Range corrections in proton halo nuclei

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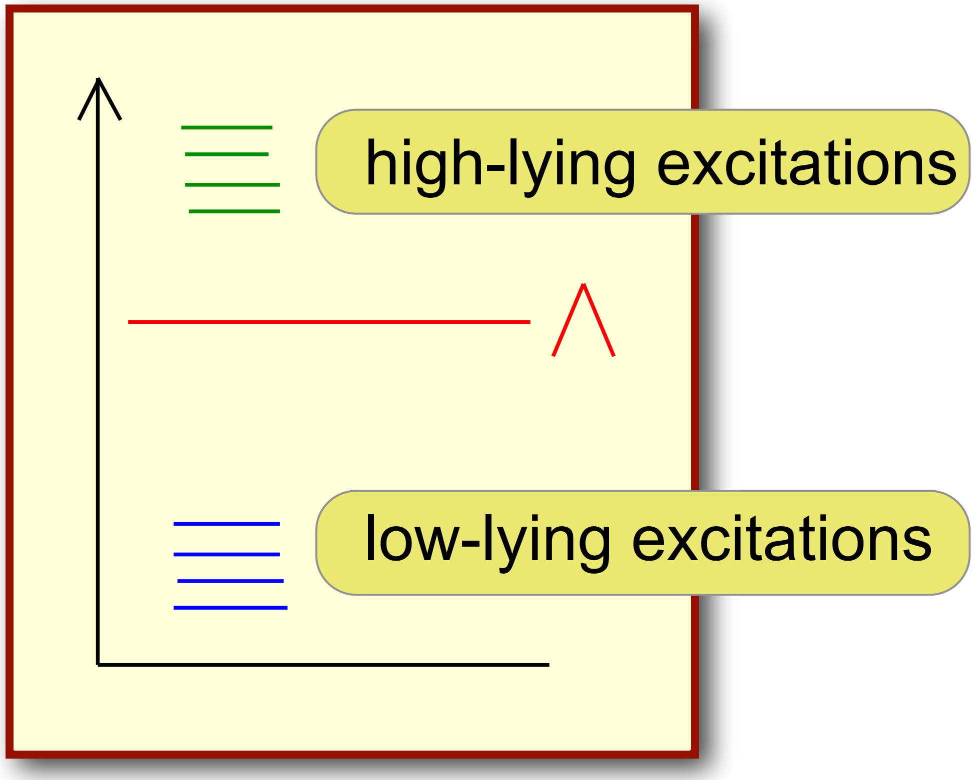
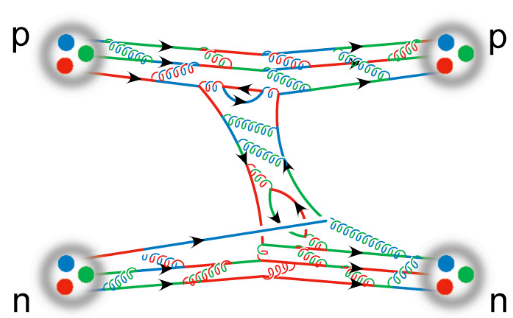
Outline

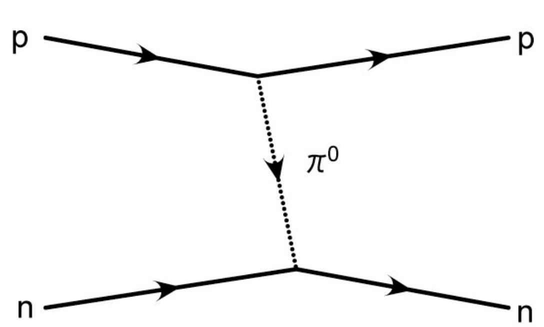
- theoretical description of effective **two-body systems**
  - proton halos
  - Fluorine-17
  - Boron-8
- universality in the presence of a finite effective range
- maybe: uncertainty estimates proton-proton fusion

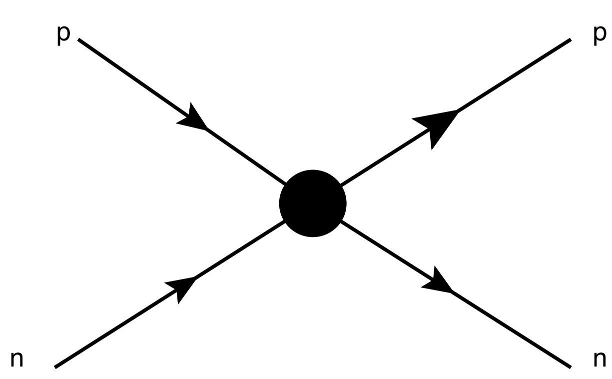
Effective Field Theory for Low-Energy Nuclear Physics

- use **effective field theory (EFT)** expansion to account for details of interaction
- use minimal set of degrees of freedom
  - depends on resolution scale
- exploit separation of scales:
  expand in **small** momentum k scale
  over **large** scale Λ
  - infinite # of coupling parameters





