Coulomb Interactions in EFT

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EMMI: The Systematic Treatment of the Coulomb Interaction in Few-Body Systems
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Motivations

Astrophysics

- Low energy reactions dominate
- Need accurate cross sections but hard to measure experimentally
- Model-independent theoretical calculations important

Theoretical

- Weakly bound systems
- First principle calculation

Use Effective Field Theory

Main Issues

- EFT power-counting
- Practical implementation

The first item is of paramount importance because you want to make error estimates even when resumming potentially sub-leading pieces for practical reasons. I concentrate on the power-counting.

EFT: the long and short of it

- Identify degrees of freedom

$$\mathcal{L} = c_0\, \mathrm{O}^{(0)} + c_1\, \mathrm{O}^{(1)} + c_2\, \mathrm{O}^{(2)} + \cdots \quad \text{expansion in } \frac{Q}{\Lambda}$$

Hide UV ignorance
- short distance

IR explicit
- long distance

- Determine c_n from data (elastic, inelastic)

- EFT : ERE + currents + relativity

Not just Ward-Takahashi identity

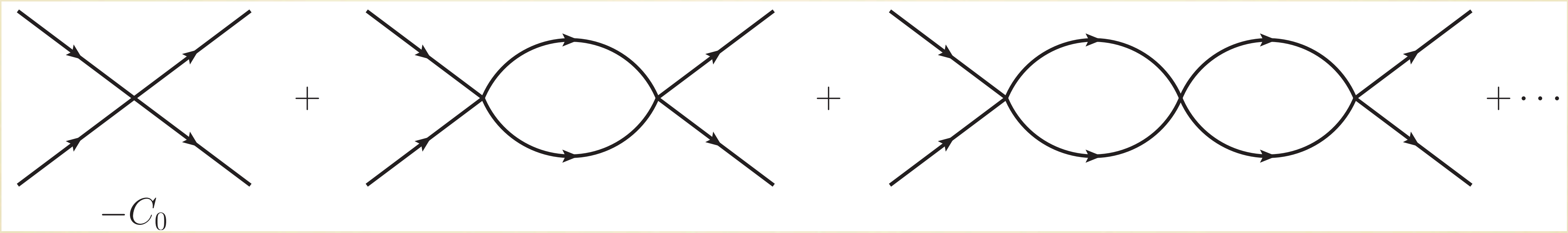
Weakly Bound Systems

$$i\mathcal{A}(p) = \frac{2\pi}{\mu}\frac{i}{p\cot\delta_0 - ip} = \frac{2\pi}{\mu}\frac{i}{-1/a + \frac{r}{2}p^2 + \cdots - ip}$$

--- Large scattering length $a >> 1/\Lambda$

$$i\mathcal{A}(p) \approx -\frac{2\pi}{\mu}\frac{i}{1/a + ip}\left[1 + \frac{1}{2}\frac{rp^2}{1/a + ip} + \cdots\right]$$

EFT non-perturbative

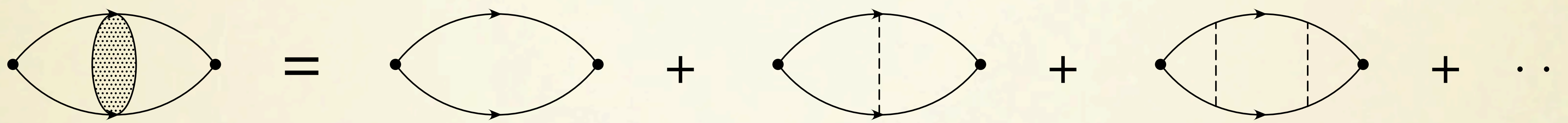


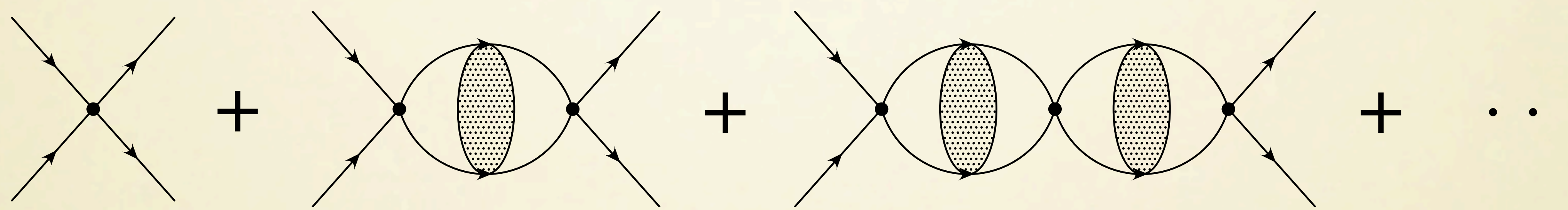
$$i\mathcal{A}(p) = \frac{-i}{\frac{1}{C_0} + i\frac{\mu}{2\pi}p} \Rightarrow C_0 = \frac{2\pi a}{\mu}$$

