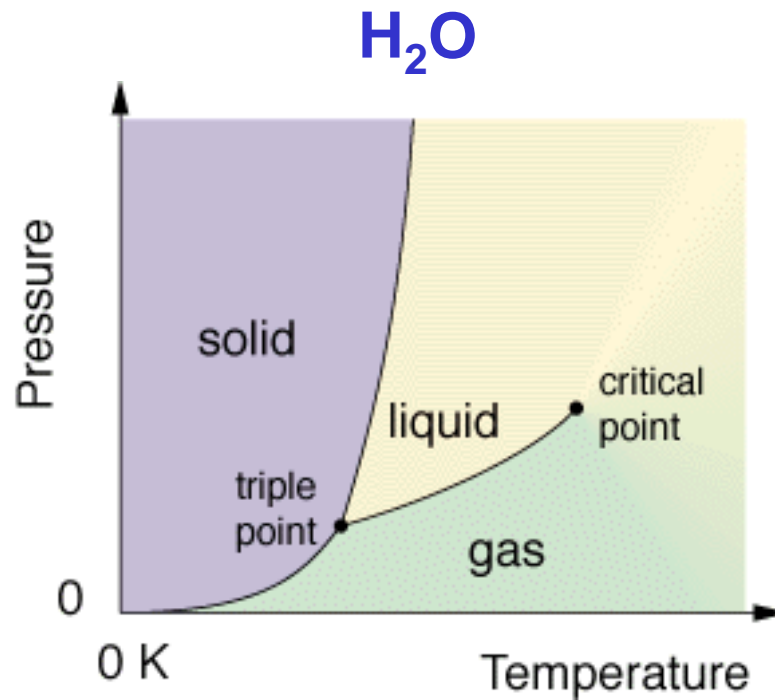


# Overview : QCD Phase Diagram

T. Hatsuda  
(Univ. Tokyo)

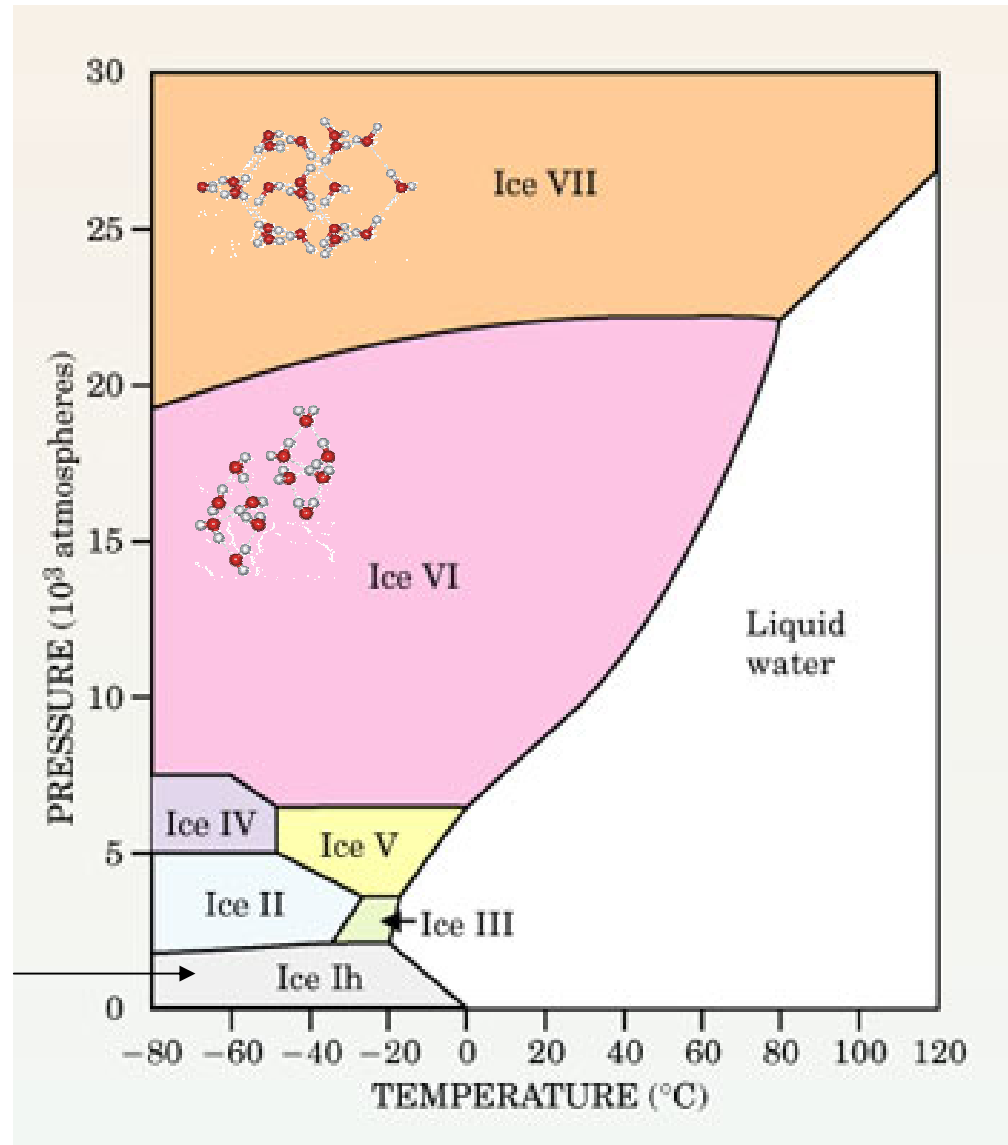
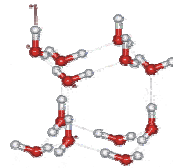
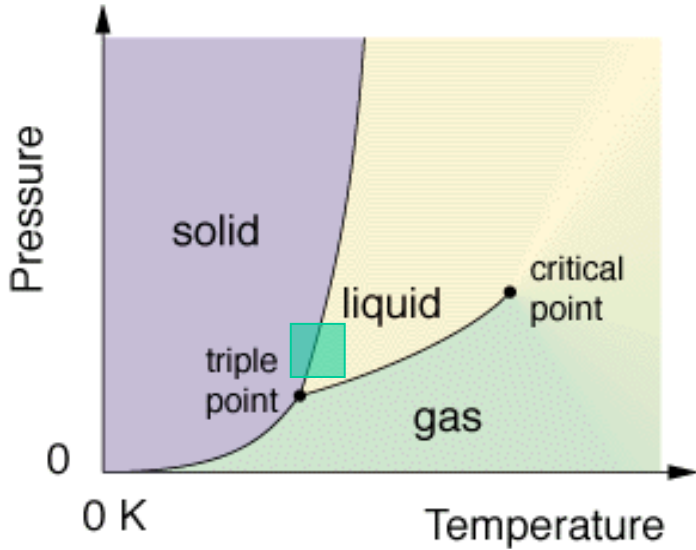


## *Outline of this talk*

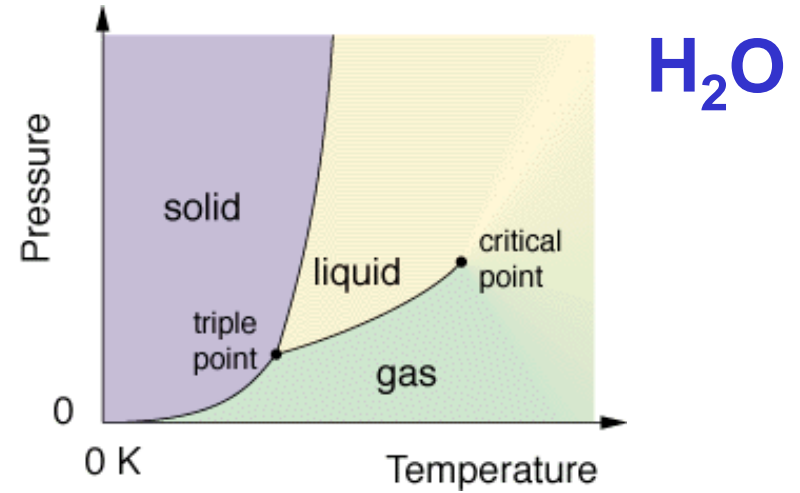
1. Possible phase structure in QCD
  - what we know and what we do not know
2. Phases in hot QCD
3. Phases in dense QCD
4. Future experimental facilities
5. Summary

# More on H<sub>2</sub>O

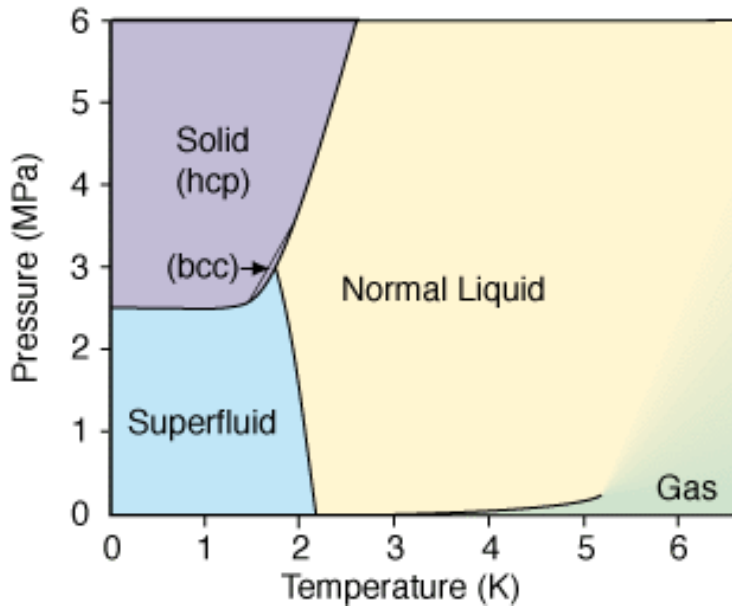
Physics Today, Dec. vol.58 (2005)  
<http://www.lsbu.ac.uk/water/phase.html>



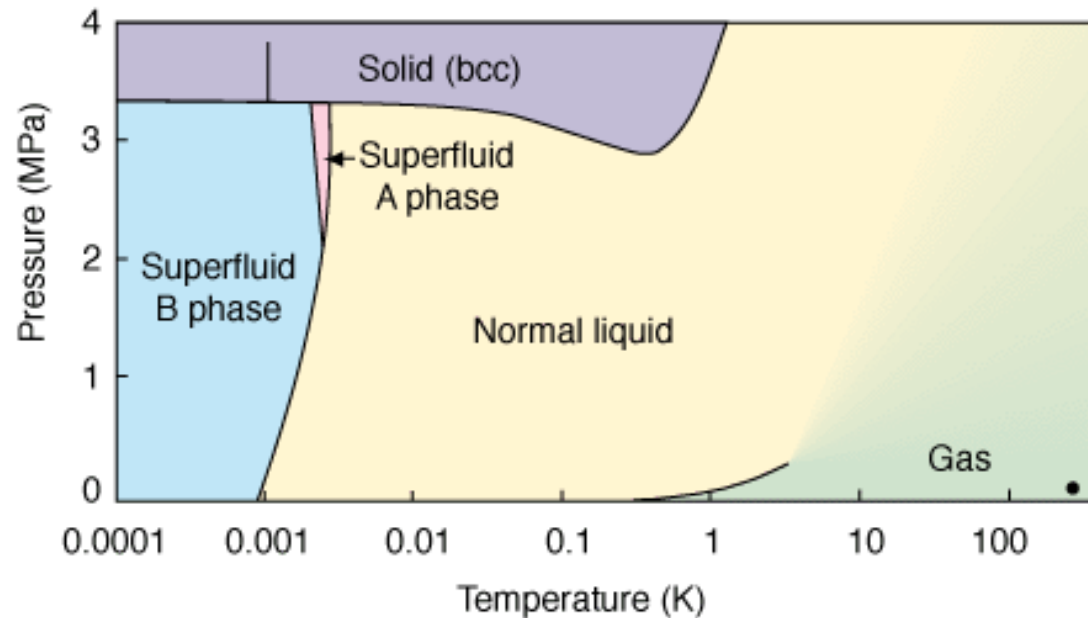
# Quantum phases of $^3\text{He}$ & $^4\text{He}$



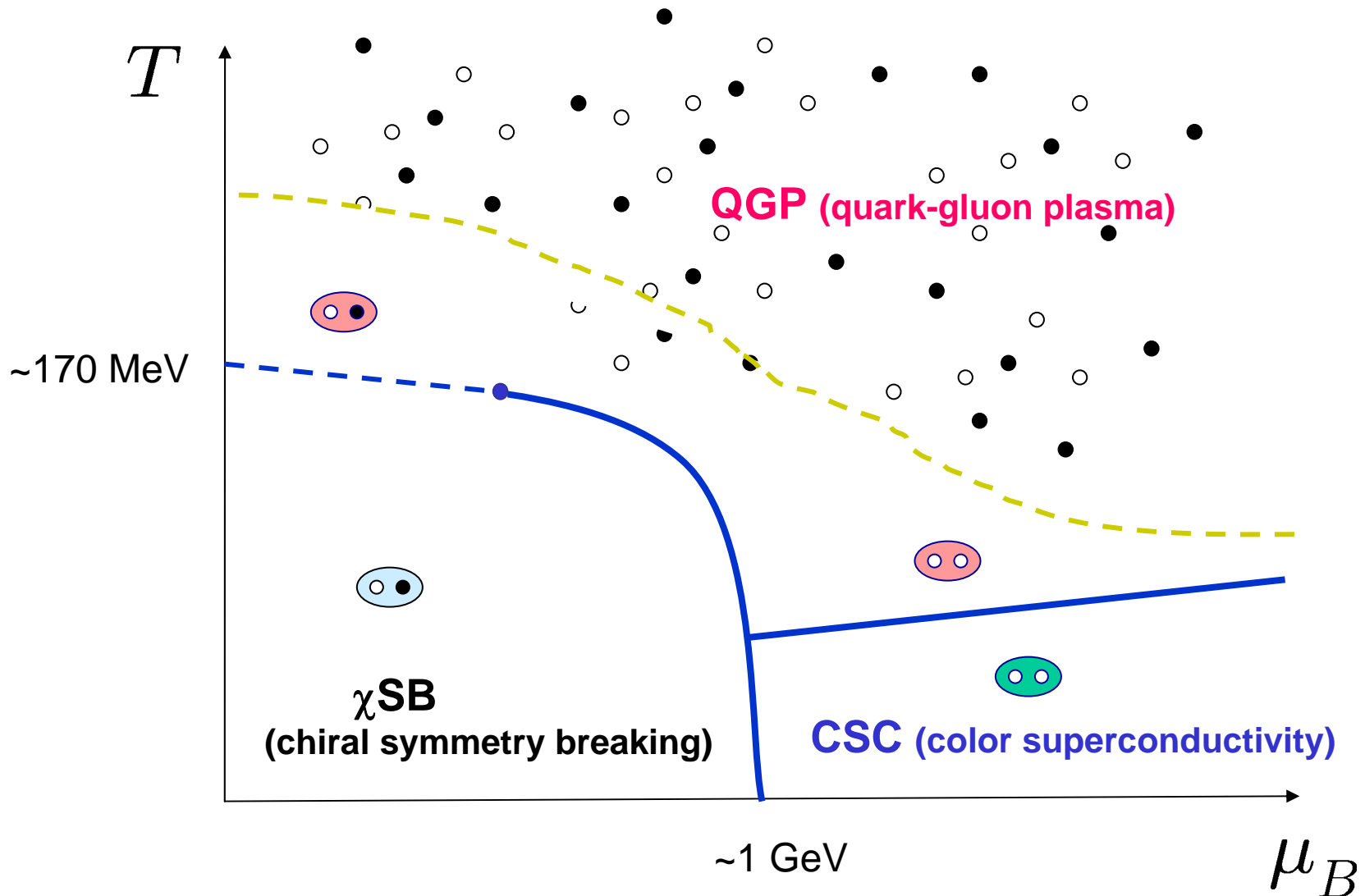
$^4\text{He}$



$^3\text{He}$



# Phases in QCD ? – a schematic picture --



# Theory behind: Quantum Chromo Dynamics

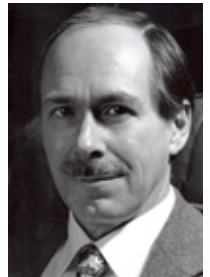
**1965**  $SU_c(3)$  YM theory as a model of strong interaction

$$L = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + \bar{q} \gamma^\mu (i \partial_\mu - g t^a A_\mu^a) q - m \bar{q} q \quad \text{Nambu ('65)}$$



**1965-1972** Precursors of asymptotic freedom

Vanyashin & Terenteev ('65), Khriplovich ('69), 't Hooft ('72)

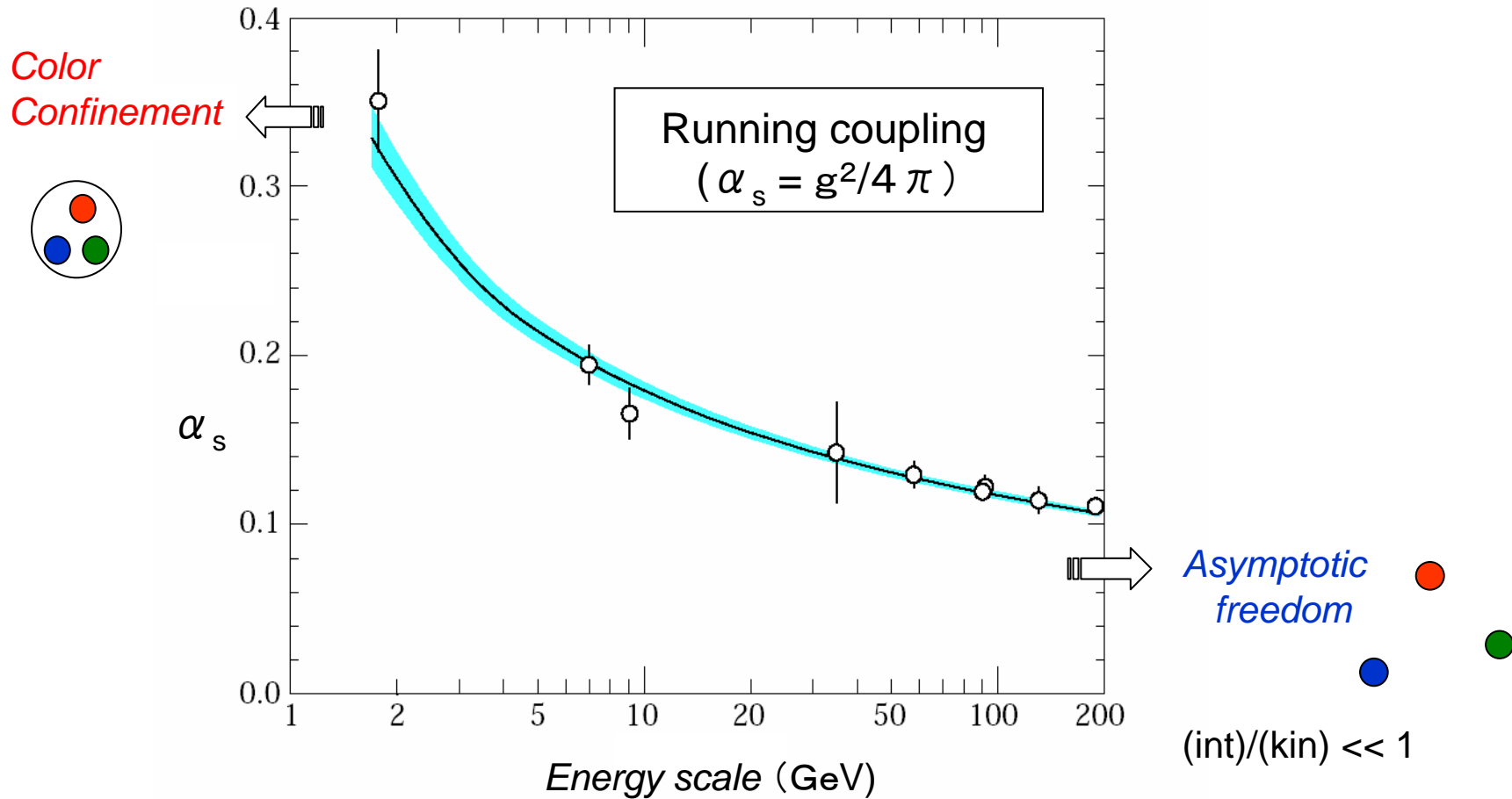


**1973** Discovery of asymptotic freedom

Gross & Wilczek, Politzer ('73)