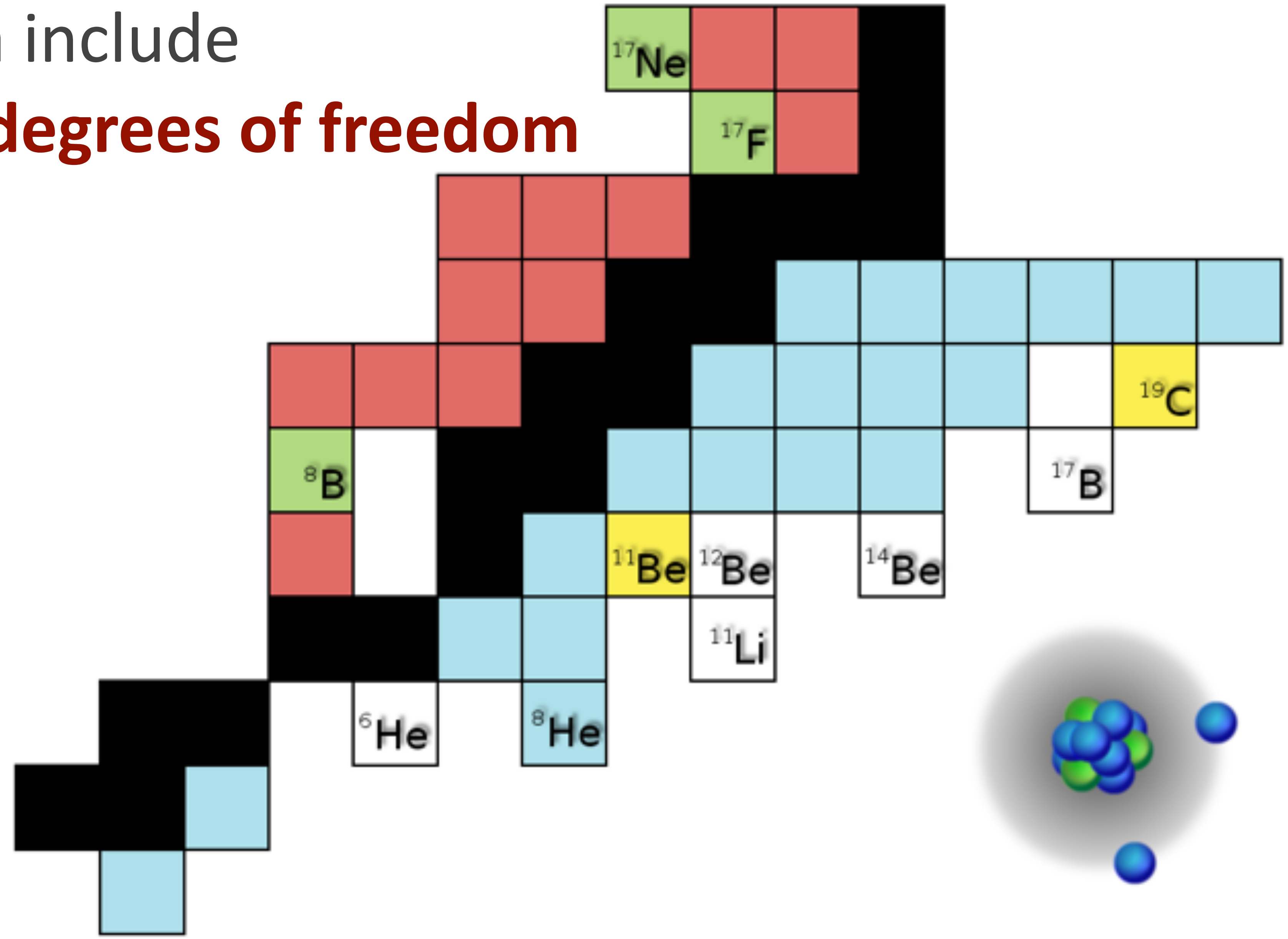




Nuclear Halos

- dripline phenomena include
  emergence of new degrees of freedom

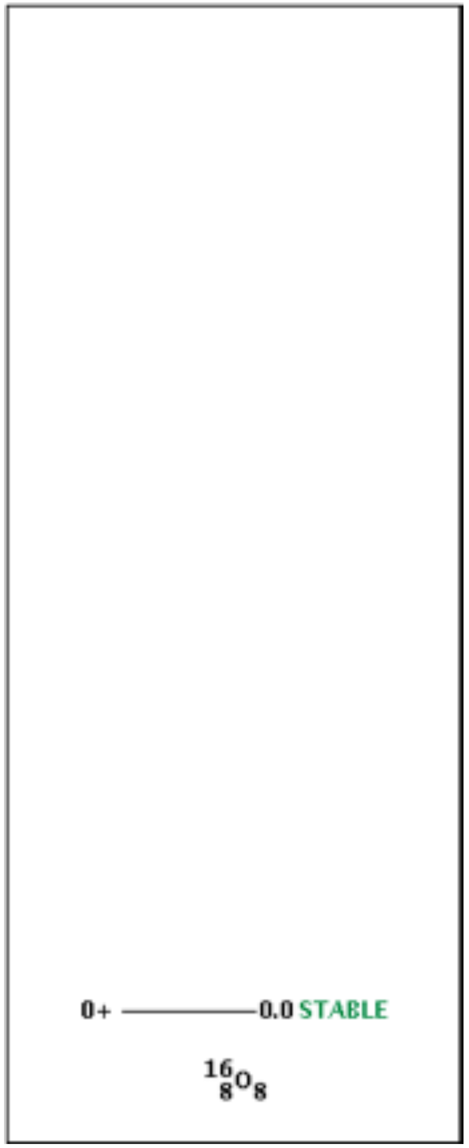
- Halo:
  tightly bound core
  + weakly bound
  spectator nucleons

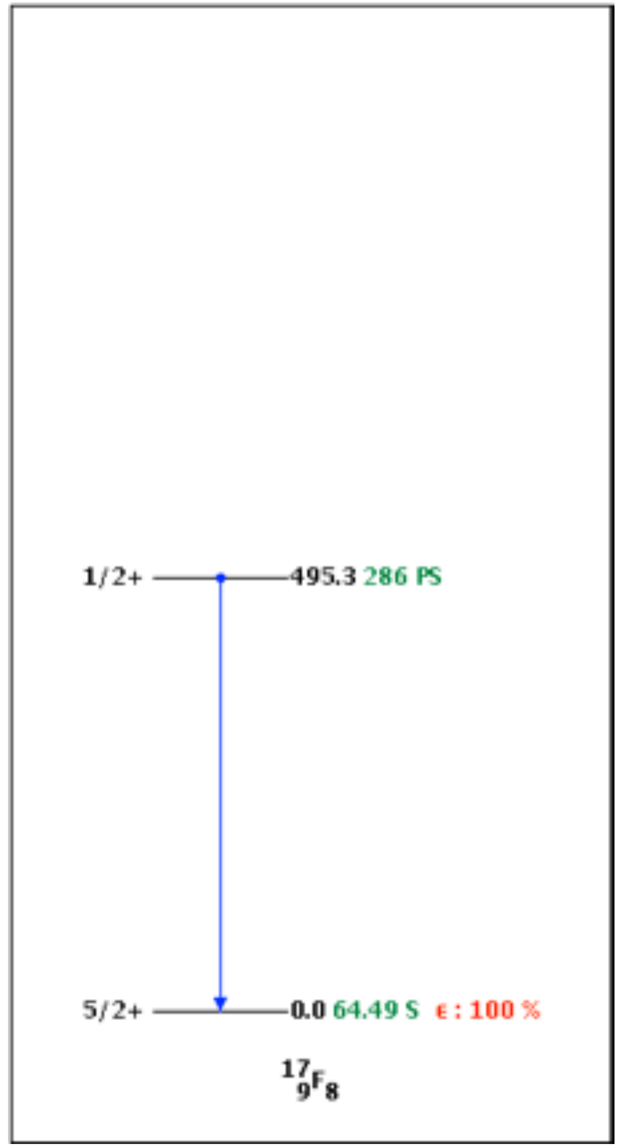


- 1-neutron halos considered in EFT by
  Bedaque, van Kolck, Hammer, Phillips, ...