large coupling

Weinberg '90
Bedaque, van Kolck '97
Kaplan, Savage, Wise '98

Proton-proton in EFT



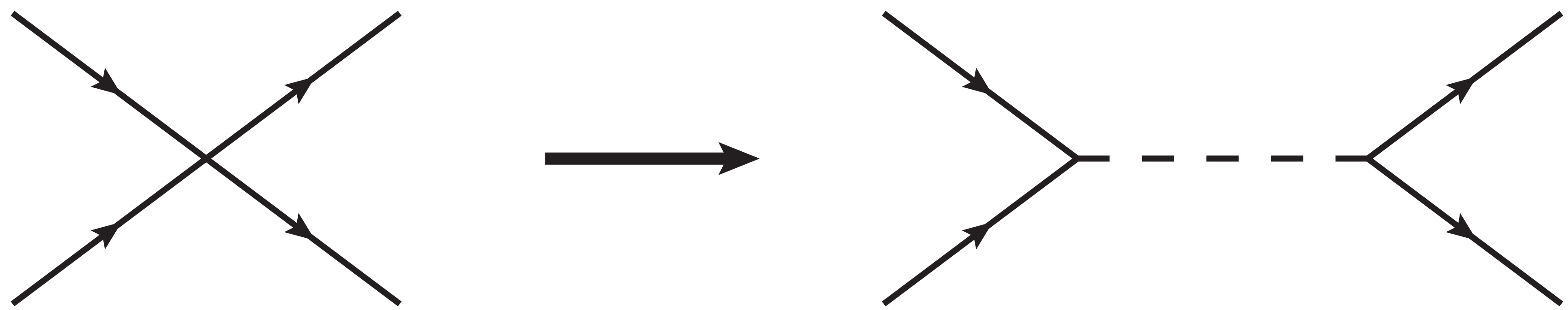


$$\frac{2\pi}{\mu C_0} = \frac{1}{a} - \frac{\lambda}{d-3} - 2\alpha\mu[\frac{1}{d-4} - \ln\frac{\mu\pi}{\alpha M} - 1 + \frac{3}{2}C_E]$$

Kong, Ravndal 1999

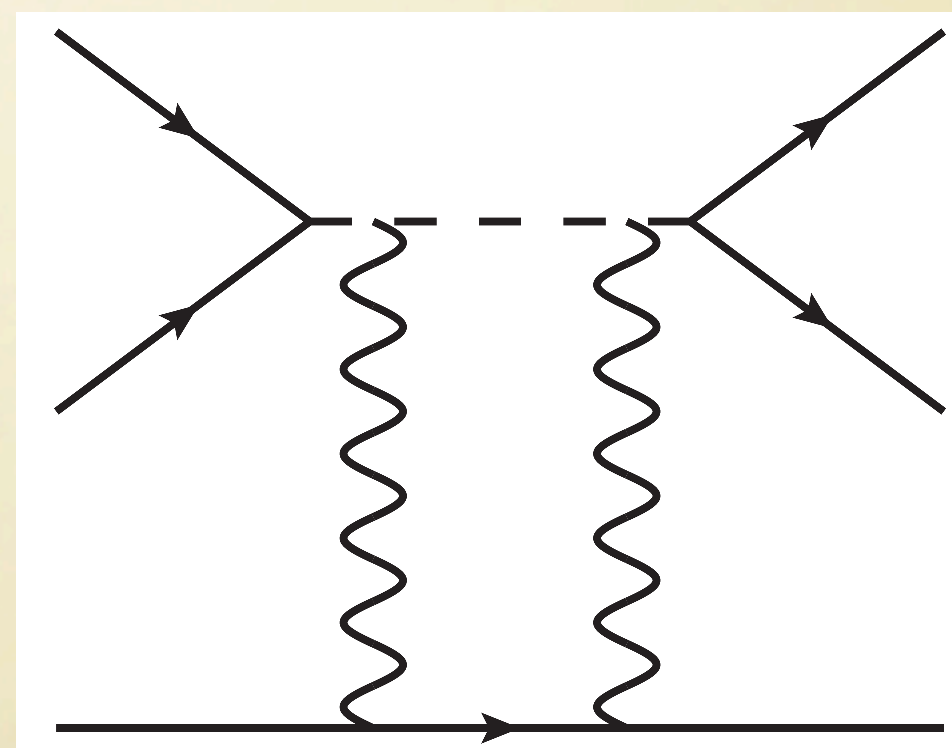
In EFT there is no way to relate coupling from n-p and p-p channel.

