N^{3}LO nd Scattering Observables in Pionless Effective Field Theory

Arman Margaryan, Jared Vanasse, Roxanne Springer

L/EFT Theory Group, Duke University Department of Physics

Relevant workshop questions

1) ``What issues arise from and can be assessed by regulator-dependence?"
[For example: How problematic is renormalising the two-nucleon amplitude before it is inserted into the 3N system? What role do regulator artifacts play? How much residual regulator dependence can be tolerated?]

2)``For which observables is non-perturbative Coulomb essential, and where not? Does that influence the answers to 1 and 2?"
[For example: Importance for S-factors, 3He-3H differences, specific breakup configurations like Space-Star, A_y problem.]

Outline

Part1

- Motivation: to study the A_y puzzle and other three body spin-polarization observables

- Method: pionless effective field theory $(\mathrm{EFT}_{\not\pi})$

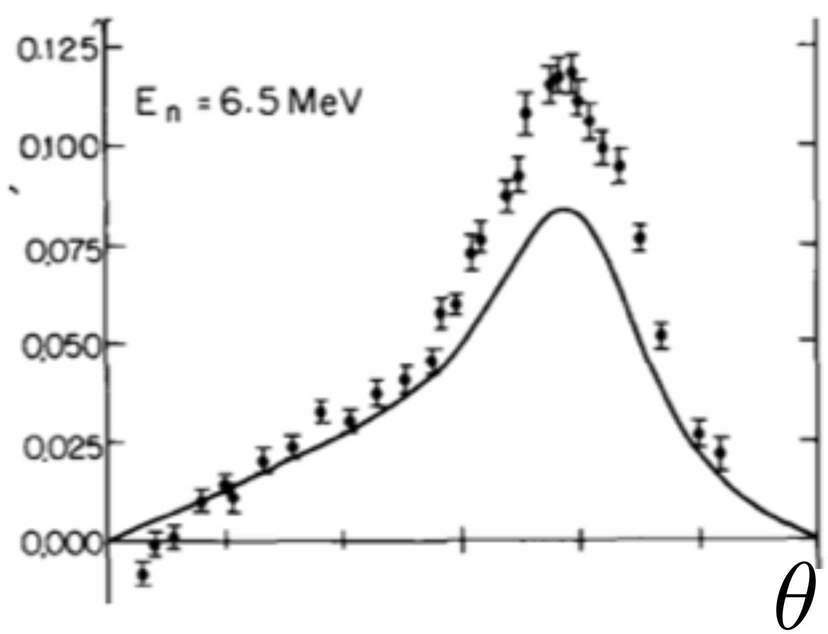
- Procedure:
  - Power counting in $\frac{Q}{\Lambda_{\not\pi}}$, where Q is the typical momentum exchange and $\Lambda_{\not\pi} < m_\pi$
  - Create all operators to given order in $\frac{Q}{\Lambda_{\not\pi}}$
  - Calculate all diagrams appearing to given order
  - Write the integral equation for scattering amplitude
  - Project onto partial waves
  - Solve integral equation
  - Calculate cross sections, phase shifts, asymmetries.

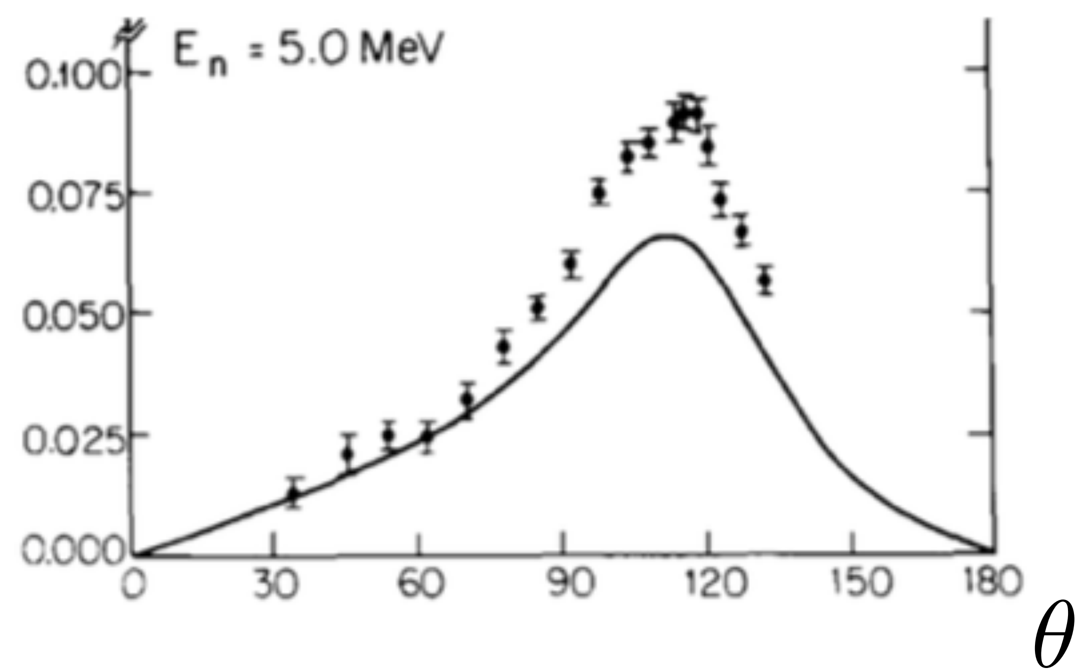
Part2

- Photon exchange diagrams for Ay in pd scattering
- Analytic calculation

Motivation

$$A_y = \frac{\frac{d\sigma}{d\Omega}|_{\uparrow} - \frac{d\sigma}{d\Omega}|_{\downarrow}}{\frac{d\sigma}{d\Omega}|_{\uparrow} + \frac{d\sigma}{d\Omega}|_{\downarrow}}$$





Neutron-deuteron elastic analyzing power A_y data at E_n=5.0 and 6.5 MeV in comparison to Faddeev calculations using the Paris NN potential. Tornow, W. et al., Phys. Lett. B 257, 273 (1991)

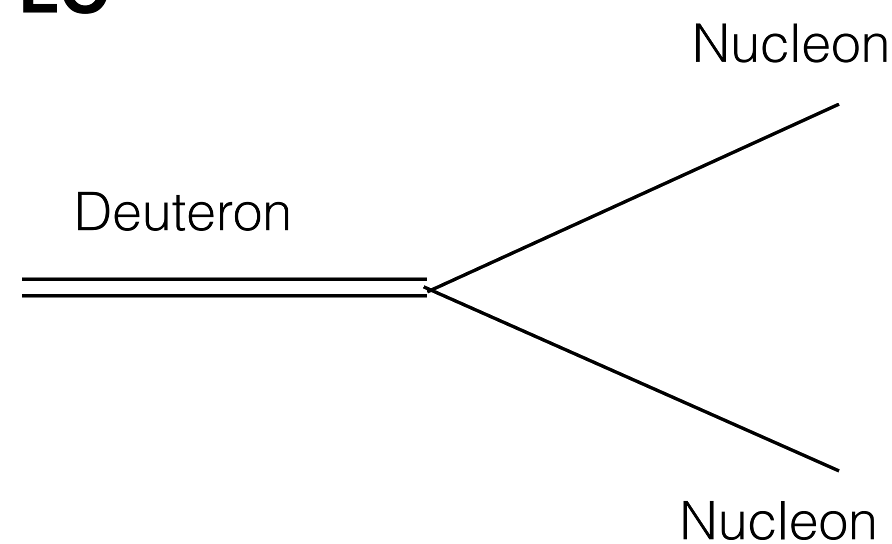
Pieces of the Lagrangian

$$N^\dagger\left(i\partial_0+\frac{\vec{\partial}^2}{2M_N}\right)N+d_t^{i\dagger}\left(\Delta_t-c_{0t}\left(i\partial_0+\frac{\vec{\partial}^2}{4M_N}+\frac{\gamma_t^2}{M}\right)-\sum_{i=1}^{\infty}c_{nt}\left(i\partial_0+\frac{\vec{\partial}^2}{4M_N}+\frac{\gamma_t^2}{M}\right)^{n+1}\right)d_t^i \longrightarrow$$

