{p}+ \rangle_{LO}
\end{aligned}
$$

Regularization & Projection.



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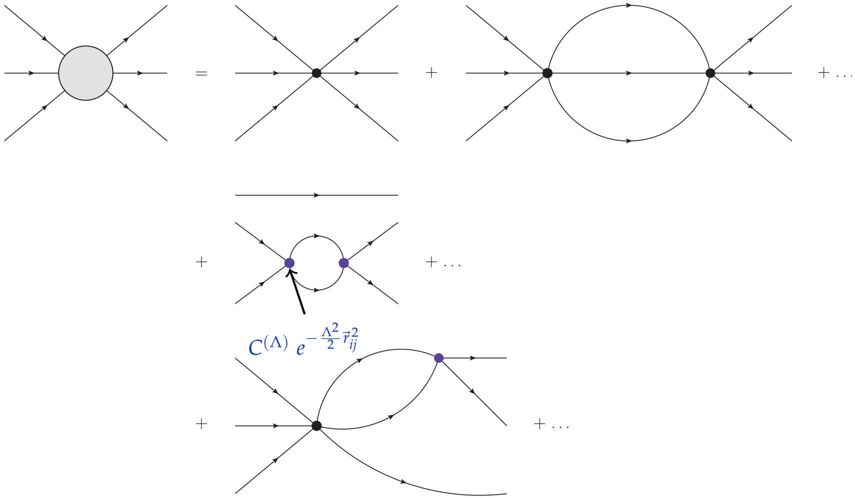

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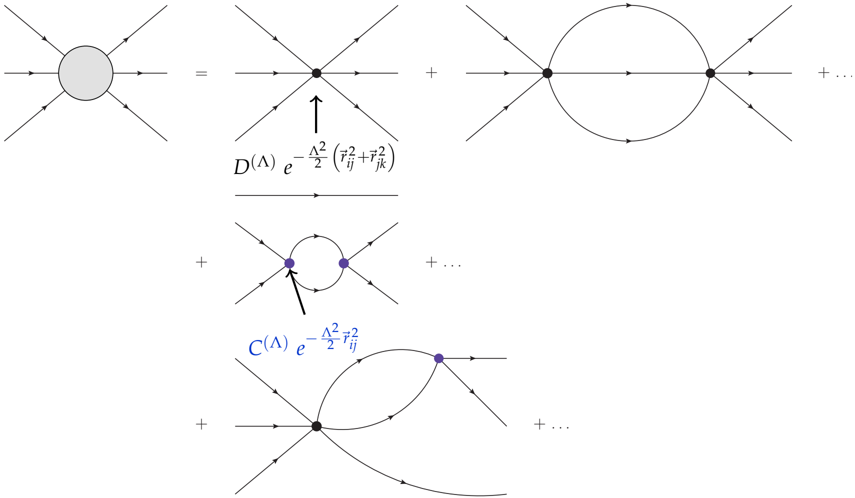

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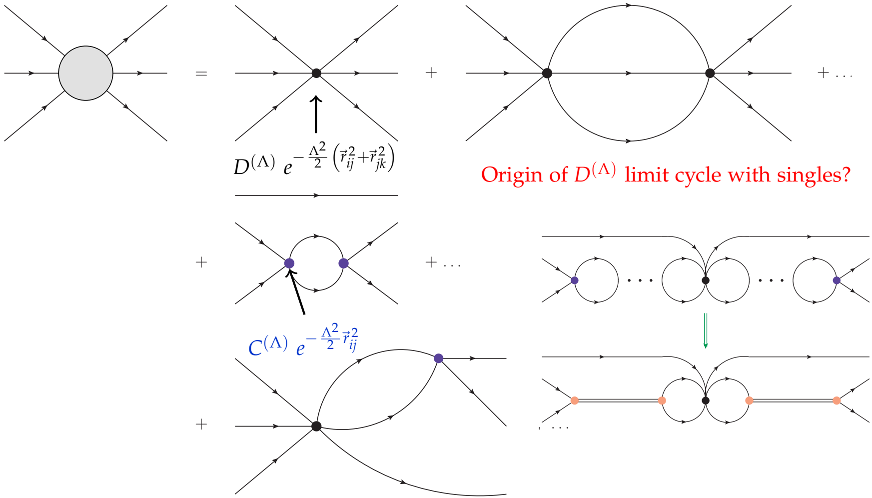

