The QCD equation of state at non-zero temperature and quark chemical potential

1) Introduction

QCD phase diagram, EoS at $\mu = 0$

2) High-T limit and Taylor expansion at non-vanishing chemical potential

perturbation theory and lattice approach;

3) Bulk thermodynamics for small values of the chemical potential

pressure, density, energy density, entropy up to $\mathcal{O}(\mu_q^6)$;

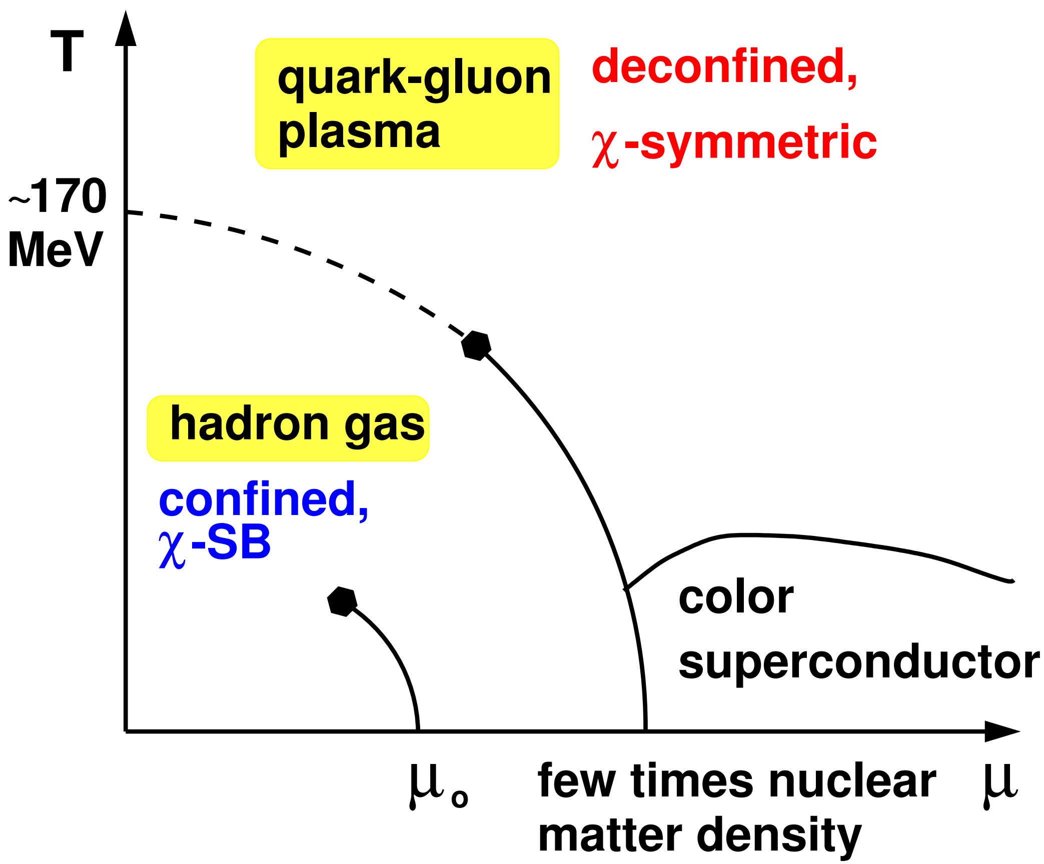
$\Rightarrow$ the isentropic equation of state

4) Hadronic fluctuations

quark number and charge fluctuations

5) Conclusions

Critical behavior in hot and dense matter: QCD phase diagram



continuous transition for small chemical potential and small quark masses at

$$T_c \simeq 170\, MeV$$
$$\epsilon_c \simeq 0.7\, GeV/fm^3$$

want accurate $T_c, \epsilon_c, \ldots$ determination to make contact to HI-phenomenology

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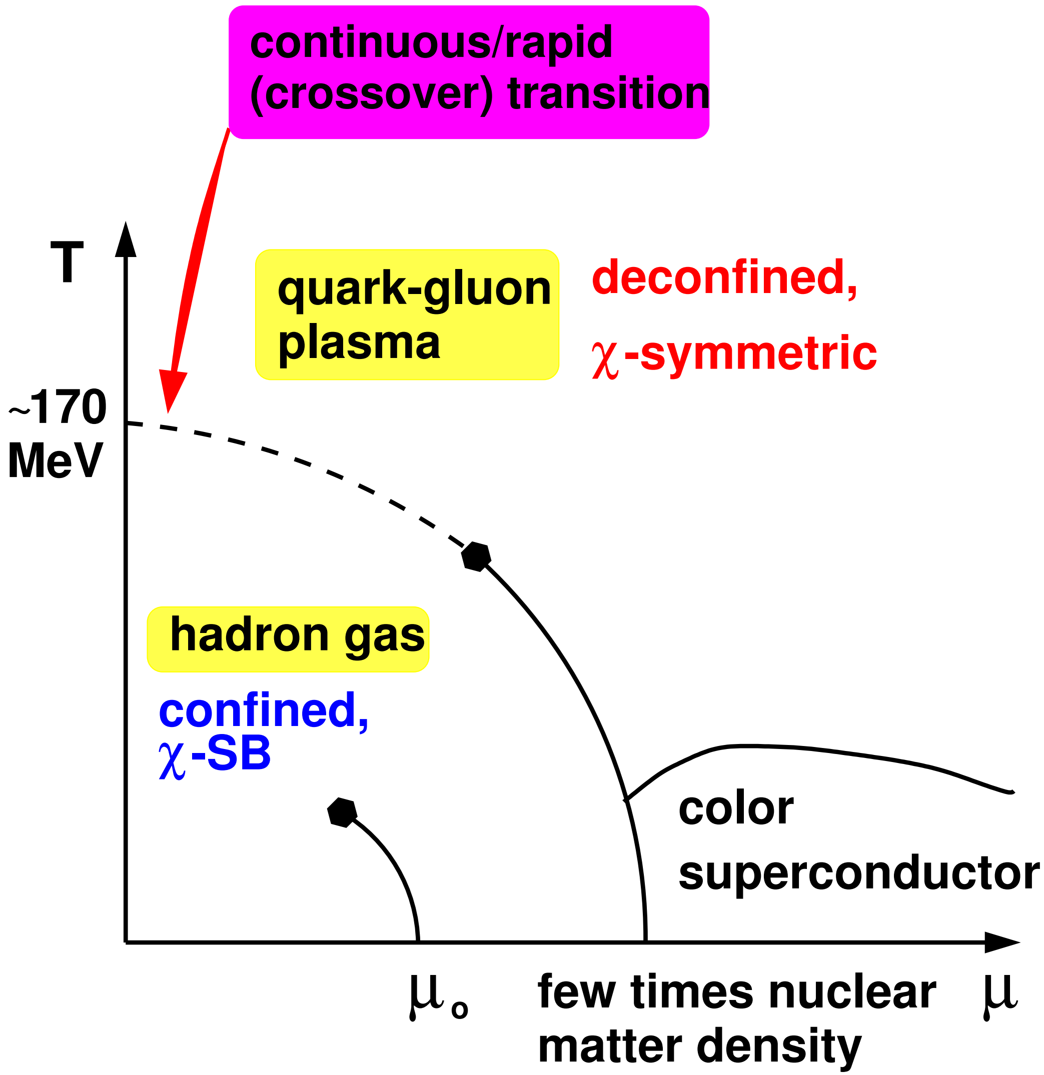

