Regularization and Renormalization in Halo Effective Field Theory

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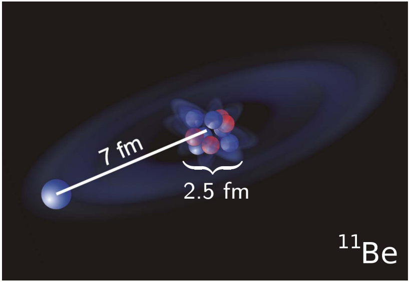
Effective Theories for Halo Nuclei

- **halo nuclei** (core + valence N)
- **separation in length scales**
  $R_{\text{core}} \ll R_{\text{halo}}$



ab initio methods

- capture dynamics inside and outside the core
- numerically expensive for loosely bound systems

halo effective field theory

- valence nucleon + core d.o.f.
- systematic expansion in $R_{\text{core}}/R_{\text{halo}}$
- capture only clustering mechanism
- numerically simpler
- complimentary to *ab initio* methods
- explain universal correlations in clustering physics

Halo Effective Field Theory

- We adopt EFT with contact interactions to describe clustering in halo nuclei

$$
\begin{aligned}
\mathscr{L} &= \mathscr{L}_1 + \mathscr{L}_2 + \mathscr{L}_3 \\
\mathscr{L}_1 &= n^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_n} \right) n + c^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_c} \right) c \\
\mathscr{L}_2 &= \eta_0 \, s^\dagger \left(i\partial_0 + \frac{\nabla^2}{4m_n} - \Delta_0 \right) s + \eta_1 \, \pi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2(m_n + m_c)} - \Delta_1 \right) \pi \\
&\quad + g_0 \left[s^\dagger (nn) + \text{h.c.} \right] + g_1 \left[\pi^{\dagger a} (nc) + \text{h.c.} \right], \\
\mathscr{L}_3 &= h \, (\pi n)^\dagger \, (\pi n)
\end{aligned}
$$

- 2-body contact (LO)

$= -i\sqrt{2}g$

$g \leftarrow$ 2-body observable

- 3-body contact (LO)

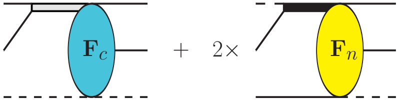
$= ih$

$h \leftarrow$ 3-body observable

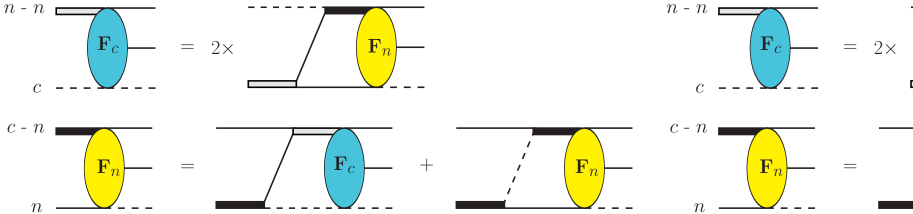
Two-Neutron Halo Nuclei

- **2n-halo wave functions**

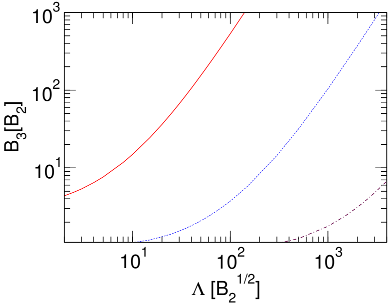
$\Psi_x(p, q) =$



- **Three-body Faddeev equation**

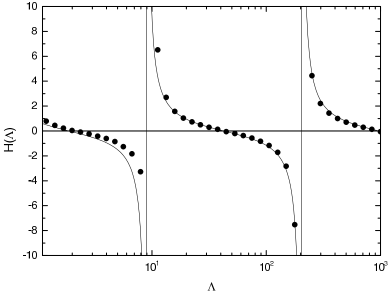


LO renormalization



- **Without 3BF:**
  - 3-body spectrum:
    cutoff dependent ($\Lambda \sim 1/\ell$)
    Platter '09