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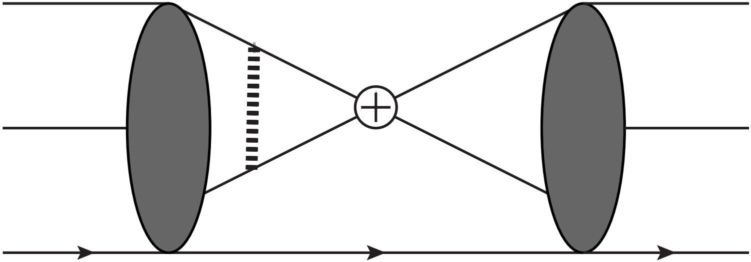

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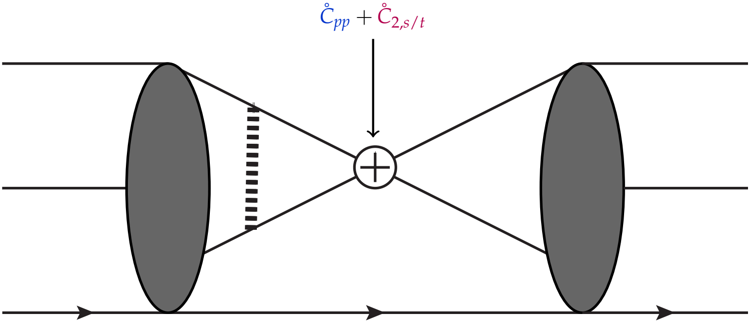

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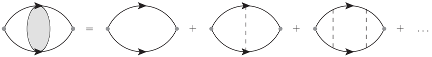
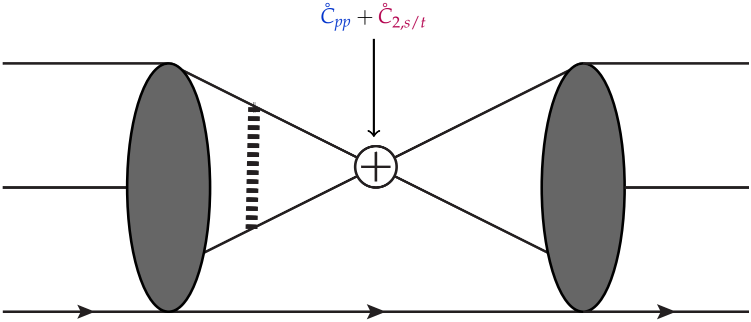


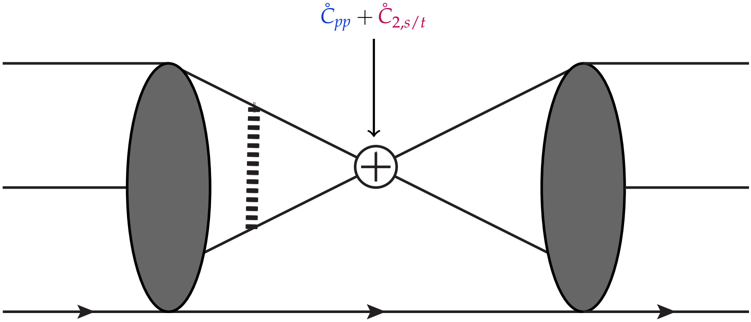
$$\frac{1}{C_1 + C_S^{pp}} = \frac{m_N}{4\pi a_{pp}} + \frac{\alpha m_N^2}{4\pi}\left[\ln\left(\frac{\mu\sqrt{\pi}}{\alpha m_N}\right) + 1 - \frac{3}{2}C_E\right] - \frac{\mu m_N}{4\pi}$$



RENORMALIZATION.

$$\text{pole momentum } \gamma = \pm i\, r^{-1}\left(1 - \sqrt{1 - 2ra^{-1}}\right) = \mp i^{7.88\ (S=0)}_{45.7\ (S=1)}\ \text{MeV} \quad .$$

$$f(p) = \left(-|\gamma| + \tfrac{r_0}{2}(\gamma^2 + p^2) - ip\right)^{-1} = A_{-1}\, p^0 + A_0\, p^2 + \mathcal{O}(p^4)$$



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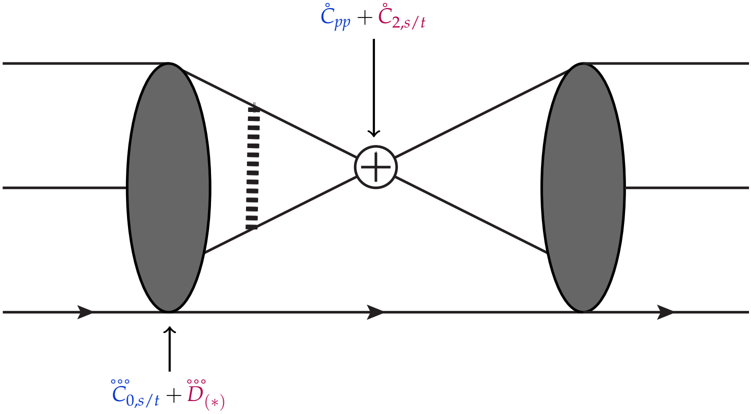

