Electroweak reactions with different coulombic power counting

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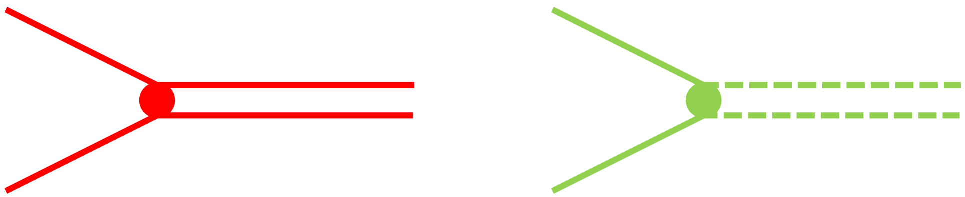
Outline

- Effective Field Theory
- ^3H and ^3He in πEFT up to NLO
- A≤3 electromagnetic interaction in πEFT.
- A≤3 electroweak Interaction in πEFT.
- Summary and outlook

Building $\not{\pi}$EFT Lagrangian

$$\mathcal{L} = N^T\left(iD_0 + \frac{\nabla^2}{2M_N}\right)N \quad - t^{i\dagger}\left(\sigma_t + iD_0 + \frac{D^2}{4M_N}\right)t^i - s^{A\dagger}\left(\sigma_s + iD_0 + \frac{D^2}{4M_N}\right)s$$

$$y_t[t^{i\dagger}(NP_t^iN) + h.c] + \; y_s[s^{A\dagger}(NP_s^AN) + h.c] + \mathcal{L}_3 + \mathcal{L}_{weak} + \mathcal{L}_{magnetic}$$



Where:

$$t^i(^3\mathrm{S}_1, \mathrm{I} = 0), \qquad \mathrm{s}^\mathrm{A}(^1\mathrm{S}_0, \mathrm{I} = 1)$$

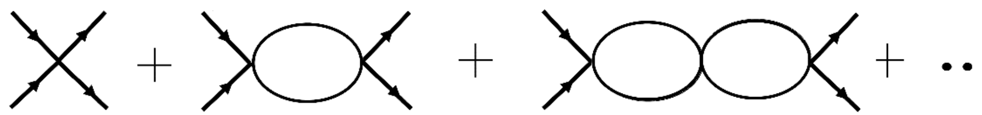
$$P_d^i = \frac{1}{\sqrt{8}}\sigma^2\sigma^i\tau^2, P_t^A = \frac{1}{\sqrt{8}}\sigma^2\tau^2\tau^A$$

$$D_\mu = \partial_\mu + iA_\mu\hat{Q}$$

S. R. Beane and M. J. Savage, 2001, P. F. Bedaque et al. 1999

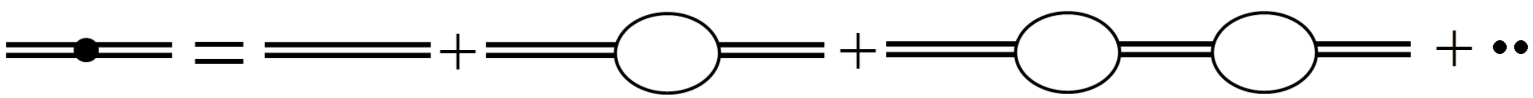
Dibaryon Propagator

- The dibaryon propagator is defined as the sum of a Dyson series:
- In LO:



$$i\mathcal{D}(q_0,q)_{t,s}^{bare} = -\frac{i4\pi}{M_N y_{t,s}^2}\frac{1}{\frac{1}{a_{t,s}}+\sqrt{\frac{q^2}{4}-M_N q_0}}$$

- Full:

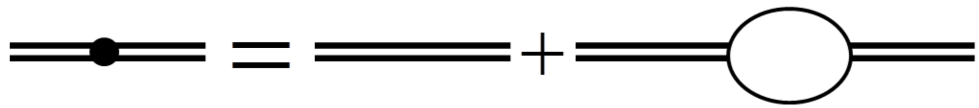


$$i\mathcal{D}(q_0,q)_{t,s}^{full} = i\mathcal{D}(q_0,q)_{t,s}^{bare}\sum_n\left[i\mathcal{D}(q_0,q)_{t,s}^{bare}\mathcal{J}_B-\left(2iy_{t,s}\right)^2\right]^n$$

$$= \frac{4\pi}{M_N}y_{t,s}^2\left[\frac{1}{a_{t,s}}-\sqrt{-M_N\,q_0+\frac{q^2}{4}}+\frac{\rho_{t,s}}{2}\left(-M_N\,q_0+\frac{q^2}{4}\right)\right]^{-1}$$

Dibaryon Propagator

- Up to NLO:



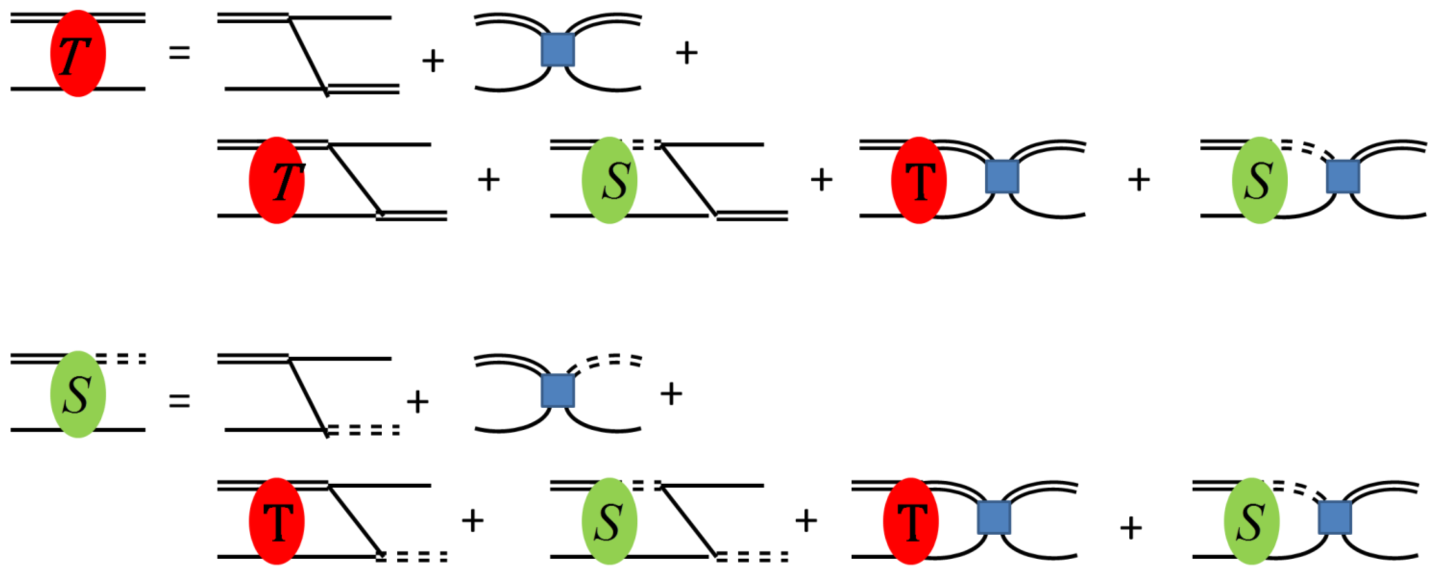
$$i\mathcal{D}(q_0,\boldsymbol{q})_t^{NLO} = i\mathcal{D}(q_0,\boldsymbol{q})_{t,s}^{bare} \times \left[1 - \frac{\rho_t}{2}\left(\sqrt{\frac{\boldsymbol{q}^2}{4} - M_N q_0} + \frac{1}{a_t}\right)\right]$$

$$i\mathcal{D}(q_0,\boldsymbol{q})_s^{NLO} = i\mathcal{D}(q_0,\boldsymbol{q})_{t,s}^{bare} \times \left[1 - \frac{\rho_s}{2}\left(\frac{\frac{\boldsymbol{q}^2}{4} - M_N q_0}{\sqrt{\frac{\boldsymbol{q}^2}{4} - M_N q_0} + \frac{1}{a_s}}\right)\right]$$

- The deuteron propagator is a result of the expansion of the full propagator near the deuteron pole,
- The singlet propagator is a result of the expansion of the full propagator near the $(\boldsymbol{q} = 0)$,

^{3}H in $\not{\pi}$EFT

- ^{3}H can be written as a result of *n-d* scattering:

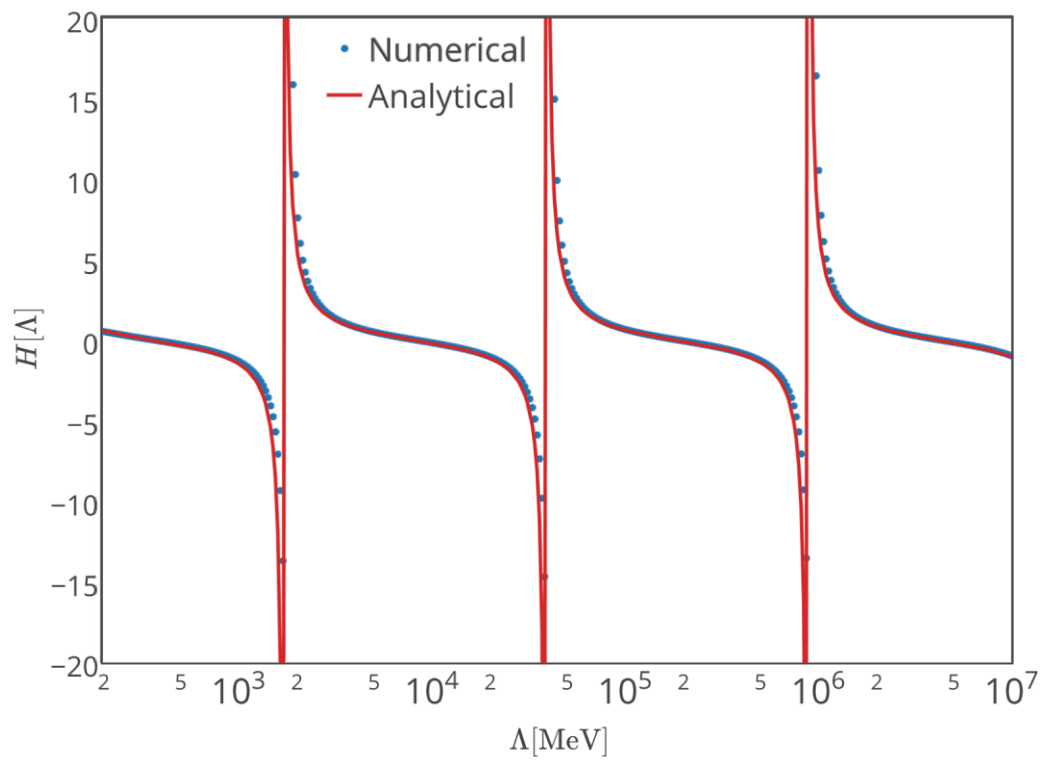


- The red bubble represents the deuteron channel, while the green bubble represents the triplet channel.
- These equations are coupled integral equations and can be solved for $E_B(^{3}\mathrm{H}) = 8.48$ MeV.

P.F. Bedaque , H.-W. Hammer and U. van Kolck 1999

Finding the 3-Body Force $H(\Lambda)$

- Solving the homogeneous parts of the Faddeev equation for ^{3}H without three body force ($H(\Lambda)$) enables to find $H(\Lambda)$ for $E_{3H} = 8.48$ MeV



P.F. Bedaque , H.-W. Hammer and U. van Kolck 1999

Power counting

- The power counting rules for two nucleons system are :

Fermion line	M_N/Q^2	D	Q
Loop	$\frac{Q^2}{4\pi M_N}$	C_N	$\frac{4\pi}{M_N \Lambda^n Q^{n+1}}$
D_0	$\frac{Q^2}{M_N}$	Photon line	$\frac{\alpha}{Q^2}$

- In the dibaryon form:

$$y^2 \to \frac{\Lambda}{M_N},\ \sigma \to \frac{Q\Lambda}{M_N}, a_{t,s,C} \sim \frac{1}{Q}, \text{LO},\ \rho_{t,s,C} \sim \frac{1}{\Lambda}, \text{NLO}$$

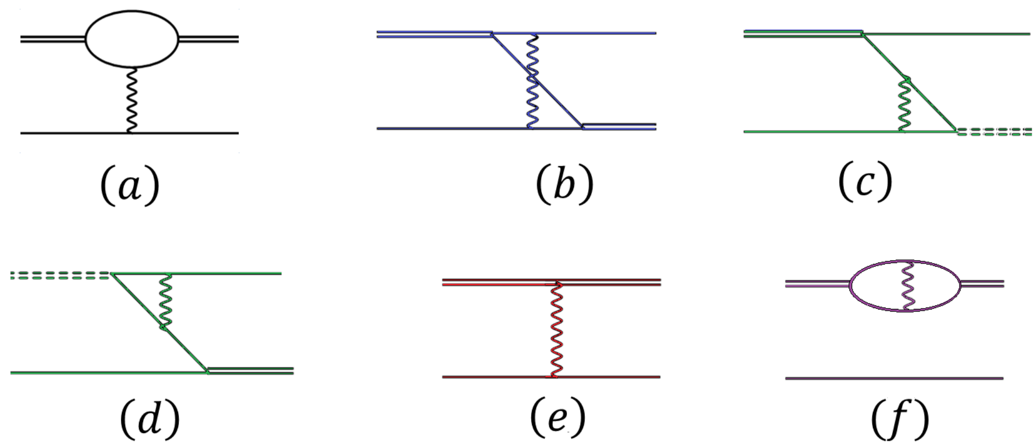
In LO ^{3}He has additional columbic diagrams and propagator.

D.B. Kaplan et al, 1996.

^{3}He in $\not{\pi}$EFT

- For the bound state the typical momentum $Q \geq \sqrt{M_N E_{3\mathrm{He}}}$ $\sim$ 85MeV], one photon exchange - $\frac{\alpha \mathrm{M_N}}{Q} \ll 1$.

The Columbic correction :



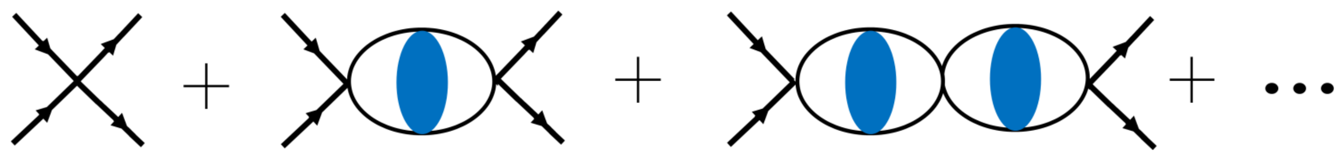
- diagrams a-d: $\sim \mathcal{O}(\mathcal{K})\frac{\alpha \mathrm{M_N}}{Q}$, diagram e$\sim \mathcal{O}(a)\frac{Q}{\Lambda}$ which is NNLO, diagram f: $\mathcal{O}(\mathcal{K})\frac{\alpha \mathrm{M_N}}{Q} Q^3 \sim \frac{M_N}{a_C}$

In the non-bound states (pp fusion etc.) $Q \geq 0, \frac{\alpha \mathrm{M_N}}{Q} \not< 1$

M. C. Birse and S.-I. Ando. 2010. X. Kong and F. Ravndal. 1999,2001. Ando et al.2008 König and H.-W. Hammer, 2011

^{3}He in $\not{\pi}$EFT

- In contrast to ^{3}H, in ^{3}He there is a Columbic interaction.
- The dibaryon *pp* propagator is Dyson series sum over *pp* interactions:



$$i\mathcal{D}(q_0, q)_{pp}^{bare} = -\frac{i4\pi}{M_N y_s^2}\frac{1}{\frac{1}{a_C} + \alpha M_N\,\Phi(\eta)}$$

Where:

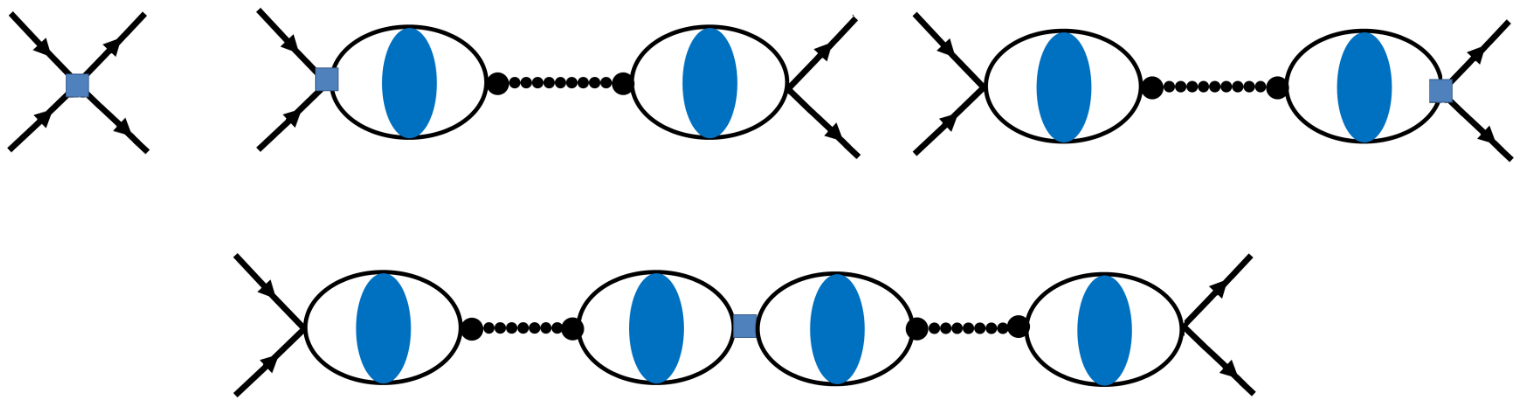
$$\Phi(x) = \psi(ix) + \frac{1}{2ix} - \log(ix), \eta = \frac{\alpha M_N}{2q'}, q' = \sqrt{\frac{q^2}{4} - M_N q_0}$$

ψ is the logarithmic derivative of Γ function.