Dimer-field

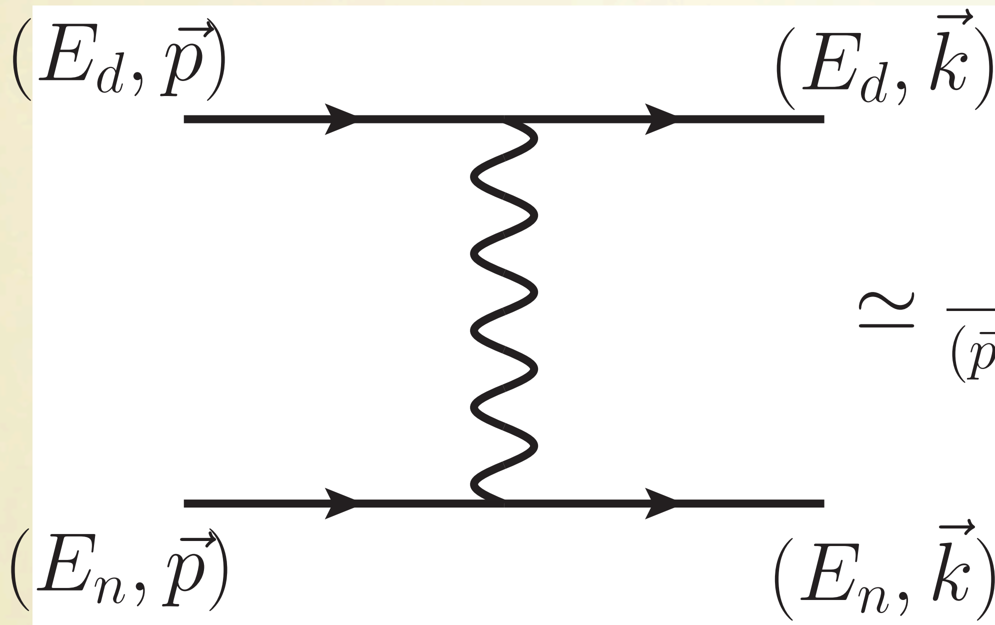


$$C_0 \to \frac{g^2}{\Delta}$$

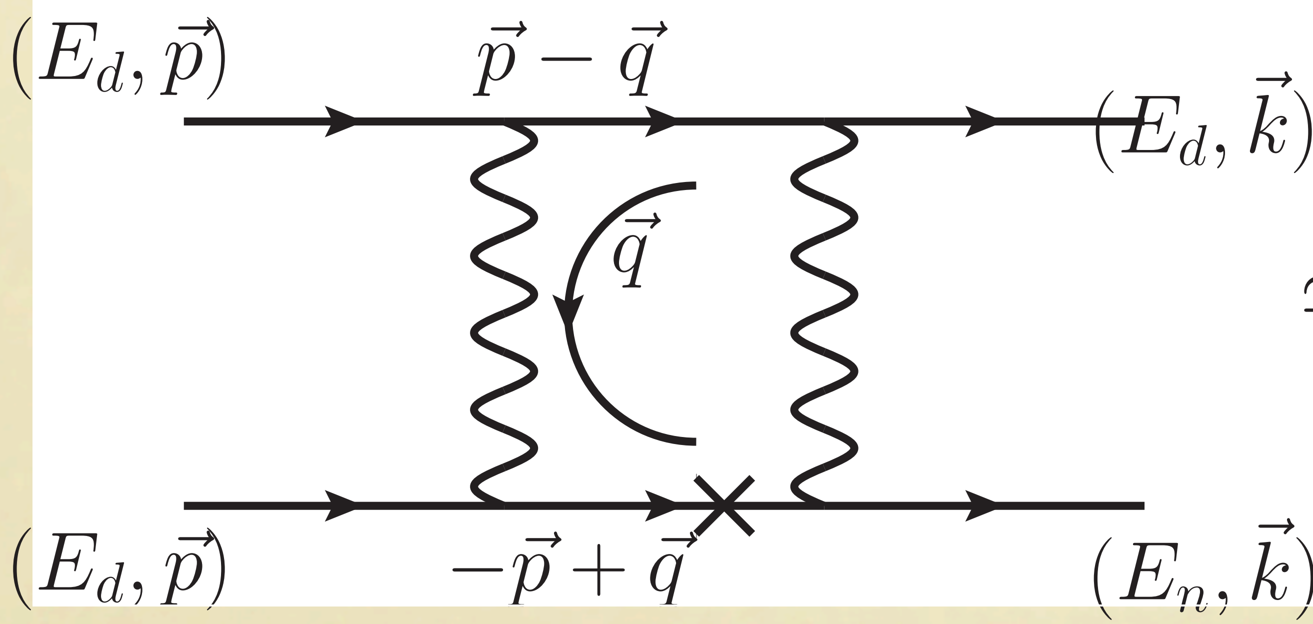
Doesn't work so nicely for p-d. No enhancement of Coulomb ladder: physics missing!



Fundamental Deuteron



$$\simeq \frac{e^2}{(\vec{p}-\vec{k})^2} \sim \frac{e^2}{p^2}$$



$$\simeq e^4 \int d^3q \frac{1}{E_T-(q-p)^2/(2\mu)} \frac{1}{q^2(q-p+k)^2}$$

$$\sim \frac{e^2}{p^2}\frac{\alpha\mu}{p}$$

N-PHOTON EXCHANGES

$$n\text{th} - \text{loop} :\sim \frac{e^2}{p^2}\left(\frac{\alpha\mu}{p}\right)^n$$