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A. Di Giacomo et al., hep-lat/0503030

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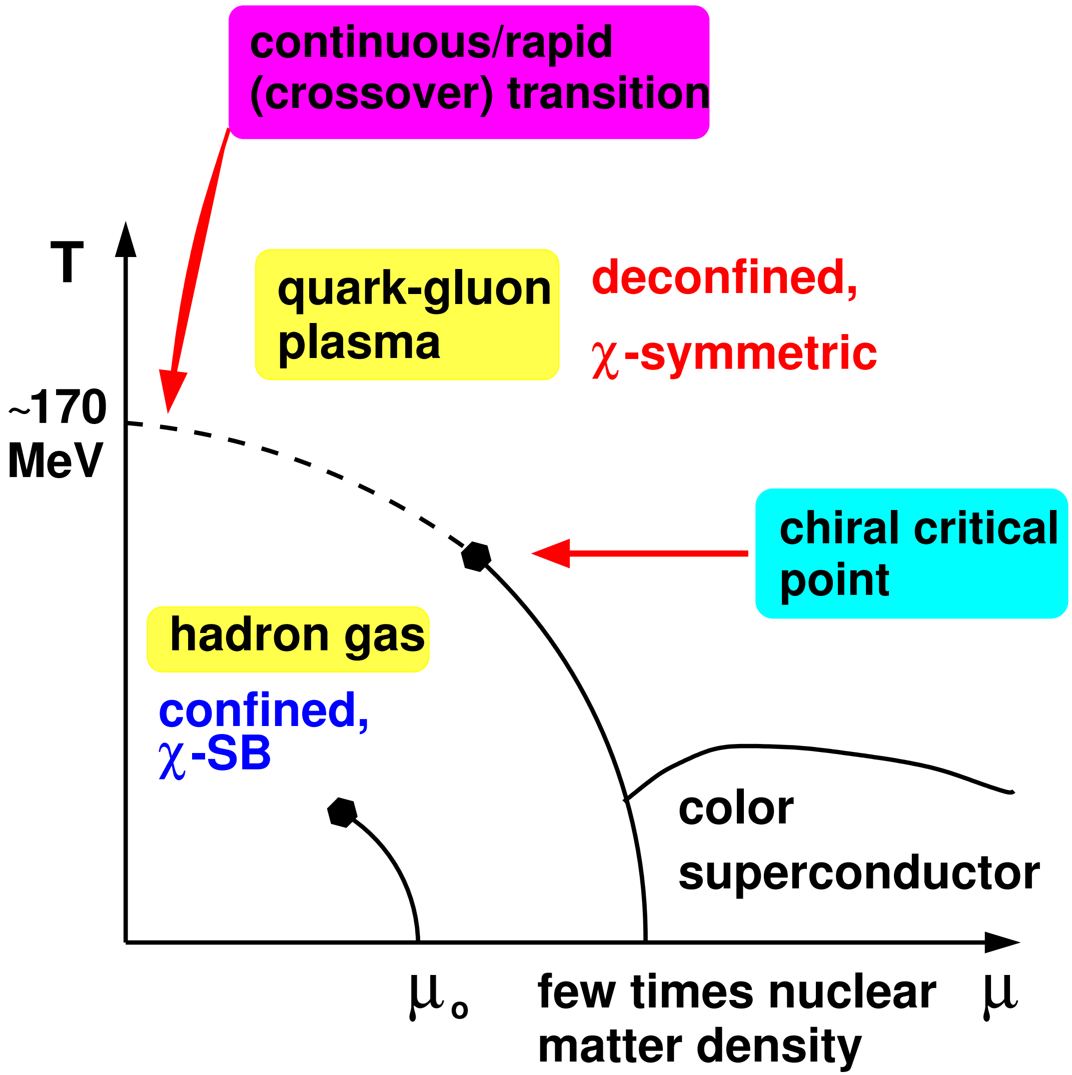

continuous transition for small chemical potential and small quark masses at

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2nd order phase transition; Ising universality class

$T_c(\mu)$ under investigation

location of CCP uncertain:
volume and quark mass dependence

recent doubts on order of transition
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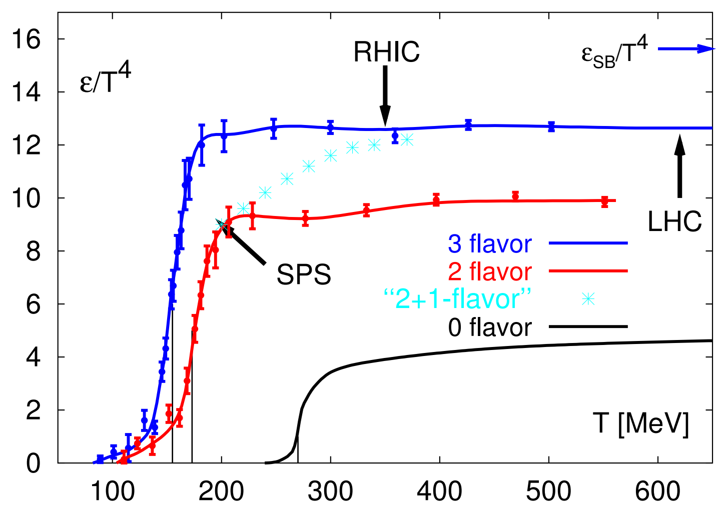
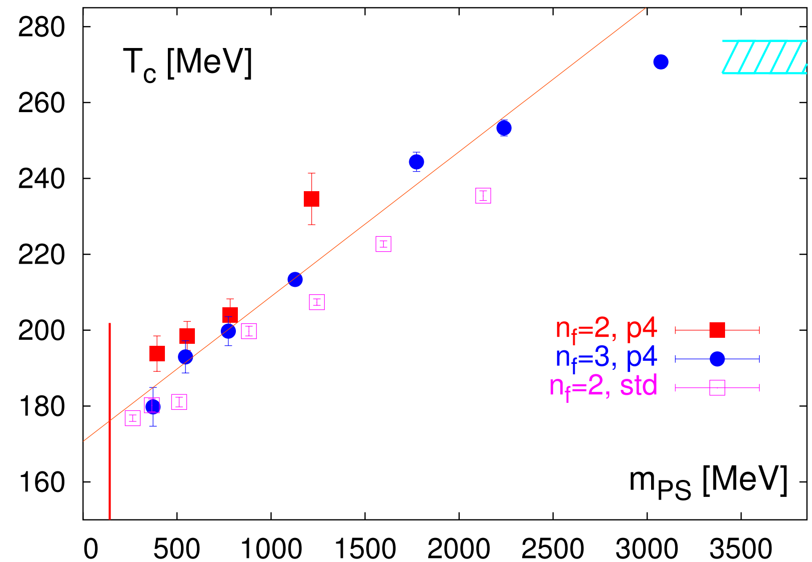
$\mu = 0$: Equation of State and T_c



