^{3}He and *pd* Scattering to Next-to-Leading Order in Pionless Effective Field Theory

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Ingredients of Pionless Effective Field Theory

- For momenta $p < m_\pi$ pions can be integrated out as degrees of freedom and only nucleons and external currents are left.
- For any effective (field) theory one writes down all terms with degrees of freedom that respect symmetries.
- Develop a power counting to organize terms by their relative importance.
- Calculate respective observables up to a given order in the power counting.

Lagrangian

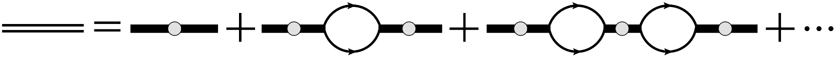
The two-body Lagrangian in $\mathrm{EFT}_{\not{\pi}}$ is

$$\begin{aligned}\mathcal{L}_2 = \hat{N}^\dagger\left(iD_0+\frac{\vec{\mathbf{D}}^2}{2M_N}\right)\hat{N} + \hat{t}_i^\dagger\left(\Delta_t - c_{0t}\left(iD_0+\frac{\vec{\mathbf{D}}^2}{4M_N}+\frac{\gamma_t^2}{M_N}\right)\right)\hat{t}_i \\ + \hat{s}_a^\dagger\left(\Delta_s - c_{0s}\left(iD_0+\frac{\vec{\mathbf{D}}^2}{4M_N}+\frac{\gamma_s^2}{M_N}\right)\right)\hat{s}_a \\ + y_t\left[\hat{t}_i^\dagger\hat{N}^TP_i\hat{N}+\mathrm{H.c.}\right] + y_s\left[\hat{s}_a^\dagger\hat{N}^T\bar{P}_a\hat{N}+\mathrm{H.c.}\right].\end{aligned}$$

The projector $P_i = \frac{1}{\sqrt{8}}\sigma_2\sigma_i\tau_2$ ($\bar{P}_a = \frac{1}{\sqrt{8}}\tau_2\tau_a\sigma_2$) projects out the spin-triplet iso-singlet (spin-singlet iso-triplet) combination of nucleons.

$$D_\mu = \partial_\mu + ieA_\mu\mathbf{Q} \qquad \mathbf{Q} = 1, (\mathbb{1}+\tau_3)/2, \mathbb{1}+I_3$$

LO dressed dibaryon propagators given by infinite sum of diagrams



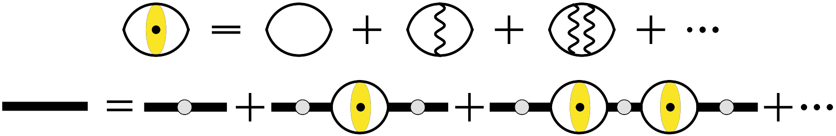
(LO)

(NLO)

$$iD_t(p_0,\vec{\mathbf{p}}) = \frac{4\pi i}{M_N y_t^2}\frac{1}{\gamma_t - \sqrt{\frac{\vec{\mathbf{p}}^2}{4} - M_N p_0 - i\epsilon}}\left[\underbrace{1}_{\text{LO}} \underbrace{- \frac{\rho_t}{2}\left(\sqrt{\frac{\vec{\mathbf{p}}^2}{4} - M_N p_0 - i\epsilon} + \gamma_t\right)}_{\text{NLO}}\right]$$

$$iD_s(p_0,\vec{\mathbf{p}}) = \frac{4\pi i}{M_N y_s^2}\frac{1}{\gamma_s - \sqrt{\frac{\vec{\mathbf{p}}^2}{4} - M_N p_0 - i\epsilon}}\left[\underbrace{1}_{\text{LO}} \underbrace{- \frac{\rho_s}{2}\frac{\frac{\vec{\mathbf{p}}^2}{4} - M_N p_0}{\gamma_s - \sqrt{\frac{\vec{\mathbf{p}}^2}{4} - M_N p_0 - i\epsilon}}}_{\text{NLO}}\right]$$

pp scattering given by (Kong and Ravndal (1999))



$$iD_{pp}(p_0,\vec{\mathbf{p}}) = \frac{4\pi i}{M_N y_s^2}\frac{1}{\frac{1}{a_C}+2\kappa H(\kappa/p')}\left[\underbrace{1}_{\text{LO}}\underbrace{-\frac{r_C}{2}\frac{\frac{\vec{\mathbf{p}}^2}{4}-M_N p_0}{\frac{1}{a_C}+2\kappa H(\kappa/p')}}_{\text{NLO}}\right],$$

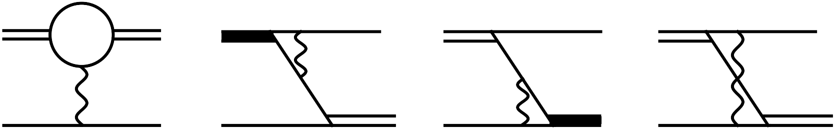
where

$$p' = i\sqrt{\frac{\vec{\mathbf{p}}^2}{4}-M_N p_0 - i\epsilon},\quad \kappa = \frac{\alpha M_N}{2},$$

and

$$H(\eta) = \psi(i\eta)+\frac{1}{2i\eta}-\log(i\eta).$$

The function ψ is the logarithmic derivative of the Γ function.



(a) (b) (c) (d)

In Rupak and Kong (Rupak and Kong (2003)) counting diagram (a) scales like

$$\alpha\Lambda_{\not{\pi}}/p^2 Q$$

and diagram (b) through (d) scale like

$$\alpha\Lambda_{\not{\pi}}/Q^3$$

$$Q \sim \gamma_t \qquad p = \text{External momentum} \qquad \Lambda_{\not{\pi}} \sim m_\pi$$

Diagram (d) projected onto the S-wave gives

$$C(q,p,E) = -2\alpha M_N \times$$
$$\left(\frac{2}{qp}F_1\left[\sqrt{2M_NE-3q^2-3p^2+i\epsilon},2\sqrt{\frac{3}{4}q^2-M_NE-i\epsilon}+2\sqrt{\frac{3}{4}p^2-M_NE-i\epsilon},\vec{\mathbf{q}}-\vec{\mathbf{p}}\right]\right.$$
$$\left.+\lambda\frac{1}{(p^2+q^2-M_NE-i\epsilon)^2-p^2q^2}+\mathcal{O}(\lambda^2)+\cdots\right).$$

- λ - photon mass (*screening method*)
- q - incoming momentum in *nd* c.m. frame
- p - outgoing momentum in *nd* c.m. frame

$$F_1[a,b,\vec{\mathbf{c}}+\vec{\mathbf{d}}]=-\frac{1}{4a}\left\{\log(z^2+a^2)\log\left(\frac{b-a}{b+a}\right)-\mathrm{Li}_2\left(-i\frac{z-ia}{b-a}\right)\right.$$
$$\left.+\mathrm{Li}_2\left(i\frac{z-ia}{a+b}\right)-\mathrm{Li}_2\left(i\frac{z+ia}{b-a}\right)+\mathrm{Li}_2\left(-i\frac{z+ia}{a+b}\right)\right\}\Bigg|_{|c-d|}^{c+d},$$

and for $\mathrm{Re}(a)>\mathrm{Re}(b)$ as

$$F_1[a,b,\vec{\mathbf{c}}+\vec{\mathbf{d}}]=\frac{1}{a}\tan^{-1}\left(\frac{z}{b}\right)\tan^{-1}\left(\frac{z}{a}\right)$$
$$+\frac{1}{4a}\left\{\log(z^2+b^2)\log\left(\frac{a-b}{b+a}\right)-\mathrm{Li}_2\left(-i\frac{z-ib}{a-b}\right)\right.$$
$$\left.+\mathrm{Li}_2\left(i\frac{z-ib}{a+b}\right)-\mathrm{Li}_2\left(i\frac{z+ib}{a-b}\right)+\mathrm{Li}_2\left(-i\frac{z+ib}{a+b}\right)\right\}\Bigg|_{|c-d|}^{c+d},$$