# Theory behind: Quantum Chromo Dynamics



# Origin of each "phase"

strong residual force  
→ **pre-formed pairs**

Hatsuda & Kunihiro,  
Phys.Rev.Lett. 55 (1985)

Asymptotic freedom  
+ Debye screening  
→ **deconfinement**

Collins & Perry,  
Phys.Rev.Lett. 34 (1975)

$T$

**QGP**

$\chi$ SB

**CSC**

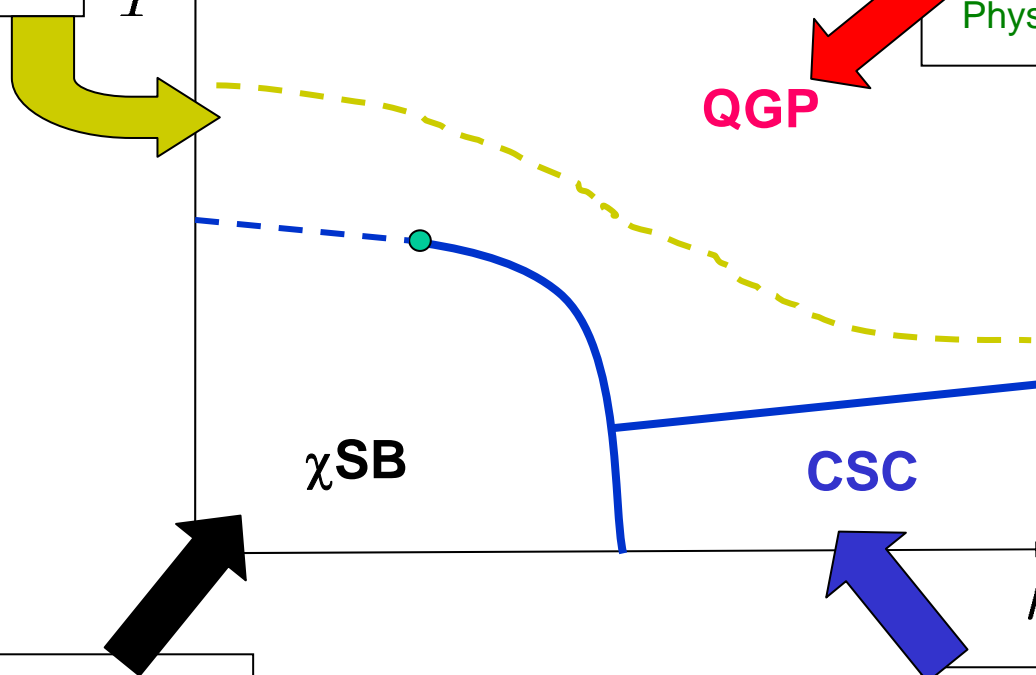
$\mu_B$

quark - anti-quark pairing  
→ **chiral instability**

Nambu & Jona-Lasinio,  
Phys.Rev. 122 (1961)

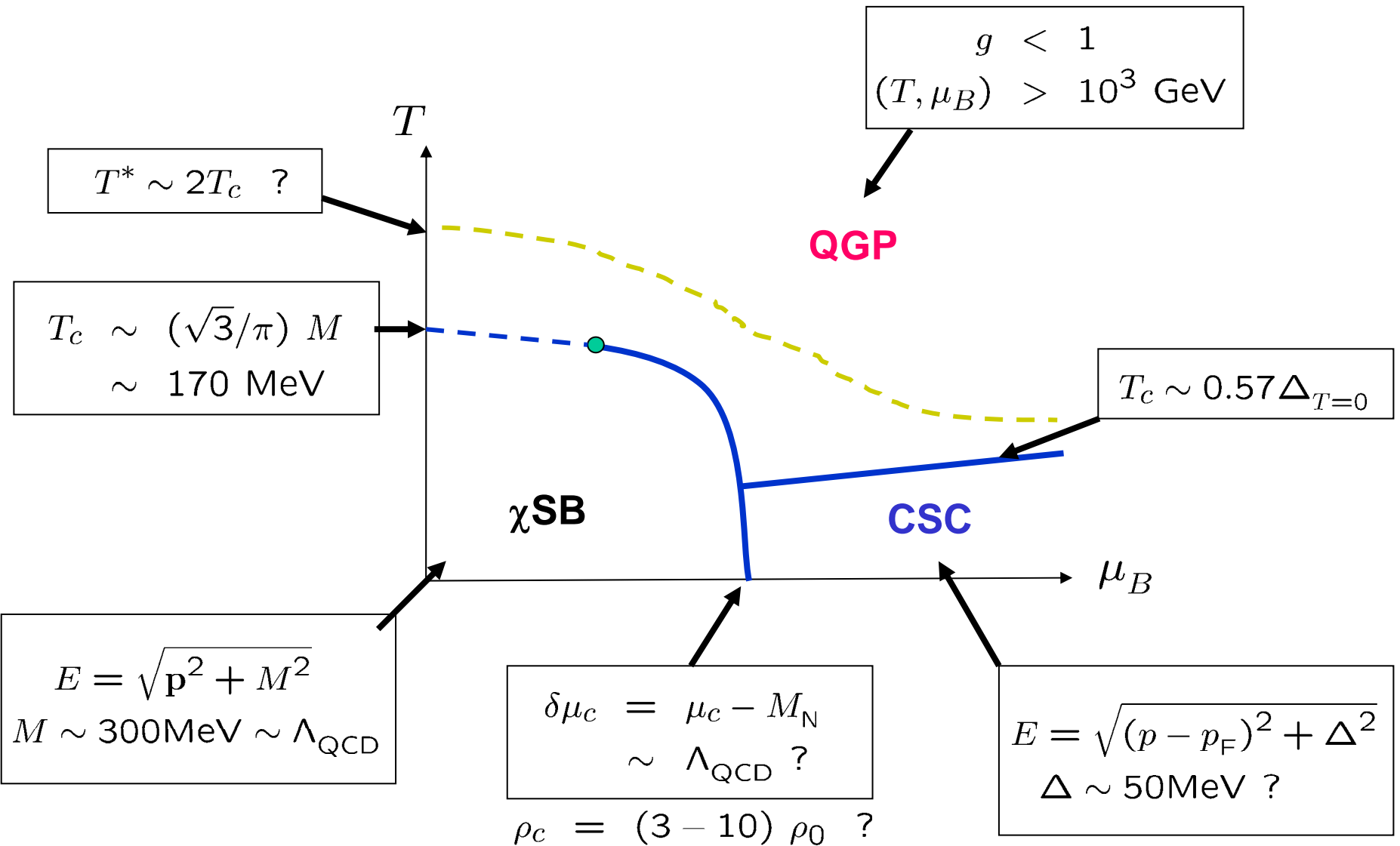
quark-quark pairing  
→ **Cooper instability**

Bailin & Love,  
Phys.Rep.107 (1984)

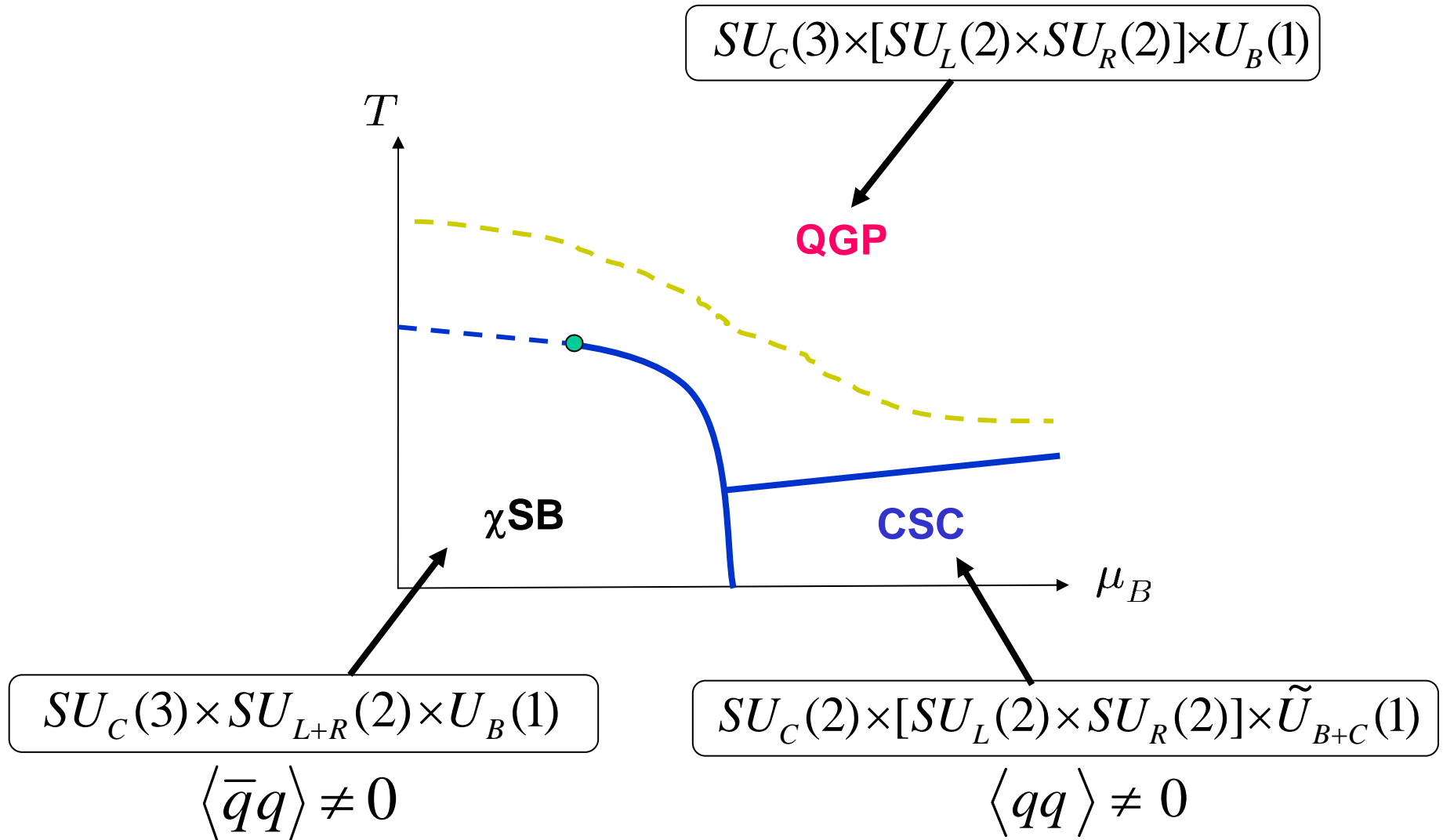




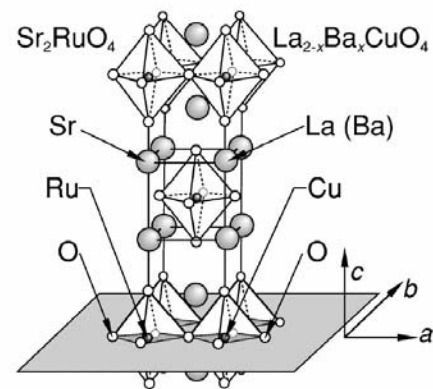
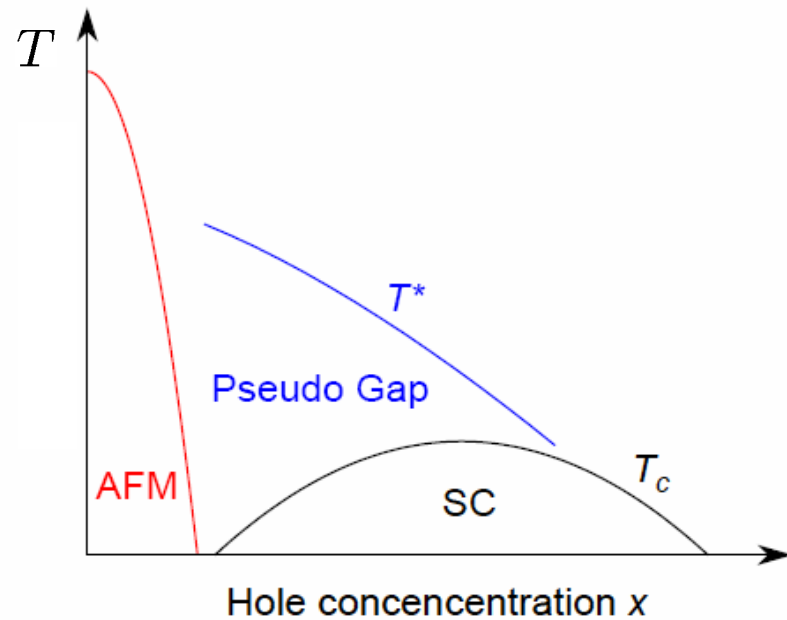
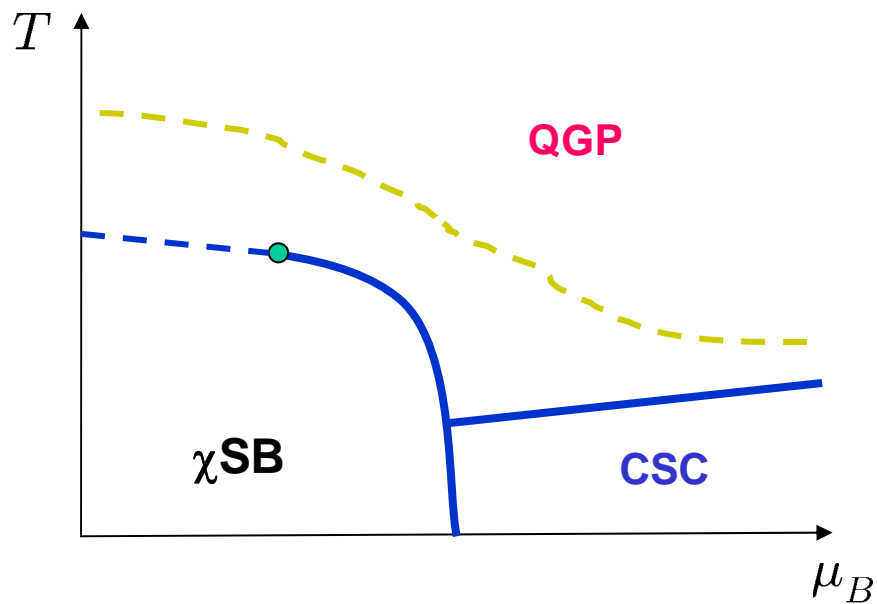
# Scale of each "phase"



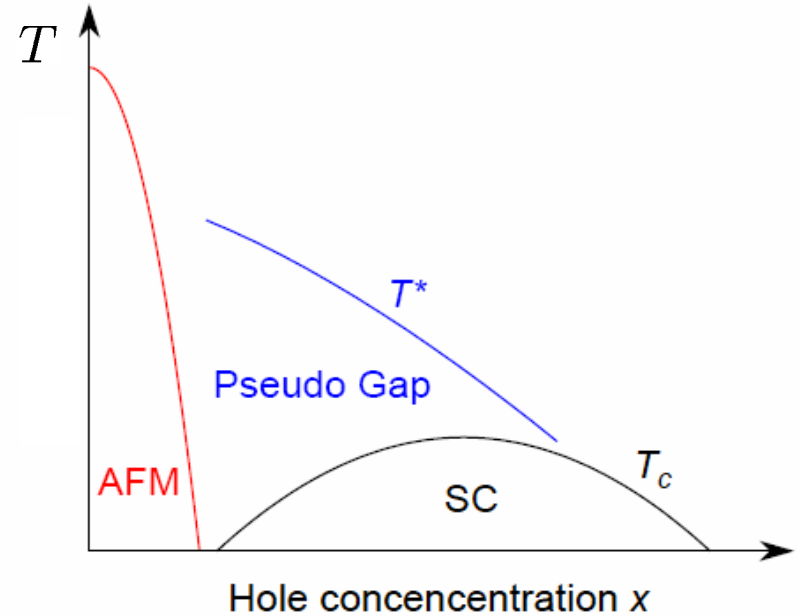
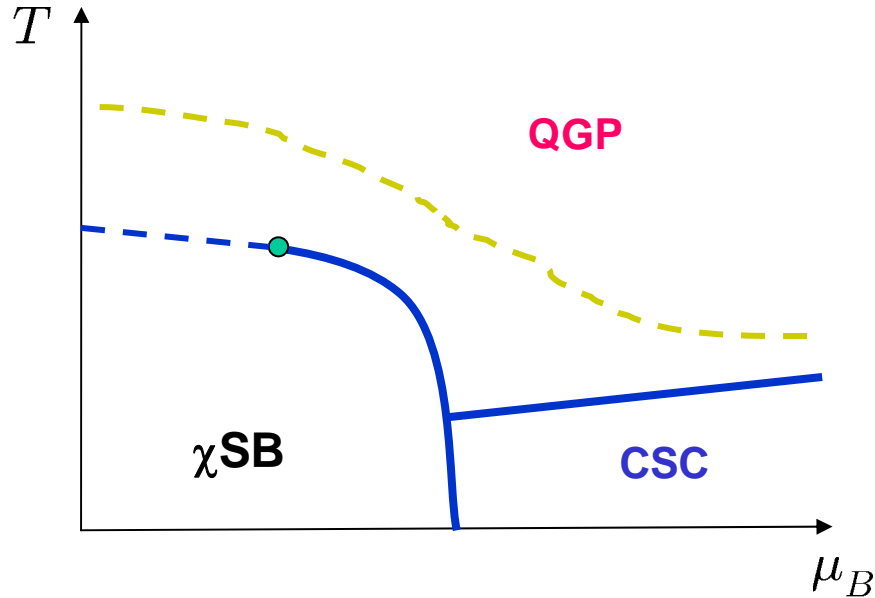
Symmetry of each “phase” (case for small  $m_{ud}$  with  $m_s = \infty$ )



# Similarity with high $T_c$ superconductors



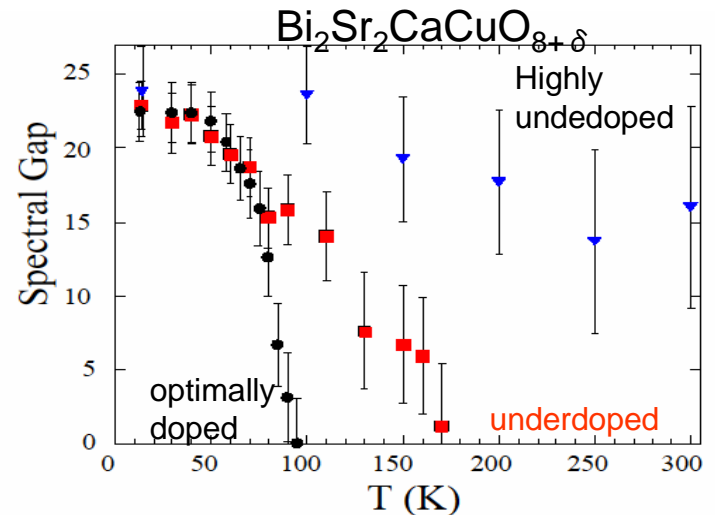
# Similarity with high $T_c$ superconductors



1. insulator – superconductor transition
2. strong coupling  $\rightarrow$  pre-formed pairs?
3. similar physics in “cold-atoms” too

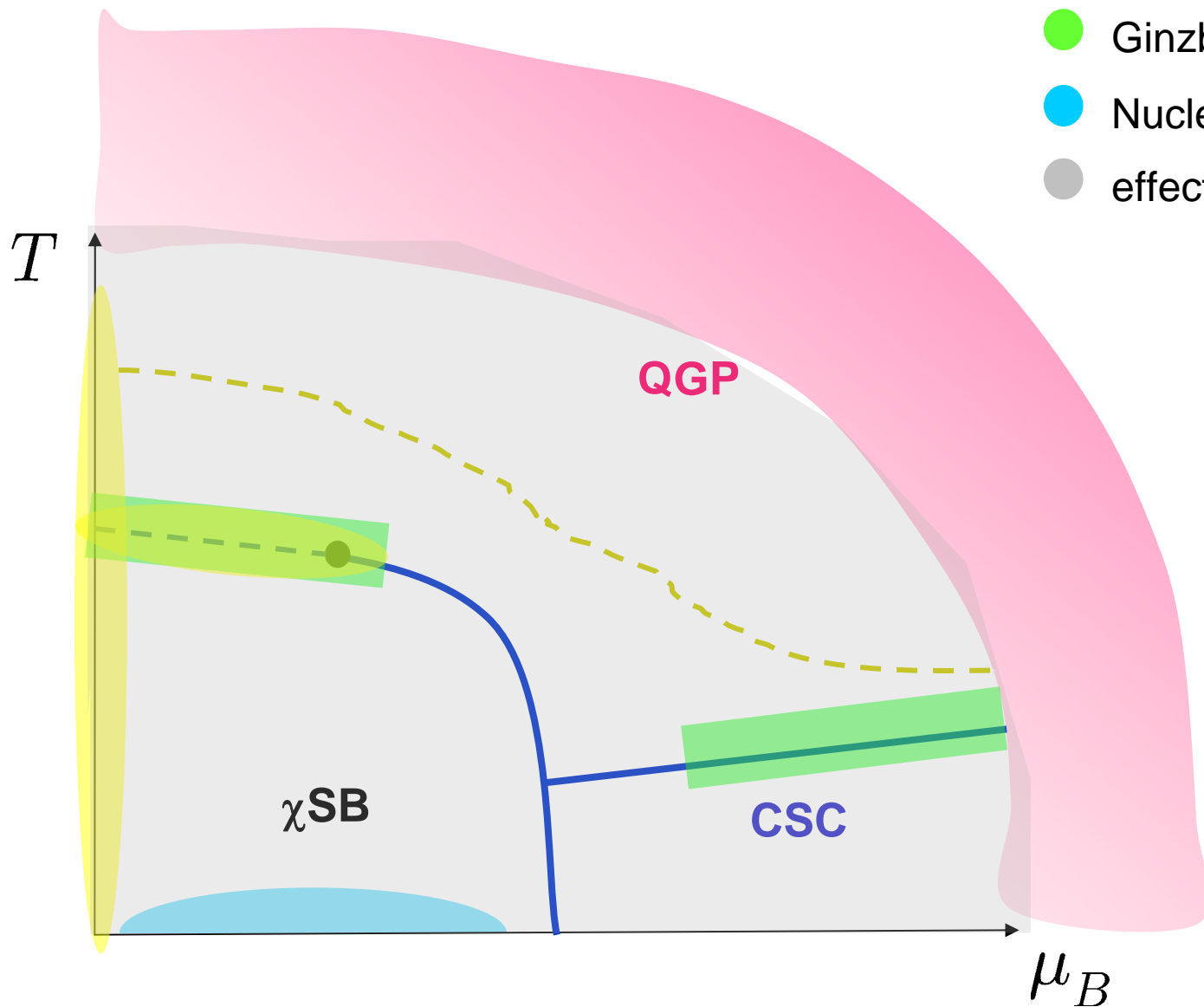
## HTS – BEC – QCD connection ?

