Strongly Interacting Matter at High Baryon Density

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thanks to:
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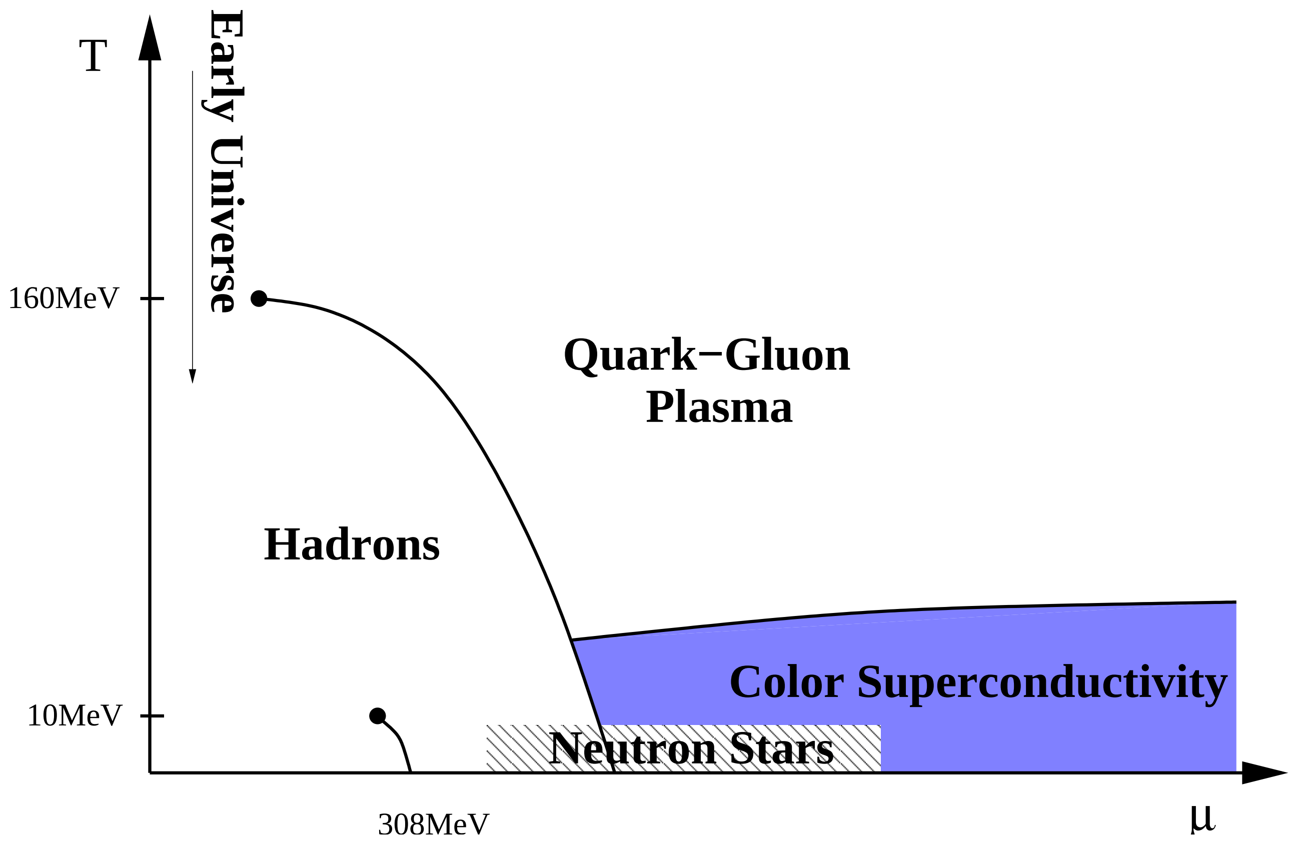
“Dense Hadronic Matter and QCD Phase Transitions”

http://theory.gsi.de/Vir-Institute/

and in particular:
M. Buballa, M. Kitazawa, D. Nickel, S. Rüster, I.A. Shovkovy, V. Werth

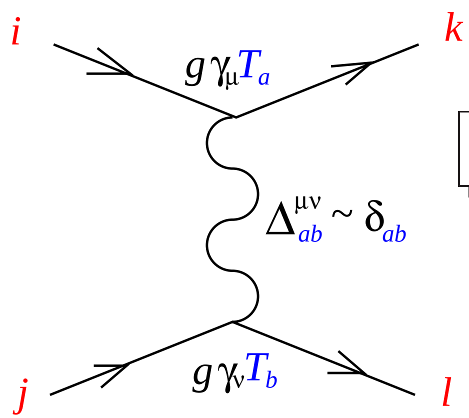
QCD phase diagram



Diquark interaction at asymptotically large μ_q

$\mu_q \gg 1$ GeV: asymptotic freedom of QCD

$\Longrightarrow$ 1-gluon exchange dominant interaction between quarks



scattering amplitude:

$$\sim (T_a)_{ki}\,(T_a)_{lj} = -\tfrac{1}{3}\,(\delta_{ki}\delta_{lj} - \delta_{kj}\delta_{li}) + \tfrac{1}{6}\,(\delta_{ki}\delta_{lj} + \delta_{kj}\delta_{li})$$

$$[3]_c \times [3]_c = [\bar{3}]_c^{\mathrm{a}} + [6]_c^{\mathrm{s}}$$

attractive! repulsive

Cooper's theorem: attractive interaction destabilizes Fermi surface

$\Longrightarrow$ condensation of quark Cooper pairs in $[\bar{3}]_c^a$ – channel

$$0 \neq \langle \psi^i \psi^j \rangle \sim \Phi^{ij} \equiv \epsilon^{ij}{}_k\, \Phi_k$$

Φ_k: order parameter for condensation

$\Longrightarrow$ cold, dense quark matter is color-superconducting!

Note: condensed matter physics: secondary interaction (phonon)

QCD: fundamental interaction (gluon)!