M. C. Birse and S.-I. Ando. 2010. X. Kong and F. Ravndal. 1999,2001. Ando et al.2008 König and H.-W. Hammer, 2011

^{3}He in $\cancel{\pi}$EFT

The NLO correction to coulomb propagator:

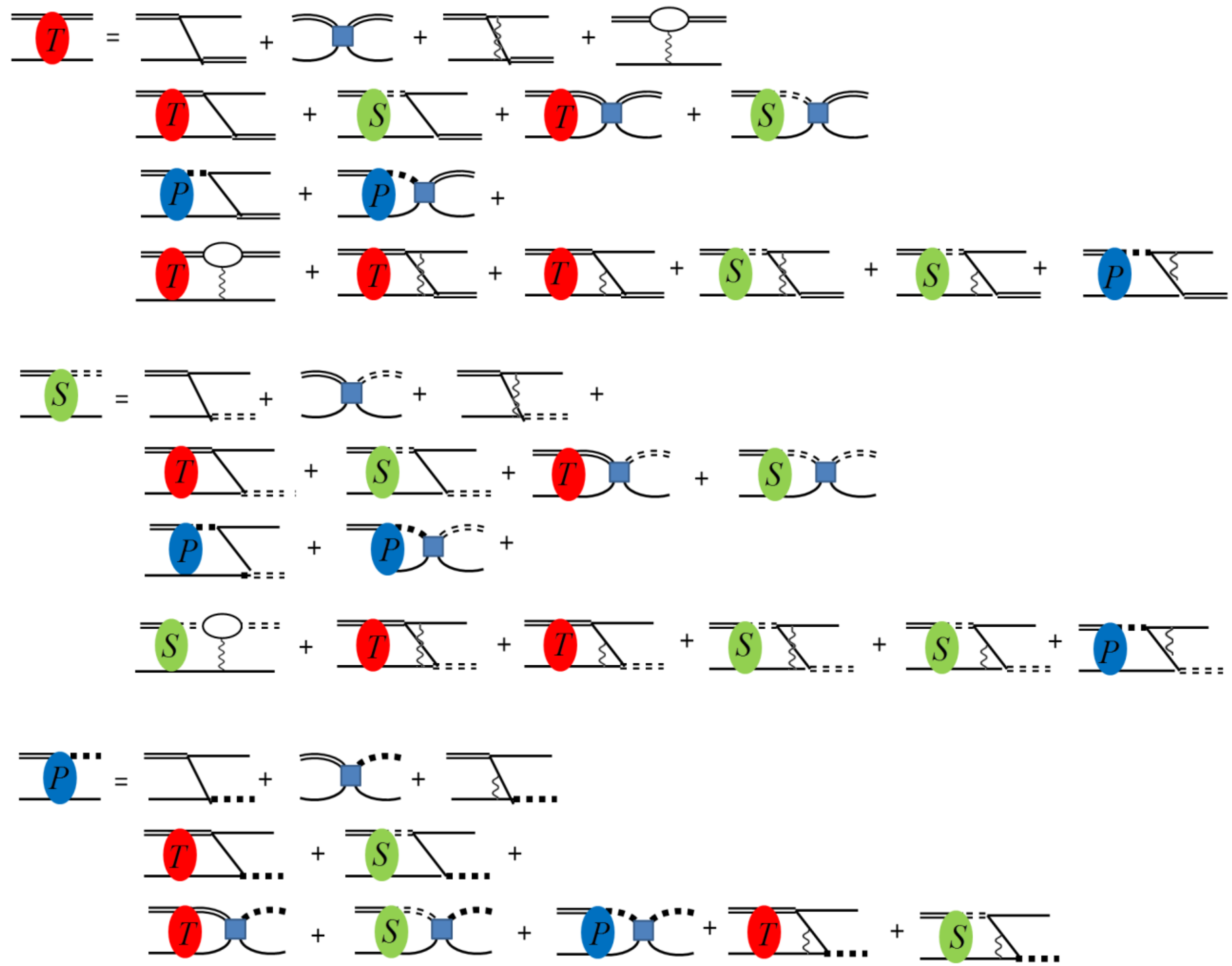


$$i\mathcal{D}(q_0, q)_{pp}^{NLO} = iD_{pp}^{LO}(q_0, q)\left[1 - \frac{\rho_C}{2}\frac{q^2 - \alpha\mu\, M_N}{\frac{1}{a_C} + H(\eta)}\right]$$

M. C. Birse and S.-I. Ando. 2010. X. Kong and F. Ravndal. 1999,2001. Ando et al.2008 S. König and H.-W. Hammer, 2011

^{3}He in $\cancel{\pi}$EFT

- ^{3}He can be written as a result of *p*-*d* scattering (in LO):



M. C. Birse and S.-I. Ando. 2010. S König and H.-W. Hammer, 2011.

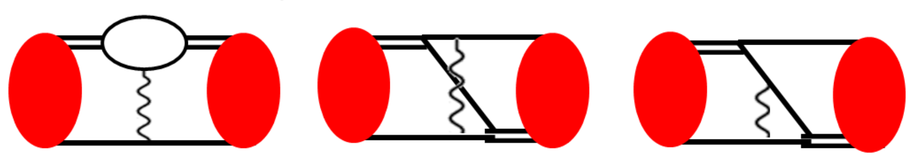
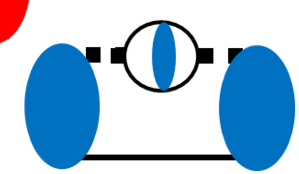
^{3}He Energy Shift matrix element

- Using the calculated wavefunctions of ^{3}H we can obtain the Coulomb-induced energy shift: $E_{^3\mathrm{He}} = E_{^3\mathrm{H}} + \Delta E.$

$$\Delta E = \langle \Gamma^{3H} | V | \Gamma^{3H} \rangle$$

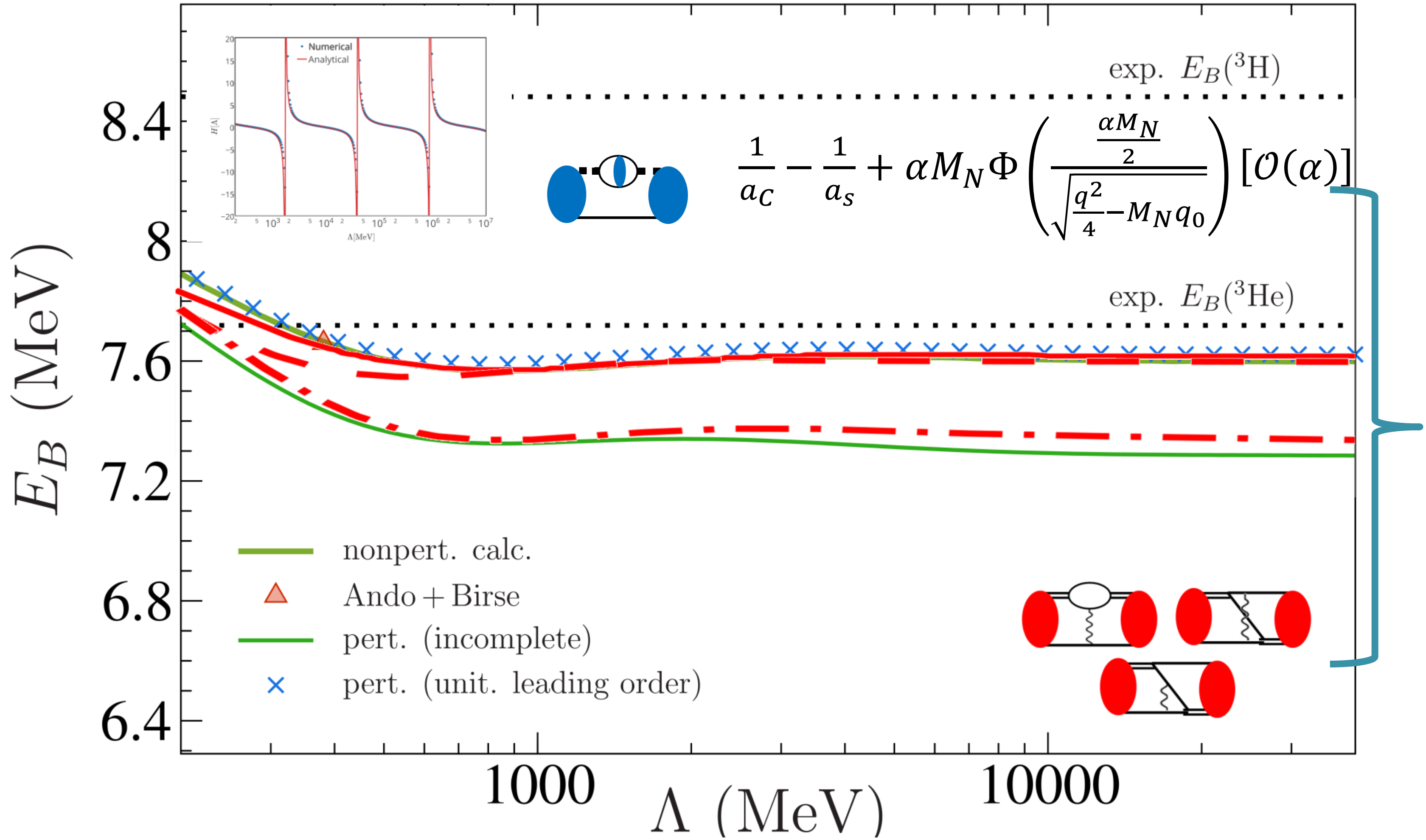
Γ^{3H}-normalized solutions to homogenous ^{3}H Faddeev equations in bound state - $E = E_{^3\mathrm{H}}$

V contains:

- α order corrections origins from one-photon exchange diagrams: 
- α order corrections origins columbic propagator: 

S. König et al 2015.

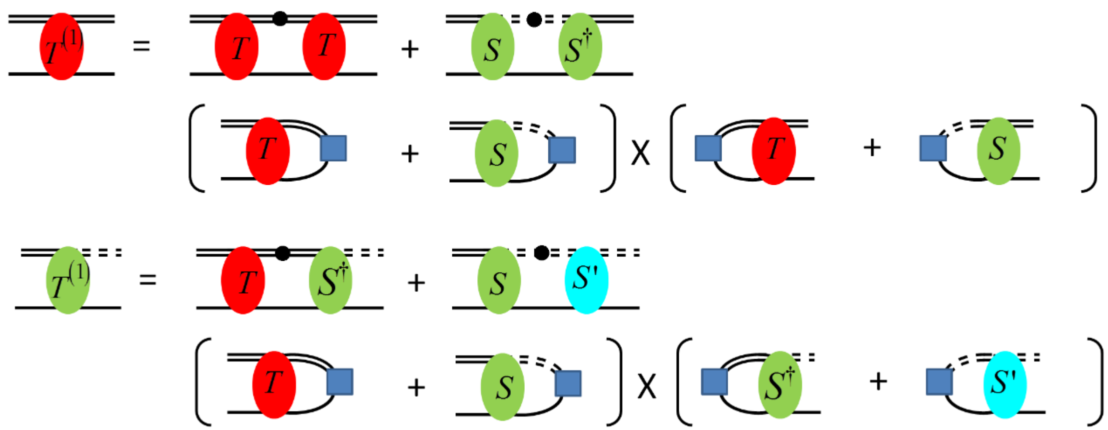
^{3}He Energy Shift matrix element



S. König et al 2015

^{3}H and ^{3}He in NLO

- In NLO for the humongous part of the ^{3}H Faddeev equations:



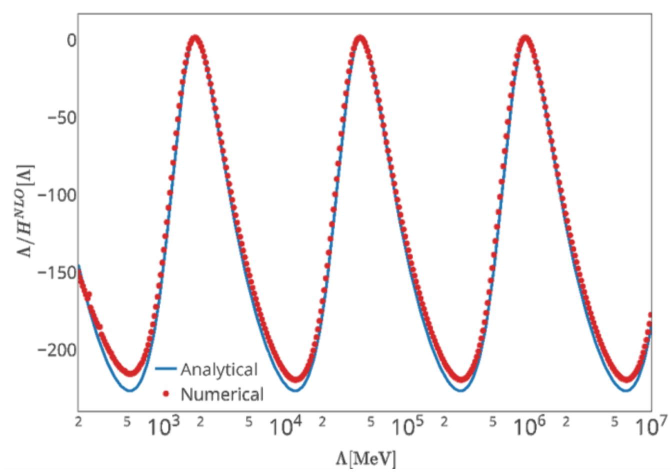
$$T^{(1)}(k,p,E_{^3\mathrm{H}}) = \frac{a(\Lambda)}{E - E_{^3\mathrm{H}}}\Gamma^{T}(k)\Gamma^{\mathrm{T}}(\mathrm{k})$$

$$\mathrm{S}^{(1)}(k,p,E_{^3\mathrm{H}}) = \frac{a(\Lambda)}{E - E_{^3\mathrm{H}}}\Gamma^{T}(k)\Gamma^{s}(\mathrm{k})$$

$$E^{NLO}_{^3\mathrm{H}} = 8.48 + B_1$$

$$a(\Lambda) = B_1 = \lim_{E \to E_{^3\mathrm{H}}} \frac{(\mathrm{E} - E_{^3\mathrm{H}})^2 t_1(k,p,E)}{Z_0} = 0$$

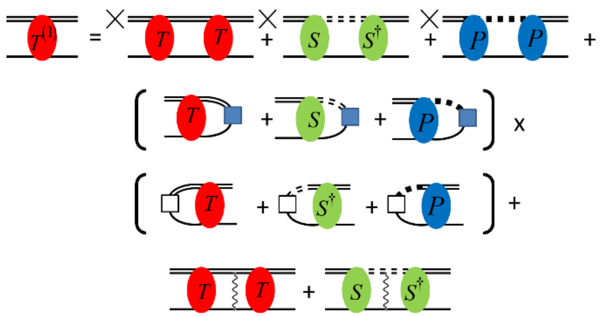
Introduce $H^{NLO}(\Lambda)$

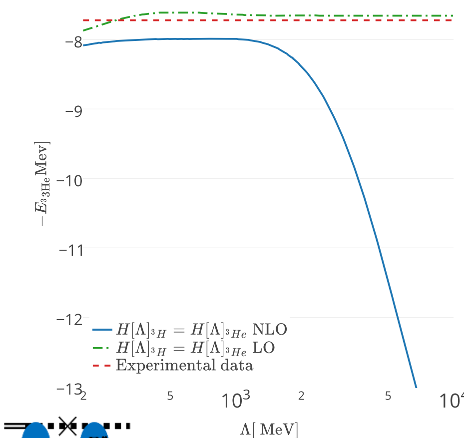


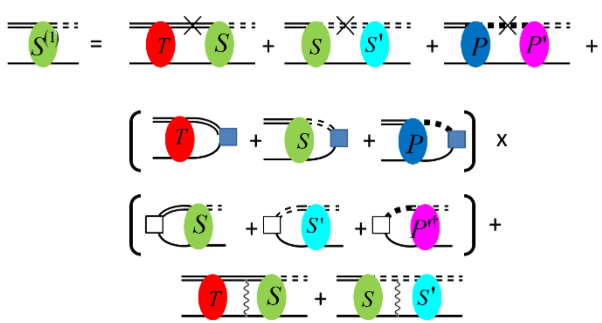
J. Vanasse et al 2014.