where the bar notation is defined as

$$f(z)\Bigg|_{|c-d|}^{c+d}=f(c+d)-f(|c-d|).$$

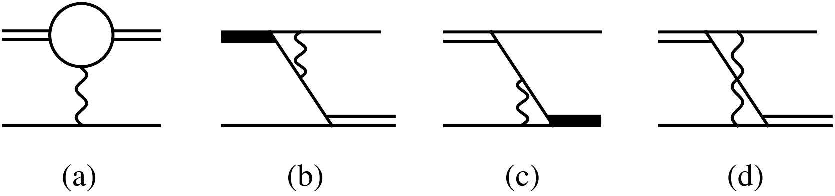


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Diagram (a) is given by

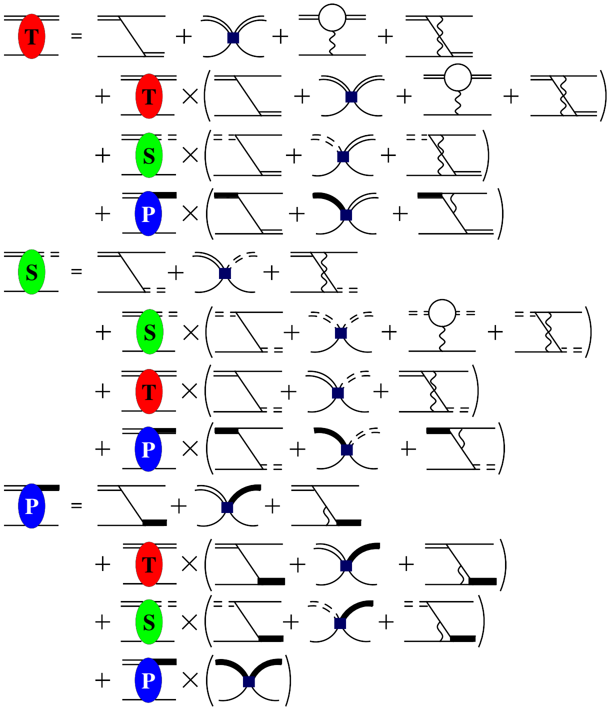
$$B(q,p,E)=\frac{4\alpha M_N}{qp}F_1\left[\lambda,2\sqrt{\frac{3}{4}q^2-M_NE-i\epsilon}+2\sqrt{\frac{3}{4}p^2-M_NE-i\epsilon},\vec{\mathbf{q}}-\vec{\mathbf{p}}\right],$$

and diagram (b) by

$$V_1(q,p,E)=\frac{4\alpha M_N}{qp}F_1\left[\sqrt{3q^2-4M_NE-i\epsilon},2\sqrt{\frac{3}{4}q^2-M_NE-i\epsilon}+2\lambda,\vec{\mathbf{q}}+2\vec{\mathbf{p}}\right].$$

Diagram (b) and (c) are related by time reversal symmetry yielding

$$V_2(q,p,E)=V_1(p,q,E).$$


T
S
P

In cluster-configuration (Grießhammer (2004)) space the LO *pd* scattering amplitude is given by

$$\mathbf{t}_0(k,p,E) = \mathbf{B}_0(k,p,E) + \mathbf{K}_0(q,p,E) \otimes \mathbf{t}_0(k,q,E).$$

where $\mathbf{t}_0(k,p,E)$ is a cluster-configuration space vector defined by

$$\mathbf{t}_0(k,p,E) = \begin{pmatrix} t_{0,Nt\to Nt}(k,p,E) \\ t_{0,Nt\to Ns}(k,p,E) \\ t_{0,Nt\to Npp}(k,p,E) \end{pmatrix},$$

and $\otimes$ is given by

$$A(q) \otimes B(q) = \frac{2}{\pi}\int_0^{\Lambda} dq q^2 A(q) B(q).$$

The kernel and inhomogeneous term is decomposed into three pieces:

$$\mathbf{B}_0(k,p,E) = \mathbf{B}_0^{(S)}(k,p,E) + \mathbf{B}_0^{(SC)}(k,p,E) + \mathbf{B}_0^{(C)}(k,p,E),$$

and

$$\mathbf{K}_0(q,p,E) = \mathbf{K}_0^{(S)}(q,p,E) + \mathbf{K}_0^{(SC)}(q,p,E) + \mathbf{K}_0^{(C)}(q,p,E).$$

- S - Strong contributions
- SC - Strong and Coulomb mixing contributions
- C - Coulomb interaction contributions (interaction only from Coulomb)

Strong contributions given by nucleon exchange and LO three-body force yielding

$$\mathbf{B}_0^{(S)}(k,p,E)=\begin{pmatrix} 2y_t^2\left[\frac{1}{pk}Q_0\left(\frac{p^2+k^2-M_NE-i\epsilon}{pk}\right)+\frac{2H_{0,0}(\Lambda)}{\Lambda^2}\right] \\ 2y_ty_s\left[\frac{1}{pk}Q_0\left(\frac{p^2+k^2-M_NE-i\epsilon}{pk}\right)+\frac{2H_{0,0}(\Lambda)}{3\Lambda^2}\right] \\ 2y_ty_s\left[\frac{2}{pk}Q_0\left(\frac{p^2+k^2-M_NE-i\epsilon}{pk}\right)+\frac{4H_{0,0}(\Lambda)}{3\Lambda^2}\right] \end{pmatrix},$$

$$\mathbf{K}_0^{(S)}(q,p,E)=\frac{M_N}{8\pi}\frac{1}{qp}Q_0\left(\frac{q^2+p^2-M_NE-i\epsilon}{qp}\right)$$
$$\times\begin{pmatrix} -y_t^2 & -3y_ty_s & -3y_ty_s \\ -y_sy_t & y_s^2 & -y_s^2 \\ -2y_sy_t & -2y_s^2 & 0 \end{pmatrix}\mathbf{D}^{(0)}\left(E-\frac{\vec{\mathbf{q}}^2}{2M_N},\vec{\mathbf{q}}\right)$$
$$+\frac{M_N}{8\pi}\frac{2H_{0,0}(\Lambda)}{\Lambda^2}\begin{pmatrix} -y_t^2 & -y_ty_s & -y_ty_s \\ -\frac{1}{3}y_sy_t & -\frac{1}{3}y_s^2 & -\frac{1}{3}y_s^2 \\ -\frac{2}{3}y_sy_t & -\frac{2}{3}y_s^2 & -\frac{2}{3}y_s^2 \end{pmatrix}\mathbf{D}^{(0)}\left(E-\frac{\vec{\mathbf{q}}^2}{2M_N},\vec{\mathbf{q}}\right).$$

The function $Q_0(a)$ is a Legendre function of the second kind given by

$$Q_0(a) = \frac{1}{2}\ln\left(\frac{a+1}{a-1}\right),$$

and $\mathbf{D}^0(E, \vec{\mathbf{q}})$ is a cluster-configuration space matrix of LO dibaryon propagators given by

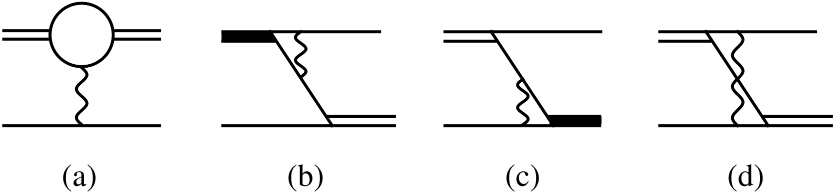
$$\mathbf{D}^0(E, \vec{\mathbf{q}}) = \begin{pmatrix} D_t^{(0)}(E, \vec{\mathbf{q}}) & 0 & 0 \\ 0 & D_s^{(0)}(E, \vec{\mathbf{q}}) & 0 \\ 0 & 0 & D_{pp}^{(0)}(E, \vec{\mathbf{q}}) \end{pmatrix}.$$

The Strong-Coulomb contributions are given by

$$\mathbf{B}_0^{(SC)}(k,p,E) = \begin{pmatrix} -y_t^2 C(k,p,E) \\ -y_t y_s C(k,p,E) \\ -2y_t y_s V_2(k,p,E) \end{pmatrix},$$

and

$$\mathbf{K}_0^{(SC)}(q,p,E) = \frac{M_N}{16\pi} \times \begin{pmatrix} y_t^2 C(q,p,E) & 3y_t y_s C(q,p,E) & 3y_t y_s V_1(q,p,E) \\ y_s y_t C(q,p,E) & -y_s^2 C(q,p,E) & y_s^2 V_1(q,p,E) \\ 2y_s y_t V_2(q,p,E) & 2y_s^2 V_2(q,p,E) & 0 \end{pmatrix} \times \mathbf{D}^{(0)}\left(E - \frac{\vec{\mathbf{q}}^2}{2M_N}, \vec{\mathbf{q}}\right).$$



