PIONLESS EFT RECOUNTED

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Outline

- Why Pionless
- Telling all about
- Counting again
- To be continued…

Why

The 60s schism: nuclear and particle physics part ways

- "too many divergences"
- "too many interactions"

⇒ fairwell field theory
data fitting eclipses consistency

The 90s re-unification: nuclear again a part of particle physics

- ✓ renormalization
- ✓ power counting

⇒ welcome effective field theory
consistency before fitting

simplest in Pionless EFT

- two-body system analytical with separable regulators
- three-body system numerically clean after two-body regulator removed
- more generally, few-nucleon physics relatively transparent
- close connection with cold-atom physics and other long-distance physics

- Renormalization and power counting playground for nuclear EFTs

naïve dimensional analysis too naïve
for non-perturbative renormalization

Cohen *et al.* '96, '97
…

but message not yet fully digested in Chiral EFT

- A theory of (real) light nuclei

emphasis today

$A \leq 6$ nuclei described up to 30% in LO,
$A \leq 3$ much better to N^2LO

Bedaque + vK '97
…

bound states and low-energy reactions, including symmetry violation

- A theory of nuclei at larger quark masses ("lattice nuclei")

Kirscher's talk?

only EFT for pion masses
in most current LQCD calculations

Barnea *et al.* '13
…

extrapolation of LQCD to heavier nuclei and to reactions

recount: to tell all about

$Q \ll M_{hi} \sim m_\pi$

- d.o.f.: nucleons
- symmetries: Lorentz, ~~P~~, ~~T~~, ~~B~~

$$\mathcal{L}_{EFT} = N^+ \left(i D_0 + \frac{D^2}{2m_N} \right) N + \sum_{I=0,1} C_{0I} \, N^+ N^+ P_I \, N \, N$$

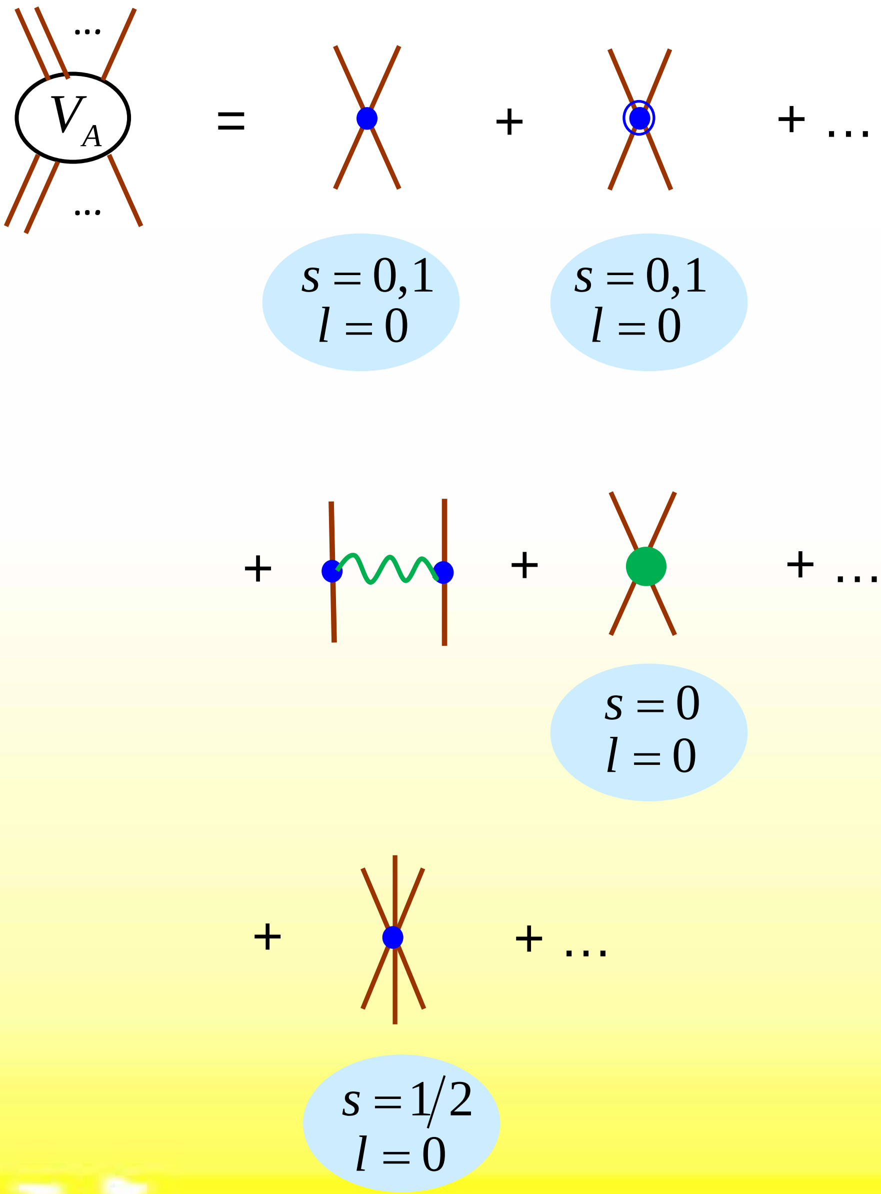
projector on isospin *I*

$$+ D_0 \, N^+ N^+ N^+ N \, N \, N$$

$$+ \Delta C_{0 I_3 = 1} \, N^+ N^+ P_{I_3=1} N \, N + \sum_{I=0,1} C_{2I} \left(N^+ N^+ P_I \, D^2 N \, N + \ldots \right)$$

$$+ \ldots$$

$$D_\mu = \partial_\mu + ie \frac{1+\tau_3}{2} A_\mu$$



$$C_{2n}Q^{2n} \sim \frac{4\pi}{m_N M_{lo}} \frac{Q^{2n}}{M_{lo}^{n} M_{hi}^{n}}$$

$$\sim \frac{4\pi}{m_N M_{lo}} \left(\frac{Q}{M_{hi}}\right)^{2n} \left(\frac{M_{lo}}{M_{hi}}\right)^{1-\#}$$

#=0,1: *s* waves

$$\frac{C_{-2}}{Q^2} \sim \frac{4\pi}{m_N M_{lo}} \frac{\alpha m_N M_{lo}}{Q^2}$$

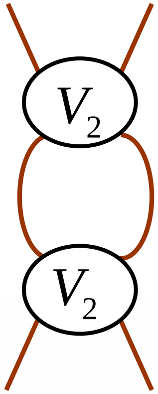
$$\Delta C_{0 I_3=1} \sim \frac{4\pi}{m_N M_{lo}} \frac{\alpha m_N}{M_{lo}}$$

$$D_0 \sim \frac{(4\pi)^2}{m_N} \frac{1}{M_{lo}^4}$$

$$D_2 Q^2 \sim \frac{(4\pi)^2}{m_N} \frac{1}{M_{lo}^4} \frac{Q^2}{M_{hi}^2}$$

$A = 2$

Bedaque + v.K. '97
v.k. '97
Kaplan, Savage + Wise '98
...