- **LO 3BF h:**
  - tune $H(\Lambda) = \Lambda^2 h / 2mg^2$:
    fix one 3-body observable
  - limit cycle:
    $H(\Lambda)$ periodic for $\Lambda \to \Lambda(\mathrm{const})^n$
    Bedaque *et al.* '00
  - $\rightarrow$ Efimov physics



EFT For $2n$ Halos

- n-core in s-wave virtual/real bound state:

  ^{11}Li, ^{12}Be, ^{20}C [Canham, Hammer, EPJA '08, NPA '10]

  ^{22}C [Yamashita, Carvalho, Frederico, Tomio, PLB '11]

  ^{22}C Acharya, C.J., Phillips, PLB 723 (2013)

- charge radius of 2n s-wave halos [Hagen, Hammer, Platter, EPJA '13]

- heaviest 2n s-wave halo:

  ^{62}Ca [Hagen, Hagen, Hammer, Platter, PRL '13]

  fit n-^{60}Ca scattering length from coupled-cluster calculations

- ^{6}**He:** n-α in p-wave resonance
  - EFT + Gamow shell model [Rotureau, van Kolck, FBS '13]
  - EFT + Faddeev Equations C.J., Elster, Phillips, PRC **90**, 044004 (2014)

^{6}He: $2n$ Halo

- **experiment in ^{6}He**
  - matter radius Tanihata *et al.* '92, Alkhazov *et al.* '97, Kislev *et al.* '05
  - charge radius Wang *et al.* '04, Mueller *et al.* '07
  - ^{6}He mass Brodeur *et al.* '12

- **cluster model**
  - separable potential Ghovanlou, Lehman '74
  - variational method Funada *et al.* '94
  - density-dependent nn contact interaction Esbensen *et al.* '97
  - Wood Saxon $n\alpha$ + GPT nn Danilin, Thompson, Vaagen, Zhukov '98

- *ab intio* **calculation**
  - no-core shell model Navrátil *et al.* '01; Sääf, Forssén '14
  - NCSM-RGM Romero-Redondo *et al.* '14
  - Green's function Monte Carlo Pieper *et al.* '01; '08
  - hyperspherical harmonics (EIHH) Bacca *et al.* '12

^{6}He: $2n$ Halo

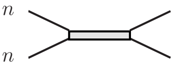
- **halo EFT**
  - explore **universal correlations** in ^{6}He
  - compare **predictions** with experiments and *ab initio* calculations

Rotureau, van Kolck Few Body Syst. **54** 725 2013

C.J., Elster, Phillips, PRC **90**, 044004 (2014)

$n - n$ interaction

- nn interaction is dominated by the $^1S_0$ state



$$= \frac{1}{4\pi^2\mu_{nn}} \frac{1}{-1/a_0 + r_0k^2/2 - ik}$$

$a_0 = -18.7$ fm, $r_0 = 2.75$ fm González Trotter *et al.* '99

- LO EFT: $r_0 \to 0$

$n - \alpha$ interaction

- $n\alpha$ interaction is dominated by the ${}^2P_{\frac{3}{2}}$ state

$$= \frac{1}{4\pi^2\mu_{n\alpha}} \frac{\vec{p}\cdot\vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3}$$

$a_1 = -62.95$ fm^3, $r_1 = -0.8819$ fm^{-1} Ardnt et al. '73

- $r_1 \not\to 0$ Nishida '12
- keep both a_1 and r_1 in LO EFT

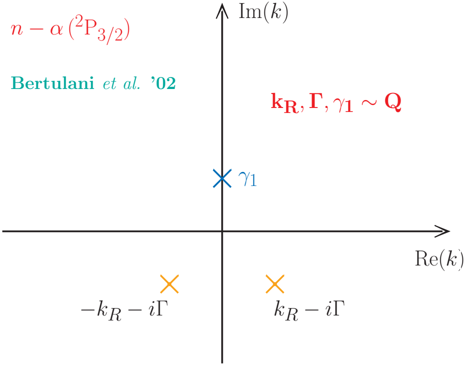
$n-\alpha$ p-wave power counting (Bertulani)