Order parameter for condensation (I)

Pauli principle $\Longrightarrow$ condensate $\langle \psi^i_f \, \Gamma^\alpha \, \psi^j_g \rangle \sim \Phi^{ij\alpha}_{fg}$ completely antisymmetric

1. Spin $J = 0$: $[1]^{\rm a}_J$

   (a) $N_f = 1$: $[1]^{\rm a}_J \times [\bar{3}]^{\rm a}_c \Longrightarrow$ condensation impossible

   (b) $N_f = 2$: $[2]_f \times [2]_f = [1]^{\rm a}_f + [3]^{\rm s}_f \Longrightarrow [1]^{\rm a}_J \times [\bar{3}]^{\rm a}_c \times [1]^{\rm a}_f \Longrightarrow \Phi^{ij}_{fg} \equiv \epsilon^{ij}{}_k \, \epsilon_{fg} \, \Phi_k$

   2SC phase: $\Phi_k \equiv \delta_{k3} \, \Phi \Longrightarrow i, j = 1, 2$
   only red and green quarks condense, blue quarks remain unpaired
   $SU(3)_c \times SU(2)_f \times U(1)_{\rm em} \to SU(2)_c \times SU(2)_f \times \tilde{U}(1)$
   $\Longrightarrow$ 5 massive, 3 massless gluons, 1 (modified) massless photon

   (c) $N_f = 3$: $[3]_f \times [3]_f = [\bar{3}]^{\rm a}_f + [6]^{\rm s}_f \Longrightarrow [1]^{\rm a}_J \times [\bar{3}]^{\rm a}_c \times [\bar{3}]^{\rm a}_f \Longrightarrow \Phi^{ij}_{fg} \equiv \epsilon^{ij}{}_k \, \epsilon_{fg}{}^h \, \Phi^h_k$

   Color-flavor-locking (CFL) phase: $\Phi^h_k = \delta^h_k \, \Phi$
   all quark colors and flavors condense
   $SU(3)_c \times SU(3)_f \times U(1)_{\rm em} \to SU(3)_{c+f} \times \tilde{U}(1)$
   $\Longrightarrow$ all 8 gluons massive, 1 (modified) massless photon

M. Alford, K. Rajagopal, F. Wilczek, NPB 537 (1999) 443

Order parameter for condensation (II)

2. Spin $J = 1$: $[3]^{s}_{J}$

(a) $N_f = 1$: $[3]^{s}_{J} \times [\bar{3}]^{a}_{c} \Longrightarrow \Phi^{ij\alpha} \equiv \epsilon^{ij}{}_{k}\, \Phi^{\alpha}_{k}$

$\Longrightarrow$ cf. superfluid He-3: $[3]^{s}_{S} \times [3]^{a}_{L}$

$\Longrightarrow$ many different phases, e.g.

i. polar phase: $\Phi^{\alpha}_{k} = \delta_{k3}\delta^{\alpha 3}\, \Phi$

ii. planar phase: $\Phi^{\alpha}_{k} = \left(\delta^{\alpha}_{k} - \delta_{k3}\delta^{\alpha 3}\right) \Phi$

iii. A phase: $\Phi^{\alpha}_{k} = \delta_{k3}\left(\delta^{\alpha 1} + i\delta^{\alpha 2}\right)$

iv. color-spin-locking (CSL) phase: $\Phi^{\alpha}_{k} = \delta^{\alpha}_{k}\, \Phi$ (He-3: B phase)

$U(1)_{\rm em}$: spin-1 color superconductors are type-1 superconductors

A. Schmitt, Q. Wang, DHR, PRL 91 (2003) 242301

Color superconductivity at $\mu_q \gg 1$ GeV (I)

1. **Gap parameter to subleading order in g:**

$$\phi_0 = 512\,\pi^4 \left(\frac{2}{N_f g^2}\right)^{5/2} \mu_q \exp\left(-\frac{3\pi^2}{\sqrt{2}g} - \frac{\pi^2+4}{8} - d - \zeta\right) [1 + O(g)]$$

D.T. Son, PRD 59 (1999) 094019

T. Schäfer, F. Wilczek, PRD 60 (1999) 114033; R.D. Pisarski, DHR, PRD 61 (2000) 051501; 074017;

D. Hong, V. Miransky, I. Shovkovy, L. Wijewardhana, PRD 61 (2000) 056001

W.E. Brown, J.T. Liu, H.-c. Ren, PRD 61 (2000) 114012; 62 (2000) 054013, 054016;

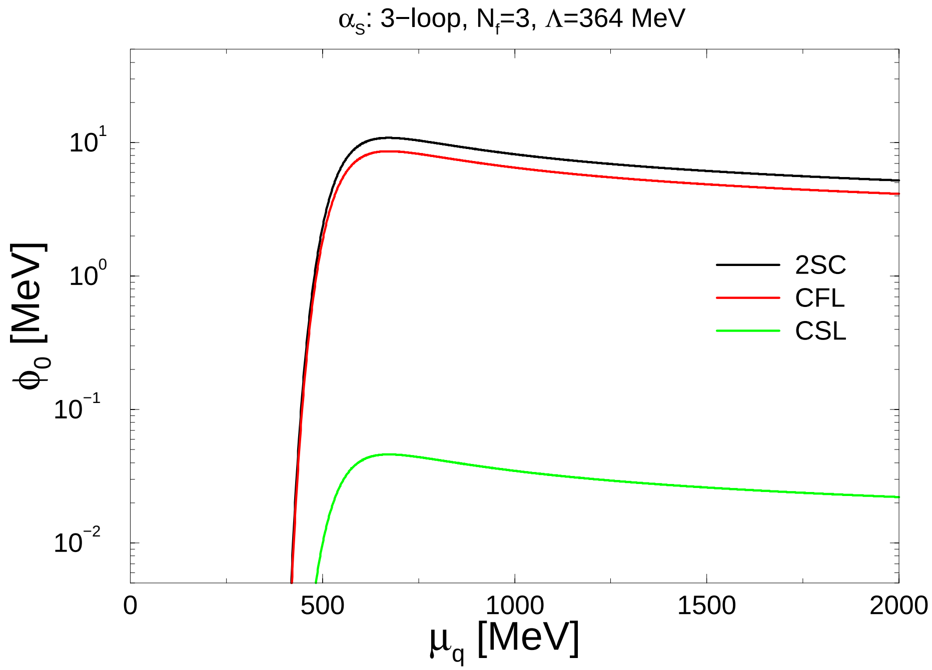
Q. Wang, DHR, PRD 65 (2002) 054005

T. Schäfer, PRD 62 (2000) 094007; A. Schmitt, Q. Wang, DHR, PRD 66 (2002) 114010

Spin 0: $d = 0$,

Spin 1: $d > 0$,

ζ depends on exp. values of order parameter in color-flavor-spin space



Color superconductivity at $\mu_q \gg 1$ GeV (II)

2. Critical temperature:

(a) mean-field level: 2nd order $\boxed{T_c = \frac{e^\gamma}{\pi}\,\phi_0\,e^\zeta \simeq 0.57\,\phi_0\,e^\zeta}$ (almost) BCS-like!

