

Fission properties of r-process nuclei

Samuel A. Giuliani[†], G. Martínez Pinedo[†], L. M. Robledo[‡]

[†]Technische Universität Darmstadt, Darmstadt, Germany

[‡]Universidad Autónoma de Madrid, Madrid, Spain

January 18, 2016

NAVI Physics Days

GSI – Darmstadt

Outline

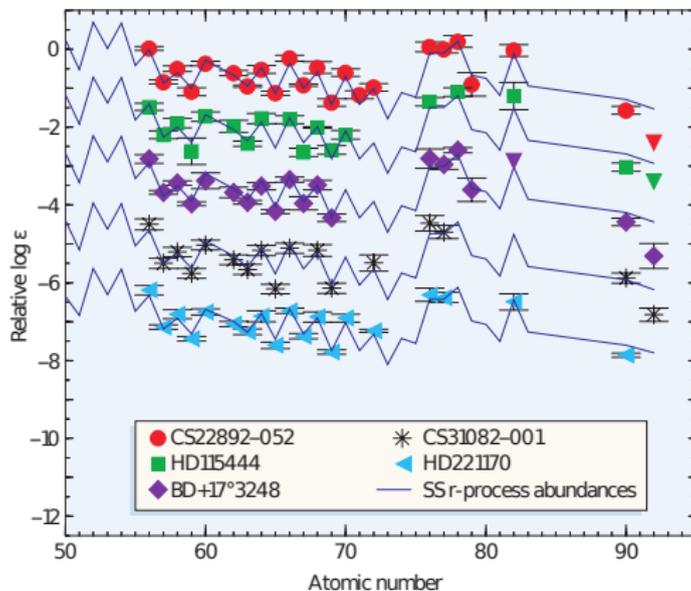
1. Fission and the r-process nucleosynthesis
2. Fission within the Energy Density Functional model
 - Potential energy surfaces and collective inertias
 - Fission observables
3. Fission properties of the BCPM EDF
 - Comparison with experimental data
 - The superheavy landscape
 - Comparison with theoretical models
4. Spontaneous fission: state-of-the-art calculations
 - A more sophisticated model for SF
 - Dynamic fission: ^{240}Pu and ^{234}U
5. Conclusions

Outline

1. Fission and the r-process nucleosynthesis
2. Fission within the Energy Density Functional model
 - Potential energy surfaces and collective inertias
 - Fission observables
3. Fission properties of the BCPM EDF
 - Comparison with experimental data
 - The superheavy landscape
 - Comparison with theoretical models
4. Spontaneous fission: state-of-the-art calculations
 - A more sophisticated model for SF
 - Dynamic fission: ^{240}Pu and ^{234}U
5. Conclusions

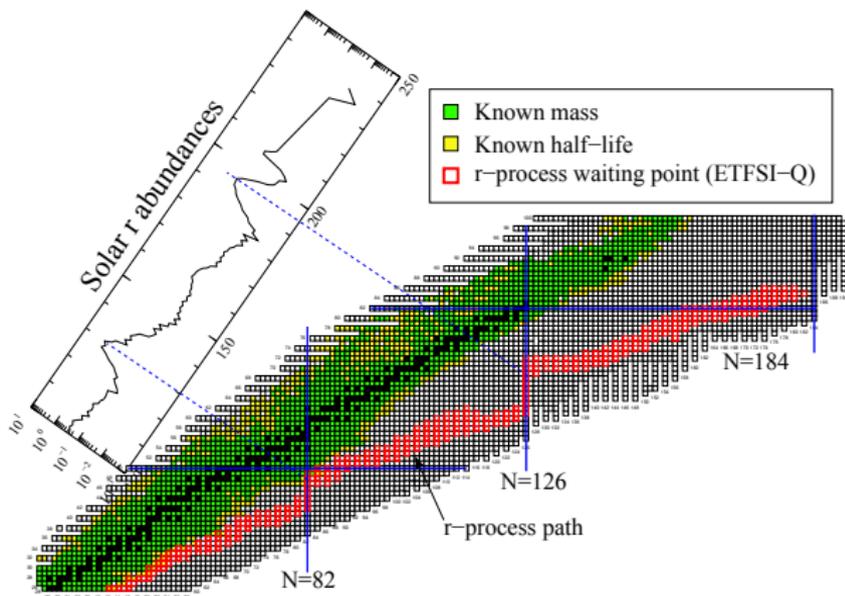
R-process abundances distribution

Cowan & Sneden, Nature 440, 1151 (2006).

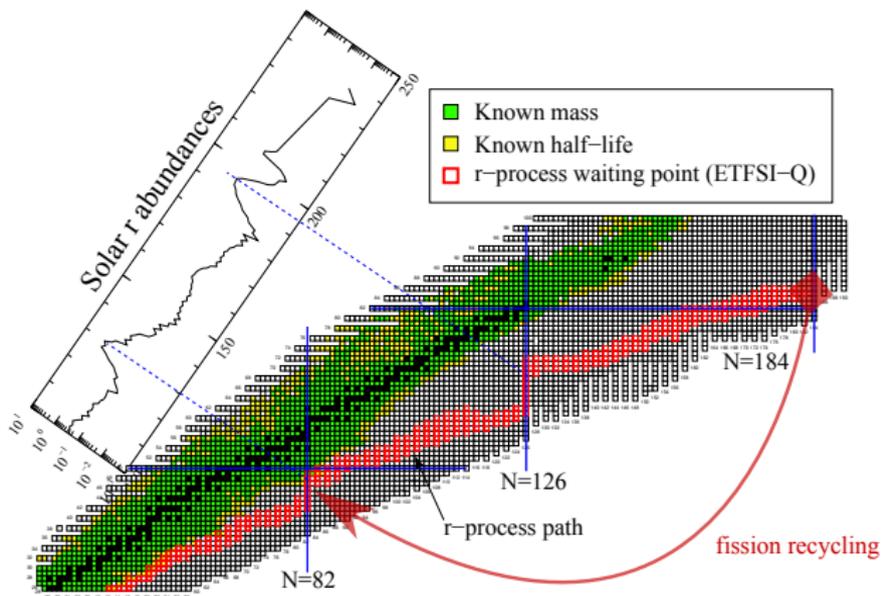


- Mechanism for a robust r-process?

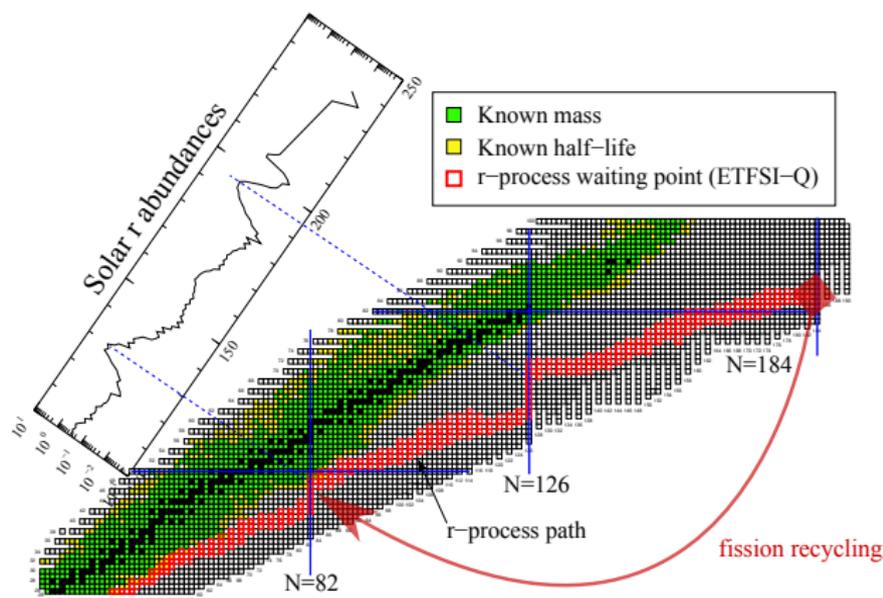
R-process & fission



R-process & fission



R-process & fission



Role of fission in NSM r-process nucleosynthesis:

- ▶ Goriely and Martínez-Pinedo, Nucl. Phys. **A944** (2015);
- ▶ Eichler et al., Astrophys. J. **808**, 30 (2015) and
- ▶ Mendoza-Temis et al., Phys. Rev. C **92**, 055805 (2015) → [M.-R. Wu talk!](#).