- $n\alpha$ EFT power counting: Bertulani, Hammer, van Kolck NPA '02
  - $a_1 \sim 1/(Q^3)$ $\quad r_1 \sim Q$
  - two fine tunings at LO

- ${}^2P_{\frac{3}{2}}$:

  shallow resonance: $k_R, \Gamma \sim Q$

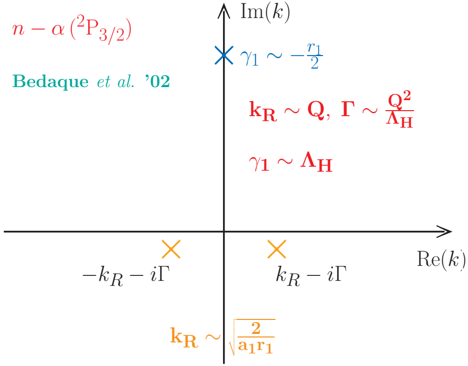
  shallow bound state: $\gamma_1 \sim Q$



$n - \alpha$ p-wave power counting (Bedaque)

- $n\alpha$ EFT power counting: Bedaque, Hammer, van Kolck PLB '02
  - $a_1 \sim 1/(Q^2\Lambda_H)$ $r_1 \sim \Lambda_H$
  - $Q/\Lambda_H \sim 0.15$
  - one fine tuning at LO

- ${}^2P_{\frac{3}{2}}$:
  shallow resonance:
  $$k_R \sim Q, \quad \Gamma \sim Q^2/\Lambda_H$$
  deep bound state: $\gamma_1 \sim \Lambda_H$

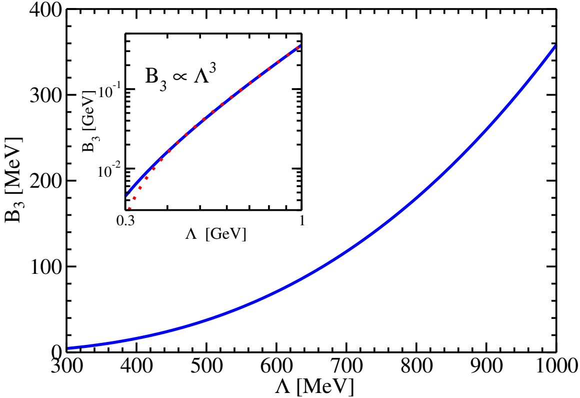


$n - \alpha\ {}^1S_{\frac{1}{2}}$ and ${}^2P_{\frac{1}{2}} \rightarrow$ beyond LO

adopted by C.J., Elster, Phillips, PRC **90**, 044004 (2014) for ^{6}He

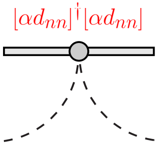
Cutoff Dependence

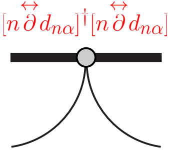
- **without $nn\alpha$ 3-body force:**
  - S_{2n} is strongly cutoff dependent: $S_{2n} \sim \Lambda^3$ $\leftarrow$ need 3body force!



$nn\alpha$ Three-Body Force (3BF)

- **candiates for $nn\alpha$ counterterms**

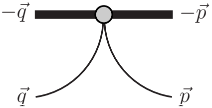




- **only needs $[n\overleftrightarrow{\partial}d_{n\alpha}]^\dagger[n\overleftrightarrow{\partial}d_{n\alpha}]$ counterterm**
  - Pauli principle
  - A similar p-wave three-body counterterm is discovered by Rotureau, van Kolck Few Body Syst. **54** 725 2013