- What about
  Proton Halos?

Separation of Scales in Fluorine-17





- 2 stable weakly-bound states

$$S_p = 600.27\,\mathrm{keV}$$
$$S_n = 16800\,\mathrm{keV}$$

- Oxygen-16 tightly bound lowest lying excitation at ~6 MeV

➡ **Excited State of Fluorine-17 is S-wave Proton Halo**

Construct Halo EFT for excited state in 17F (Ryberg, Hammer, Forssen & LP arXiv 2015)

- simple Lagrangian that includes only excited state dynamics
- core and proton treated as pointlike particles

$$\mathcal{L} = \sum_{k=0,1} \psi_k^\dagger \left[i\mathrm{D}_0 + \frac{\mathbf{D}^2}{2m_k} \right] \psi_k + d^\dagger \left[\Delta + \nu \left(i\mathrm{D}_0 + \frac{\mathbf{D}^2}{2M_{\mathrm{tot}}} \right) \right] d$$
$$- g \left[\psi_1^\dagger \psi_0^\dagger d + \mathrm{h.c.} \right]$$

- We take Coulomb into account non-perturbatively

Coulomb Interaction

- We include the Coulomb interaction by using the QM Green's function

$$\langle \mathbf{r} | G_{\mathrm{C}}(E) | \mathbf{r} \rangle = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{\psi_{\mathbf{p}}(\mathbf{r}) \psi^*_{\mathbf{p}}(\mathbf{r}')}{E - \mathbf{p}^2/(2m_{\mathrm{R}}) + i\varepsilon}$$

- Coulomb wave functions are partial wave decomposed

$$\psi_{\mathbf{p}}(\mathbf{r}) = \sum_{l=0}^{\infty} (2l+1) i^l \exp\left(i\sigma_l\right) \frac{F_l(\eta,\rho)}{\rho} P_l(\hat{\mathbf{p}} \cdot \hat{\mathbf{r}})$$

- Feynman rules have to be adjusted to take this feat into account

Input Parameters

- Use binding energy and asymptotic normalization coefficient (ANC) A for renormalization

$$w_l(r) = A W_{i\eta, l+1/2}(2\gamma r)$$

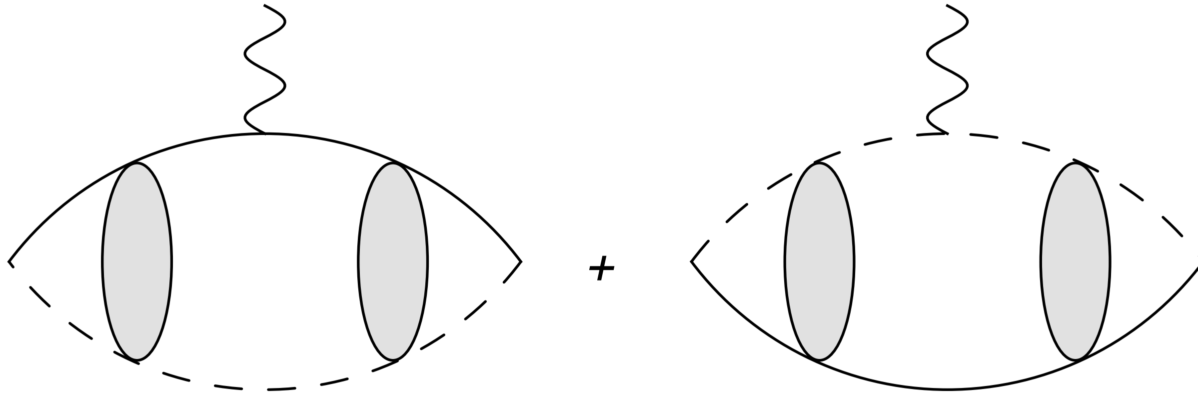
- this way, fix Z-factor and pole position of halo propagator
- sometimes: observables don't change from one order to another

$$\mathcal{Z} \propto \frac{\pi}{m_{\mathrm{R}}^2 [\Gamma(1 + k_C/\gamma)]^2} A^2$$

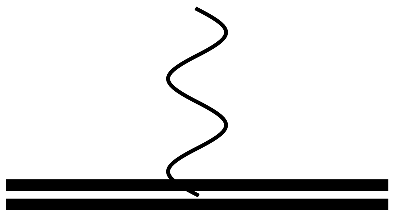
Charge Radius of excited state of ^{17}F

- Feynman diagrams lead to simple expressions

$$i\Gamma^0_{\rm LO}(\mathbf{Q}) = ieZ_{\rm core}\int {\rm d}^3r(0|G_{\rm C}(-B)|\mathbf{r})\exp\left(if\mathbf{Q}\cdot\mathbf{r}\right)(\mathbf{r}|G_{\rm C}(-B)|0) + \left[(f\to 1-f),\ (Z_{\rm core}\to 1)\right]$$