Kinetic terms and effective range expansion (ERE) insertions

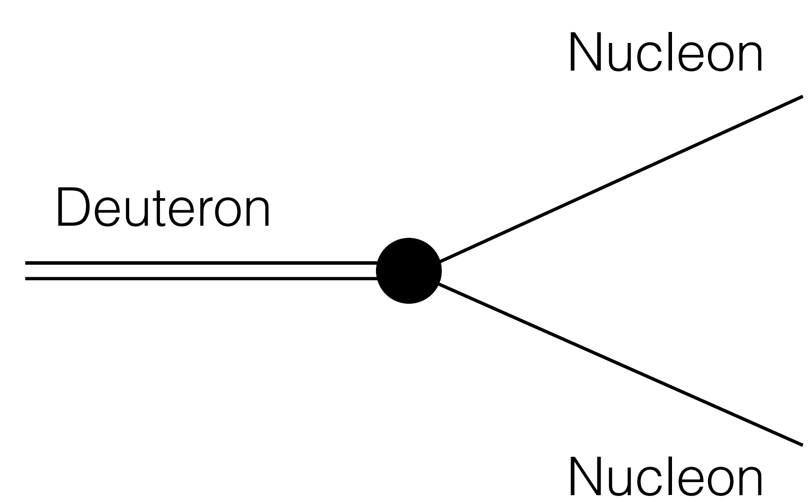
Two-body force, LO

$$-y_t\left(d_t^{i\dagger}(N^T P^i N)+h.c.\right) \longrightarrow$$



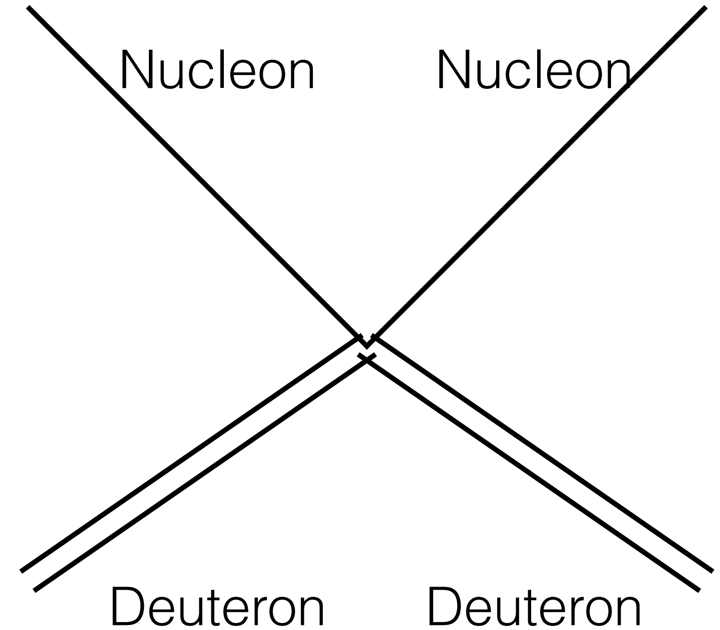
S-D mixing, N2LO

$$+\quad y_{SD}d_t^{i\dagger}\left(N^T((\overrightarrow{\partial}-\overleftarrow{\partial})^i(\overrightarrow{\partial}-\overleftarrow{\partial})^j-\frac{1}{3}\delta^{ij}(\overrightarrow{\partial}-\overleftarrow{\partial})^2)P^jN\right)+h.c. \longrightarrow$$



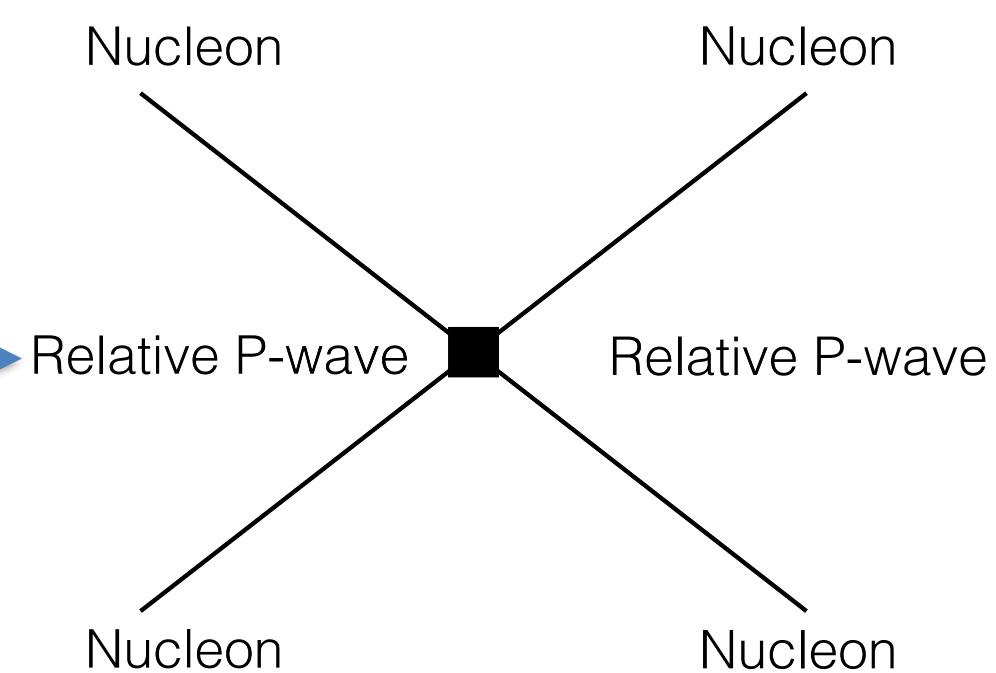
Three-body force, LO and N2LO

$$C_3 d_t^{i\dagger} N^\dagger \mathcal{O}_{ij} d_t^j N \longrightarrow$$

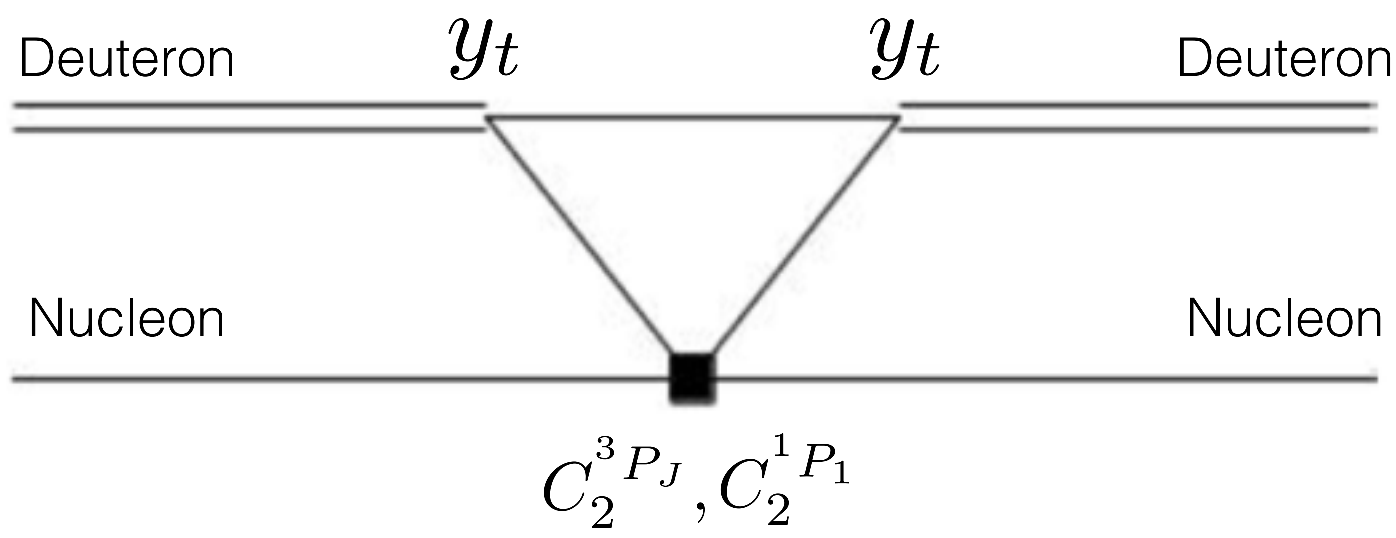


Two-body P-wave, N3LO

$$\left(C_2^{^3P_0}\delta^{xy}\delta^{wz} + C_2^{^3P_1}[\delta^{xw}\delta^{yz} - \delta^{xz}\delta^{yw}] + C_2^{^3P_2}[2\delta^{xw}\delta^{yz} + 2\delta^{xz}\delta^{yw} - \frac{4}{3}\delta^{xy}\delta^{wz}]\right)$$
$$\times\frac{1}{4}(N^T\mathcal{O}_{xy}^{(1,P)}N)^\dagger(N^T\mathcal{O}_{wz}^{(1,P)}N) \longrightarrow$$
$$+C_2^{^1P_1}\frac{1}{4}(N^T\mathcal{O}_{x}^{(0,P)}N)^\dagger(N^T\mathcal{O}_{x}^{(0,P)}N)$$