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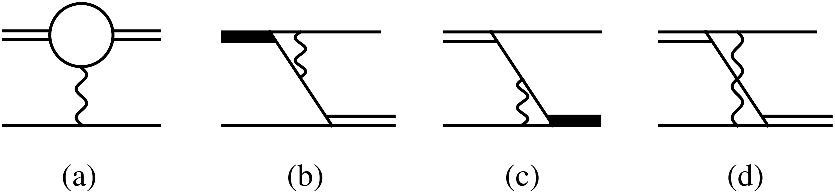
Finally the Coulomb only contributions are given by

$$\mathbf{B}_0^{(C)}(k,p,E) = \begin{pmatrix} -y_t^2 B(k,p,E) \\ 0 \\ 0 \end{pmatrix},$$

and

$$\mathbf{K}_0^{(C)}(q,p,E) = \frac{M_N}{16\pi} \begin{pmatrix} y_t^2 B(q,p,E) & 0 & 0 \\ 0 & y_s^2 B(q,p,E) & 0 \\ 0 & 0 & 0 \end{pmatrix} \times \mathbf{D}^{(0)}\left(E - \frac{\vec{\mathbf{q}}^2}{2M_N}, \vec{\mathbf{q}}\right).$$



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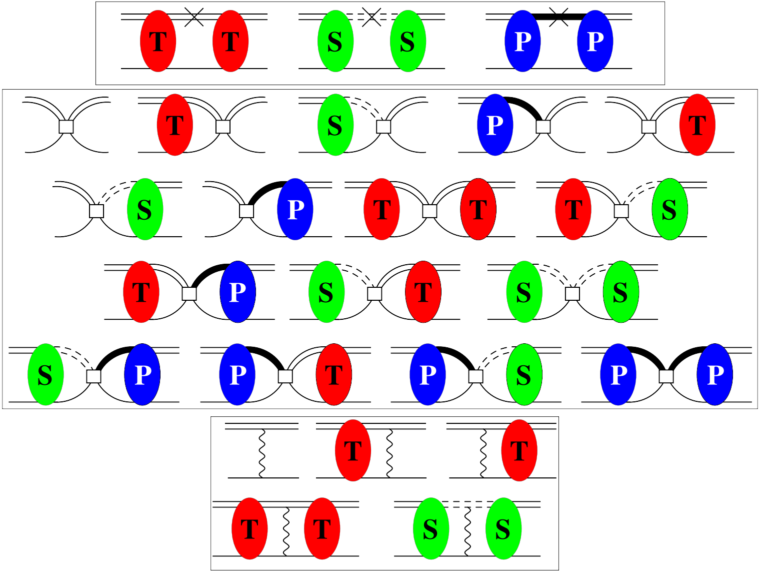
NLO pd Scattering



NLO *pd* scattering amplitude has three contributions

$$t_{1,Nt\to Nt}(k,p,E)=t_1^{(ER)}(k,p,E)+t_1^{(3B)}(k,p,E)+t_1^{(DK)}(k,p,E),$$

effective range corrections given by

$$t_1^{(ER)}(k,p,E)=\frac{\rho_t}{4\pi}\int_0^\Lambda dqq^2\left(t_{0,Nt\to Nt}(k,q)\right)^2\frac{\sqrt{\frac{3}{4}q^2-M_NE-i\epsilon}+\gamma_t}{\sqrt{\frac{3}{4}q^2-M_NE-i\epsilon}-\gamma_t}$$

$$+\frac{3\rho_s}{4\pi}\int_0^\Lambda dqq^2\left(t_{0,Nt\to Ns}(k,q)\right)^2\frac{\frac{3}{4}q^2-M_NE}{\left(\sqrt{\frac{3}{4}q^2-M_NE-i\epsilon}-\gamma_s\right)^2}$$

$$+\frac{3r_C}{8\pi}\int_0^\Lambda dqq^2\left(t_{0,Nt\to pp}(k,q)\right)^2\frac{\frac{3}{4}q^2-M_NE}{\left(2\kappa H\left(\frac{\kappa}{\sqrt{\frac{3}{4}q^2-M_NE-i\epsilon}}\right)+\frac{1}{a_C}\right)^2}.$$

Contribution from three-body forces factorizes to give

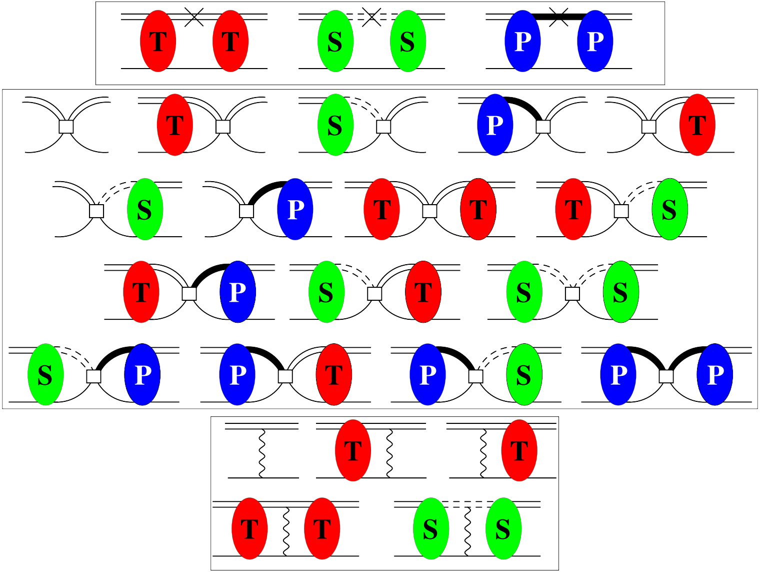
$$t_1^{(3B)}(k,p,E) = \frac{4(H_1(\Lambda)+H_1^{(\alpha)}(\Lambda))}{\Lambda^2}$$

$$\times\left[1+\frac{1}{2\pi}\int_0^\Lambda dqq^2\, t_{0,Nt\to Nt}(k,q)\frac{1}{\sqrt{\frac{3}{4}q^2-M_NE-i\epsilon}-\gamma_t}\right.$$

$$+\frac{1}{2\pi}\int_0^\Lambda dqq^2\, t_{0,Nt\to Ns}(k,q)\frac{1}{\sqrt{\frac{3}{4}q^2-M_NE-i\epsilon}-\gamma_s}$$

$$\left.+\frac{1}{2\pi}\int_0^\Lambda dqq^2\, t_{0,Nt\to Npp}(k,q)\frac{1}{-\frac{1}{a_C}-2\kappa H\left(\frac{\kappa}{\sqrt{\frac{3}{4}q^2-M_NE-i\epsilon}}\right)}\right]^2.$$

NLO pd Scattering



Contributions from gauging dibaryon kinetic term

$$t_1^{(DK)}(k,p,E) = -\frac{\alpha M_N \rho_t}{k^2} Q_0\left(\frac{2k^2+\lambda^2}{-2k^2}\right)$$

$$-\frac{\alpha M_N \rho_t}{\pi}\int_0^{\Lambda} dq q^2 t_{0,Nt\to Nt}(k,q)\frac{1}{\sqrt{\frac{3}{4}q^2 - M_N E - i\epsilon} - \gamma_t}\frac{1}{qk}Q_0\left(\frac{k^2+q^2+\lambda^2}{-2qk}\right)$$

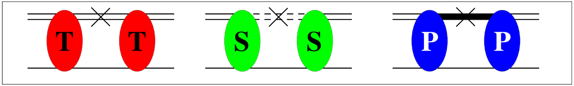
$$-\frac{\rho_t \alpha M_N}{4\pi^2}\int_0^{\Lambda} dq q^2 \int_0^{\Lambda} d\ell \ell^2 t_{0,Nt\to Nt}(k,q) t_{0,Nt\to Nt}(k,\ell)$$

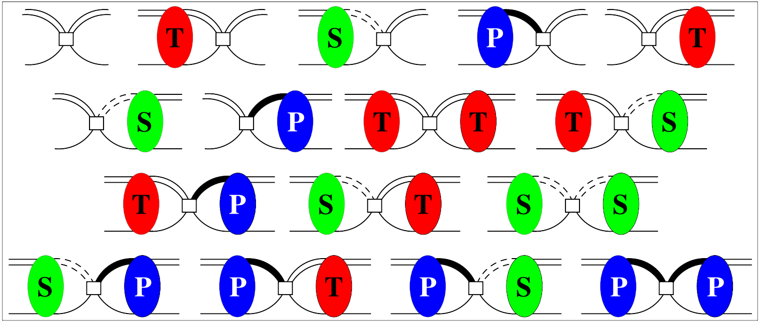
$$\times \frac{1}{\sqrt{\frac{3}{4}q^2 - M_N E - i\epsilon} - \gamma_t}\frac{1}{\sqrt{\frac{3}{4}\ell^2 - M_N E - i\epsilon} - \gamma_t}\frac{1}{q\ell}Q_0\left(\frac{-q^2-\ell^2-\lambda^2}{2q\ell}\right)$$