$$i\Gamma^0_{\rm NLO} = i\nu e(Z_{\rm core}+1)\ .$$

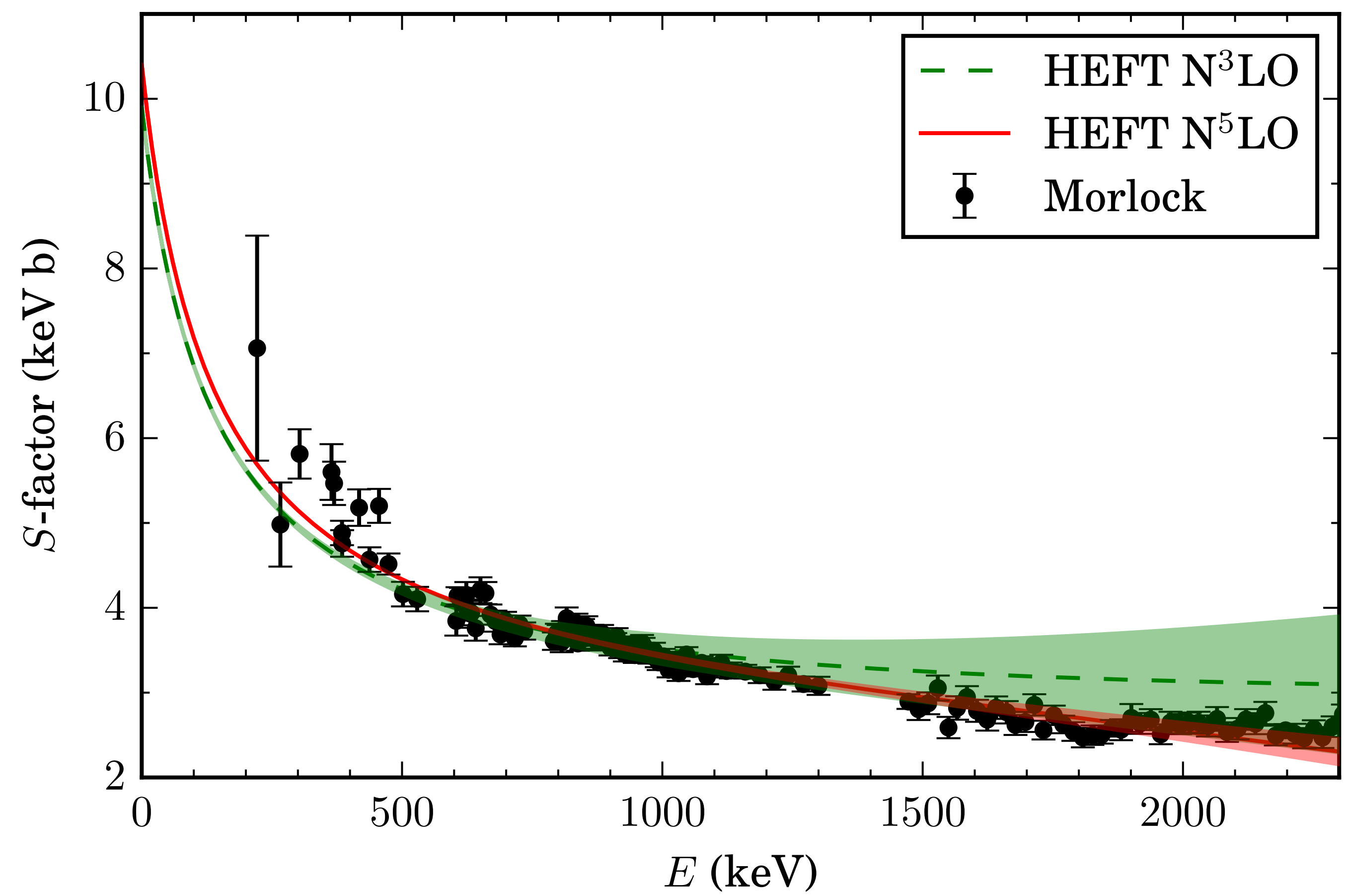


- Result for charge radius (relative to core) using ANC and S_p

$$r_{\rm C,NLO} = (2.20\pm 0.04\ ({\rm EFT})\pm 0.11\ ({\rm ANC}))\ {\rm fm}$$

Determine ANCs from EFT

- fit ANC (Z-factor) to experimental data
- obtain ANCs with error estimate for different orders