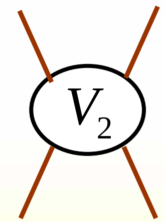
$$\sim \frac{m_N}{4\pi} C_{2n} C_{2n'} \left\{ \# \Lambda^{2(n+n')+1} + \ldots + k^{2(n+n')} \Lambda + i k^{2(n+n')+1} + \mathcal{O}\left(\frac{k^{2(n+n'+1)}}{\Lambda} \right) \right\}$$

absorbed in same or lower order

non-analytic in E

absorbed in higher order

$$\sim \frac{m_N Q}{4\pi} V_2 \times$$



C_0 : series in Q/M_{lo} ➡ non-perturbative for $Q \gtrsim M_{lo}$

$C_{2n>0}$: expansion in Q/M_{hi}

C_{-2} : series in $\alpha m_N / Q$ ➡ non-perturbative for $Q \lesssim \alpha m_N$

expansion in $\alpha m_N M_{lo} / Q^2$

$\Delta C_{0 I_3=1}$: expansion in $\alpha m_N / M_{lo}$

pole of T_A at

$Q_A \sim M_{lo}$

N.B. Perturbative treatment of subLOs not in general optional

1) Except for regular interactions, it can break RG invariance

e.g. iterating C_2 ⇒ $r_2 < 0$ Wigner bound Cohen *et al.* '96, '97

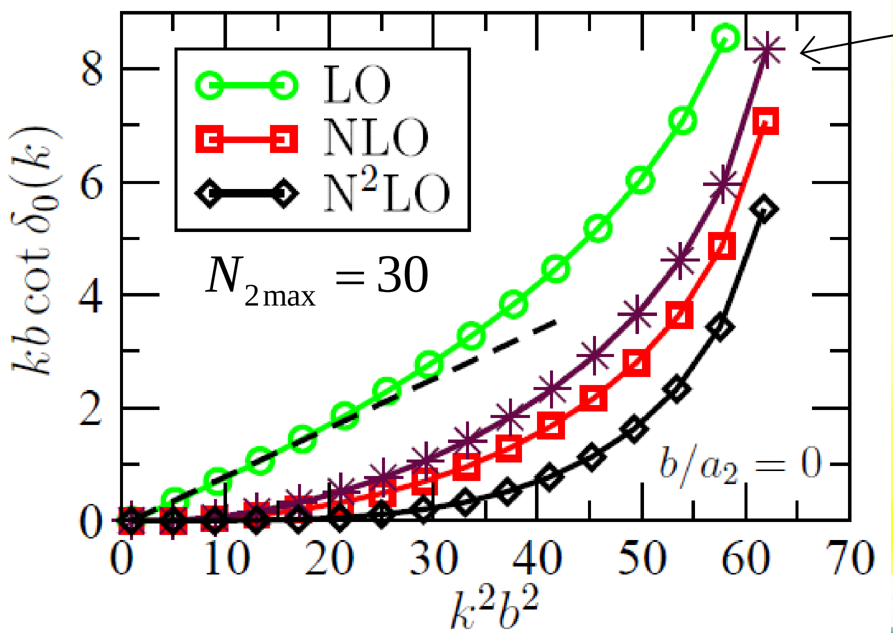
RG invariance

2) Even at fixed cutoff, it can give worse results

e.g.

Rotureau, Stetcu, Barrett + v.K. '10

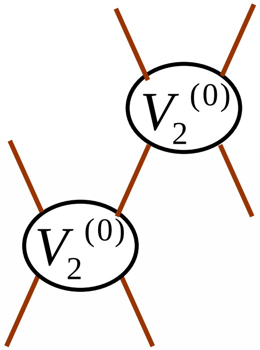
two spin-1/2 fermions at unitarity, in a harmonic oscillator of length *b* and N_{2max} shells



N²LO Hamiltonian fully diagonalized: worse than NLO!

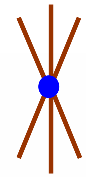
Bedaque, Hammer + v.K. '99 '00
...

$A = 3$



$$\frac{m_N C_0^2}{Q^2} \sim \frac{(4\pi)^2}{m_N} \frac{1}{M_{lo}^2 Q^2}$$

$Q_A \sim M_{lo}$



$$D_0 \sim \frac{(4\pi)^2}{m_N} \frac{1}{M_{lo}^4}$$

$S_{1/2}$

For sharp cutoff $\Lambda_3 \ll \Lambda_2$

$$T_{2+1}^{(0)}(\Lambda_3 \gg p \gg M_{lo}; D_0 = 0) \approx A \cos\left(s_0 \ln \frac{p}{\Lambda_3} + \delta \right)$$

approximate
scale invariance

$s_0 = 1.0064\ldots$

$$\Rightarrow \frac{\Lambda_3}{T_{2+1}^{(0)}} \frac{\partial T_{2+1}^{(0)}}{\partial \Lambda_3} (p \sim M_{lo}; D_0 = 0) \sim 1$$

