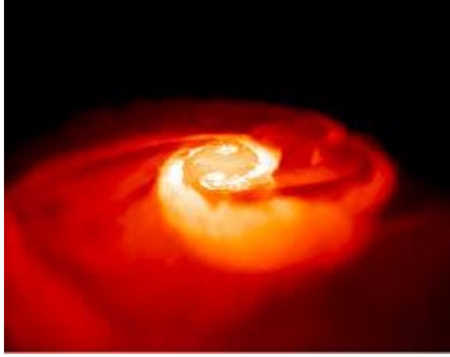

Neutrino-matter interactions for core-collapse supernovae – Precision at subleading order

NAVI Physics Days 2016 – January 18– GSI

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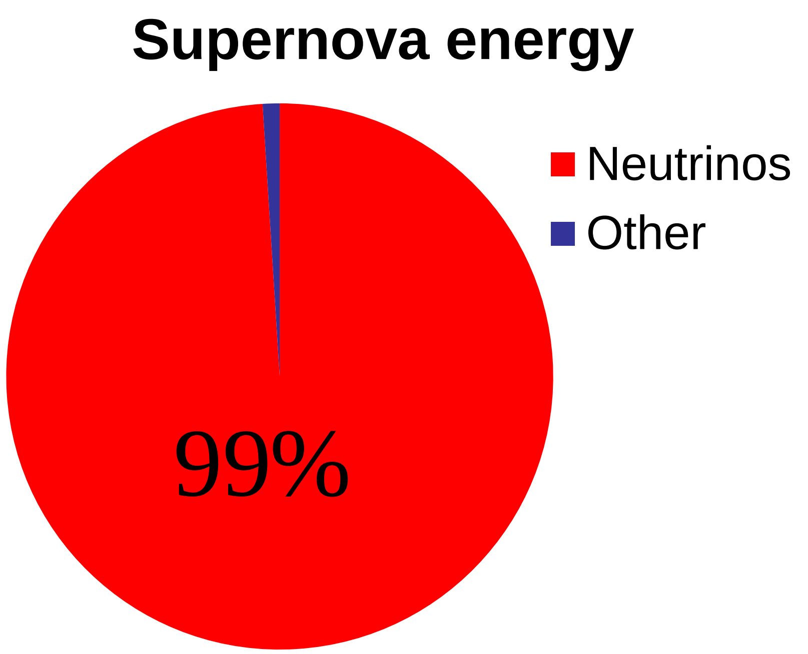
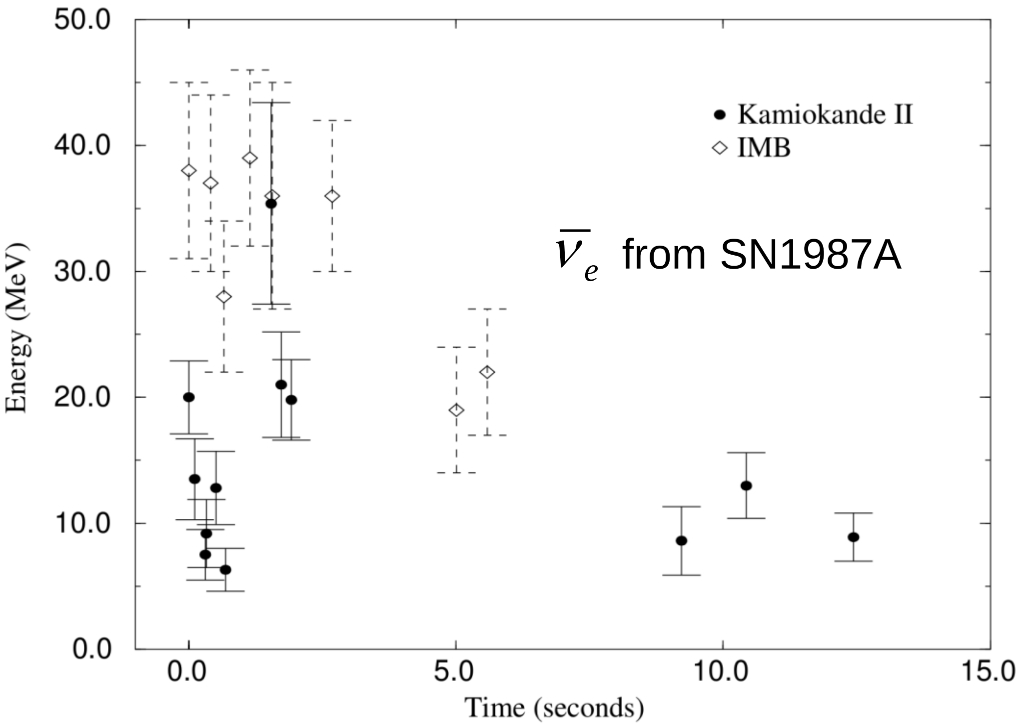
Neutrinos in Supernovae

Core collapse supernovae release huge amount of energy.





Neutrino spectra and interactions with matter are maior determinants of nucleosynthesis conditions.

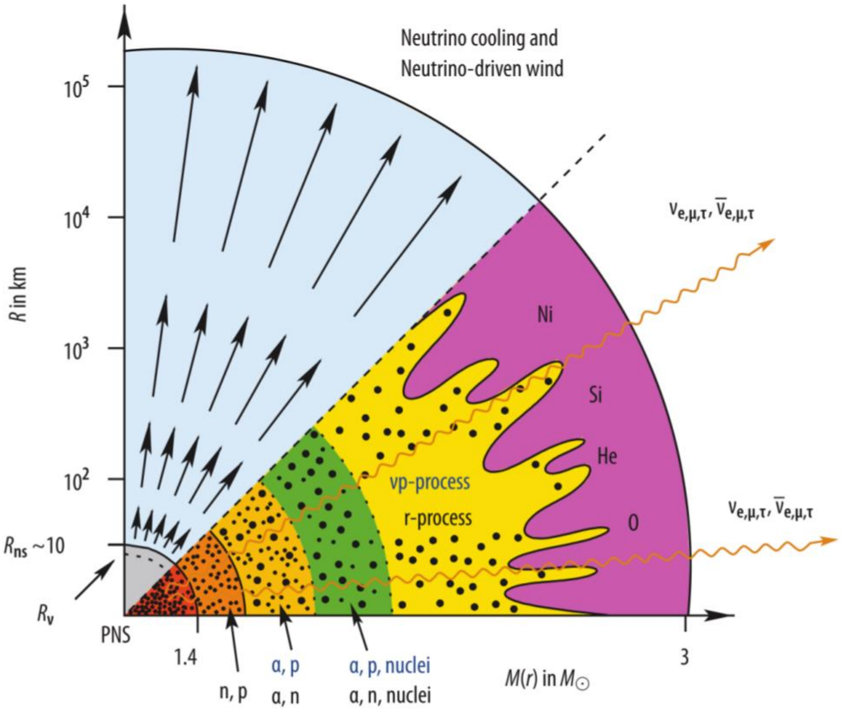
Neutrino-Interactions: Two Regimes

Interior of the neutron star:

Neutrino spectra formation

$$p + e^- \rightleftarrows \nu_e + n$$

$$n + e^+ \rightleftarrows \bar{\nu}_e + p$$



$$\langle E_{\bar{\nu}_e} \rangle - \langle E_{\nu_e} \rangle$$

$$\Downarrow$$

$$Y_e$$

Neutrino Driven Wind Ejecta:

Neutrino absorption ejects matter

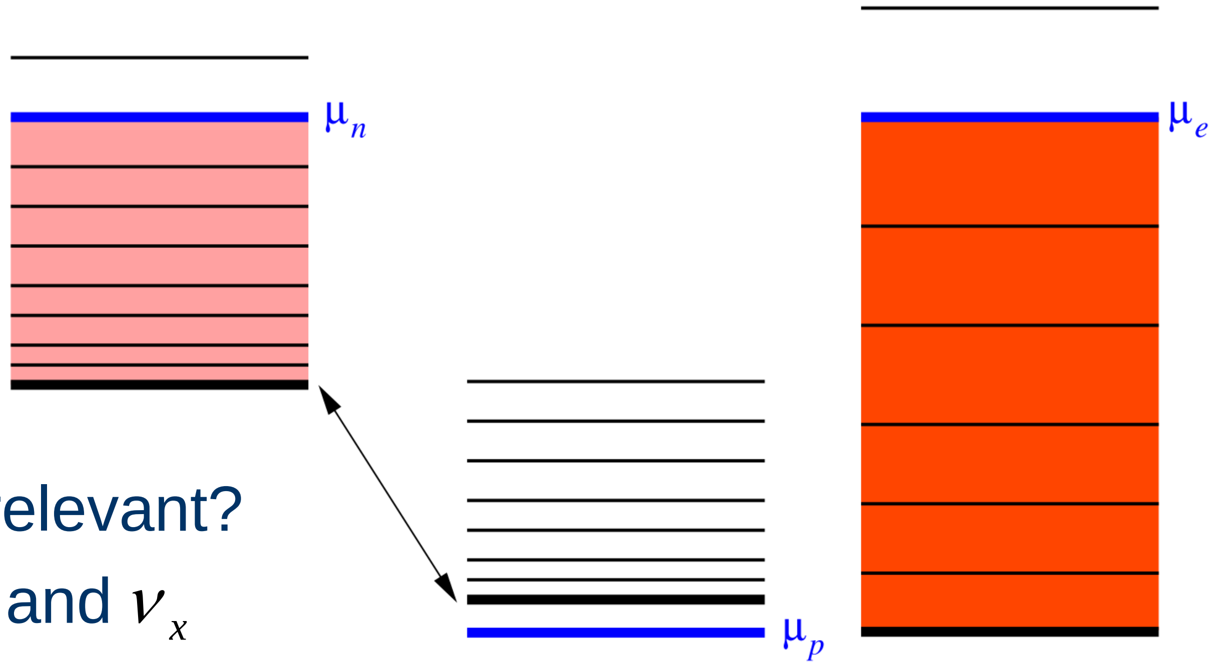
Spectrum determines composition

$$\nu_e + n \rightarrow p + e^-$$

$$\bar{\nu}_e + p \rightarrow n + e^+$$

Uncertainties in Neutrino Physics

What is the correct Equation of state?



Which reactions are relevant?

- Not obvious for $\bar{\nu}_e$ and ν_x
- Answer may vary for different SNe

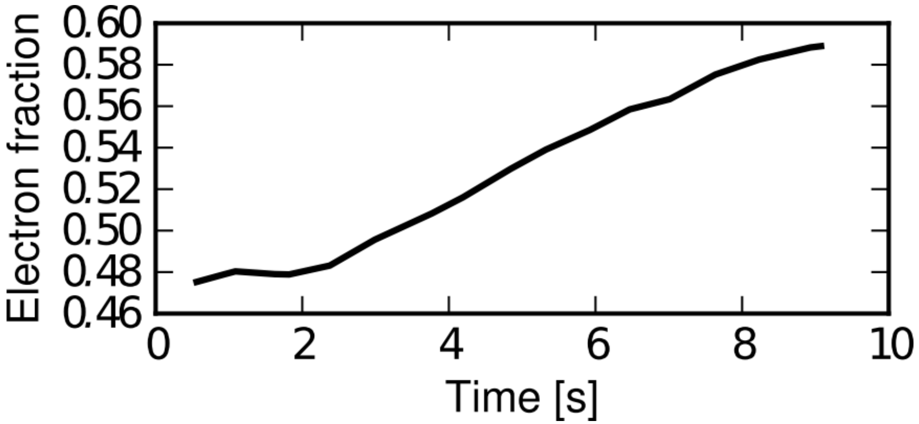
How to compute neutrino interactions?

-inelasticity, relativity, medium effects, weak magnetism …

Nucleosynthesis in Neutrino Driven Wind

Production of heavy elements depends on (Y_e,S,v_{Exp})

$$Y_e = \frac{r_{\nu_e,n}}{r_{\nu_e,n} + r_{\bar{\nu}_e,p}}$$



- Long term simulations indicate proton rich late NDW
- Ye≤0.5 possible during first seconds
- No full r-process, but weak r-process possible?