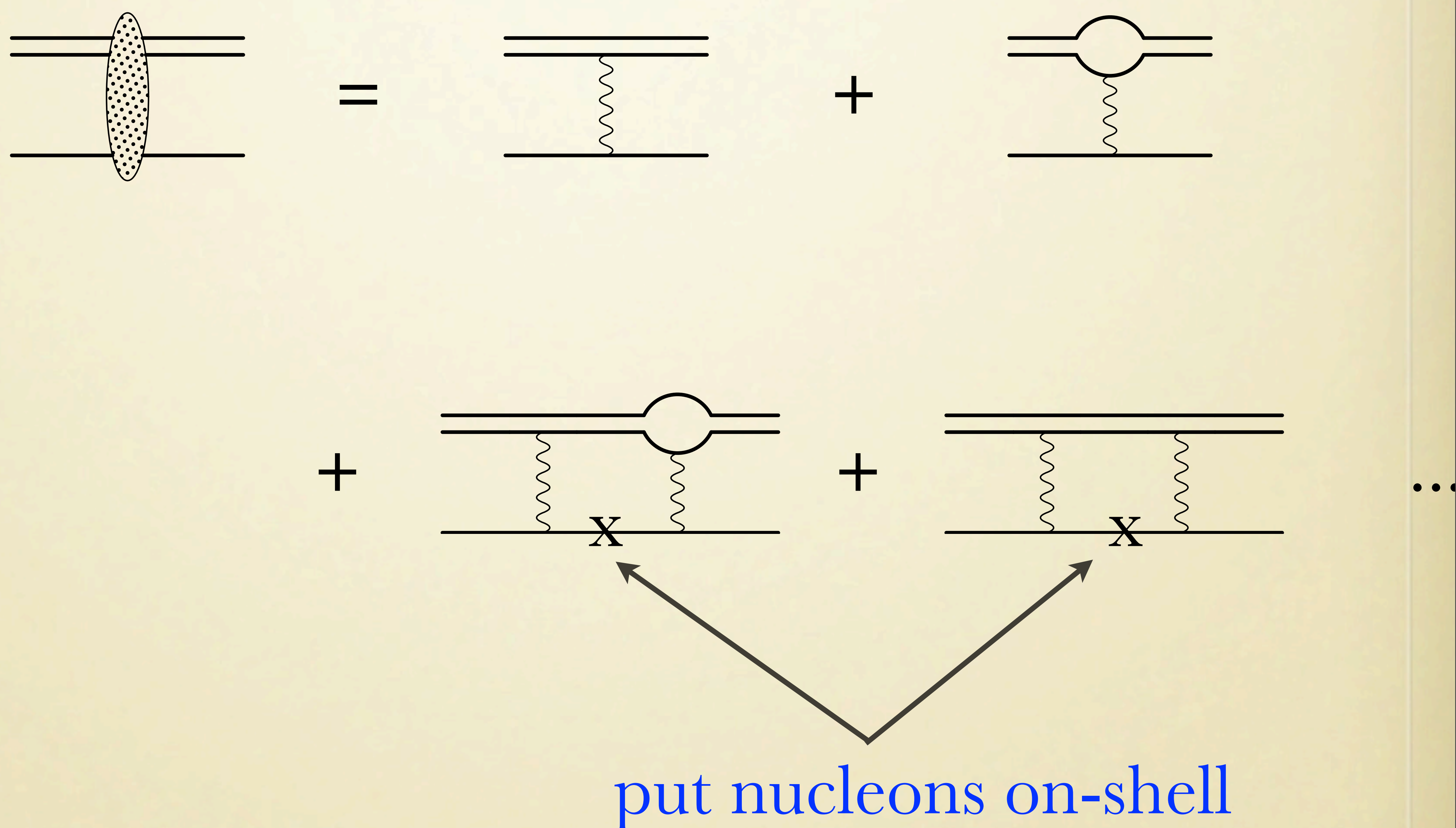
It is clear that we will have the right physics if the in the EFT the dimer propagator has the right physical cuts. Use the dressed dimer propagator

$$D(p_0, \vec{p}) = \frac{1}{\Delta} \to \frac{2\pi}{\mu g^2}\frac{1}{-\gamma + \sqrt{p^2/4 - \mu p_0}}$$

$$D(E_d + q_0, \vec{p}) \sim \frac{2\gamma}{(q^2 - p^2)/4 - M_N q_0} \sim \frac{\gamma}{q^2 - p^2}$$

Put nucleon on pole, IR enhancement.
Coulomb ladder recovered

Coulomb Ladder



But ... what happens if we also include nucleon-exchange?