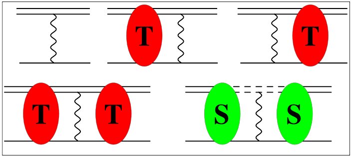
$$-\frac{3\rho_s \alpha M_N}{4\pi^2}\int_0^{\Lambda} dq q^2 \int_0^{\Lambda} d\ell \ell^2 t_{0,Nt\to Ns}(k,q) t_{0,Nt\to Ns}(k,\ell)$$

$$\times \frac{1}{\sqrt{\frac{3}{4}q^2 - M_N E - i\epsilon} - \gamma_s}\frac{1}{\sqrt{\frac{3}{4}\ell^2 - M_N E - i\epsilon} - \gamma_s}\frac{1}{q\ell}Q_0\left(\frac{-q^2-\ell^2-\lambda^2}{2q\ell}\right).$$

NLO pd Scattering







The LO binding energy B_0 fulfills the condition

$$\det(1 - \mathbf{K}_0(q, p, B_0)) = 0.$$

Scattering amplitude near bound state pole can be written as

$$\mathbf{t}_0(k, p, E) + \mathbf{t}_1(k, p, E) = \frac{\mathbf{Z}_0(k, p) + \mathbf{Z}_1(k, p)}{E + B_0 + B_1} + \mathbf{R}_0(k, p, E) + \mathbf{R}_1(k, p, E),$$

LO residue function is

$$\mathbf{Z}_0(k, p) = \lim_{E \to -B_0} (E + B_0)\mathbf{t}_0(k, p, E).$$

NLO correction to binding energy is

$$B_1 = -\lim_{E \to -B_0} \frac{(E + B_0)^2 [\mathbf{t}_1]_n(k, p, E)}{[\mathbf{Z}_0]_n(k, p)}.$$

LO residue function can be factorized as

$$\mathbf{Z}_0(k,p) = \begin{pmatrix} \Gamma_{Nt}(k)\Gamma_{Nt}(p) \\ \Gamma_{Nt}(k)\Gamma_{Ns}(p) \\ \Gamma_{Nt}(k)\Gamma_{Npp}(p) \end{pmatrix},$$

where $\mathbf{\Gamma}_0(p)$ is solution to homogeneous equation

$$\mathbf{\Gamma}_0(p) = \mathbf{K}_0(q,p,B_0)\otimes\mathbf{\Gamma}_0(q), \qquad \mathbf{\Gamma}_0(p) = \begin{pmatrix} \Gamma_{Nt}(p) \\ \Gamma_{Ns}(p) \\ \Gamma_{Npp}(p) \end{pmatrix}.$$

Can be used to derive an expression for B_1 that has no momentum dependence.

$$t_1(k,p,E)=\int_0^{\Lambda}dqq^2t_0(k,q)t_0(p,q)f(q)$$

$$t_0(k,q)=\frac{\Gamma(k)\Gamma(q)}{E+B_0}$$

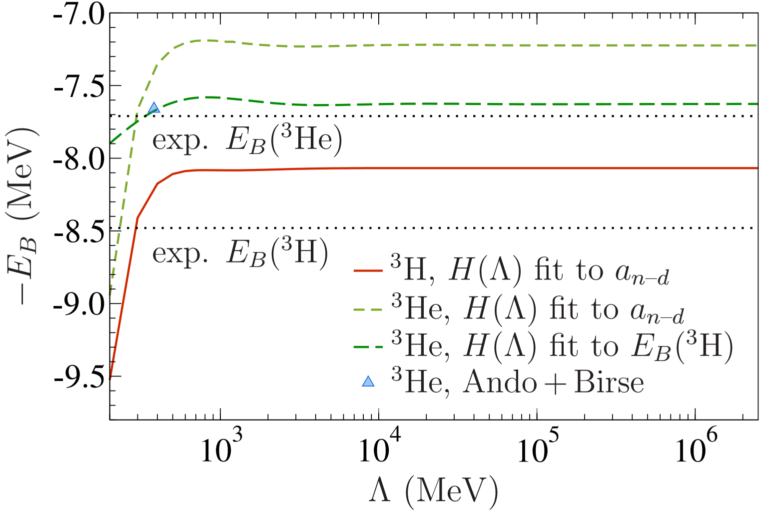
$$t_1(k,p,E)=\int_0^{\Lambda}dqq^2\frac{\Gamma(k)\Gamma(p)(\Gamma(q))^2}{(E+B_0)^2}f(q)$$

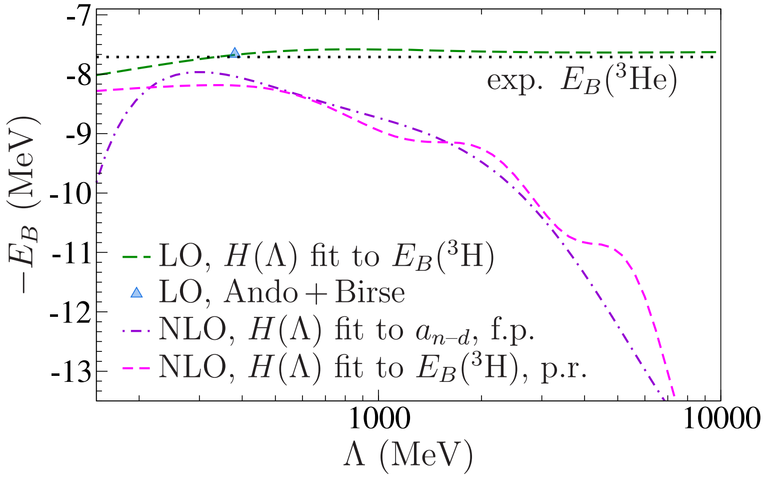
$$B_1=-\lim_{E\to-B_0}\frac{(E+B_0)^2t_1(k,p,E)}{Z_0(k,p)},$$

$$B_1=-\lim_{E\to-B_0}(E+B_0)^2\frac{1}{\Gamma(k)\Gamma(p)}\int_0^{\Lambda}dqq^2\frac{\Gamma(k)\Gamma(p)(\Gamma(q))^2}{(E+B_0)^2}f(q)$$

$$B_1=\int_0^{\Lambda}dqq^2(\Gamma(q))^2f(q)$$

$$
\begin{aligned}
B_1 =& \frac{\rho_t}{4\pi} \int_0^\Lambda dq q^2 \left(\Gamma_{Nt}(q)\right)^2 \frac{\sqrt{\frac{3}{4}q^2 - M_N E} + \gamma_t}{\sqrt{\frac{3}{4}q^2 - M_N E} - \gamma_t} \\
&+ \frac{3\rho_s}{4\pi} \int_0^\Lambda dq q^2 \left(\Gamma_{Ns}(q)\right)^2 \frac{\frac{3}{4}q^2 - M_N E}{\left(\sqrt{\frac{3}{4}q^2 - M_N E} - \gamma_s\right)^2} \\
&+ \frac{3r_C}{8\pi} \int_0^\Lambda dq q^2 \left(\Gamma_{Npp}(q)\right)^2 \frac{\frac{3}{4}q^2 - M_N E}{\left(2\kappa H\left(\frac{\kappa}{\sqrt{\frac{3}{4}q^2 - M_N E}}\right) + \frac{1}{a_C}\right)^2} \\
&+ \frac{(H_1(\Lambda) + H_1^{(\alpha)}(\Lambda))}{\pi^2\Lambda^2} \left[\int_0^\Lambda dq q^2 \Gamma_{Nt}(q) \frac{1}{\sqrt{\frac{3}{4}q^2 - M_N E} - \gamma_t}\right. \\
&\left. + \int_0^\Lambda dq q^2 \Gamma_{Ns}(q) \frac{1}{\sqrt{\frac{3}{4}q^2 - M_N E} - \gamma_s} + \int_0^\Lambda dq q^2 \Gamma_{Npp}(q) \frac{1}{-\frac{1}{a_C} - 2\kappa H\left(\frac{\kappa}{\sqrt{\frac{3}{4}q^2 - M_N E}}\right)}\right]^2 \\
&- \frac{\rho_t \alpha M_N}{4\pi^2} \int_0^\Lambda dq q^2 \int_0^\Lambda d\ell \ell^2 \Gamma_{Nt}(q)\Gamma_{Nt}(\ell) \frac{1}{\sqrt{\frac{3}{4}q^2 - M_N E} - \gamma_t} \\
&\qquad \times \frac{1}{\sqrt{\frac{3}{4}\ell^2 - M_N E} - \gamma_t} \frac{1}{q\ell} Q_0\left(\frac{-q^2 - \ell^2 - \lambda^2}{2q\ell}\right) \\
&- \frac{3\rho_s \alpha M_N}{4\pi^2} \int_0^\Lambda dq q^2 \int_0^\Lambda d\ell \ell^2 \Gamma_{Ns}(q)\Gamma_{Ns}(\ell) \frac{1}{\sqrt{\frac{3}{4}q^2 - M_N E} - \gamma_s} \\
&\qquad \times \frac{1}{\sqrt{\frac{3}{4}\ell^2 - M_N E} - \gamma_s} \frac{1}{q\ell} Q_0\left(\frac{-q^2 - \ell^2 - \lambda^2}{2q\ell}\right).
\end{aligned}
$$