A more detailed picture requires (among other things) inclusion of subleading order effects in neutrino rates

Mean Free Path for Neutrino Absorption

Elastic Approximation

- Lowest order expression for nonrelativistic nucleons
- Analytic formula for $\lambda(E_\nu)$
- Can be corrected to include recoil, weak magnetism, …

Nucleons as quasi-free fermions – Hartree response

- Relativistic kinematics, „full“ matrix element, no correlations
- Mostly 2-D numerical integrals to obtain $\lambda(E_\nu)$

Structure function from RPA / Linear response theory

- Fully consistent with RMF-EOS, correlations (can be) included
- Requires 3-D numerical integrals to obtain $\lambda(E_\nu)$

Elastic Approximation for Neutrino Absorption

Mean-free path for (quasi-) free particles:

$$\lambda(E_\nu)^{-1} \sim \int d^3p_e \left[1 - f_e(E_e)\right] \int d^3p_n \int d^3p_p \frac{\left\langle |M|^2 \right\rangle}{16 E_\nu E_n E_e E_p} f_n(E_n) \left[1 - f_p(E_p)\right] \delta^4$$

Assume non-relativistic nucleons and elastic collision:

$$E_{n,p} \simeq m_{n,p} \Rightarrow \frac{\left\langle |M|^2 \right\rangle}{16 E_\nu E_n E_e E_p} \simeq G_A^2 (3 - x) + G_V^2 (1 + x)$$

$$E_n - E_p \simeq m_n - m_p + U_n - U_p$$

Mean-free path reduces to

$$\lambda(E_\nu)^{-1} \sim \left(3G_A^2 + G_V^2\right) (E_\nu + \Delta m + \Delta U)^2 \left[1 - f_e(E_\nu + \Delta m + \Delta U)\right] \frac{n_n - n_p}{1 - \exp\left[(\eta_p - \eta_n)/T\right]}$$

Recoil and Weak Magnetism Corrections

[Horowitz, PRD 65 (2002) 043001] pointed out:

- “Elastic Approximation“ is more simplified than necessary
- Kinematics/Recoil can be treated relativistically

$$E_n = m_n \Rightarrow E_e = \frac{E_\nu}{1 + \frac{E_\nu}{m_n}(1-x)}$$

- To include in phase space factor and matrix element
- Gives rise to analytic correction factor for cross-section

$$R = \left\{ G_V^2 \left(1 + 4e + \frac{16}{3}e^2\right) + 3G_A^2 \left(1 + \frac{4}{3}e\right)^2 \pm 4G_A (G_V + F_2) e \left(1 + \frac{4}{3}e\right) + \frac{8}{3}G_V F_2 e^2 + \frac{1}{3}F_2^2 e^2 (5 + 2e) \right\} / \left[(1+2e)^3 \left(G_V^2 + 3G_A^2\right) \right]$$

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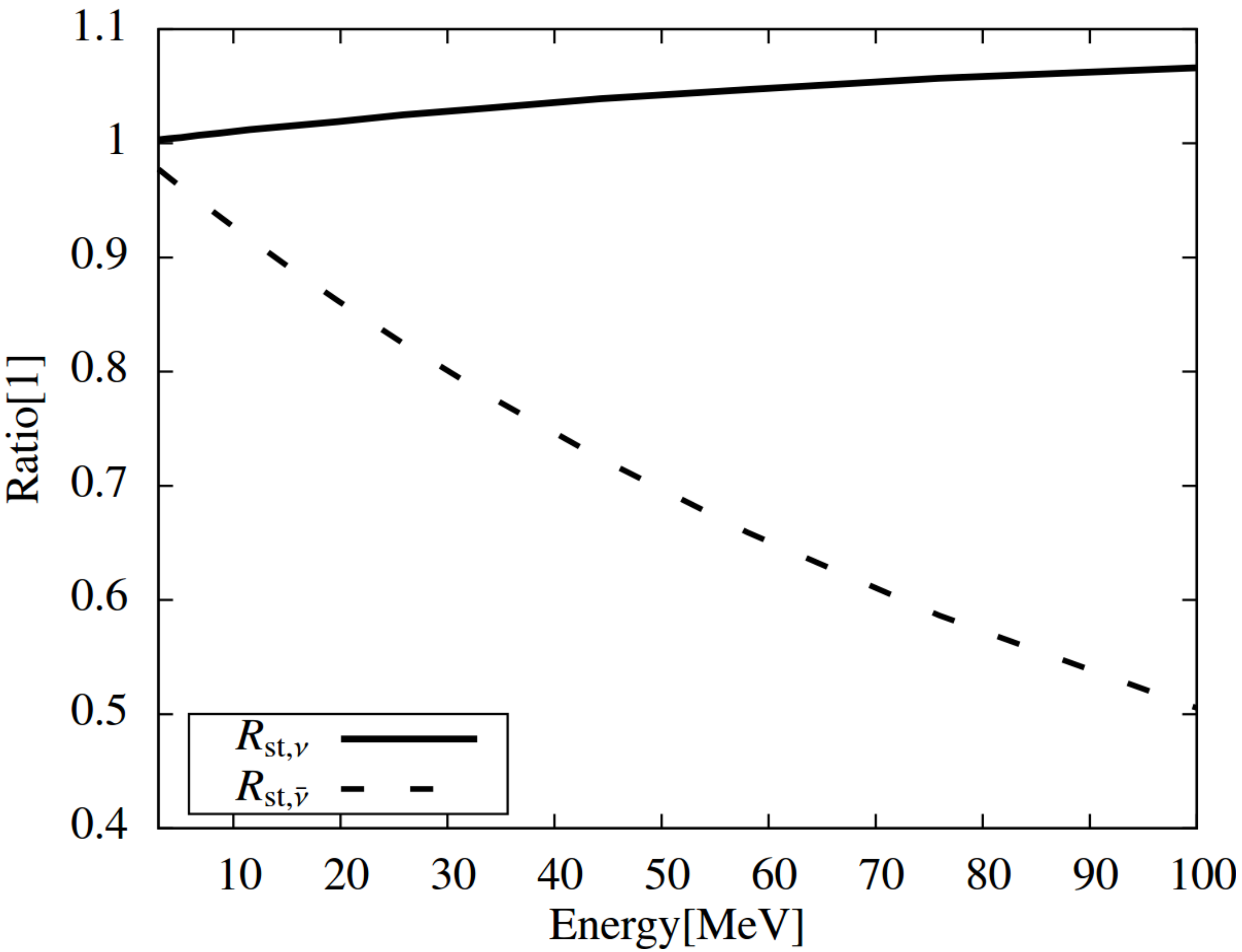
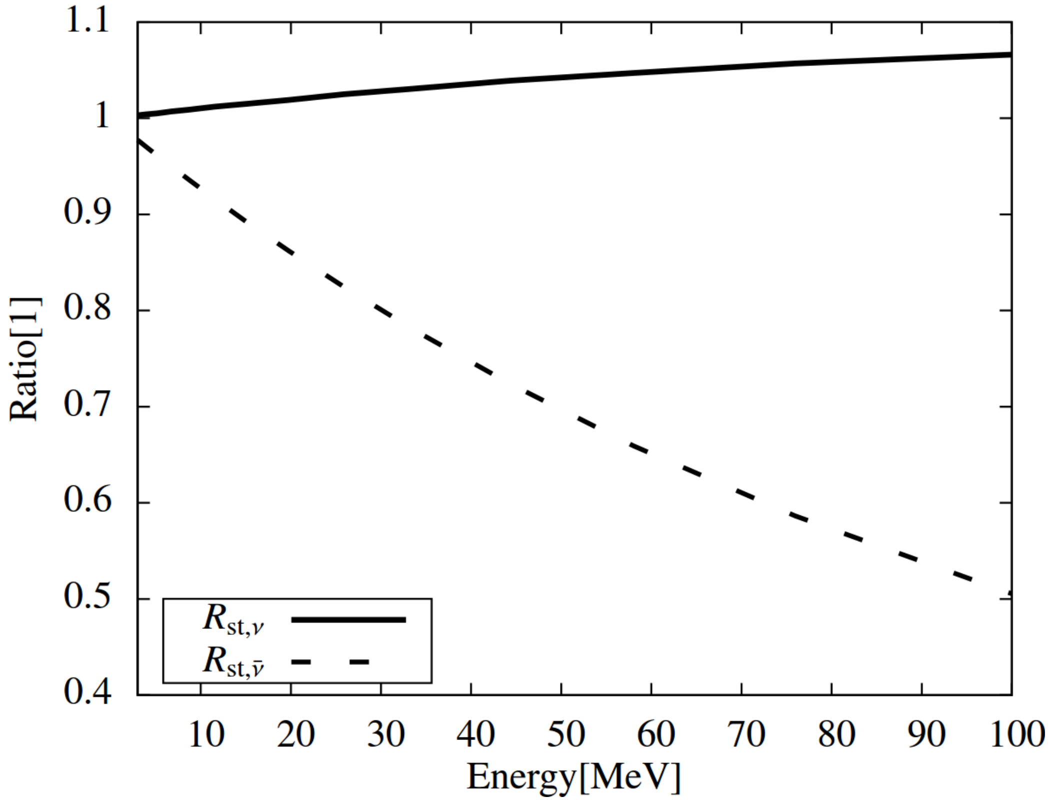
Correction factors are ratios of vacuum cross sections with exact and approximated matrix element and phase space, assuming the target nucleon at rest.

Correction Factor for Cross-Section



Correction Factor for Cross-Section



Two (contrasting) effects

- In NDW, equilibrium shifted towards neutrino absorption
- In decoupling region, neutrino sphere moves inwards for antineutrinos, resulting in higher average energy

Improvement: Consider Mass and Potential Differences

- Masses and strong interaction potentials of nucleons differ
- At large densities effective masses decrease

$$E_e = \frac{E_\nu + \frac{M_*^2 - m_p^{*2}}{2M_*}}{1 + \frac{E_\nu}{M_*}(1-x)} \qquad M_* = m_n^* + U_n - U_p$$

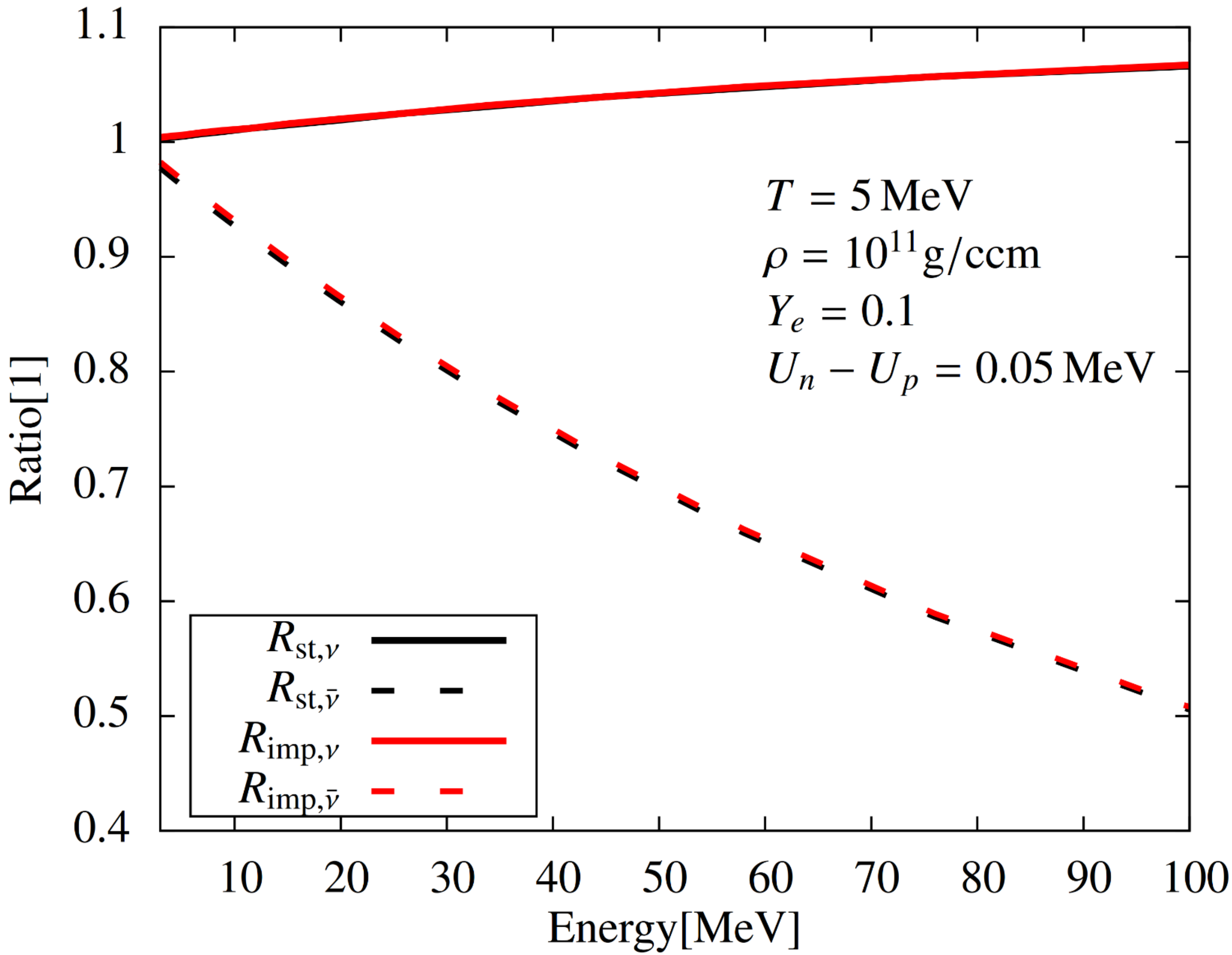
- Analytic correction factor can still be derived the same way
- In the matrix element, additional terms can be included
- For neutrino scattering, only difference is exchange of rest mass with effective mass