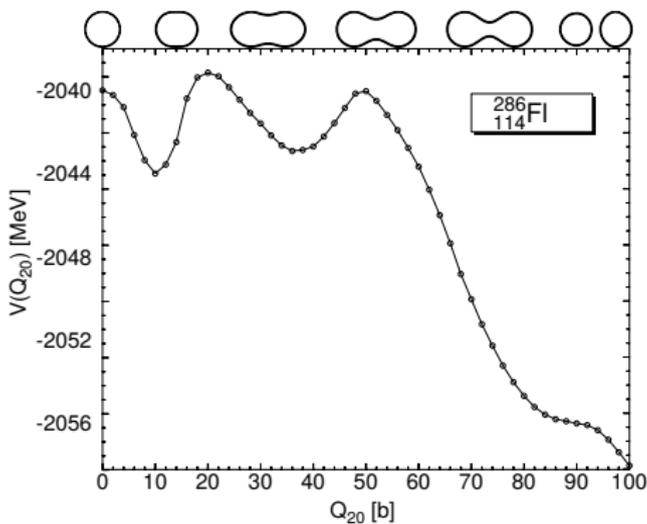
Outline

1. Fission and the r-process nucleosynthesis
2. Fission within the Energy Density Functional model
 - Potential energy surfaces and collective inertias
 - Fission observables
3. Fission properties of the BCPM EDF
 - Comparison with experimental data
 - The superheavy landscape
 - Comparison with theoretical models
4. Spontaneous fission: state-of-the-art calculations
 - A more sophisticated model for SF
 - Dynamic fission: ^{240}Pu and ^{234}U
5. Conclusions

Fission within the Energy Density Functional approach

Potential Energy Surface

Collective inertias

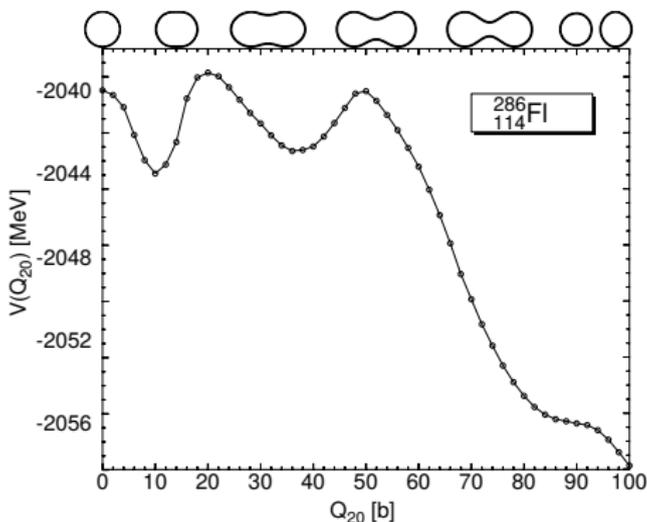


Fission within the Energy Density Functional approach

Potential Energy Surface

Energy evolution from the ground state to the scission point.

Collective inertias



Fission within the Energy Density Functional approach

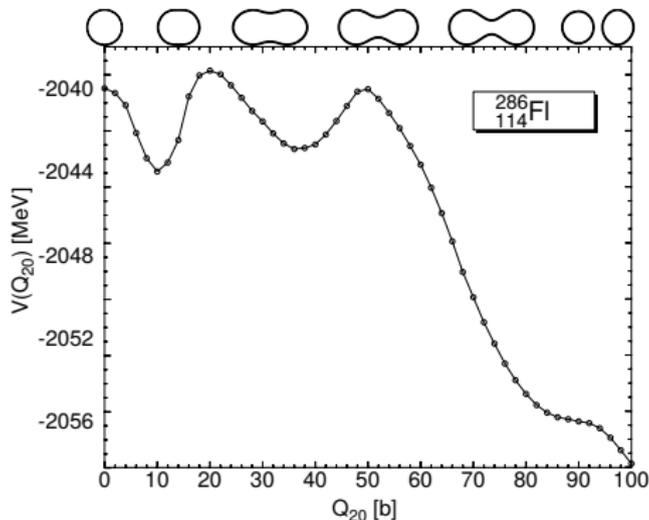
Potential Energy Surface

Energy evolution from the ground state to the scission point.

Relevant degrees of freedom?

Collective inertias

SAG, Robledo and Rodríguez-Guzmán, PRC 90 (2014).



Fission within the Energy Density Functional approach

Potential Energy Surface

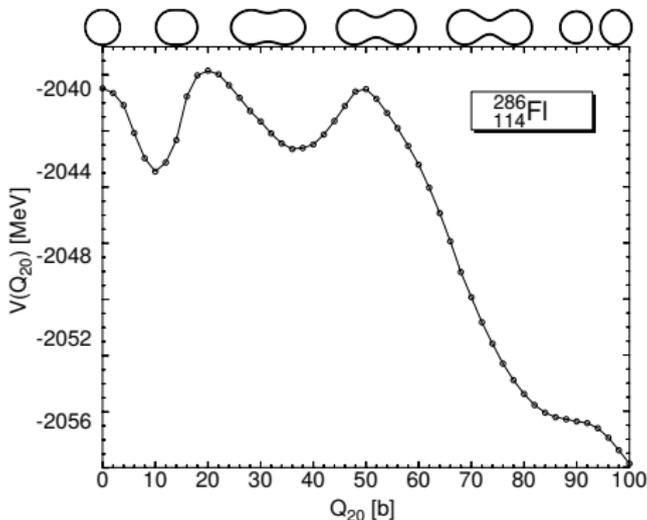
Energy evolution from the ground state to the scission point.

Relevant degrees of freedom?

Collective inertias

Resistance of the nucleus against the deformation forces.

SAG, Robledo and Rodríguez-Guzmán, PRC 90 (2014).



Fission within the Energy Density Functional approach

Potential Energy Surface

Energy evolution from the ground state to the scission point.

Relevant degrees of freedom?

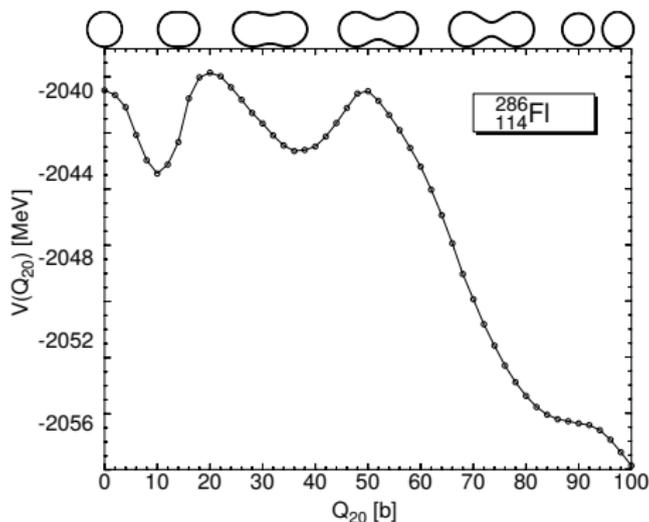
SAG, Robledo and Rodríguez-Guzmán, PRC 90 (2014).