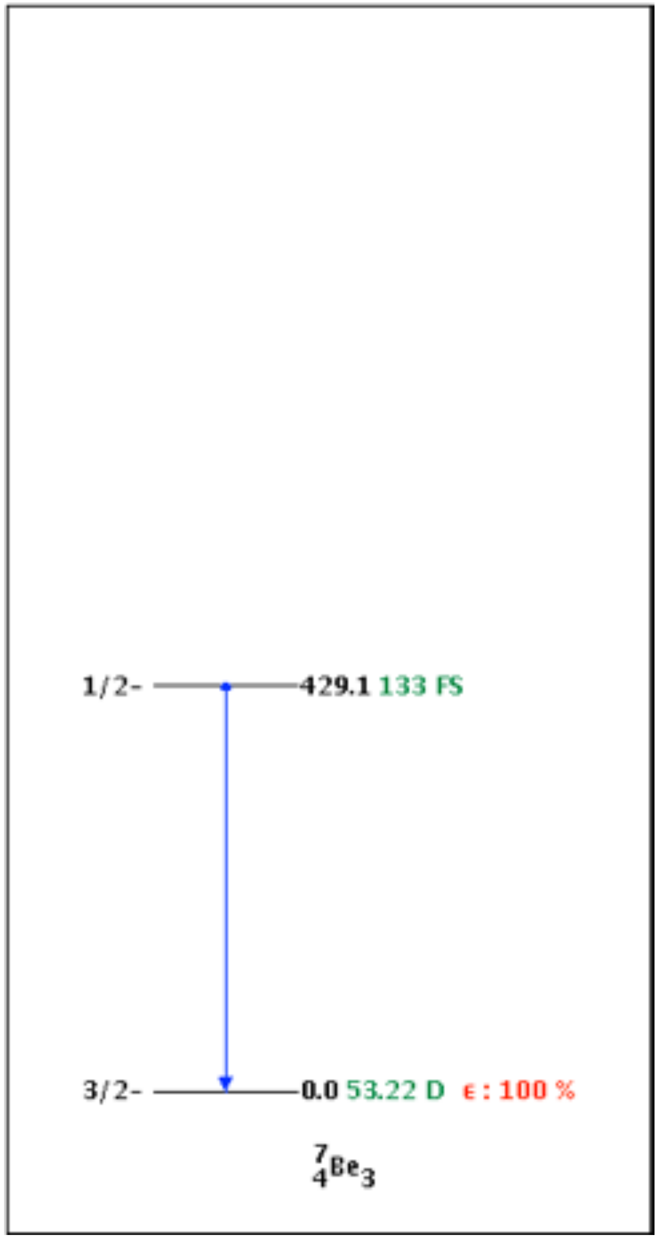


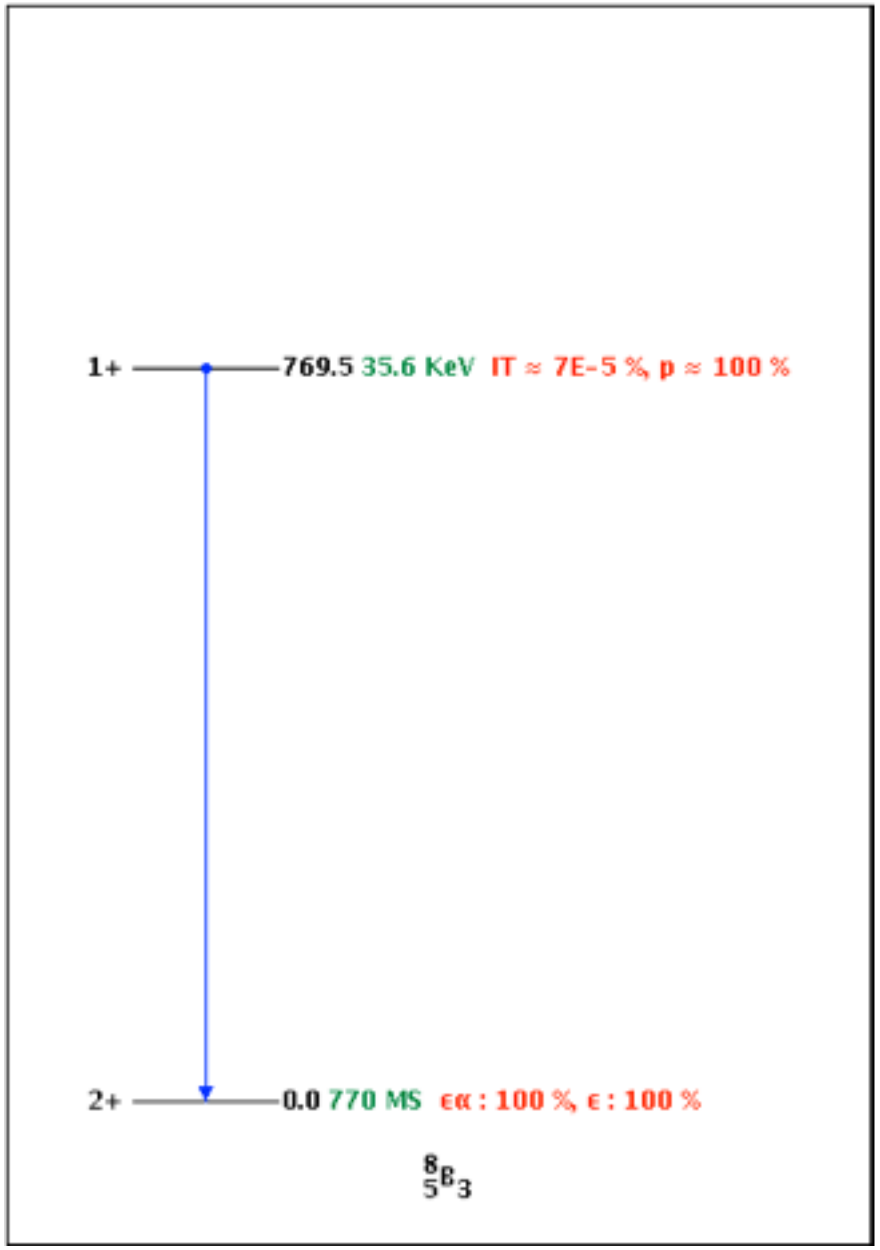
$$A = \begin{cases} \left(76.9 \pm 4.3\ (\text{fit}) \pm 7.7\ (\text{norm.})\right)\ \text{fm}^{-1/2}\ , & \text{N}^3\text{LO} \\ \left(78.9 \pm 4.8\ (\text{fit}) \pm 7.9\ (\text{norm.})\right)\ \text{fm}^{-1/2}\ , & \text{N}^5\text{LO} \end{cases}$$

$$S = \begin{cases} \left(9.9 \pm 0.1\ (\text{fit}) \pm 1.0\ (\text{norm.})\right)\ \text{keV b}\ , & \text{N}^3\text{LO} \\ \left(10.4 \pm 0.1\ (\text{fit}) \pm 1.0\ (\text{norm.})\right)\ \text{keV b}\ , & \text{N}^5\text{LO} \end{cases}$$

compares well with Huang et al. and Gagliardi et al.

Scale Separation in Boron-8
(Ryberg, Forssen, Hammer & LP EPJA 2015)





- Boron-8 p-wave proton halo

$$S_p = 137.5\,\mathrm{keV}$$
$$S_n = 13020\,\mathrm{keV}$$

- relevant for nuclear astrophysics ➞ solar neutrinos

Setting up the EFT

- Two Spin Channels (S=1 & S=2)