Improved Correction Factor

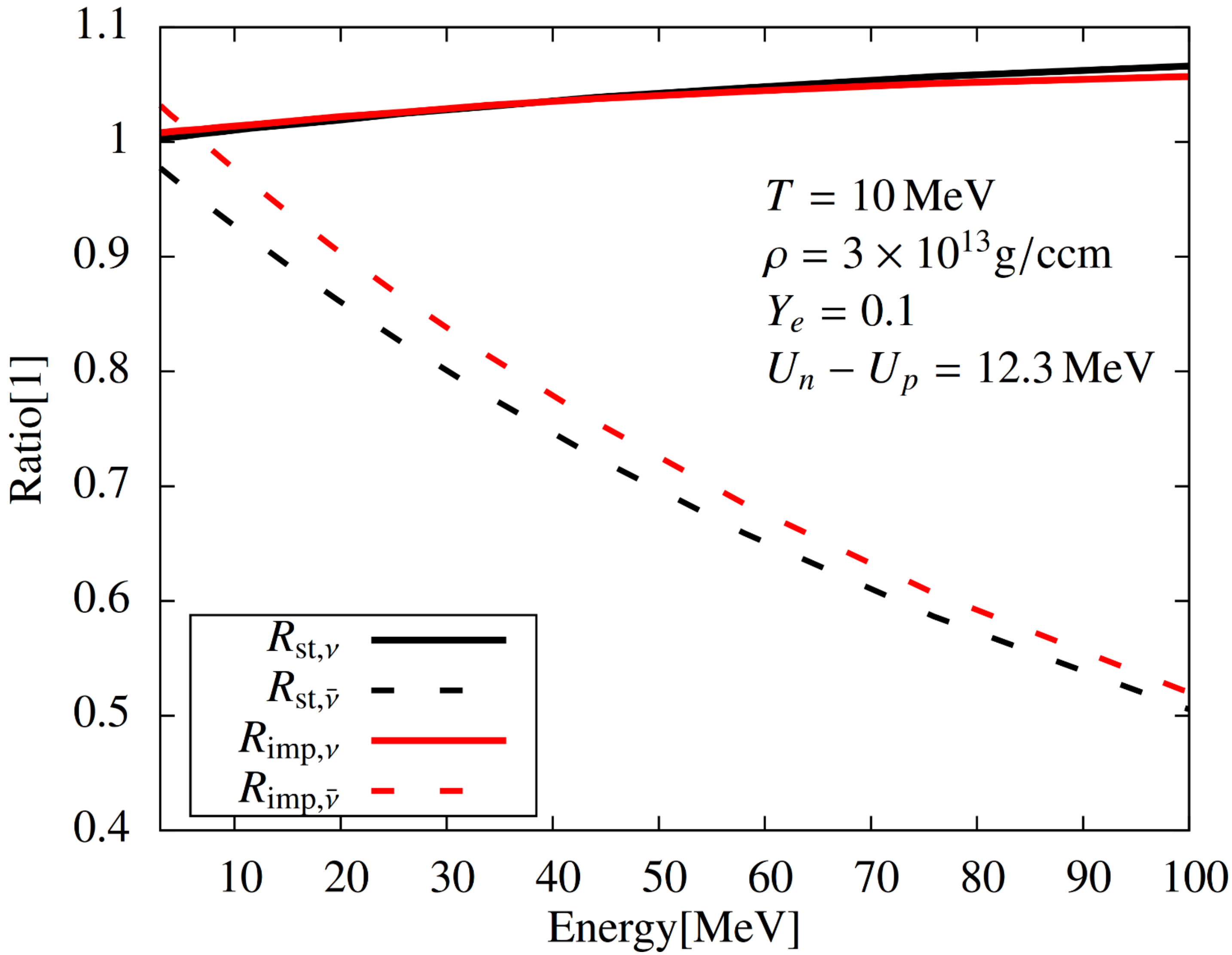
$$R = \left\{ G_V^2 \left[1 + 4e_* + \frac{16}{3} e_*^2 + \frac{4}{3} e_* \xi + \left(1 + \frac{2}{3} e_*\right) (\xi - q_*) \right] \right.$$
$$+ G_A^2 \left[3 + 8e_* + \frac{16}{3} e_*^2 - \frac{4}{3} e_* \xi - \left(1 + \frac{2}{3} e_*\right) (\xi + q_*) \right]$$
$$\pm G_A \left[G_V + F_2 \frac{M_*}{m_N} \left(1 - \frac{\xi}{2}\right) \right] \left[4e_* + \frac{16}{3} e_*^2 + q_* \left(2 + \frac{4}{3} e_*\right) \right]$$
$$+ G_V F_2 \frac{M_*}{m_N} \left[\left(1 + \frac{q_*}{e_*} - \frac{\xi}{2}\right) \frac{8}{3} e_*^2 + \xi q_* \left(1 + 2e_* + \frac{4}{3} e_*^2\right) \right]$$
$$\left. + F_2^2 \frac{M_*^2}{m_N^2} \left[\frac{5}{3} e_*^2 + \frac{2}{3} e_*^3 + \left(\frac{1}{2} + e_*\right) \tilde{A} + \left(\frac{1}{2} + \frac{1}{3} e_*\right) \tilde{B} + \frac{2}{3} e_* \tilde{C} \right] \right\}$$
$$/ \left[(1 + 2e)^3 \left(G_V^2 + 3G_A^2\right) \right]$$

$$\xi = \frac{\Delta m^* + \Delta U}{M_*}, \quad q = \frac{m_n^{*2} - m_p^{*2}}{2M_*^2}, \quad q_* = \frac{M_*^2 - m_p^{*2}}{2M_*^2}$$

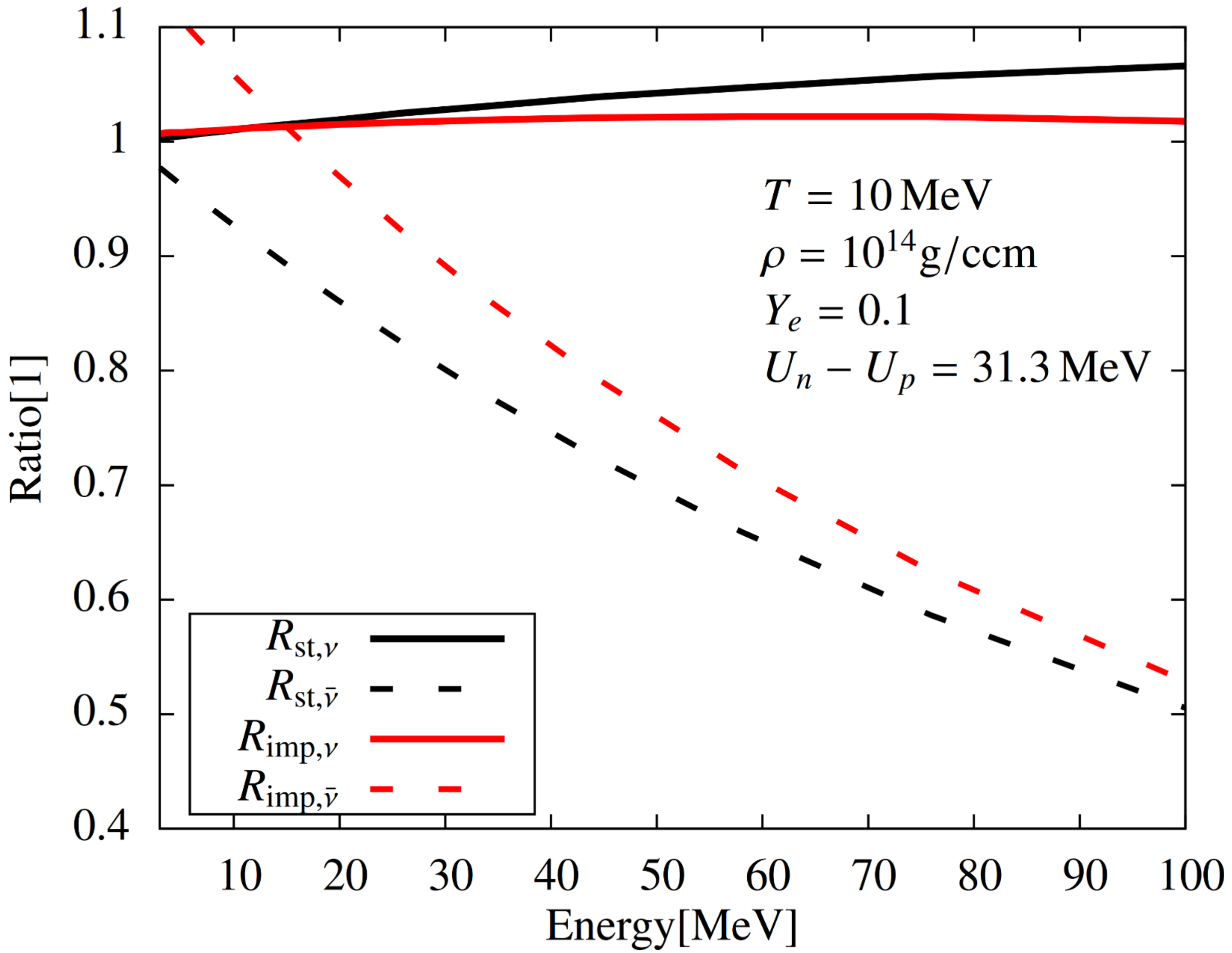
Improved Correction Factor at Low Densities



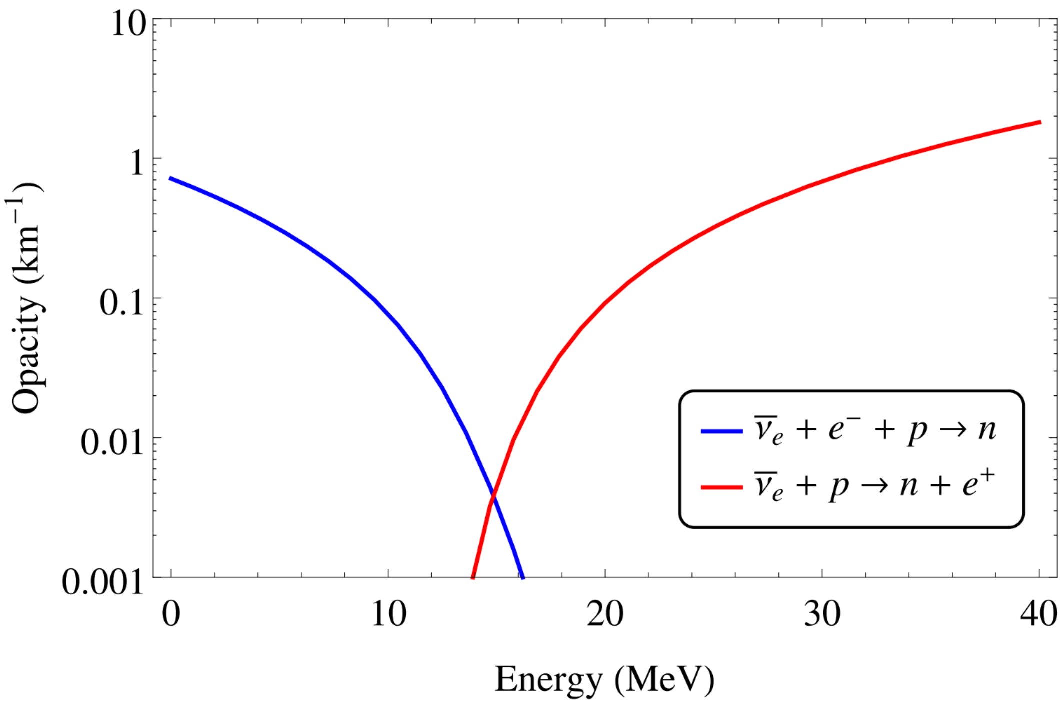
Improved Correction Factor at High Densities