R.D. Pisarski, DHR, PRD 61 (2000) 051501; 074017; A. Schmitt, Q. Wang, DHR, PRD 66 (2002) 114010

(b) incl. gluonic fluctuations: 1st order!

T. Matsuura, K. Iida, T. Hatsuda, G. Baym, PRD 69 (2004) 074012

I. Giannakis, D.-f. Hou, H.-c. Ren, DHR, PRL 93 (2004) 232301

J. Noronha, H.-c. Ren, I. Giannakis, D.-f. Hou, DHR, in preparation

Ginzburg-Landau free energy $\boxed{\Omega = \Omega_n + \Omega_{\text{cond}} + \Omega_{\text{fluc}}}$

Ω_n: free energy of normal phase

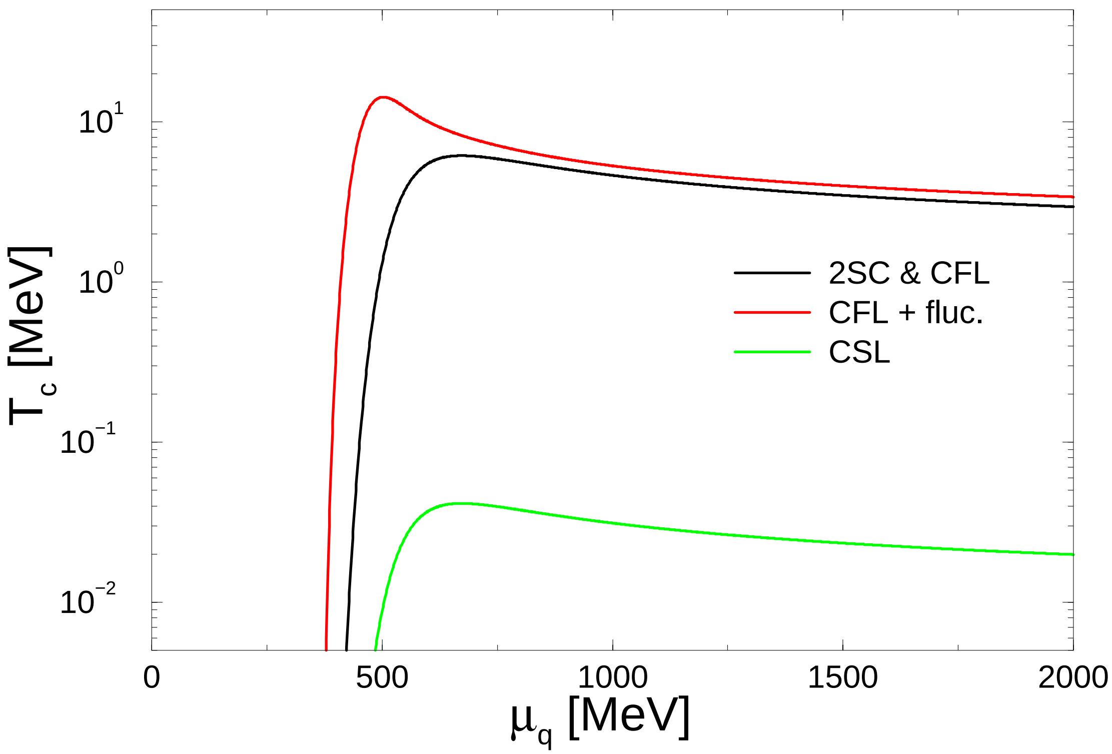
$$\Omega_{\text{cond}} = \frac{6\mu_q^2}{\pi^2}\,\frac{T-T_c}{T_c}\,\phi^2 + \frac{21\zeta(3)}{4\pi^4}\,\left(\frac{\mu_q}{T_c}\right)^2 \phi^4 \qquad \Longrightarrow \quad T_c$$

$$\Omega_{\text{fluc}} \simeq -32\pi T_c^4\, z\,(\ln z + const.)\;, \quad z \equiv \frac{g^2\mu_q^2}{384\pi^3 T_c^4}\,\phi^2 \gg 1 \qquad \Longrightarrow \quad \text{Pippard limit!}$$

Color superconductivity at $\mu_q \gg 1$ GeV (III)

$\Longrightarrow$ **Transition temperature of the first-order transition:**

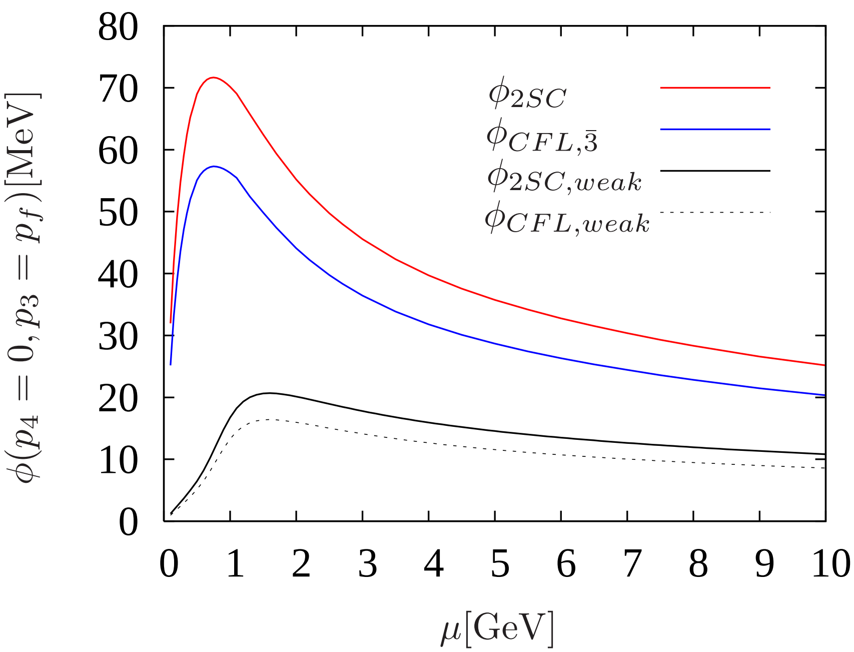
$$T_c^* = T_c\left(1+\frac{\pi^2}{12\sqrt{2}}\,g + O(g^2)\right) > T_c \ !!$$