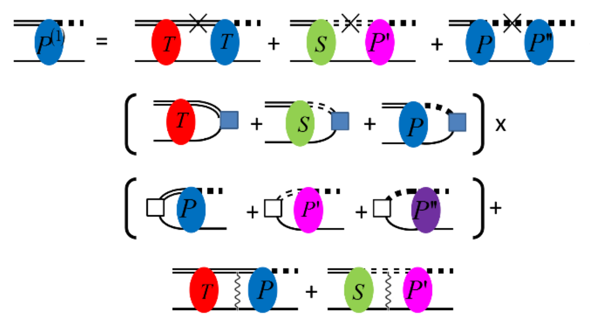
^{3}H and ^{3}He in NLO

- In NLO for the humongous part of the ^{3}H Faddeev equations contains additional columbic corrections:









$$E^{NLO}_{^3\mathrm{He}}(\Lambda) = E^{LO}_{^3\mathrm{He}}(\Lambda) + B_1 = [E^{\mathrm{LO}}_{^3\mathrm{He}}(\Lambda), 7.72\mathrm{MeV}]$$

$$a(\Lambda) = B_1$$

Introduce $H^{\alpha}(\Lambda)$

Vanasse et al 2014.

Electromagnetic Interaction in πEFT

- For $A < 4$ there are four electromagnetic interaction with two body LEC L_1 and L_2.

$A = 2$:

- $\sigma_{np}: n + p \to d + \gamma$ – radiative capture cross section.
- The deuteron magnetic moment - $\langle \mu_d \rangle$.

Well measured and calculated.

$A = 3$

- ^{3}H and ^{3}He magnetic moment - $\langle \mu_{^3\mathrm{H}} \rangle, \langle \mu_{^3\mathrm{He}} \rangle$

Well measured and never calculated using πEFT.

- Examine the consistency of πEFT between from $A = 2$ to $A = 3$ and vice-versa.
- Determining the effect of the position and residue of the deuteron pole ($Z - Parametrisation$)

Magnetic Interaction in EFT

- The one body Lagrangian of the magnetic is given by:

$$\mathcal{L}^{1}_{\text{magnetic}} = \frac{e}{2M_N} N^{\dagger} \left(\kappa_0 + \kappa_1 \tau_3\right) \sigma \cdot B N$$

- The two body Lagrangian of the magnetic is given by:

$$\mathcal{L}^{2}_{\text{magnetic}} = e\left[L_1 (N^T P_s^A N)^{\dagger} (N^T P_t^i N) B_i - L_2 \left(N^T P_t^i\right)^{\dagger} \left(N^T P_t^j N\right) B_k + h.c\right]$$

- The nucleon magnetic moments are given by:

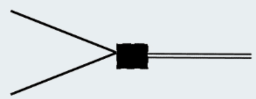
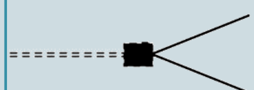
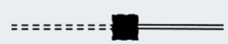
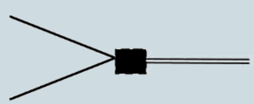
$$\kappa_0 = \frac{1}{2}\left(\kappa_p + \kappa_n\right), \ \ \kappa_1 = \frac{1}{2}\left(\kappa_p - \kappa_n\right)$$

Kaplan, Savage, Wise 1998; Ando and Hyun 2004

Magntic Interaction in EFT

- In the dibaryon frame of work with EXACT Hubbard-Stratonovich transformation:

$$e\left[L_1(N^TP_s^AN)^\dagger(N^TP_t^iN)B_i - L_2\left(N^TP_t^i\right)^\dagger\left(N^TP_t^jN\right)B_k + h.c\right] \to$$

$L_1(NP_s^AN)^\dagger t^i$ $/L_1 t^{i\dagger}(NP_s^AN)$		$\pi\sqrt{\frac{\rho_t}{2\pi}}\frac{1}{\mu-\frac{1}{a_s}}(\kappa_n-\kappa_p)-L_1\frac{1}{\sqrt{2\pi\rho_t}}\left(\mu-\frac{1}{a_t}\right)$
$L_1\left(NP_t^iN\right)^\dagger s^A$ $/L_2 s^{A\dagger}\left(NP_t^iN\right)$		$\pi\sqrt{\frac{\rho_s}{2\pi}}\frac{1}{\mu-\frac{1}{a_t}}\left(\kappa_n-\kappa_p\right)-L_1\frac{1}{\sqrt{2\pi\rho_s}}\left(\mu-\frac{1}{a_s}\right)$
$L_1 t^{i\dagger}\, s^A$ $/L_1 s^{A\dagger}t^i$		$\pi\frac{\rho_t+\rho_s}{2\sqrt{\rho_t\rho_s}}(\kappa_n-\kappa_p)-L_1\frac{1}{2\pi\rho_t\rho_s}\left(\mu-\frac{1}{a_t}\right)\left(\mu-\frac{1}{a_s}\right)$
$L_2\left(NP_t^iN\right)^\dagger t^j$ $/L_2 t^{i\dagger}\left(NP_t^jN\right)$		$\pi\sqrt{\frac{\rho_t}{2\pi}}\frac{1}{\mu-\frac{1}{a_t}}(\kappa_n+\kappa_p)-L_2\frac{1}{\sqrt{2\pi\rho_t}}\left(\mu-\frac{1}{a_t}\right)$
$L_2 t^{i\dagger}\, t^j$		$\pi\frac{\rho_t+\rho_s}{2\sqrt{\rho_t\rho_s}}(\kappa_n+\kappa_p)-L_2\frac{1}{2\pi\rho_t\rho_s}\left(\mu-\frac{1}{a_t}\right)^2$

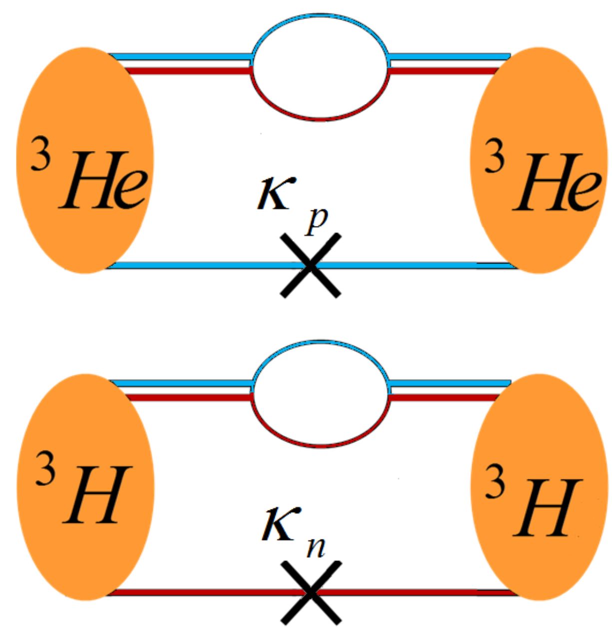
A=3 magnetic Interaction in EFT

- The magnetic moments of ^{3}H and ^{3}He:

$$\langle \mu_3 \rangle = \frac{\left\langle \frac{1}{2}, I_z, \frac{1}{2}, m_f \middle\| \mathcal{L}_{\text{magntic}} \middle\| \frac{1}{2}, I_z, \frac{1}{2} m_i \right\rangle}{\sqrt{2J+1}}$$

- I_z is 3-nucleons isospin projection $\left(\frac{1}{2} \text{ for } {}^3\text{H}, -\frac{1}{2} \text{ for } {}^3\text{He}\right)$.
- m_f and m_i the finial and initial spin projections.

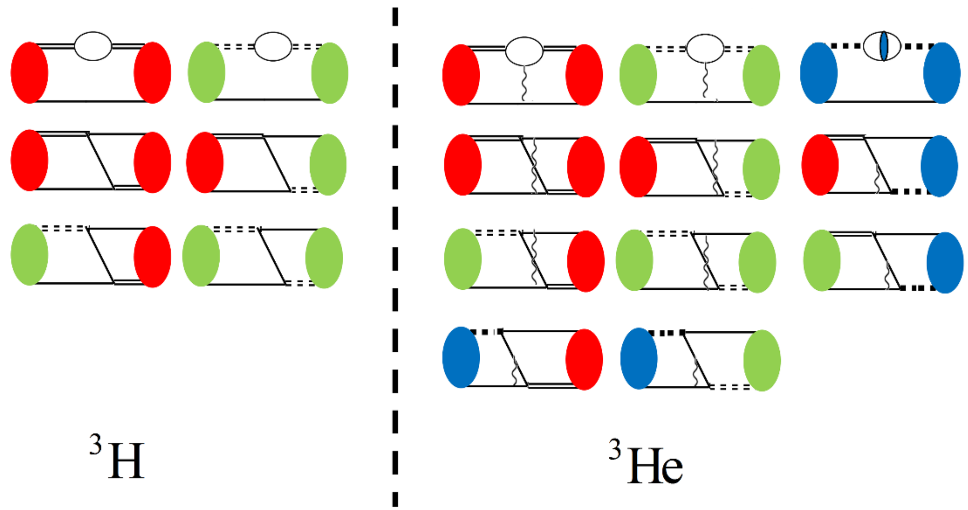
$$\langle \mu_3 \rangle = Z_3 \int \frac{d^3 q}{2\pi^3} \Gamma_A(q, E_3) D_A(E_3, q) y_t^2 \times \frac{M_N^2}{4\,\pi\,\sqrt{3\,q^2 - 4 M_N\, E_3}} \Gamma_A(q, E_3) D_B(E_3, q) \times \frac{\langle m_f | \sigma^k | m_i \rangle}{\left\langle \frac{1}{2}, m_i, 1, k \middle| \frac{1}{2}, m_f \right\rangle}$$



H. D and D. Gazit. In preparation

3-nucleons normalization

- Z_3 is the three-nucleon normalization:



$$Z_3 = \left(\hat{\mathcal{D}}\vec{\mathcal{B}}\right)^T \otimes \frac{d}{dE}\left(\hat{I} - K\right)|_{E=E_B} \otimes \left(\hat{\mathcal{D}}\vec{\mathcal{B}}\right)$$

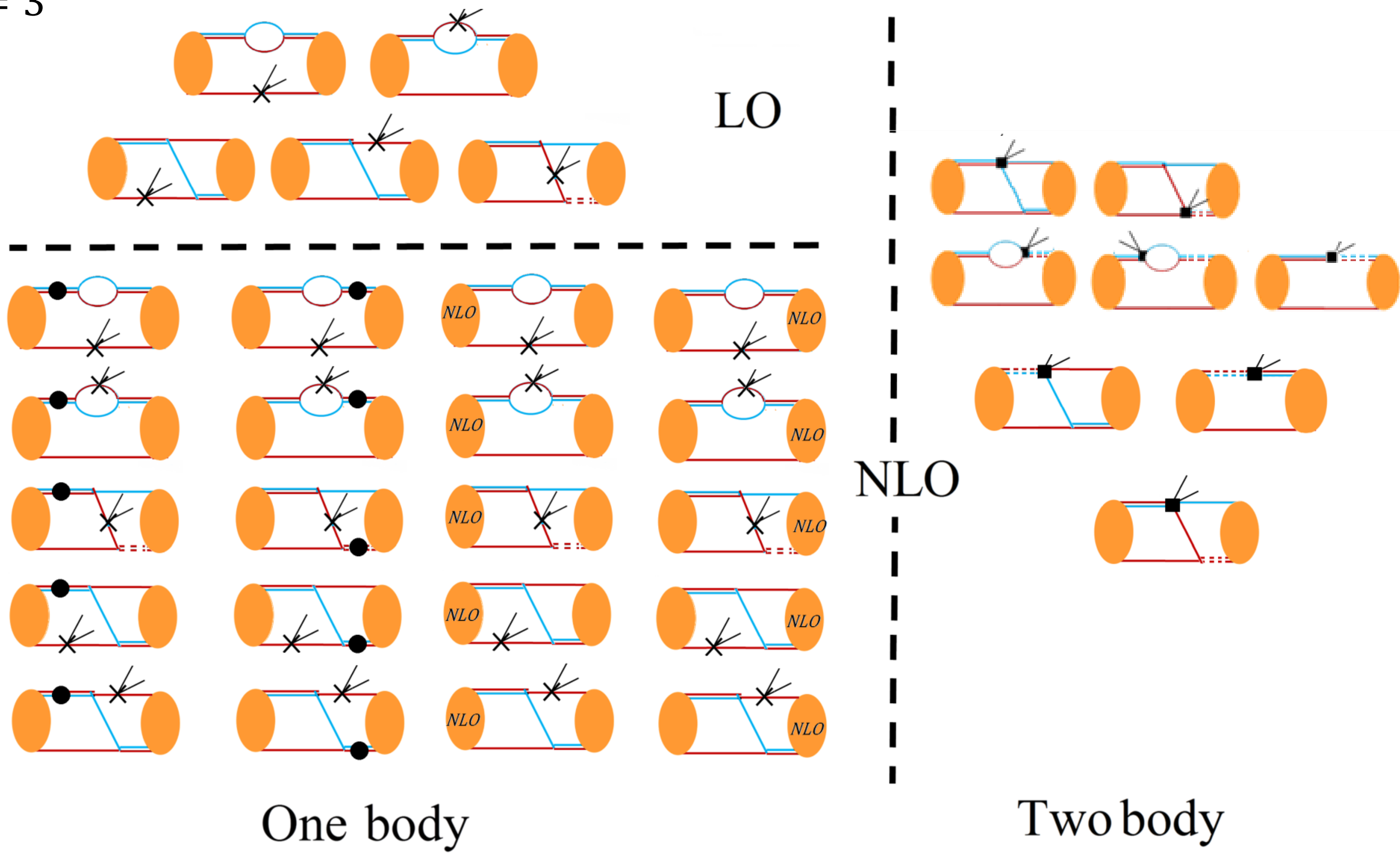
$$\hat{I} = diag(I_t, I_s), \qquad I_{t,s}(q, q', E) = \frac{2\pi^2}{q^2}\delta(q - q')D_{t,s}(E, q)^{-1}.$$

$$\vec{\mathcal{B}} = \begin{pmatrix} \Gamma_T \\ \Gamma_S \end{pmatrix}, \qquad K(q, q', E) = \frac{M_N}{2}\frac{1}{qq'}Q_0\left(\frac{q^2 + q'^2 - M_N E}{qq'}\right)\begin{pmatrix} y_t^2 & -3y_t y_s \\ -3y_t y_s & y_s^2 \end{pmatrix}$$