- Babaev, PRD ('00)
- Abuki, Itakura & Hatsuda, PRD ('02)
- Kitazawa, Koide, Kunihiro & Nemoto, PRD ('02)
- Chen, Stajic, Tan & Levin, Phys. Rep. ('05)

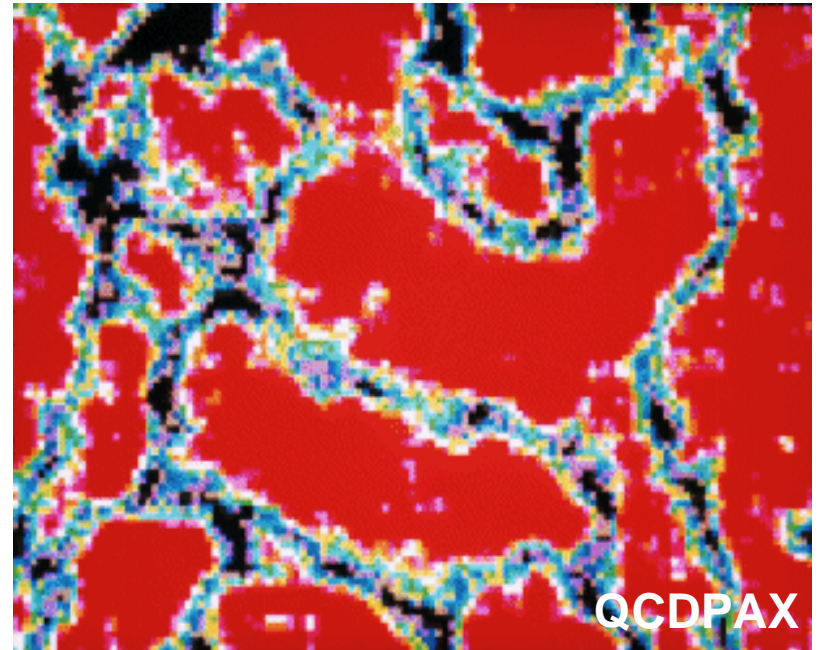
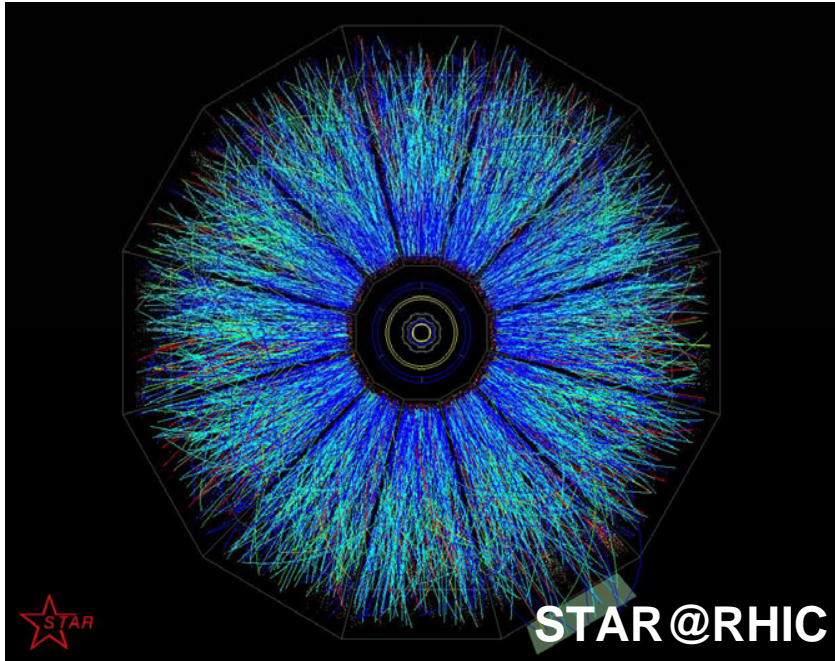


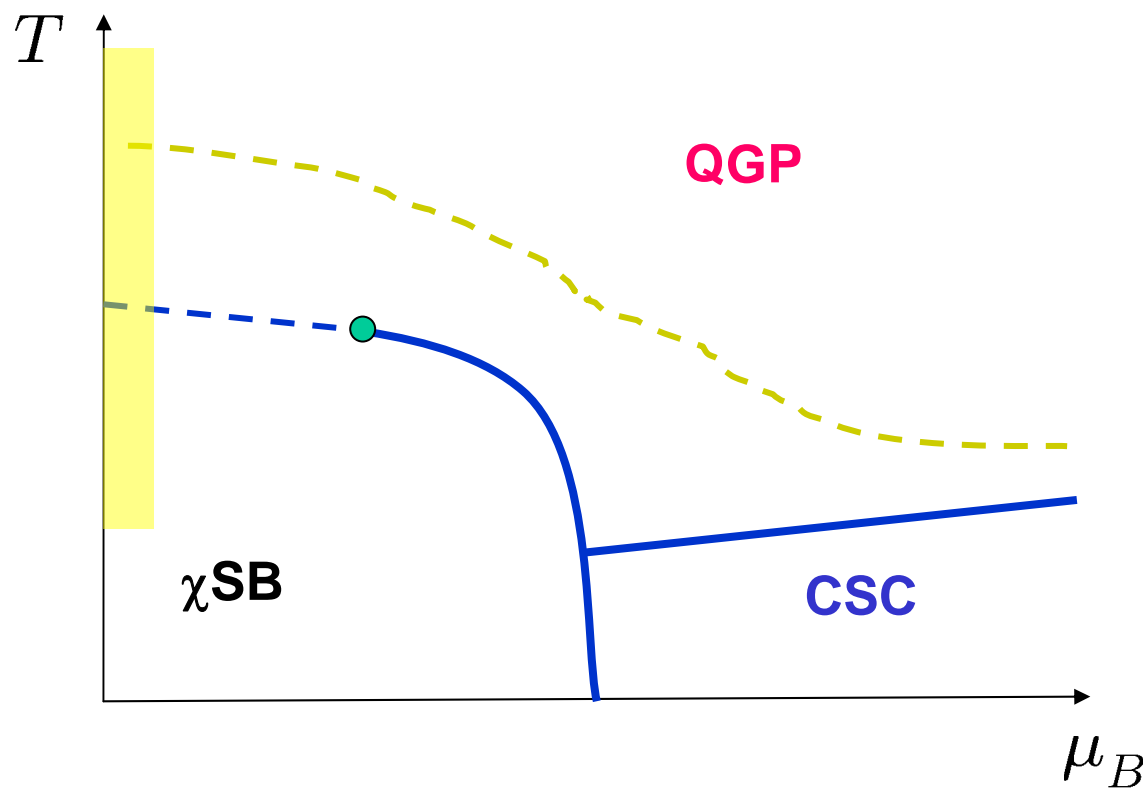
# Theoretical status

- Lattice QCD
- Perturbative QCD
- Ginzburg-Landau + RG
- Nuclear theory
- effective models

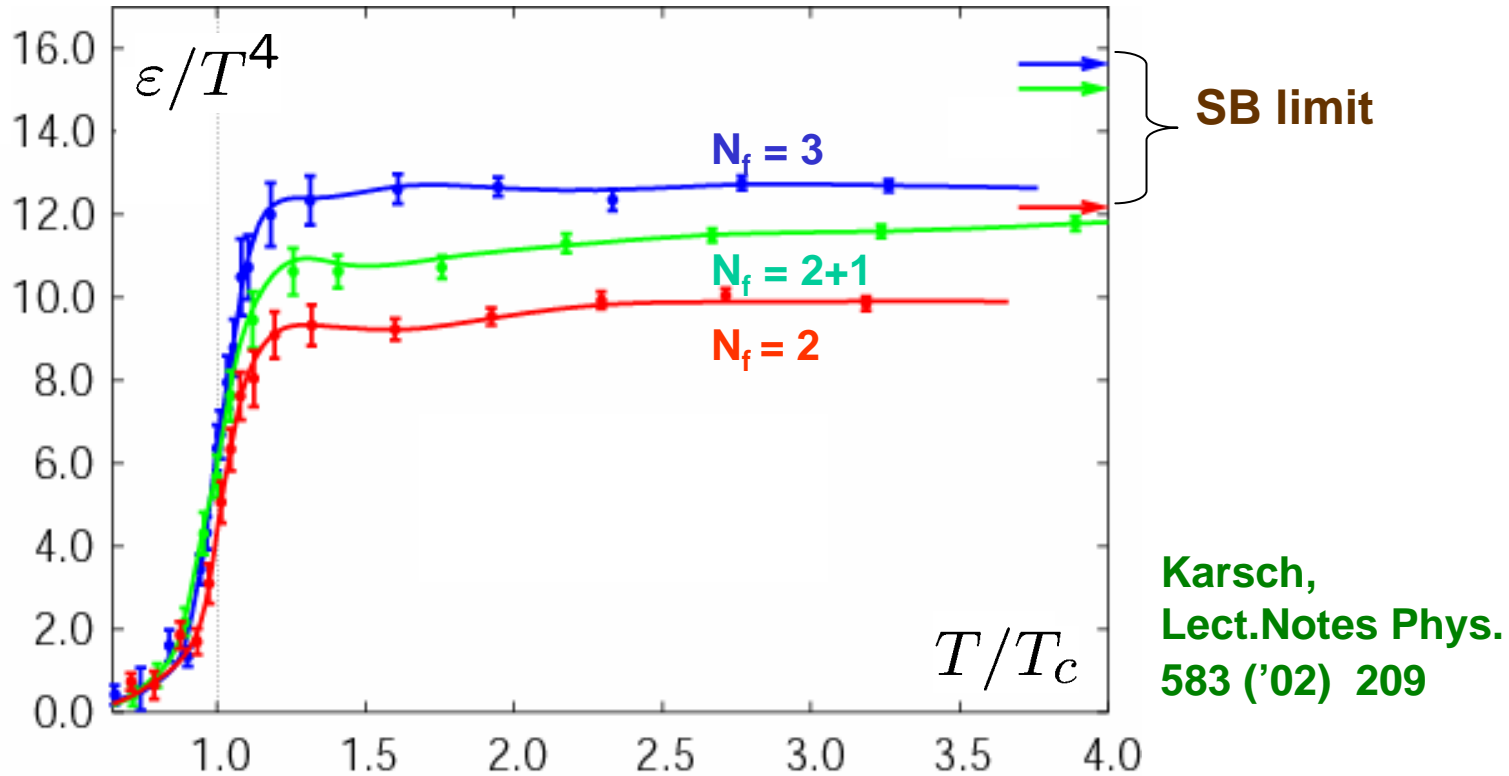
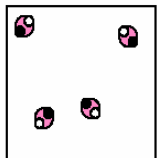
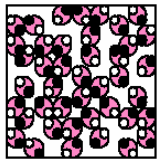
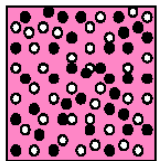


# *Phases in hot QCD*





# Equation of State ( $\mu=0$ )



Karsch,  
Lect.Notes Phys.  
583 ('02) 209



$T_c = 173 \pm 10$  MeV ( $N_f=2$ )  
 $T_c = 154 \pm 10$  MeV ( $N_f=3$ )



Karsch's talk



# Order of the thermal transition ( $\mu=0$ )

Svetitsky & Yaffe, NPB210 ('82)

Pisarski and Wilczek, PRD29 ('84)

## O(4) GL theory

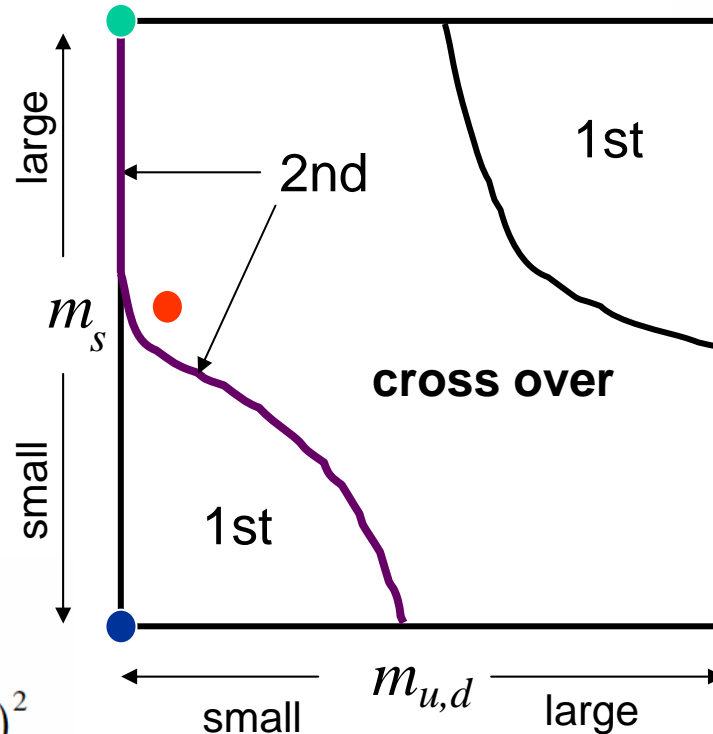
$$L_{\text{GL}} = \frac{1}{2}(\partial\vec{\phi})^2 + \frac{a(m_s, T)}{2}\vec{\phi}^2 + \frac{b(m_s, T)}{4!}(\vec{\phi}^2)^2 + \frac{c}{6!}(\vec{\phi}^2)^3 - h\phi_0$$

$$L_{\text{GL}} = \frac{1}{2}(\nabla L^*)(\nabla L) + \frac{a}{2}L^*L - \frac{c}{3}\text{Re}(L^3) + \frac{b}{4}(L^*L)^2 - h\text{Re}(L)$$

## Z(3) GL theory

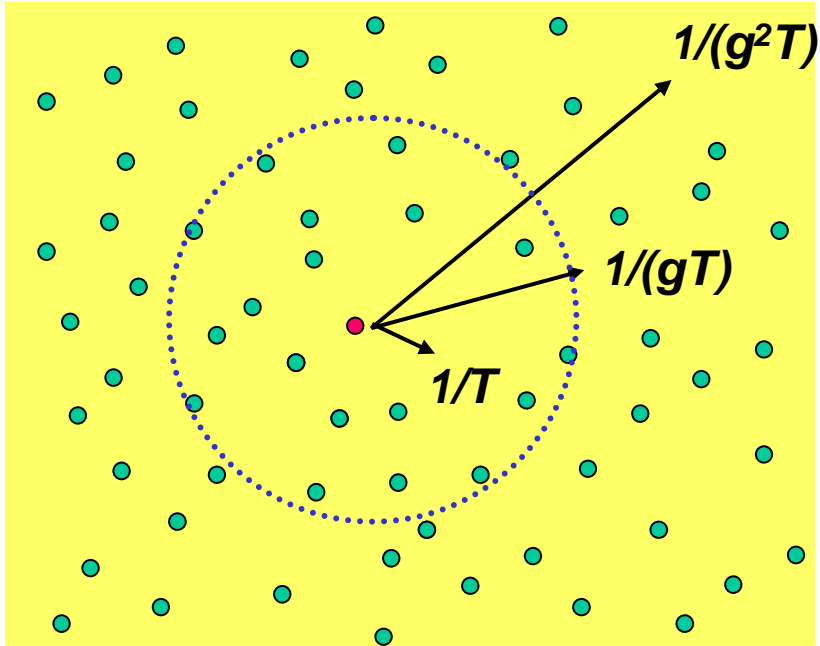
## SU<sub>L</sub>(3)xSU<sub>R</sub>(3) GL theory

$$L_{\text{GL}} = \frac{1}{2}\text{tr}\partial\Phi^\dagger\partial\Phi + \frac{a}{2}\text{tr}\Phi^\dagger\Phi + \frac{b_1}{4!}(\text{tr}\Phi^\dagger\Phi)^2 + \frac{b_2}{4!}\text{tr}(\Phi^\dagger\Phi)^2 - \frac{c}{2}(\det\Phi + \det\Phi^\dagger) - \frac{1}{2}\text{tr}h(\Phi + \Phi^\dagger)$$



Unified treatment  
**Weise's talk**

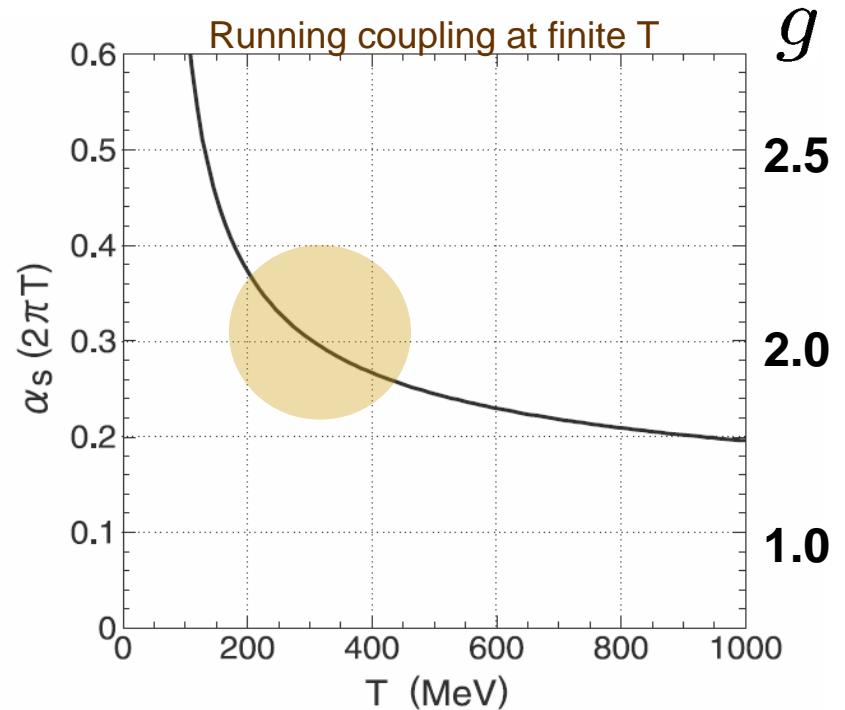
# Scale degeneracy near $T_c$



$T^{-1}$  Inter-particle distance

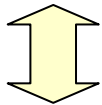
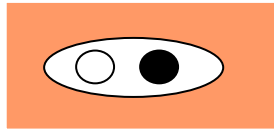
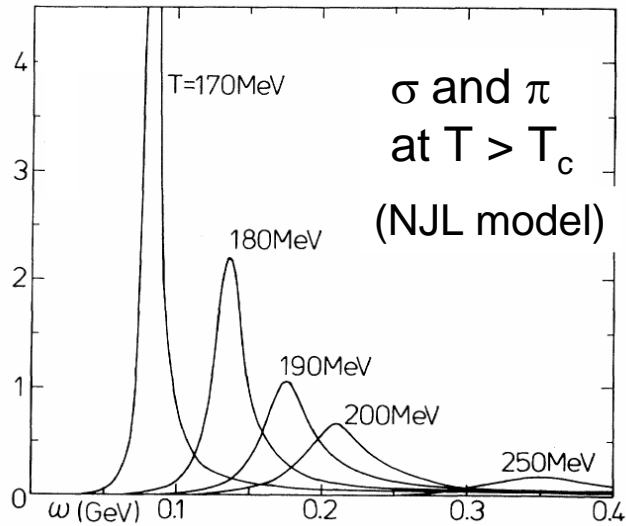
$(gT)^{-1}$  Electric screening length