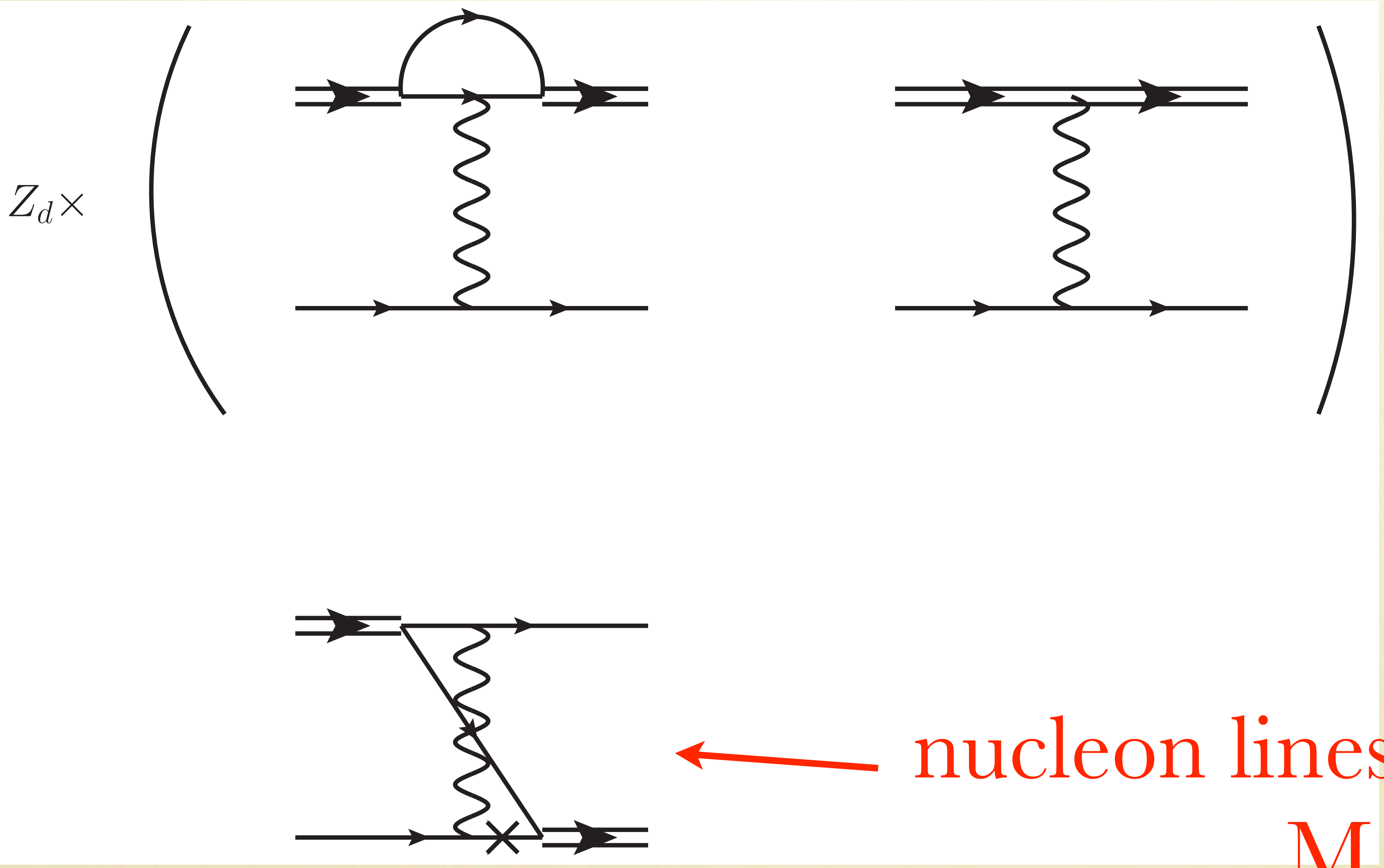
Key Insight

In p-d scattering we either have have a deuteron-nucleon or 3-nucleon state. The latter is off-shell by deuteron breakup and for these diagrams the loop momentum need not scale with p but with $\gamma \sim Q$

Photon iteration again

This loop is not IR enhanced but larger by 1/Q

Rupak, Kong 2001



$$= \frac{4\pi\alpha}{p^2}$$

nucleon lines always scale as M/Q^2

Power-Counting

After we put a single nucleon in every loop on-shell, either no no nucleon line or two nucleon lines exists.