3-rd order contribution



$$64t = \int \frac{d^3q}{(2\pi)^3} \frac{1}{\frac{k^2}{4M} - \frac{\gamma^2}{M} - \frac{q^2}{2M} - \frac{(\vec{k}-\vec{q})^2}{2M}} \frac{1}{\frac{k^2}{4M} - \frac{\gamma^2}{M} + \epsilon - \frac{q^2}{2M} - \frac{(\vec{p}-\vec{q})^2}{2M}} \frac{1}{4}(2\vec{k} - \vec{q})_w (2\vec{p} - \vec{q})_x$$

$$\times \big(\sigma_y \sigma_i \sigma_j \sigma_z$$

$$(C_2^{^3P_0} \delta^{xy} \delta^{wz}$$

$$+ C_2^{^3P_1} [\delta^{xw}\delta^{yz} - \delta^{xz}\delta^{yw}]$$

$$+ C_2^{^3P_2} [2\delta^{xw}\delta^{yz} + 2\delta^{xz}\delta^{yw} - \frac{4}{3}\delta^{xy}\delta^{wz}])$$

$$+ C_2^{^1P_1} \delta^{xw} \sigma_i \sigma_j \big)$$

Master integrals

$$\int \frac{d^3q}{(2\pi)^3} \frac{1}{(\vec{q} - \frac{\vec{k}}{2})^2 + a^2} \frac{1}{(\vec{q} - \frac{\vec{p}}{2})^2 + b^2} q_k q_l$$

$$\int \frac{d^3q}{(2\pi)^3} \frac{1}{(\vec{q} - \frac{\vec{k}}{2})^2 + a^2} \frac{1}{(\vec{q} - \frac{\vec{p}}{2})^2 + b^2} q_k$$

$$\int \frac{d^3q}{(2\pi)^3} \frac{1}{(\vec{q} - \frac{\vec{k}}{2})^2 + a^2} \frac{1}{(\vec{q} - \frac{\vec{p}}{2})^2 + b^2} = \frac{\pi^2}{(2\pi)^3} 4 \frac{\tan^{-1}(\alpha)}{|\vec{k} - \vec{p}|}$$

Sample calculation

$$\int \frac{d^3q}{(2\pi)^3} \frac{1}{(\vec{q} - \frac{\vec{k}}{2})^2 + a^2} \frac{1}{\vec{q}^2 + b^2} q_k q_l = A\delta_{kl} + Bk_k k_l$$

$$\left\{ \begin{array}{r} \displaystyle\int \frac{d^3q}{(2\pi)^3} \frac{1}{(\vec{q} - \frac{\vec{k}}{2})^2 + a^2} \frac{1}{\vec{q}^2 + b^2} q^2 = 3A + Bk^2 \\ \displaystyle\int \frac{d^3q}{(2\pi)^3} \frac{1}{(\vec{q} - \frac{\vec{k}}{2})^2 + a^2} \frac{1}{\vec{q}^2 + b^2} (\vec{q} \cdot \vec{k})^2 = Ak^2 + Bk^4 \end{array} \right.$$

$$\vec{q} \cdot \vec{k} = -((\vec{q} - \frac{\vec{k}}{2})^2 + a^2) + (\vec{q}^2 + b^2) + \frac{k^2}{4} + a^2 - b^2$$

Troublesome integral

Changing integration variable

$$\int \frac{d^3q}{(2\pi)^3} \frac{1}{(\vec{q} - \frac{\vec{k}}{2})^2 + a^2} \vec{q} \cdot \vec{k} = \frac{1}{4\pi^2} k^2 \Lambda - \frac{ak^2}{8\pi}$$

Direct calculation

$$\int \frac{d^3q}{(2\pi)^3} \frac{1}{(\vec{q} - \frac{\vec{k}}{2})^2 + a^2} \vec{q} \cdot \vec{k} = \frac{1}{6\pi^2} k^2 \Lambda - \frac{ak^2}{8\pi}$$

regulator-dependence

Lemma

Conditions:

Function f $\rightarrow$ $\left\{ \begin{array}{l} \text{One variable} \\ \text{Certain integrability conditions} \\ \text{Analyticity near infinity} \end{array} \right.$

Claim:

The difference does not have the constant term

$$\int_{-\Lambda}^{\Lambda} f(x)dx - \int_{-\Lambda+a}^{\Lambda+a} f(x) = \int_{-\Lambda}^{\Lambda} (f(x) - f(x+a))dx$$

$$= ... + \frac{a_{-1}}{\Lambda} + \underbrace{a_0}_{=0} + a_1\Lambda + ...$$

Proof

$$\int_{-\Lambda}^{\Lambda} f(x)dx - \int_{-\Lambda+a}^{\Lambda+a} f(x)dx$$
$$= \int_{-\Lambda}^{-\Lambda+a} f(x)dx + \int_{-\Lambda+a}^{\Lambda} f(x)dx - \int_{-\Lambda+a}^{\Lambda} f(x)dx - \int_{\Lambda}^{\Lambda+a} f(x)dx$$
$$= \int_{-\Lambda}^{-\Lambda+a} f(x)dx - \int_{\Lambda}^{\Lambda+a} f(x)dx$$

Analyticity of f $\longrightarrow$ $f(x) = \sum_{n=-\infty}^{\infty} a_n x^n$

Proof

$$\int_{-\Lambda}^{\Lambda} f(x)dx - \int_{-\Lambda+a}^{\Lambda+a} f(x)dx$$

$$= \int_{-\Lambda}^{-\Lambda+a} \sum_{n=-\infty}^{\infty} a_n x^n dx - \int_{\Lambda}^{\Lambda+a} \sum_{n=-\infty}^{\infty} a_n x^n dx$$

$$= a_{-1} \int_{-\Lambda}^{-\Lambda+a} \frac{1}{x} dx - a_{-1} \int_{\Lambda}^{\Lambda+a} \frac{1}{x} dx$$

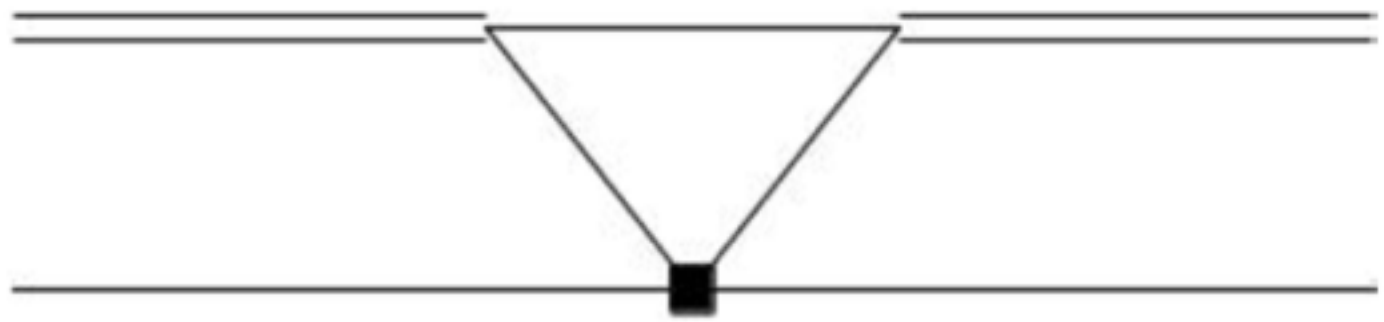
$$+ \sum_{n\neq-1} a_n \int_{-\Lambda}^{-\Lambda+a} x^n dx - \sum_{n\neq-1} a_n \int_{\Lambda}^{\Lambda+a} x^n dx$$

$$\sum_{n\neq-1} a_n \int_{-\Lambda}^{-\Lambda+a} x^n dx - \sum_{n\neq-1} a_n \int_{\Lambda}^{\Lambda+a} x^n dx$$

$$= \sum_{n<-1} \frac{a_n}{n+1}((-\Lambda+a)^{n+1} - (-\Lambda)^{n+1} - (\Lambda+a)^{n+1} + \Lambda^{n+1})$$

$$+ \sum_{n>-1} \frac{a_n}{n+1}((-\Lambda+a)^{n+1} - (-\Lambda)^{n+1} - (\Lambda+a)^{n+1} + \Lambda^{n+1})$$