+ some changes of NDA in other channels

Grießhammer '05

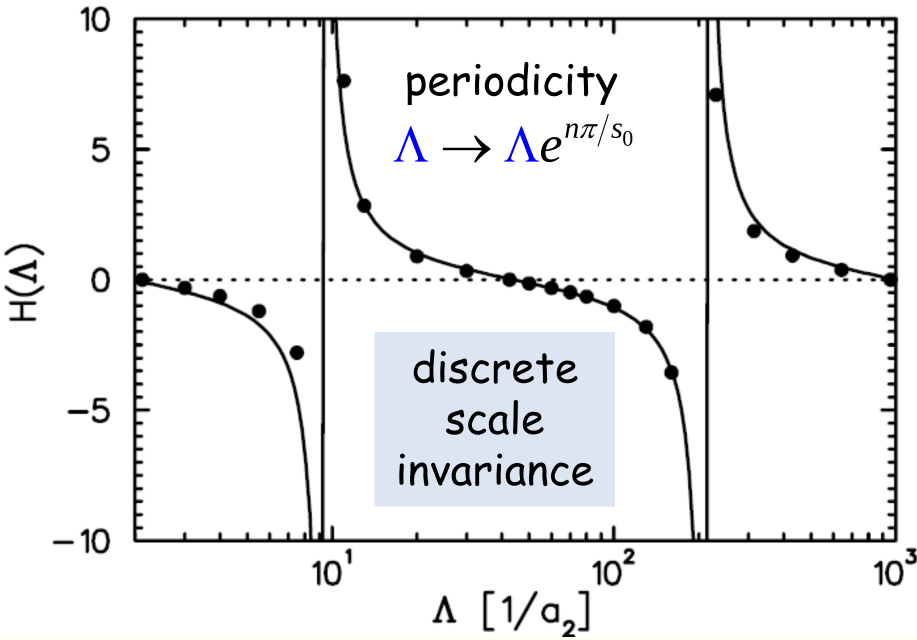
Bedaque, Hammer + v.K. '99 '00

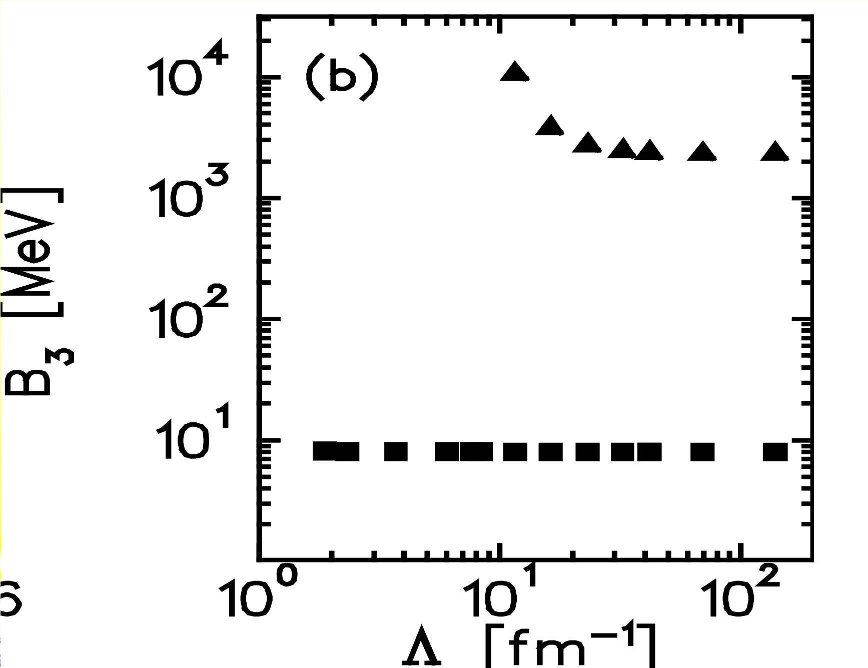
RG invariance:

$$H(\Lambda) \equiv \frac{\Lambda^2 D_0(\Lambda)}{m_N C_0^2(\Lambda)} \simeq \frac{\sin\left(\ln(\Lambda/\Lambda_*) + \arctan(1/s_0)\right)}{\sin\left(\ln(\Lambda/\Lambda_*) - \arctan(1/s_0)\right)}$$

dimensionful parameter
(dimensional transmutation)

RG limit cycle!





outside EFT
$Q_3 \sim M_{hi}$

triton
$Q_3 \sim M_{lo}$

Efimov state

Role of deep bound states in $A \geq 4$ calculations?

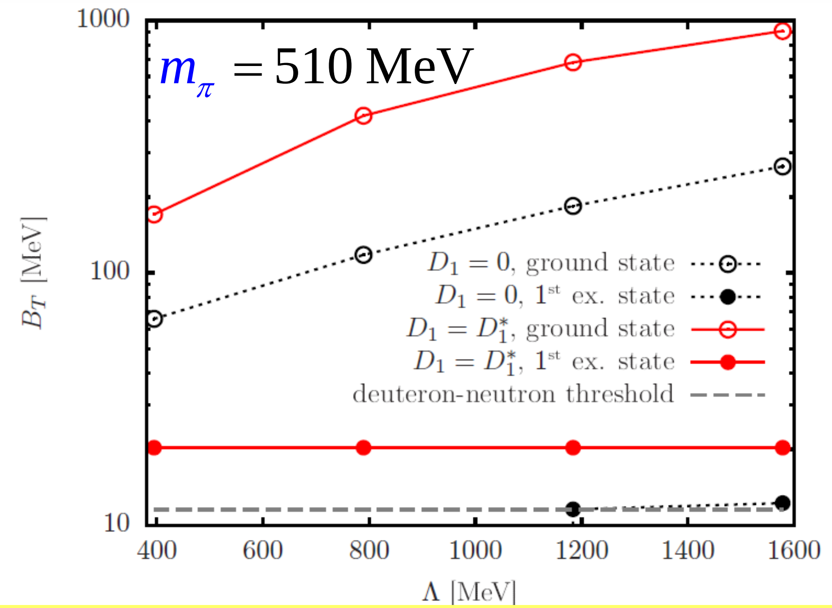
Role of cutoff procedure?

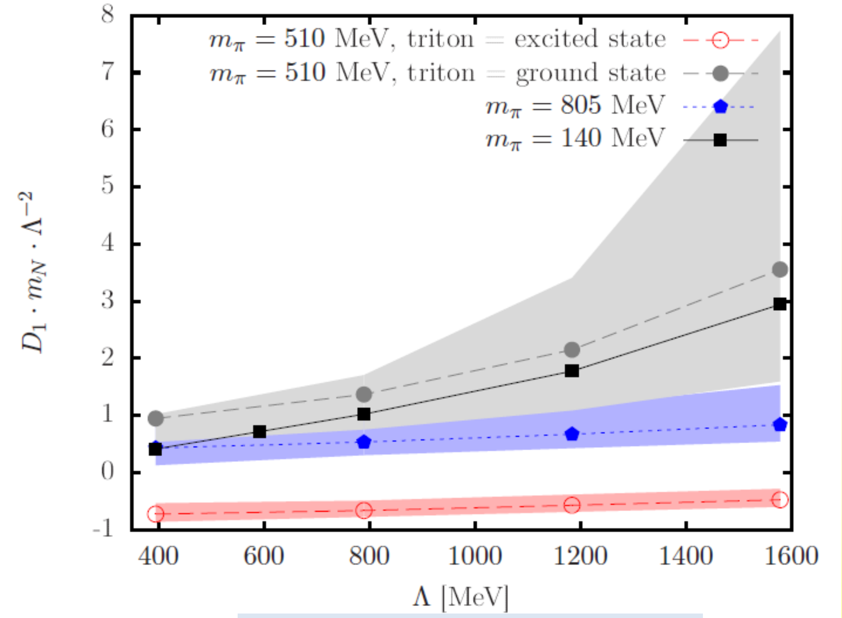
e.g.
$$\begin{cases} \Lambda_2 = \Lambda_3 \\ \delta^{(3)}(\vec{r}) \to \frac{\Lambda^3}{16\pi^{3/2}} e^{-\Lambda^2 r^2/4} \end{cases}$$