S. König and H.-W. Hammer, 2011

Electromagnetic Interaction in EFT

$A = 3$



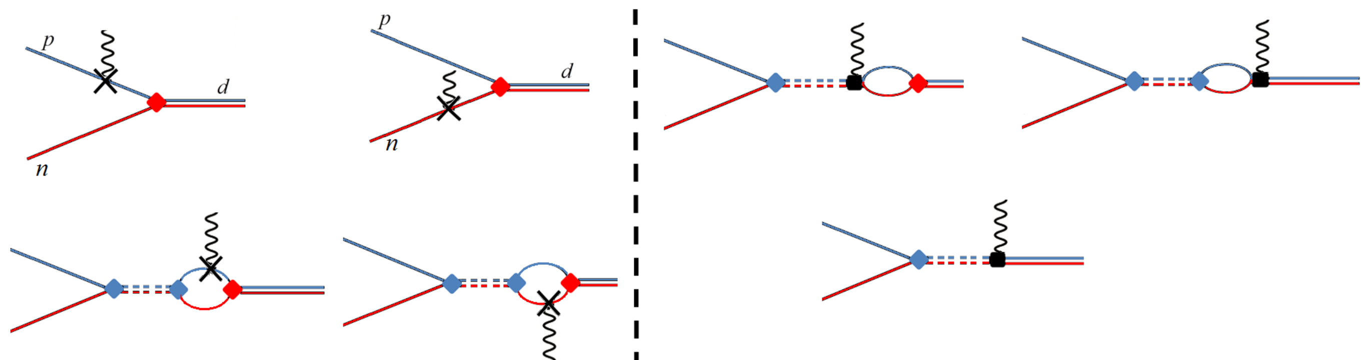
H. D and D. Gazit. In preparation

^{3}H

Electromagnetic Interaction in EFT

- For $A < 3$ there are measured two electromagnetic processes:

np radiatve capture: $n + p \rightarrow \ d + \gamma$



One body

Two body

$$\sigma_{np} = \frac{\alpha\,(\gamma_t^2 + p^2)^3}{M_N^2 p} Y^2$$

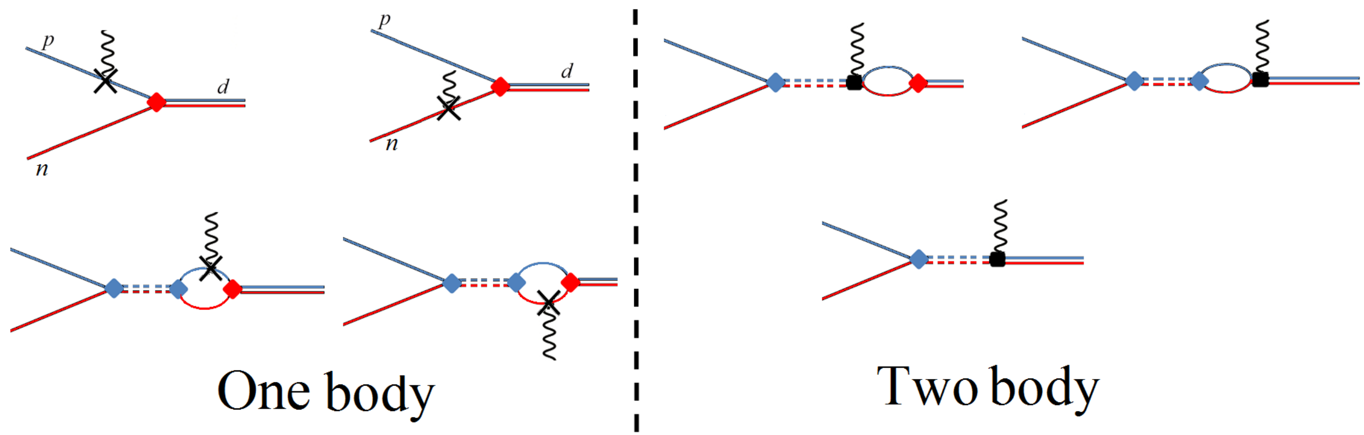
$$Y = \sqrt{\frac{\pi}{\gamma_t} Z_d}\,\frac{(\kappa_p - \kappa_n) a_s}{M_N}\left[\left(1 - \frac{1}{\gamma_t a_s}\right) + \frac{\gamma_t}{4}(\rho_t + \rho_s)\right.$$

Ando and Hyun 2004

Electromagnetic Interaction in EFT

- For $A < 3$ there are measured two electromagnetic processes:

$$\langle \mu_d \rangle$$



$$\langle \mu_d \rangle = Z_d \frac{e}{2M_N}(\kappa_n + \kappa_p)\left[1 - \gamma_t \rho_t + L_2 \frac{M_N}{2\pi\rho_t(\kappa_p - \kappa_n)}\left(\mu - \frac{1}{a_t}\right)^2\right]$$

Ando and Hyun 2004

Deuteron normalization

$$Z_d = \frac{1}{1-\gamma_t\rho_t} = \underbrace{1}_{\text{LO}} + \underbrace{\gamma_t\rho_t}_{\text{NLO}} + \underbrace{(\gamma_t\rho_t)^2}_{\text{N}^2\text{LO}} + \underbrace{(\gamma_t\rho_t)^3}_{\text{N}^3\text{LO}} + \ldots$$

which are equivalent to Q expansion around $k = 0$

$$C_2^t = 2\pi\frac{Z_d^{NLO} - 1}{M_N\gamma_t(\mu - \gamma_t)^2}$$

expansion around the deuteron pole: $Z_d = \frac{1}{1-\gamma_t\rho_t} = 1.69$

$$Z_d = \frac{1}{1-\gamma_t\rho_t} = \underbrace{1}_{\text{LO}} + \underbrace{Z_d - 1}_{\text{NLO}} + \underbrace{0}_{\text{N}^2\text{LO}} + \underbrace{0}_{\text{N}^3\text{LO}} + \ldots$$

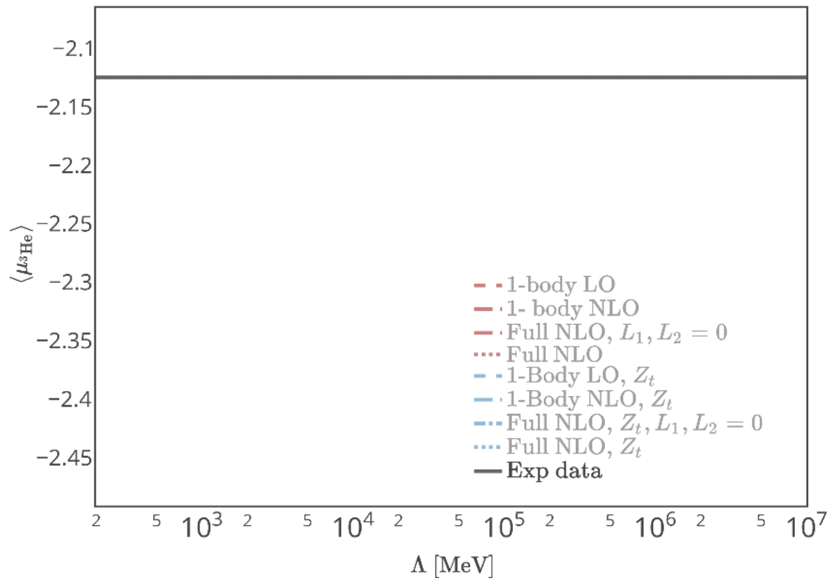
And therefore:

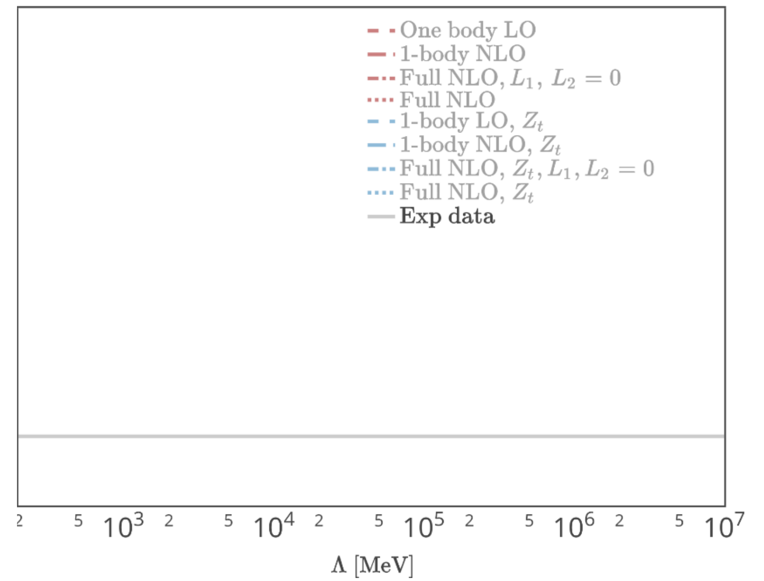
$$C_2^t = 2\pi\frac{Z_d^{NLO} - 1}{M_N\gamma_t(\mu - \gamma_t)^2} = 2\pi\frac{0.69}{M_N\gamma_t(\mu - \gamma_t)^2}$$

$$y_t^2 = \frac{(C_0^t)^2}{C_2^t} = \frac{8\pi}{M_N 0.69}$$

X. Kong and F. Ravndal. 1999,2001, H. W. Grießhamme 2004

Electromagnetic Interaction in EFT





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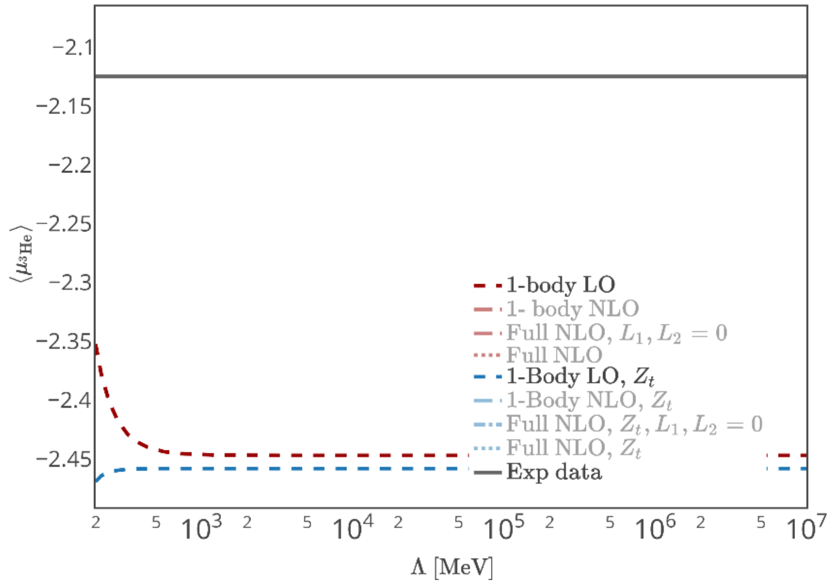
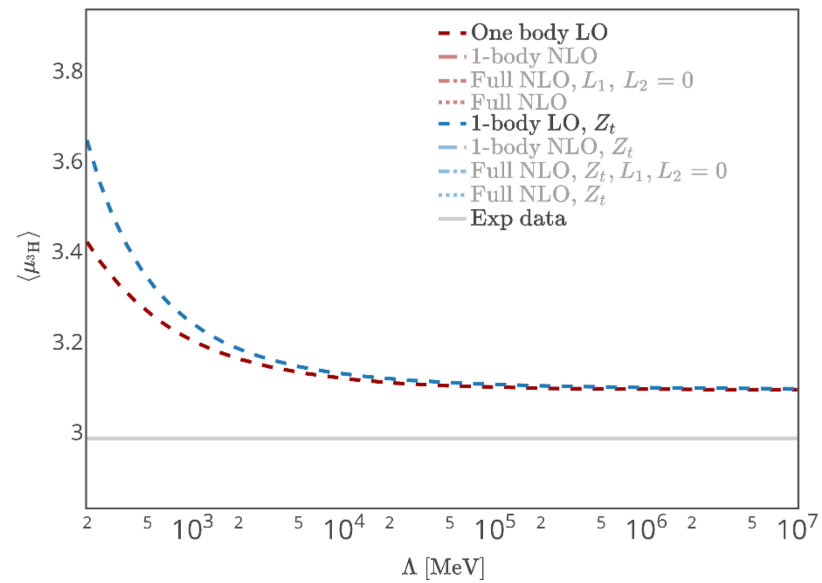
H. D and D. Gazit. In preparation

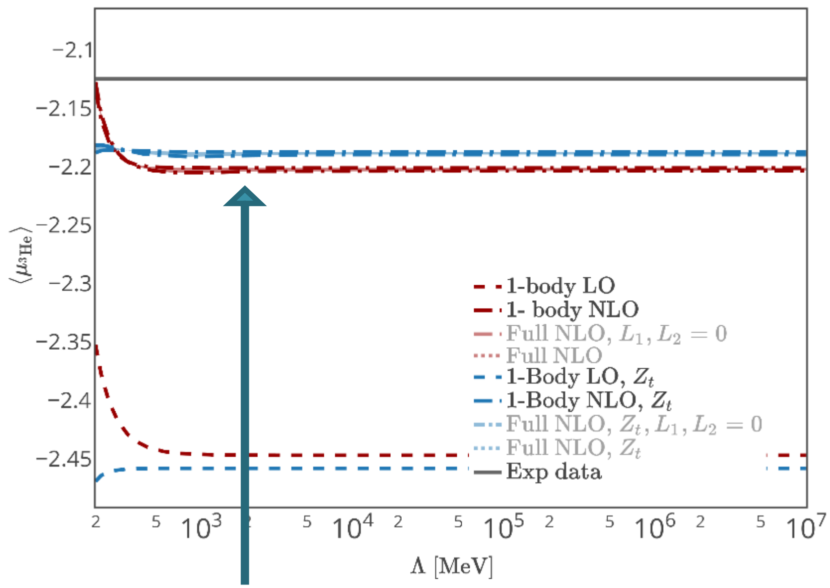
Electromagnetic Interaction in EFT

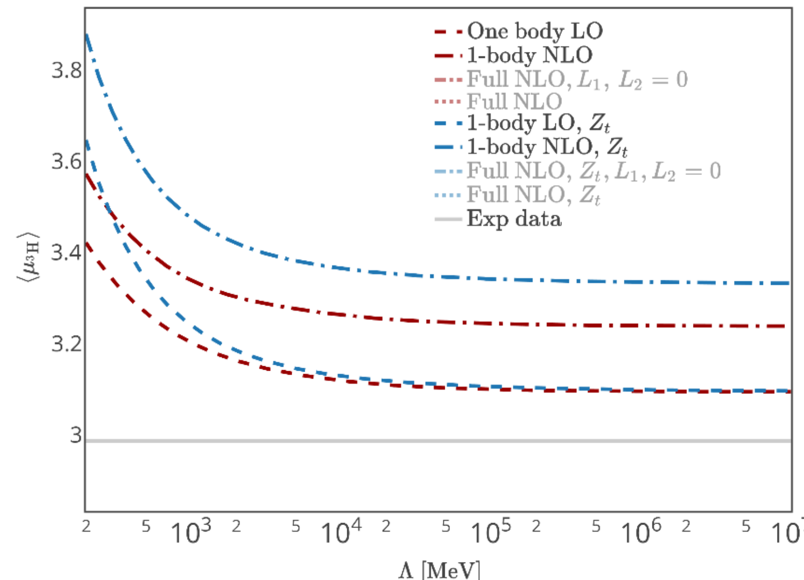




H. D and D. Gazit. In preparation

Electromagnetic Interaction in EFT





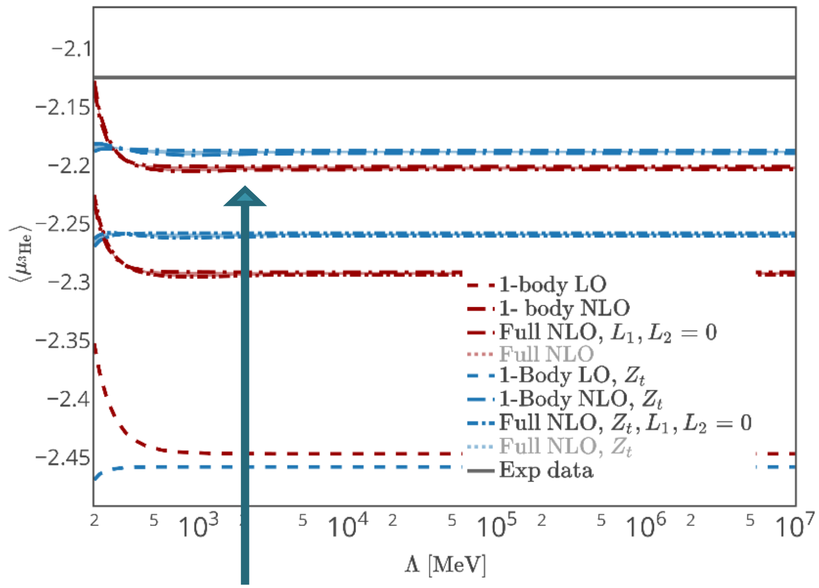
- The thickness in $\langle \mu_{3He} \rangle_{NLO}$ comes from:

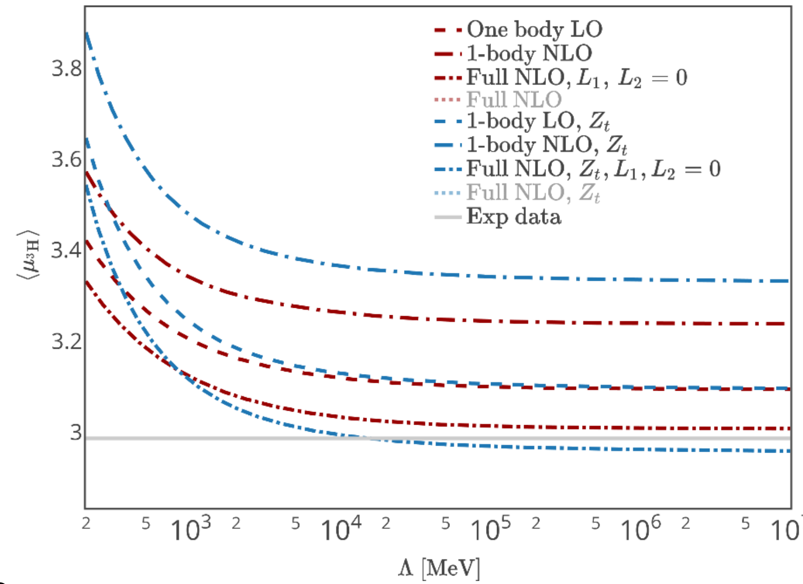
$$E^{\mathrm{NLO}}_{^3\mathrm{He}}(\Lambda) = E^{\mathrm{NLO}}_{^3\mathrm{He}}(\Lambda) - 7.72\mathrm{MeV}$$