-7.0
-7.5
-8.0
-8.5
-9.0
-9.5
$-E_B$ (MeV)
exp. $E_B(^3\text{He})$
exp. $E_B(^3\text{H})$
^{3}H, $H(\Lambda)$ fit to a_{n-d}
^{3}He, $H(\Lambda)$ fit to a_{n-d}
^{3}He, $H(\Lambda)$ fit to $E_B(^3\text{H})$
^{3}He, Ando + Birse
10^3
10^4
10^5
10^6
Λ (MeV)


-7
-8
-9
-10
-11
-12
-13
$-E_B$ (MeV)
1000
10000
Λ (MeV)
exp. $E_B(^3\text{He})$
LO, $H(\Lambda)$ fit to $E_B(^3\text{H})$
LO, Ando + Birse
NLO, $H(\Lambda)$ fit to a_{n-d}, f.p.
NLO, $H(\Lambda)$ fit to $E_B(^3\text{H})$, p.r.

To look at asymptotic behavior we define the basis

$$t_{+}(k,p) = t_{0,Nt\to Nt}(k,p) + t_{0,Nt\to Ns}(k,p) + t_{0,Nt\to Npp}(k,p)$$

$$t_{-}(k,p) = t_{0,Nt\to Nt}(k,p) - t_{0,Nt\to Ns}(k,p) - t_{0,Nt\to Npp}(k,p)$$

$$t_{\emptyset}(k,p) = t_{0,Nt\to Ns}(k,p) - \frac{1}{2}t_{0,Nt\to Npp}(k,p)$$

and the corresponding dibaryon propagators

$$D_{+}(E,\vec{\mathbf{q}}) = \left(y_t^2 D_t^{(0)}(E,\vec{\mathbf{q}}) + \frac{1}{3}y_s^2 D_s^{(0)}(E,\vec{\mathbf{q}}) + \frac{2}{3}y_s^2 D_{pp}^{(0)}(E,\vec{\mathbf{q}})\right)$$

$$D_{-}(E,\vec{\mathbf{q}}) = \left(y_t^2 D_t^{(0)}(E,\vec{\mathbf{q}}) - \frac{1}{3}y_s^2 D_s^{(0)}(E,\vec{\mathbf{q}}) - \frac{2}{3}y_s^2 D_{pp}^{(0)}(E,\vec{\mathbf{q}})\right)$$

$$D_{\emptyset}(E,\vec{\mathbf{q}}) = y_s^2\left(D_s^{(0)}(E,\vec{\mathbf{q}}) - D_{pp}^{(0)}(E,\vec{\mathbf{q}})\right)$$

In the limit $q \sim p \gg k, E, \gamma_t, \gamma_s, \gamma_C$ we find $t_+(p)$ is given by

$$\begin{aligned}
t_+(p) =& \frac{4}{\sqrt{3}\pi}\frac{1}{p}\int_0^\infty dq \ln\left(\frac{q^2+pq+p^2}{q^2-pq+p^2}\right)t_+(q) \\
&+ \frac{4}{3\pi}\left(\gamma_t + \frac{1}{3}\gamma_s + \frac{2}{3}\gamma_C\right)\frac{1}{p}\int_0^\infty dq \ln\left(\frac{q^2+pq+p^2}{q^2-pq+p^2}\right)\frac{1}{q}t_+(q) \\
&+ \frac{16\kappa}{9\pi}\int_0^\infty dq \ln\left(\frac{q^2+pq+p^2}{q^2-pq+p^2}\right)\frac{\ln(q)}{q}t_+(q) \\
-& \frac{32\kappa}{3\sqrt{3}\pi}\int_0^\infty dqq(\tilde{C}(q,p,0) + \tilde{V}_1(q,p,0) + \tilde{V}_2(q,p,0) + \frac{1}{2}\tilde{B}(q,p,0))t_+(q) + \cdots ,
\end{aligned}$$

where

$$\tilde{B}(q,p,0) = \frac{1}{8\kappa}B(q,p,0).$$

We use the ansatz $t_+(p) = Cp^{s-1} + A_+p^{s-2} + B_+\ln(p)p^{s-2}$

The integrals necessary for asymptotic amplitude are

$$I(s)=\frac{4}{\sqrt{3}\pi}\int_0^\infty dx\ln\left(\frac{x^2+x+1}{x^2-x+1}\right)x^{s-1}=\frac{8}{\sqrt{3}s}\frac{\sin(\frac{\pi s}{6})}{\cos(\frac{\pi s}{2})}.$$

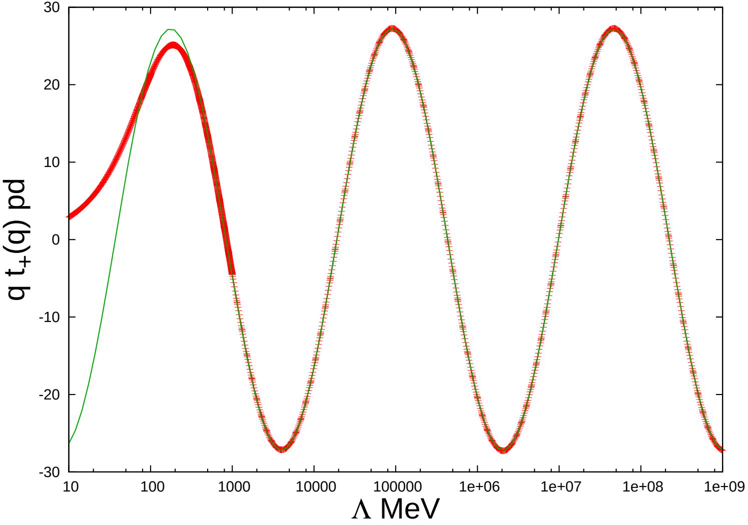
$$\int_0^\infty dx\ln(x)\ln\left(\frac{x^2+x+1}{x^2-x+1}\right)x^{s-1}=\frac{\partial}{\partial\alpha}\int_0^\infty dxx^\alpha\ln\left(\frac{x^2+x+1}{x^2-x+1}\right)x^{s-1}\Big|_{\alpha=0}$$
$$=\frac{\partial}{\partial\alpha}I(s+\alpha)\Big|_{\alpha=0}=I'(s).$$

$$\int_0^\infty dqq^2q^s\tilde{B}(q,p,0)=\sum_{n=1}^\infty\mathcal{J}_B(s-n)p^{s-1-n},$$