Kirscher, Barnea, Gazit, Pederiva + v.K. '15

$$B_t = 20.3 \pm 4.5 \text{ MeV (LQCD)}$$

$$D_1(\Lambda) \equiv -\frac{\Lambda^3}{16\pi^{3/2}} D_0(\Lambda)$$





no visible cycle yet

D_2 : expansion in Q^2/M_{hi}^2

Bedaque, Hammer + v.K. '99 '00

…

Vanasse '13

…

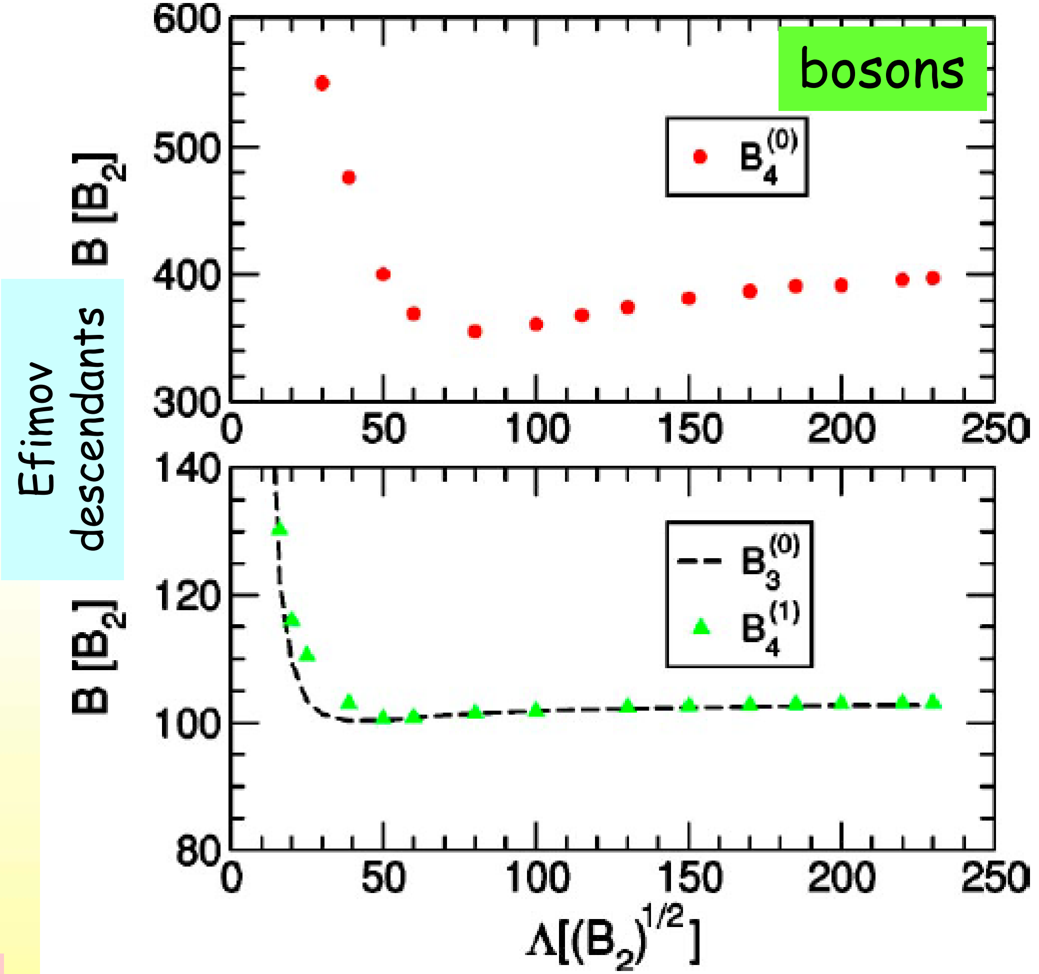
Analogous for
higher derivatives?

$A = 4$

Hammer, Meißner + Platter '04, '05
Hammer + Platter '07

At what order a four-body force?

Not at LO:



Not at NLO*

Kirscher, Shukla, Grießhammer + Hofmann '09

$$E_0 \sim \frac{(4\pi)^2}{m_N}\frac{1}{M_{hi}^7} \quad \text{(NDA)} \quad ?$$

Standard Power Counting

$$M_{lo} \sim \frac{1}{|a_{2,I=0}|} \sim \frac{1}{|a_{2,I=1,I_3}|} \sim \alpha m_N$$

- treat *d* and *d** the same way

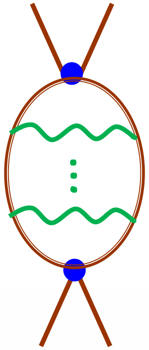
Chen, Rupak + Savage '99
Bedaque, Hammer + v.K. '99
...

- Coulomb is non-perturbative for bound states

Kirscher, Shukla, Grießhammer + Hofmann '09
Ando + Birse '10
...

- quark mass splitting effects?

Kirscher + Phillips '11
König + Hammer '14

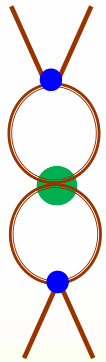


$$\left(\frac{m_N Q}{4\pi}\right)^L \left(\frac{4\pi\alpha}{Q^2}\right)^{L-1} C_{0,I=1}^2 \left(\delta_{L,2} \ln \frac{\Lambda}{\alpha m_N} + \mathcal{O}(1)\right) P_{I_3=1}$$

Kong + Ravndal '00

...

RG invariance



$$\left(\frac{m_N}{4\pi}\right)^2 \Delta C_{I_3=1} \left(\Lambda + Q\right)^2 C_{0,I=1}^2 P_{I_3=1}$$

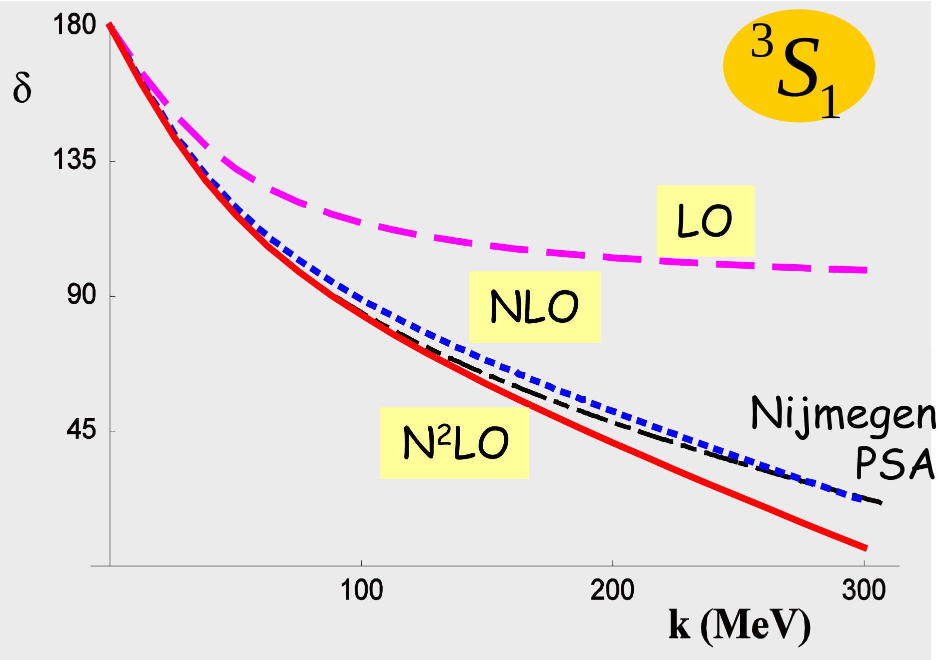
$$\Delta C_{0 I_3=1}(\Lambda) \sim 4\pi\alpha \frac{\ln\left(\alpha m_N / \Lambda\right)}{\Lambda^2}$$