Collective inertias

Resistance of the nucleus against the deformation forces.

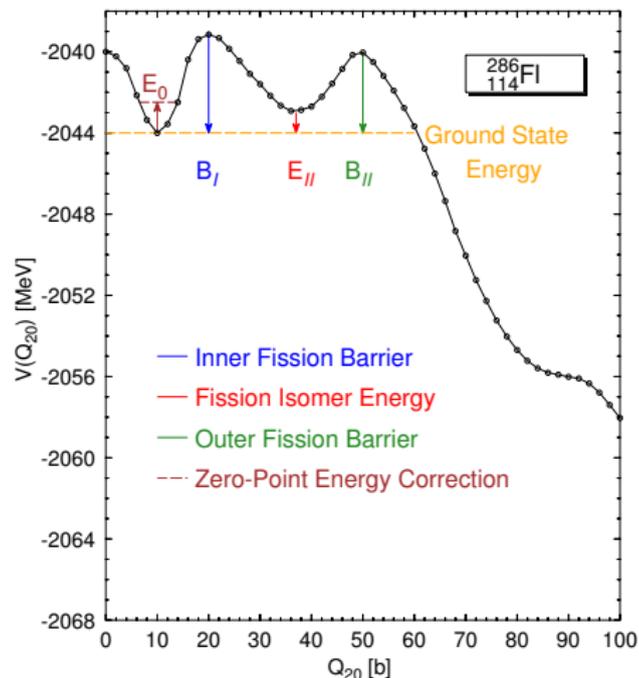
Hard to compute (exactly).

Baran, Sheikh, Dobaczewski et al., PRC 84, (2011).



Fission observables

- ▶ Parameters defining the potential energy surface:
 - inner and outer fission barrier heights,
 - isomer excitation energy.
- ▶ Fission lifetimes t_{sf} :
 - probability of tunneling under the fission barrier.



Theory of spontaneous fission lifetimes

Semiclassical approach given by the WKB formalism:

$$t_{\text{sf}} = 2.86 \times 10^{-21} (1 + \exp(2S)).$$

Action along the fission path $L(s) = L(Q_{20})$:

$$S(L) = \int_a^b dQ_{20} \sqrt{2 \times B(Q_{20}) [E(Q_{20}) - E_0]}.$$

- Collective inertias
- Potential energy
- Zero-Point Energy correction

Fission path given by:

- ▶ minimization of the energy (static approach).

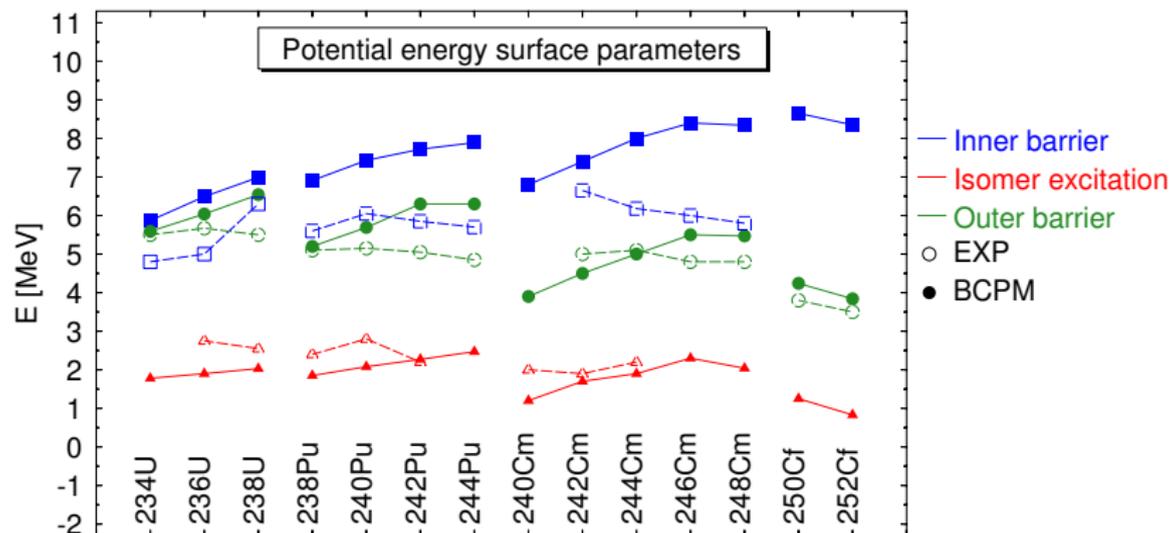
Outline

1. Fission and the r-process nucleosynthesis
2. Fission within the Energy Density Functional model
 - Potential energy surfaces and collective inertias
 - Fission observables
3. Fission properties of the BCPM EDF
 - Comparison with experimental data
 - The superheavy landscape
 - Comparison with theoretical models
4. Spontaneous fission: state-of-the-art calculations
 - A more sophisticated model for SF
 - Dynamic fission: ^{240}Pu and ^{234}U
5. Conclusions

BCPM barrier heights and isomer energy

BCPM: Baldo et al., PRC87 (2013)

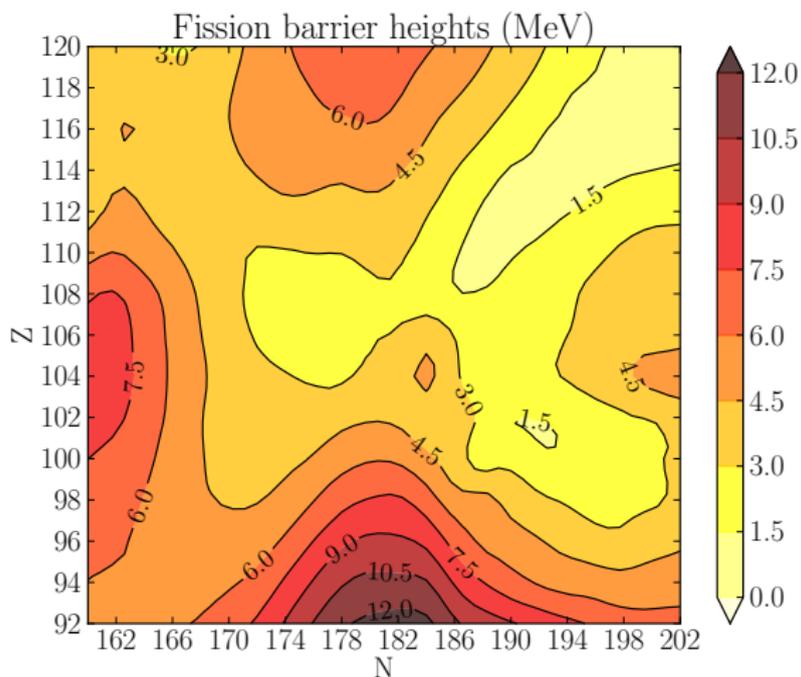
Exp: Sing et al., Nucl. Data Sheets **97**, 241 (2002); R. Capote et al., Nucl. Data Sheets **110**, 3107 (2009).