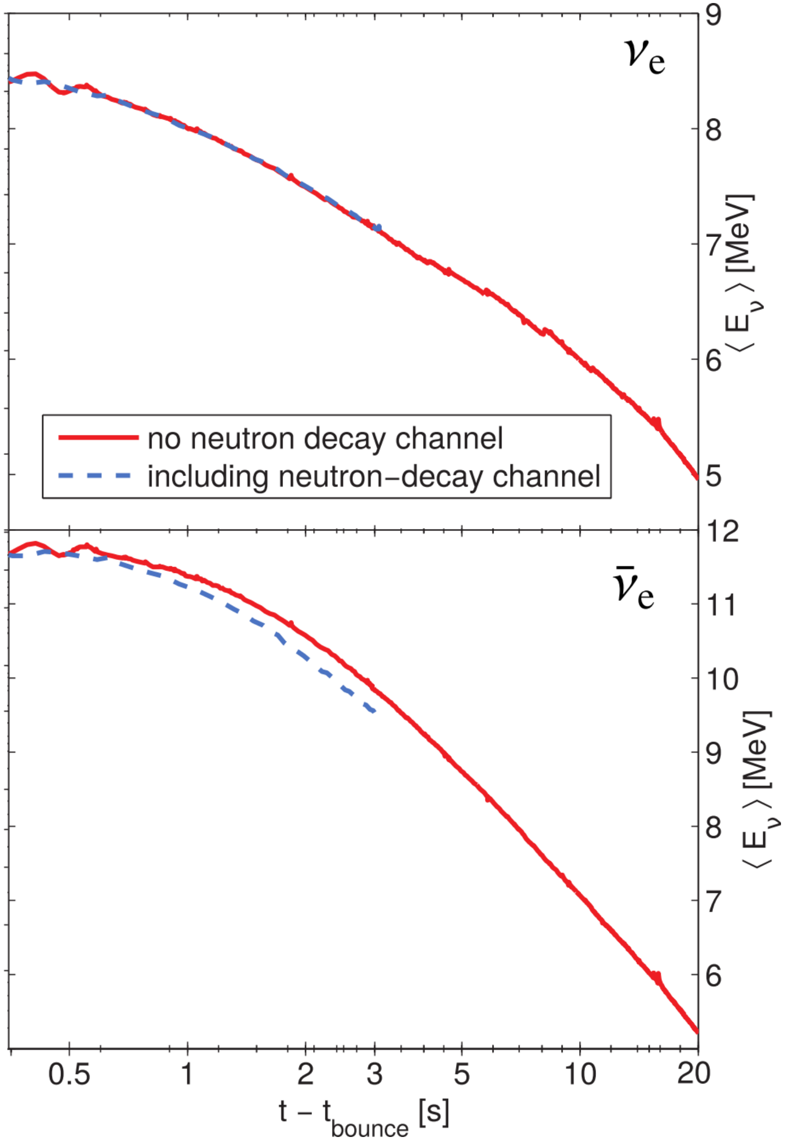


Improved Correction Factor at High Densities



Neutron decay at high density





Reaction for low energy antineutrinos

- Decreasing average energy of $\overline{\nu}_e$
- Spectral change can affect nucleosynthesis yields

Elastic Approximation and Corrections for Neutron Decay

Elastic approximation for neutron decay similar to absorption

$$\lambda(E_\nu)^{-1} \sim \left(3G_A^2 + G_V^2\right)\left(\Delta m + \Delta U - E_\nu\right)^2 f_e(\Delta m + \Delta U - E_\nu)\frac{n_p - n_n}{1 - \exp\left[\left(\eta_n - \eta_p\right)/T\right]}$$

Kinematic relation for inverse neutron decay, assuming proton at rest

$$E_e = \frac{-E_{\bar{\nu}} + \frac{M_*^2 - M_f^2}{2M_*}}{1 + \frac{E_{\bar{\nu}}}{M_*}(1 - x)}$$

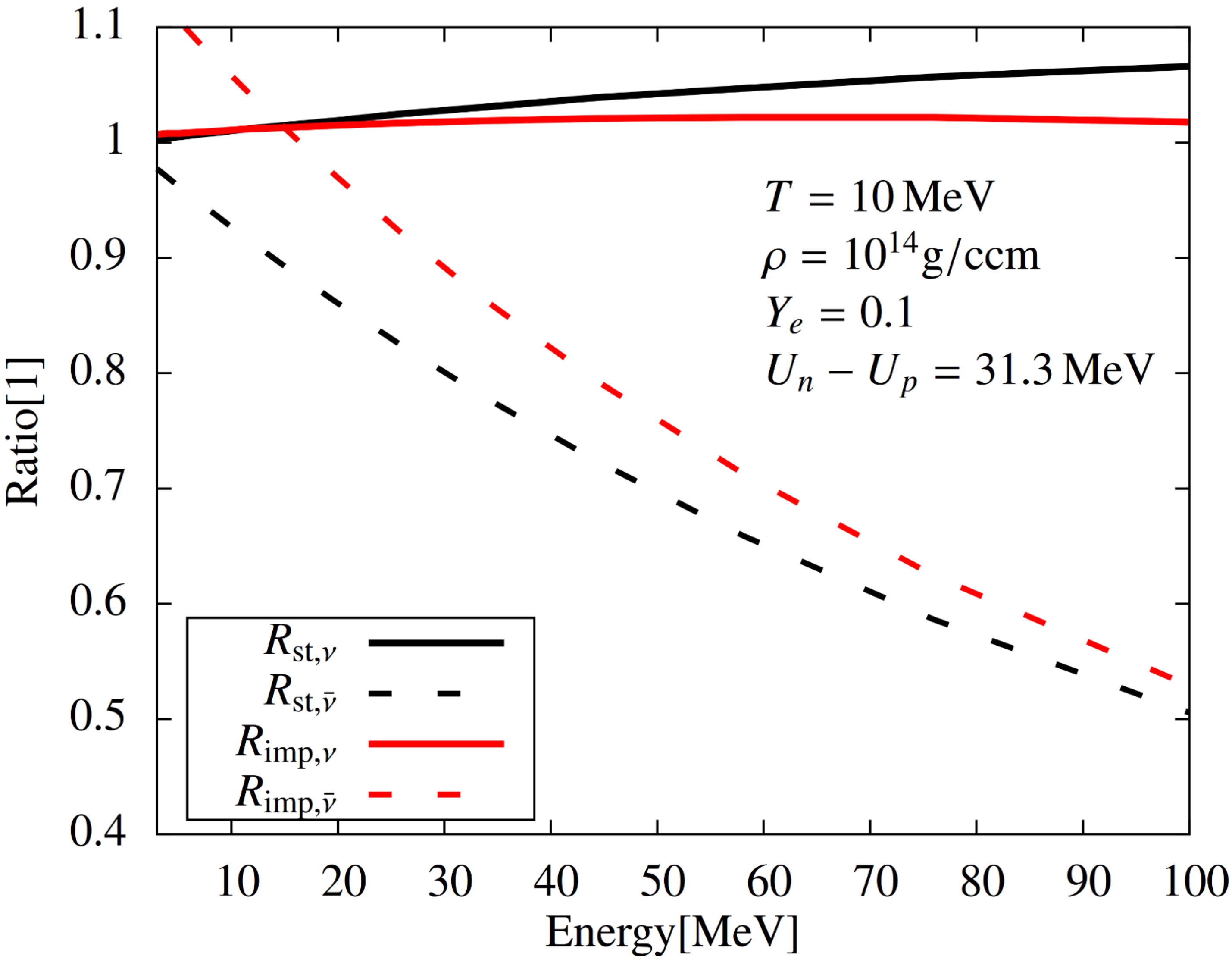
Physical meaning only for different nucleon masses and/or potentials

Elastic Approximation and Corrections for Neutron Decay

Result: Correction factor for neutron decay is EXACTLY the same as for antineutrino absorption

- Neutron decay and antineutrino absorption refer to different neutrino energies
- Absorption: $m_e + \Delta m_* + \Delta U < E_{\bar{\nu}} < \infty$
- Decay: $0 < E_{\bar{\nu}} < -m_e + \Delta m_* + \Delta U$
- For decay, corrections increase or decrease rate, depending on antineutrino energy

Improved Correction Factor at High Densities



Limit of Approximations

- Elastic opacities with weak magnetism corrections should be very good for zero temperature vacuum

- How good are the approximations for finite T and ρ?
  - at conditions of NDW?
  - at conditions of neutrinosphere?

- Compare approximations with „exact“ opacity

- How to compute „exact“ opacity?

Computing „exact“ neutrino opacities in CCSN

Hartree approximation for nucleon response:

- nucleon-nucleon interaction described by RMF-potentials and effective masses
- nucleons are quasi-free particles with modified energy

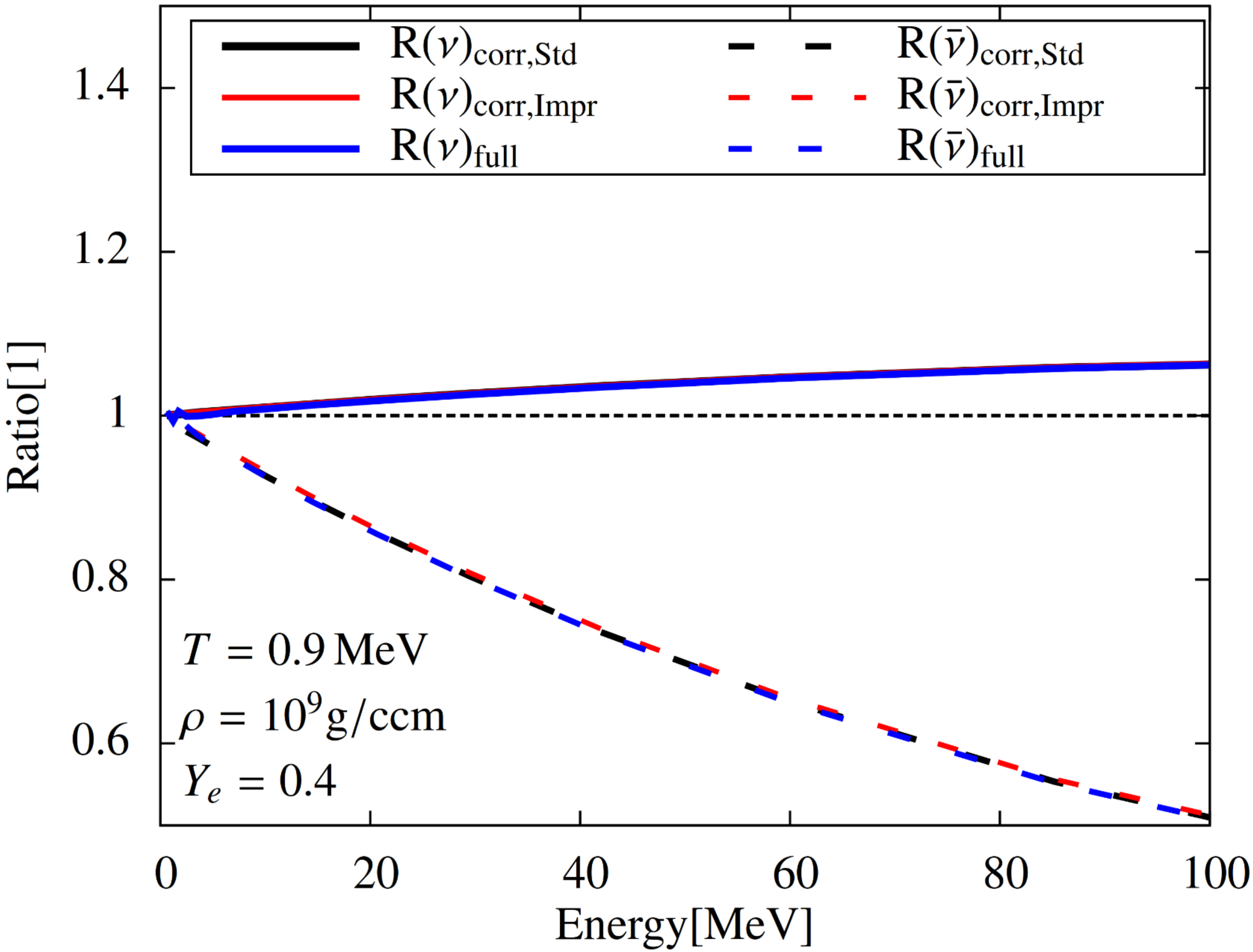
$$E_{n,p} = \sqrt{\mathbf{p}^2 + m_{n,p}^{*2}} + U_{n,p}$$