QCD EoS



transition temperature

- ϵ/T^4 for $m_\pi \simeq$ 770 MeV;
  $(m_\pi/m_\rho \simeq 0.7, TV^{1/3} = 4)$
  $\epsilon_c/T_c^4 = 6 \pm 2$ $\Rightarrow$
- $T_c = (173 \pm 8 \pm sys)$ MeV
  (T_c for $m_\pi \gtrsim$ 300 MeV)
  $\epsilon_c = (0.3 - 1.3)$GeV/fm^3

- improved staggered fermions but still on rather coarse lattices:
  $N_\tau = 4$, i.e. $a^{-1} \simeq 0.8$ GeV

FK, E. Laermann, A. Peikert, Nucl. Phys. B605 (2001) 579

Critical temperature, equation of state and the resonance gas

$\mathbf{Hagedorn\,spectrum:}\quad \rho(m_H) \sim c\, m_H^a\, \mathrm{e}^{m_H/T_H}$

$$\ln Z(T,\mu_B) \;=\; \int \mathrm{d}m_H\; \rho(m_H)\; \ln Z_{m_H}(T,\mu_B)$$

- $\int \;\Rightarrow\; \sum \;\sim$ 1000 exp. known resonance d.o.f.

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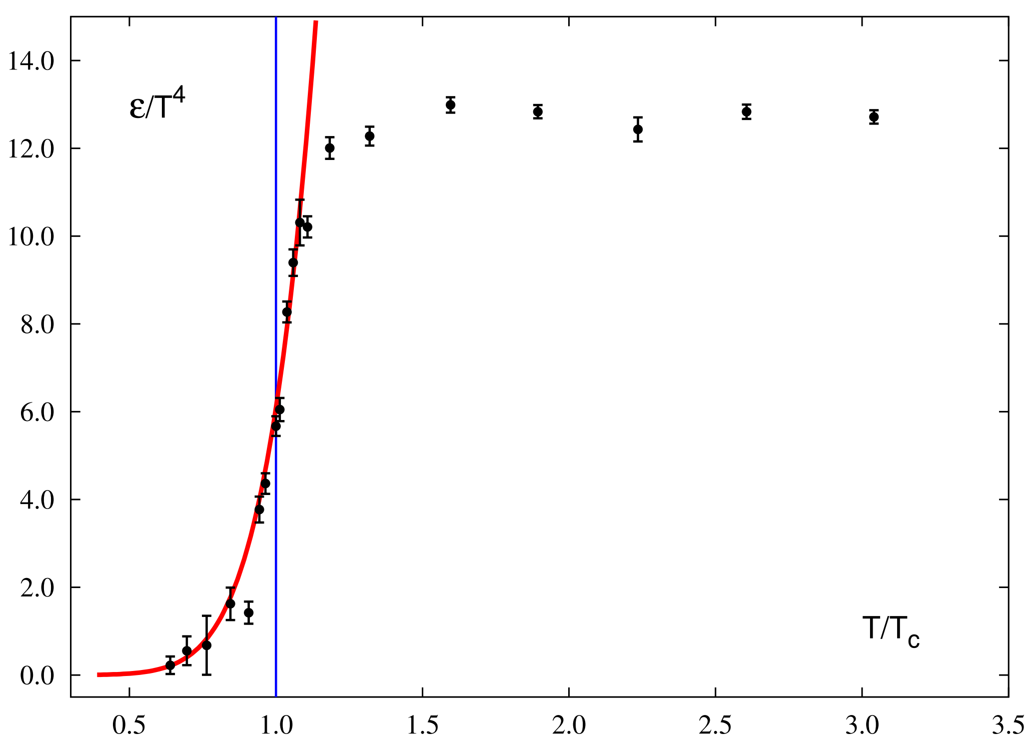

resonance gas:

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vs.

lattice calculation:

(2+1)-flavor QCD, $m_q/T = 0.4$

resonances give large contribution at T_c

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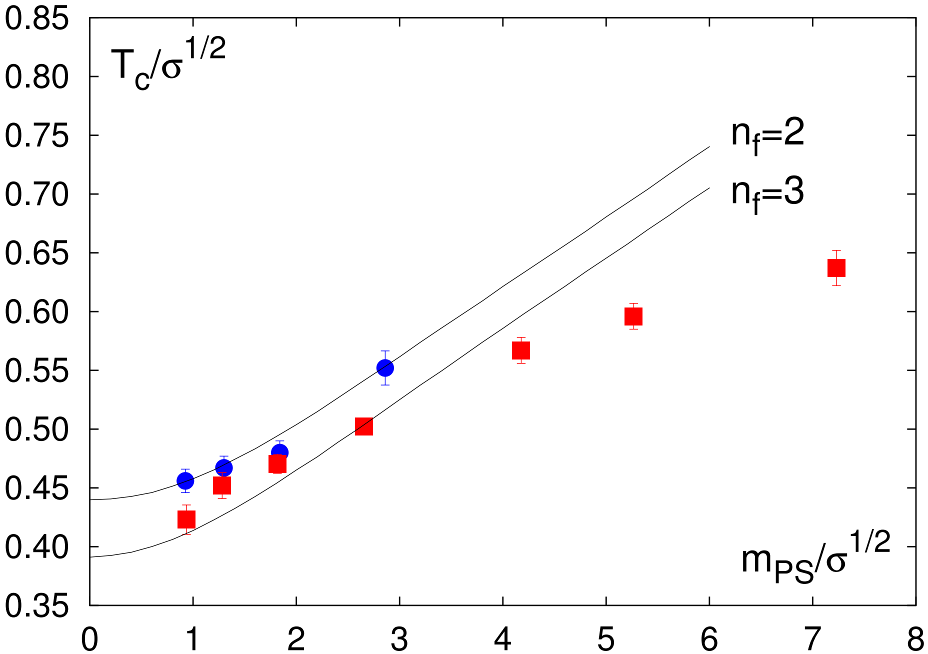

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and explain quark mass dependence of T_c