$(g^2T)^{-1}$  Magnetic screening length



# Pre-formed pairs (PFP) for $T_c < T < T^*$

Hatsuda & Kunihiro, PRL ('85)



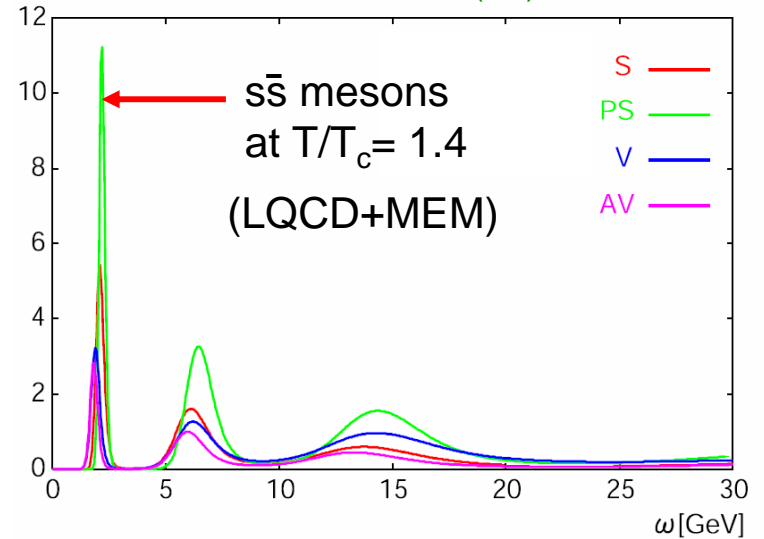
$T_c$  : absence of coherence

$T^*$  : dissociation of PFP

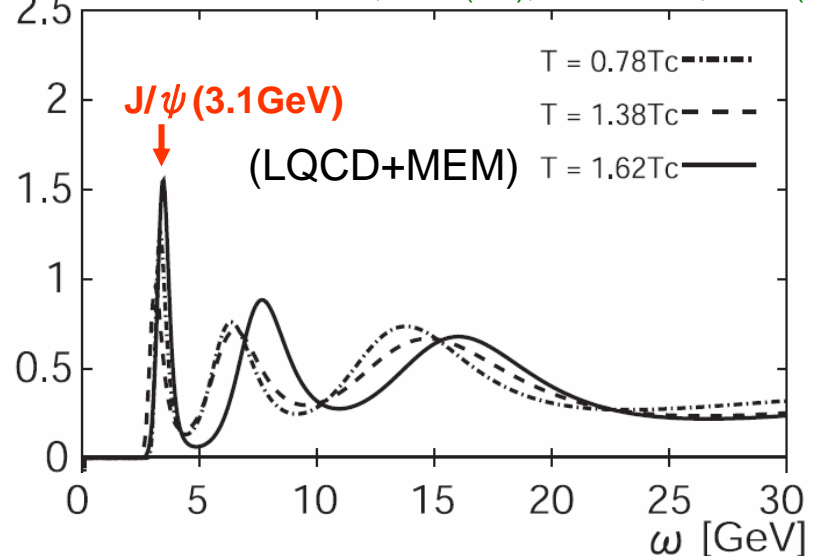
BEC-BCS crossover:

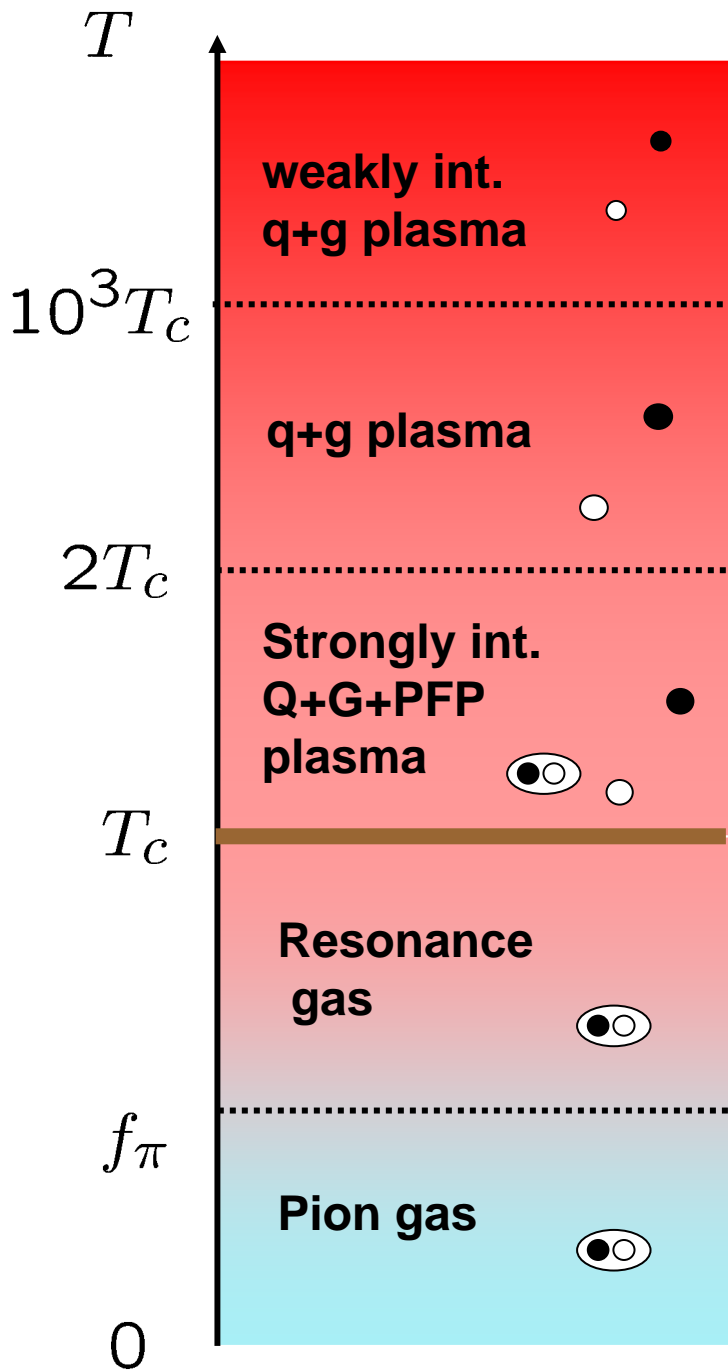
Leggett ('80), Nozieres & Schmitt-Rink ('85)

Asakawa & Hatsuda, PTP ('04)



Asakawa & Hatsuda, PRL ('04), Datta et al., PRD ('04)

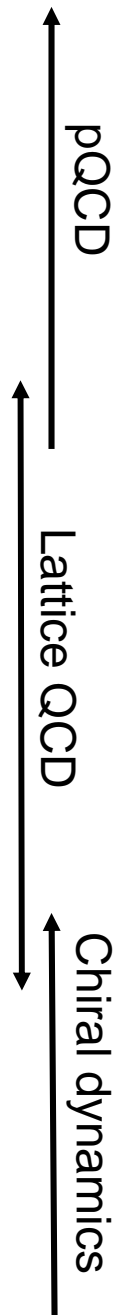




viscous fluid

perfect fluid

viscous fluid



A modern  
"picture"  
of hot QCD

# Critical end point at finite $\mu$

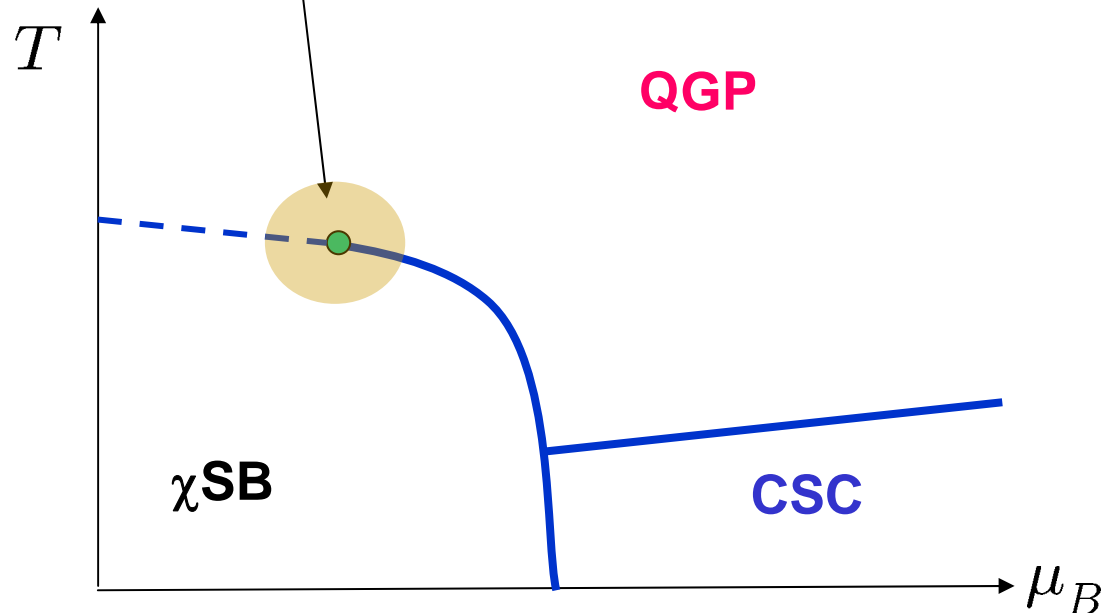
- First evidence in some models: Asakawa & Yazaki, NPA ('89), Barducci et al., PLB ('89)  
See however, Klimt, Lutz and Weise, PLB ('90)
- General properties in Ginzburg-Landau+RG  
Halasz et al., PRD ('98), Hatta and Ikeda, PRD ('03)

$$\mathcal{L}_{\text{GL}} = \frac{1}{2}(\partial\vec{\phi})^2 + \frac{a(\mu, T)}{2}\vec{\phi}^2 + \frac{b(\mu, T)}{4!}(\vec{\phi}^2)^2 + \frac{c}{6!}(\vec{\phi}^2)^3 - h\phi_0$$

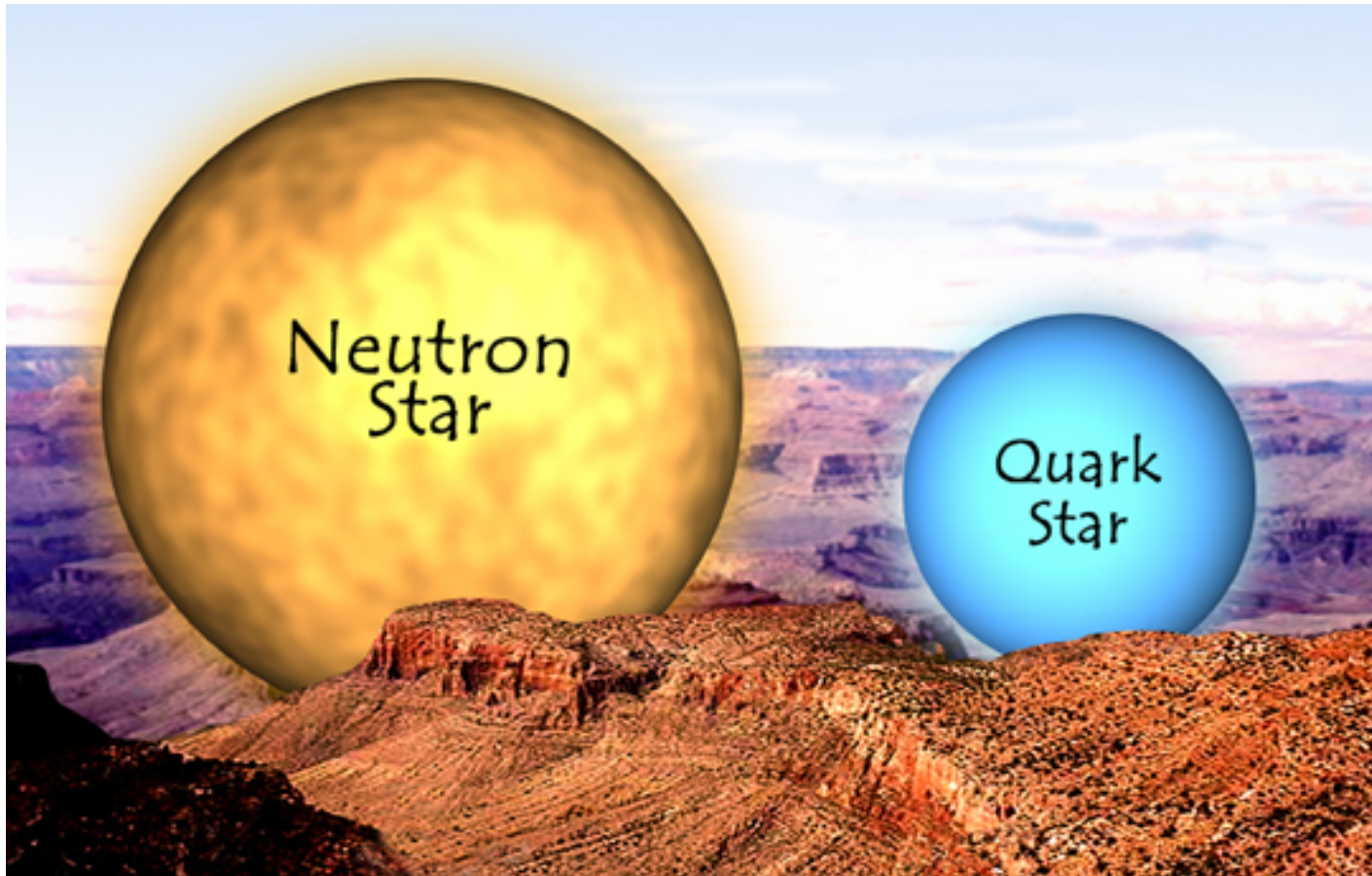
- Lattice QCD study



Karsch's talk  
Fodor's talk

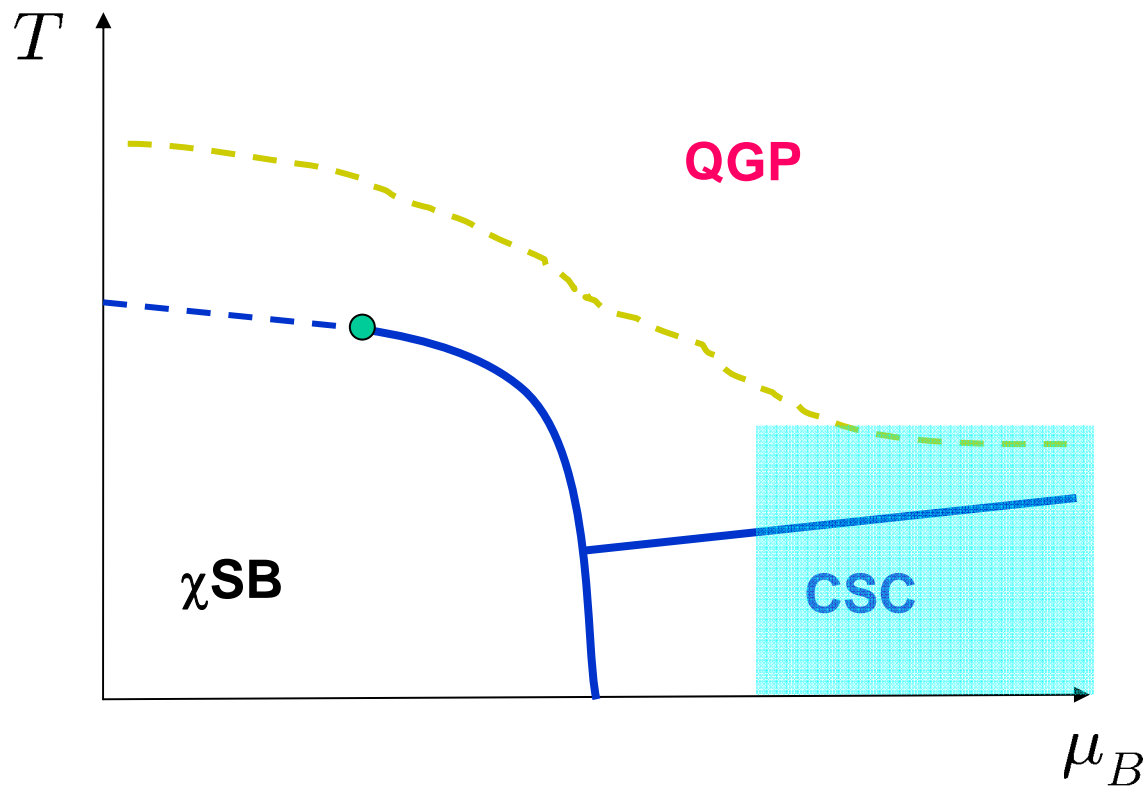


# *Phases in dense QCD*



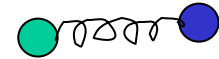
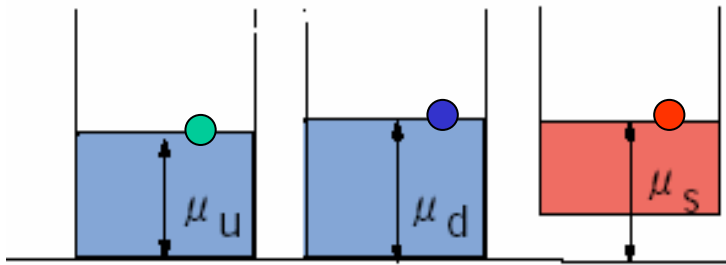
Baade-Zwicky ('34)

N. Itoh ('70), E. Witten ('84)



Bailin & Love, Phys. Rep. 107 (1984)  
Iwasaki & Iwado, PL B350 (1995)  
Alford, Rajagopal & Wilczek, PL B422 (1998)  
Rapp, Schafer, Shuryak & Velkovsky, PRL81 (1998)

# Origin of Color Superconductivity (CSC)



$$\Delta_{i\alpha} = \epsilon_{ijk} \epsilon_{\alpha\beta\gamma} \langle q_{j,\beta} C \gamma_5 q_{k,\gamma} \rangle$$

$\swarrow$  flavor     $\swarrow$  color

## Major differences from BCS

1. Highly relativistic  
Long range magnetic int.

$$|\Delta| \sim \varepsilon_F e^{-c/\sqrt{\alpha_s}}$$



**High T<sub>c</sub> superconductor**

$$T_c/\varepsilon_F \sim 0.1$$

**Compact Cooper pair**

$$\text{size} \sim 1-10 \text{ fm}$$

2. Color-flavor entanglement

$$\Delta_{i\alpha} = \begin{pmatrix} \Delta_1 & 0 & 0 \\ 0 & \Delta_2 & 0 \\ 0 & 0 & \Delta_3 \end{pmatrix}$$