$$\Delta C_{0 I_3=1} \sim \frac{4\pi}{m_N M_{lo}} \frac{\alpha m_N}{M_{lo}}$$

NLO: no new two-body counterterm since Coulombless argument does not change

new three-body force at NLO or not?

Kirscher's and Vanasse's talks?



$A = 2$

Chen, Rupak + Savage '99

$a_1 = 5.42$ fm (exp) ➡ $C_{0,I=0}$

$r_1 = 1.75$ fm (exp) ➡ $C_{2,I=0}$

predict

$B_d = 1.91$ MeV (NLO)

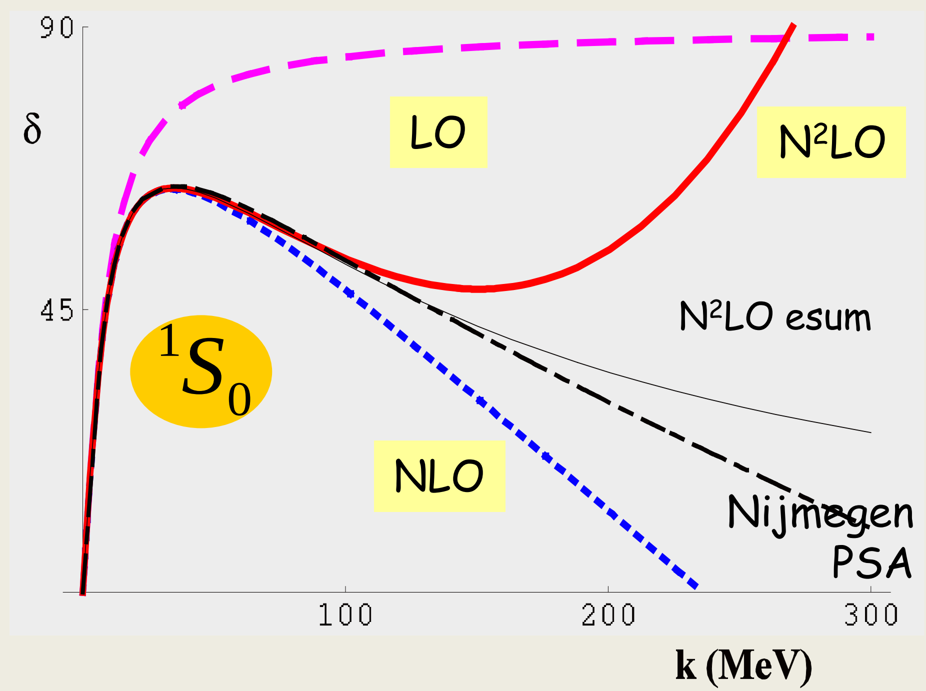
$B_d = 2.22$ MeV (exp)

$a_0 = -20.0$ fm (exp) ➡ $C_{0,I=1}$

$r_0 = 2.78$ fm (exp) ➡ $C_{2,I=1}$

predict

$B_{d^*} = 0.09$ MeV (NLO)