(1) Loop : $\int d^3q \sim Q^3 \text{ or } p^3$

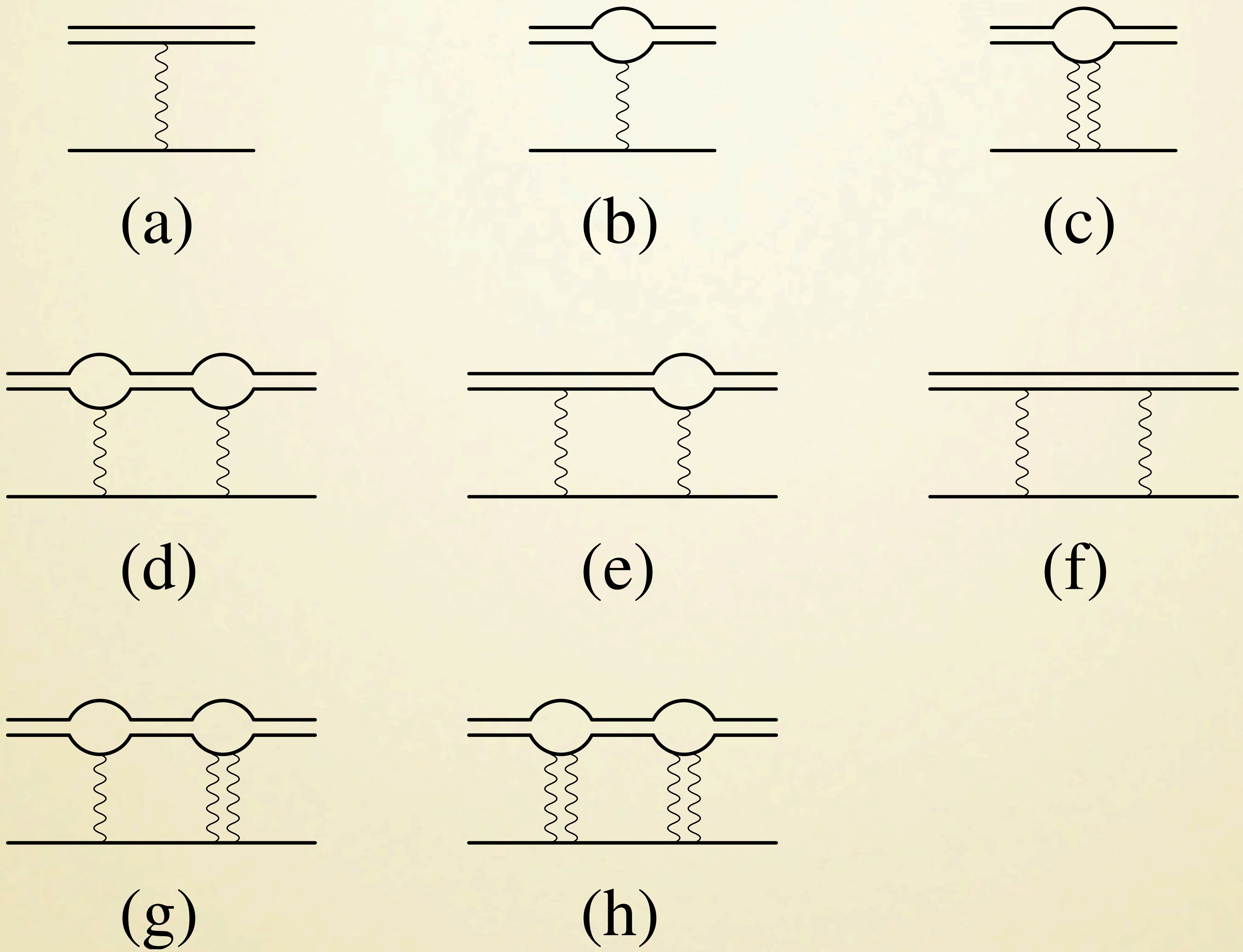
(2) Nucleon propagator : M_N/Q^2

(3) Dressed dimer : Q/q^2

(4) photon : $1/q^2$

Usual one when p~Q

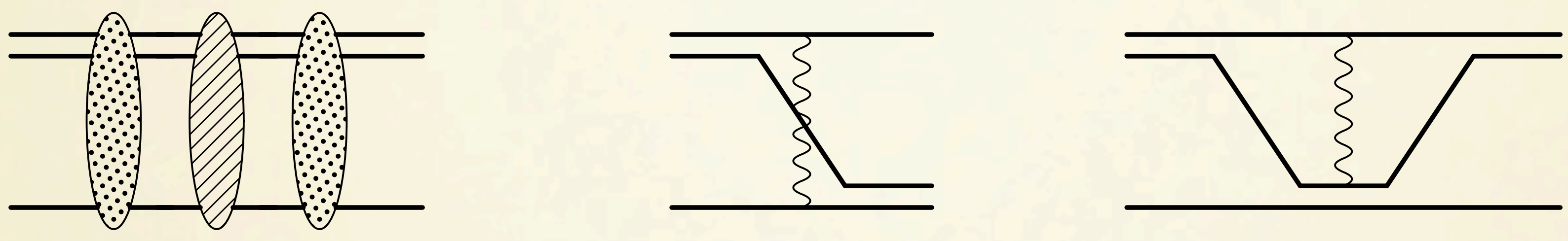
Examples



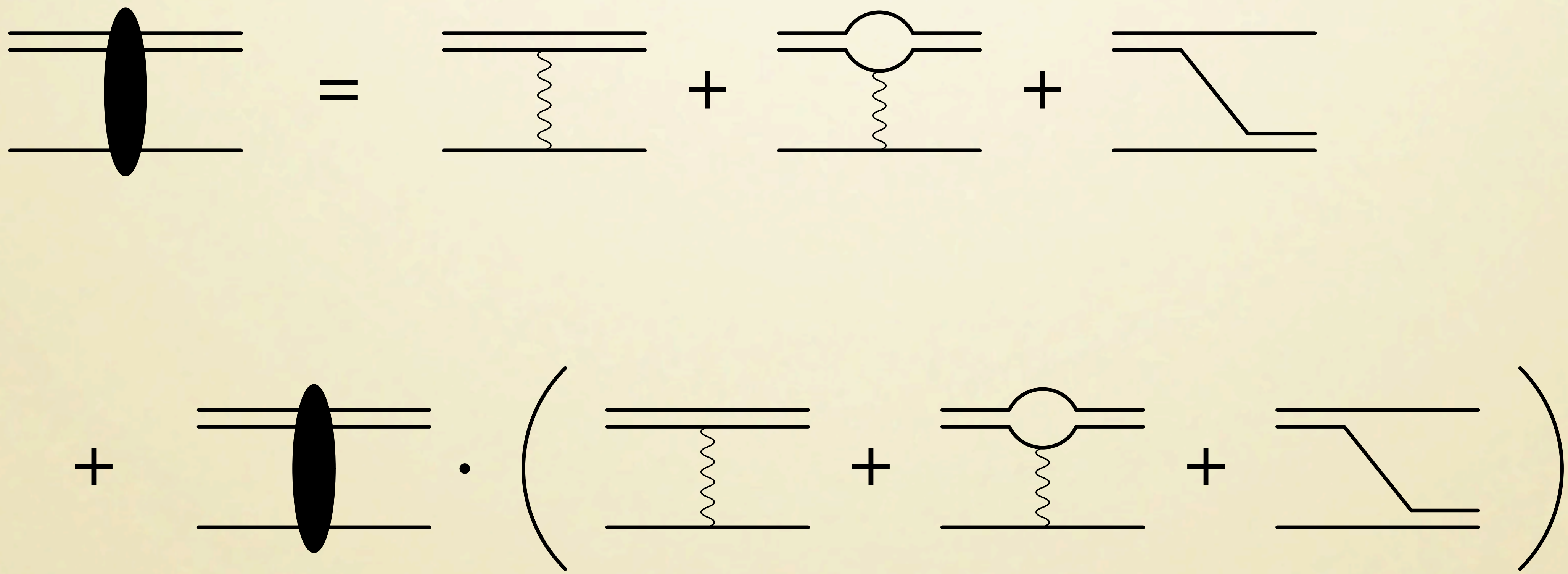
Rupak, Kong NPA 717, 73 (2003)