SAG and Robledo, Phys. Rev. C **88**, 054325 (2013)

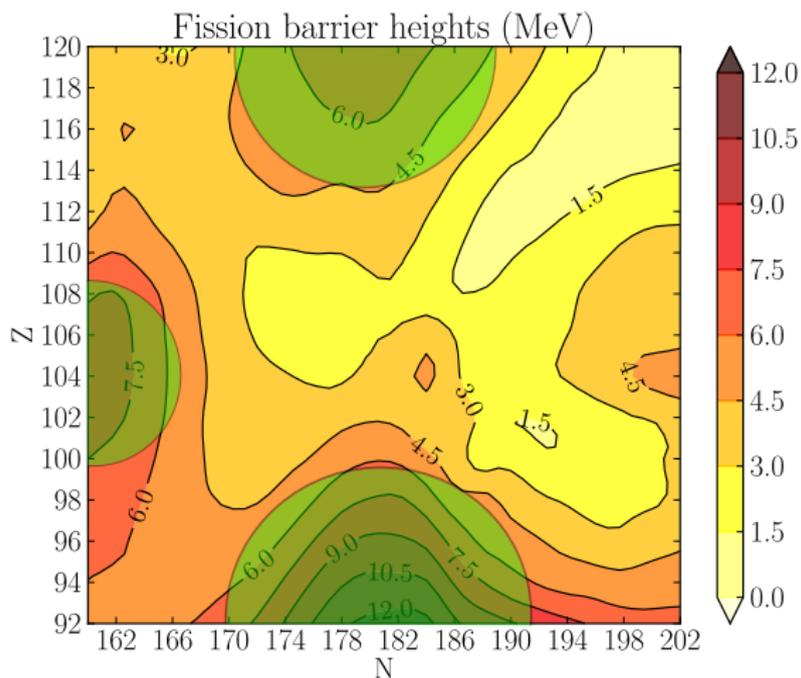
- Outer barriers and isomer energies quite well reproduced for all nuclei.
- Inner barriers are reduced when triaxiality is allowed (Erler+(2012), Guzmán+(2014)).

The superheavy nuclear landscape: fission properties



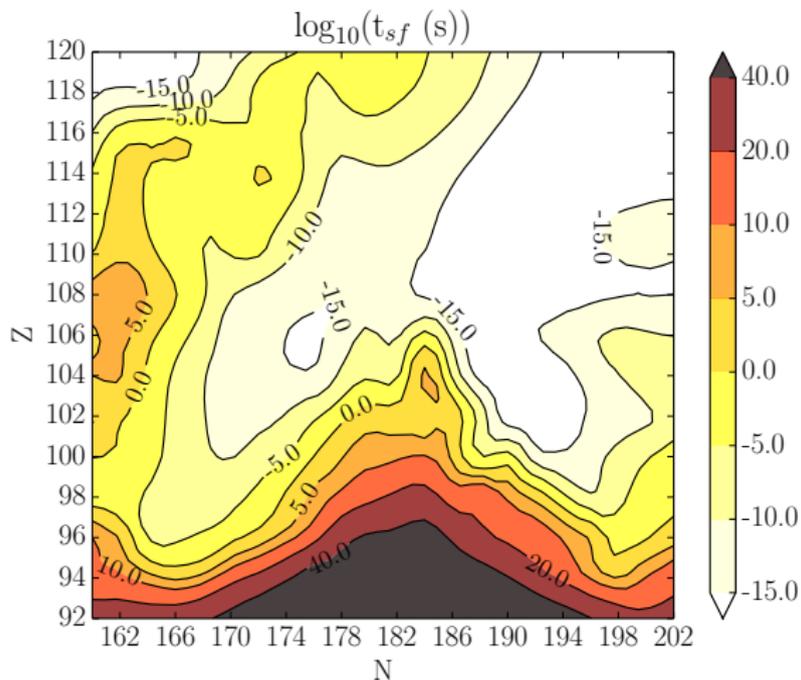
SAG, Martínez-Pinedo and Robledo, PoS (NIC XIII) 095, (2014).

The superheavy nuclear landscape: fission properties



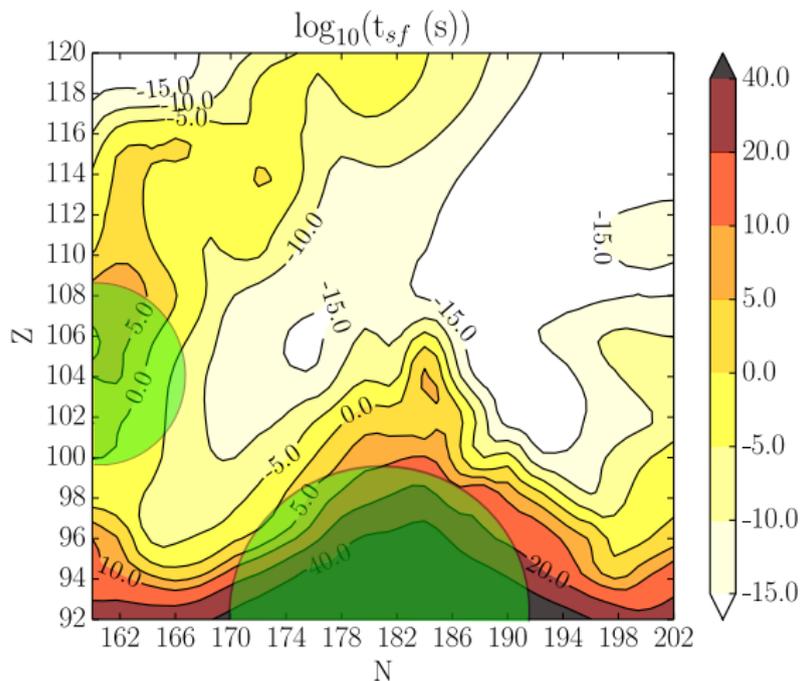
SAG, Martínez-Pinedo and Robledo, PoS (NIC XIII) 095, (2014).

The superheavy nuclear landscape: fission properties



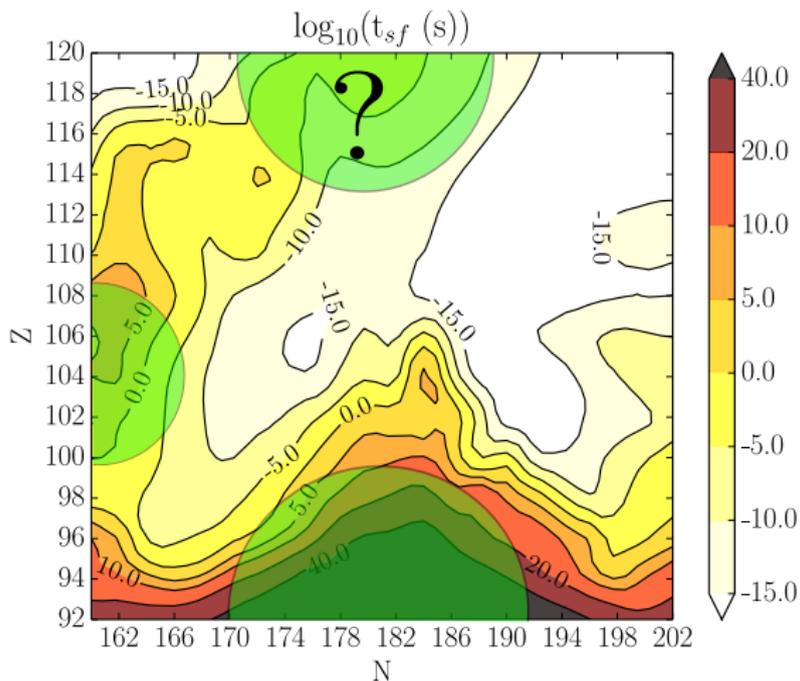
SAG, Martínez-Pinedo and Robledo, PoS (NIC XIII) 095, (2014).