Color superconductivity at $\mu_q \sim 1$ GeV (I)

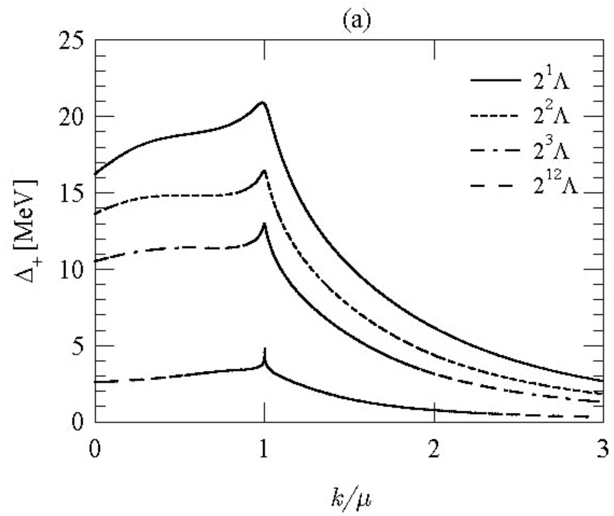
$g(\mu_q \sim 1\,\text{GeV}) \sim 1 \Longrightarrow$ **no control parameter** $\Longrightarrow$ **non-perturbative methods**

$\Longrightarrow$ **Dyson-Schwinger equations:**



D. Nickel, R. Alkofer, J. Wambach, in preparation

Why is gap larger than extrapolated weak-coupling result?



H. Abuki, T. Hatsuda, K. Itakura, PRD 65 (2002) 074014

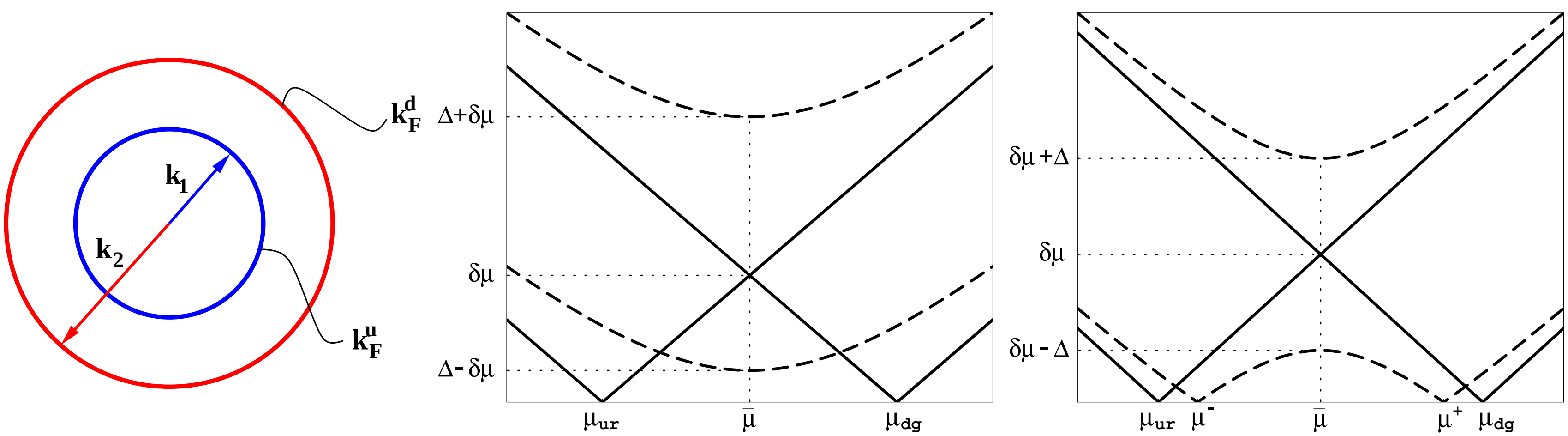
$\Longrightarrow$ **At realistic densities, gap function extends further into Fermi sea, partially compensates factor $\exp(-\frac{\pi^2+4}{8}) \sim 1/6$**

Color superconductivity at $\mu_q \sim 1$ GeV (II)

Compact stellar objects are

- electrically neutral: $\Sigma_{c,f} q_f \, n_f^c = 0$, $c = r, g, b$, $f = u, d, \ldots$
- color neutral: $\Sigma_{c,f} q^c \, n_f^c = 0$, $c = r, g, b$, $f = u, d, \ldots$
- in β equilibrium: $\mu_d^c = \mu_u^c + \mu_e$, $c = r, g, b$

$\Longrightarrow$ Fermi momenta of different quark colors and flavors differ



M. Huang, I.A. Shovkovy, NPA 729 (2003) 835

$\Longrightarrow$ gapless color superconductivity!

Color superconductivity at $\mu_q \sim 1$ GeV (III)

$\Longrightarrow$ New phases:

M. Alford, C. Kouvaris, K. Rajagopal, PRL 92 (2004) 222001; PRD 71 (2005) 054009

S.B. Rüster, I.A. Shovkovy, DHR, NPA 743 (2004) 127

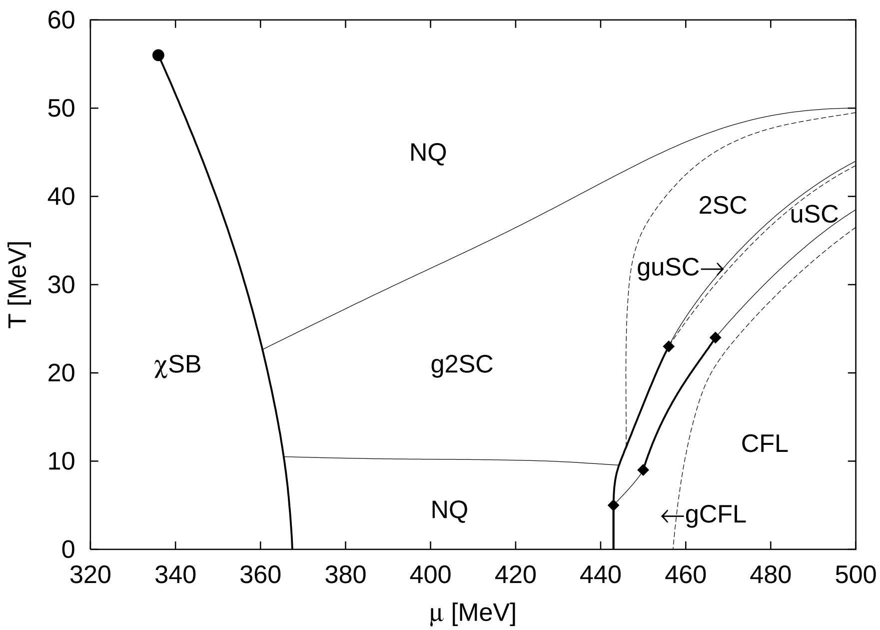
K. Iida, T. Matsuura, M. Tachibana, T. Hatsuda, PRD 71 (2005) 054003

S.B. Rüster, V. Werth, M. Buballa, I.A. Shovkovy, DHR, PRD 72 (2005) 034004

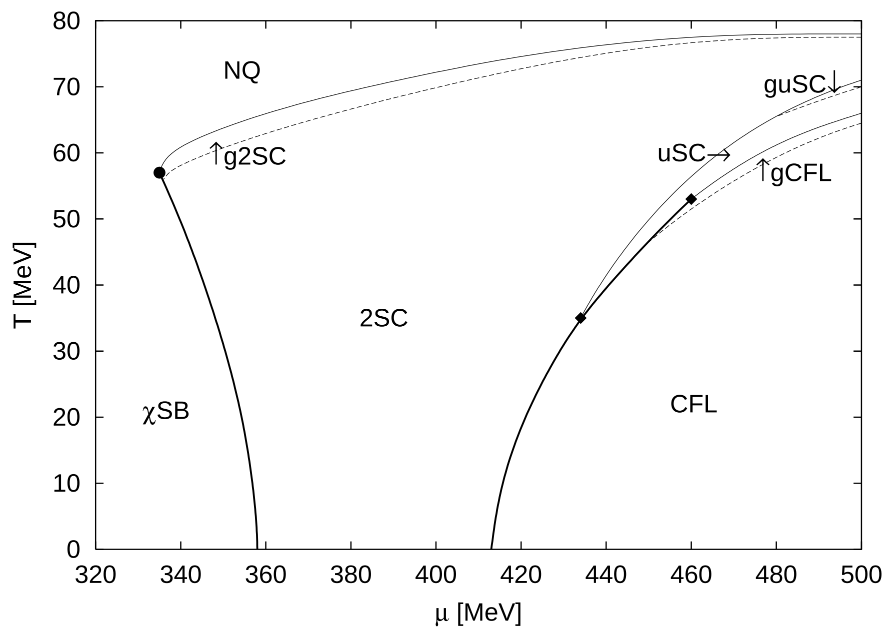
D. Blaschke, S. Fredriksson, H. Grigorian, A.M. Oztas, F. Sandin, PRD 72 (2005) 065020

NJL model: $\bar{q}q$ coupling strength G_S, qq coupling strength G_D

$G_D = \frac{3}{4} G_S$ (from Fierz transf.)



$G_D = G_S$

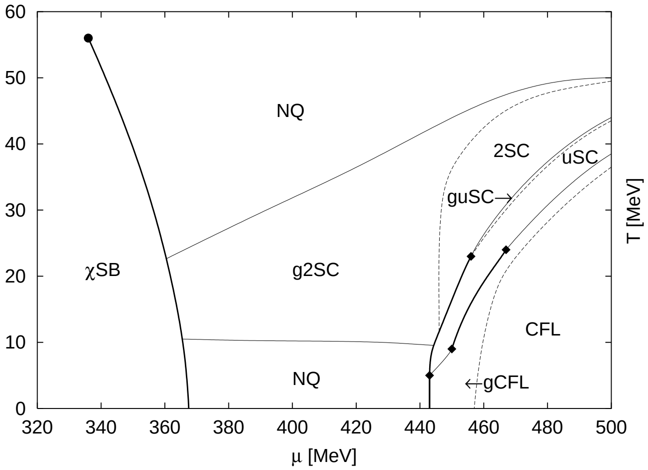


Color superconductivity at $\mu_q \sim 1$ GeV (IV)

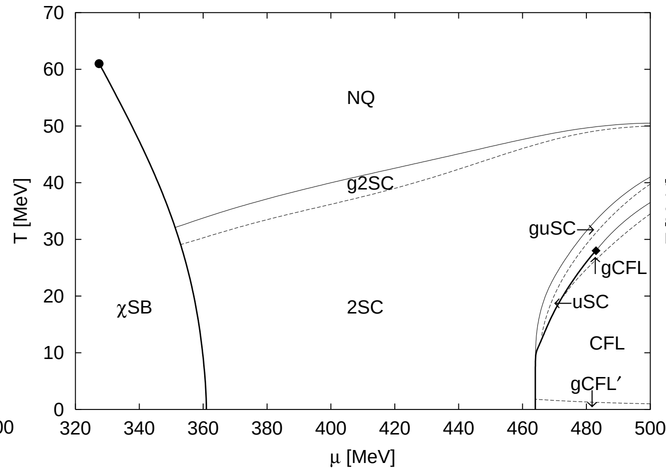
Young neutron stars before deleptonization contain neutrinos, $\mu_{\nu_e} \neq 0$