H. D and D. Gazit. In preparation

Electromagnetic Interaction in EFT

- For $A = 3$





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H. D and D. Gazit. In preparation

Electromagnetic Interaction in EFT

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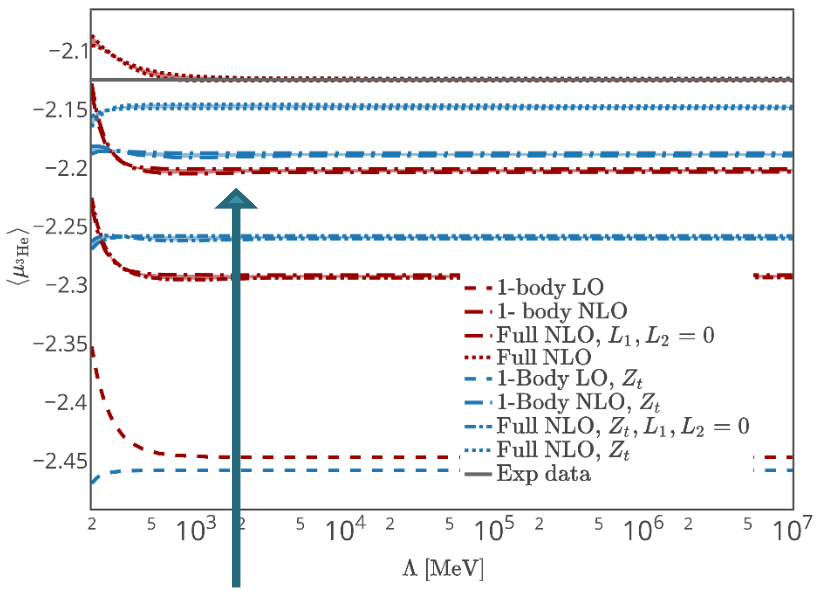
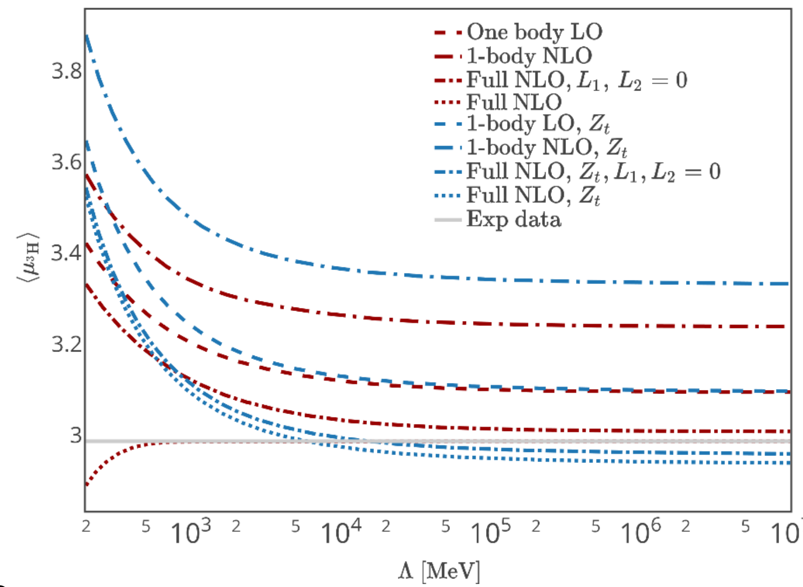




- The thickness in $\langle\mu_{3He}\rangle_{NLO}$ comes from:

$$E_{^3\mathrm{He}}^{\mathrm{NLO}}(\Lambda) = [E_{^3\mathrm{He}}^{\mathrm{NLO}}(\Lambda), 7.72\mathrm{MeV}]$$

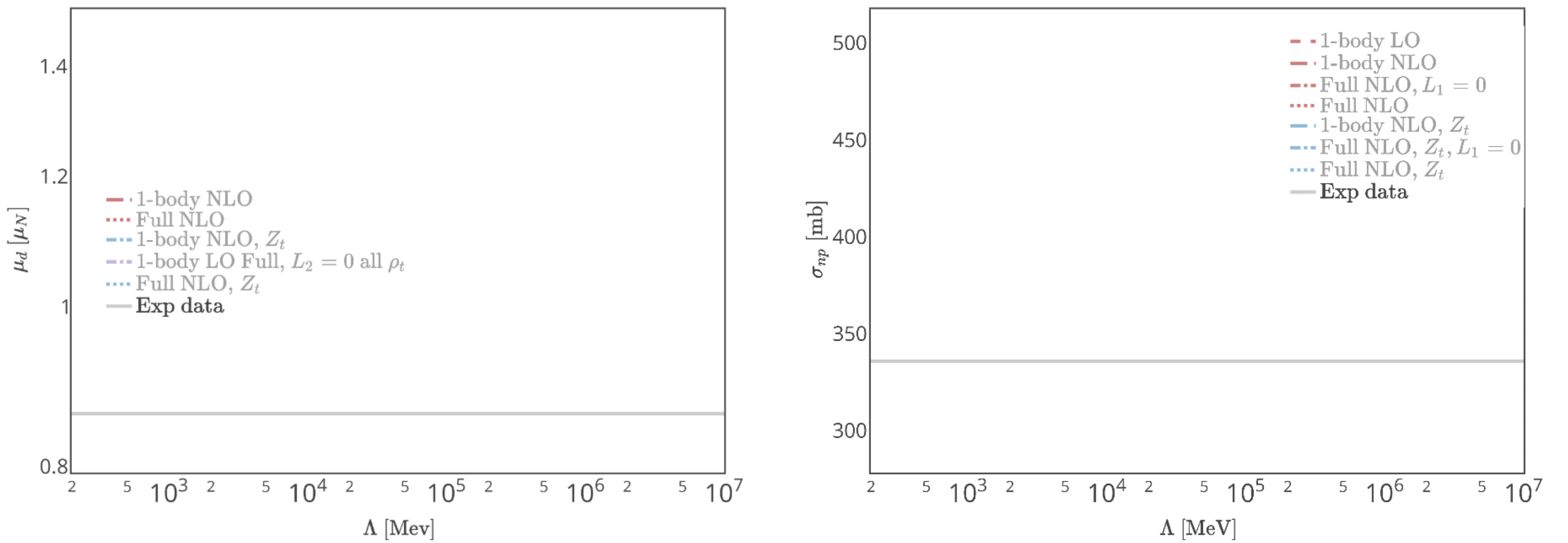
- Full NLO was calculated using $A = 2$ calibrated LEC:

$$L_1(\sigma_{np}),\ L_2(\langle\mu_d\rangle)$$

H. D and D. Gazit. In preparation

Electromagnetic Interaction in EFT

- For $A = 2$



H. D and D. Gazit. In preparation

Electromagnetic Interaction in EFT

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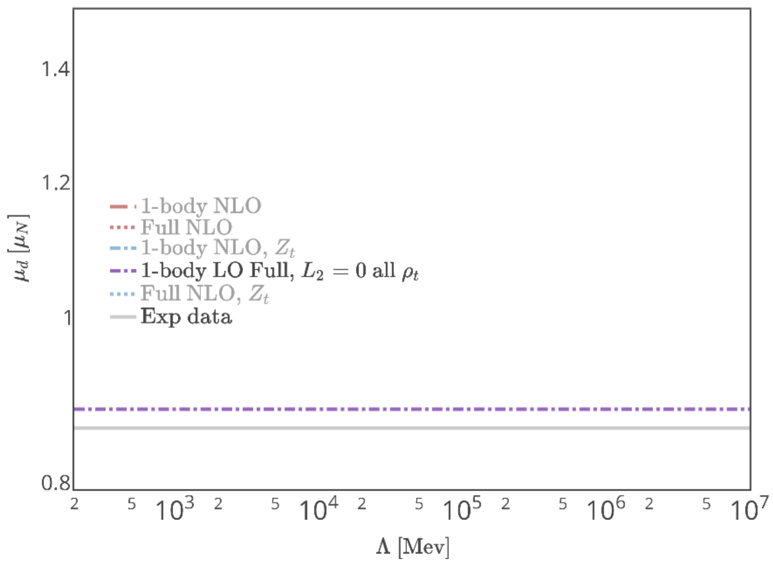

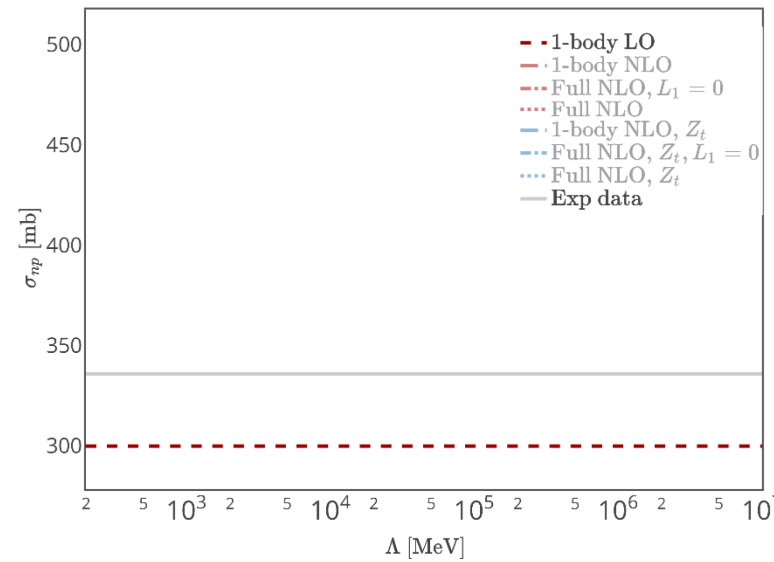

H. D and D. Gazit. In preparation

Electromagnetic Interaction in EFT

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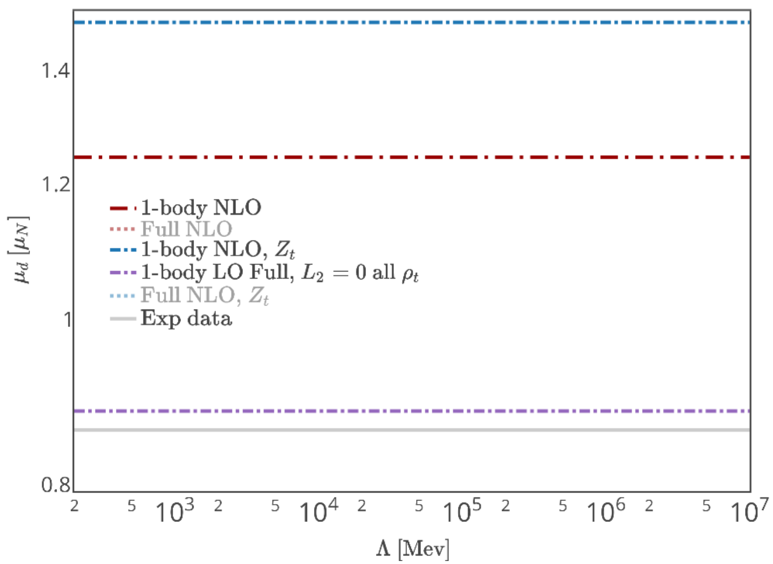

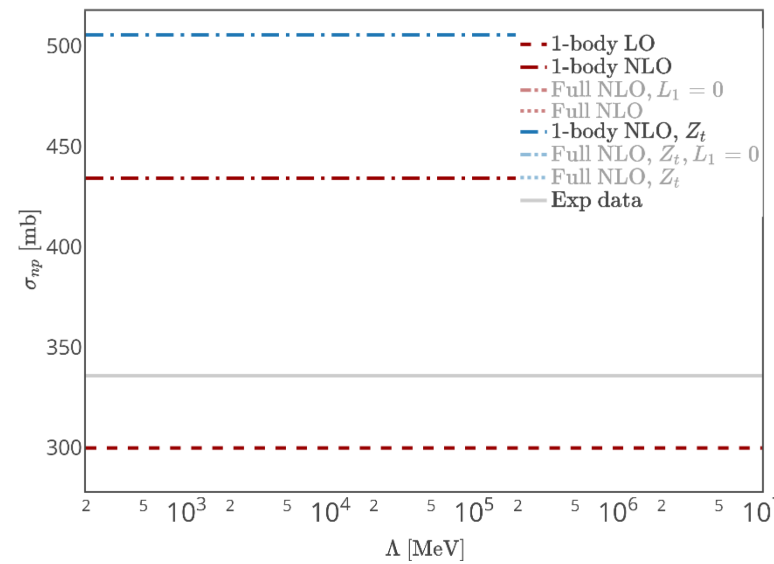

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Electromagnetic Interaction in EFT

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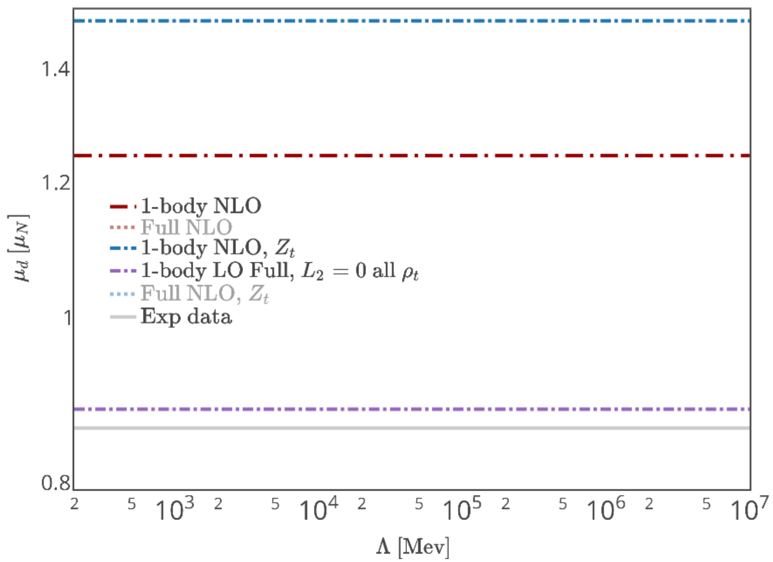