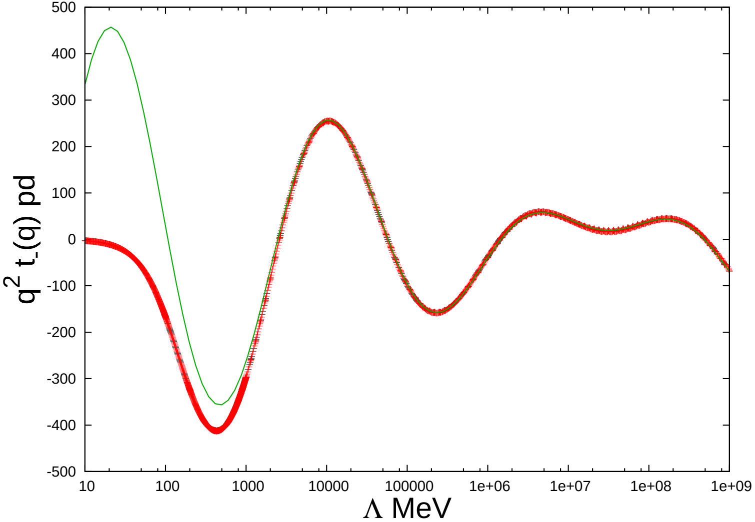
The value for s in the ansatz is

$$I(s)=1,\qquad s=\pm is_0,\qquad s_0\simeq 1.00624$$

$$t_+(q) = C\left\{\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)\right)}{q}\right.$$
$$+\frac{1}{\sqrt{3}}\left(\gamma_t+\frac{1}{3}\gamma_s+\frac{2}{3}\gamma_C\right)|B_{-1}|\sin\frac{\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(B_{-1})\right)}{q^2}$$
$$+\frac{4\kappa}{3\sqrt{3}}|C_{-1}|\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(C_{-1})\right)}{q^2}$$
$$+\frac{4\kappa}{3\sqrt{3}}|D_{-1}|\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(D_{-1})\right)}{q^2}$$
$$+\frac{4\kappa}{3\sqrt{3}}|B_{-1}|\ln(q)\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(B_{-1})\right)}{q^2}$$
$$\left.-\frac{16\kappa}{\sqrt{3}\pi}|E_{-1}|\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(E_{-1})\right)}{q^2}\cdots\right\},$$


30
20
10
0
-10
-20
-30
q t+(q) pd
10
100
1000
10000
100000
1e+06
1e+07
1e+08
1e+09
Λ MeV

$$t_-(q) = C\left\{-\frac{1}{2\sqrt{3}}\left(\gamma_t - \frac{1}{3}\gamma_s - \frac{2}{3}\gamma_C\right)|\tilde{B}_{-1}|\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(\tilde{B}_{-1})\right)}{q^2}\right.$$
$$+\frac{2\kappa}{3\sqrt{3}}|\tilde{C}_{-1}|\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(\tilde{C}_{-1})\right)}{q^2}$$
$$-\frac{\kappa}{3\sqrt{3}}|\tilde{D}_{-1}|\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(\tilde{D}_{-1})\right)}{q^2}$$
$$+\frac{2\kappa}{3\sqrt{3}}|\tilde{B}_{-1}|\ln(q)\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(\tilde{B}_{-1})\right)}{q^2}$$
$$\left.-\frac{16\kappa}{\sqrt{3}\pi}|\tilde{E}_{-1}|\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(\tilde{E}_{-1})\right)}{q^2}\cdots\right\},$$


500
400
300
200
100
0
-100
-200
-300
-400
-500
q^2 t_(q) pd
10
100
1000
10000
100000
1e+06
1e+07
1e+08
1e+09
Λ MeV

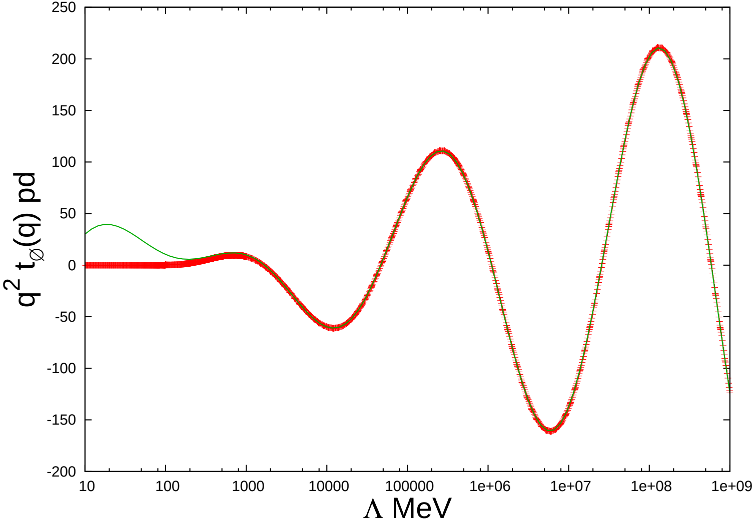
$$t_{\emptyset}(q) = C\left\{-\frac{1}{6\sqrt{3}}\left(\gamma_s-\gamma_C\right)|\tilde{B}_{-1}|\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(\tilde{B}_{-1})\right)}{q^2}\right.$$

$$+\frac{\kappa}{3\sqrt{3}}|\tilde{C}_{-1}|\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(\tilde{C}_{-1})\right)}{q^2}$$

$$-\frac{\kappa}{6\sqrt{3}}|\tilde{D}_{-1}|\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(\tilde{D}_{-1})\right)}{q^2}$$

$$+\frac{\kappa}{3\sqrt{3}}|\tilde{B}_{-1}|\ln(q)\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(\tilde{B}_{-1})\right)}{q^2}$$

$$\left.-\frac{8\kappa}{\sqrt{3}\pi}|\tilde{E}_{-1}|\frac{\sin\left(s_0\ln\left(\frac{q}{\Lambda^*}\right)+\mathrm{Arg}(\tilde{E}_{-1})\right)}{q^2}\cdots\right\}.$$


250
200
150
100
50
0
-50
-100
-150
-200
10
100
1000
10000
100000
1e+06
1e+07
1e+08
1e+09
Λ MeV

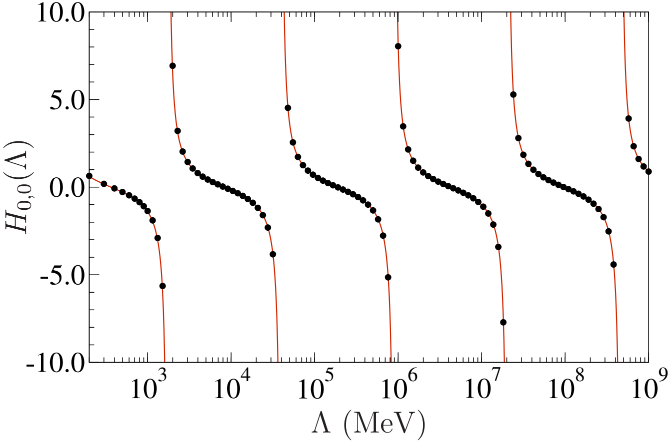
LO Three-body Force

LO three-body force is given by (Bedaque, Hammer, and Kolck (1999))

$$H_{0,0}(\Lambda) = c\frac{\sin\left(s_0 \ln\left(\frac{\Lambda}{\Lambda^*}\right) + \arctan s_0\right)}{\sin\left(s_0 \ln\left(\frac{\Lambda}{\Lambda^*}\right) - \arctan s_0\right)}$$

$c \simeq 0.877 \pm 0.003$

(Braaten, Kang, and Platter (2011))



$$
\begin{aligned}
B_1 =& \frac{\rho_t}{4\pi} \int_0^\Lambda dq q^2 \, (\Gamma_{Nt}(q))^2 \, \frac{\sqrt{\frac{3}{4} q^2 - M_N E} + \gamma_t}{\sqrt{\frac{3}{4} q^2 - M_N E} - \gamma_t} \\
&+ \frac{3\rho_s}{4\pi} \int_0^\Lambda dq q^2 \, (\Gamma_{Ns}(q))^2 \, \frac{\frac{3}{4} q^2 - M_N E}{\left(\sqrt{\frac{3}{4} q^2 - M_N E} - \gamma_s\right)^2} \\
&+ \frac{3r_C}{8\pi} \int_0^\Lambda dq q^2 \, \left(\Gamma_{Npp}(q)\right)^2 \, \frac{\frac{3}{4} q^2 - M_N E}{\left(2\kappa H\left(\frac{\kappa}{\sqrt{\frac{3}{4} q^2 - M_N E}}\right) + \frac{1}{a_C}\right)^2} \\
&+ \frac{(H_1(\Lambda) + H_1^{(\alpha)}(\Lambda))}{\pi^2 \Lambda^2} \left[\int_0^\Lambda dq q^2 \Gamma_{Nt}(q) \frac{1}{\sqrt{\frac{3}{4} q^2 - M_N E} - \gamma_t} \right. \\
&\left. + \int_0^\Lambda dq q^2 \Gamma_{Ns}(q) \frac{1}{\sqrt{\frac{3}{4} q^2 - M_N E} - \gamma_s} + \int_0^\Lambda dq q^2 \Gamma_{Npp}(q) \frac{1}{-\frac{1}{a_C} - 2\kappa H\left(\frac{\kappa}{\sqrt{\frac{3}{4} q^2 - M_N E}}\right)} \right]^2 \\
&- \frac{\rho_t \alpha M_N}{4\pi^2} \int_0^\Lambda dq q^2 \int_0^\Lambda d\ell \ell^2 \Gamma_{Nt}(q) \Gamma_{Nt}(\ell) \frac{1}{\sqrt{\frac{3}{4} q^2 - M_N E} - \gamma_t} \\
&\qquad \times \frac{1}{\sqrt{\frac{3}{4} \ell^2 - M_N E} - \gamma_t} \frac{1}{q\ell} Q_0\left(\frac{-q^2 - \ell^2 - \lambda^2}{2q\ell}\right) \\
&- \frac{3\rho_s \alpha M_N}{4\pi^2} \int_0^\Lambda dq q^2 \int_0^\Lambda d\ell \ell^2 \Gamma_{Ns}(q) \Gamma_{Ns}(\ell) \frac{1}{\sqrt{\frac{3}{4} q^2 - M_N E} - \gamma_s} \\
&\qquad \times \frac{1}{\sqrt{\frac{3}{4} \ell^2 - M_N E} - \gamma_s} \frac{1}{q\ell} Q_0\left(\frac{-q^2 - \ell^2 - \lambda^2}{2q\ell}\right) .
\end{aligned}
$$