The superheavy nuclear landscape: fission properties



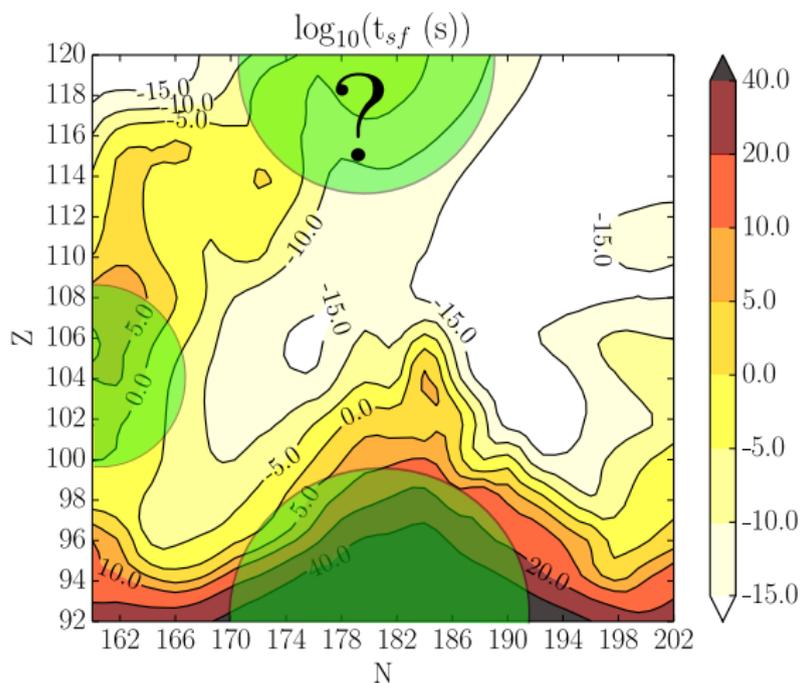
SAG, Martínez-Pinedo and Robledo, PoS (NIC XIII) 095, (2014).

The superheavy nuclear landscape: fission properties



SAG, Martínez-Pinedo and Robledo, PoS (NIC XIII) 095, (2014).

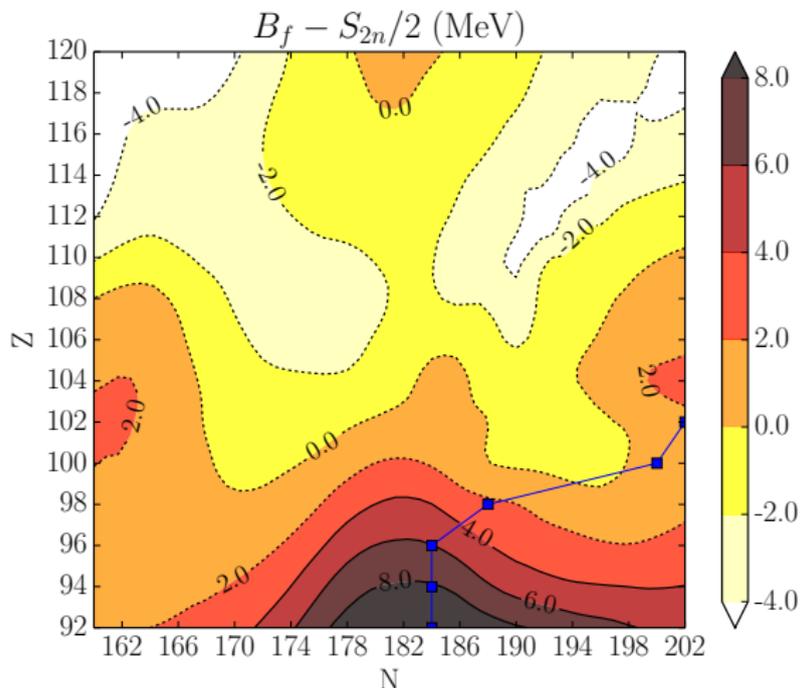
The superheavy nuclear landscape: fission properties



SAG, Martínez-Pinedo and Robledo, PoS (NIC XIII) 095, (2014).

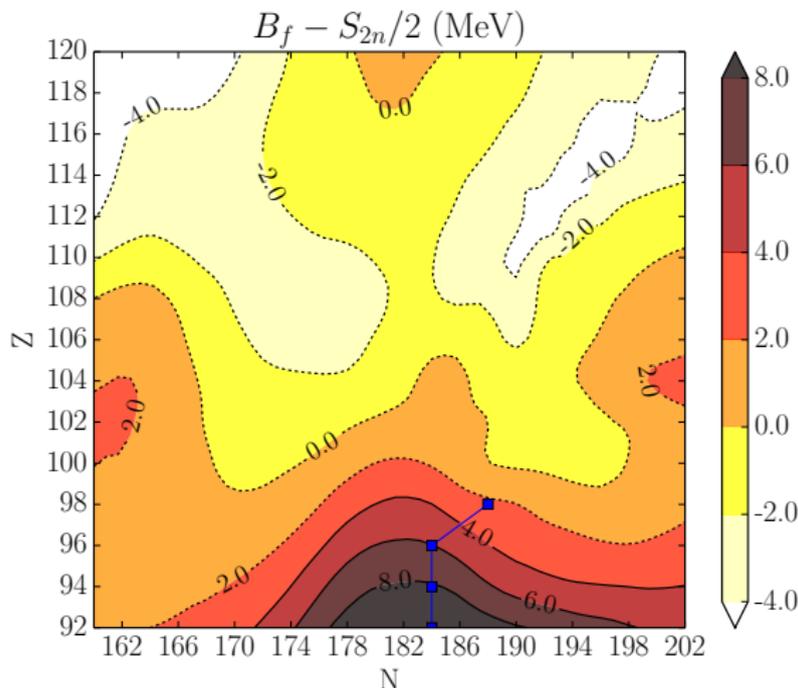
Fission barriers are not everything!

The superheavy nuclear landscape: fission properties



SAG, Martínez-Pinedo and Robledo, PoS (NIC XIII) 095, (2014).

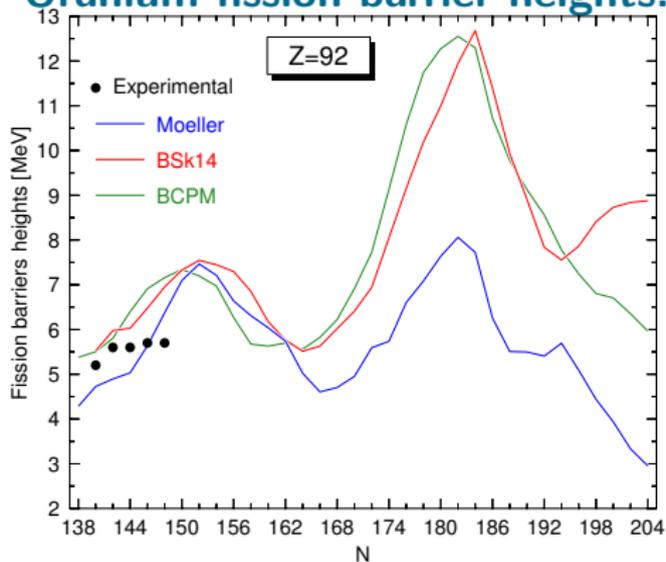
The superheavy nuclear landscape: fission properties



SAG, Martínez-Pinedo and Robledo, PoS (NIC XIII) 095, (2014).