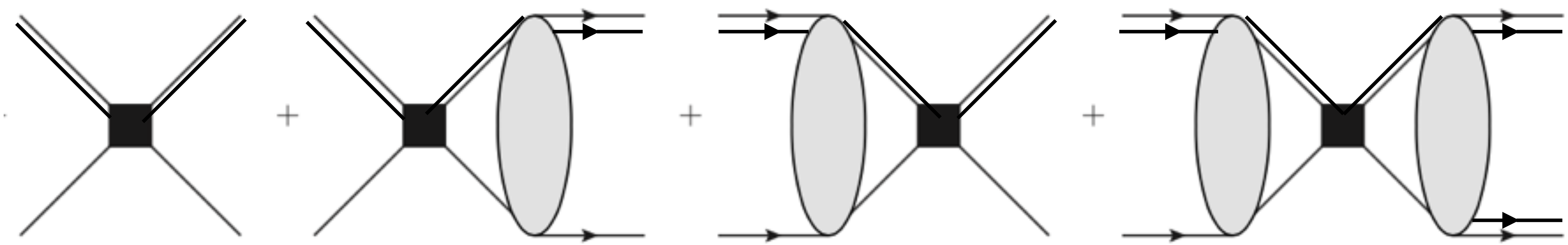
From the diagram



- Significance of the lemma: 1)easier calculation and 2) importance for other integrals
- The infinity in the quartet channel and issues with Pauli principle

Acknowledgment: Matthias Schindler

Resolution of issues with Pauli principle

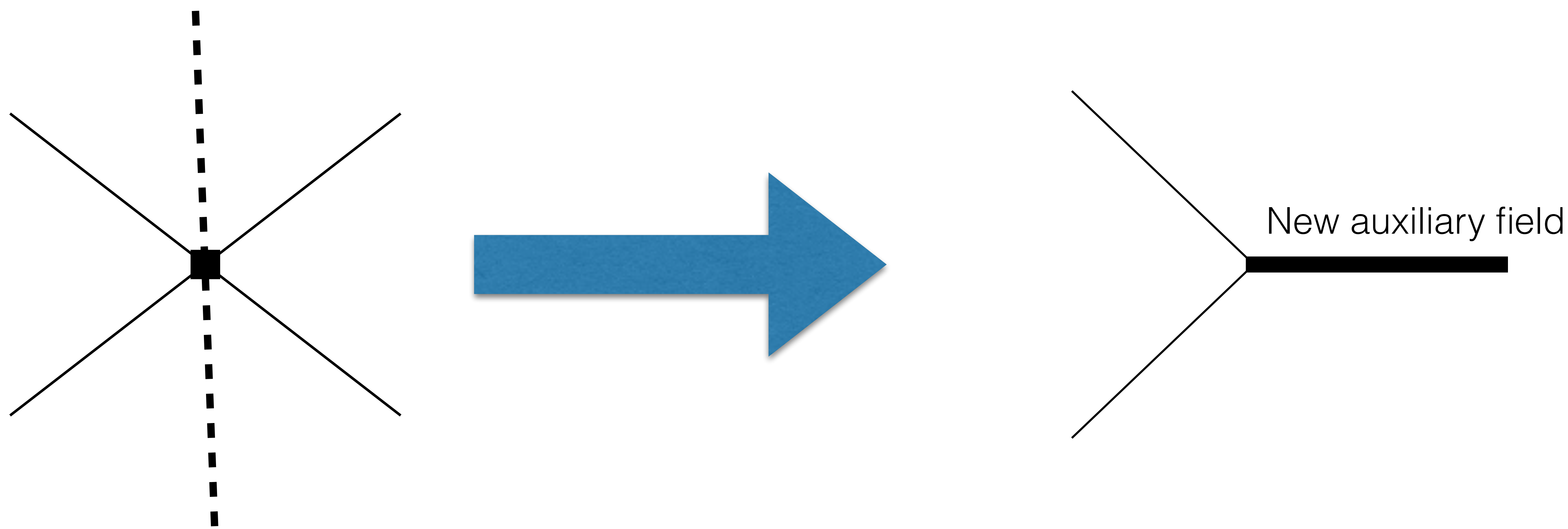


$$\sim \left(1 + \frac{1}{2\pi}\int_0^{\Lambda} dq q^2 t_{Nt\to Nt}(k,q)\frac{1}{\sqrt{\frac{3}{4}q^2 - M_N E - i\epsilon} - \gamma_t}\right)^2$$

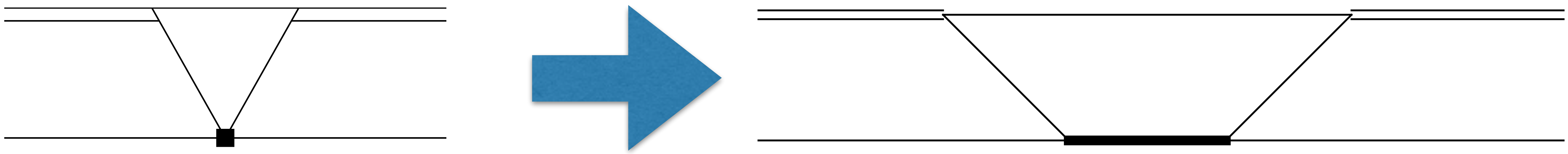
New interaction

$$
\begin{aligned}
\mathcal{L}_2^P = &-\hat{\mathcal{P}}_{0A}^{{}^3\mathrm{P}_0\dagger}\Delta^{({}^3\mathrm{P}_0)}\hat{\mathcal{P}}_{0A}^{{}^3\mathrm{P}_0} - \hat{\mathcal{P}}_{iA}^{{}^3\mathrm{P}_1\dagger}\Delta^{({}^3\mathrm{P}_1)}\hat{\mathcal{P}}_{iA}^{{}^3\mathrm{P}_1} - \hat{\mathcal{P}}_{iA}^{{}^3\mathrm{P}_2\dagger}\Delta^{({}^3\mathrm{P}_2)}\hat{\mathcal{P}}_{iA}^{{}^3\mathrm{P}_2} - \hat{\mathcal{P}}_{i}^{{}^1\mathrm{P}_1\dagger}\Delta^{({}^1\mathrm{P}_1)}\hat{\mathcal{P}}_{i}^{{}^1\mathrm{P}_1} \\
&+ \frac{1}{2}\sum_{J=0}^{2} y^{{}^3\mathrm{P}_J}\left[C_{1,1,J}^{i,j,k}\left(\hat{\mathcal{P}}_{kA}^{{}^3\mathrm{P}_J}\right)^{\dagger}\hat{N}^T i\mathcal{O}_{jiA}^{(1,P)}\hat{N} + \mathrm{H.c.}\right] \\
&+ \frac{1}{2} y^{{}^1\mathrm{P}_1}\left[\left(\hat{\mathcal{P}}_{i}^{{}^1\mathrm{P}_1}\right)^{\dagger}\hat{N}^T i\mathcal{O}_{i}^{(0,P)}\hat{N} + \mathrm{H.c.}\right].
\end{aligned}
$$