$$
\begin{aligned}
\mathcal{L} = {} & \mathrm{p}_\sigma^\dagger\left(i\mathrm{D}_t + \frac{\mathbf{D}^2}{2m}\right)\mathrm{p}_\sigma + \mathrm{c}_a^\dagger\left(i\mathrm{D}_t + \frac{\mathbf{D}^2}{2M}\right)\mathrm{c}_a \\
& + \tilde{\mathrm{c}}_\sigma^\dagger\left(i\mathrm{D}_t + \frac{\mathbf{D}^2}{2M} - E^*\right)\tilde{\mathrm{c}}_\sigma + \mathrm{d}_\alpha^\dagger\left[\Delta + \nu\left(i\mathrm{D}_t + \frac{\mathbf{D}^2}{2M_{\mathrm{tot}}}\right)\right]\mathrm{d}_\alpha \\
& - g_1\left[\mathrm{d}_\alpha^\dagger \mathcal{C}_{jk}^{\alpha}\mathcal{C}_{a\sigma}^{j}\mathrm{c}_a\left((1-f)i\overrightarrow{\nabla}_k - fi\overleftarrow{\nabla}_k\right)\mathrm{p}_\sigma + \mathrm{h.c.}\right] \\
& - g_2\left[\mathrm{d}_\alpha^\dagger \mathcal{C}_{\beta k}^{\alpha}\mathcal{C}_{a\sigma}^{\beta}\mathrm{c}_a\left((1-f)i\overrightarrow{\nabla}_k - fi\overleftarrow{\nabla}_k\right)\mathrm{p}_\sigma + \mathrm{h.c.}\right] \\
& - g_*\left[\mathrm{d}_\alpha^\dagger \mathcal{C}_{jk}^{\alpha}\mathcal{C}_{\sigma\chi}^{j}\tilde{\mathrm{c}}_\chi\left((1-f)i\overrightarrow{\nabla}_k - fi\overleftarrow{\nabla}_k\right)\mathrm{p}_\sigma + \mathrm{h.c.}\right] + \ldots\ ,
\end{aligned}
$$

- p is proton field,
  c is spin 3/2 ground state core field,
  c~ spin-1/2 excited state core field
- $g=g_1^2+g_2^2$ always appear as sum; only as one parameter

Renormalization conditions

- We use asymptotic normalization coefficients and binding/excitation energies

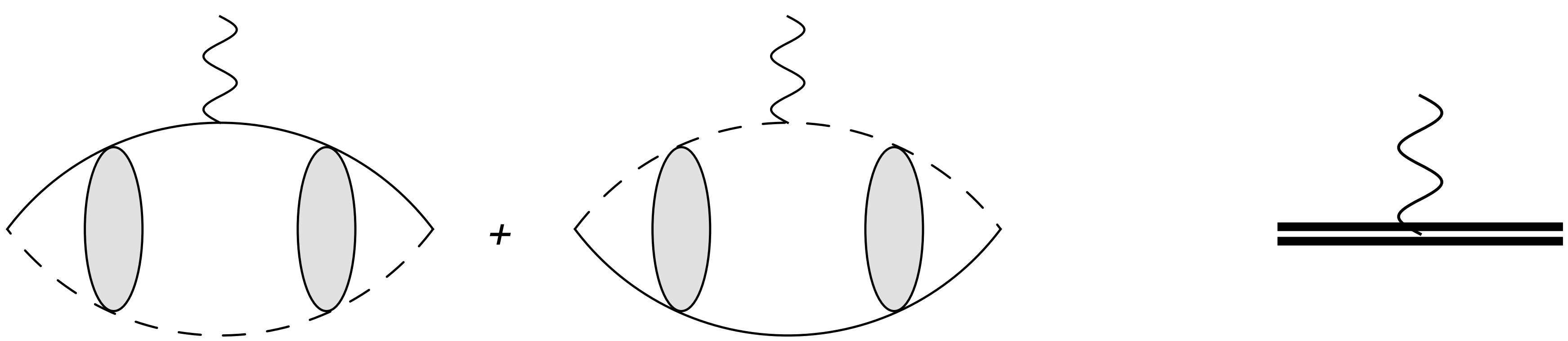
$$A_1^2 + A_2^2 = 2\gamma^2\Gamma(2 + k_{\rm C}/\gamma)^2\left[-r_1 + \frac{2k_{\rm C}}{m_{\rm R}}\frac{\rm d}{\rm dE}\left(h_1(\eta) + \frac{g_*^2}{g^2}h_1(\eta_*)\right)\Big|_{E=-B}\right]^{-1},$$

$$A_*^2 = 2\gamma_*^2\Gamma(2 + k_{\rm C}/\gamma_*)^2\left[-\frac{g^2}{g_*^2}r_1 + \frac{2k_{\rm C}}{m_{\rm R}}\frac{\rm d}{\rm dE}\left(\frac{g^2}{g_*^2}h_1(\eta) + h_1(\eta_*)\right)\Big|_{E=-B}\right]^{-1}$$

- also allows us to extract also the effective range

Charge Radius

- charge radius will get measured (Noertersheuser, Mueller, ...)
- Feynman diagrams for Charge radius relative to Beryllium-7



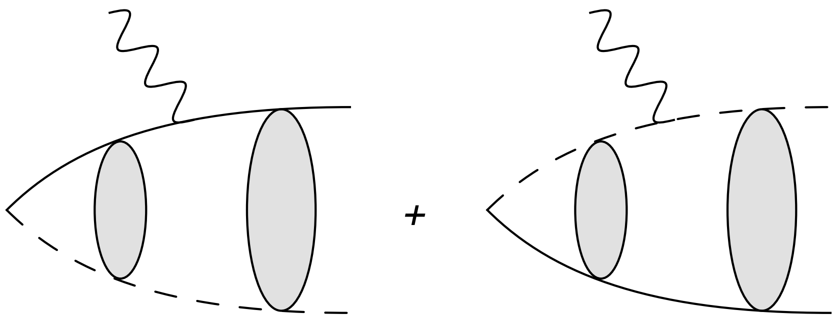
- parameters fixed from S_p and ab initio ANCs

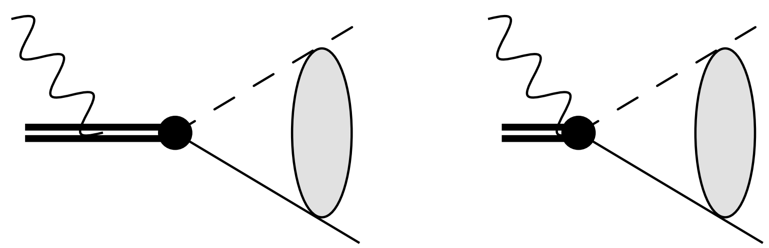
$$r_C^2 = \begin{cases} (2.56 \pm 0.08\ \text{fm})^2 & (\text{Nollett ANCs}) \\ (2.50\ \text{fm})^2 & (\text{Navrátil ANCs}) \\ (2.41 \pm 0.18\ \text{fm})^2 & (\text{Tabacaru ANCs}) \end{cases}$$