S.B. Rüster, V. Werth, M. Buballa, I.A. Shovkovy, DHR, hep-ph/0509073

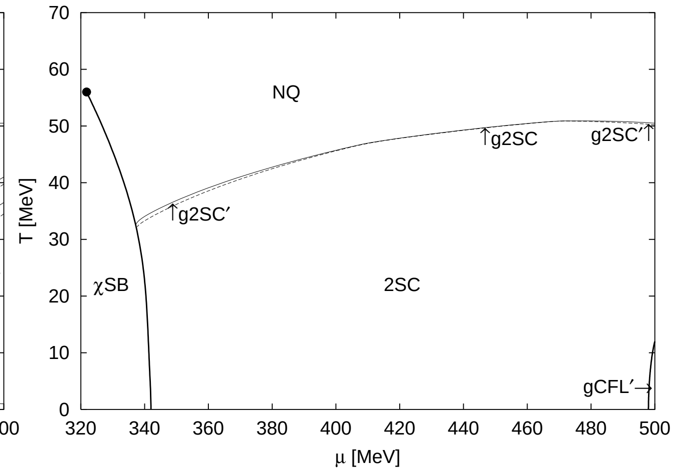
$\mu_{\nu_e} = 0$



$\mu_{\nu_e} = 200$ MeV



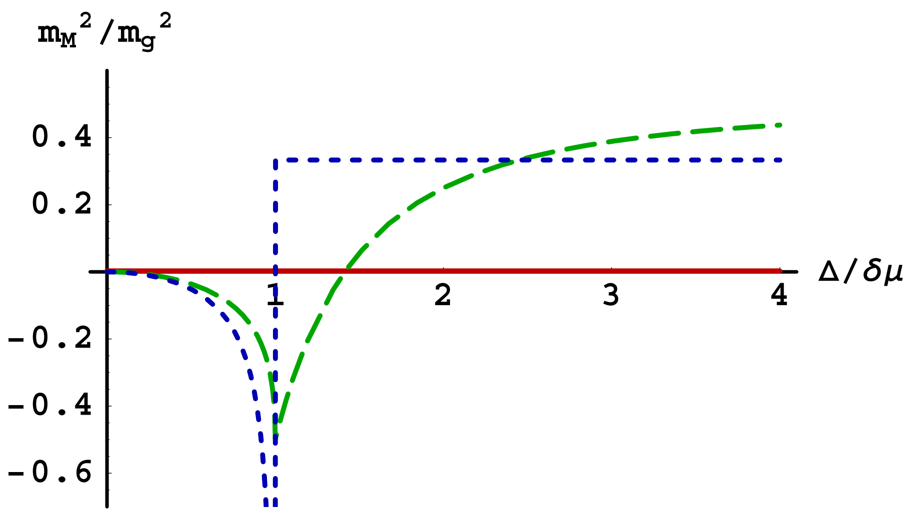
$\mu_{\nu_e} = 400$ MeV



$\Longrightarrow$ **neutrinos favor 2SC phase!**

Color superconductivity at $\mu_q \sim 1$ GeV (V)

Problem: Gapless phases magnetically unstable, gluon Meissner masses $m_a^2 < 0$



$a = 1, 2, 3$

$a = 4, 5, 6, 7$

$a = 8$

2SC:

M. Huang, I.A. Shovkovy, PRD 70 (2004) 051501

CFL:

R. Casalbuoni, R. Gatto, M. Mannarelli, G. Nardulli, M. Ruggieri, PLB 605 (2005) 362; 615 (2005) 297

What is the true, i.e., energetically favored, state?

1. Crystalline color superconductivity (LOFF)?

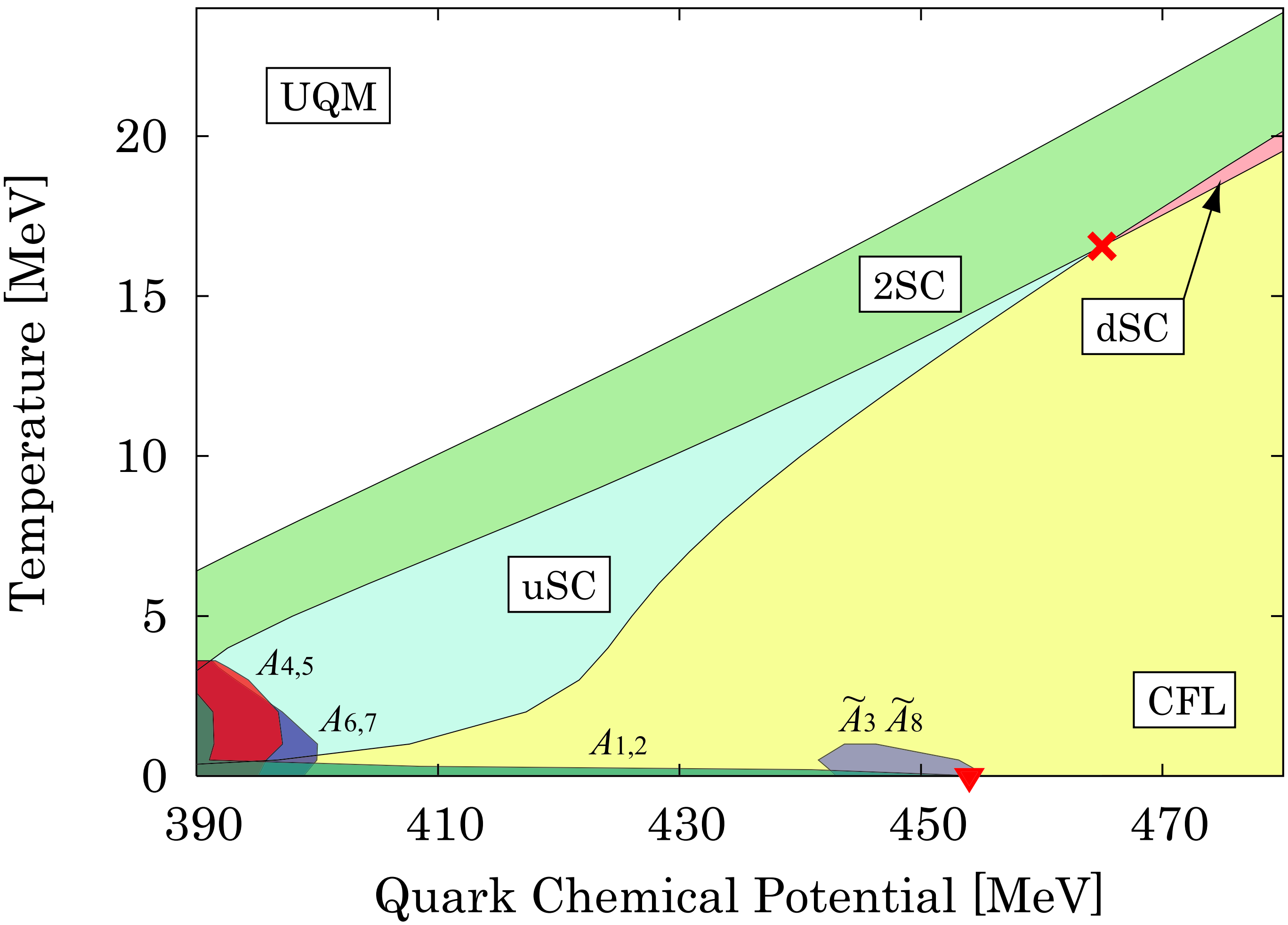
   M.G. Alford, J.A. Bowers, K. Rajagopal, PRD 63 (2001) 074016
   I. Giannakis, H.-c. Ren, PLB 611 (2005) 137; NPB 723 (2005) 255
   I. Giannakis, D.-f. Hou, H.-c. Ren, hep-ph/0507306
   R. Casalbuoni, R. Gatto, N. Ippolito, G. Nardulli, M. Ruggieri, PLB 627 (2005) 89