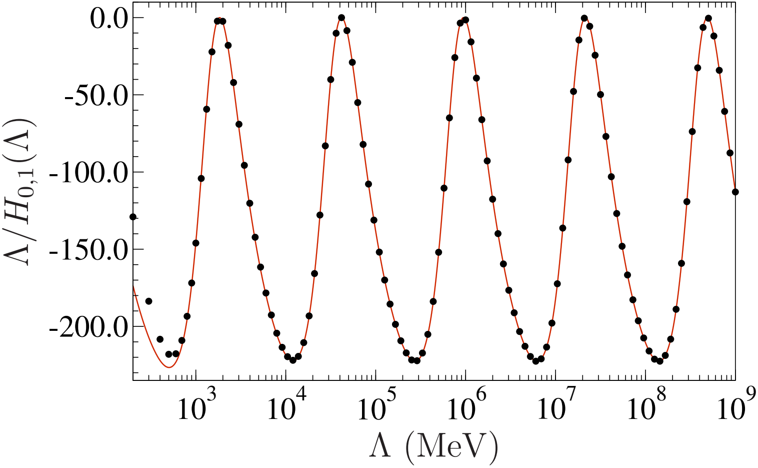
$$\begin{aligned}
[B_1]_{\text{UV-div}} &= \frac{1}{4\pi}\left(\frac{1}{4}\rho_t+\frac{1}{12}\rho_s+\frac{1}{6}r_C\right)\int^\Lambda dq q^2(\Gamma_+^{(-1)}(q))^2\\
&+\frac{1}{2\pi}\left(\frac{1}{4}\rho_t+\frac{1}{12}\rho_s+\frac{1}{6}r_C\right)\int^\Lambda dq q^2\Gamma_+^{(-1)}(q)\Gamma_+^{(-2)}(q)\\
&+\frac{1}{4\pi}\left(\frac{1}{2}\rho_t-\frac{1}{6}\rho_s-\frac{1}{3}r_C\right)\int^\Lambda dq q^2\Gamma_+^{(-1)}(q)\Gamma_-^{(-2)}(q)+\frac{1}{4\pi}\left(\frac{2}{3}\rho_s-\frac{2}{3}r_C\right)\int^\Lambda dq q^2\Gamma_+^{(-1)}(q)\Gamma_0^{(-2)}(q)\\
&+\frac{1}{\sqrt{3}\pi}\left(\frac{1}{4}\rho_t\gamma_t+\frac{1}{12}\rho_s\gamma_s+\frac{1}{6}r_C\gamma_C\right)\int^\Lambda dq q(\Gamma_+^{(-1)}(q))^2+\frac{\kappa r_C}{3\sqrt{3}\pi}\int^\Lambda dq q\ln(q)(\Gamma_+^{(-1)}(q))^2\\
&+\frac{4(H_{0,1}(\Lambda)+H_{0,1}^{(\alpha)}(\Lambda))}{3\pi^2\Lambda^2}\left\{\left(\int^\Lambda dq q\Gamma_+^{(-1)}(q)\right)^2+2\int^\Lambda dq q\Gamma_+^{(-1)}(q)\int^\Lambda dq\ell\Gamma_+^{(-2)}(\ell)\right.\\
&+\sqrt{\frac{4}{3}}\left(\gamma_t+\frac{1}{3}\gamma_s+\frac{2}{3}\gamma_C\right)\int^\Lambda dq q\Gamma_+^{(-1)}(q)\int^\Lambda d\ell\Gamma_+^{(-1)}(\ell)\\
&\left.+\sqrt{\frac{4}{3}}\frac{4\kappa}{3}\int^\Lambda dq q\Gamma_+^{(-1)}(q)\int^\Lambda d\ell\ln(\ell)\Gamma_+^{(-1)}(\ell)\right\}\\
&-\frac{\alpha M_N}{3\pi^2}\left(\frac{1}{4}\rho_t+\frac{1}{12}\rho_s\right)\int^\Lambda dq\int^\Lambda d\ell\Gamma_+^{(-1)}(k,q)\Gamma_+^{(-1)}(k,l)Q_0\left(\frac{-q^2-\ell^2-\lambda^2}{2q\ell}\right),
\end{aligned}$$

NLO three-body force for *nd* scattering

$$h_{10}(\Lambda) = -\frac{3\pi(1+s_0^2)}{128}(\rho_t+\rho_s)\frac{\left(1-\frac{1}{\sqrt{1+4s_0^2}}\sin\left(2s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\tan^{-1}\left(\frac{1}{2s_0}\right)\right)\right)}{\sin^2\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)-\tan^{-1}(s_0)\right)}$$

$$\begin{aligned}H_{0,1}(\Lambda) = \Lambda h_{10}(\Lambda) - \frac{3\pi(1+s_0^2)}{64}&\left\{\frac{1}{\sqrt{3}}(\rho_t+\rho_s)(\gamma_t+\gamma_s)|B_{-1}|\mathcal{G}_1(B_{-1})\right.\\ &-\frac{1}{2\sqrt{3}}(\rho_t-\rho_s)(\gamma_t-\gamma_s)|\tilde{B}_{-1}|\mathcal{G}_1(\tilde{B}_{-1})\\ &\left.+\frac{2}{\sqrt{3}}(\rho_t\gamma_t+\rho_s\gamma_s)\mathcal{G}_1(0)+f\right\}/\sin^2\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)-\arctan(s_0)\right)\end{aligned}$$

$$\mathcal{G}_1(x) = \cos(\mathrm{Arg}(x))\ln(\Lambda) - \frac{1}{2s_0}\sin\left(2s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\mathrm{Arg}(x)\right).$$


0.0
-50.0
-100.0
-150.0
-200.0
$\Lambda/H_{0,1}(\Lambda)$
10^3 10^4 10^5 10^6 10^7 10^8 10^9
Λ (MeV)

NLO isospin breaking contribution to *pd* scattering

$$h_l^{(\alpha)}(\Lambda) = -\frac{3\pi(1+s_0^2)}{16}\times$$
$$\left\{\frac{1}{12}(r_C-\rho_s)\Lambda\left[1-\frac{1}{\sqrt{1+4s_0^2}}\sin\left(2s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\arctan\left(\frac{1}{2s_0}\right)\right)\right]\right.$$
$$+\frac{1}{3\sqrt{3}}\left(\frac{1}{2}(\rho_t+\rho_s)(\gamma_C-\gamma_s)+\frac{1}{2}(r_C-\rho_s)(\gamma_t+\gamma_s)+\frac{1}{3}(r_C-\rho_s)(\gamma_C-\gamma_s)\right)|B_{-1}|\mathcal{G}_1(B_{-1})$$
$$-\frac{1}{12\sqrt{3}}\left(\frac{4}{3}(r_C-\rho_s)(\gamma_C-\gamma_s)-(\rho_t-\rho_s)(\gamma_C-\gamma_s)-(r_C-\rho_s)(\gamma_t-\gamma_s)\right)|\tilde{B}_{-1}|\mathcal{G}_1(\tilde{B}_{-1})$$
$$+\frac{1}{3\sqrt{3}}(r_C\gamma_C-\rho_s\gamma_t)\mathcal{G}_1(0)-\frac{64}{9\sqrt{3}\pi s_0\sqrt{1+s_0^2}}h_{10}(\Lambda)\sin\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)-\tan^{-1}(s_0)\right)(\gamma_C-\gamma_s)$$
$$\left.\times\left[|B_1|\mathcal{G}_3(B_{-1})+\mathcal{G}_3(0)\right]+f\right\}/\sin^2\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\arctan(s_0)\right).$$

$$\mathcal{G}_3(x)=\cos\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\mathrm{Arg}(x)\right)$$