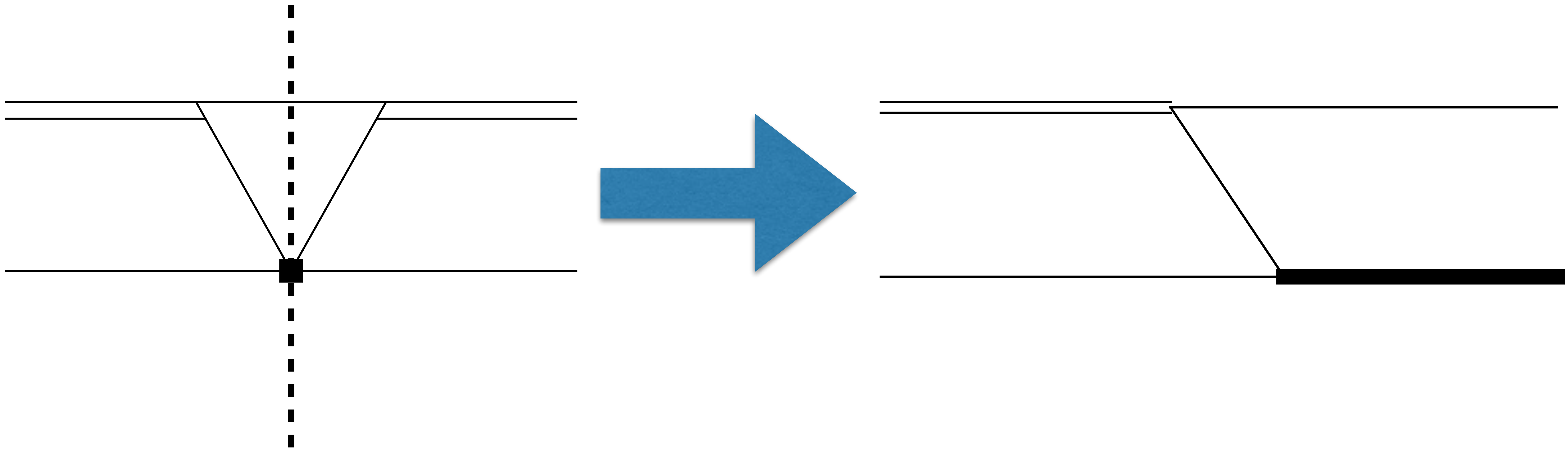
New vertex



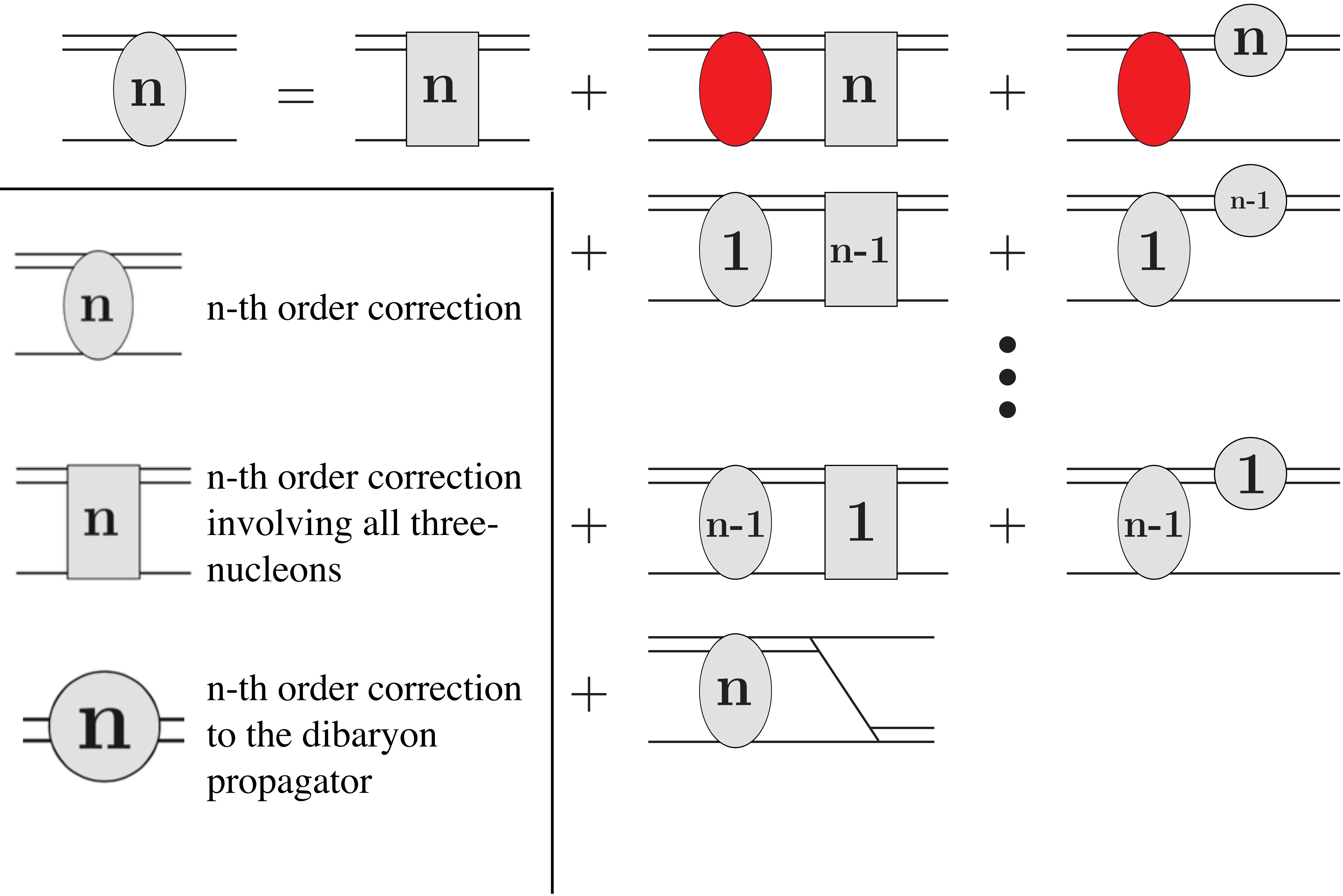
Equivalence



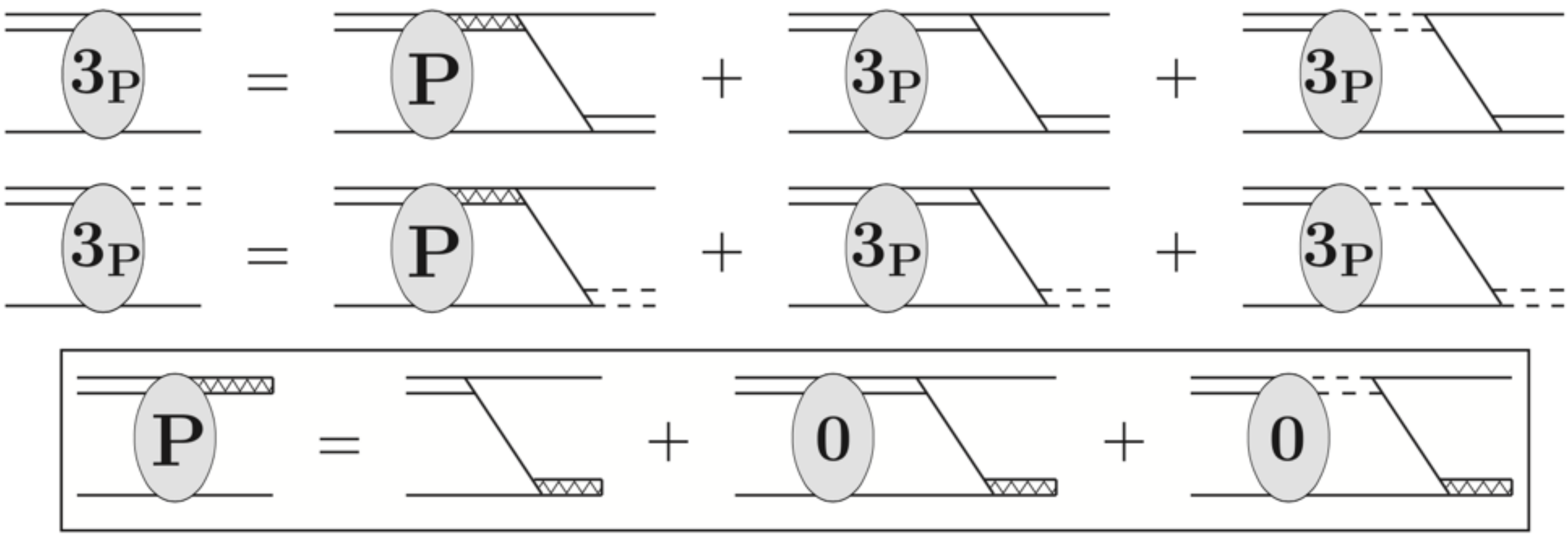
New 3-rd order contribution