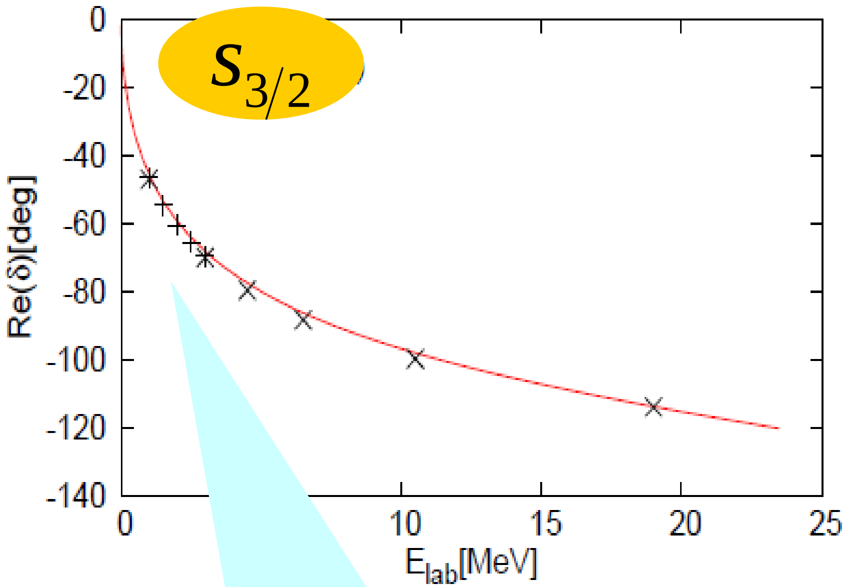


Also *P* waves (N³LO) Vanasse *et al.* '15

Vanasse '13

$A=3$ fully perturbative calculation cf. Ji *et al.* '12

predict

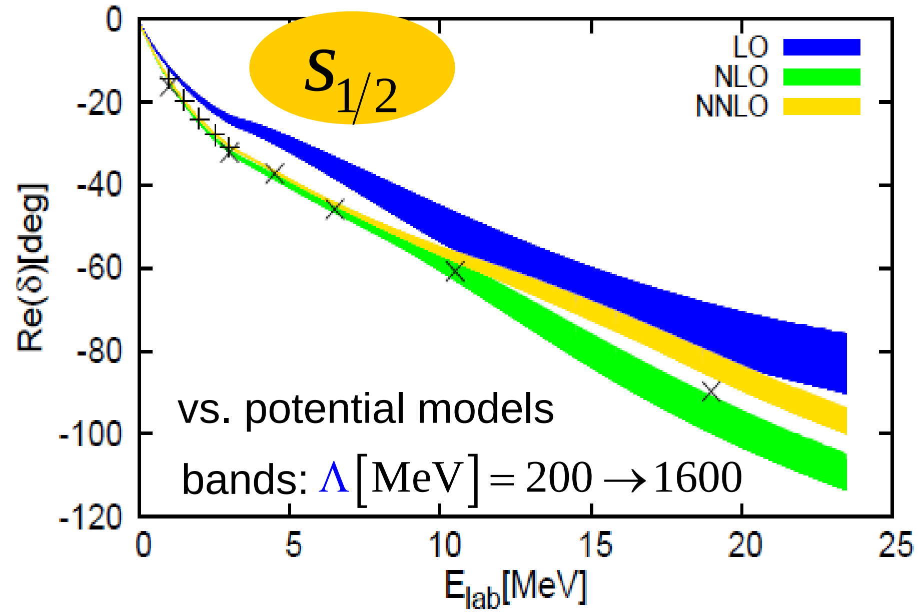


$$a_{3,3/2} = 6.19 \pm 0.03 \text{ fm } (\text{N}^2\text{LO})$$
$$a_{3/2} = 6.35 \pm 0.02 \text{ fm } (\text{exp})$$

$a_{3,s=1/2} = -0.65$ fm (exp) ⇨ D_0

$B_t = -8.48$ fm (exp) ⇨ D_2

predict



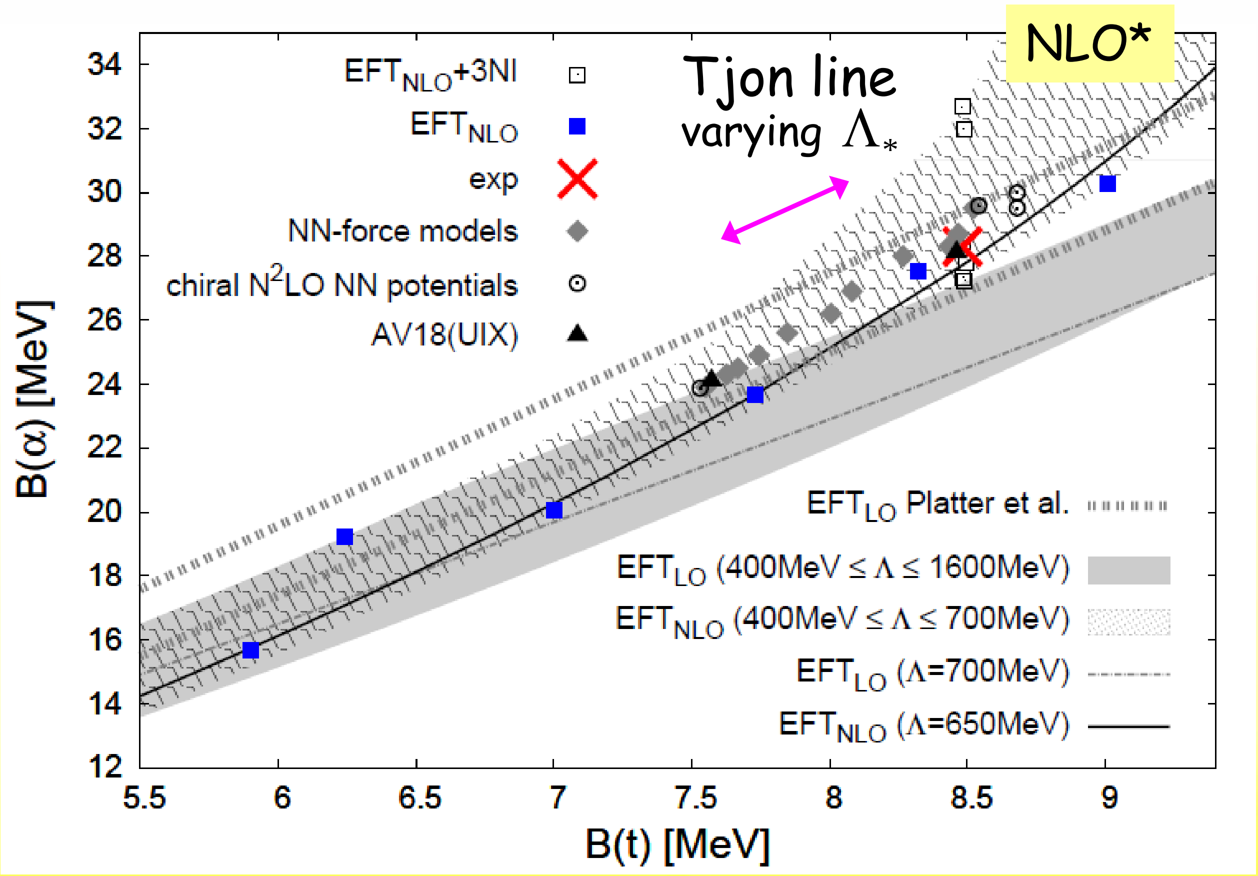
Also higher waves...

...
Vanasse '13
...

Hammer, Meissner + Platter '04
Kirscher, Shukla, Griesshammer + Hofmann '07

$A = 4$

NLO iterated?
NLO LECs?
Coulomb counterterm?