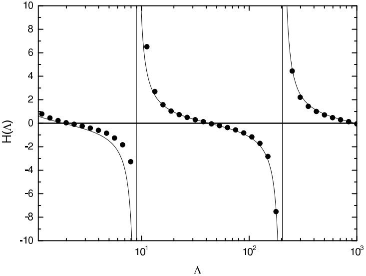
Running of 3BF Coupling

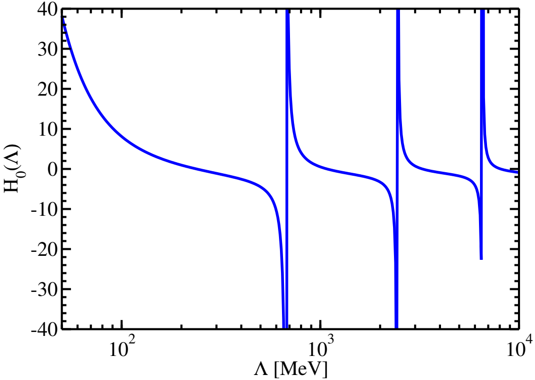
- p-wave 3BF:
  reproduce $S_{2n} = 0.973$ MeV



$$= M_n \, qp \, \frac{H(\Lambda)}{\Lambda^2}$$

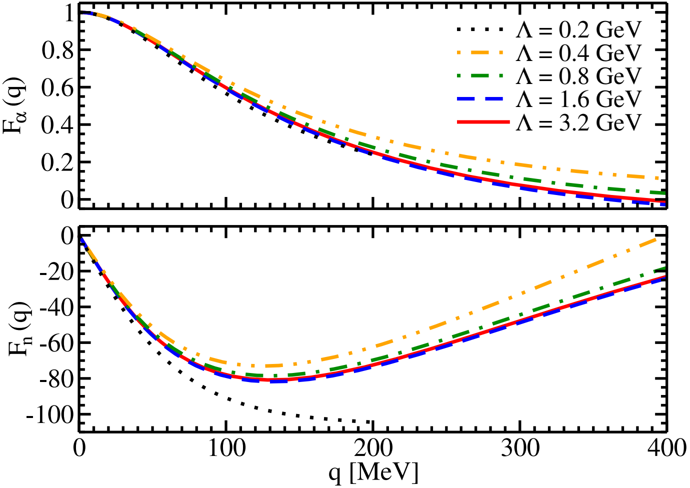
- log oscillation
- No limit cycle
  (c.f. 3-body in S-wave)





Renormalized Faddeev Components

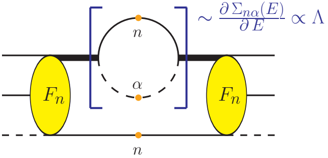
$F_\alpha(\alpha, nn)$ and $F_n(n, \alpha n)$ are cutoff independent



C.J., Elster, Phillips, PRC **90**, 044004 (2014)

Normalization

- The norm is defined by $\langle\Psi|\Psi\rangle = \sum_{ij}\langle\psi_i|\psi_j\rangle$

$\langle\psi_n|\psi_n\rangle =$ 

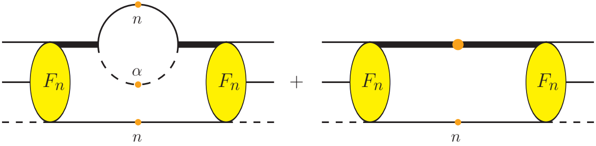
- Non-hermitian Hamiltonian
  - $\langle\vec{p}|V_{n\alpha}|\vec{p}'\rangle = (\lambda_0 + \lambda_1 mE)\,\vec{p}\cdot\vec{p}'$
  - $1/\lambda_0 \propto \Lambda^3_{n\alpha} + \frac{3\pi}{2a_1}$
  - $\lambda_1/\lambda_0^2 \propto \frac{4}{\pi}\Lambda + r_1$

c.f. Rotureau, van Kolck, FBS '13: $nn\alpha$ in Gamow shell basis

Normalization

- 3-body normalization (with p-wave): $\langle\Psi|\Psi\rangle - \langle\Psi|\frac{dV}{dE}|\Psi\rangle = 1$