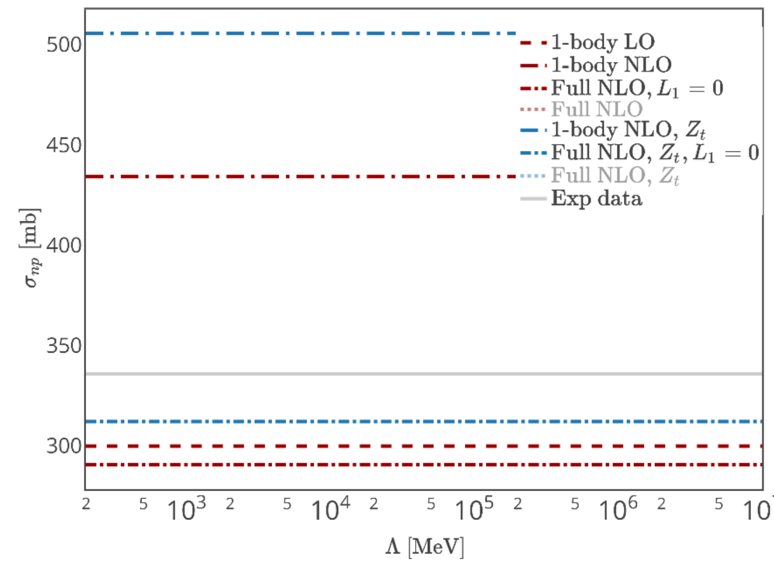

H. D and D. Gazit. In preparation

Electromagnetic Interaction in EFT

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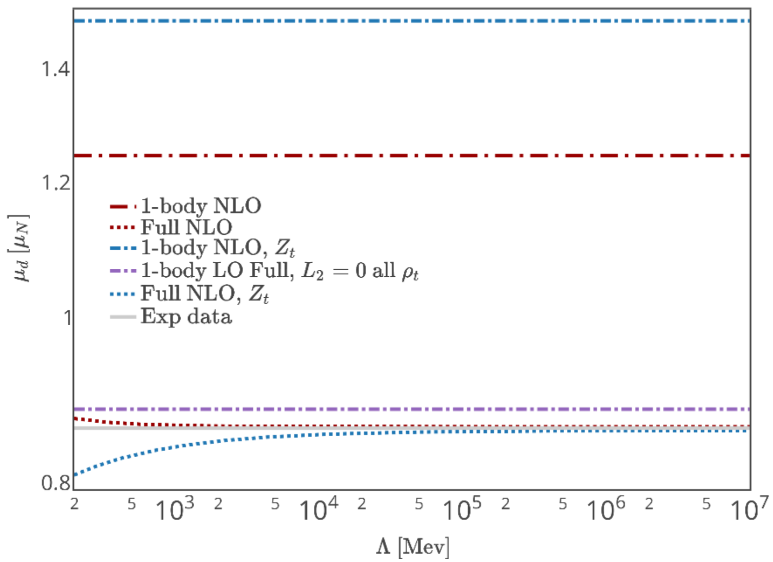
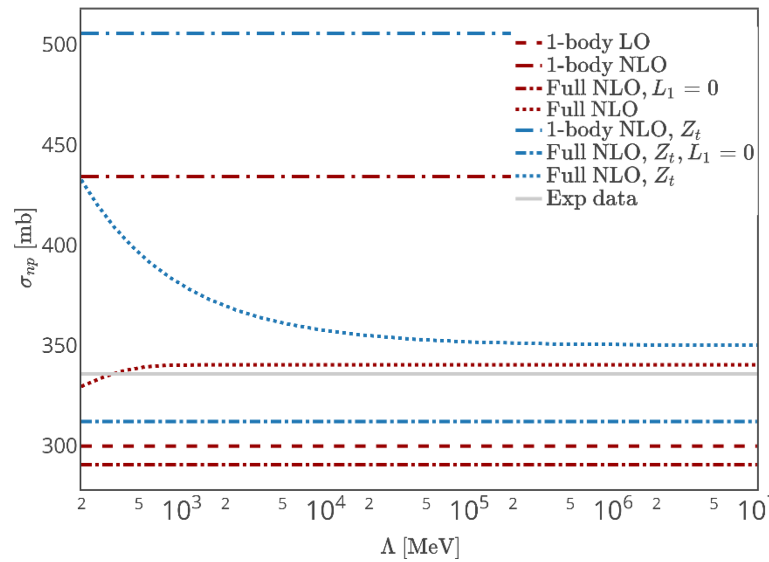




- The full NLO was calculated using $A = 3$ calibrated LEC (simultaneously):

$$L_1(\langle\mu_{^3\mathrm{H}}\rangle, \langle\mu_{^3\mathrm{He}}\rangle)$$
$$L_2(\langle\mu_{^3\mathrm{H}}\rangle, \langle\mu_{^3\mathrm{He}}\rangle)$$

H. D and D. Gazit. In preparation

Electromagnetic Interaction in EFT

	$\langle \mu_{^3\mathrm{H}} \rangle$	$\langle \mu_{^3\mathrm{He}} \rangle$	σ_{np}	$\langle \mu_d \rangle$
Full NLO	2.980 ± 0.002	-2.127 ± 0.001	338.8 ± 0.1	0.8592 ± 0.001
Full NLO Z_t	2.93 ± 0.01	-2.150 ± 0.003	347.8 ± 0.1	0.8547 ± 0.001
Exp data	2.9789	−2.12762	334.2 ± 0.5	0.8574
Δ	0.04%	0.03%	1%	0.2%
$\Delta\ \mathrm{Z_t}$	2%	1%	4%	0.3%

- In the case of expansions around $Q = 0$ the results show good consistency between A=2 and A=3.
- In the case of expansions around deuteron pole the results show good consistency between A=2 and A=3 only in the bound state.

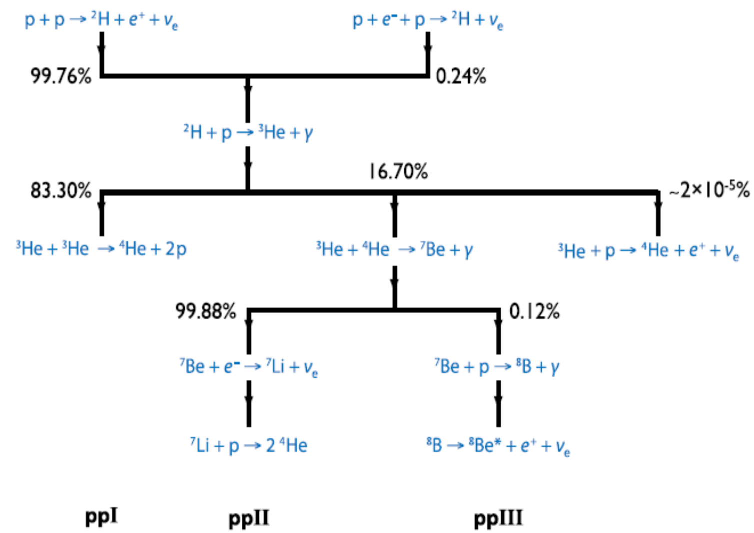
H. D and D. Gazit. In preparation

pp Fusion

- The energy generated in the Sun comes from an exothermic set of reactions, *pp* chain:

  $$4p \rightarrow {}^{4}He + 2e^{+} + 2\nu_{\mathrm{e}}$$

- The leading reaction (~99%) is *pp* fusion: $p + p \rightarrow d + e^{+} + \nu_{\mathrm{e}}$



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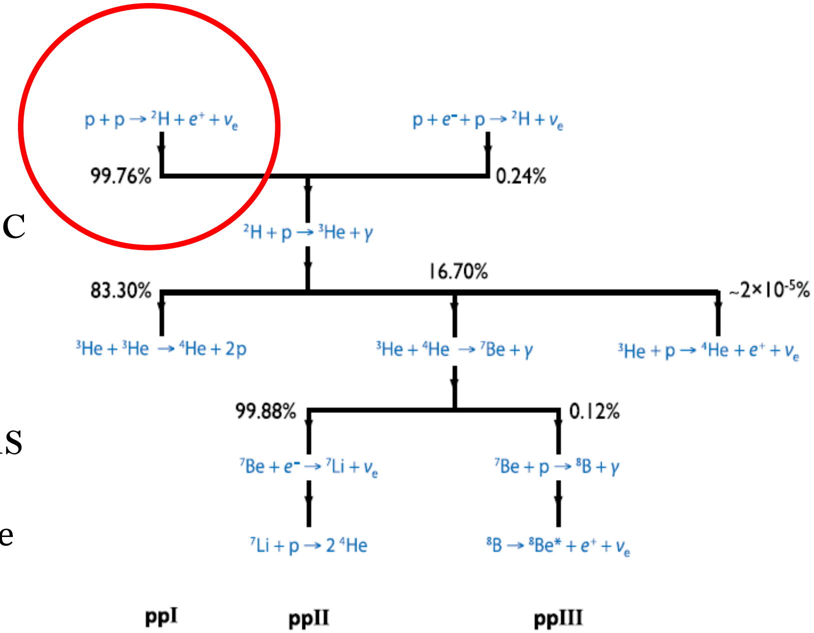

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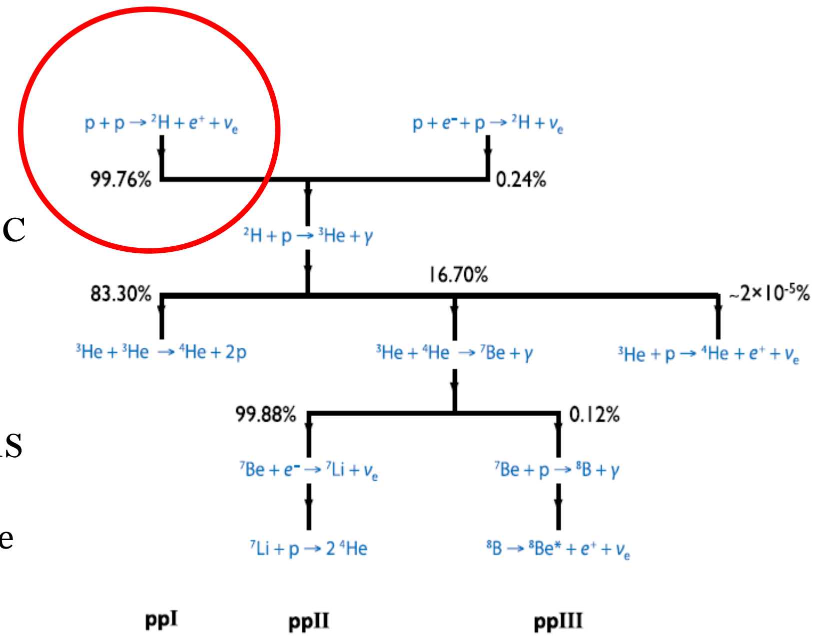


- This reaction is governed by the weak interaction which makes it the slowest reaction in the whole chain ($\tau \sim 10^9$ years) and therefore it determines the Sun's lifetime.
- The typical energy transfer is ~10 keV (depending in the Sun's temperature).
- Measurement of its cross section is impossible, so it must be calculated from the fundamental theory of physics.

pp Fusion in EFT

- L.E Marcucci et al. have calculated $S(E)$ and the next derivatives using χEFT.

The disadvantages of χEFT:

- Cannot change the cutoff continuously.
- The nuclear Hamiltonian contains more parameters than πEFT.
- The typical energies of the fusion: $B_d = 2.25\text{MeV}, \text{E}_{\text{transfer}} \sim 10\text{keV}$, suitable for πEFT.
- Power counting issue.
- Many parameters to fit.

L.E. Marcucci et al 2013

Weak Interaction in EFT

- The Lagrangian of the week interaction for low energies ($E \ll W, Z$) is given by: $\mathcal{L}_{weak} = -\frac{G_F}{\sqrt{2}} l_+^{\mu} j_{\mu}^- + h.c$
- l_+^{μ} is the leptonic current and j_{μ}^- is the hadronic current.
- j_{μ}^- contains two parts : polar-vector and axial vector.

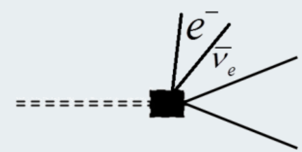
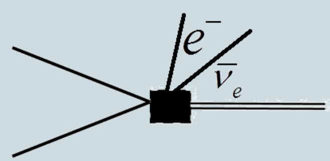
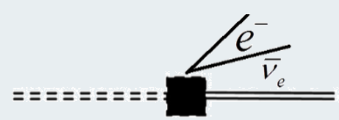
$$j_{\mu} = V_{\mu} - A_{\mu}$$

- The polar-vector part: $V_{\mu} = \frac{N^{\dagger} v_{\mu} \tau^{+} N}{2}$
- The axial -vector part: $A_{\mu} = \frac{g_A}{2} N^{\dagger} \tau^{+} N + L_{1A} (N P_t^i N)^{\dagger} (N P_s^A N)$
- g_A: axial coupling constant, known from neutron β decay.
- L_{1A}: two body analog to g_A, unknown, NLO.

X. Kong and F. Ravndal. 1999,2001.

Weak Interaction in EFT

- In the dibaryon frame of work with EXACT Hubbard-Stratonovich transformation: $L_{1A}\left[\left(NP_t^iN\right)^\dagger\left(NP_s^AN\right)+h.c\right]\rightarrow$

$L_{1A}\left(NP_t^iN\right)^\dagger s^A$ $/L_{1A}s^{A\dagger}\left(NP_t^iN\right)$		$\pi\sqrt{\frac{\rho_t}{2\pi}}\frac{1}{\mu-1/a_s}g_A$ $-L_{1A}\frac{1}{\sqrt{2\pi\rho_t}}\left(\mu-\frac{1}{a_t}\right)$
$L_{1A}(NP_s^AN)^\dagger t^i$ $/L_{1A}t^{i\dagger}(NP_s^AN)$		$\pi\sqrt{\frac{\rho_s}{2\pi}}\frac{1}{\mu-1/a_t}g_A$ $-L_{1A}\frac{1}{\sqrt{2\pi\rho_s}}\left(\mu-\frac{1}{a_s}\right)$
$L_{1A}t^{i\dagger}\,s^A$ $/L_{1A}s^{A\dagger}t^i$		$\pi\frac{\rho_t+\rho_s}{2\sqrt{\rho_t\rho_s}}g_A$ $-L_{1A}\frac{1}{2\pi\rho_t\rho_s}\left(\mu-\frac{1}{a_t}\right)\left(\mu-\frac{1}{a_s}\right)$

H. D, L. Platter and D. Gazit. In preparation

L_{1A} Calibration

- Chen et al have calibrated L_{1A} using Muon capture:
$$\mu^- + d \rightarrow \nu_\mu + n + n$$
- The cross section in known from the MuSun experiment, but the energy transfer might be to high for πEFT.
- The accuracy is 1% only.
- The cutoff dependence require the introduction of additional coupling constants.

J.-W. Chen et al , 2005

L_{1A} Calibration

- ^{3}H β decay into ^{3}He:

$$^3\mathrm{H} \rightarrow \overline{\nu_e} + e^- + {}^3\mathrm{He}$$

The cross section is well know and the energy transfer is suitable for πEFT.

The challenges:

- Weak interaction process that includes three nucleons was never described using πEFT.
- L_{1A} is at NLO, which require a full derivation of the 3-noucliun scatting amplitude up to NLO, in order to keep consistency.
- The description of 3-noucloen system with coulomb interaction is not well define.

H. D, L. Platter and D. Gazit. In preparation

^{3}H β-decay into ^{3}He

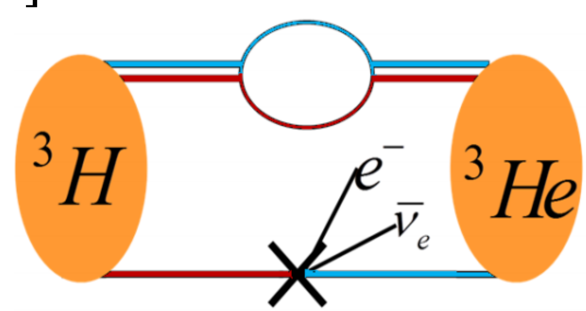
- The half life time of ^{3}H β decay is given by:

$$\frac{K/G_V}{f_V\langle F\rangle^2 + f_A g_A^2 \langle GT\rangle^2}$$

Where:

$$\langle GT\rangle = \frac{\langle ^3\mathrm{H}||\boldsymbol{\sigma}\tau^-||^3\mathrm{H}e\rangle}{\sqrt{2J+1}}, \qquad \langle F\rangle = \frac{\langle ^3\mathrm{H}||\tau^-||^3\mathrm{H}e\rangle}{\sqrt{2J+1}}$$

$$\langle GT\rangle$$
$$= \sqrt{Z_{^3H} Z_{^3He}} \int \frac{d^3q}{2\pi^3} \Gamma_A(q, E_3) D_A(E_3, q) y_t^2$$
$$\times \frac{M_N^2}{2\pi\left[\sqrt{3\,q^2 - 4M_N\,E_{3\mathrm{H}}} + \sqrt{3\,q^2 - 4M_N\,(E_{3\mathrm{H}} + l_0)}\right]}$$
$$\Gamma_A(q, E_{3\mathrm{H}} + l_0) D_B(E_{3\mathrm{H}} + l_0\,, q)$$
$$\times \frac{\langle m_f|\sigma^k|m_i\rangle}{\left\langle \frac{1}{2}, m_i, 1, k \middle| 1/2, m_f \right\rangle}$$