- Beryllium radius added in quadrature (comp w/ Pastore -> $(2.6\ \text{fm})^2$

Capture reaction

- We can also evaluate capture diagrams





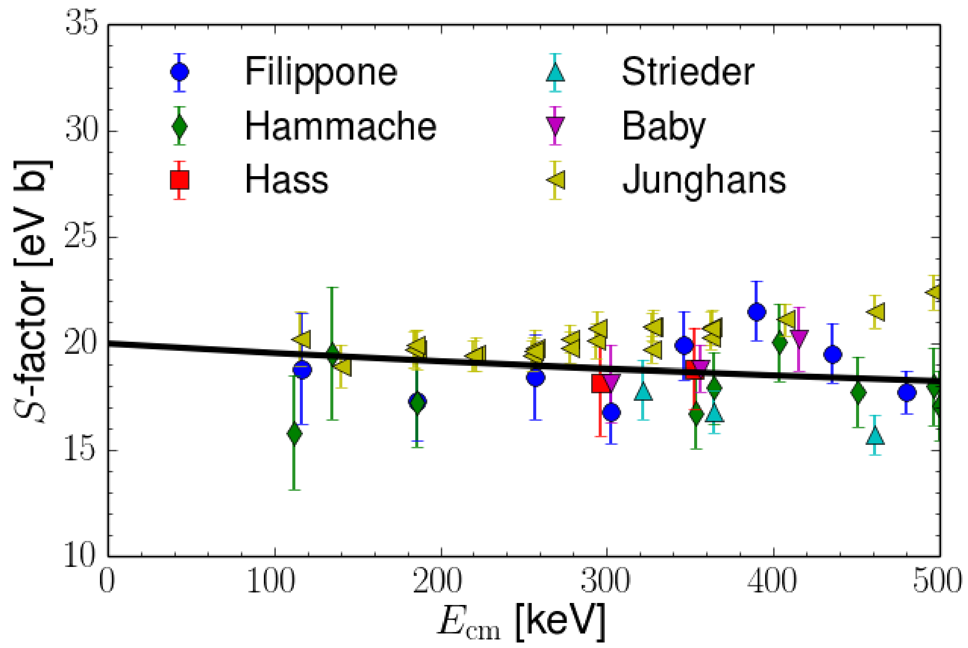
(a) (b)

- and obtain the threshold S-factor

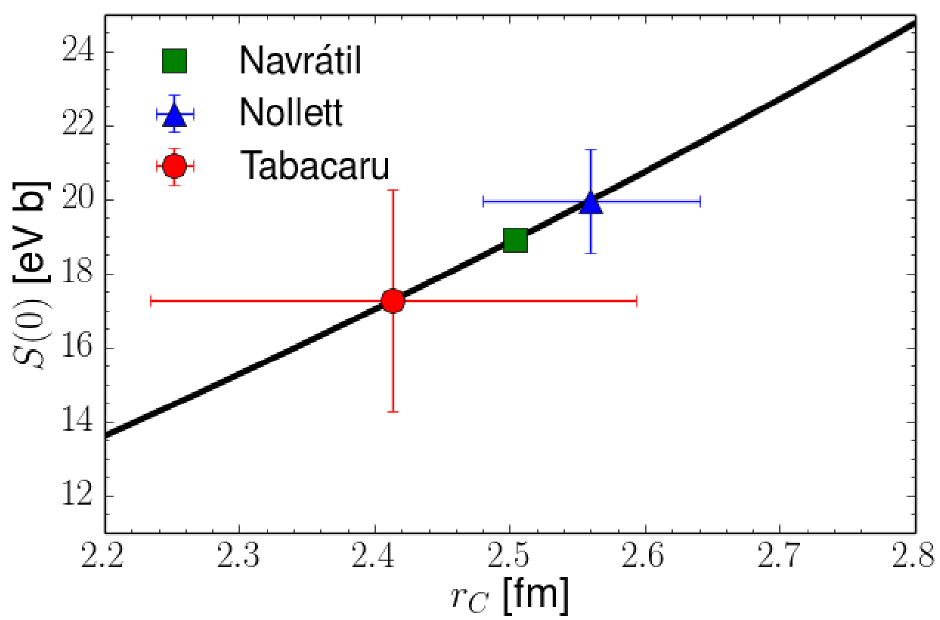
$$S(0) = \begin{cases} (20.0 \pm 1.4)\ \text{eV b} & (\text{Nollett ANCs}) \\ 18.9\ \text{eV b} & (\text{Navrátil ANCs}) \\ (17.3 \pm 3.0)\ \text{eV b} & (\text{Tabacaru ANCs}) \end{cases}$$

S-factor (compare Zhang, Nollett & Phillips 2014)

- good agreement w/ experiment

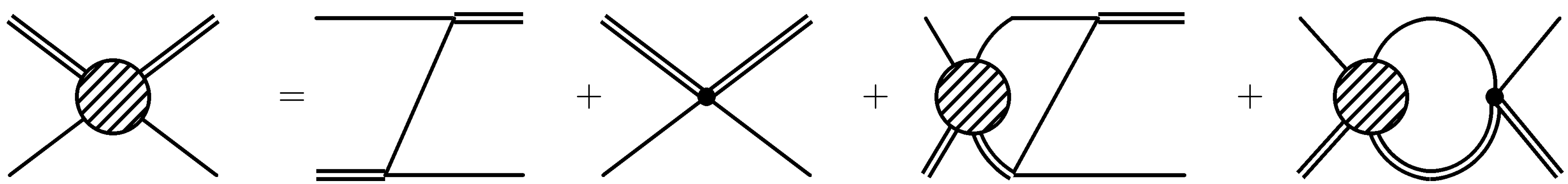


- universal correlation btw radius and S-factor



A slightly different topic: NLO universality

- STM equation *contains* the Efimov effect



w/o 3-body force: Skorniakov & Ter-Martirosian '56

- in QFT find that predictions require 3-body force (Bedaque, Hammer, and van Kolck 1999)
- Efimov: New type of universality: discrete scale invariance, scattering length and one three-body input determine everything else