**Variety of phases  
(such as ice and <sup>3</sup>He)**

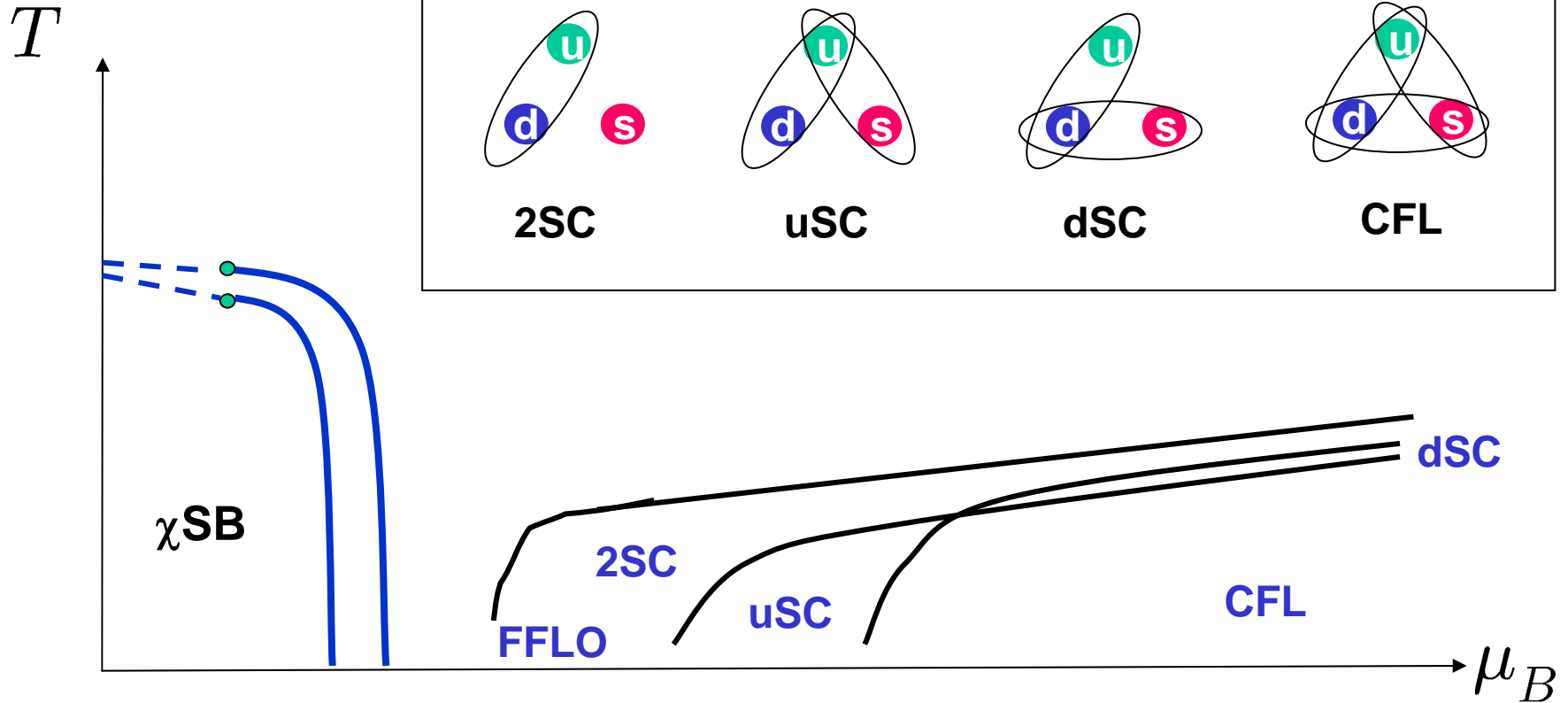
CFL, 2SC, dSC, uSC, etc



# Phase structure relevant to compact stars

with charge-neutrality &  $\beta$ -equilibrium

$$n_d > n_u > n_s$$



**2SC**: Bailin and Love, Phys. Rep. ('84)

**CFL**: Alford, Rajagopal and Wilczek, NPB ('99)

**dSC**: Iida, Matsuura, Tachibana and Hatsuda, PRL ('04)

**uSC**: Ruster, Werth, Buballa, Shovkovy and Rischke, PRD ('05)

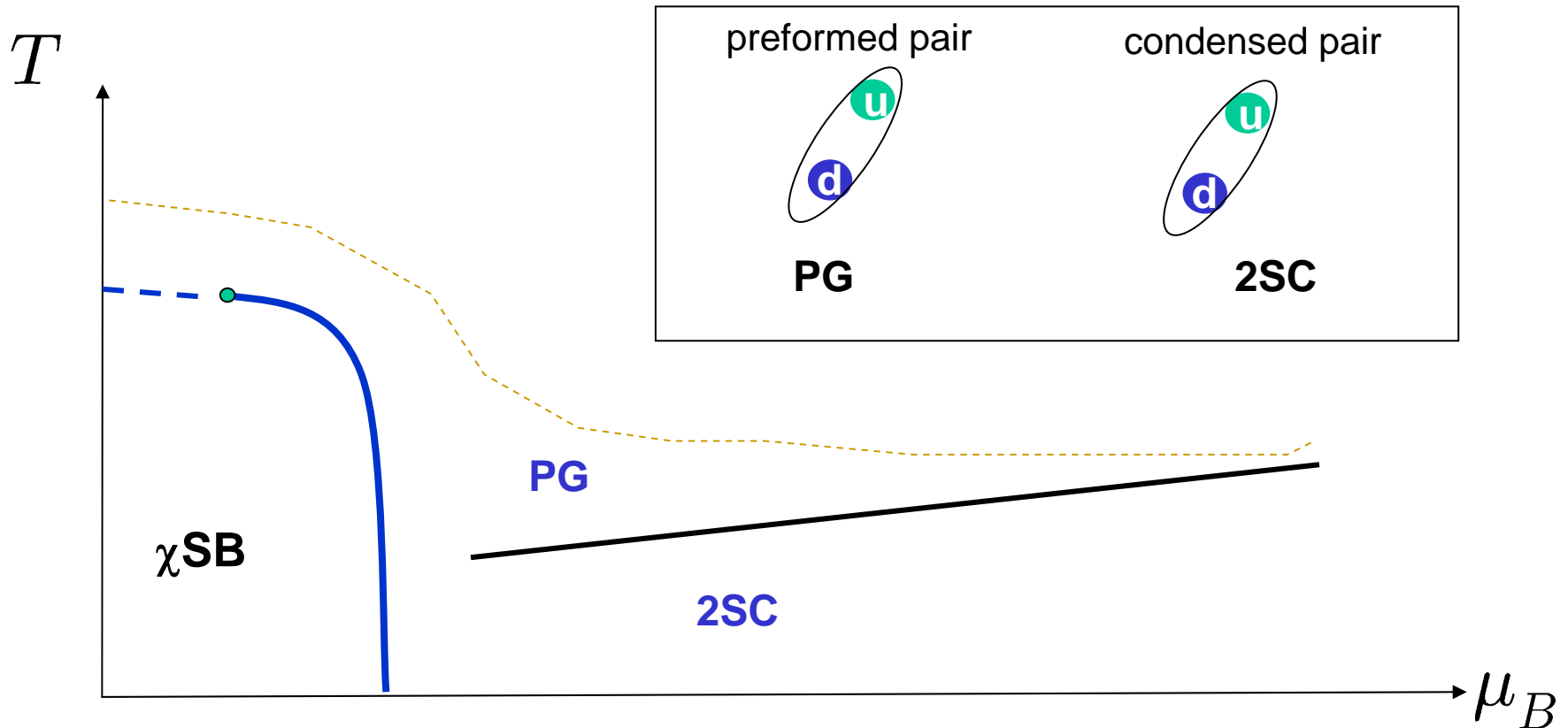
**FFLO, gapless phase, CSL, K-cond. etc**



**Rischke's talk**

# Phase structure relevant to heavy ion collisions

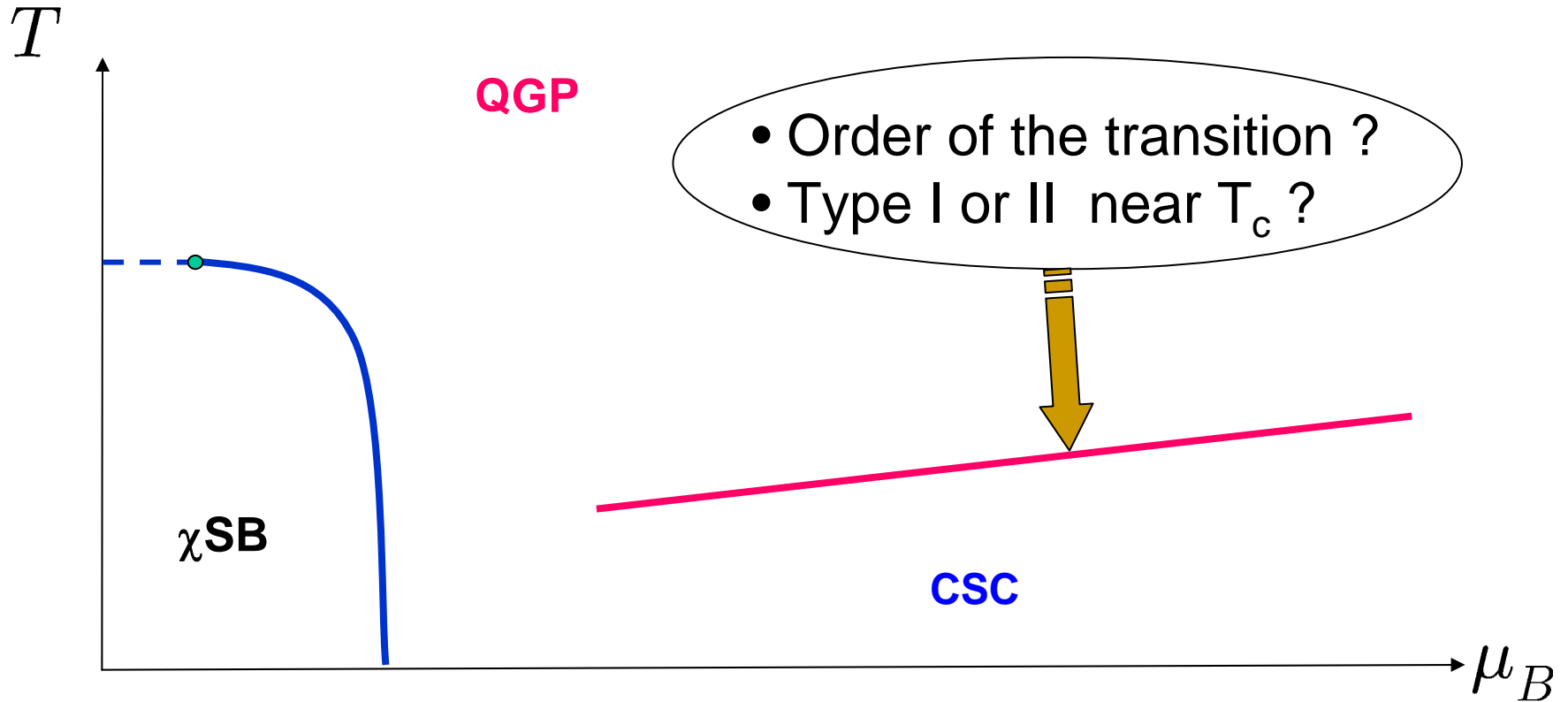
no charge-neutrality &  $\beta$ -equilibrium  $n_d = n_u, n_s = 0$



2SC: Bailin and Love, Phys. Rep. ('84)

PG: Kitazawa, Koide, Kunihiro and Nemoto, PRD ('04)

# Thermal phase transition of CSC



# Ginzburg-Landau theory at $T \sim T_c$

$SU_c(3) \times SU_{L+R}(3)$  gauged Higgs model in 3d :

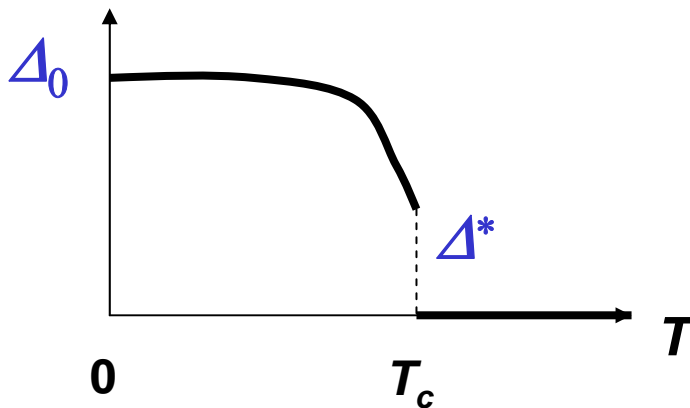
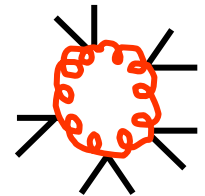
Iida & Baym, PRD63 ('01)

$$A_\mu^a, \quad \Delta_{i\alpha} \quad (= \Delta_{i\alpha}^L = \Delta_{i\alpha}^R)$$

- Weak coupling analyses Matsuura, Iida, Hatsuda & Baym, PRD 69 ('04)  
Giannakis, Hou, Ren & Rischke, PRL 93 ('04)
- Lattice simulations : Digal, Hatsuda & Ohtani, hep-lat/0511018



**First order transition at high density : Coleman-Weinberg type**

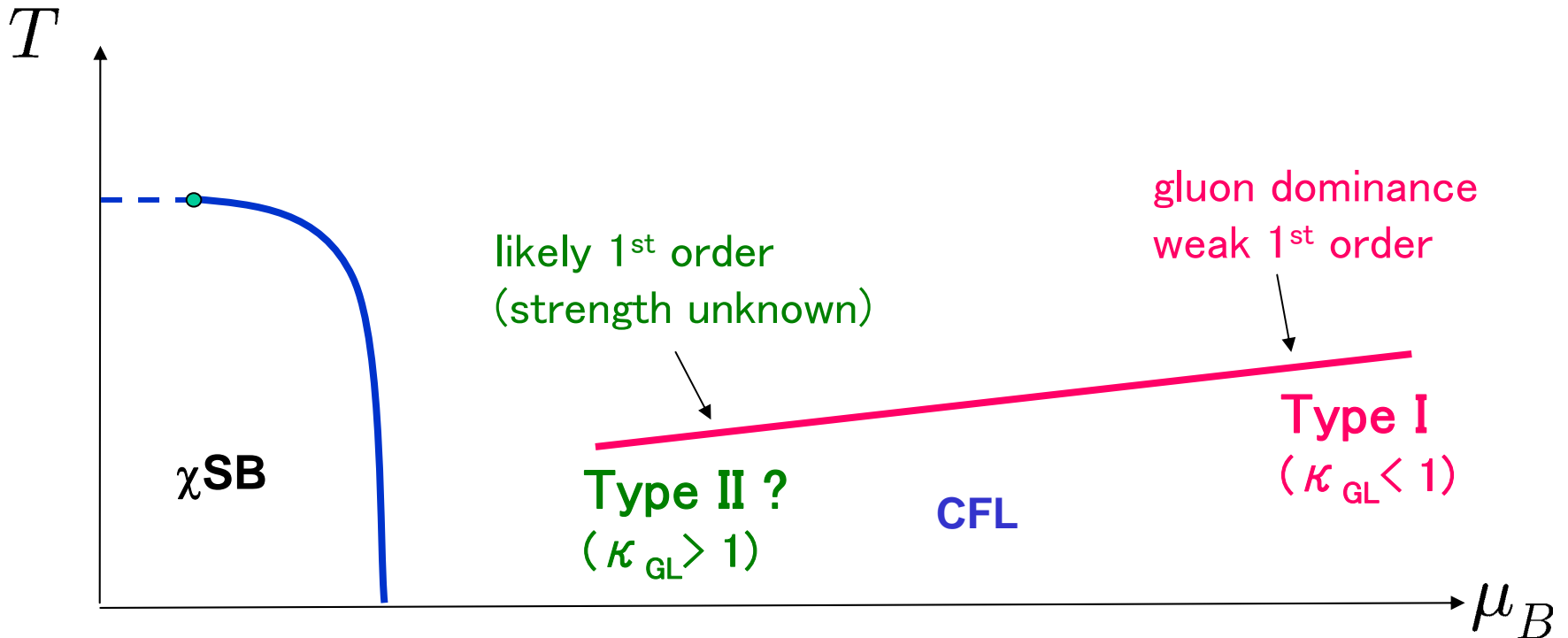


$$\frac{\Delta^*}{\Delta_0} \Big|_{\text{WC}} \sim \frac{g}{4}$$

# Thermal transition : CFL $\rightarrow$ normal

Ginzburg-Landau parameter ::

$$\kappa_{\text{GL}} = \frac{\text{color penetration depth}}{\text{color coherence length}} = \frac{m_{\Delta}}{m_A} \sim \frac{1}{\sqrt{\alpha_s}} \left( \frac{T_c}{\mu_B} \right)$$

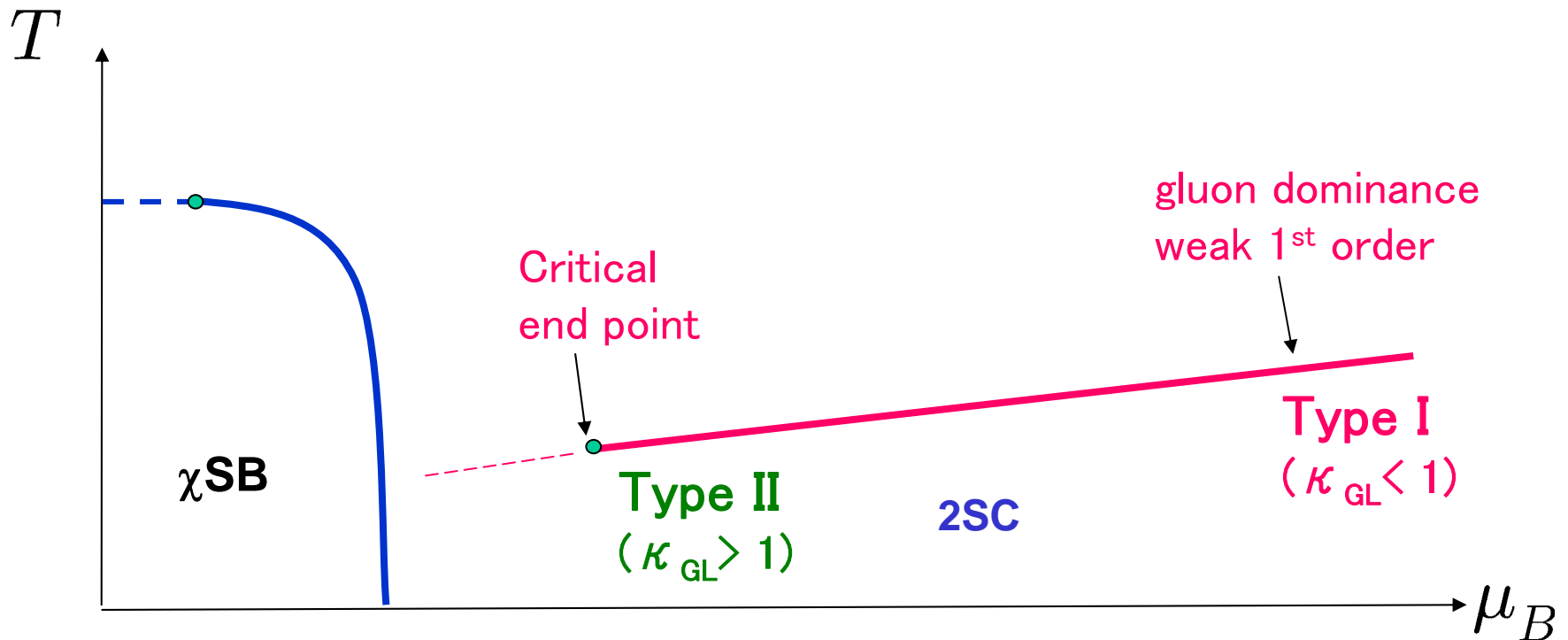


Case for  $m_{uds}=0$

# Thermal transition : 2SC $\rightarrow$ normal

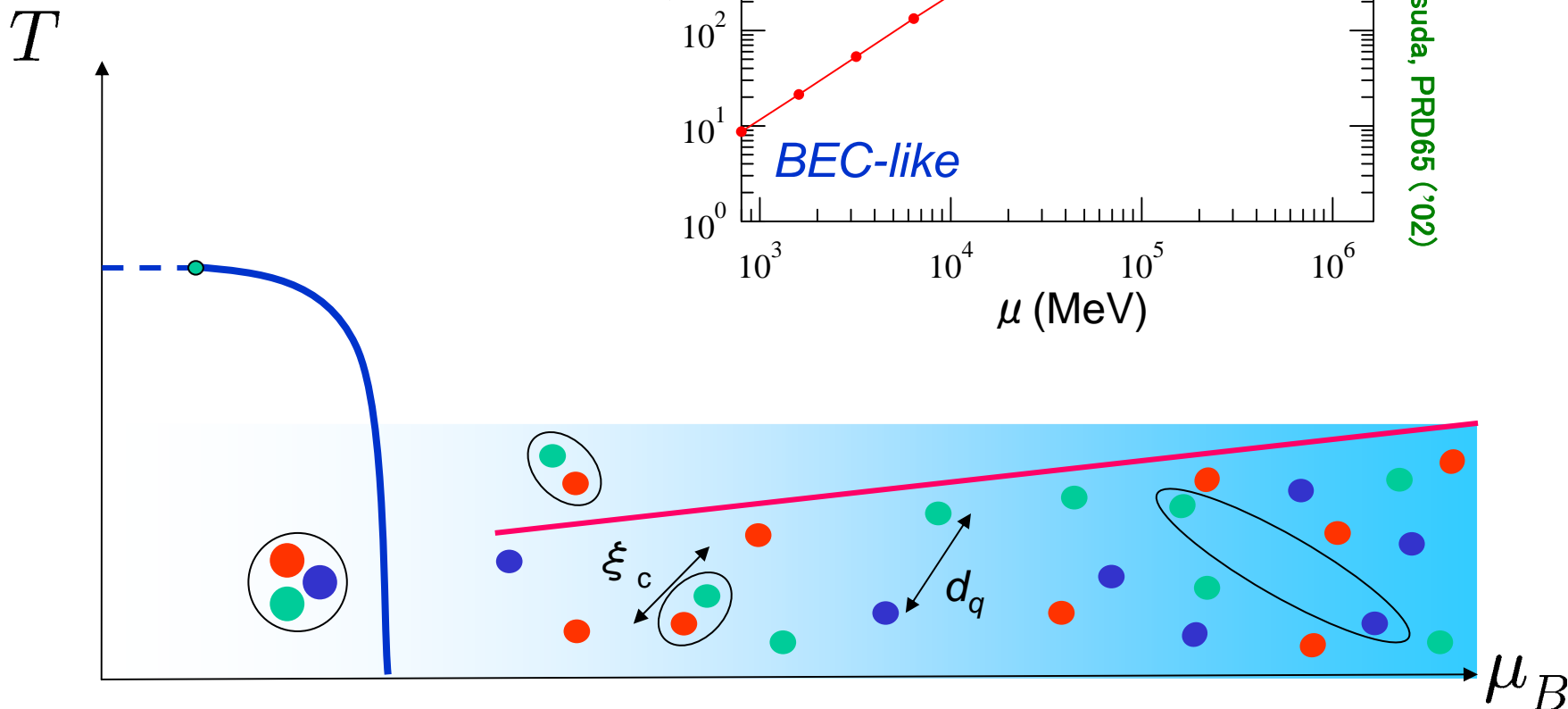
Ginzburg-Landau parameter ::