FK, K. Redlich, A. Tawfik, hep-ph/0303108

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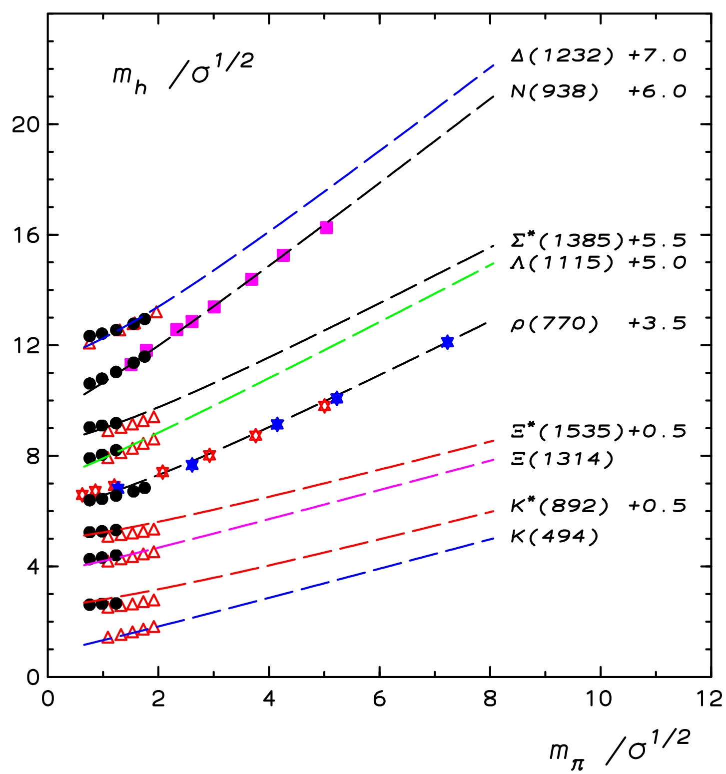
$\Uparrow$



hadron	$m_q \sim 0$	$m_q \to \infty$
pion rho	$m_\pi \sim \sqrt{m_q}$ $m_\rho \sim 770\ \mathrm{MeV}\ +\ c_\rho m_q$	$m_\pi \sim 2m_q$ $m_\rho \sim 2m_q$
...higher meson resonances...		
nucleon	$m_N \sim 940\ \mathrm{MeV}\ +\ c_N m_q$	$m_N \sim 3m_q$
...higher baryon resonances...		

adjust hadron spectrum to conditions realized on the lattice

Critical temperature, equation of state and the resonance gas

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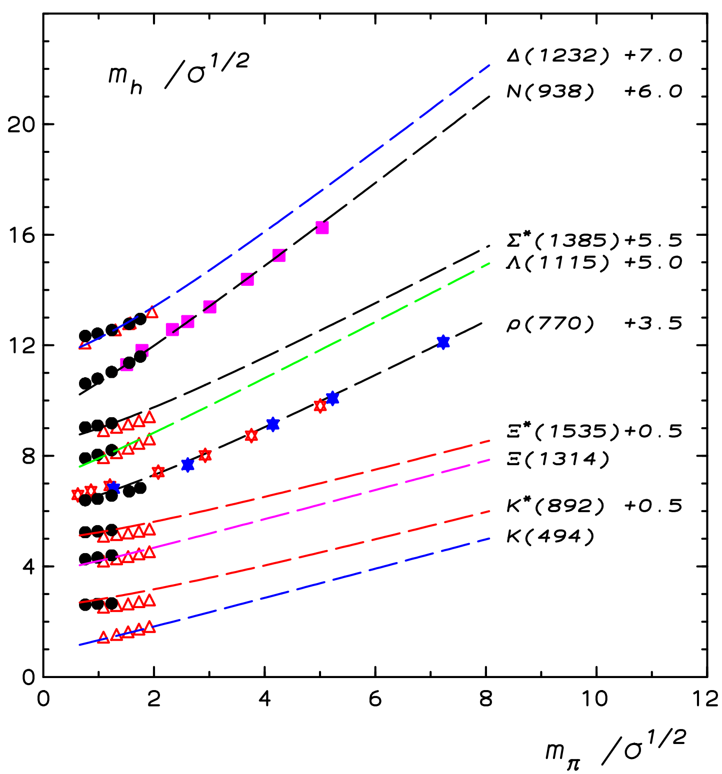

Future:

need lattice calculations with realistic quark masses in order to

- perform more direct comparisons
- check (un)importance of light pions
- finally determine T_c and ϵ_c

recent results on T_c

attempt to extrapolate to chiral and continuum limit:

$$\Lambda T_c = c_0 (m_\pi / m_\rho)^d + c_2 (a T_c)^2$$

C. Bernard et al., Phys. Rev. D71 (2005) 034504

$\mathcal{O}(a^2)$ improved staggered fermions, (2+1)-flavor, $N_\tau = 6$, $TV^{1/3} = 2$
$m_\pi / m_\rho \gtrsim 0.3$, $\Lambda \equiv r_1 = 0.317(7)$fm,

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$$\Rightarrow \quad T_c = 169(12)(4) \;\; (d = 2/\beta\delta = 1.08)$$
$$T_c = 174(11)(4) \;\; (d = 2)$$

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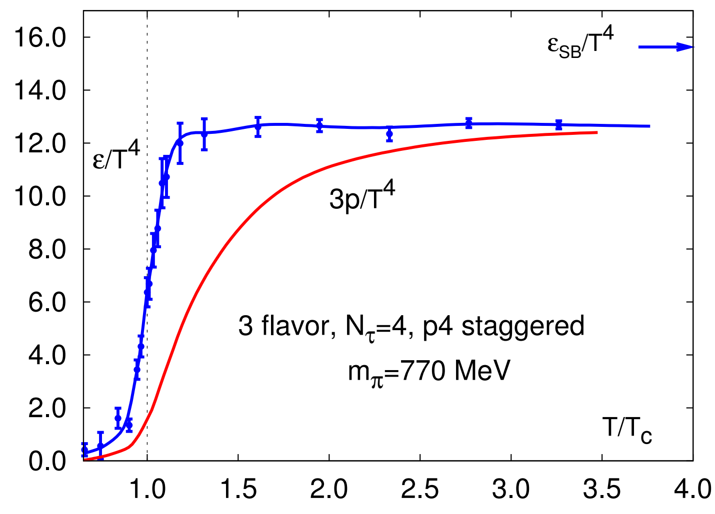
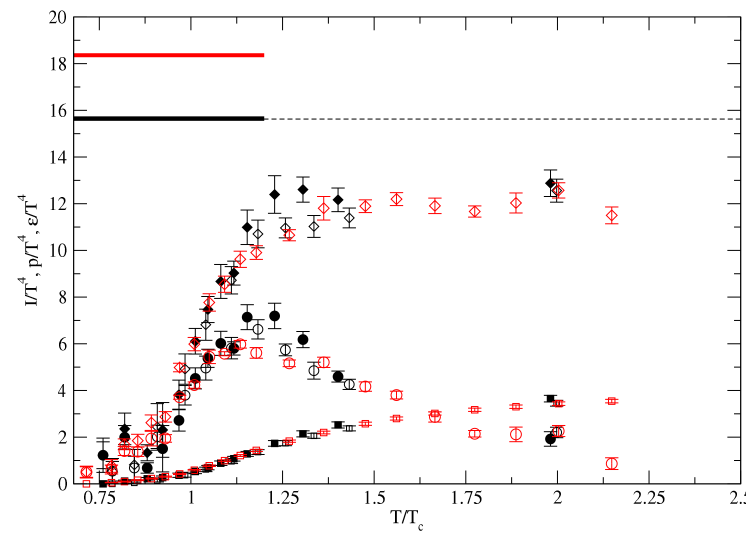
$$T_c = 173(3) \;\; (d = 2)$$