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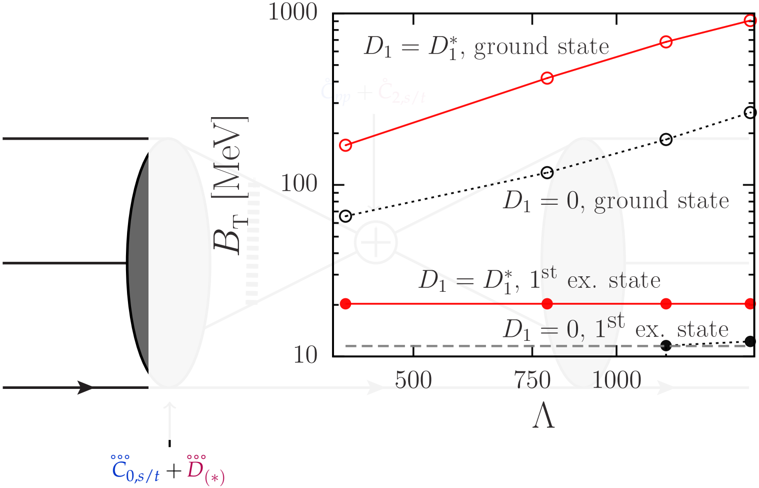
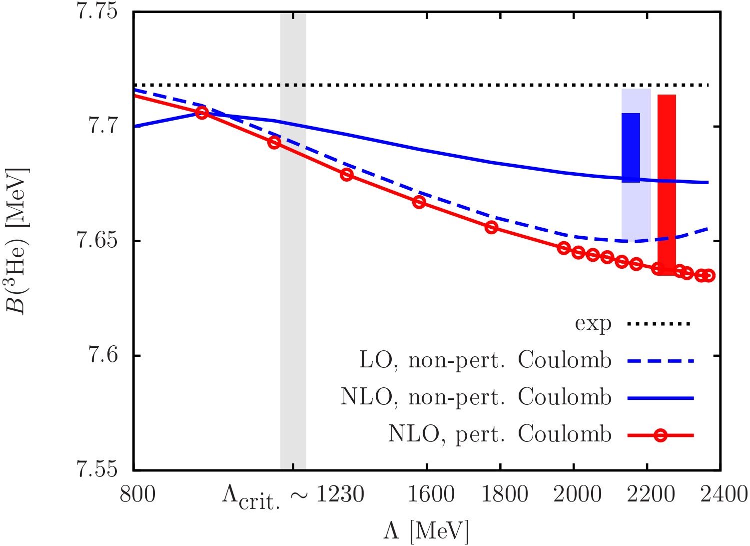
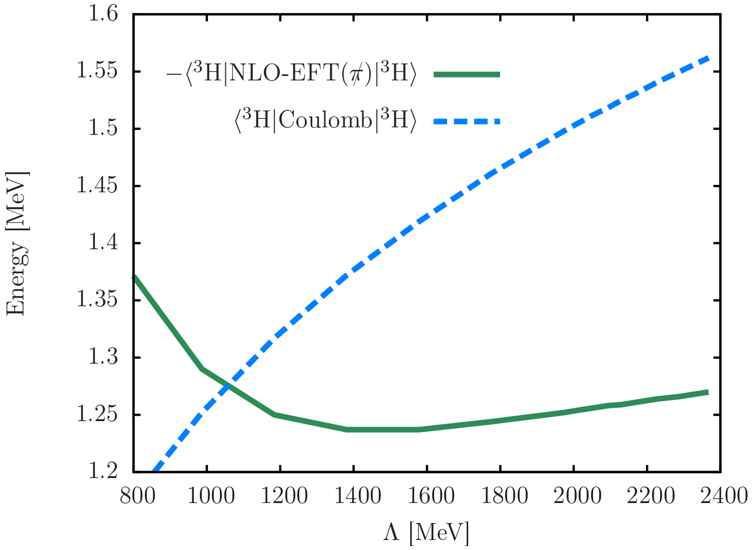


Helion results.



Two perturbations.



What physics do we learn from a seemingly technical detail?

- Which (unknown) part of the physical parameter space do we want to explore with “Coulombic” EFT($\not\pi$) ?
- How sensitive is the power counting with respect to the regulator type (sharp Wilson, vertex, mixed)?
- Which approach (single or married) is appropriate to analyze the RG flow of the underlying theory?
- How do we define EFT($\not\pi$) ?
- Promising ansatz for $A > 3$?
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