Nuclei with $B_f - 0.5 \times S_{2n} \leq 2$ MeV will fission after capturing a neutron

Uranium fission barrier heights: theoretical predictions

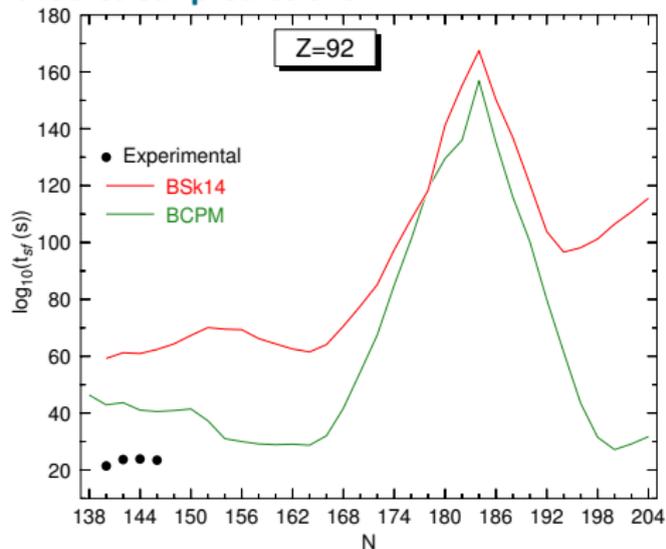
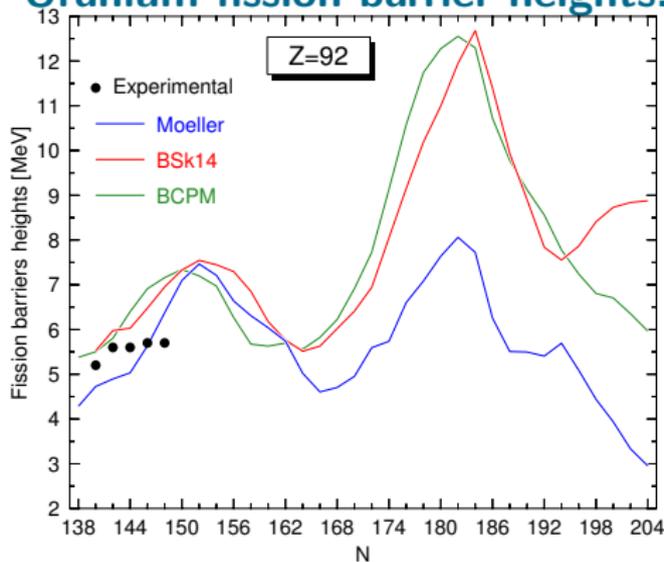


Enhancement around $N = 184$ also predicted by other models.

BSk14: S. Goriely et al., Phys. Rev. **C75**, 064312 (2007).

Möller: P. Möller et al., Phys. Rev. **C91**, 024310 (2015).

Uranium fission barrier heights: theoretical predictions



Collective inertias

BSk14:

$$B = 0.054A^{5/3} \text{ MeV}^{-1}$$

BCPM:

$$B(Q_{20}) = \frac{1}{2} \frac{(M_{-2})^2}{(M_{-1})^3} \quad \text{with} \quad M_{(-n)} = \sum_{\alpha > \beta} \frac{|\langle \alpha \beta | Q_{20} | 0 \rangle|^2}{(E_\alpha + E_\beta)^n}$$

Outline

1. Fission and the r-process nucleosynthesis
2. Fission within the Energy Density Functional model
 - Potential energy surfaces and collective inertias
 - Fission observables
3. Fission properties of the BCPM EDF
 - Comparison with experimental data
 - The superheavy landscape
 - Comparison with theoretical models
4. Spontaneous fission: state-of-the-art calculations
 - A more sophisticated model for SF
 - Dynamic fission: ^{240}Pu and ^{234}U
5. Conclusions

A more sophisticated model for SF

$$t_{SF} \sim \exp(2S) \quad \Leftrightarrow \quad S(L) = \int_a^b ds \sqrt{2 \times B(s) [E(s) - E_0]}$$

Expand the multidimensional PES: relevant d.o.f. in s ?

- ▶ Deformation multipoles: $Q_{20} \Rightarrow Q_{20}, Q_{22}, Q_{30}, \dots$
- ▶ Pairing correlations Δ .

How to determine the fission path $L(s)$?

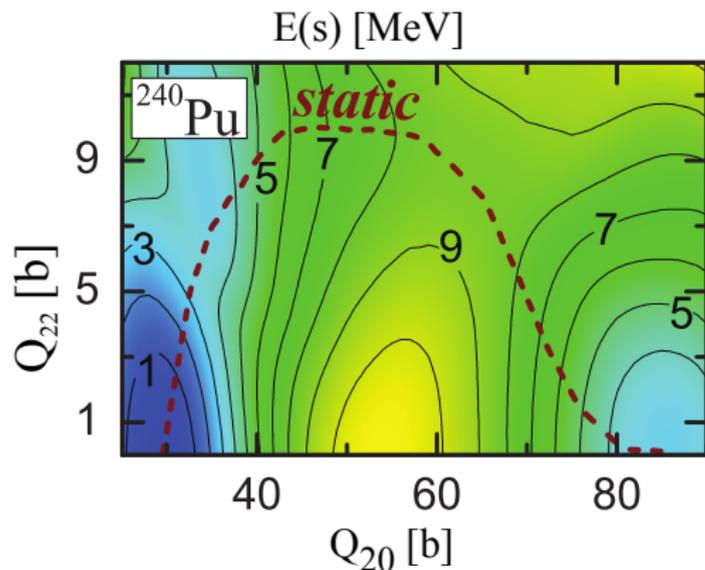
- ▶ minimizing the energy $E(s)$: *static approximation*, or
- ▶ minimizing the action $S(L)$: *dynamic approach*.

State-of-the-art SF calculations:

Sadhukhan et al, PRC88(2013) and PRC90(2014); SAG et al, PRC90(2014); Yao et al, PRC92(2015).

Static vs dynamic fission: ^{240}Pu and ^{234}U

Triaxial case: ^{240}Pu - SkM* interaction



from Shadukhan et al., PRC90(2014).

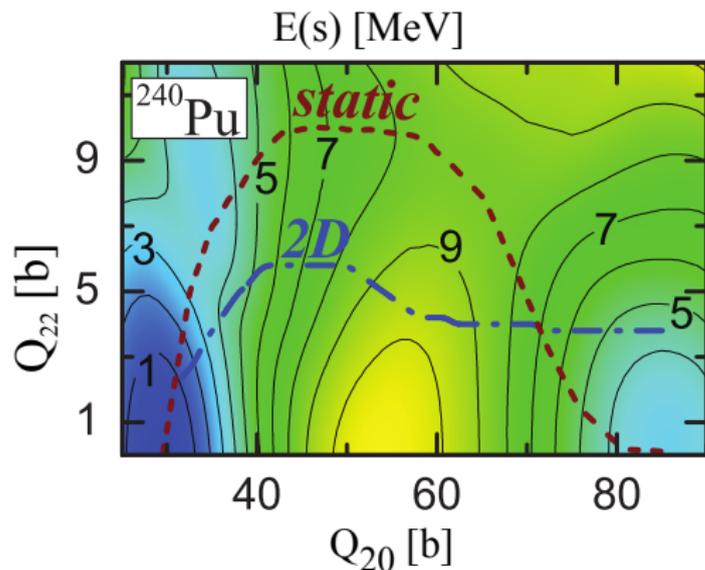
dynamic paths:

$$2\text{D: } s = \{Q_{20}, Q_{22}\}$$

$$3\text{D: } s = \{Q_{20}, Q_{22}, \Delta N^2\}$$

Static vs dynamic fission: ^{240}Pu and ^{234}U

Triaxial case: ^{240}Pu - SkM* interaction



from Shadukhan et al., PRC90(2014).