NLO Coulomb contribution to *pd* scattering

$$h_{\kappa}^{(\alpha)} = -\frac{\sqrt{3}\kappa\pi(1+s_0^2)}{48}\left\{\left(\rho_t+\frac{1}{3}\rho_s+\frac{2}{3}r_C\right)\left[\,|C_{-1}|\mathcal{G}_1(C_{-1})+|D_{-1}|\mathcal{G}_1(D_{-1})\right.\right.$$

$$\left.-\frac{12}{\pi}|E_{-1}|\mathcal{G}_1(E_{-1})+\frac{1}{2}|B_{-1}|\mathcal{G}_2(B_{-1})\right]+r_C\mathcal{G}_2(0)$$

$$+\frac{1}{2}\left(\rho_t+\frac{1}{3}\rho_s-\frac{4}{3}r_c\right)\left[|\tilde{C}_{-1}|\mathcal{G}_1(\tilde{C}_{-1})-\frac{1}{2}|\tilde{D}_{-1}|\mathcal{G}_1(\tilde{D}_{-1})-\frac{24}{\pi}|\tilde{E}_{-1}|\mathcal{G}_1(\tilde{E}_{-1})+\frac{1}{2}|\tilde{B}_{-1}|\mathcal{G}_2(\tilde{B}_{-1})\right]$$

$$-\frac{128}{3\pi s_0\sqrt{1+s_0^2}}h_{10}(\Lambda)\sin\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)-\tan^{-1}(s_0)\right)\left[\cdot|C_{-1}|\mathcal{G}_3(C_{-1})+|D_{-1}|\mathcal{G}_3(D_{-1})\right.$$

$$\left.\left.-\frac{1}{s_0}|B_{-1}|\mathcal{G}_4(B_{-1})-12|E_{-1}|\mathcal{G}_3(E_{-1})-\frac{1}{s_0}\mathcal{G}_4(0)\right]+\Psi(\Lambda)\right\}/$$

$$\sin^2\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\arctan s_0\right).$$

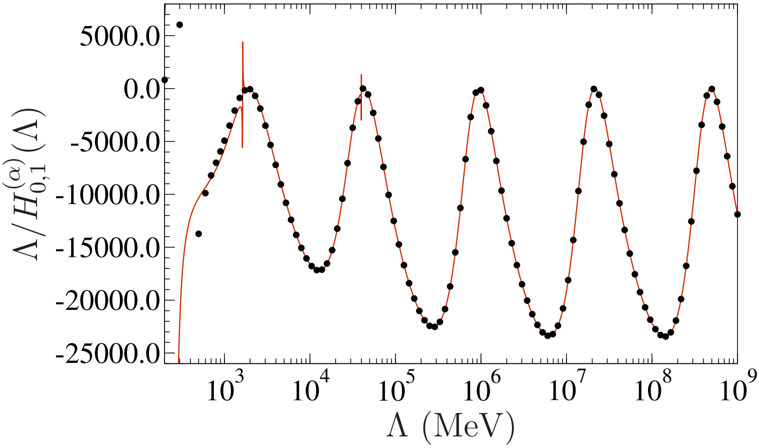
$$\mathcal{G}_2(x)=\cos(\mathrm{Arg}(x))\ln^2(\Lambda)-\frac{1}{2s_0^2}\cos\left(2s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\mathrm{Arg}(x)\right)$$

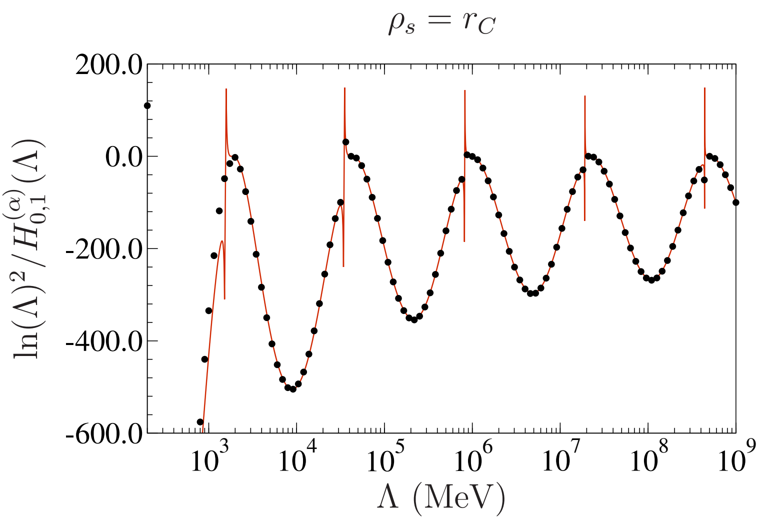
$$-\frac{1}{s_0}\ln(\Lambda)\sin\left(2s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\mathrm{Arg}(x)\right)$$

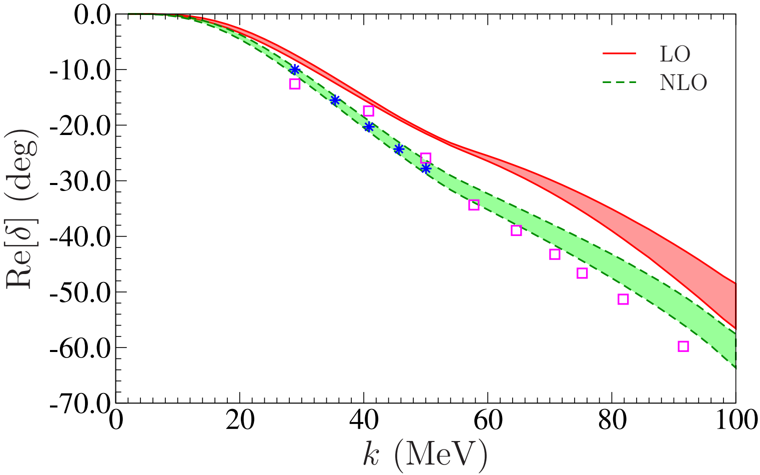
$$\mathcal{G}_4(x)=\sin\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\mathrm{Arg}(x)\right)-s_0\ln(\Lambda)\cos\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\mathrm{Arg}(B_{-1})\right)$$

$$\Psi(\Lambda)=a\ln(\Lambda)+b\sin\left(2s_0\left(\frac{\Lambda}{\Lambda^*}\right)+c\right)+d$$

$\rho_s \neq r_C$


5000.0
0.0
-5000.0
-10000.0
-15000.0
-20000.0
-25000.0
$\Lambda/H_{0,1}^{(\alpha)}(\Lambda)$
10^3
10^4
10^5
10^6
10^7
10^8
10^9
Λ (MeV)


$\rho_s = r_C$
200.0
0.0
-200.0
-400.0
-600.0
$\ln(\Lambda)^2/H_{0,1}^{(\alpha)}(\Lambda)$
10^3
10^4
10^5
10^6
10^7
10^8
10^9
Λ (MeV)



0.0
-10.0
-20.0
-30.0
-40.0
-50.0
-60.0
-70.0
0
20
40
60
80
100
Re[δ] (deg)
k (MeV)
LO
NLO

Conclusions

- Can use same LO three-body force for *nd* and *pd* systems.
- Cannot use same NLO three-body force for *nd* and *pd* systems.
- NNLO calculation of *pd* scattering will require two three-body forces.
- NNLO three-body calculation of *pd* scattering is necessary to investigate polarization observables such as A_y.
- Regularization procedure used in two and three-body sectors is different
- Coulomb is treated perturbatively in three-body sector and non-perturbatively in two-body sector.
- Photon has finite mass.

The deuteron wavefunction renormalization is given by the residue of the spin-triplet dibaryon

$$Z_D = \frac{2\gamma_t}{M_N}\left[\underbrace{1}_{\text{LO}} + \underbrace{\rho_t\gamma_t}_{\text{NLO}} + \cdots\right]$$

We define

$$Z_{LO} = \frac{2\gamma_t}{M_N}$$

The on-shell energy for *nd* scattering in the c.m. frame is

$$E = \frac{k^2}{2M_N} + \frac{k^2}{4M_N} - \frac{\gamma_t^2}{M_N} = \frac{3}{4}\frac{k^2}{M_N} - \frac{\gamma_t^2}{M_N}$$