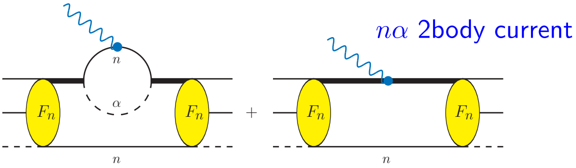
  c.f. ^{11}Be p-wave form factor (Hammer, Phillips NPA 2011)



- $\frac{dV}{dE} \sim \frac{4\Lambda}{\pi} - r_1$
- $\langle\Psi|\frac{dV}{dE}|\Psi\rangle$ does not require additional 3-body parameters

^{6}He Form Factors

- 3-body form factor (with p-wave n-core interactions)

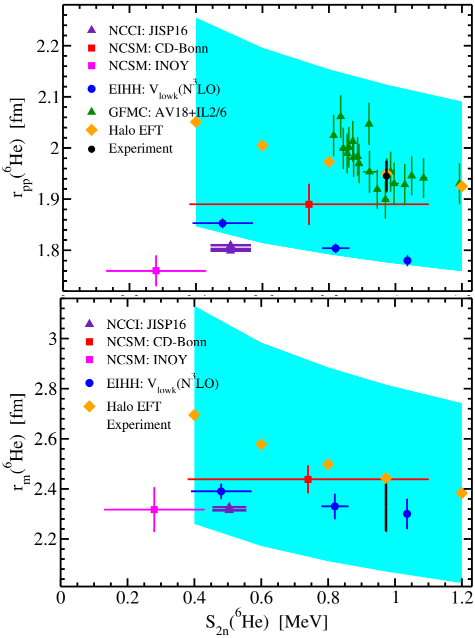


- The $n\alpha$ two-body current counterterm is fixed by r_1
- It does not require an additional 3-body input
- matter/charge radii

$$F_{c(m)}(q^2) = 1 - \frac{1}{6}\langle r^2_{c(m)}\rangle q^2 + \cdots$$

- current stage: $\Lambda_{ff} = -\frac{\pi}{4}r_1 \approx 140$ MeV ($\lambda_1 \to 0$)

^{6}He Radii



[Preliminary]

- He-6 point-proton radius
- He-6 matter radius

compare with

NCCI: Caprio, Maris, Vary, PRC '14

NSCM: Caurier, Navratil, PRC '06

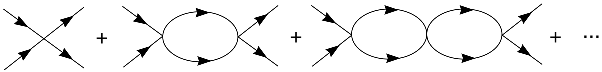
GFMC: Pieper, RNC '08

EIHH: Bacca, Barnea, Schwenk, PRC '12

Halo EFT: preliminary (uncertainty)

Clustering EFT in potential forms

- energy-dependent coupling



  - s-wave:

$$\langle p|V_0|p'\rangle = -\frac{\pi^2}{\mu}\, g\left(\frac{p}{\Lambda}\right)(\lambda_0 + \lambda_1 E)\, g\left(\frac{p'}{\Lambda}\right)$$

  - p-wave:

$$\langle p|V_1|p'\rangle = -\frac{\pi^2}{\mu} g\left(\frac{p}{\Lambda}\right)(\lambda_0 + \lambda_1 E)\, g\left(\frac{p'}{\Lambda}\right) \vec{p}\cdot\vec{p}\,'$$

  - regulator

$$g\left(\frac{p}{\Lambda}\right) = \exp\left[-\left(\frac{p}{\Lambda}\right)^{2m}\right] \quad m = 1, 2, 3, \ldots$$

  - reproduce a_0 & r_0 (a_1 & r_1); cutoff $\Lambda \to \infty$
  - non-hermitian Hamiltonian

Clustering EFT in potential forms

- momentum-dependent coupling
  - s-wave:

$$\langle p|V_0|p'\rangle = -\frac{\pi^2}{\mu}\, g\left(\frac{p}{\Lambda}\right)\left[\lambda_0 + \lambda_1(p^2 + p'^2)\right] g\left(\frac{p}{\Lambda}\right)$$