- relativistic kinematics, „full“ matrix element, weak magnetism included

$$\lambda(E_\nu)^{-1} \sim \int d^3p_e \left[1 - f_e(E_e)\right] \int d^3p_n \int d^3p_p \frac{\left\langle |M|^2 \right\rangle}{16 E_\nu E_n E_e E_p} f_n(E_n) \left[1 - f_p(E_p)\right] \delta^4$$

- No correlations, but always better than elastic approximation

Comparing Approximations and Exact Opacities



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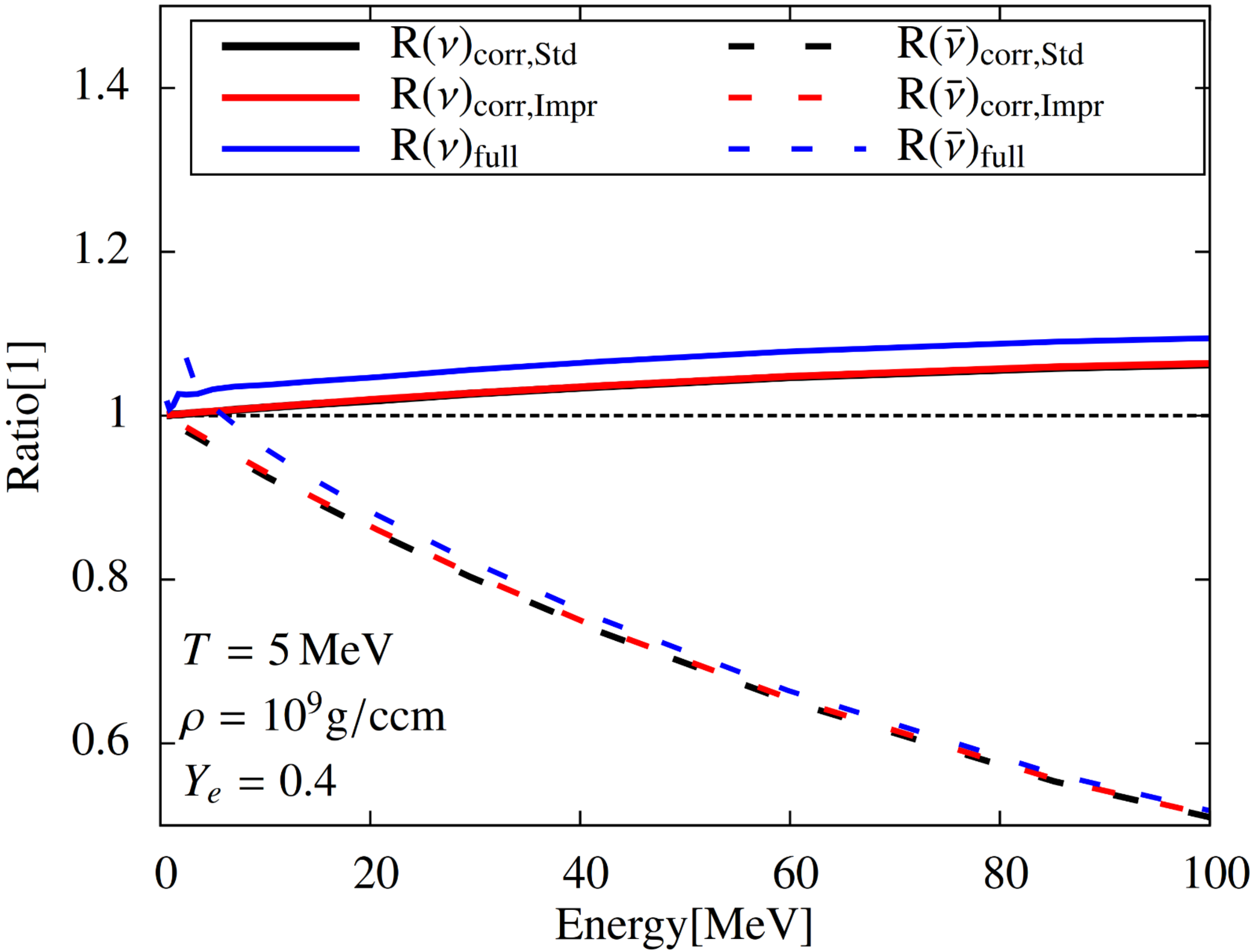

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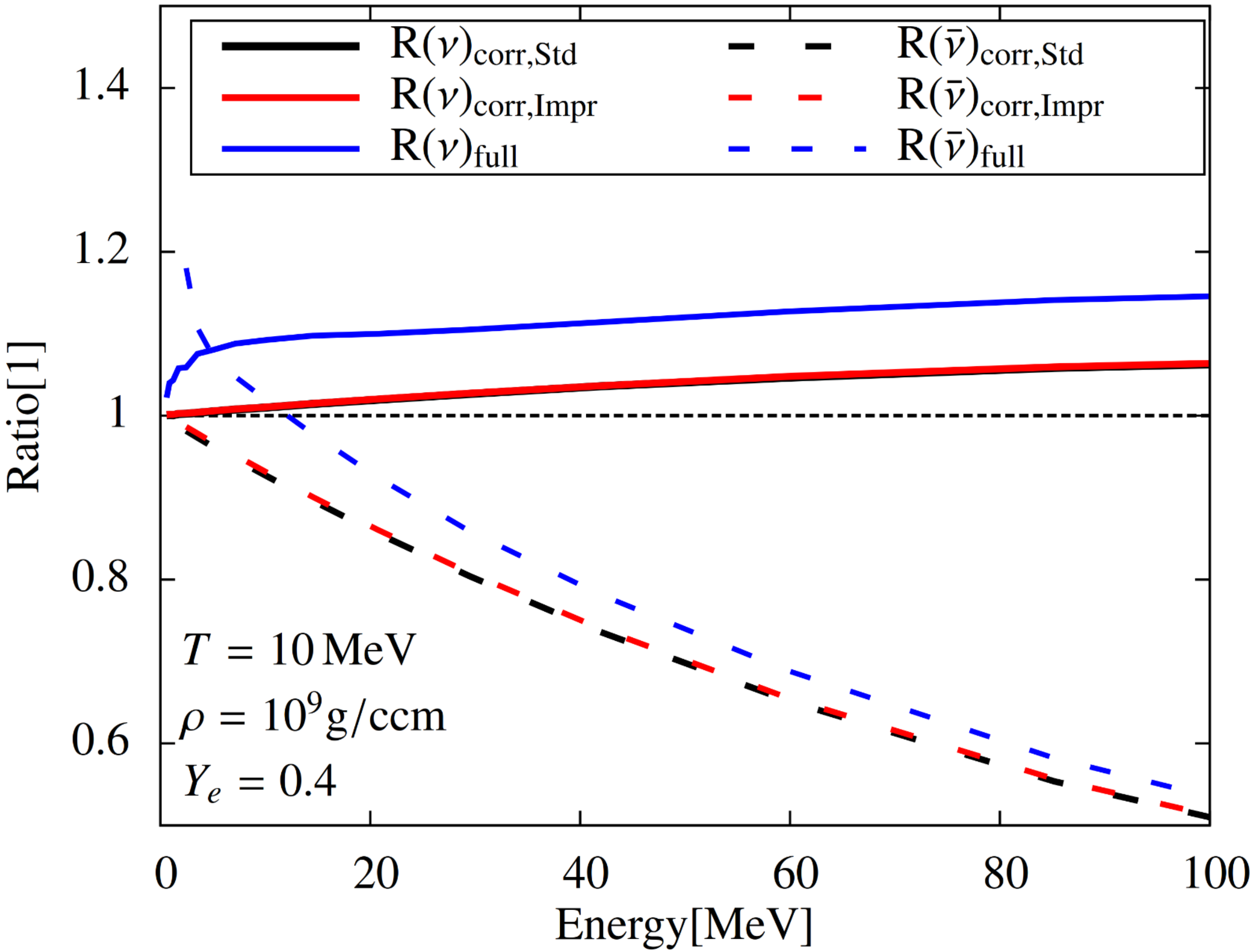

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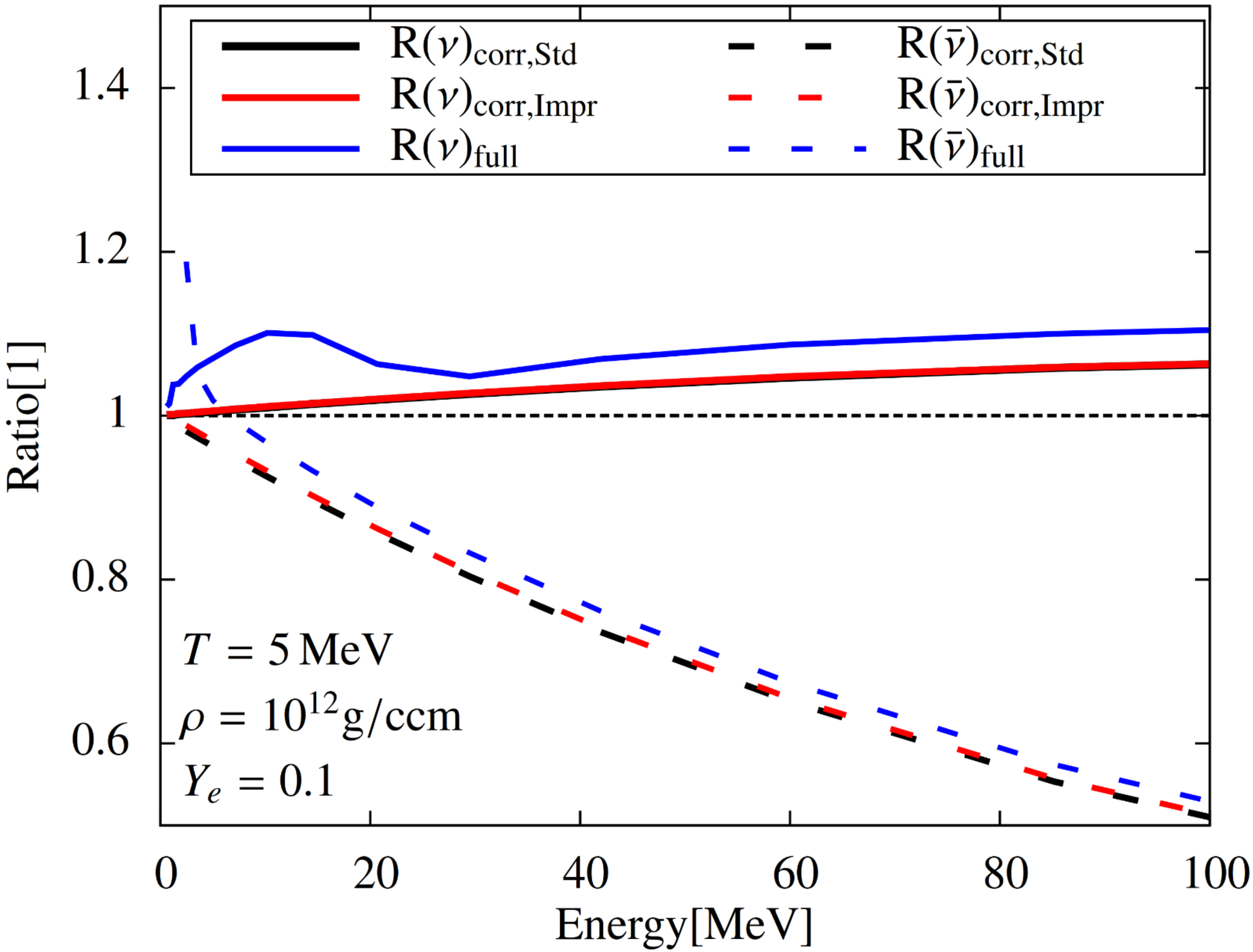