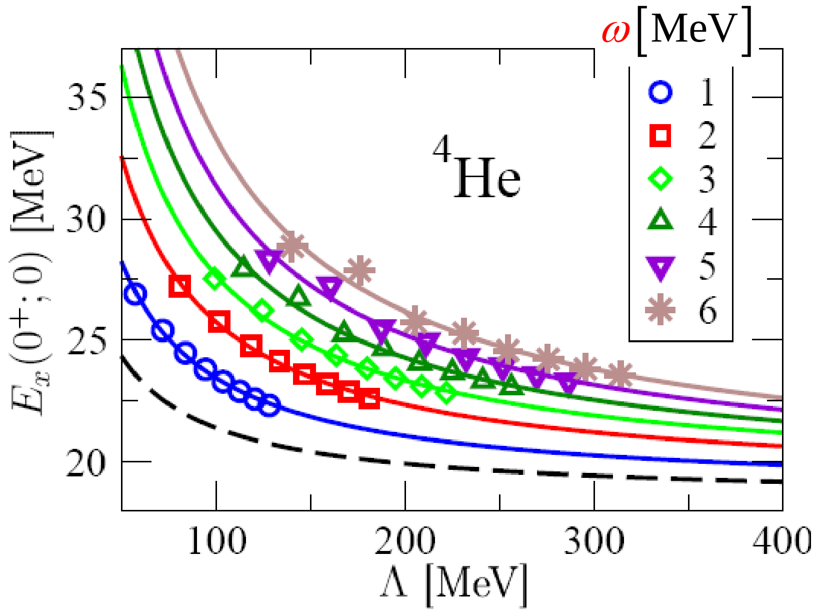


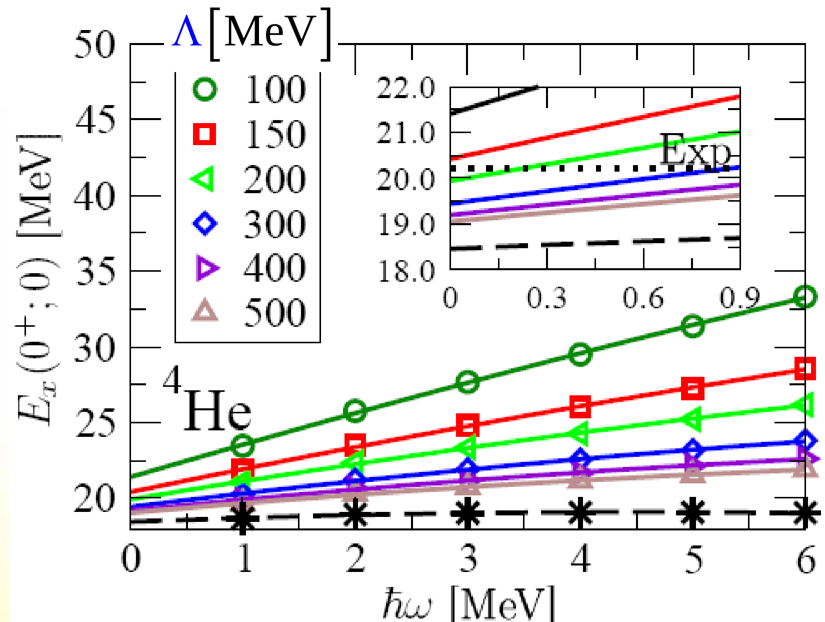
harmonic oscillator basis of frequency ω and N_{max} shells $\Lambda \sim \sqrt{N_{max} m_N \omega}$

LO
- parameters fitted to *d*, *t*, α ground-state energies
- Coulomb assumed higher order

Stetcu, Barrett + v.K. '06

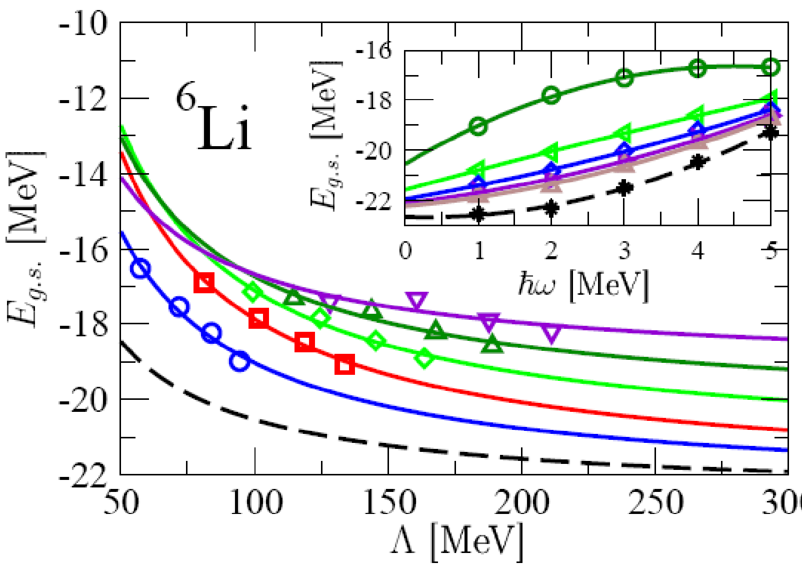
$A = 4$





within ~10%

$A = 6$



within ~30%

How far in *A* can we push Pionless EFT?

recount: to count again

ground states

A	$Q_A \simeq \sqrt{2m_N\ B_A/A}$ (MeV)
2	45
3	70
4	115
5	100
6	100
$\ldots$	$\ldots$
∞	125

$\left\|a_{2,I=1,I_3=0}\right\|^{-1}$	8
αm_N	7
$\left\|a_{2,I=1,I_3=+1}\right\|^{-1} - \left\|a_{2,I=1,I_3=0}\right\|^{-1}$	17
$\left\|a_{2,I=1,I_3=-1}\right\|^{-1} - \left\|a_{2,I=1,I_3=0}\right\|^{-1}$	3

$$Q_A \sim M_{lo}$$

$$\left|a_{2,I=1,I_3=0}\right|^{-1} \equiv \aleph_0 \sim \frac{M_{lo}^2}{M_{hi}}$$

$$\left|a_{2,I=1,I_3=+1}\right|^{-1} - \left|a_{2,I=1,I_3=0}\right|^{-1} \sim \alpha m_N \sim \aleph_0$$

$$\left|a_{2,I=1,I_3=-1}\right|^{-1} - \left|a_{2,I=1,I_3=0}\right|^{-1} \sim \frac{M_{lo}^3}{M_{hi}^2}$$

consistent with

$$\left|a_{2,I=1,I_3=+1}\right|^{-1} - \left|a_{2,I=1,I_3=0}\right|^{-1} = \mathcal{O}\left(\alpha m_N, m_d - m_u\right)$$

$$\left|a_{2,I=1,I_3=-1}\right|^{-1} - \left|a_{2,I=1,I_3=0}\right|^{-1} = \mathcal{O}\left(m_d - m_u\right)$$

while

$$m_n - m_p = \mathcal{O}\left(\frac{\alpha m_N}{4\pi}, m_d - m_u\right)$$

König, Grießhammer, Hammer + v.K. '15

- treat ground states as usual

  but two-nucleon spin-singlet *S* wave expanded around unitarity

- Coulomb is perturbative for bound states,

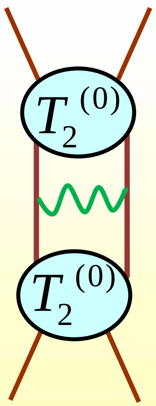
  and needs to be resummed only near scattering thresholds

cf. Hammer, Meißner + Platter '05
Stetcu, Barrett + v.K. '07

- smaller quark mass splitting effects smaller

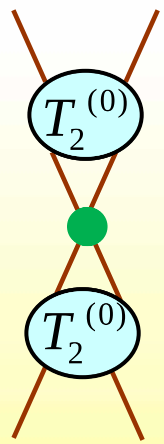
also cf. Kirscher + Gazit '15

Kirscher's talk?



$$\left(\frac{m_N}{4\pi}\right)^2 4\pi\alpha\left(\ln\frac{\Lambda}{\alpha m_N}+\mathcal{O}(1)\right)T^{(0)2}P_{I_3=1}$$

RG invariance



$$\left(\frac{m_N}{4\pi}\right)^2 \Delta C_{I_3=1}\left(\Lambda+Q\right)^2 T^{(0)2}P_{I_3=1}$$