On blackboard

More Diagrams



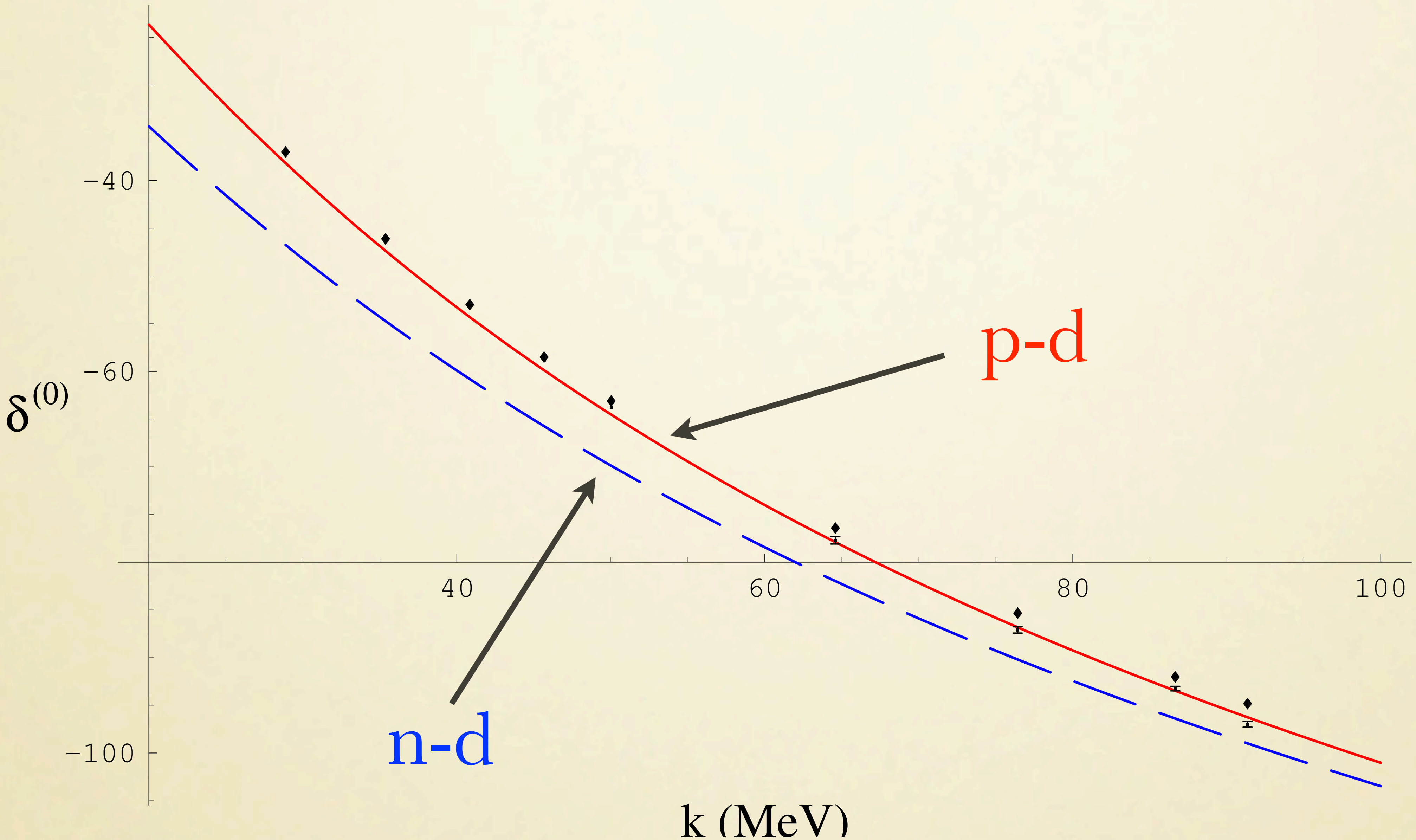
(a) (b) (c)



(d)