$$\kappa_{\text{GL}} = \frac{\text{color penetration depth}}{\text{color coherence length}} = \frac{m_{\Delta}}{m_A} \sim \frac{1}{\sqrt{\alpha_s}} \left( \frac{T_c}{\mu_B} \right)$$

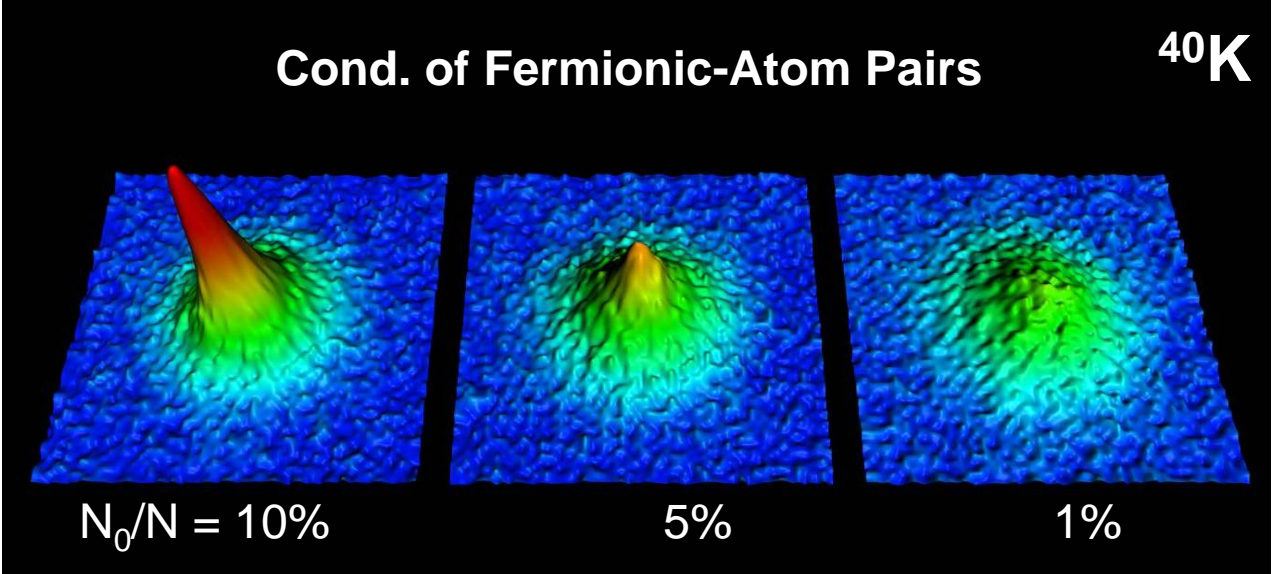


Case for  $m_s = \infty$

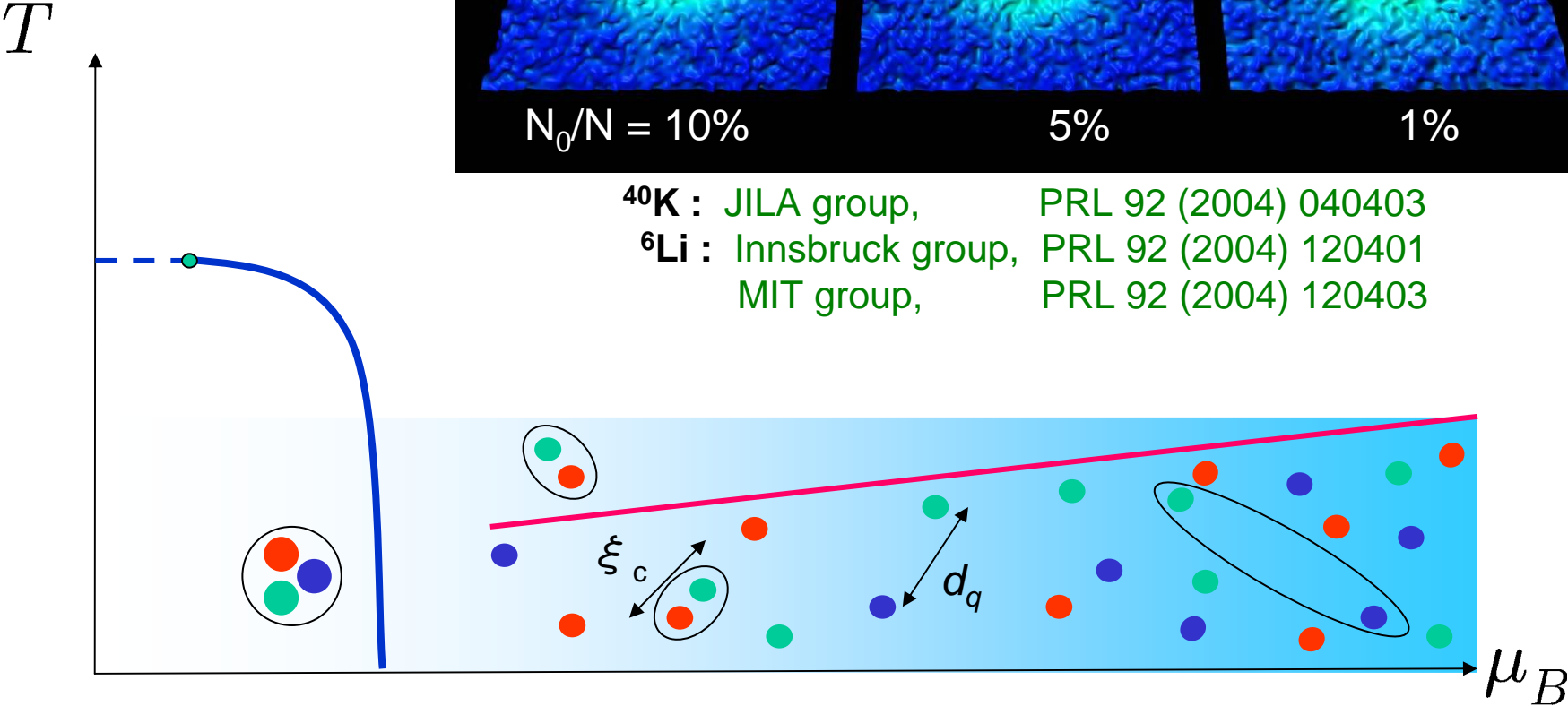
# BCS-BEC crossover ?



# BCS-BEC crossover ?



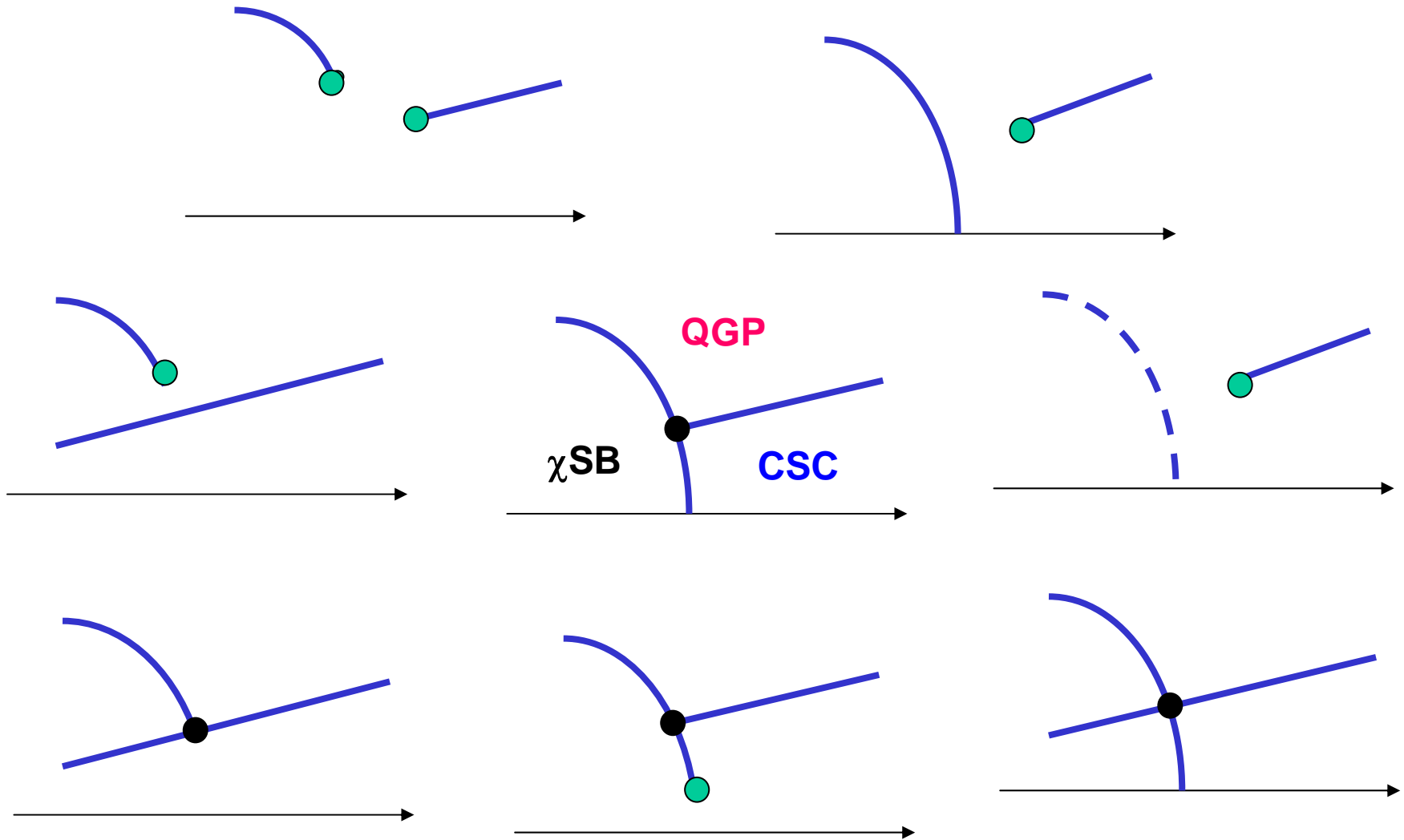
<sup>40</sup>K : JILA group, PRL 92 (2004) 040403  
<sup>6</sup>Li : Innsbruck group, PRL 92 (2004) 120401  
 MIT group, PRL 92 (2004) 120403



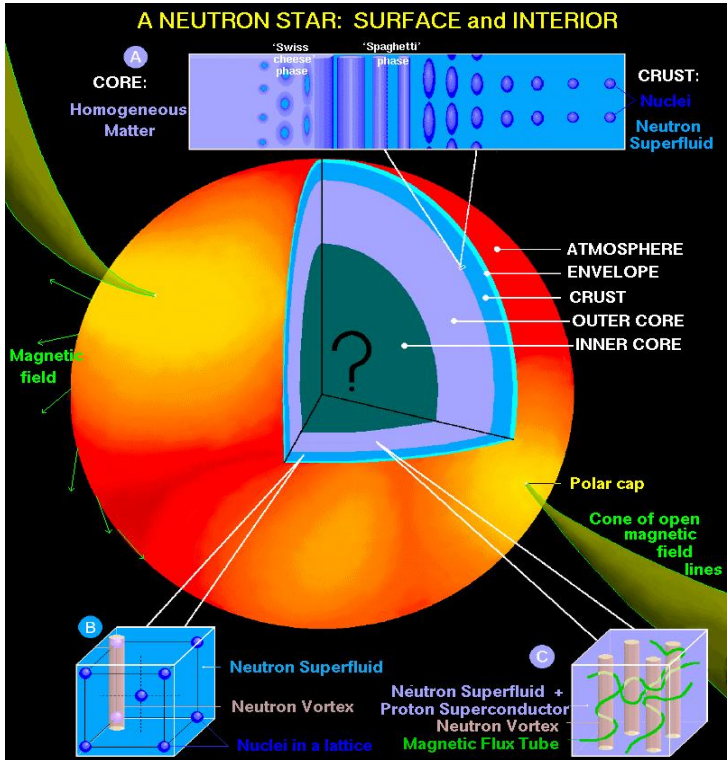


# Possible phase structure

$$n_d = n_u, n_s = 0$$



# Probing Dense QCD



J-PARC



SIS100/300



Weber's talk

# Future Experimental Facilities for hot/dense QCD

## LHC (2008-)



2.8 TeV/A

- Hottest matter
- Precision QGP

## J-PARC (2008-)



50 GeV PS

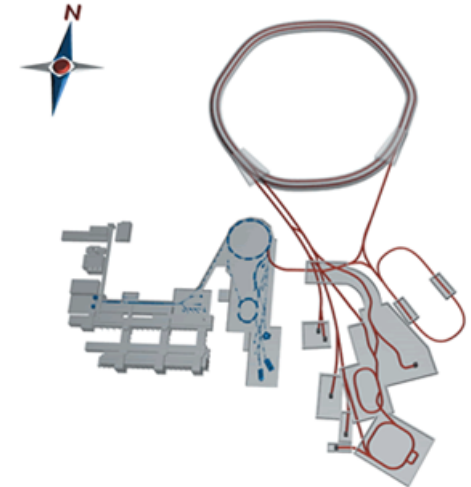
### Phase I

- Dense mesic nuclei
- Exotic hadrons

### Phase II

- Primary beam phys.

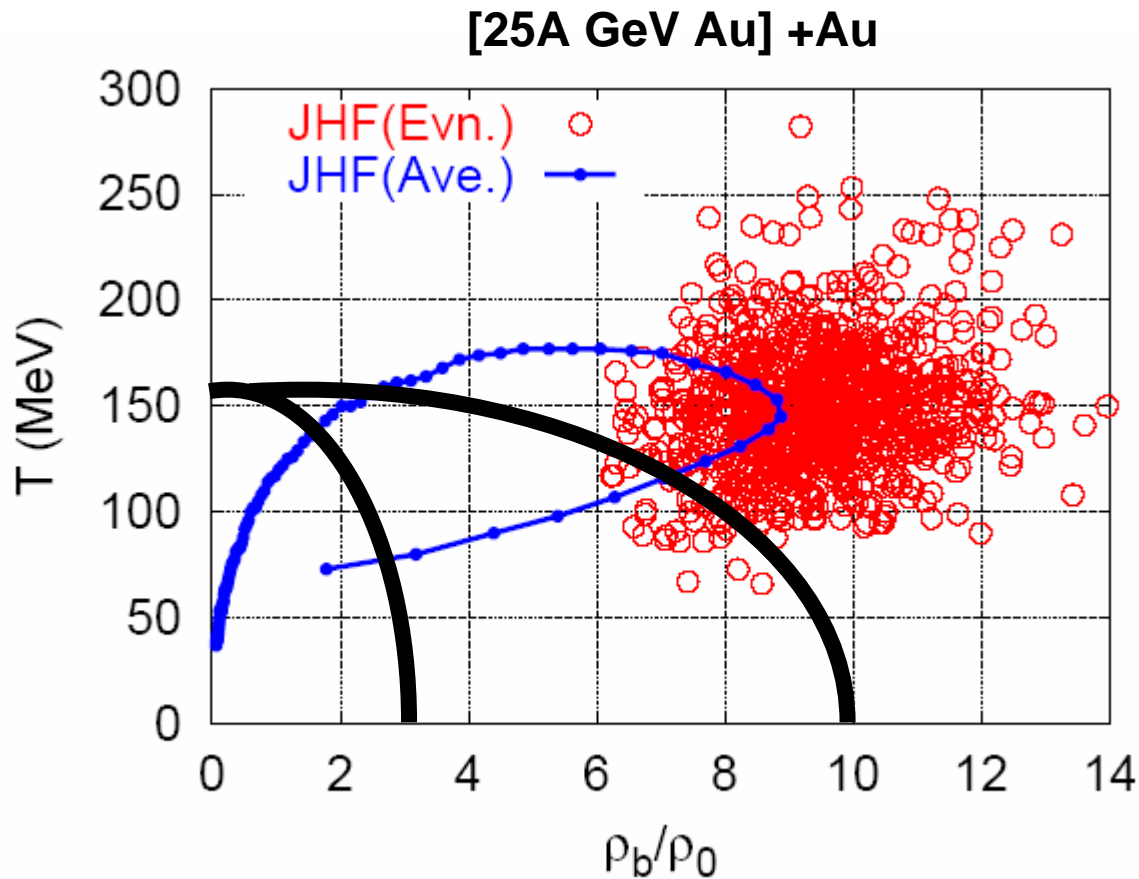
## SIS100/300 (201? -)



90 GeV PS

- Densest matter
- In-medium hadrons

# Heavy ion collision at J-PARC & SIS energies



JAM  
(Hadronic cascade model)  
Y. Nara et al, PRC61 (2000)