H. D, L. Platter and D. Gazit. In preparation

^{3}H β-decay into ^{3}He

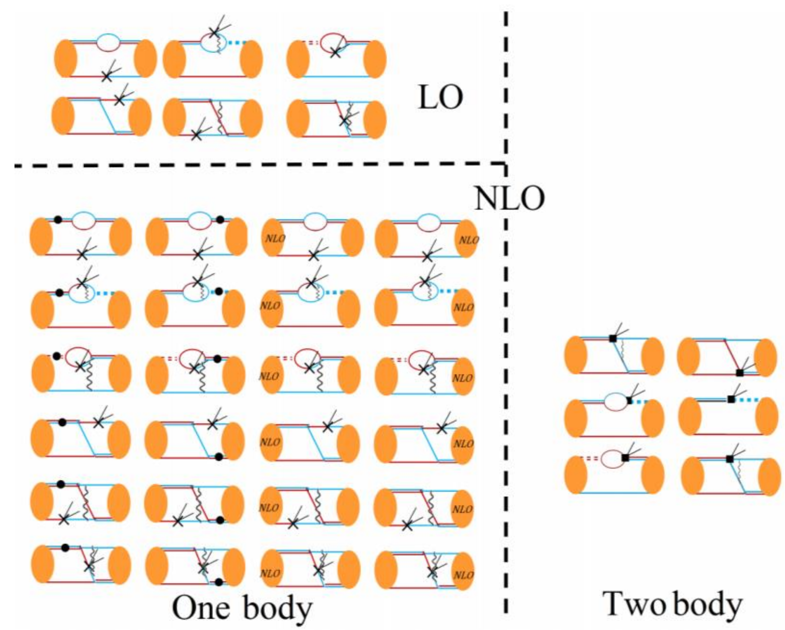
- The half life time of 3H β decay is given by:

$$\frac{K/G_V}{f_V\langle F\rangle^2 + f_A g_A^2 \langle GT\rangle^2}$$

Where:

$$\langle GT\rangle = \frac{\langle {}^3\mathrm{H}||\boldsymbol{\sigma}\tau^-||{}^3\mathrm{He}\rangle}{\sqrt{2J+1}}, \qquad \langle F\rangle = \frac{\langle {}^3\mathrm{H}||\tau^-||{}^3\mathrm{He}\rangle}{\sqrt{2J+1}}$$

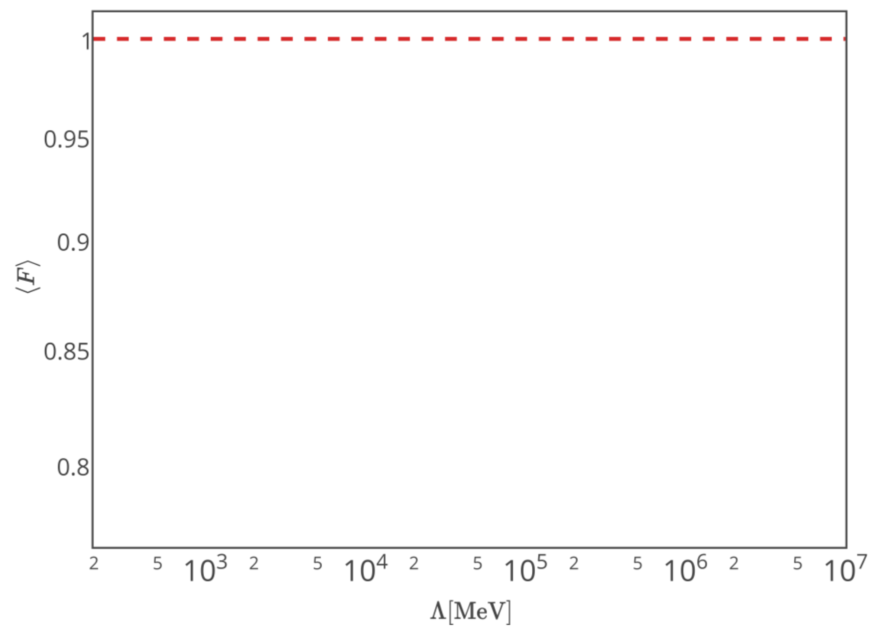
- The LHS of each diagram is ^{3}H.
- The RHS is ^{3}He.
- The blue line represents proton, while the red line represents neutron.



H. D, L. Platter and D. Gazit. In preparation

Fermi transition

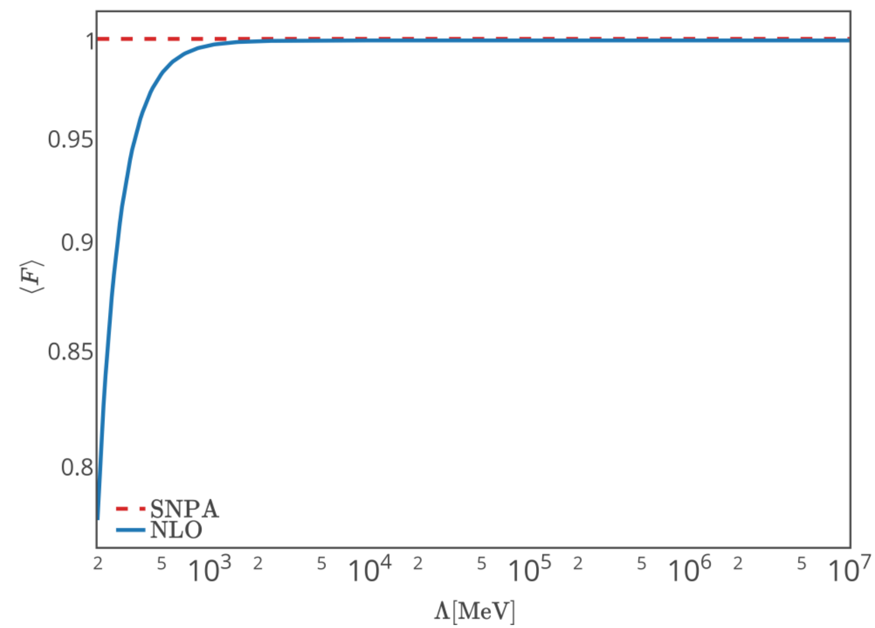
- **Red** –Former calculation of the Fermi reduced matrix elements for total isospin, $\langle F \rangle = 0.993$

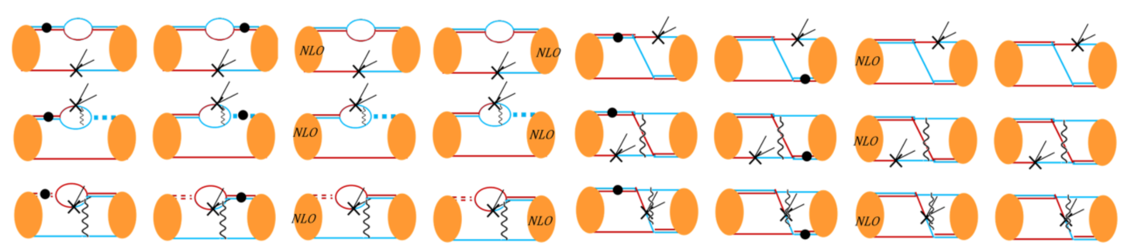


T.-Y. Saito, et al 1990. H. D, L. Platter and D. Gazit. In preparation

Fermi transition

- **Red** –Former calculation of the Fermi reduced matrix elements for total isospin, $\langle F \rangle = 0.993$
- **Blue** – calculated τ^- transition (contains only the one body diagrams). $\langle \mathrm{F} \rangle = 0.9985 \pm 0.0005$





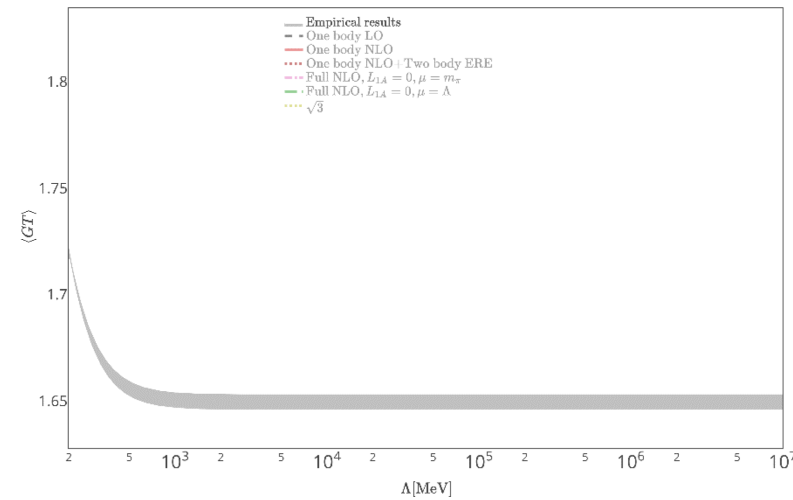
One body NLO

T.-Y. Saito, et al 1990. H. D, L. Platter and D. Gazit. In preparation

Gamow -Teller transition

- Gray – empirical results:

$$\langle GT \rangle^2 = \frac{K/G_V - \langle F \rangle^2 f_V (FT_{1/2})_t}{f_A g_A^2 (FT_{1/2})_t}$$



T.-Y. Saito, et al 1990. H. D, L. Platter and D. Gazit. In preparation

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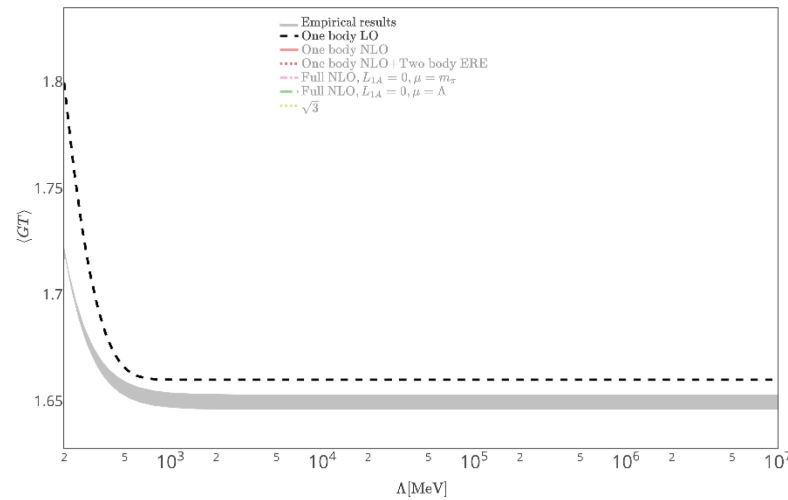
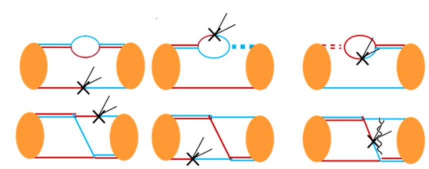
- **Black** – one body LO





LO

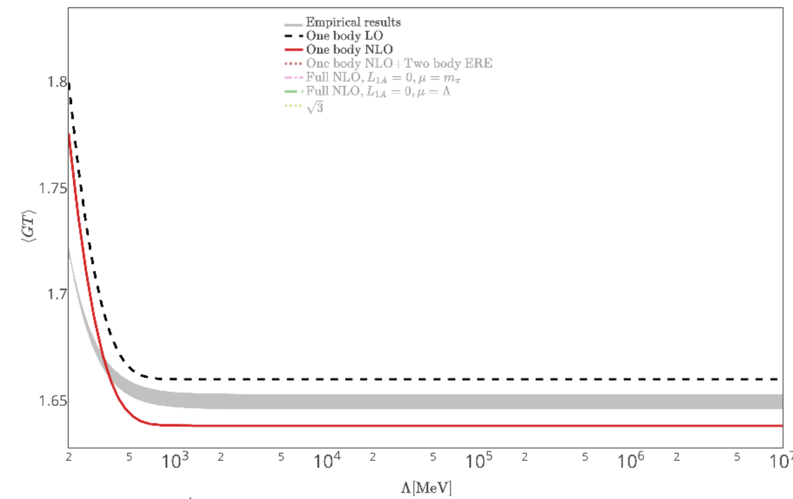
T.-Y. Saito, et al 1990. H. D, L. Platter and D. Gazit. In preparation

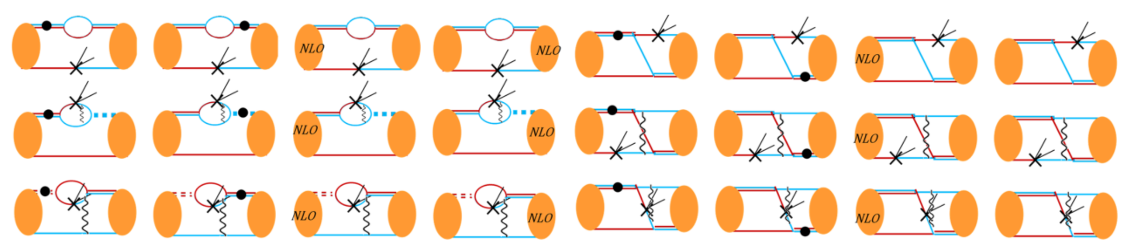
τ⁻ transition

- Gray – empirical results:

$$\langle GT\rangle^2 = \frac{K/G_V - \langle F\rangle^2 f_V (FT_{1/2})_t}{f_A g_A^2 (FT_{1/2})_t}$$

- **Black** – one body LO
- **Red** –one body NLO





One body NLO

T.-Y. Saito, et al 1990. H. D, L. Platter and D. Gazit. In preparation

Gamow -Teller transition

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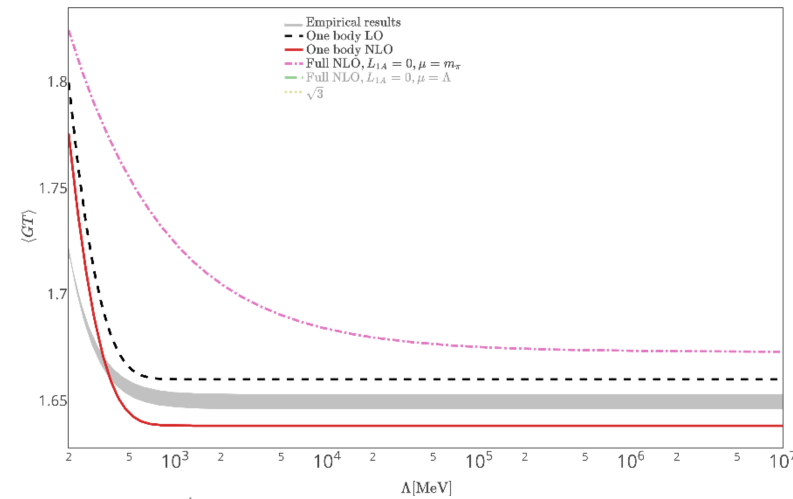
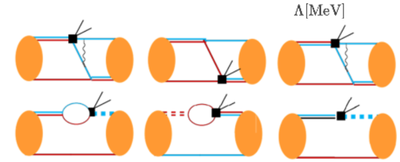
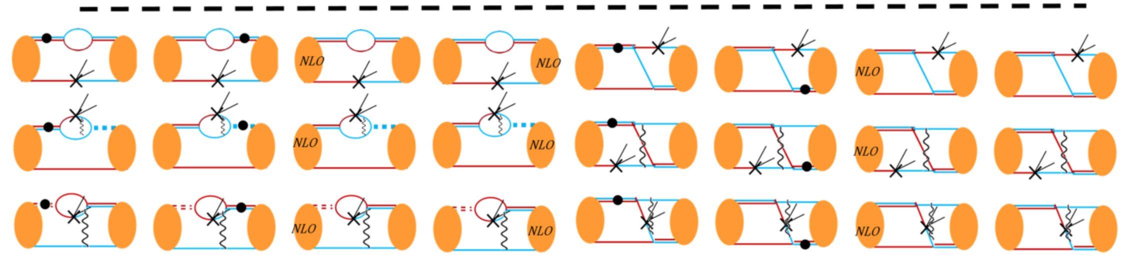
  $\langle GT \rangle^2 = \frac{K/G_V - \langle F \rangle^2 f_V (FT_{1/2})_t}{f_A g_A^2 (FT_{1/2})_t}$
- **Black** – one body LO
- **Red** –one body NLO
- **Pink** – full NLO with $L_{1A} = 0, \mu = m_\pi$