2. Vector gluon condensation?

   M. Huang, hep-ph/0504235; D.K. Hong, hep-ph/0506097
   E.V. Gorbar, M. Hashimoto, V.A. Miransky, hep-ph/0507303; hep-ph/0509334
   A. Kryjevski, hep-ph/0508180; T. Schäfer, hep-ph/0508190

Color superconductivity at $\mu_q \sim 1$ GeV (VI)

How serious is chromomagnetic instability?



K. Fukushima, hep-ph/0510299

Bound states

$N_f = 2$ NJL model

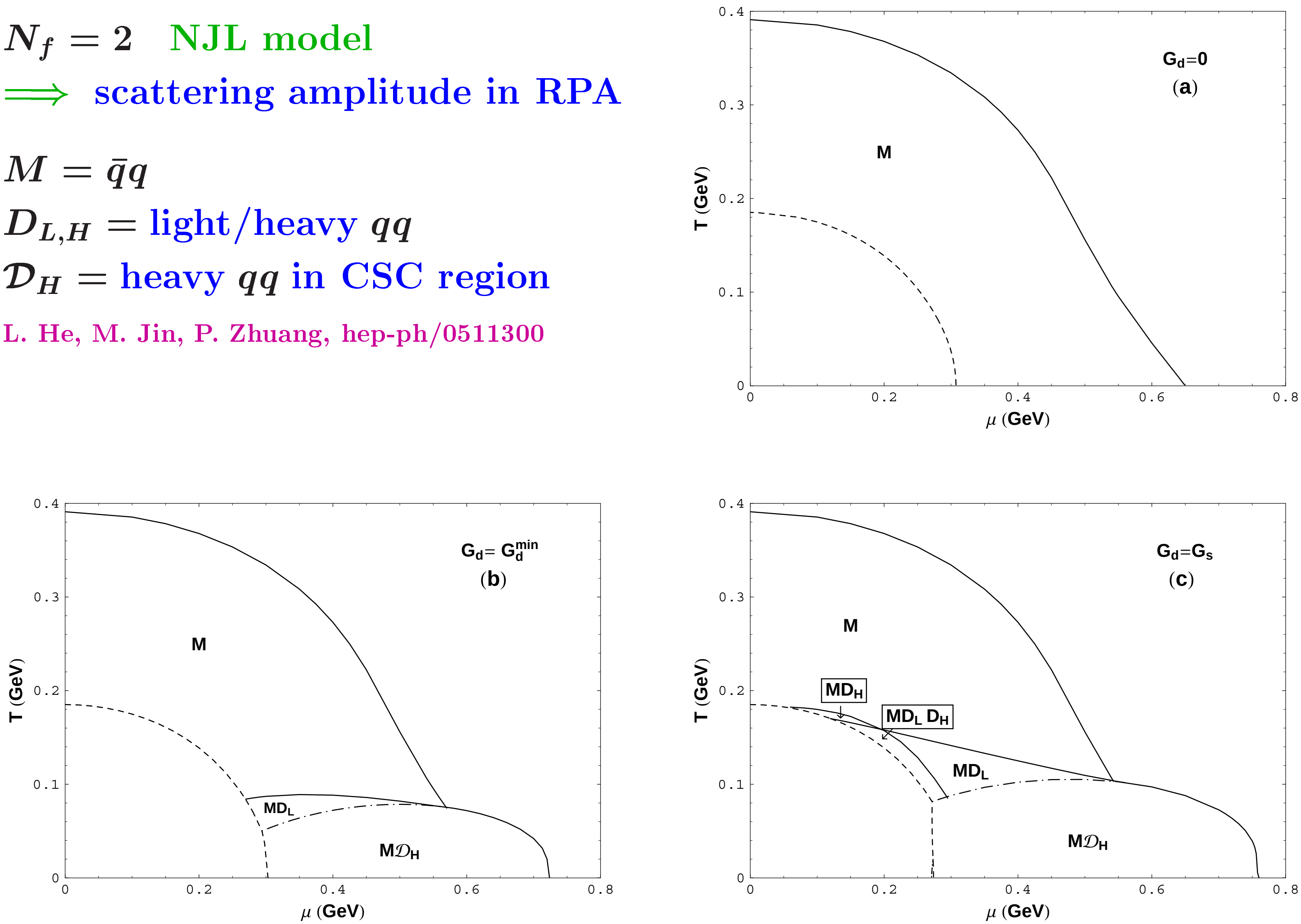
$\Longrightarrow$ scattering amplitude in RPA

$M = \bar{q}q$

$D_{L,H}$ = light/heavy qq

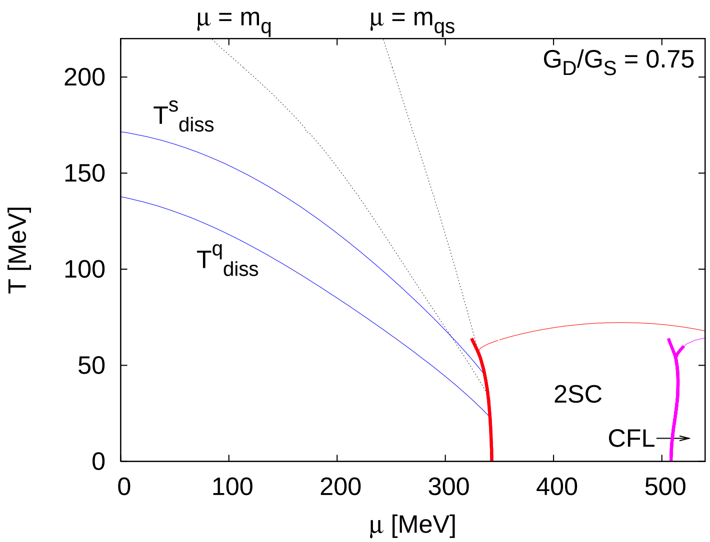
$\mathcal{D}_H$ = heavy qq in CSC region