NOTE systematic errors: $\sim 10\%$ in scale setting and chiral extrapolation

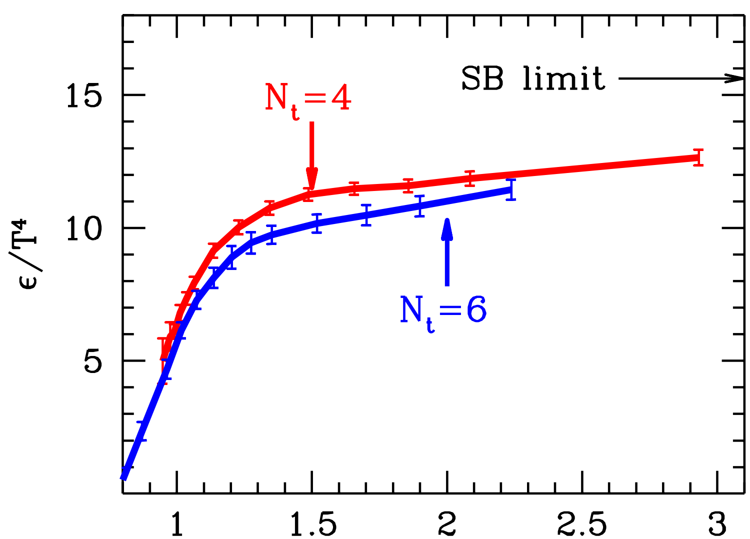
recent results on QCD EoS



old Bielefeld result, 2001
improved staggered (p4), $N_\tau = 4$
3-flavor, $m_\pi \simeq 770$ MeV



MILC-collaboration, hep-lat/0509053
$\mathcal{O}(a^2)$ improved staggered, $N_\tau = 4,\ 6$
(2+1)-flavor, $m_\pi \gtrsim 250$ MeV



$\epsilon_c/T_c^4 \simeq 6$ insensitive to m_π and a^{-1}
HOWEVER: thermodynamic limit?? $TV^{1/3} \simeq 2$
cut-off effects?? $\Rightarrow$ improved actions

Y. Aoki et al., hep-lat/0510084
standard staggered, $N_\tau = 4,\ 6$
(2+1)-flavor, $m_\pi \rightarrow 140$ MeV (extrap.)
ϵ/T^4 rescaled with $(\epsilon_{SB}/T^4)(N_\tau)$

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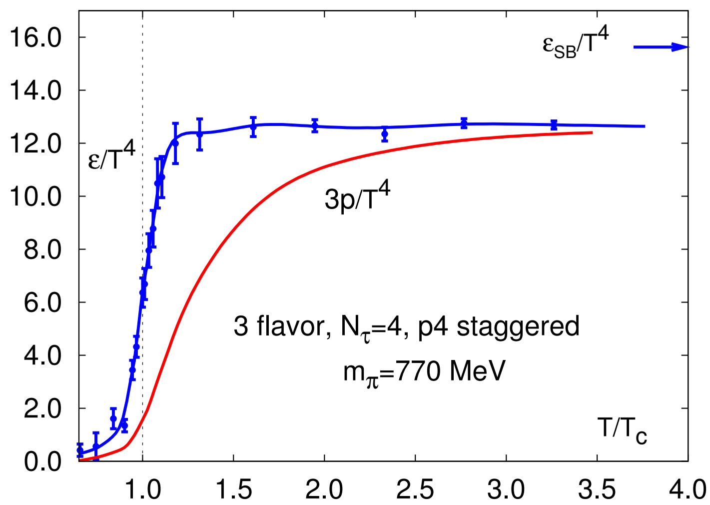

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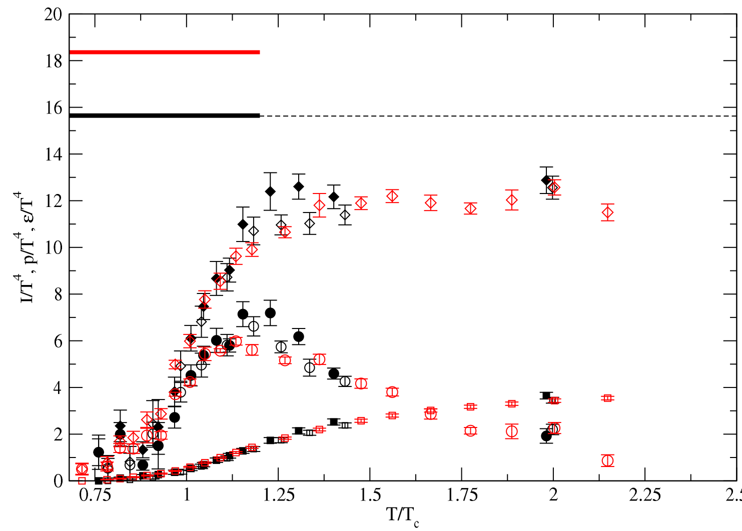

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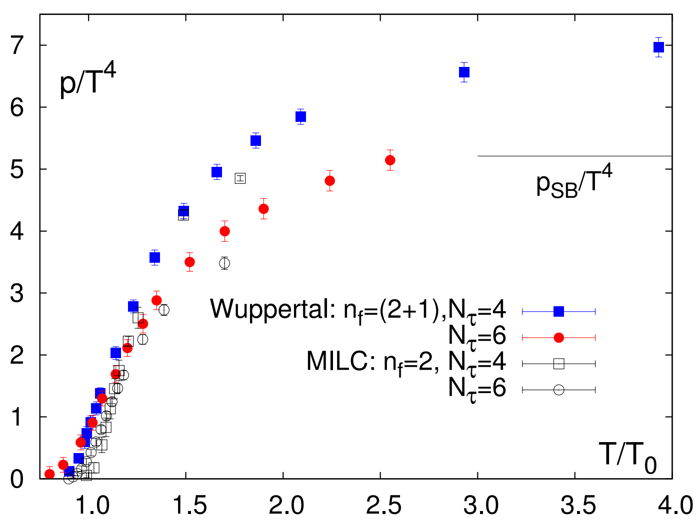

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Bulk thermodynamics with non-vanishing chemical potential

$$Z(V, T, \mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E(V,T,\mu)}$$
$$= \int \mathcal{D}\mathcal{A}\ \left[det\ M(\mu)\right]^f\ \mathrm{e}^{-S_G(V,T)}$$