Universal relations at leading order

- knowledge of one feature gives info on all others:
  - ➡ binding energies in the unitary limit

$$E_{T,n} \longrightarrow \lambda^{-2n}\kappa_*^2/m$$

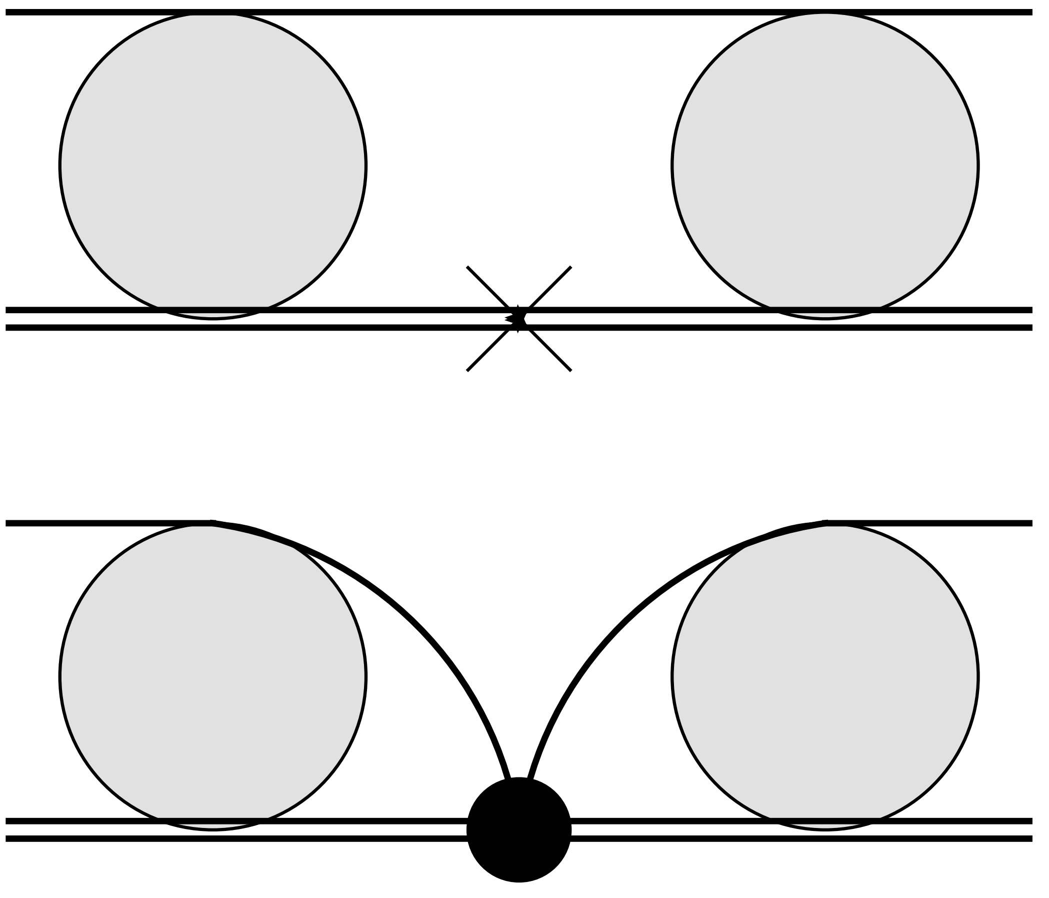
  - ➡ recombination features, e.g. recombination maxima

$$a_{-,n} \longrightarrow \theta_- \lambda^n \kappa_*^{-1}$$

Including the effective range

Hammer&Mehen 2001, Bedaque et al. 2002, Ji&Phillips&LP 2011

- use perturbation theory to include effective range
- find that new counterterm is required when variable scattering length is considered
- regulator dependence of new 3-body force can be derived explicitly



Universal relations at next-to-leading order (Ji, Braaten, Phillips & LP PRA 2015)

- recombination features can be expressed as

$$a_{i,n} = \lambda^n \theta_i \kappa_*^{-1} + (J_i - n\sigma) r_s$$

- J_i is non-universal, however (J_i-J_j) is universal
- relation above can be modified to give ratio of 2 features as function of 2 other feature

Running Efimov Parameter

- form of NLO 3-body force suggest running 3-body parameter

$$\bar{\kappa}_*(Q,a) = (Q/\kappa_*)^{-\gamma r_s/a}\kappa_*$$

- leads to Renormalization-Group improved universal relations

$$a_{i,n} = \lambda^n\theta_i(\lambda^n|\theta_i|)^{-\gamma r_s\kappa_*/(\lambda^n\theta_i)}\kappa_*^{-1} + \tilde{J}_i r_s$$

- can be written as equation for ratios depending on 2 known ratios

Benchmarking the relations

- Compare the universal relations to calculations that employ finite range interaction
- **Deltuva:** Momentum space calculations with short-range separable interaction
- **Schmidt, Rath & Zwerger:** Two-channel model that with 2 parameters and a form factor that gives

Deltuva PRA 2012

$$(a_{-,n+1}/a_{-,n})/\lambda$$

n	0	1	2	3	4
Deltuva 2012	0.7822	0.9665	0.9976	0.9999	1.0000
NLO	0.7822	0.9665	0.9975	0.9998	1.0000
RG-NLO	0.7822	0.9665	0.9975	0.9998	1.0000

Schmidt, Rath & Zwerger

$(a_{-,n+1}/a_{-,n})/\lambda$

n	0	1	2
Schmidt et al.	0.753	0.962	0.998
NLO	<u>0.753</u>	<u>0.962</u>	0.997
RG-NLO	<u>0.753</u>	<u>0.962</u>	0.997
Schmidt et al.	1.008	0.998	0.9998
NLO	<u>1.008</u>	<u>0.998</u>	0.9998
RG-NLO	<u>1.008</u>	<u>0.998</u>	0.9998

$(a_{*,n+1}/a_{*,n})/\lambda$

0	1	2
0.175	1.764	1.029
-8.1	1.150	1.032
0.0002	1.206	1.034
0.757	0.983	1.001
-0.431	0.986	1.002
0.240	0.986	1.002

Summary

- Proton Halos are more fine-tuned
  - weakly bound systems in the presence of the repulsive Coulomb interaction
- obtain input information from *ab initio* calculations
- RG improvement corresponds to some *iteration* of the range
- new RG improved universal relations at NLO