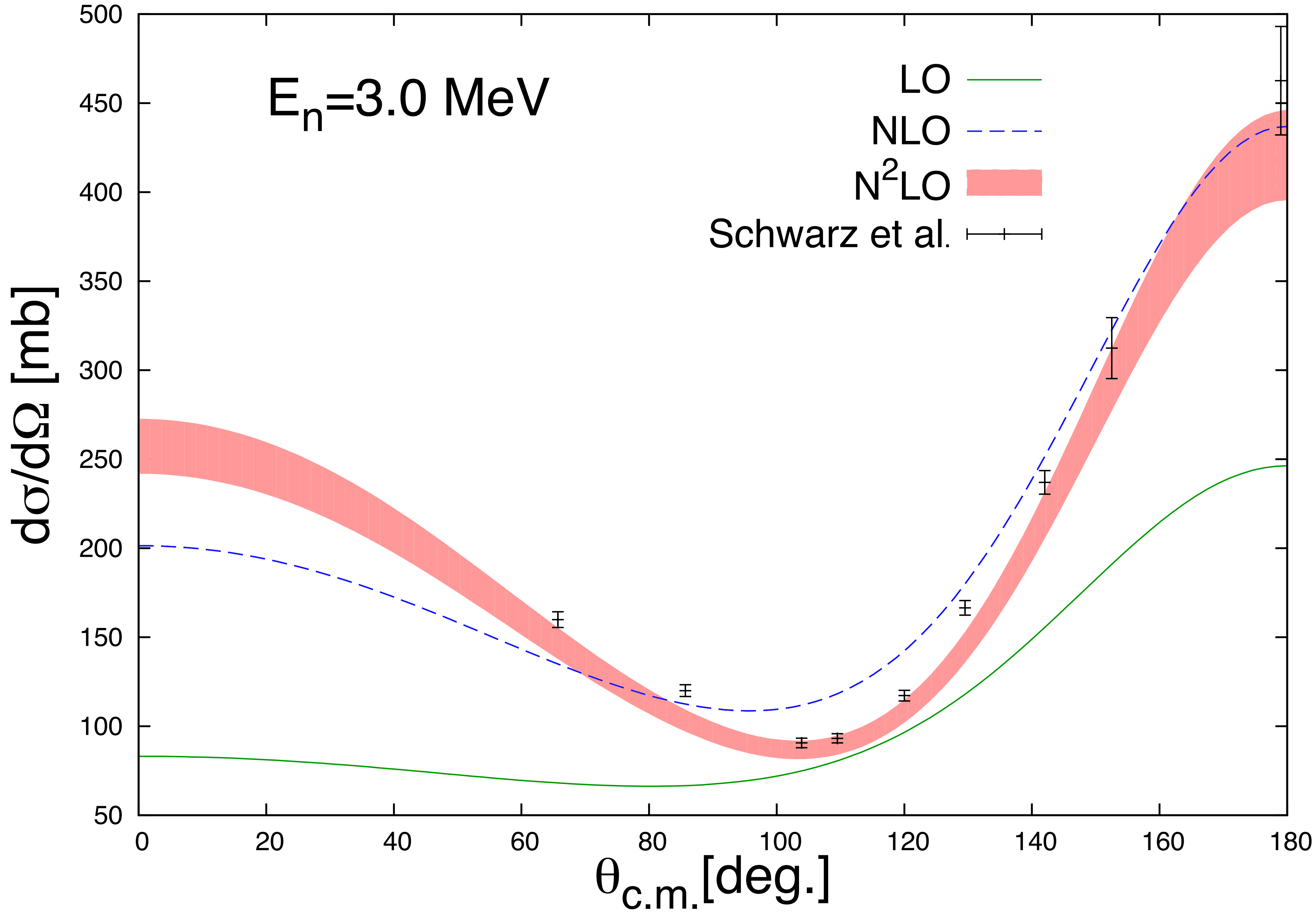
Integral equation



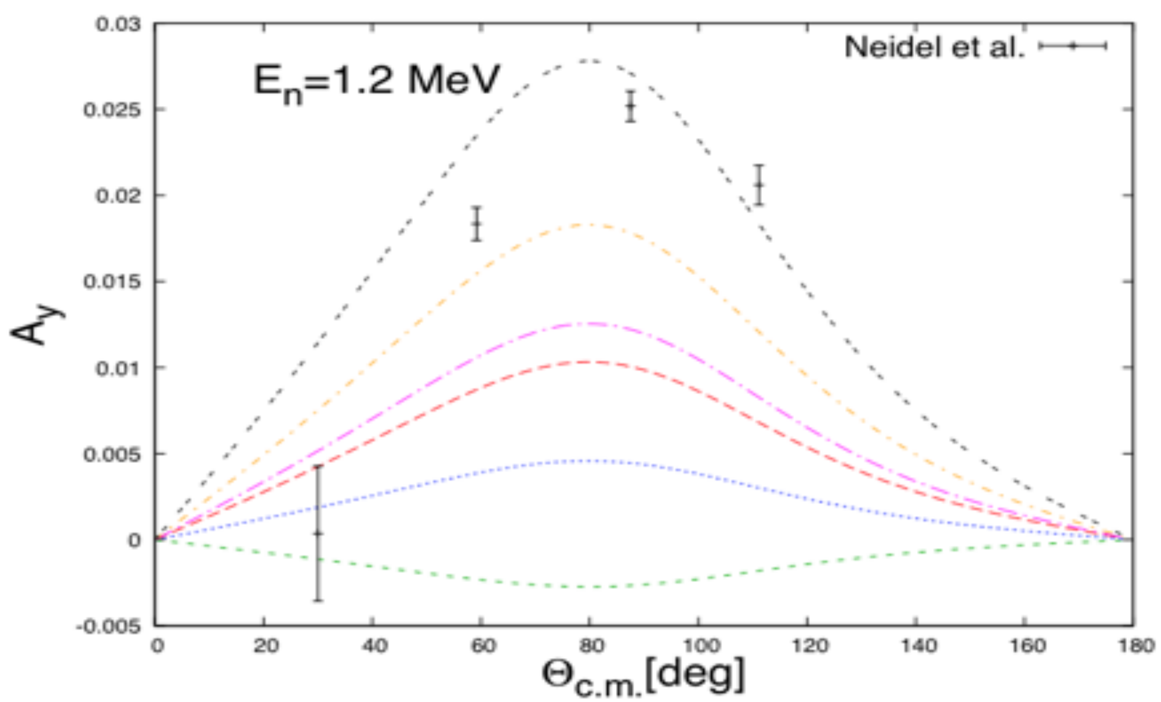
Three-body contribution from two-body P-wave contact interactions