⇑complex fermion determinant;

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ways to circumvent this problem:

- reweighting: works well on small lattices; requires exact evaluation of $detM$
  Z. Fodor, S.D. Katz, JHEP 0203 (2002) 014
- Taylor expansion around $\mu = 0$: works well for small μ;
  C. R. Allton et al. (Bielefeld-Swansea), Phys. Rev. D66 (2002) 074507
  R.V. Gavai, S. Gupta, Phys. Rev. D68 (2003) 034506
- imaginary chemical potential: works well for small μ; requires analytic continuation
  Ph. deForcrand, O. Philipsen, Nucl. Phys. B642 (2002) 290
  M. D'Elia and M.P. Lombardo, Phys. Rev. D64 (2003) 014505

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$\Uparrow$ complex fermion determinant;

recent progress;

- reweighting: larger lattices; smaller quark mass;
  Z. Fodor, S.D. Katz, JHEP 0404 (2004) 050
- Taylor expansion: higher orders; larger volumes;
  C. R. Allton et al., Phys. Rev. D71 (2005) 054508
  R.V. Gavai, S. Gupta, Phys. Rev. D71 (2005) 114014
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searches for the CCP:

μ_B sensitive to V (and m_q)
$\mu_B \sim 360$ MeV

no clear-cut evidence

$\mu_B \sim 180$ MeV
(might be ~ 230 MeV)?

no evidence

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still an open question

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⇑complex fermion determinant;

⇓Taylor expansion;

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu)$$

$$\equiv \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n$$

$$= c_0 + c_2 \left(\frac{\mu}{T}\right)^2 + c_4 \left(\frac{\mu}{T}\right)^4 + \mathcal{O}((\mu/T)^6)$$

$$\mu = 0 \quad \Rightarrow \quad \frac{p}{T^4} \equiv c_0(T)$$

High-T limit and Taylor expansion at non-vanishing chemical potential

- p/T^4 is a polynomial in μ_q/T (good starting point for a Taylor expansion) (notation: $\mu \equiv \mu_q \equiv \mu_B/3$)

Infinite temperature limit $\Leftrightarrow$ ideal gas $\qquad p/T^4 = (VT^3)^{-1} \ln Z(V,T,\mu_q)$

$$\frac{p}{T^4}\Big|_\infty = \frac{n_\mathrm{f}}{2\pi T^3}\left(\int_0^\infty dk\, k^2 \ln\left(1 + z \exp\{-\varepsilon(k)/T\}\right)\right.$$
$$\left. + \int_0^\infty dk\, k^2 \ln\left(1 + z^{-1} \exp\{-\varepsilon(k)/T\}\right)\right) \quad , \quad z = \exp(\mu_q/T)$$

holds true in QCD at $\mathcal{O}(g^2)$

$$\frac{p}{T^4}\Big|_\infty = n_f\left(\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_q}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_q}{T}\right)^4\right)$$
$$-g^2\,\frac{n_f}{2\pi^2}\left(\frac{5\pi^2}{36} + \frac{1}{2}\left(\frac{\mu_q}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_q}{T}\right)^4\right)$$

...some remarks on higher order perturbation theory

- all perturbatively calculable contributions (up to $\mathcal{O}(g^6 \ln 1/g)$) have been calculated also for $\mu > 0$;

  A. Vuorinen, Phys.Rev. D68 (2003) 054017

- electric mass contributes in $\mathcal{O}(g^3)$; also shows up in Taylor expansion

$$\Omega^{(3)}(T,\mu) = \frac{1}{6\pi}\left(\frac{m_E(T,\mu^2)}{gT}\right)^3 = \frac{1}{6\pi}\left(1 + \frac{n_f}{6} + \frac{1}{2\pi^2}\mu^2\right)^{3/2}$$

$$= \frac{1}{6\pi}\left(\frac{m_E(T,0)}{gT}\right)^3 \left(1 + \frac{9}{(12+2n_f)\pi^2}\mu^2 + \mathcal{O}(\mu^4)\right)$$

- first contribution to $\mathcal{O}(\mu^6)$ is $\mathcal{O}(g^3)$ and arises from expansion of electric mass

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- electric mass contributes in $\mathcal{O}(g^3)$; also shows up in Taylor expansion;

- leading contributions in higher orders are all $\mathcal{O}(g^3)$, arise from an expansion of electric mass and thus oscillate in sign

Bulk thermodynamics for small μ_q/T on $16^3 \times 4$ lattices

- Taylor expansion of pressure up to $\mathcal{O}\left((\mu_q/T)^6\right)$

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln \mathcal{Z} = \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu_q}{T}\right)^n \simeq c_0 + c_2\left(\frac{\mu_q}{T}\right)^2 + c_4\left(\frac{\mu_q}{T}\right)^4 + c_6\left(\frac{\mu_q}{T}\right)^6$$

quark number density $$\frac{n_q}{T^3} = 2c_2\frac{\mu_q}{T} + 4c_4\left(\frac{\mu_q}{T}\right)^3 + 6c_6\left(\frac{\mu_q}{T}\right)^5$$

quark number susceptibility $$\frac{\chi_q}{T^2} = 2c_2 + 12c_4\left(\frac{\mu_q}{T}\right)^2 + 30c_6\left(\frac{\mu_q}{T}\right)^4$$

an estimator for the radius of convergence

$$\left(\frac{\mu_q}{T}\right)_{crit} = \lim_{n\to\infty}\left|\frac{c_{2n}}{c_{2n+2}}\right|^{1/2}$$