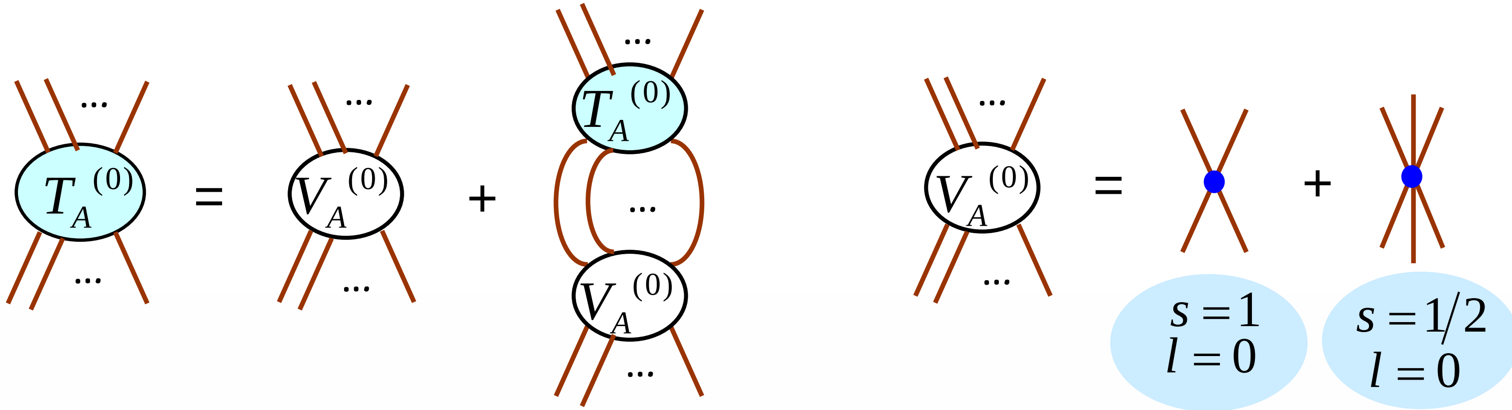
$$\Delta C_{0I_3=1}(\Lambda)\sim 4\pi\alpha\frac{\ln\left(\alpha m_N/\Lambda\right)}{\Lambda^2}$$

$$\Delta C_{0I_3=1}\sim\frac{4\pi}{m_N M_{lo}}\frac{\alpha m_N}{M_{lo}}$$

+ no three-body LEC: can predict 3He up to NLO

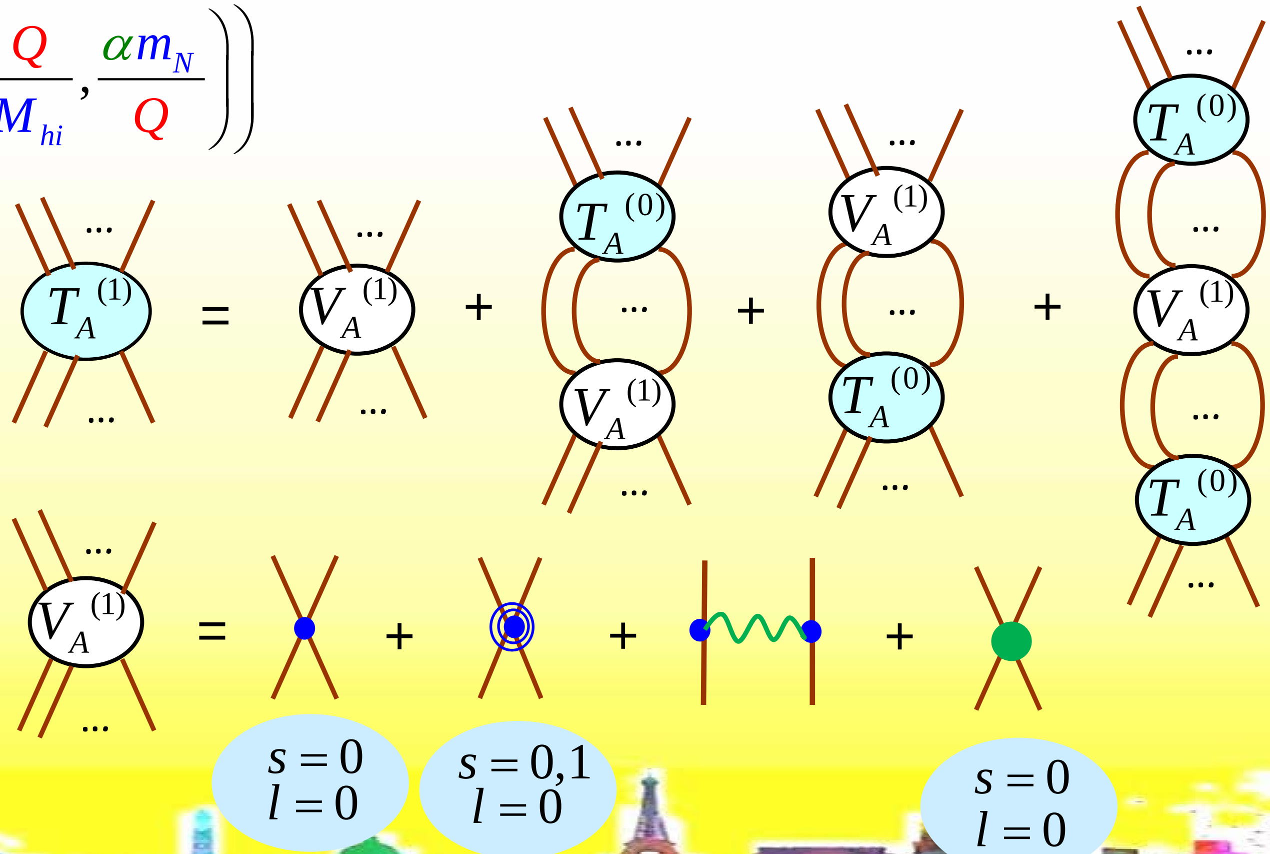
LO

$$\mathcal{O}\left(\frac{4\pi}{m_N M_{lo}}\right)$$



NLO

$$\mathcal{O}\left(\frac{4\pi}{m_N M_{lo}} \times \left(\frac{Q}{M_{hi}}, \frac{\alpha m_N}{Q}\right)\right)$$



etc.

point:
smaller number of LECs at each order,
more predictive power

For more, see

Sebastian's talk

Conclusions

- Convergence and standard power counting without Coulomb well understood up to N^2LO for $A \leq 3$, needs to be further studied for $A \geq 4$
- "Technical" issues remain about structure of three-body force and effects on $A \geq 3$
- Convergence and standard power counting with Coulomb not settled for $A \geq 3$
- New power counting more predictive for bound states using expansion around unitarity in two-nucleon spin singlet

... to be revised at the end of the workshop!