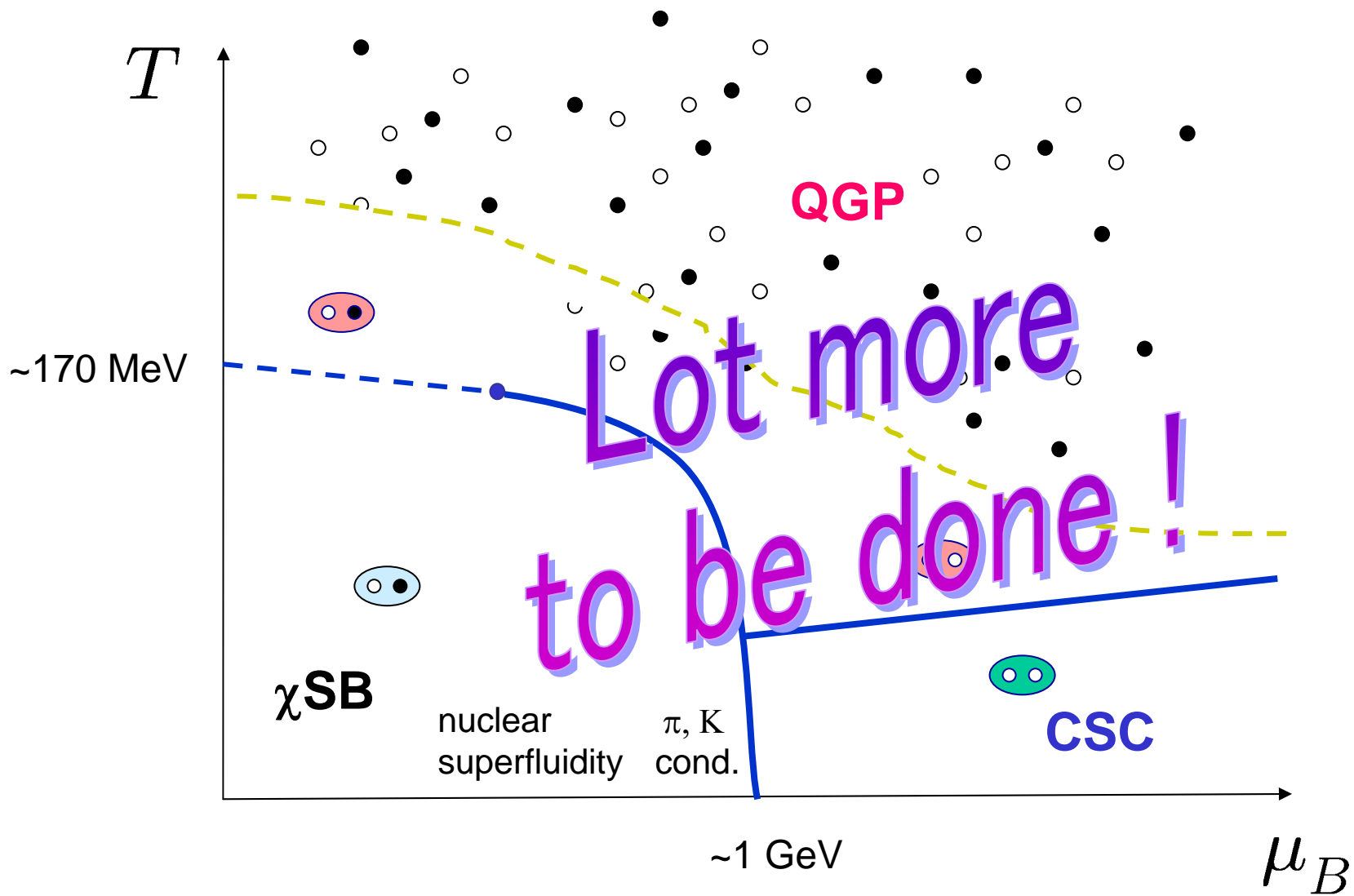
$\rho_B > 6 \rho_0$   
for about 3 fm/c

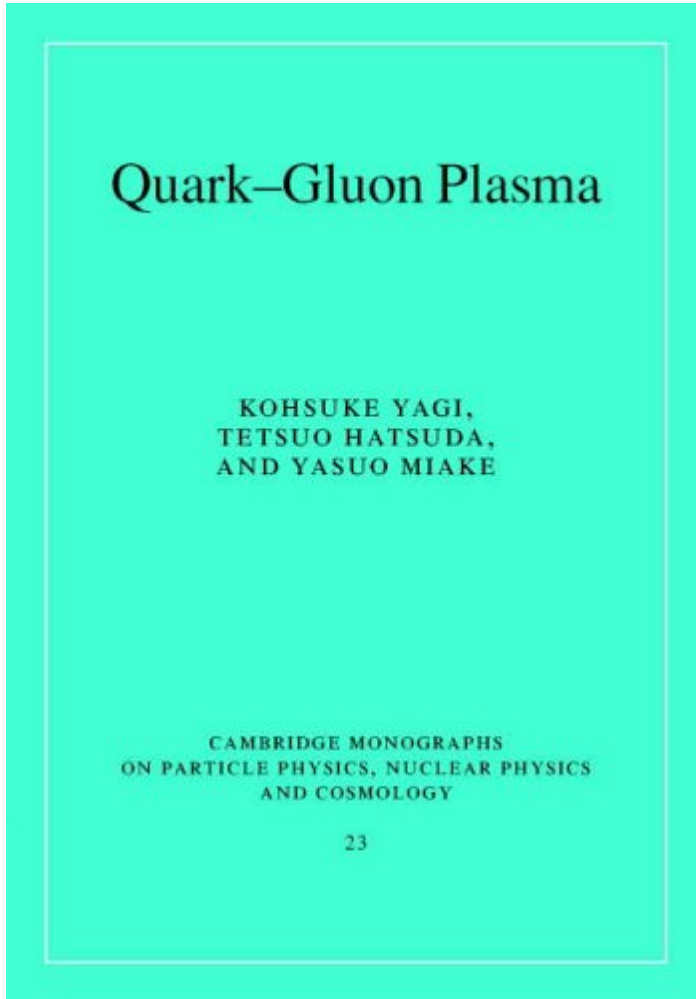


Ivanov's talk

# Summary

“phase”	theory	exp./obs.
$\chi$ <b>SB</b> (low T & low $\mu$ )	<u>Mature (precision physics)</u> lattice QCD effective theories	variety of data
<b>QGP</b> (high T)	<u>Developing</u> lattice QCD effective theories	data accumulation RHIC $\rightarrow$ LHC
<b>QM and CSC</b> (low T & high $\mu$ )	<u>Developing</u> effective theories Need lattice inputs	<ul style="list-style-type: none"> <li>• Neutron stars (M-R)</li> <li>• SIS, J-PARC, Nuclotron</li> </ul>
<b>“PG”</b> (intermediate T & $\mu$ )	<u>Not- fully explored</u> Interesting connection to HTS, cold atoms	Relevant region to RHIC, SIS, J-PARC, Nuclotron ?





published, Dec. 15, 2005

1. What is quark-gluon plasma

## **Part I. Basic Concept of Quark-Gluon Plasma:**

2. Introduction to QCD
3. Physics of quark-hadron phase transition
4. Field theory at finite temperature
5. Lattice gauge approach to QCD phase transitions
6. Chiral phase transition
7. Hadronic states in hot environment

## **Part II. QGP in Astrophysics:**

8. QGP in the early universe
9. Compact stars

## **Part III. QGP in Relativistic Heavy Ion Collisions:**

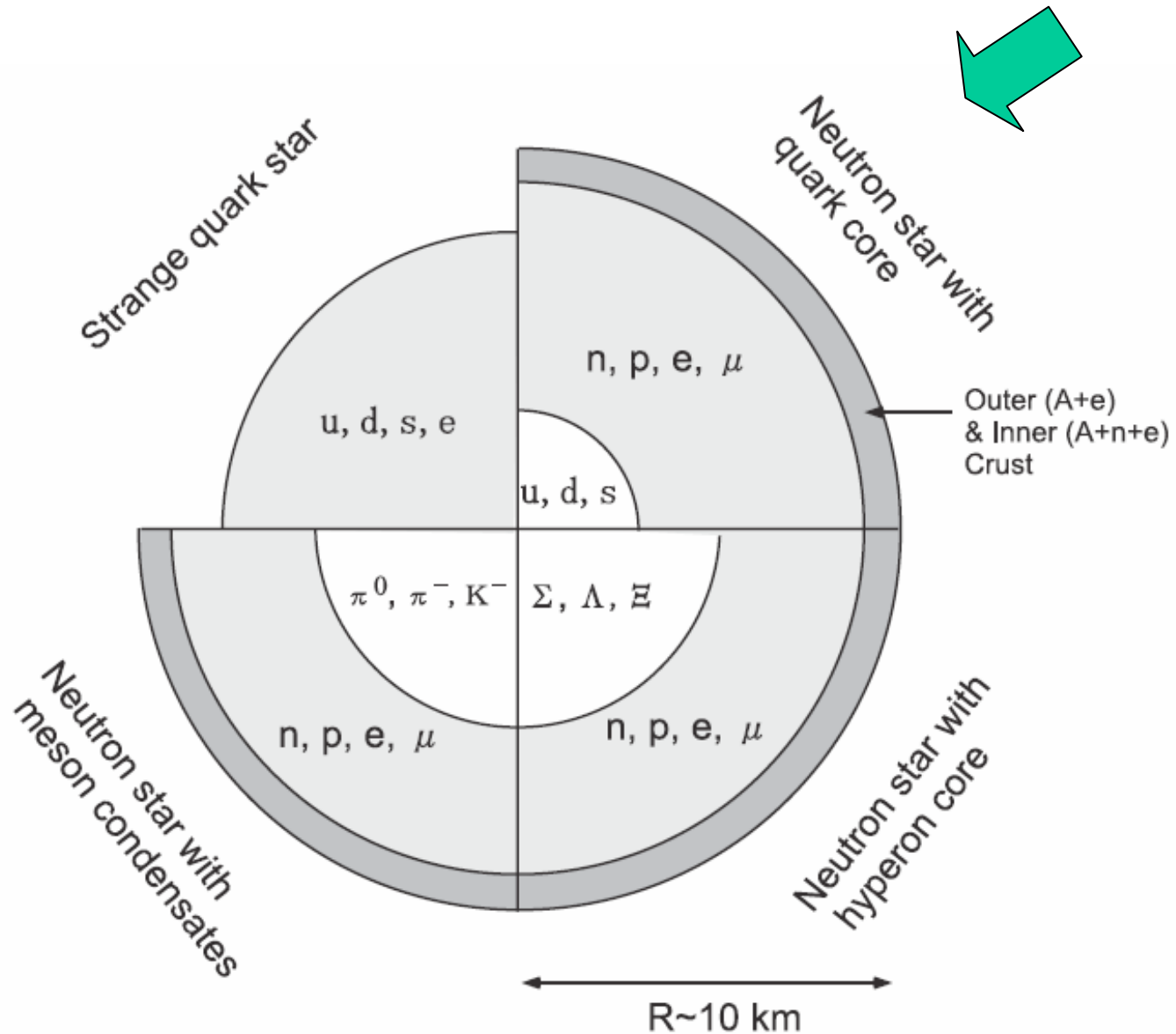
10. Introduction to relativistic heavy ion collisions
11. Relativistic hydrodynamics for heavy ion collisions
12. Transport theory for pre-equilibrium process
13. Formation and evolution of QGP
14. Fundamentals of QGP diagnostics
15. Results from CERN-SPS experiments
16. First results from BNL-RHIC
17. Detectors in relativistic heavy ion experiments

**Appendices A-H:  
120 Exercises**

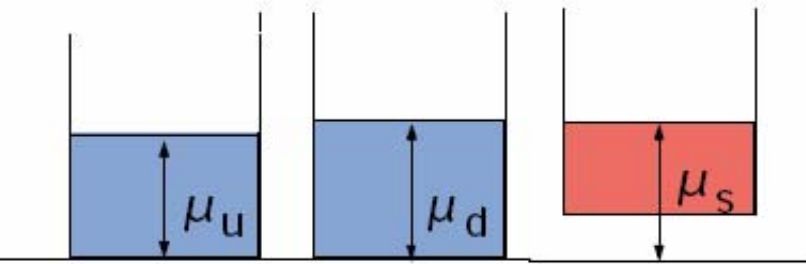
Back up slides



# Neutron Star Structure



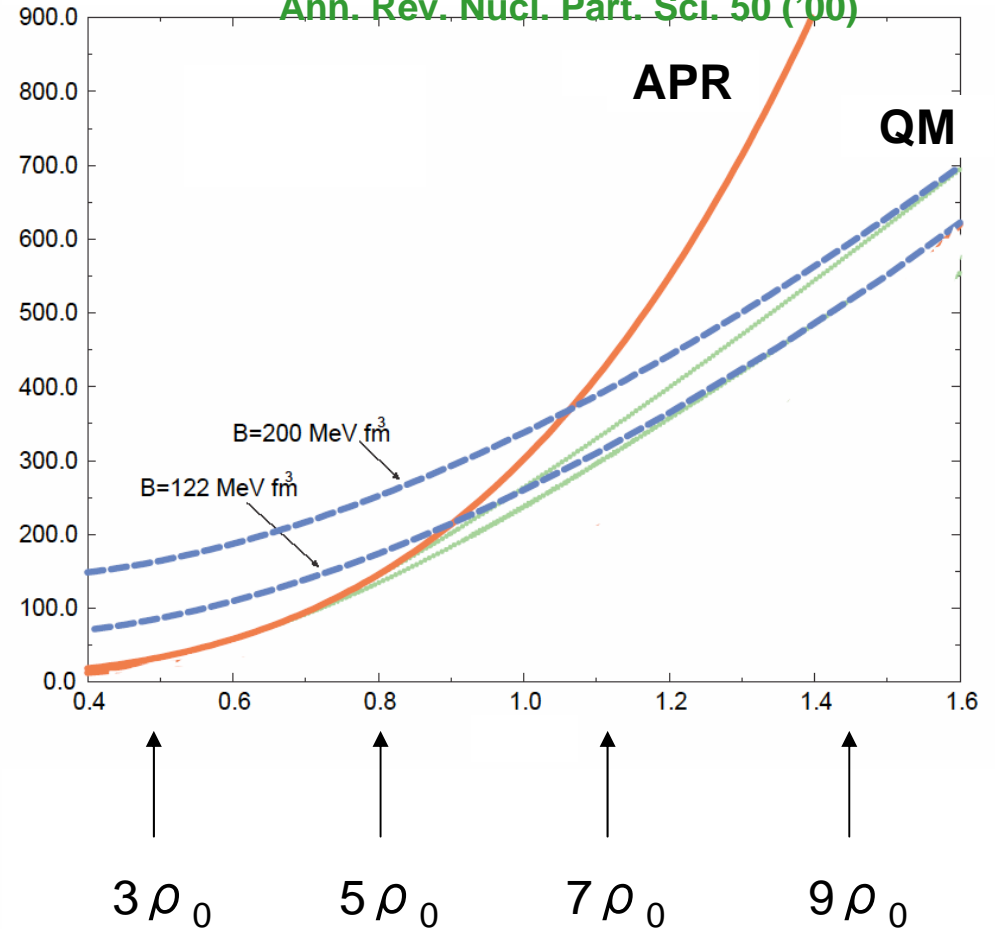
Quark matter :  $u, d, s, e^-$  with  $d \rightleftharpoons u + e^-, d \rightleftharpoons u + e^-, s \rightleftharpoons d$



Remarks

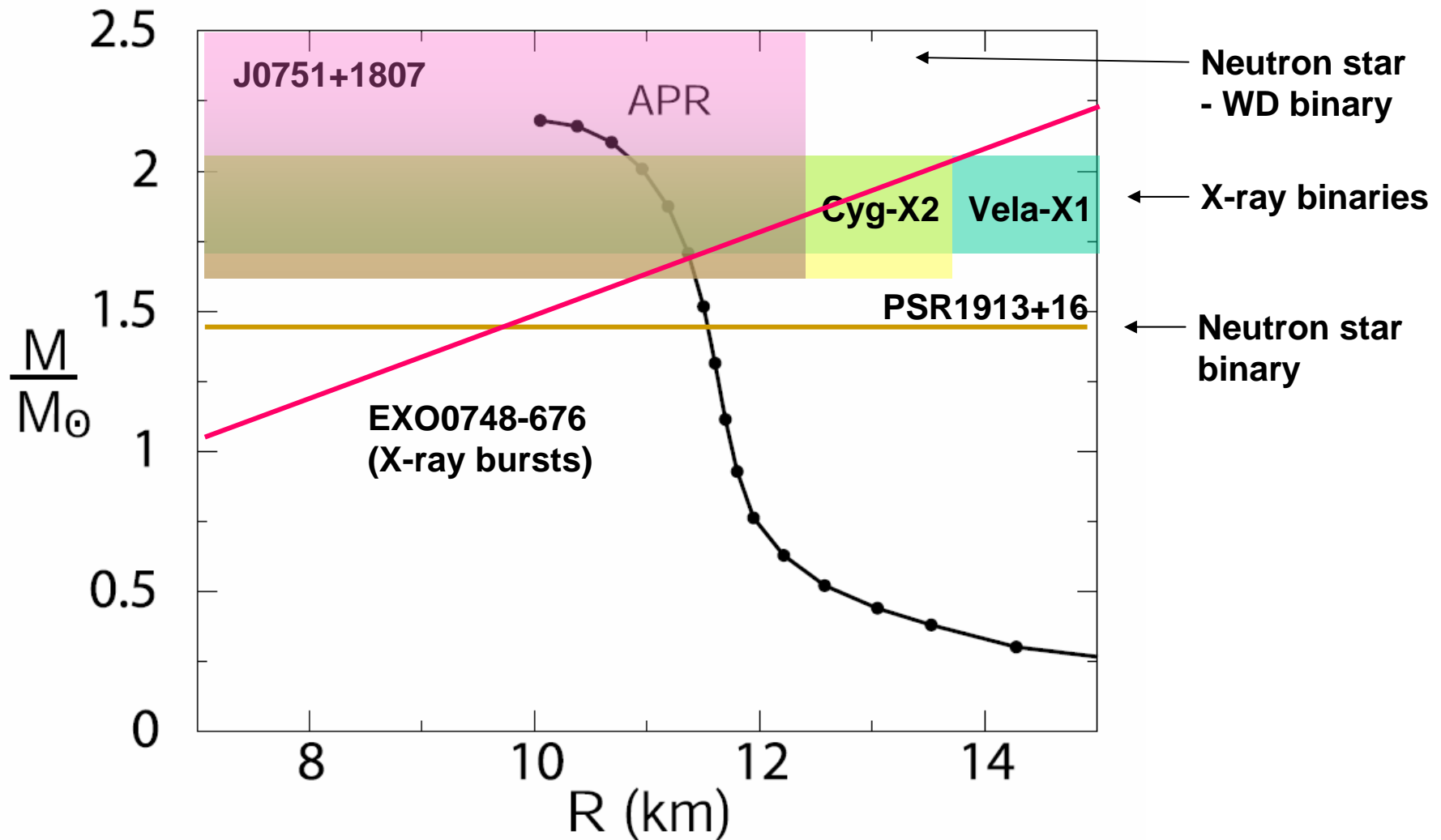
- Mixed phase may start even at  $\sim 3 \rho_0$
- sensitive to quark matter EOS

Heiselberg & Pandharipande,  
Ann. Rev. Nucl. Part. Sci. 50 ('00)

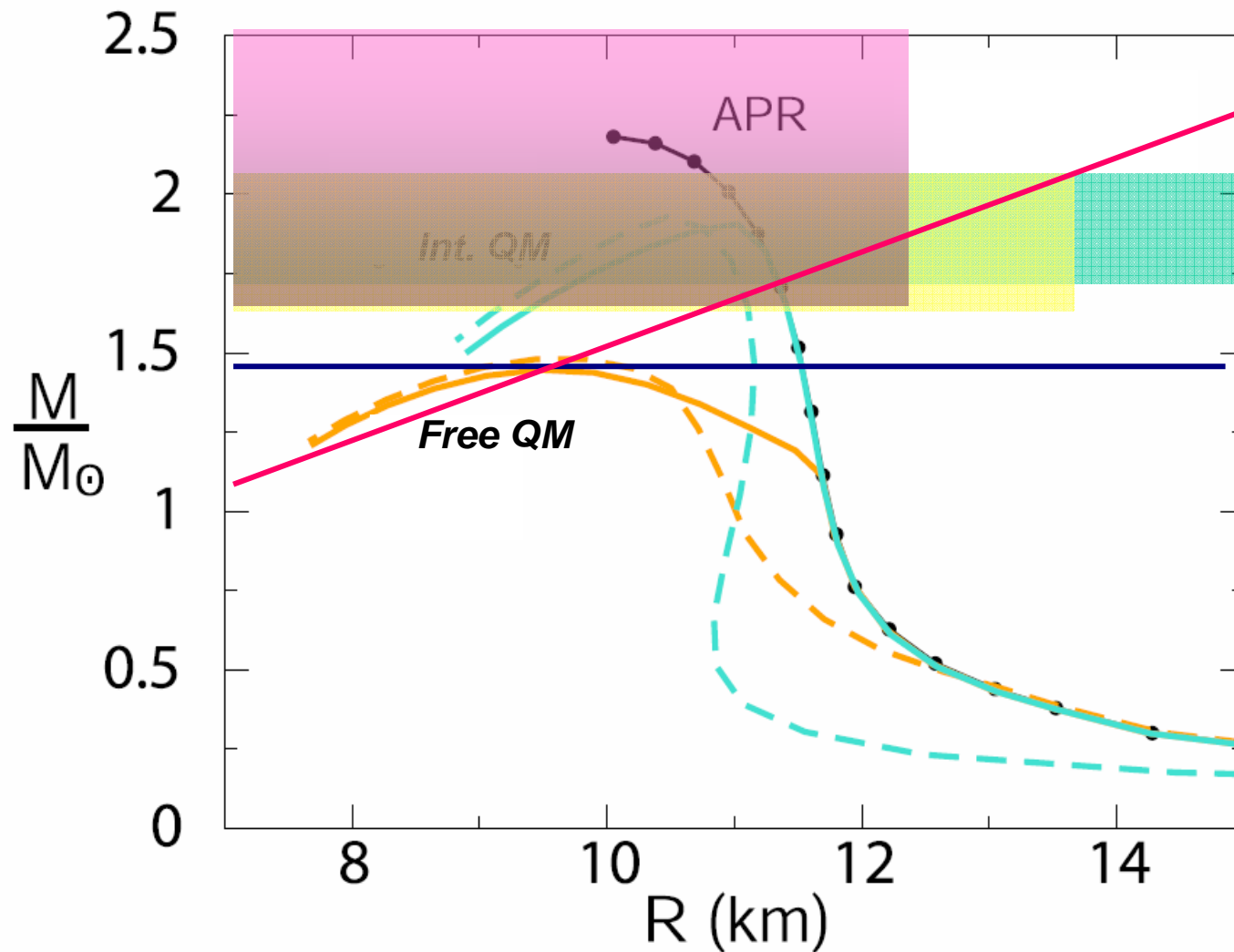


# M-R relation in APR EoS

( $\rho_{\max} \sim 6\rho_0$ )