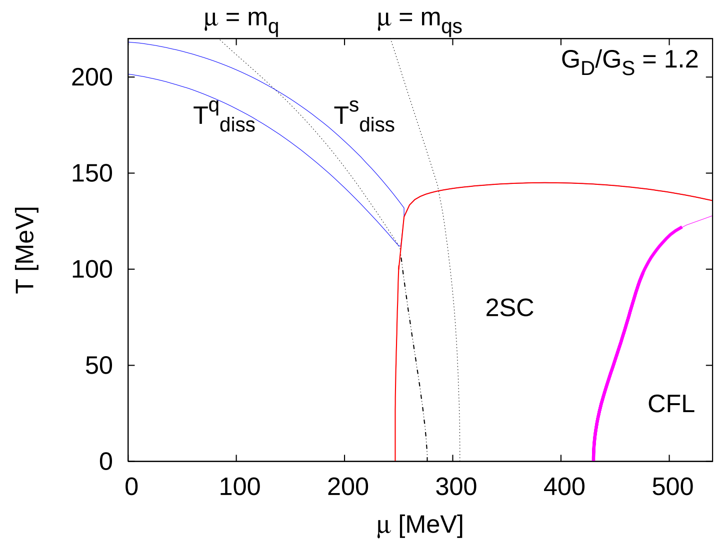
L. He, M. Jin, P. Zhuang, hep-ph/0511300

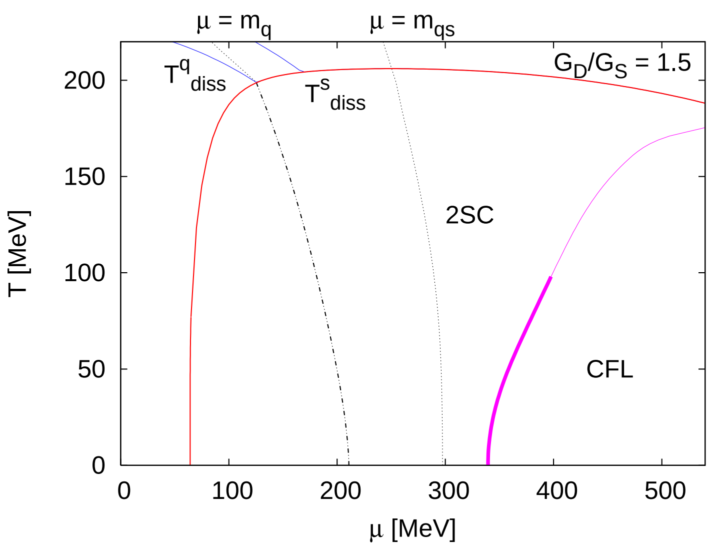


Bose–Einstein condensation (BEC)

Is there BEC of diquark molecules in the QCD phase diagram?







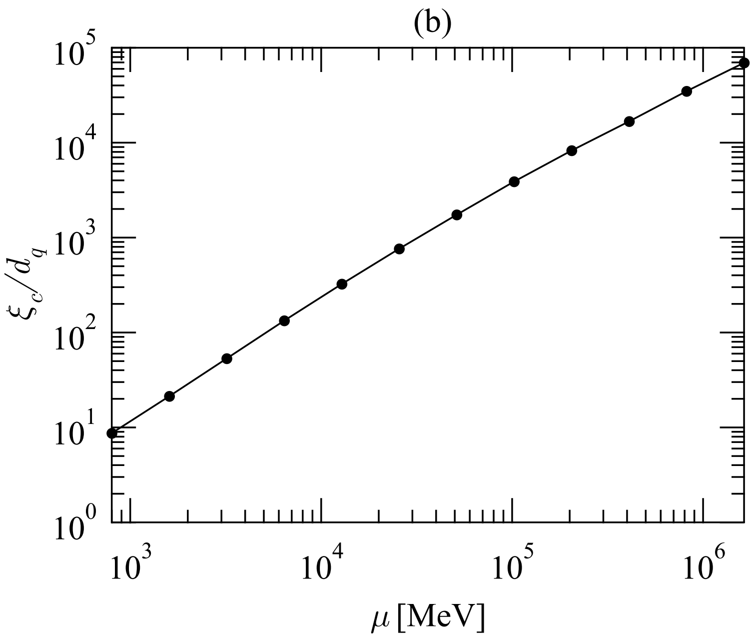
$\Longrightarrow$ **BEC only for large diquark coupling strength G_D**

M. Kitazawa, I.A. Shovkovy, DHR, in preparation

BCS–BEC transition

$\mu_q \searrow \implies g \nearrow \implies$ Cooper pairs become more localized, form diquark molecules

$\implies$ ratio correlation length/interquark distance $\xi_c/d_q \searrow$



H. Abuki, T. Hatsuda, K. Itakura, PRD 65 (2002) 074014

$\implies$ BCS–BEC transition is smooth!

P. Nozières, S. Schmitt-Rink, J. Low Temp. Phys. 59 (1985) 195

Conclusions and open questions

1. Sufficiently large μ_q and sufficiently small T:
   quark matter is a color superconductor

2. Color superconductivity at asymptotically large μ_q:
   - QCD gap parameter $\phi_0 \sim \mu_q \exp(-1/g) \sim 10$ MeV
   - mean-field $T_c \simeq 0.57\, \phi_0\, e^{\zeta}$, (almost) like in BCS theory!
   - fluctuation-induced first-order transition $T_c^* \sim 2T_c$!

3. Color superconductivity at realistically large μ_q:
   - $\phi_0 \sim 70$ MeV (gap function extends further into Fermi sea)
   - color and electric charge neutrality $\Longrightarrow$ new (gapless) phases!
   - large neutrino fraction favors 2SC phase
   - chromomagnetic instability of gapless phases
   - true ground state?

4. Diquark molecules and BEC