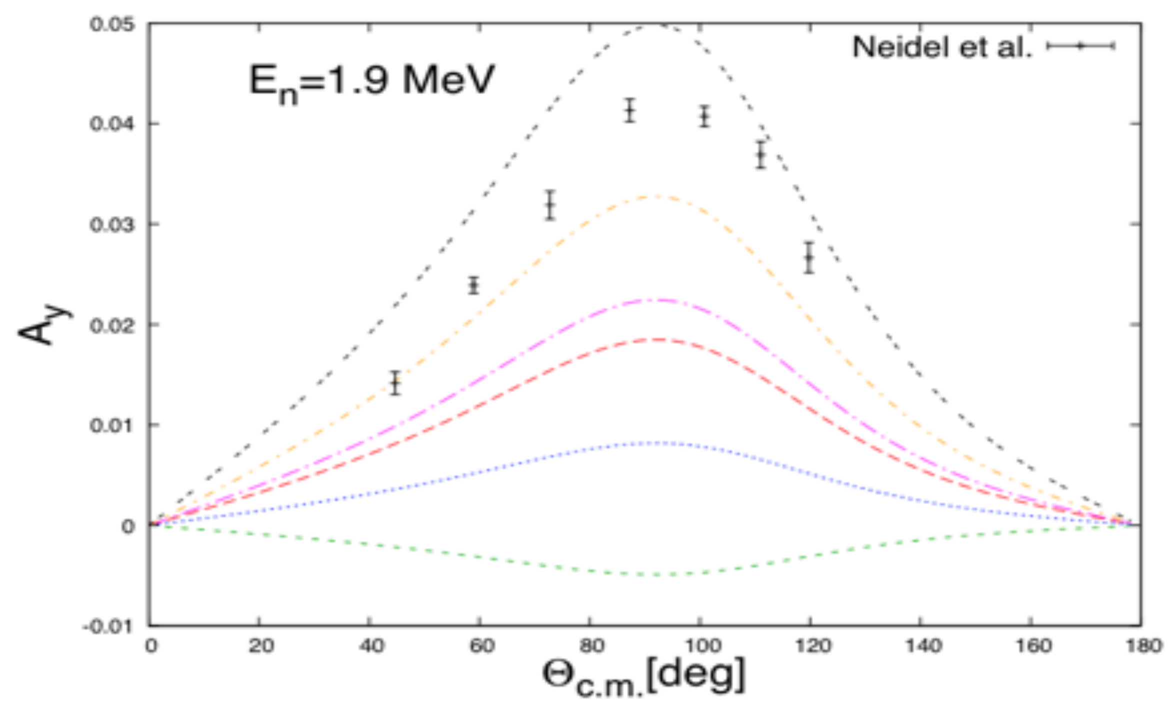


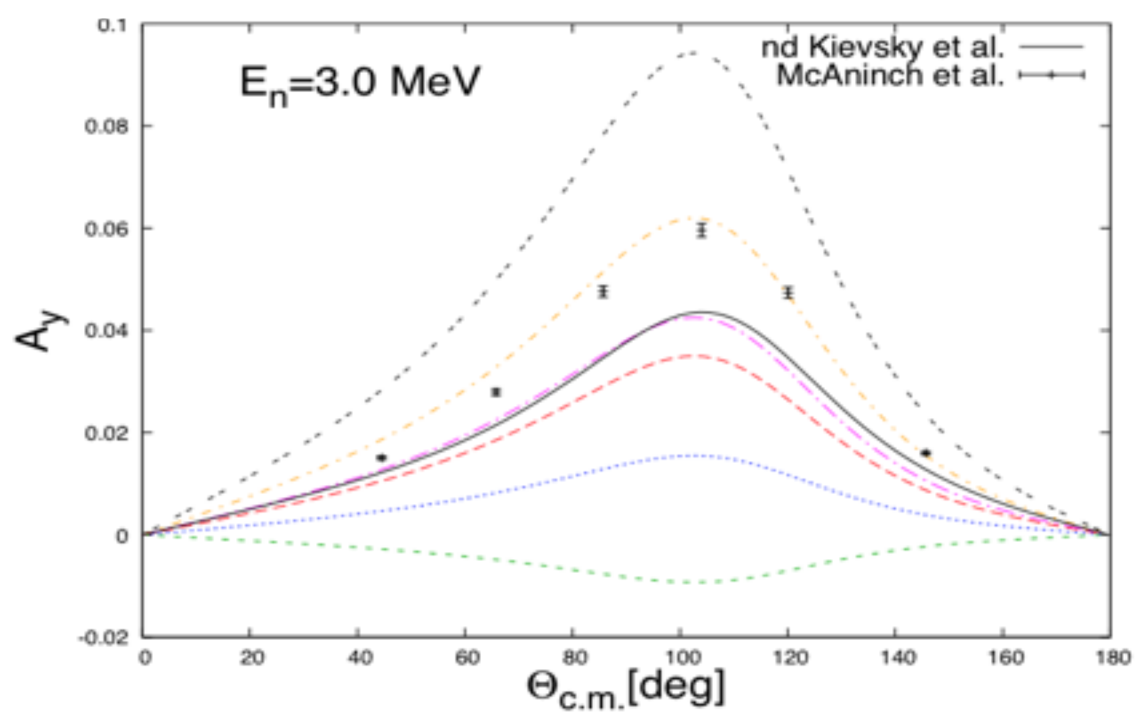
Result for cross-section



Results for Ay



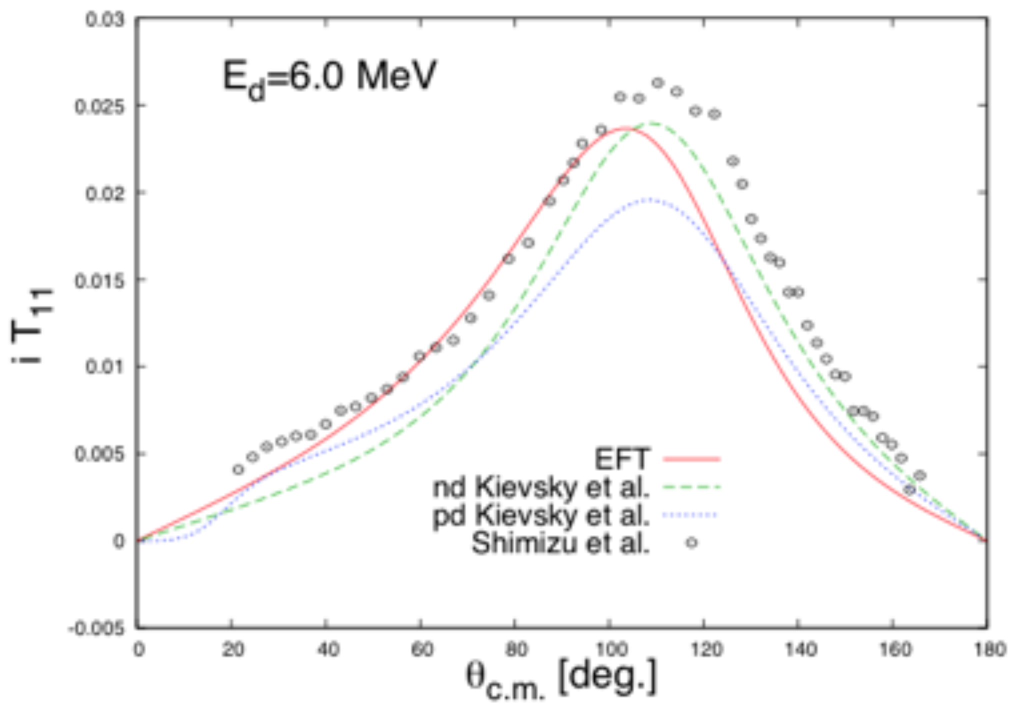


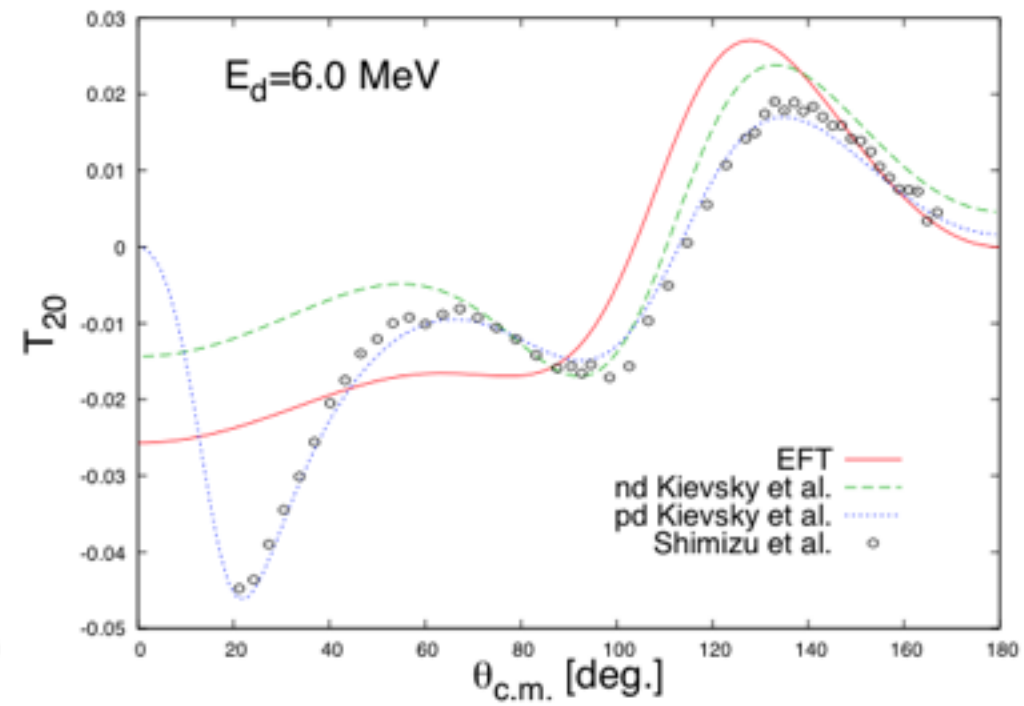


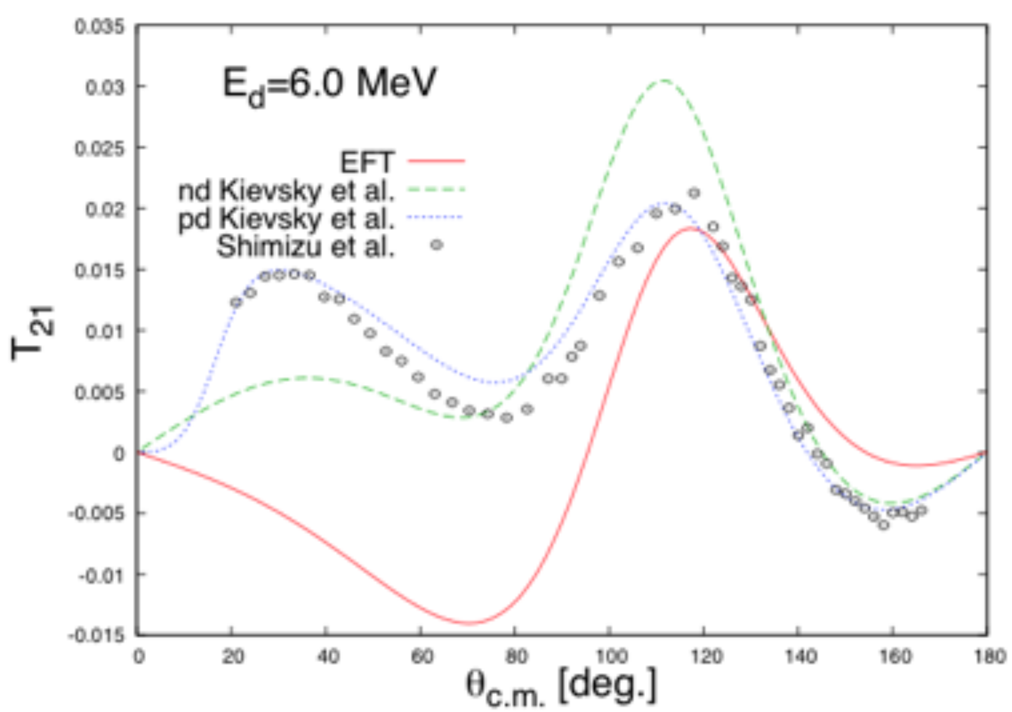
	$^3P_0$	$^3P_1$	$^3P_2$
double-dot (black)	—	+	+
short-dash-dot (orange)	—	0	0
long-dash-dot (purple)	0	0	+
long dash (red)	0	0	0
small dots (blue)	+	0	+
big dots (green)	+	—	+