# M-R relation in APR EoS + CFL quark matter



# Cooling of neutron stars

	Name	Processes	Emissivity
<b>Standard</b>	Modified Urca	$n + n \rightarrow n + p + e^- + \bar{\nu}_e$ $n + p + e^- \rightarrow n + n + \nu_e$	$\sim 10^{20} T_9^8$
	Direct Urca	$n \rightarrow p + e^- + \bar{\nu}_e$ $p + e^- \rightarrow n + \nu_e$	$\sim 10^{27} T_9^6$
<b>Exotic</b>	Quark modified Urca	$d + u + e^- \rightarrow d + d + \nu_e$ $u + u + e^- \rightarrow u + d + \nu_e$ $d + u + e^- \rightarrow d + s + \nu_e$ $u + u + e^- \rightarrow u + s + \nu_e$	$\sim 10^{20} T_9^8$
	Quark direct Urca	$d \rightarrow u + e^- + \bar{\nu}_e$ $u + e^- \rightarrow d + \nu_e$ $s \rightarrow u + e^- + \bar{\nu}_e$ $u + e^- \rightarrow s + \nu_e$	$\sim 10^{26} T_9^6$
	$\pi^-$ condensate	$n + \langle \pi^- \rangle \rightarrow n + e^- + \bar{\nu}_e$	$\sim 10^{26} T_9^6$
	$K^-$ condensate	$n + \langle K^- \rangle \rightarrow n + e^- + \bar{\nu}_e$	$\sim 10^{26} T_9^6$
<b>quenching</b>	<i>n superfluidity</i> <i>Q color super</i>	$\exp(-\Delta/T)$	

# Cooling of neutron stars

Standard

Name

Modified Urca

Direct Urca

Quark modified Urca

Exotic

Quark direct Urca

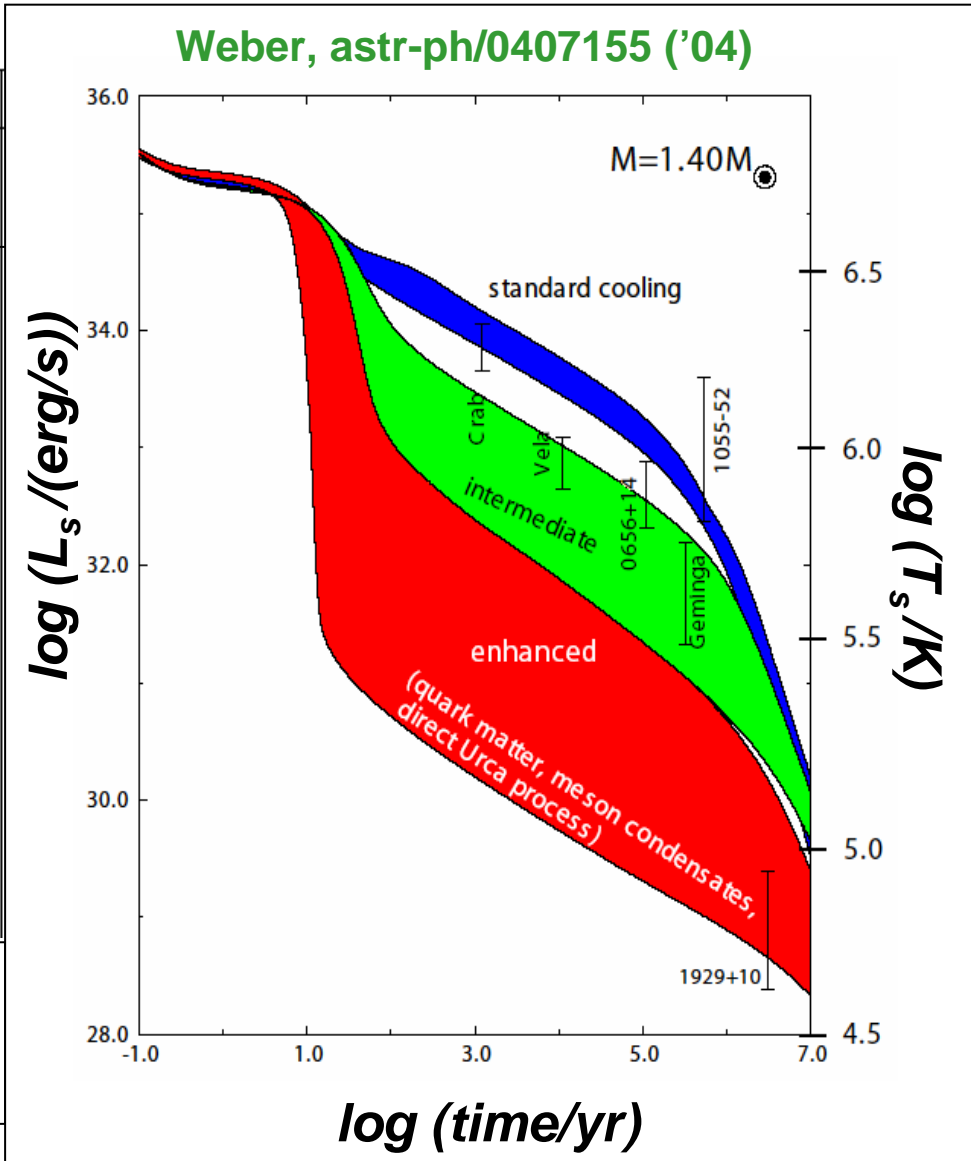
$\pi^-$  condensate

$K^-$  condensate

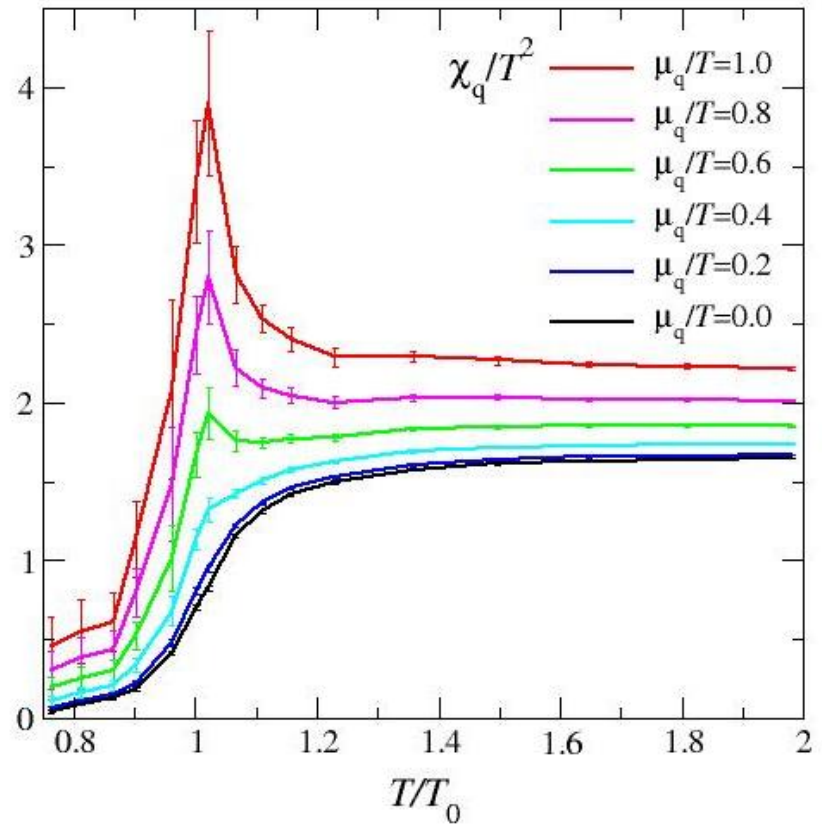
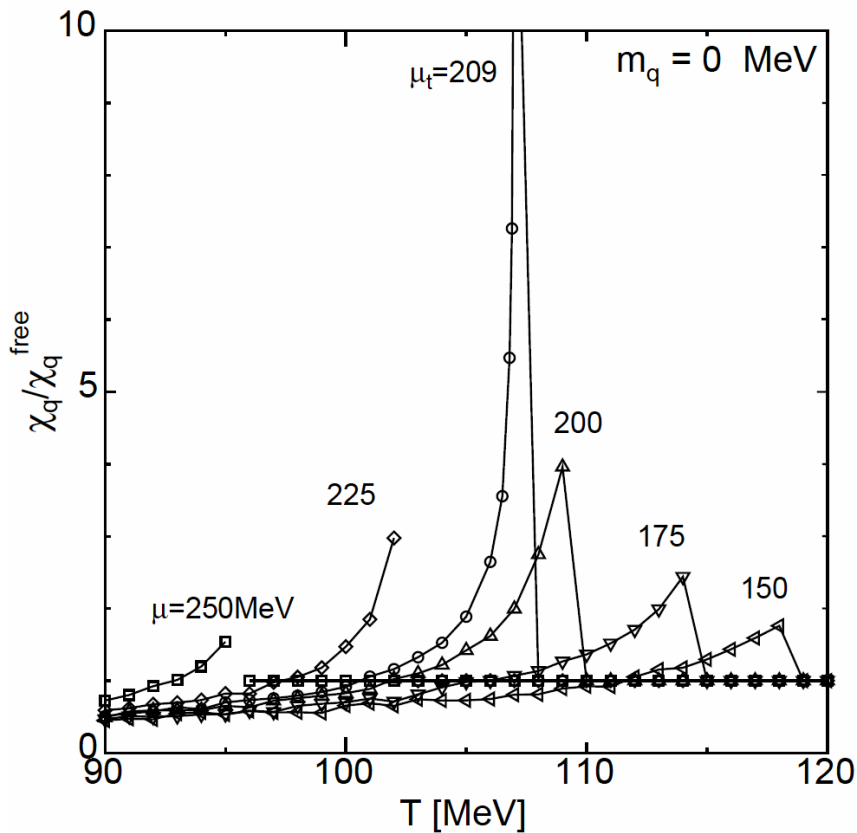
quenching

*n* superfluidity

*Q* color super



# Quark number susceptibility



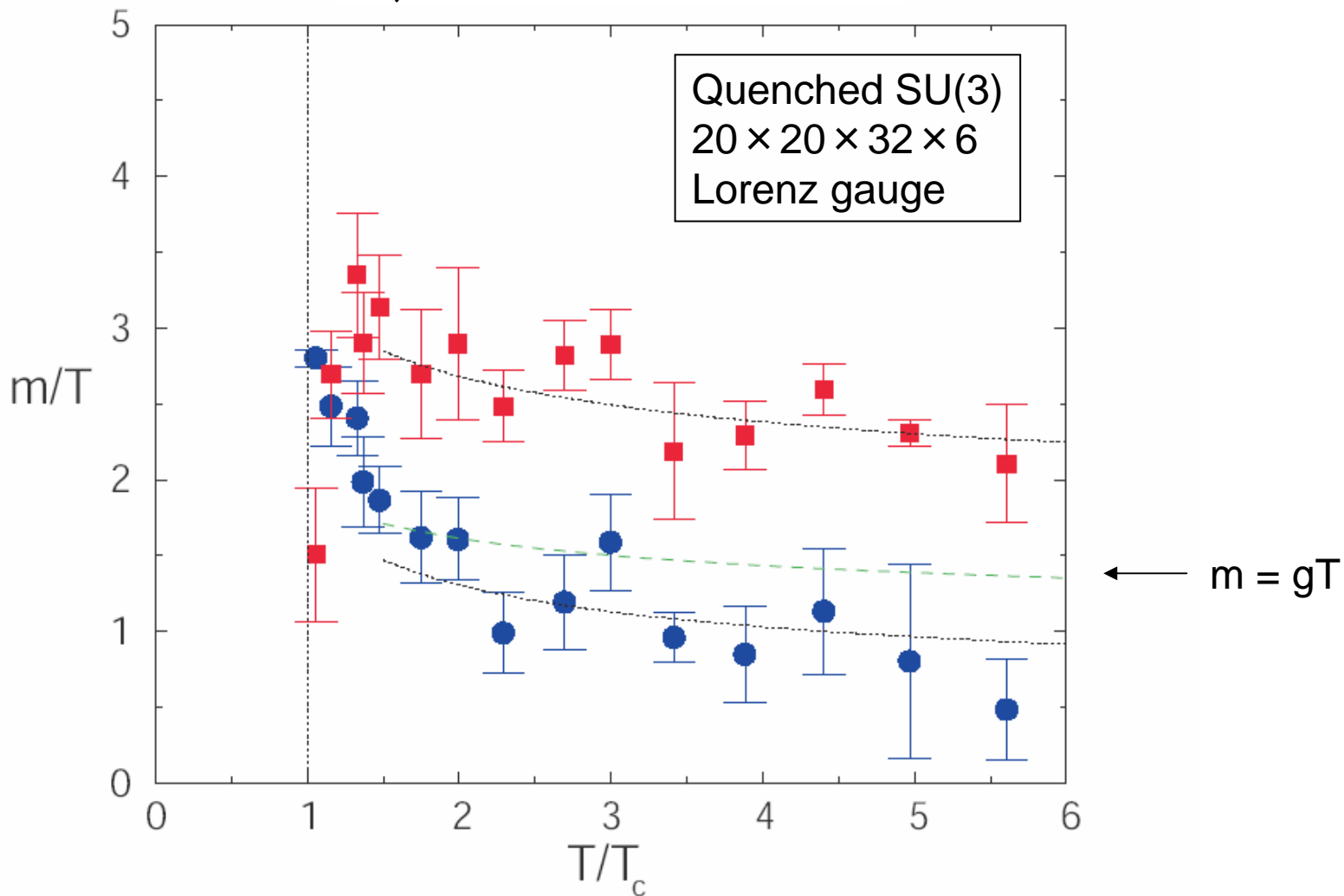
$$\frac{\chi_q}{T^4}(\mu_q) = +2c_2 + 12c_4 \left( \frac{\mu_q}{T} \right)^2 + O(\mu_q^4)$$

Kunihiro, Phys. Lett. B271 ('92) 395  
 Hatta-Ikeda, Phys.Rev.D67 ('03) 014028

Ejiri et al. (Bielefeld-Swansea Coll.)

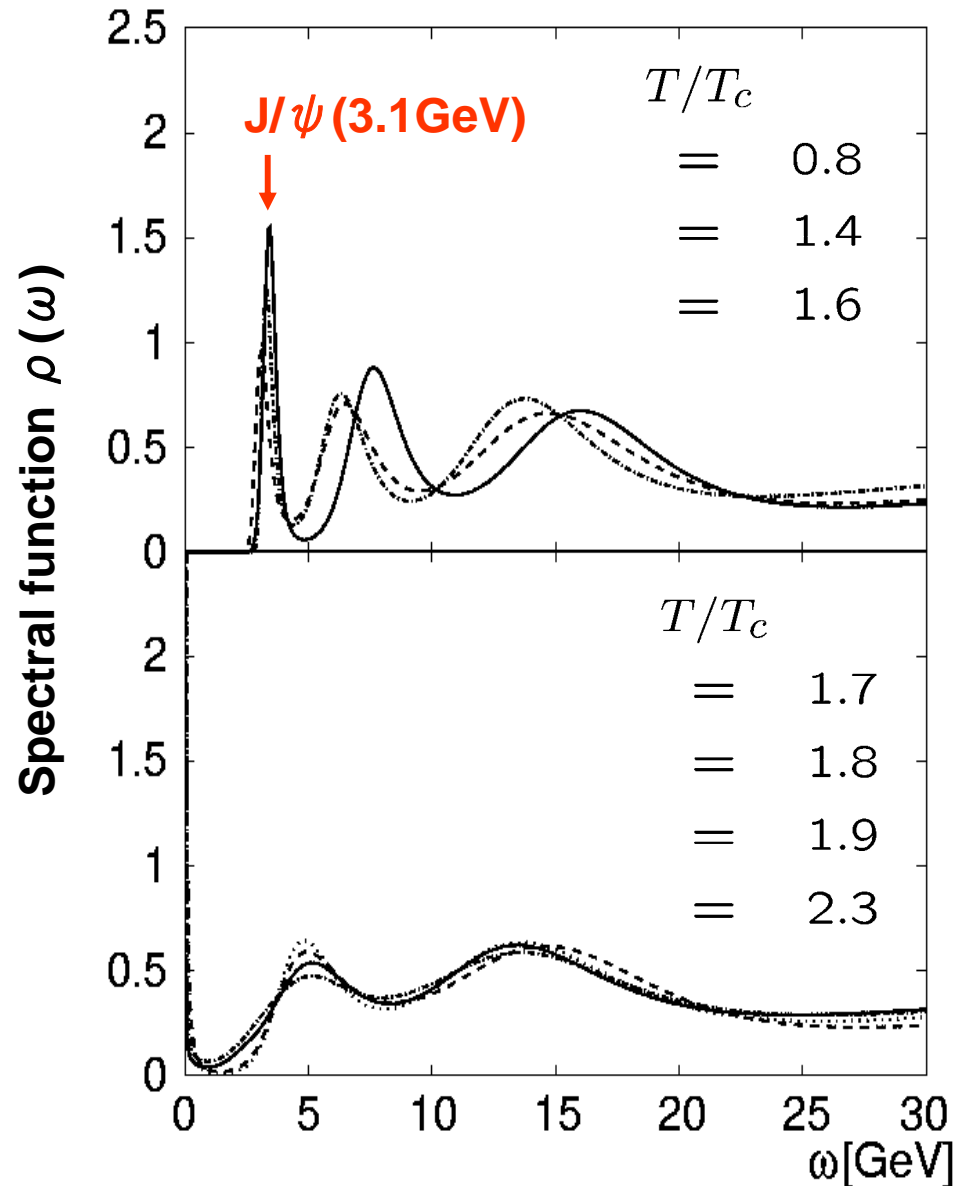
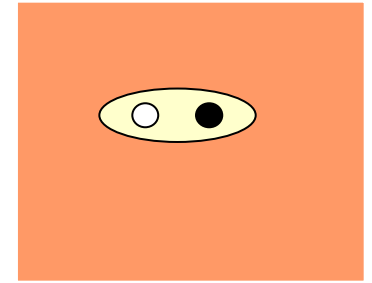
# Screening masses at $T > T_c$ on the lattice

$$\langle A_\mu(\mathbf{x}) A_\nu(\mathbf{0}) \rangle \sim e^{-m|\mathbf{x}|}$$





# $c\bar{c}$ bound state above $T_c$ (quenched)



1.  $J/\psi$  survives  
up to  $1.6 T_c$

2.  $J/\psi$  disappears  
in  $1.6 T_c < T < 1.7 T_c$

Asakawa & T.H., PRL 92 ('04) 012001

see also,

Umeda et al, hep-lat/0401010

Datta et al., PRD 69 ('04) 094507