Two body NLO



One body NLO

T.-Y. Saito, et al 1990. H. D, L. Platter and D. Gazit. In preparation

Gamow -Teller transition

- Gray – empirical results:
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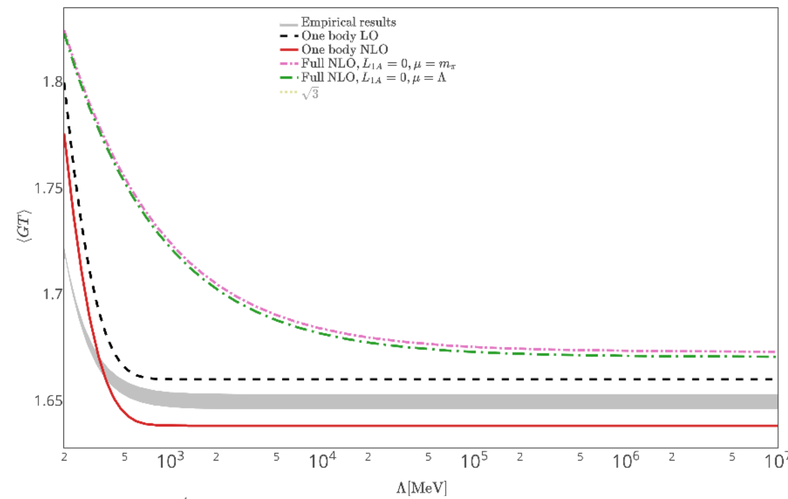
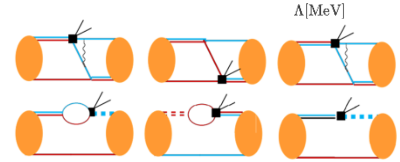
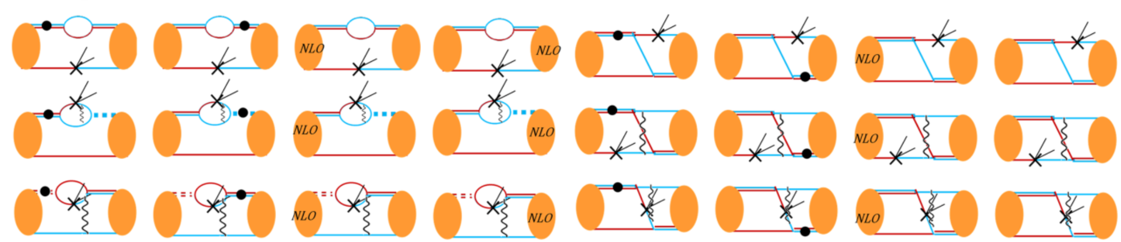
- **Black** – one body LO
- **Red** –one body NLO
- **Pink** – full NLO with $L_{1A} = 0, \mu = m_\pi$
- **Green** - full NLO with $L_{1A} = 0, \mu = \Lambda$





Two body NLO

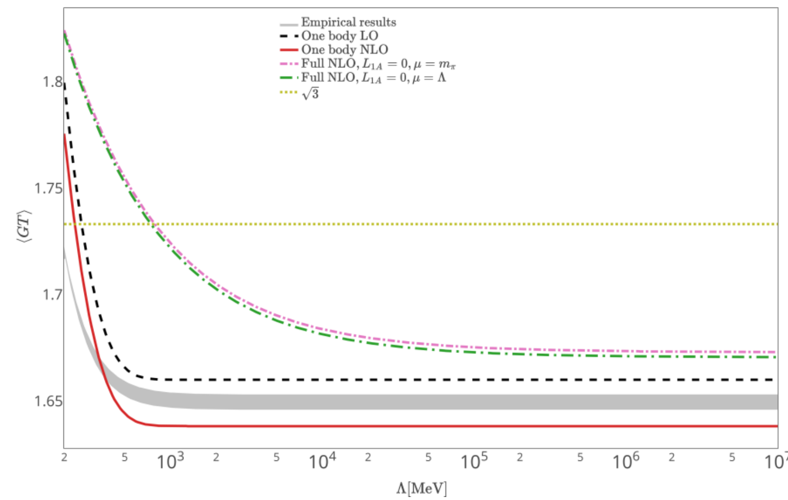


One body NLO

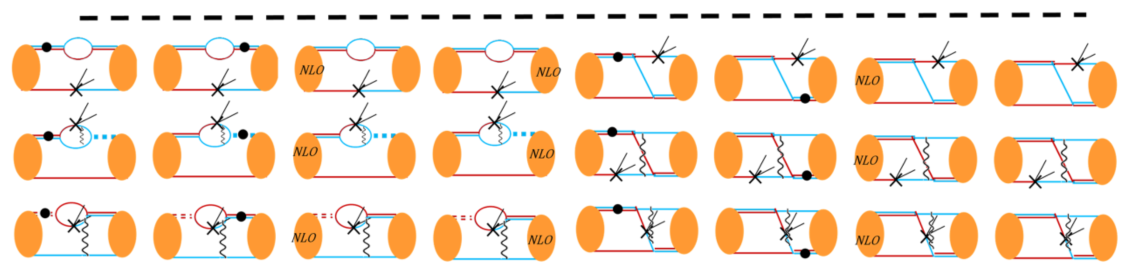
T.-Y. Saito, et al 1990. H. D, L. Platter and D. Gazit. In preparation

Gamow -Teller transition

- Gray – empirical results:
  $$\langle GT\rangle^2 = \frac{K/G_V - \langle F\rangle^2 f_V (FT_{1/2})_t}{f_A g_A^2 (FT_{1/2})_t}$$
- **Black** – one body LO
- **Red** –one body NLO
- **Pink** – full NLO with $L_{1A} = 0, \mu = m_\pi$
- **Green** - full NLO with $L_{1A} = 0, \mu = \Lambda$
- **Yellow** - $\sqrt{3}$



Two body NLO



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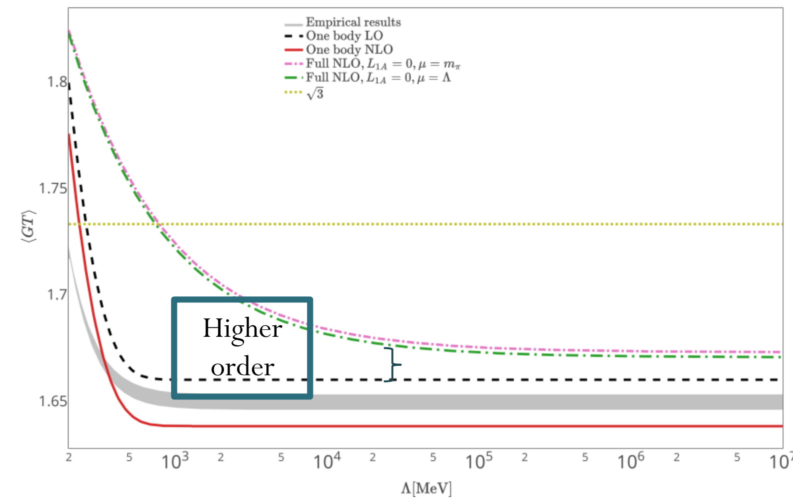
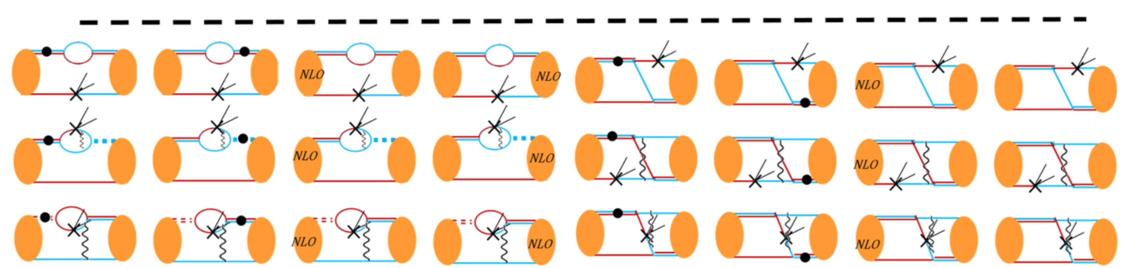
  (LO with $\alpha = 0$)



Two body NLO



One body NLO

T.-Y. Saito, et al 1990. H. D, L. Platter and D. Gazit. In preparation

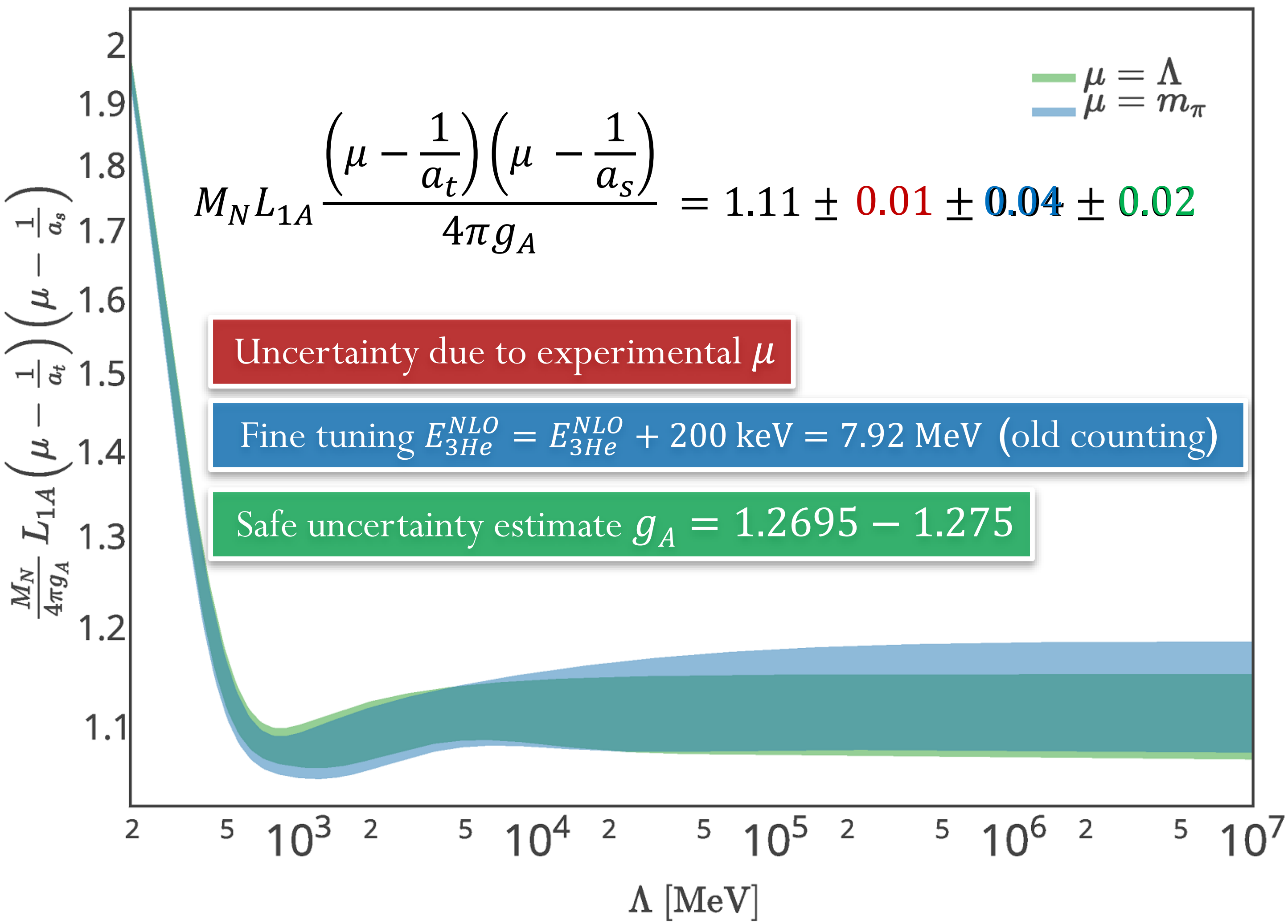
L_{1A} calibration

$$M_N L_{1A} \frac{\left(\mu - \frac{1}{a_t}\right)\left(\mu - \frac{1}{a_s}\right)}{4\pi g_A} = 1.11 \pm 0.01 \pm 0.04 \pm 0.02$$

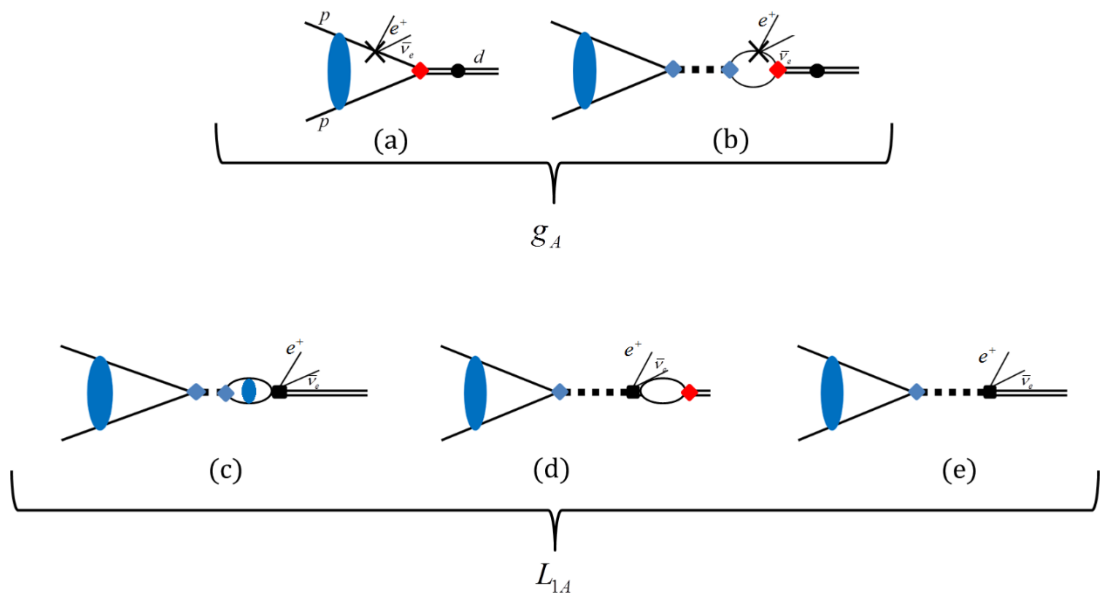
Uncertainty due to experimental μ

Fine tuning $E_{3He}^{NLO} = E_{3He}^{NLO} + 200$ keV $= 7.92$ MeV (old counting)

Safe uncertainty estimate $g_A = 1.2695 - 1.275$



pp fusion in πEFT



$$\Lambda(0)^{NLO} = \sqrt{\frac{\gamma_d^3}{8\pi C_\eta^2}}\,|T_{fi}(0)| = \left(1+\frac{\gamma\rho_t}{2}\right)\left[e^{\chi} - a_p\gamma\chi\; I(\chi)\right] + \frac{a_p\gamma^2}{4}\left[(\rho_t+\rho_C) - \frac{\mathrm{L}_{1A}}{\pi g_A}\left(\mu-\frac{1}{a_t}\right)\left(\mu-\frac{1}{a_{C(\mu)}}\right)\right]$$

$$\Lambda^{\mathrm{NLO}}(0) = 2.650 \pm 0.005$$

$$\Lambda^{NLO}(0)^2 = 7.02 \pm 0.03$$

pp fusion in πEFT

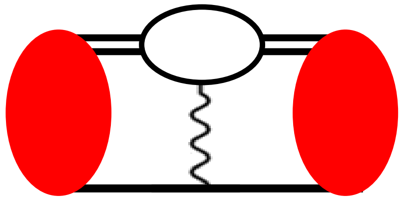
- Previous calculations of pp fusion matrix element using EFT:

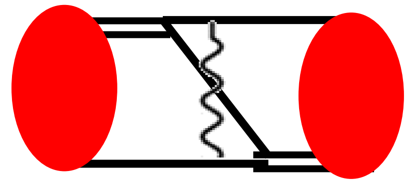
	BM (ERE) (1969)	KR (NLO) (2001)	BC (N^4LO) (2005)	Ando et. al (2008)	Marcucci et al (2013)	Our result:
$\Lambda(0)^2$	7.08	7.04~7.70	6.71-7.03	7.07-7.11	7.04−7.08	6.99−7.05 With Z_t ~ 7.5

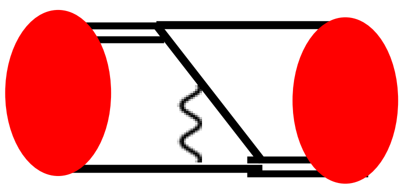
- Bahcall and May have used full ERE.
- NLO by Kong and Ravndal with rough estimate of L_{1A}.
- N^4LO by Butler and Chen with rough estimate of L_{1A}.
- Full ERE with dibaryon field by Ando et al with L_{1A} fixed from χEFT
- χEFT by Marcucci et al with $g_A = 1.2695$
- With g_A uncertainty $6.98 - 7.08$

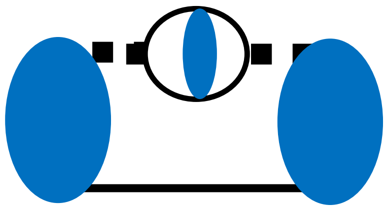
Summary

- A description of 3-body electroweak interaction using πEFT is possible.
- πEFT shows good consistency between A=2 and A=3 for both expansion ($q = 0$, deuteron pole) for bound states.
- In the case of non-bound state the $q = 0$ expansion is better.
- L_{1A} can be fixed from fundamental physics.









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$\Lambda(0)^2$	7.04~7.70	7.04~7.70	6.71-7.03	7.07-7.11	7.03−7.08	6.99−7.05

- The πEFT calculation include L_{1A} which was never calibrated.
- It is essential to find a well measured weak interaction in πEFT regime.

X. Kong and F. Ravndal. 1999,2001. Ando et al.2008. M. Butler and J.W Chen (2001)