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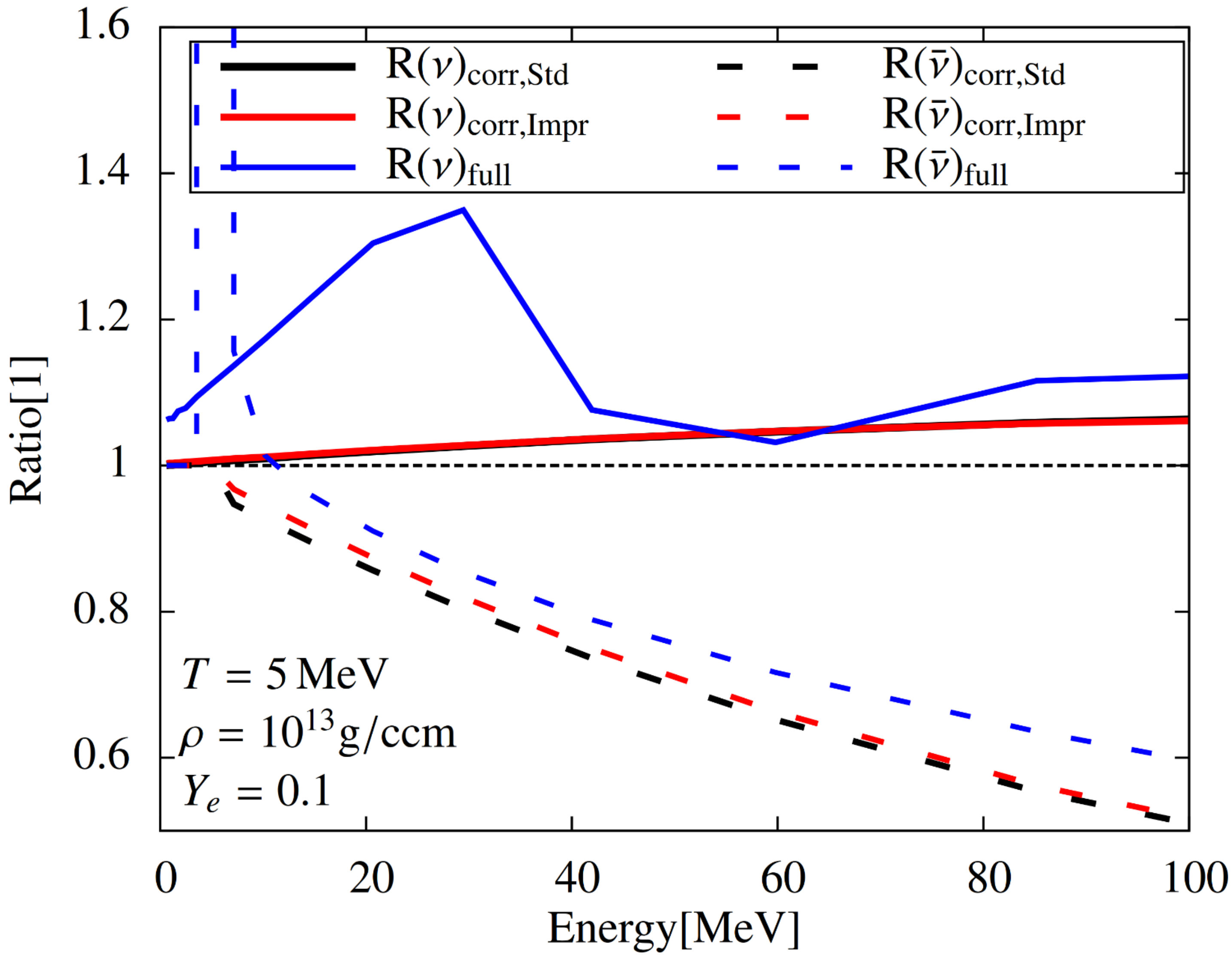
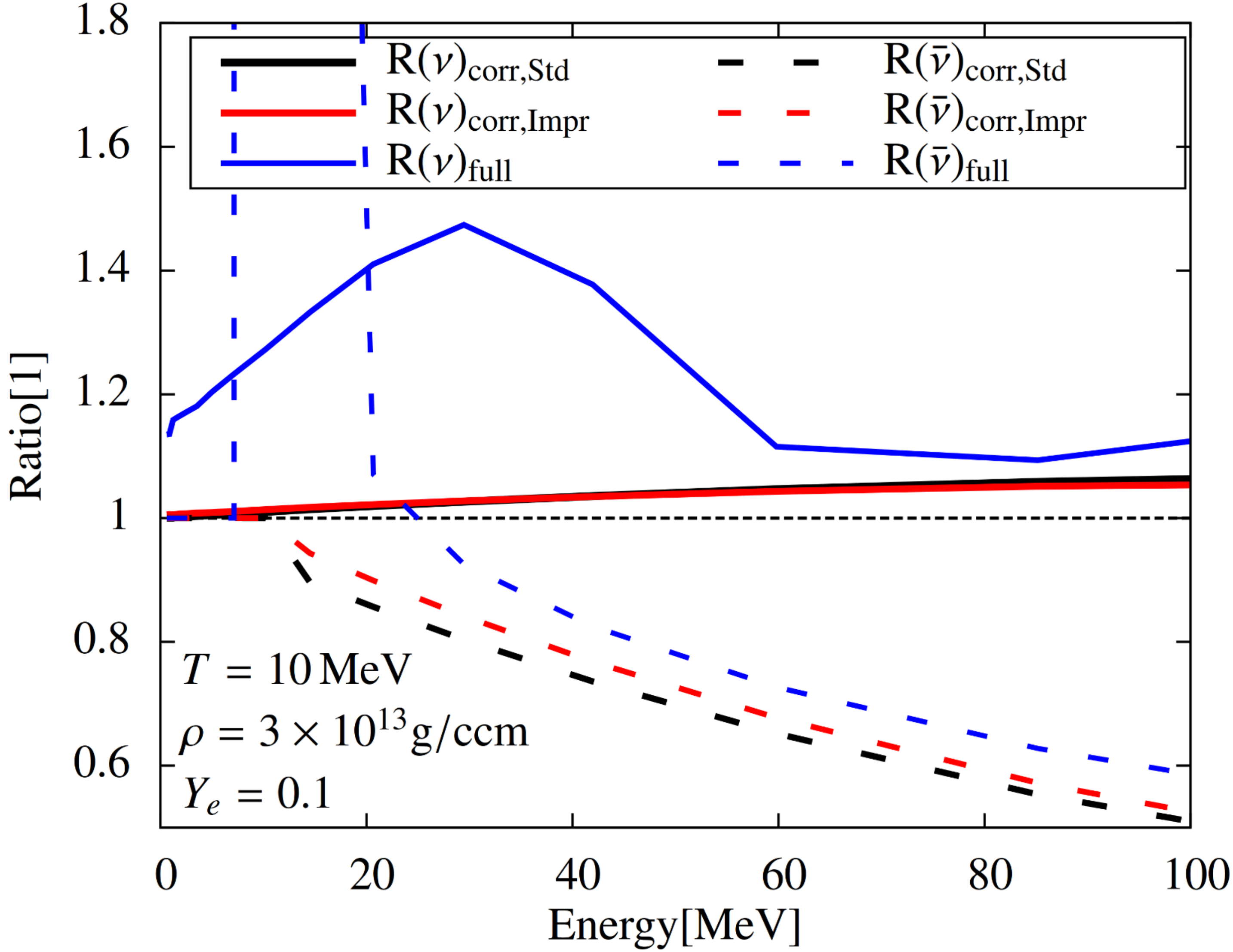


Comparing Approximations and Exact Opacities



Limit of Approximations

- Elastic opacities with weak magnetism corrections are indeed very good for zero temperature vacuum

- For temperatures of several MeV, approximation underestimates opacities (~10%)
- At neutrinosphere, additional significant deviations for neutrino energies of several 10 MeV

- Approximation „fails“ at the level of weak magnetism corrections for higher densities/temperatures
  - What is the reason for the failure? (target at rest; inelasticity; relativity)
  - Is there a „cure“?

Alternativ Approximations and Corrections

Reminder:

Correction factors are ratios of vacuum cross sections with exact and approximated matrix element and phase space, assuming the target nucleon at rest.

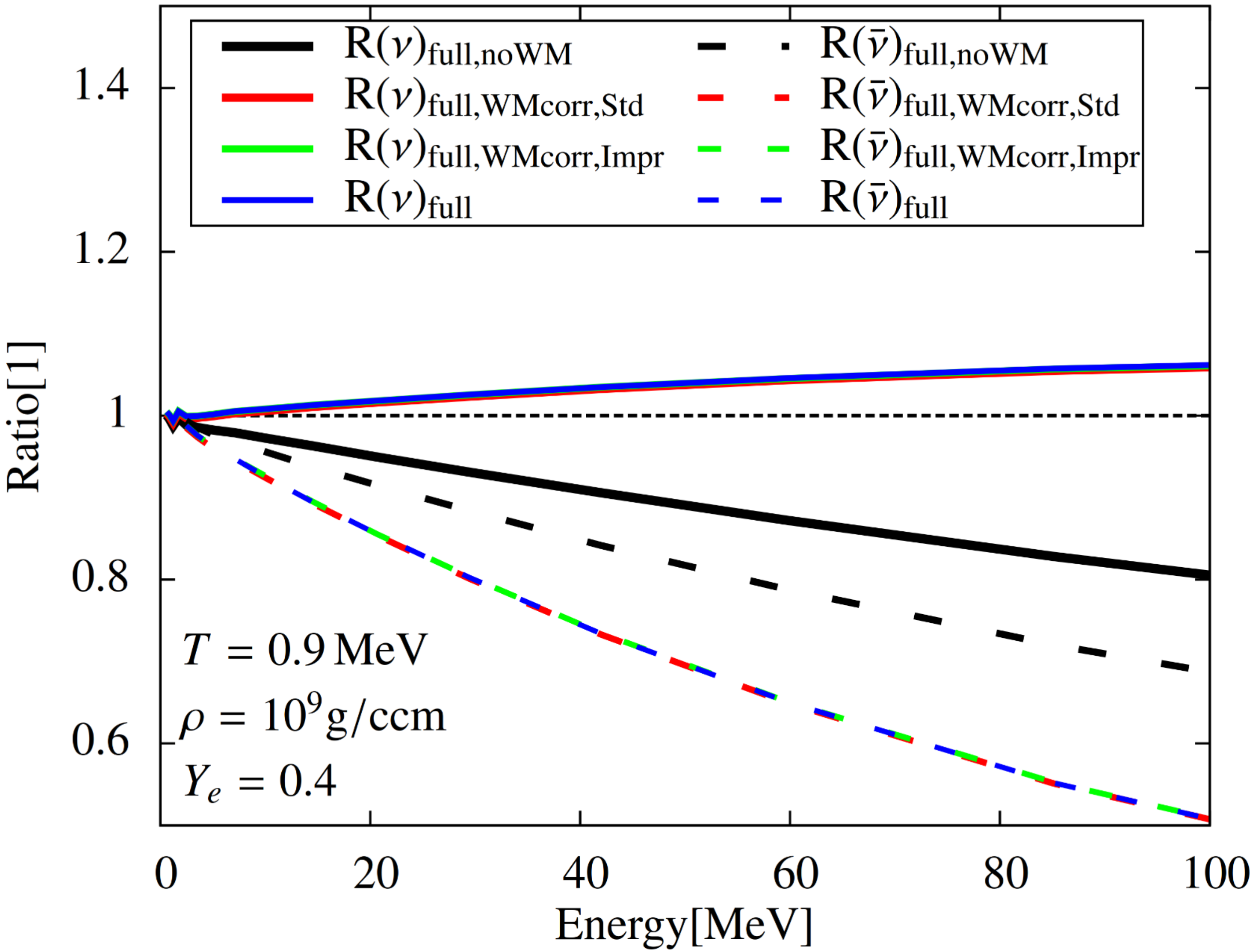
Idea:

- Study correction factors isolated from elastic approximation
- Choose different/lesser approximations of matrix element

Example 1:

- Calculate „exact“ opacity, but with coupling constant $F_2=0$
- Correct by corresponding ratio of vacuum cross sections

Comparing Approximations and Exact Opacities



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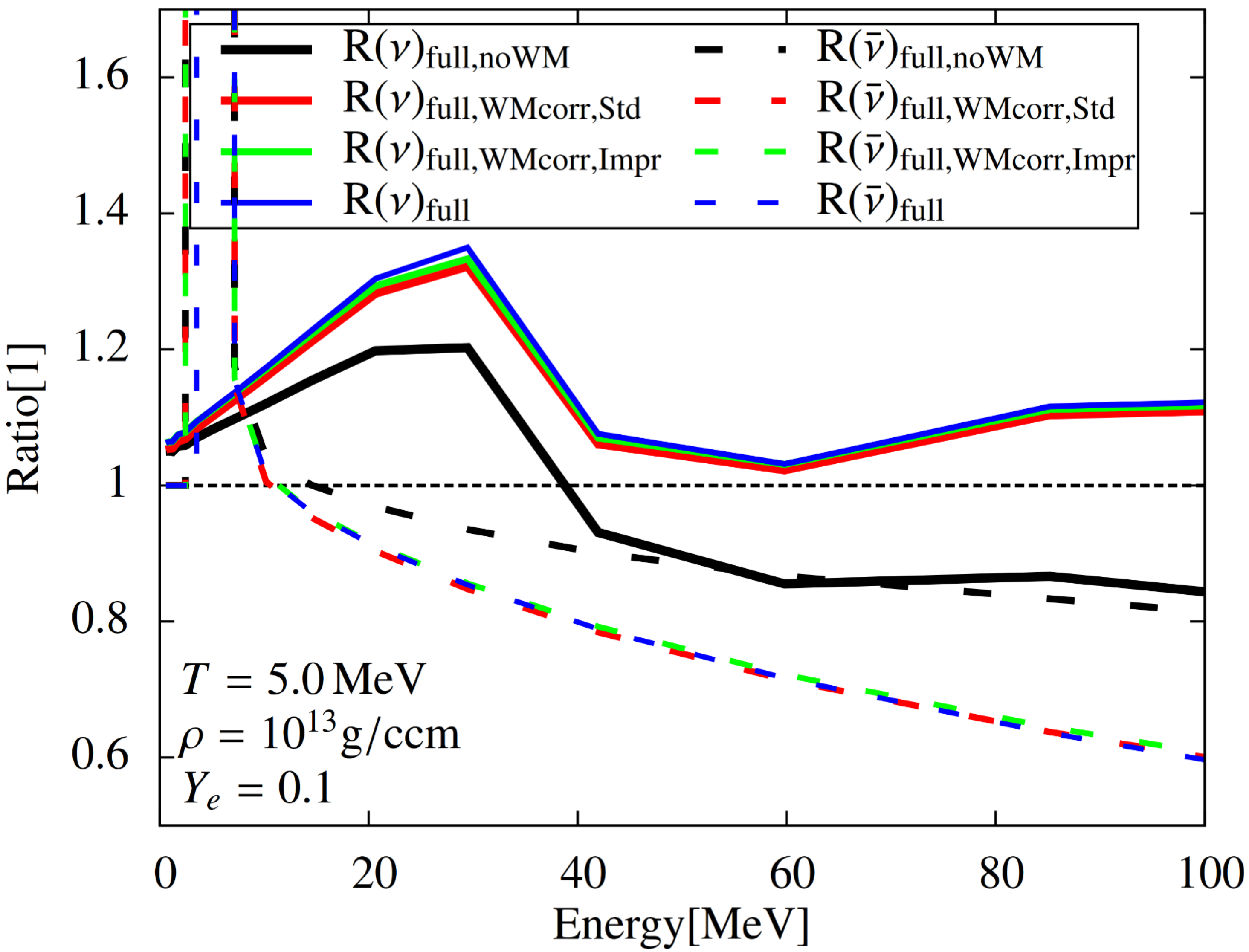

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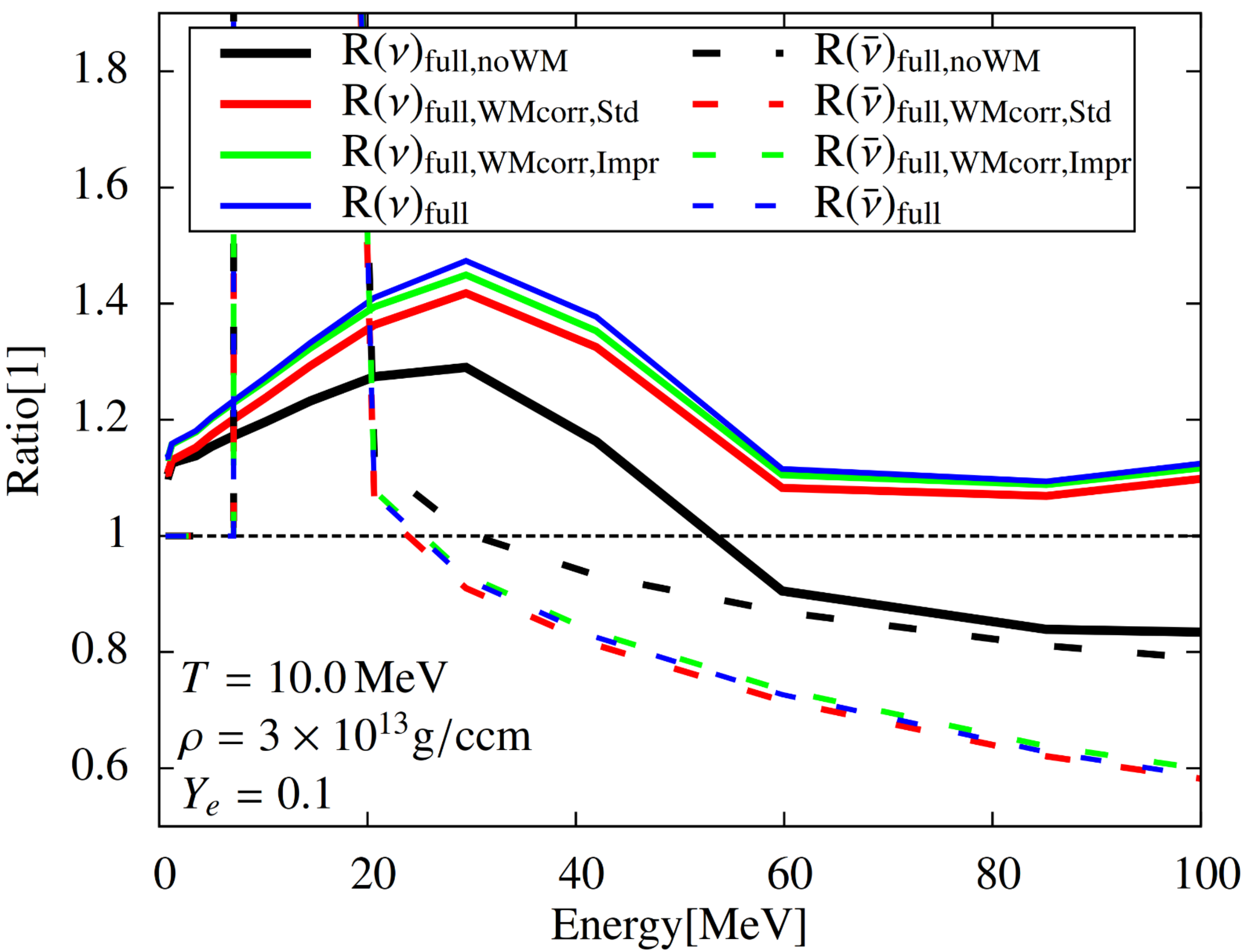


Alternativ Approximations and Corrections

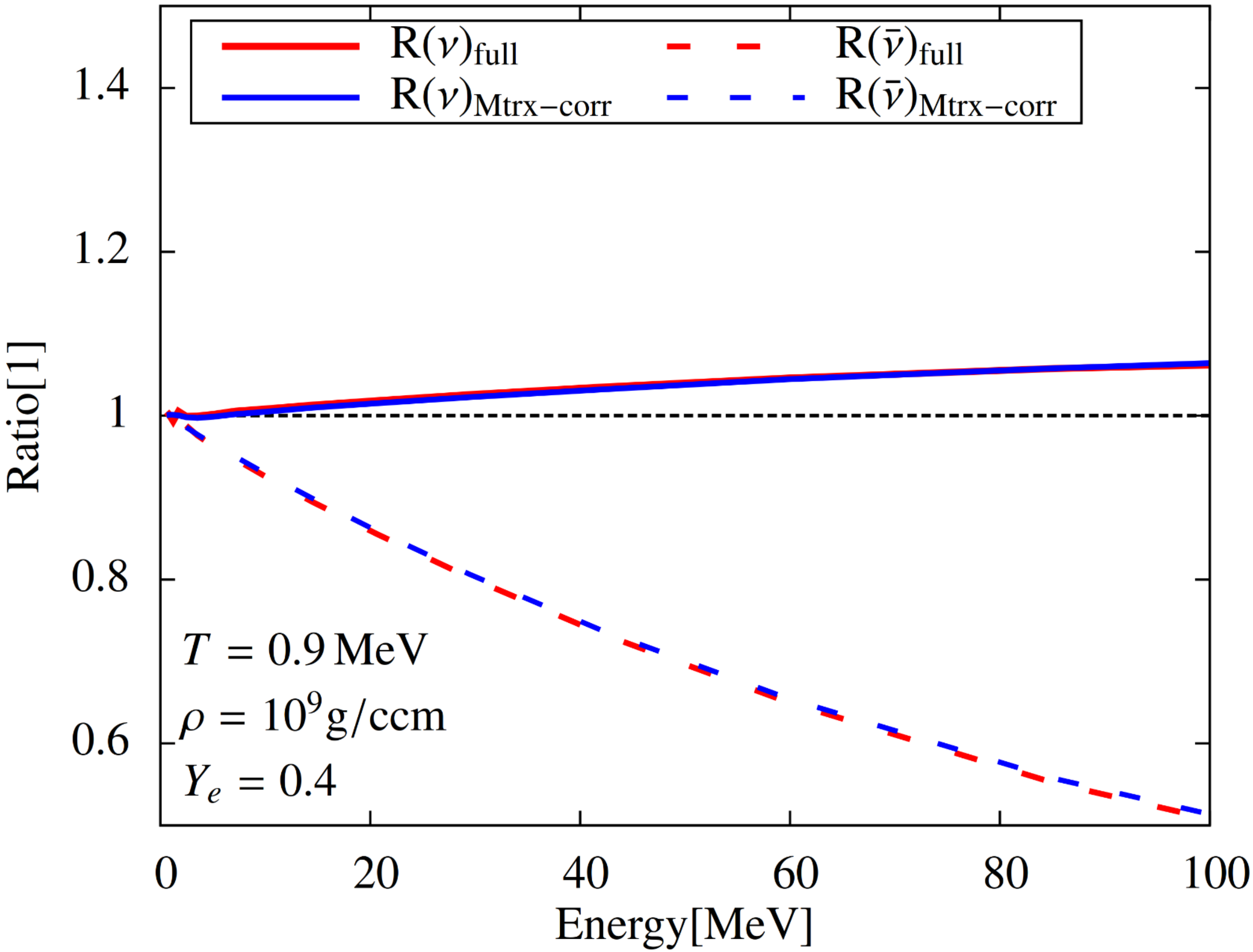
Example 1:

- Calculate „exact“ opacity, but with coupling constant F_2=0
- Correct by corresponding ratio of vacuum cross sections
- ➢ Correction factors precise also at higher densities and temperatures

Example 2:

- Inelastic, nonrelativistic opacity as base
- Most simple matrix element ~ $(G_V^2+3G_A^2)$
- Similar approaches currently applied in some simulations