$c_n > 0$ for all n;
singularity for real μ

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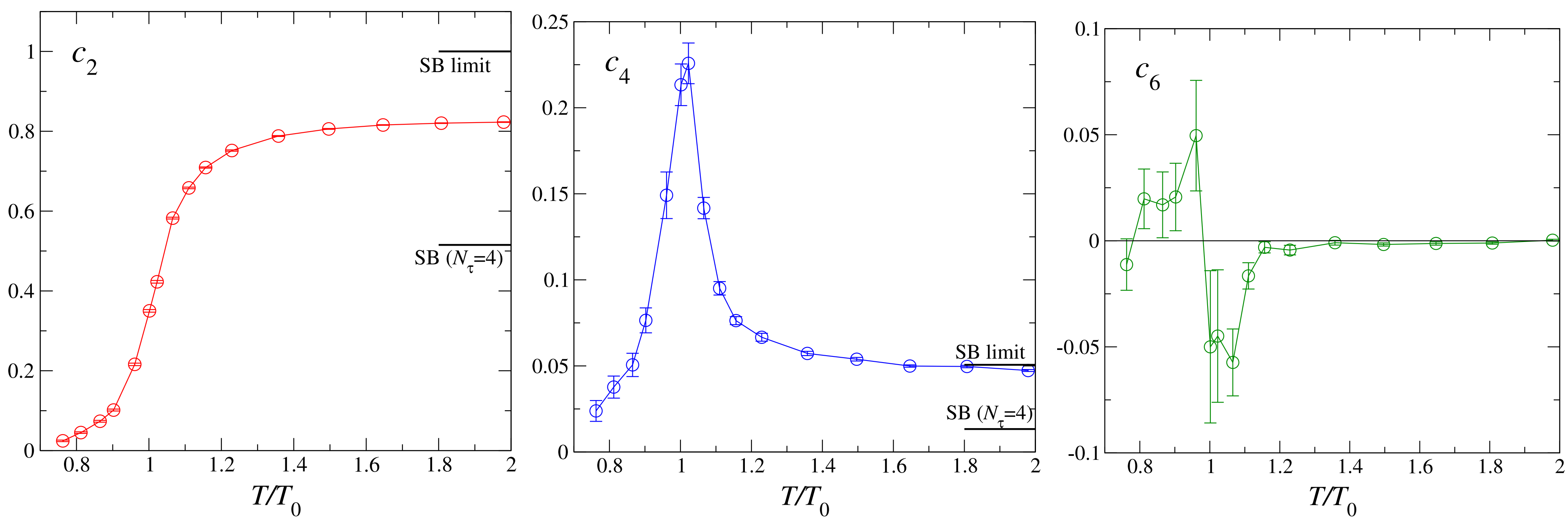

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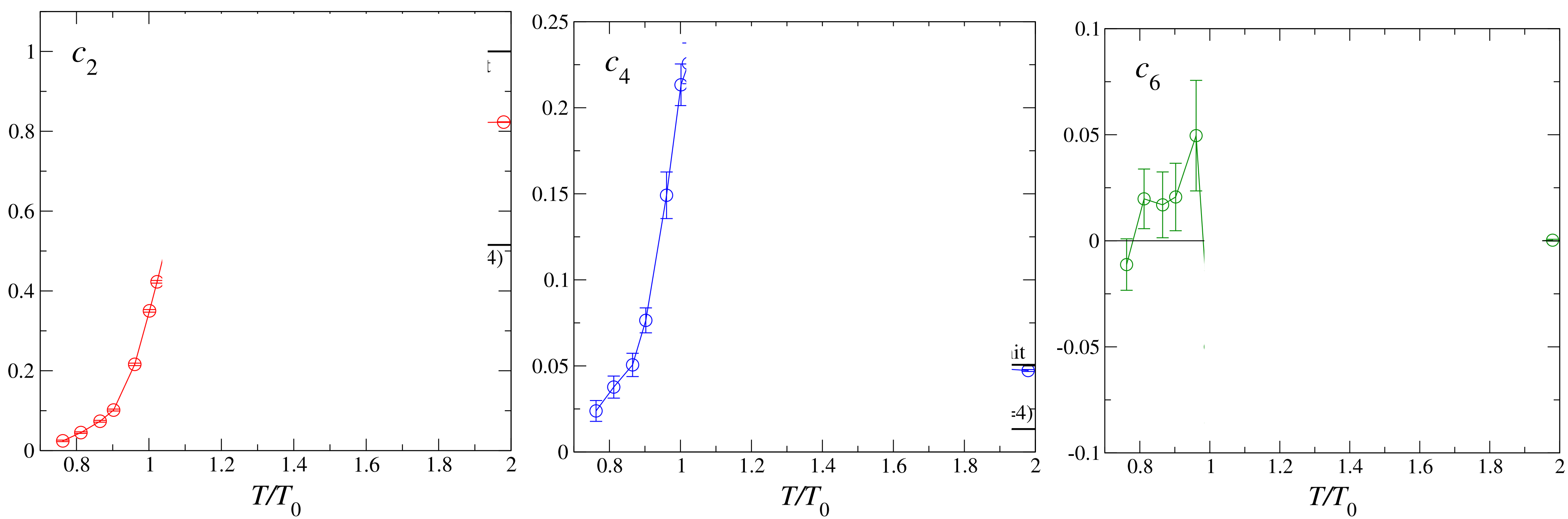

$c_n > 0$ for all n and $T \lesssim 0.95\ T_c$ $\Leftrightarrow$ singularity for real μ (positive μ^2)