  - p-wave:

$$\langle p|V_1|p'\rangle = -\frac{\pi^2}{\mu} g\left(\frac{p}{\Lambda}\right)\left[\lambda_0 + \lambda_1(p^2 + p'^2)\right] g\left(\frac{p}{\Lambda}\right) \vec{p}\cdot\vec{p}\,'$$

- cutoff limited by the Wigner bound if λ_1 is not treated perturbatively
  - NN: $a_0 \to$ LO $\quad r_0 \to$ NLO
  - $n-\alpha$ p-wave resonance: $a_1, r_1 \to$ LO
  - $\alpha-\alpha$ resonance: $a_0, r_0 \to$ LO

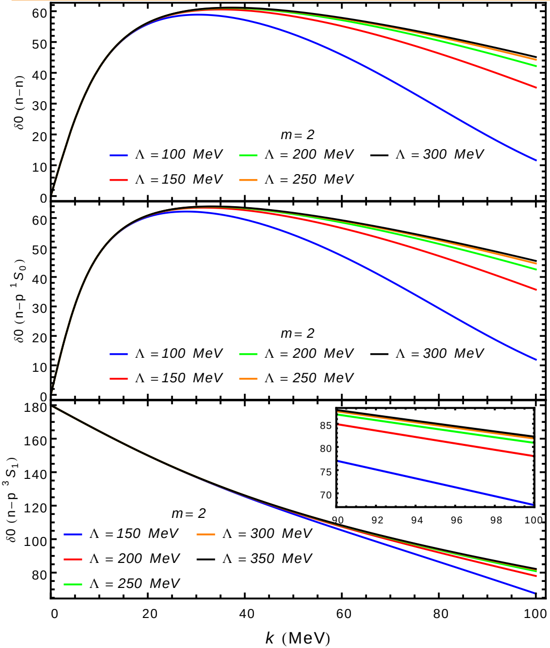
Wigner bound

- in the limit $\Lambda \to +\infty$
  - $r_0 < 0$
  - $r_1 \to -\infty$

- in physical systems:
  - in NN systems: $r_0 \sim 1/m_\pi$
  - in $n\alpha$ system: $r_1 \sim -m_\pi$
  - requires a finite regulation cutoff

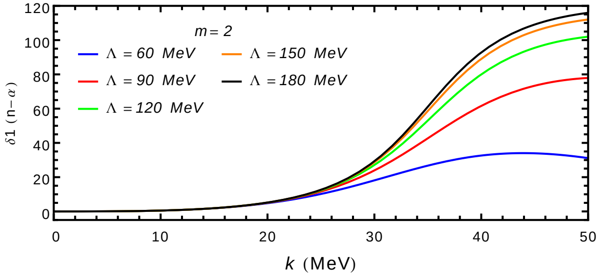
- naive dimensional analysis:
  - in s-wave: $\lambda_0 = c_0/\Lambda$; $\lambda_1 = c_1/\Lambda^3$
  - in p-wave: $\lambda_0 = c_0/\Lambda^3$; $\lambda_1 = c_1/\Lambda^5$
  - $c_0, c_1 \sim \mathcal{O}(1)$

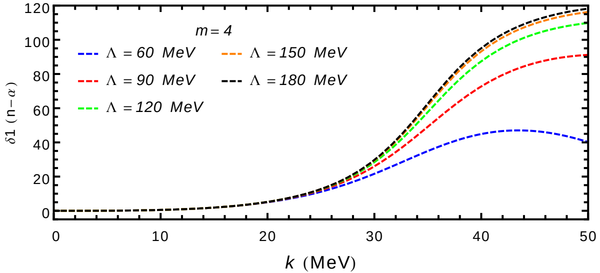
s-wave phase shifts



- nn system
- np singlet
- np triplet

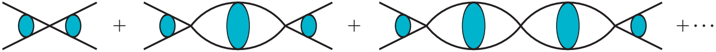
p-wave phase shift





$n\alpha$ system

$\alpha - \alpha$ resonance



- Three momentum scale:

  $k_R \approx 14$ MeV
  $\Lambda_s \approx 180$ MeV
  $k_c = 2\alpha_e m_\alpha = 54$ MeV

- non-perturbative Coulomb: $\Lambda \gg k_c \gg k_R$

- refit counter term $c_0 \to \tilde{c}_0 = c_0 - \frac{8\alpha\mu}{\pi\Lambda} c_1$

Power counting

- Coulomb corrected phase shift in ERE

$$T_{CS} = -\frac{2\pi}{\mu}\frac{e^{2i\sigma_0}}{k\cot\delta_0^c - ik} = -\frac{2\pi}{\mu}\frac{C_\eta^2 e^{2i\sigma_0}}{-\frac{1}{a_0} + \frac{r_0}{2}k^2 - \frac{\mathcal{P}_0}{4}k^4 + \cdots - 2k_c H(\eta)}$$

- for resonance physics ($\sim k_R$) one can expand in k/Λ_s and k/k_c

$$T_{CS} = -\frac{2\pi}{\mu}C_\eta^2 e^{2i\sigma_0}\left[\frac{1}{\tilde{r}_0(k^2-k_R^2)/2 - ikC_\eta^2} + \frac{\tilde{P}_0}{4}\frac{k^4-k_R^4}{(\tilde{r}_0(k^2-k_R^2)/2 - ikC_\eta^2)^2}\right]$$

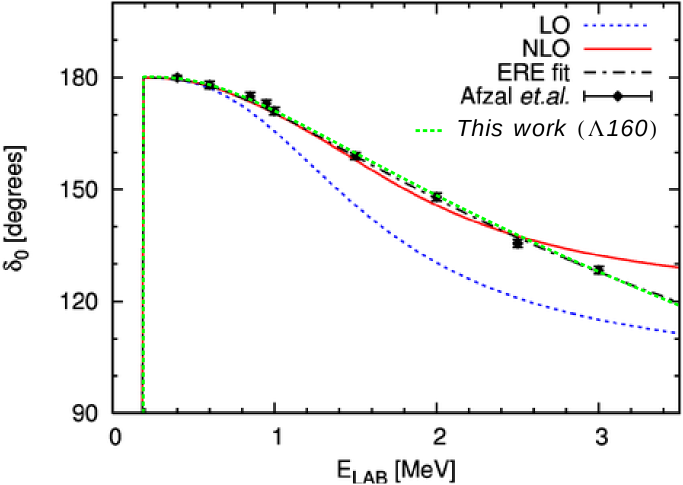
$$\tilde{r}_0 = r_0 - 1/3k_c; \quad \tilde{\mathcal{P}}_0 = \mathcal{P}_0 + 1/15k_c^3$$

$$k_R^2 = \frac{2}{a_0\tilde{r}_0}\left(1 - \tilde{\mathcal{P}}_0/a_0\tilde{r}_0^2\right)$$

LO+NLO: Higa, Hammer, van Kolck NPA '08

- to extend beyond k_R: expand only in k/Λ_s
  - reproduce $T_{CS} = -\frac{2\pi}{\mu}\frac{C_\eta^2 e^{2i\sigma_0}}{-\frac{1}{a_0}+\frac{r_0}{2}k^2-2k_c H(\eta)}$

$\alpha - \alpha$ phase shift



Summary

- Halo EFT describes structure/reaction in halo nuclei in a systematic expansion of R_{core}/R_{halo}
- Halo EFT rejuvenate cluster models with a systematic uncertainty estimates
- $2n$ halo with p-wave n-core interactions
  - renormalization: p-wave 3BF
  - form factors: minimal substitution
- cluster EFT in few- and many-body calculations
  - construct contact potentials
  - in $n-\alpha$ and $\alpha-\alpha$ both a_0 and r_0 enter at LO
  - potential interaction: Wigner bound $\rightarrow$ finite cutoff
  - peturbative / non-perturbative Coulomb regime