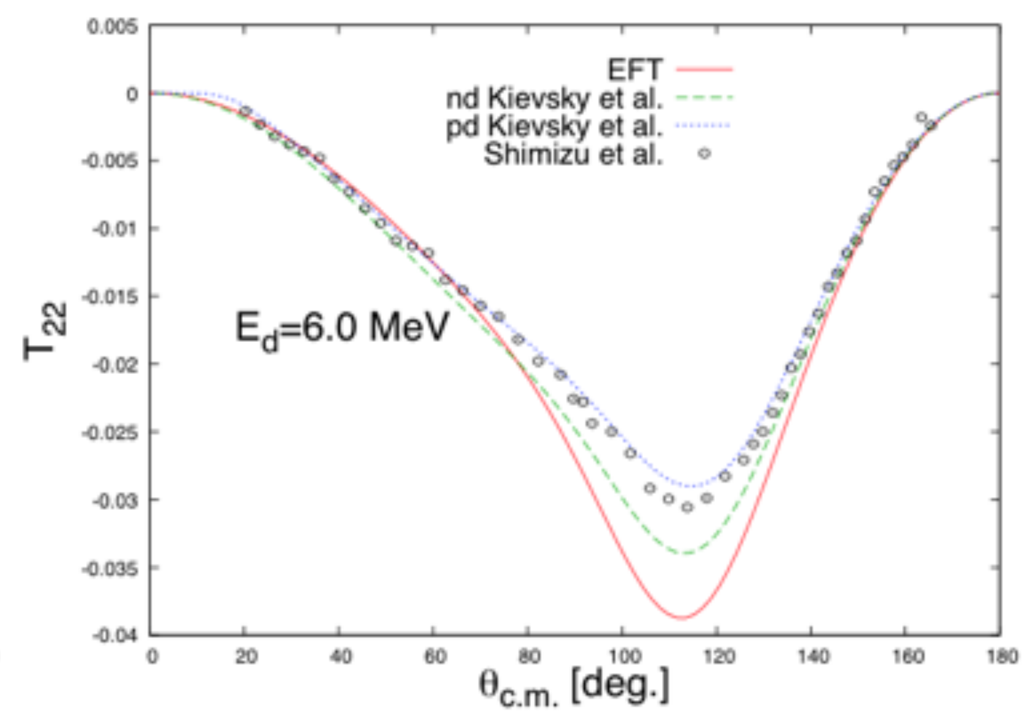
Top Left: E_n = 1.2 MeV, experimental data from Neidel et al.. Top Right: E_n = 1.9 MeV, experimental data from Neidel et al. . Bottom: E_n = 3.0 MeV, the solid line is a PMC calculation from Kievsky et al. using AV-18+UR, with experimental data from McAninch et al..

Deuteron polarization observables

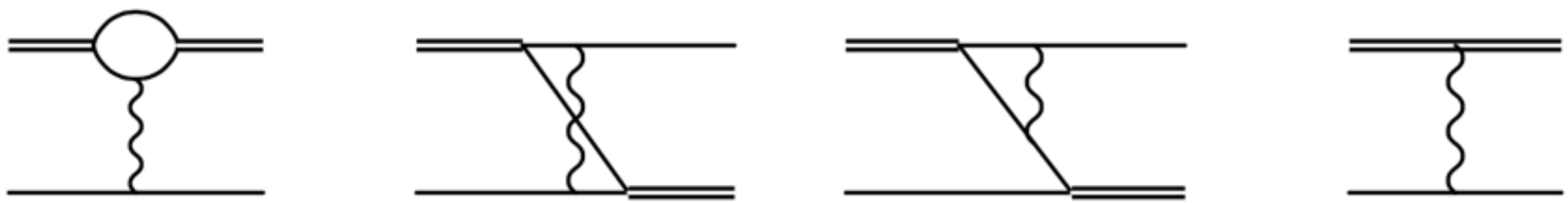








pd vs nd:
photon exchange diagrams



(a) (b) (c) (d)

Diagram b)

New master integrals

$$\int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2} \frac{1}{(\vec{q} - \frac{\vec{k}}{2})^2 + a^2} \frac{1}{(\vec{q} - \frac{\vec{p}}{2})^2 + b^2} q_k q_l$$

$$\int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2} \frac{1}{(\vec{q} - \frac{\vec{k}}{2})^2 + a^2} \frac{1}{(\vec{q} - \frac{\vec{p}}{2})^2 + b^2}$$

Old master integrals

$$\int \frac{d^3q}{(2\pi)^3} \frac{1}{(\vec{q} - \frac{\vec{k}}{2})^2 + a^2} \frac{1}{(\vec{q} - \frac{\vec{p}}{2})^2 + b^2} = \frac{\pi^2}{(2\pi)^3} 4 \frac{\tan^{-1}(\alpha)}{|\vec{k} - \vec{p}|}$$

Diagram b) calculation

Change of integration variable $\longrightarrow$ $\vec{l} = \frac{\hat{q}}{|\vec{q}|}$

$$\kappa_1^2 = a^2 + \frac{k^2}{2}, \kappa_2^2 = b^2 + \frac{p^2}{2}$$

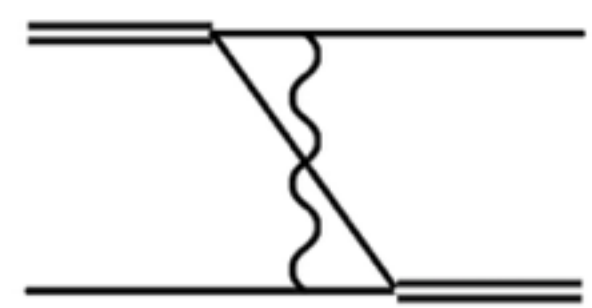
$$\int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2} \frac{1}{(\vec{q} - \frac{\vec{k}}{2})^2 + a^2} \frac{1}{(\vec{q} - \frac{\vec{p}}{2})^2 + b^2} = \frac{1}{\kappa_1^2 \kappa_2^2} \int \frac{d^3l}{(2\pi)^3} \frac{1}{(\vec{l} - \frac{\vec{k}}{2\kappa_1^2})^2 + \frac{a^2}{\kappa_1^4}} \frac{1}{(\vec{l} - \frac{\vec{p}}{2\kappa_2^2})^2 + \frac{b^2}{\kappa_2^4}}$$

$$= \frac{1}{2\pi \kappa_1^2 \kappa_2^2} \frac{1}{|\frac{\vec{k}}{\kappa_1^2} - \frac{\vec{p}}{\kappa_2^2}|} \tan^{-1}\left(\frac{|\frac{\vec{k}}{\kappa_1^2} - \frac{\vec{p}}{\kappa_2^2}|}{2(\frac{a}{\kappa_1^2} + \frac{b}{\kappa_2^2})}\right)$$

Summary of the results

- We treat two-body P-wave contact interaction using a P-wave auxiliary field
- We reproduce the cross-section well at N2LO
- The position of the maximum of Ay is determined by the minimum of the cross-section
- The magnitude of Ay is determined by the two-body P-wave interactions; contribution from S-D mixing is negligible

Summary of the relevant points

- Regulator dependence; the lemma
- Closed form for diagram: 
- Full N3LO calculation of Ay for pd scattering

Thank You