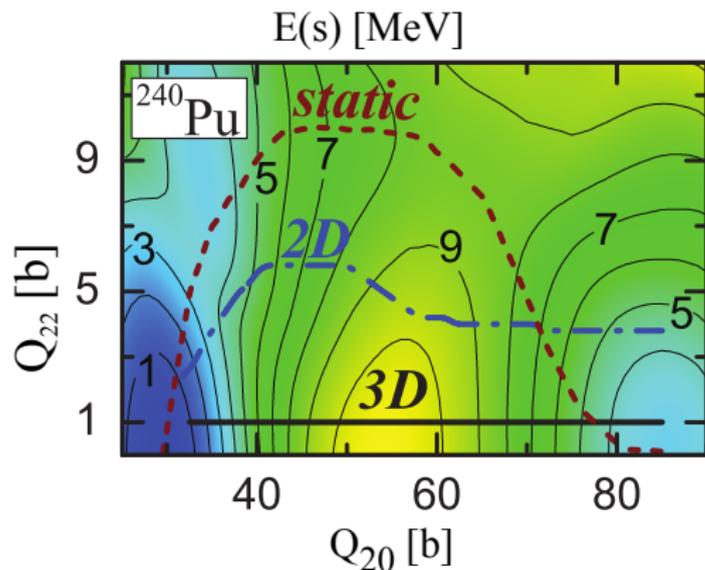
dynamic paths:

$$2\text{D}: s = \{Q_{20}, Q_{22}\}$$

$$3\text{D}: s = \{Q_{20}, Q_{22}, \Delta N^2\}$$

Static vs dynamic fission: ^{240}Pu and ^{234}U

Triaxial case: ^{240}Pu - SkM* interaction



from Shadukhan et al., PRC90(2014).

dynamic paths:

$$2\text{D}: s = \{Q_{20}, Q_{22}\}$$

$$3\text{D}: s = \{Q_{20}, Q_{22}, \Delta N^2\}$$

Pairing fluctuations **restore** the **axial symmetry** (artifact?)!

Static vs dynamic fission: ^{240}Pu and ^{234}U

Axial case: ^{234}U - BCPM interaction

Method	t_{sf} (s)
E_{min} (static)	0.81×10^{43}
$S_{\text{min}}(Q_{20}, Q_{30})$	0.44×10^{42}
$S_{\text{min}}(Q_{20}, Q_{40})$	0.12×10^{43}
$S_{\text{min}}(Q_{20}, \Delta N^2)$	0.18×10^{23}

SAG, Robedo and Guzmán-Rodríguez PRC90(2014).

Pairing fluctuations decrease the t_{sf} (20 OM!).

Static vs dynamic fission: ^{240}Pu and ^{234}U

Axial case: ^{234}U - BCPM interaction

Method	t_{sf} (s)
E_{min} (static)	0.81×10^{43}
$S_{\text{min}}(Q_{20}, Q_{30})$	0.44×10^{42}
$S_{\text{min}}(Q_{20}, Q_{40})$	0.12×10^{43}
$S_{\text{min}}(Q_{20}, \Delta N^2)$	0.18×10^{23}

SAG, Robedo and Guzmán-Rodríguez PRC90(2014).

Pairing fluctuations decrease the t_{sf} (20 OM!).

Conclusion

Fission properties strongly modified by pairing fluctuations!

Outline

1. Fission and the r-process nucleosynthesis
2. Fission within the Energy Density Functional model
 - Potential energy surfaces and collective inertias
 - Fission observables
3. Fission properties of the BCPM EDF
 - Comparison with experimental data
 - The superheavy landscape
 - Comparison with theoretical models
4. Spontaneous fission: state-of-the-art calculations
 - A more sophisticated model for SF
 - Dynamic fission: ^{240}Pu and ^{234}U
5. Conclusions

Conclusions

- ▶ Fission plays a crucial role in r-process nucleosynthesis.
- ▶ EDFs give a good **qualitative description** of the fission process.
- ▶ However, theoretical predictions affected from several uncertainties (choice of the d.o.f., collective inertias. . .).
- ▶ BCPM superheavy landscape:
 - peak of stability around $N = 184$ and $Z = 104$,
 - production of SH nuclei inhibited by neutron-induced fission.
- ▶ **Take-away messages:**
 - ▶ Fission barriers are not everything!
 - ▶ Pairing fluctuations strongly modify fission properties!
- ▶ **Future work:**
 - Computation of **fission rates and fragments distribution**.
 - Improve computation of collective masses.

THANK YOU



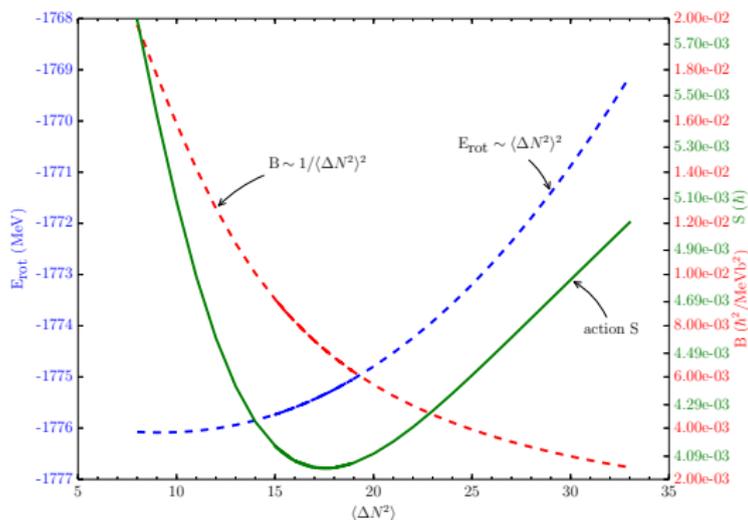
TECHNISCHE
UNIVERSITÄT
DARMSTADT



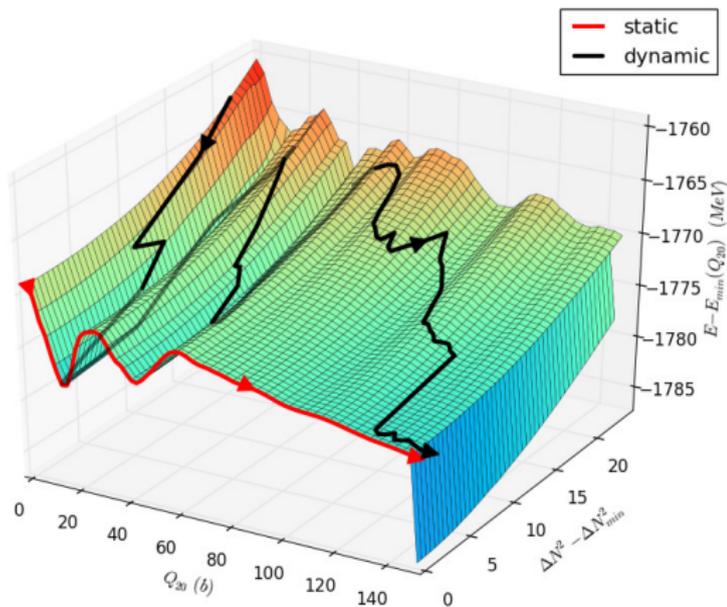
HIC | **FAIR**
for
Helmholtz International Center