On blackboard

Quartet Channel



see König's and Vanasse's talk

Go All The Way

-- go beyond counting IR enhancement

-- explicitly identify three scales $Q, \ p, \ \alpha\mu$

see van Kolck's talk

Reactions in lattice EFT

- Consider: $a(b,\gamma)c$
- Need effective "cluster" Hamiltonian -- acts in cluster coordinates, spins,etc.
- Calculate reaction with cluster Hamiltonian. Many possibilities --- traditional methods, continuum EFT, lattice method

Rupak & Lee, PRL 2013, arXiv:1302.4158

Pine, Lee, Rupak, EPJA 2013, arXiv:1309.2616

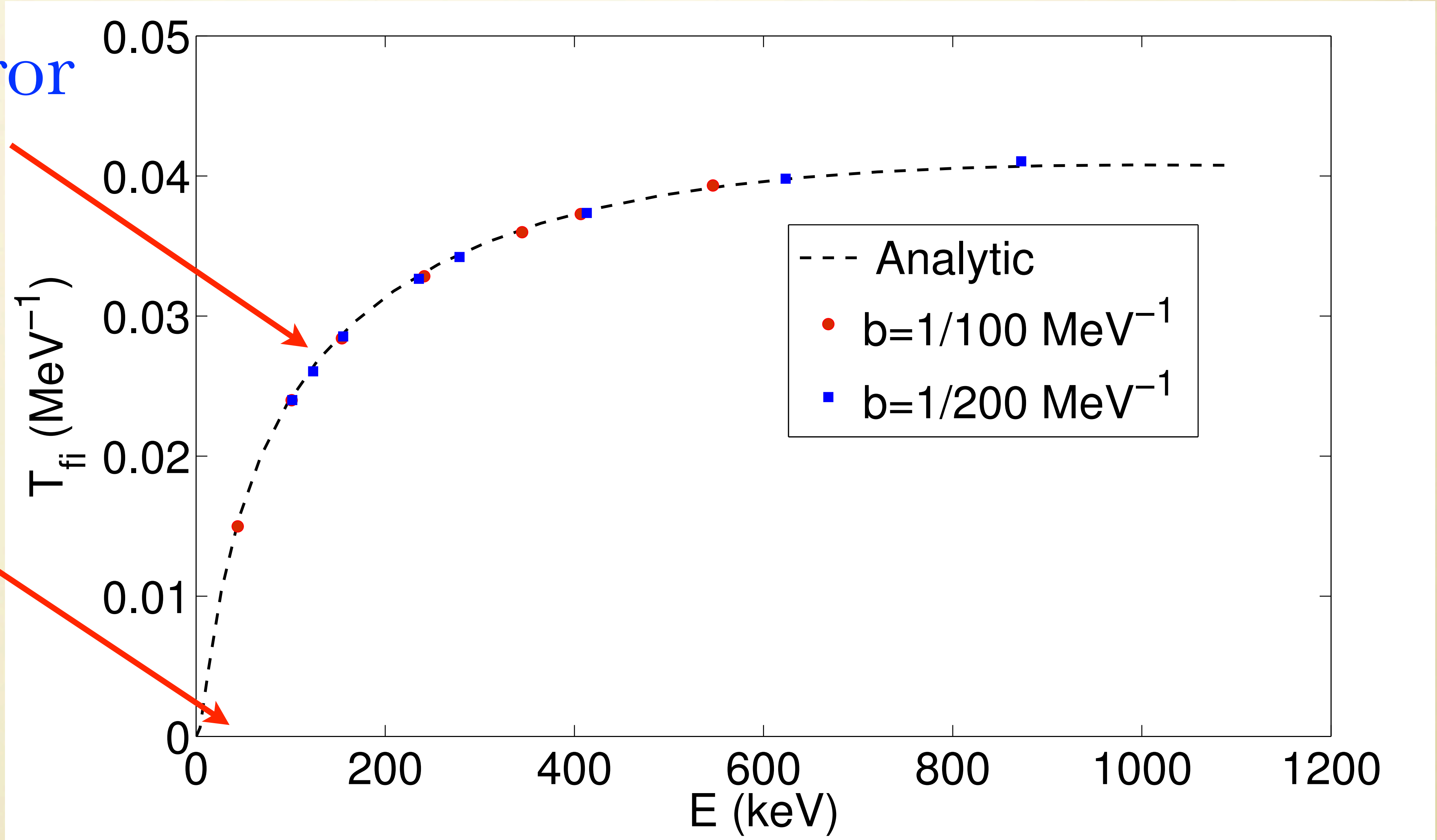
Rupak & Ravi, PLB 2014, arXiv:1411.2436

Elhatisari et al, Nature 528, 111 (2015), arXiv:1506.03513

Spherical Wall

3% fitting error propagates

Gamow peak 6 keV



$$\Lambda(p) = \sqrt{\frac{\gamma^2}{8\pi C_\eta^2}}|T_{fi}(p)|$$

$\Lambda_{EFT}(0) \approx 2.51$ Kong, Ravndal 1999

Lattice fit : $\Lambda(0) \approx 2.49 \pm 0.02$ Rupak, Ravi PLB 2014

Conclusions

- Power-counting loop momenta in p-d
- Identify powers of three scales
- Iterating sub-leading pieces could be trouble
- Non-analytic behavior at very low momentum
- Big picture

Thank you