Comparing Approximations and Exact Opacities



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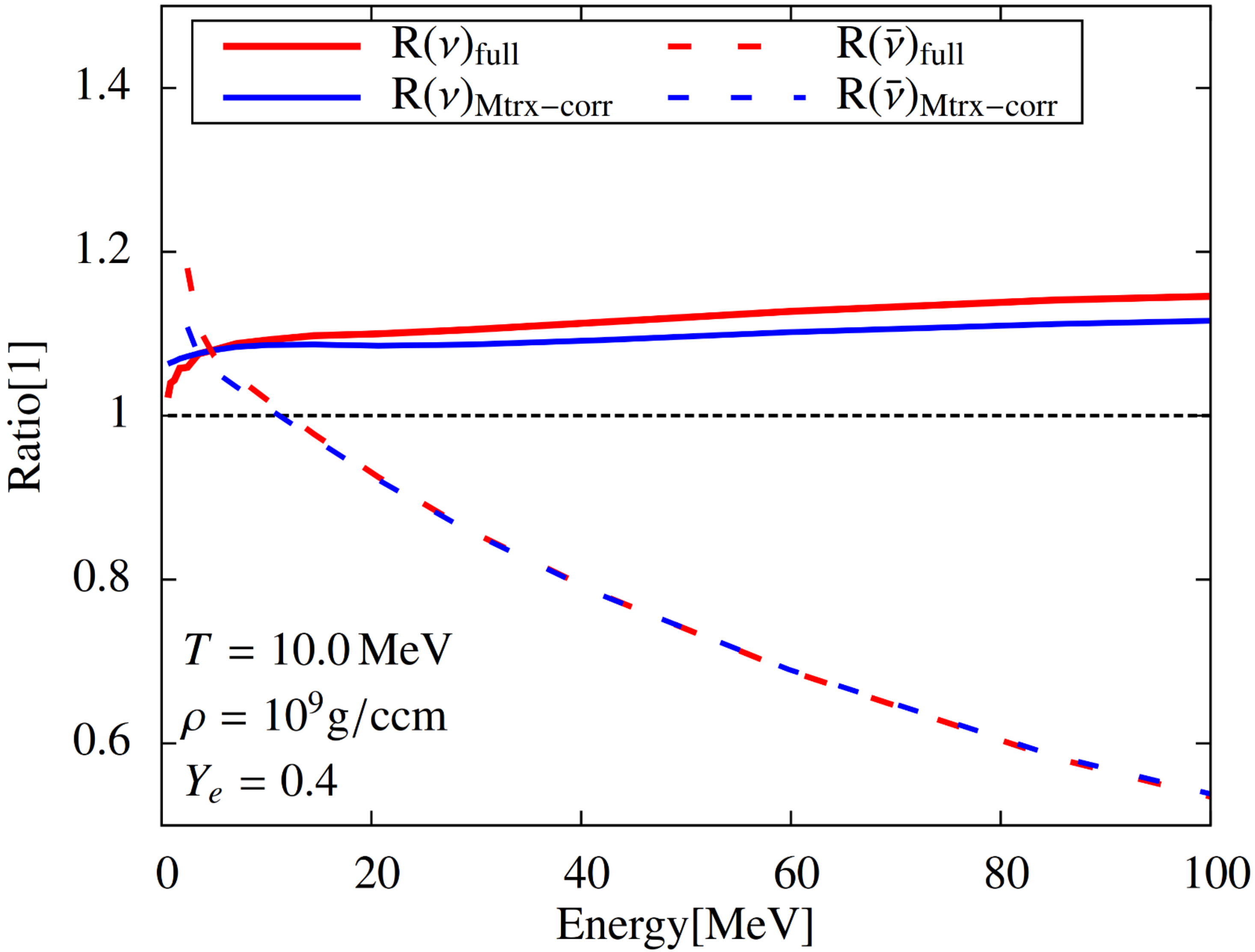

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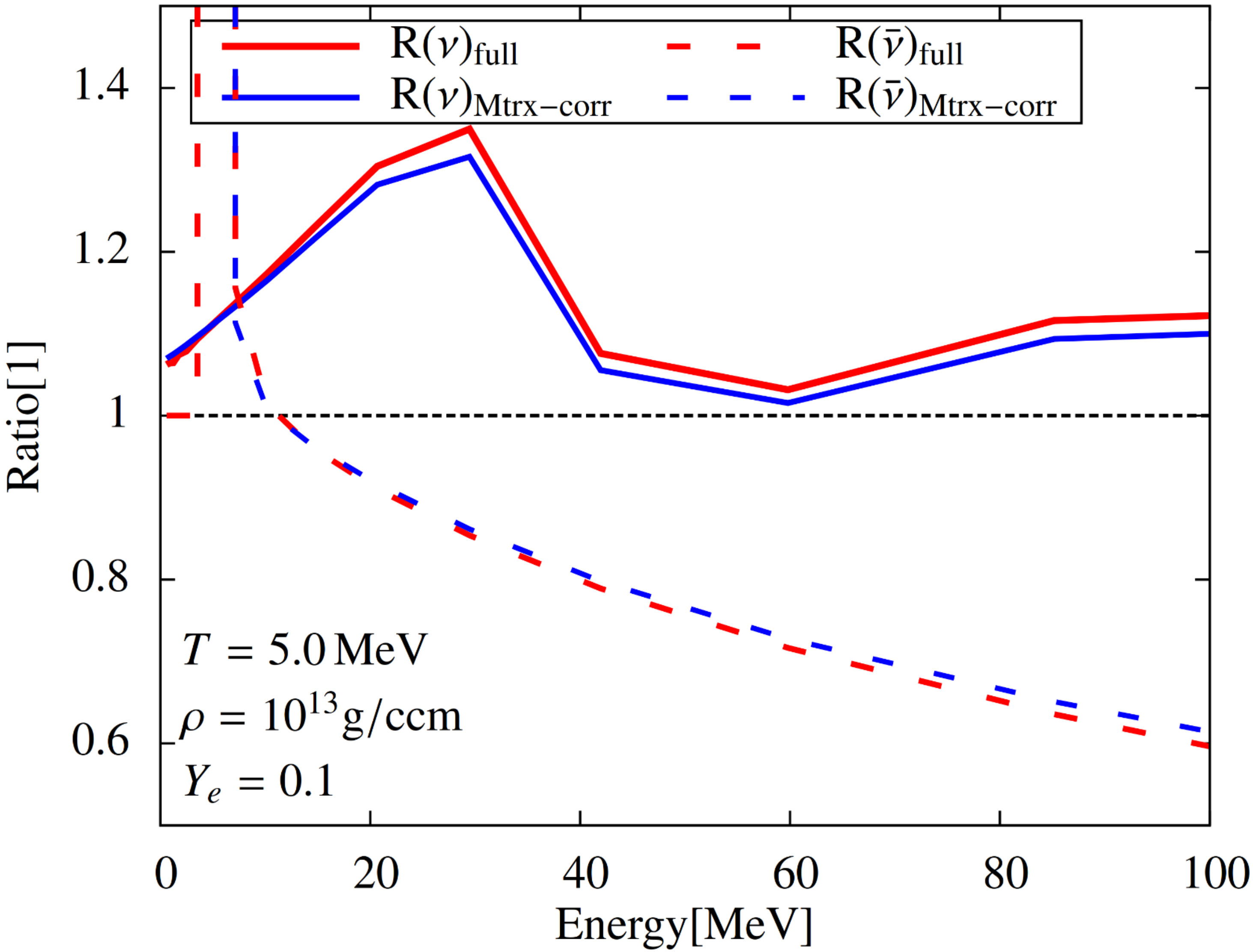

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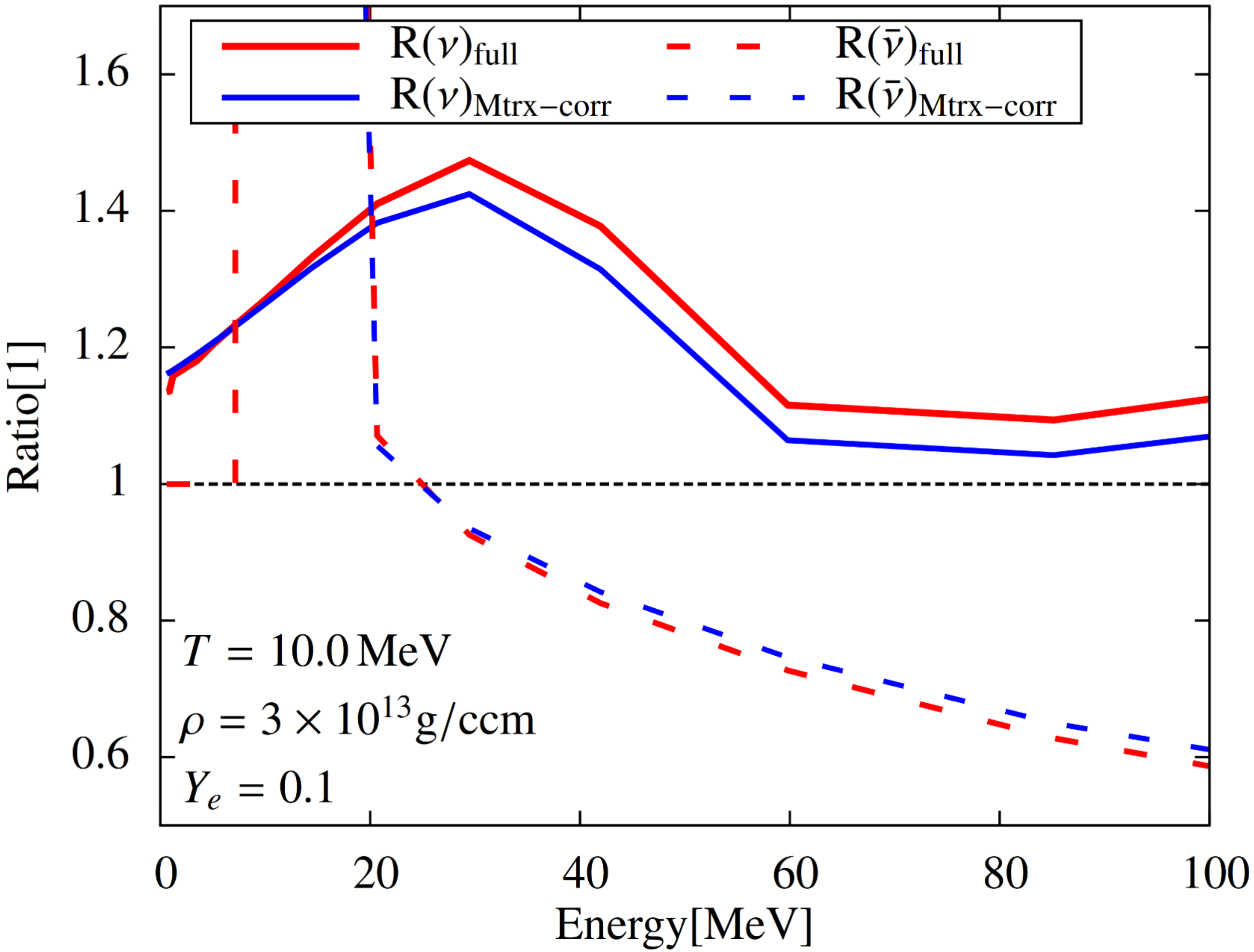

Limit of Approximations for Neutrino Opacities

- For densities up to NDW-conditions and temperatures below several MeV, exact neutrino opacities can be reproduced by elastic approximation + correction factors.
- For higher temperatures or for neutrinosphere densities, the approximation „fails“ at the level of the correction.
- For inelastic opacities, „good“ corrections can be found also at higher densities and temperatures.
- Calculation of inelastic opacities equally demanding as „exact“ opacities
  - For precision at 10% level, „exact“ opacity generally favourable over elastic approximation
  - When interested in correlations, inelastic but approximated opacity + corrections can be suitable

Summary and Conclusion

Correction Factors for neutrino-nucleon interactions:

- Can be extended to include strong interaction potentials and effective masses, and to describe neutron decay
- With elastic appoximation, validity of corrections restricted to to „lower“ densities and temperatures
- For inelastic opacities, precise corrections exist also at neutrinospheres (decoupling region)
- „Exact“ opacity preferable when interested in precise neutrino spectra

- Outlook: Include momentum dependence of coupling constants into „exact“ opacity