Minimizing the action: $B(\Delta N^2)$ vs $E(\Delta N^2)$ - ^{234}U

$$S = \int_a^b ds \sqrt{2 \times B(s) [E(s) - E_0]}$$



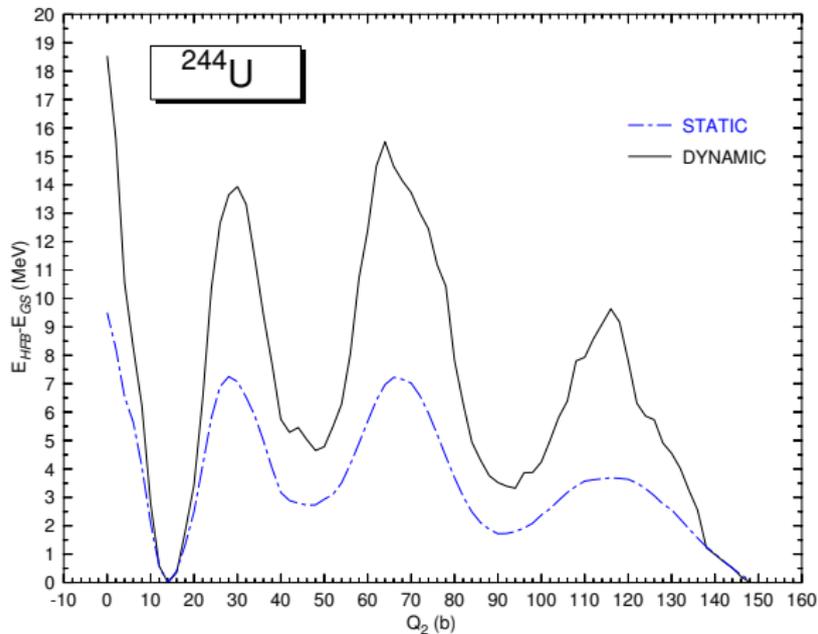
The least action path



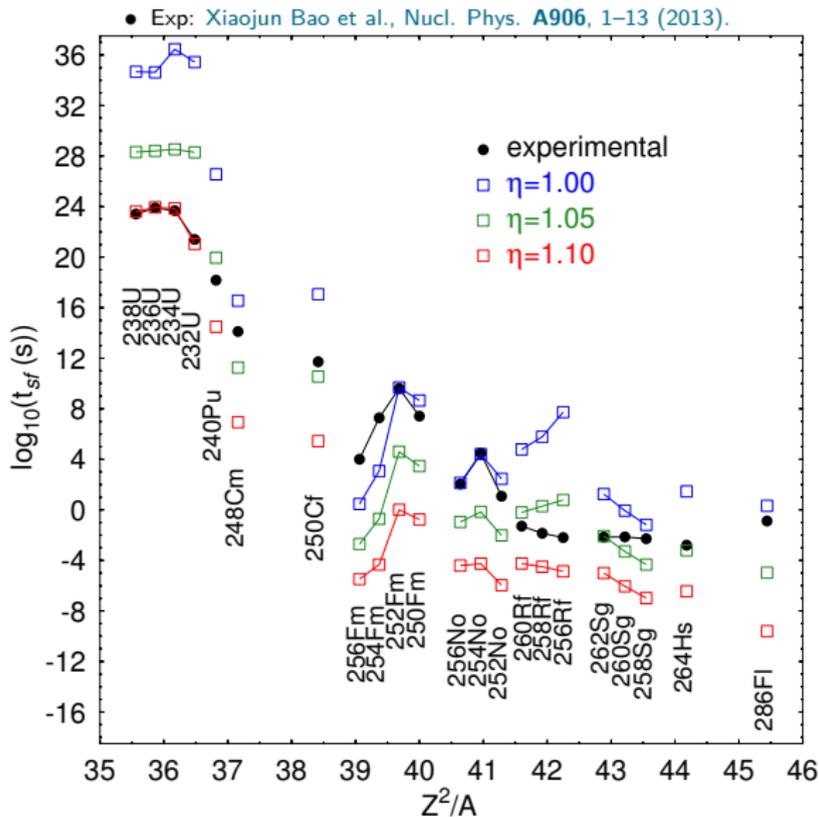
- ▶ The least action path (black) strongly differ from the least energy one (red)!

Fission barriers and E_0 - ^{244}U

$$S = \int_a^b ds \sqrt{2 \times B(s) [E(s) - E_0]}$$



Spontaneous fission lifetimes (BCPM results)



The BCPM functional

The energy of a finite nucleus is given by

$$E = T_0 + E_{int}^{\infty} + E_{int}^{FR} + E^{s.o.} + E_C + E_{pair}$$

$$E_{int}^{\infty}[\rho_p, \rho_n] = \int d\vec{r} [P_s(\rho)(1 - \beta^2) + P_n(\rho)\beta^2] \rho$$

with $\rho(\vec{r}) = \rho_n(\vec{r}) + \rho_p(\vec{r})$ and $\beta(\vec{r}) = (\rho_n(\vec{r}) - \rho_p(\vec{r}))/\rho(\vec{r})$.

P_s and P_n are polynomial fits to reproduce microscopic EoS in nuclear matter.

► Phenomenological surface contribution

$$E_{int}^{FR}[\rho_n, \rho_p] = \frac{1}{2} \sum_{t,t'} \iint d\vec{r} d\vec{r}' \rho_t(\vec{r}) v_{t,t'}(\vec{r} - \vec{r}') \rho_{t'}(\vec{r}')$$

with $v_{t,t'}(r) = V_{t,t'} e^{-r^2/r_0 t^2}$; $V_{n,n} = V_{p,p} = V_L = 2\tilde{b}_1/(\pi^{3/2} r_{0L}^3 \rho_0)$;
 $V_{n,p} = V_{p,n} = V_U = (4a_1 - 2\tilde{b}_1)/(\pi^{3/2} r_{0U}^3 \rho_0)$.

Remaining contributions to the EDF

- ▶ Coulomb

$$\text{Direct } E_C^H = (1/2) \iint d\vec{r} d\vec{r}' \rho_p(\vec{r}) |\vec{r} - \vec{r}'|^{-1} \rho_p(\vec{r}')$$

$$\text{Exchange: } E_C^{ex} = -(3/4)(3/\pi)^{1/3} \int d\vec{r} \rho_p(\vec{r})^{4/3}$$

- ▶ Spin-Orbit

$$\hat{v}_{ij}^{so} = iW_{LS}(\vec{\sigma}_i + \vec{\sigma}_j) \cdot [\vec{k}' \times \delta(\vec{r}_i - \vec{r}_j)\vec{k}]$$

Free parameters

W_{LS} and r_{0L}, r_{0U}

- ▶ Pairing Correlations (E. Garrido et al. Phys. Rev. C **60**, 064312 (1999))

Zero-range interaction,

$$v^{pp}(\rho(\vec{r})) = \eta \times \frac{v_0}{2} \left[1 - \gamma \left(\frac{\rho(\vec{r})}{\rho_0} \right)^\alpha \right], \quad \rho_0 = \frac{2}{3\pi^2} k_F^3.$$

$\eta \equiv$ multiplicative parameter setting the pairing strength...