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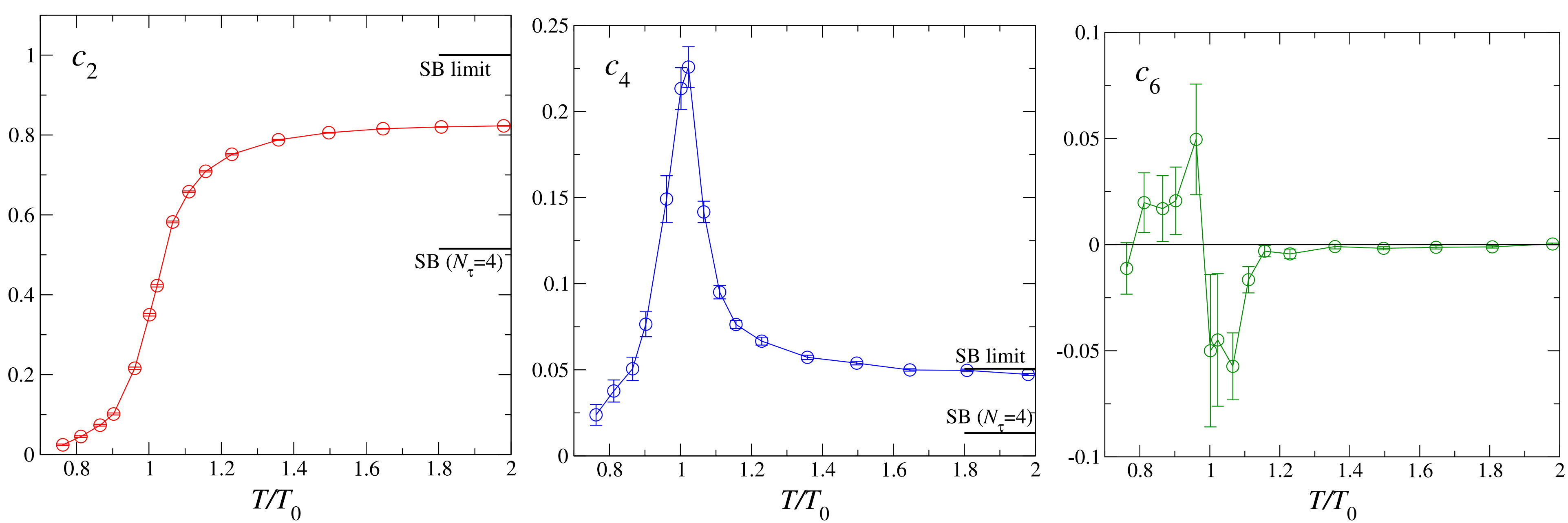
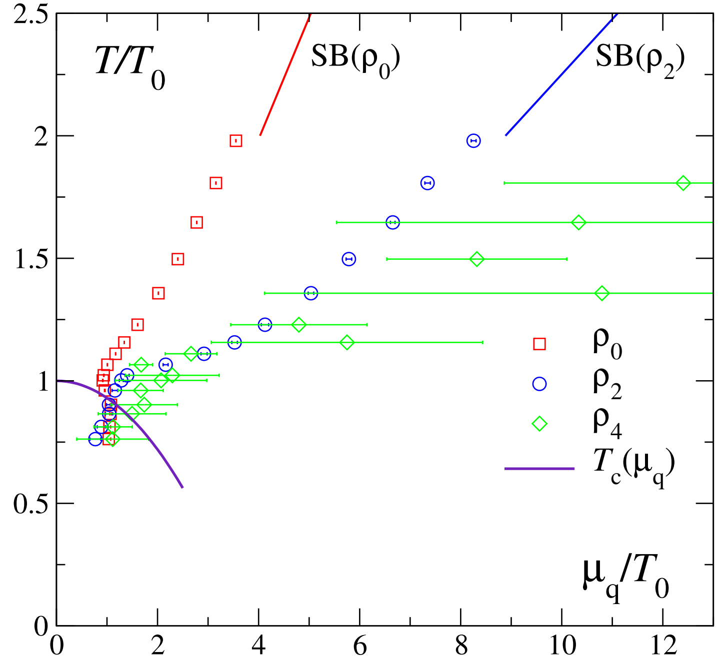
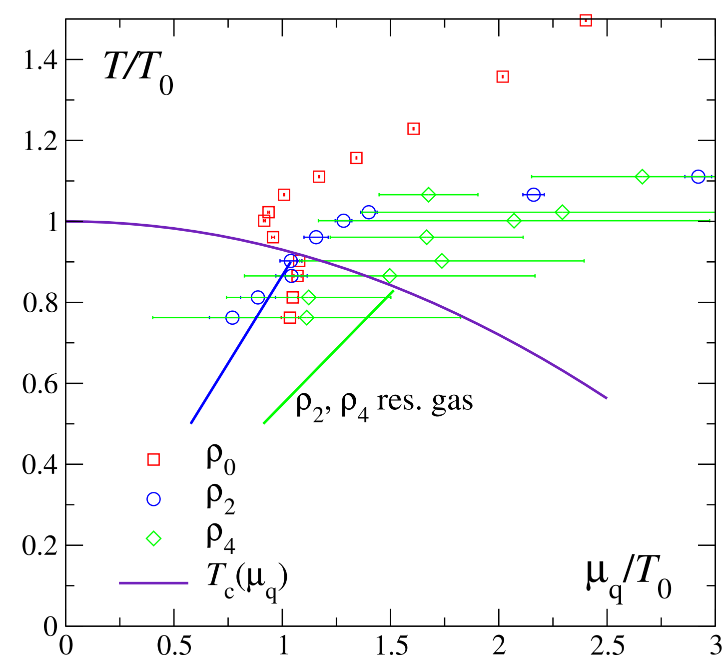


irregular sign of c_n for $T \gtrsim T_c$ $\Leftrightarrow$ singularity in complex plane

Radius of convergence: lattice estimates vs. resonance gas

- Taylor expansion $\Rightarrow$ estimates for radius of convergence $\rho_{2n} = \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}$

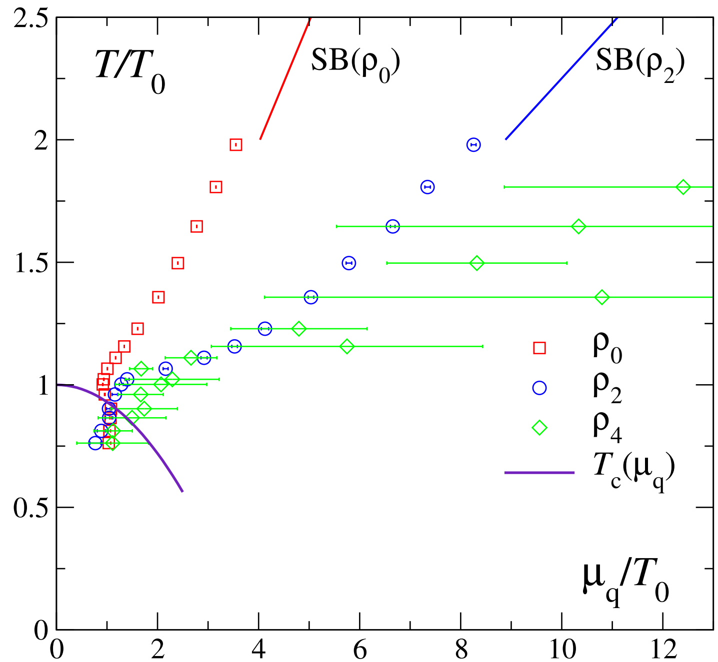


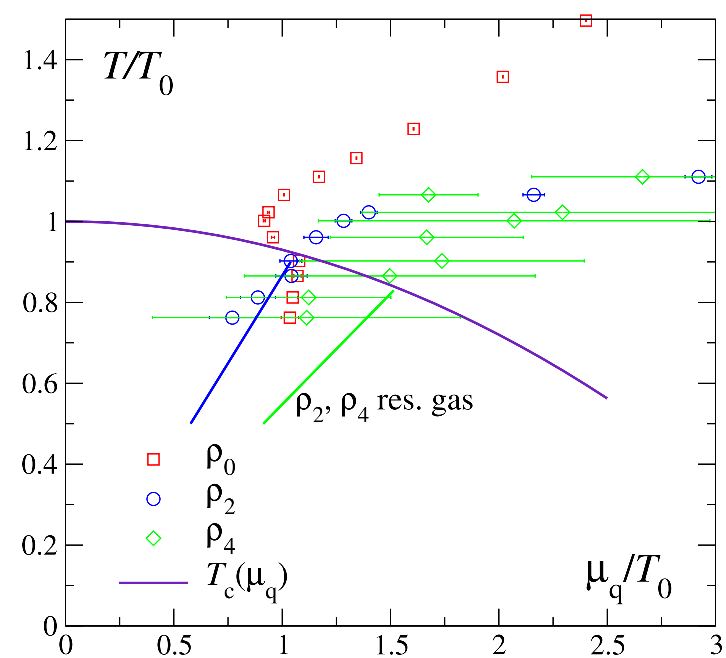


$T < T_0$: $\rho_n \simeq 1.0$ for all n $\Rightarrow$ $\mu_B^{crit} \simeq 500$ MeV

Radius of convergence:
lattice estimates vs. resonance gas

- Taylor expansion $\Rightarrow$ estimates for radius of convergence $\rho_{2n} = \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}$





$T < T_0$: $\rho_n \simeq 1.0$ for all n $\Rightarrow$ $\mu_B^{crit} \simeq 500$ MeV

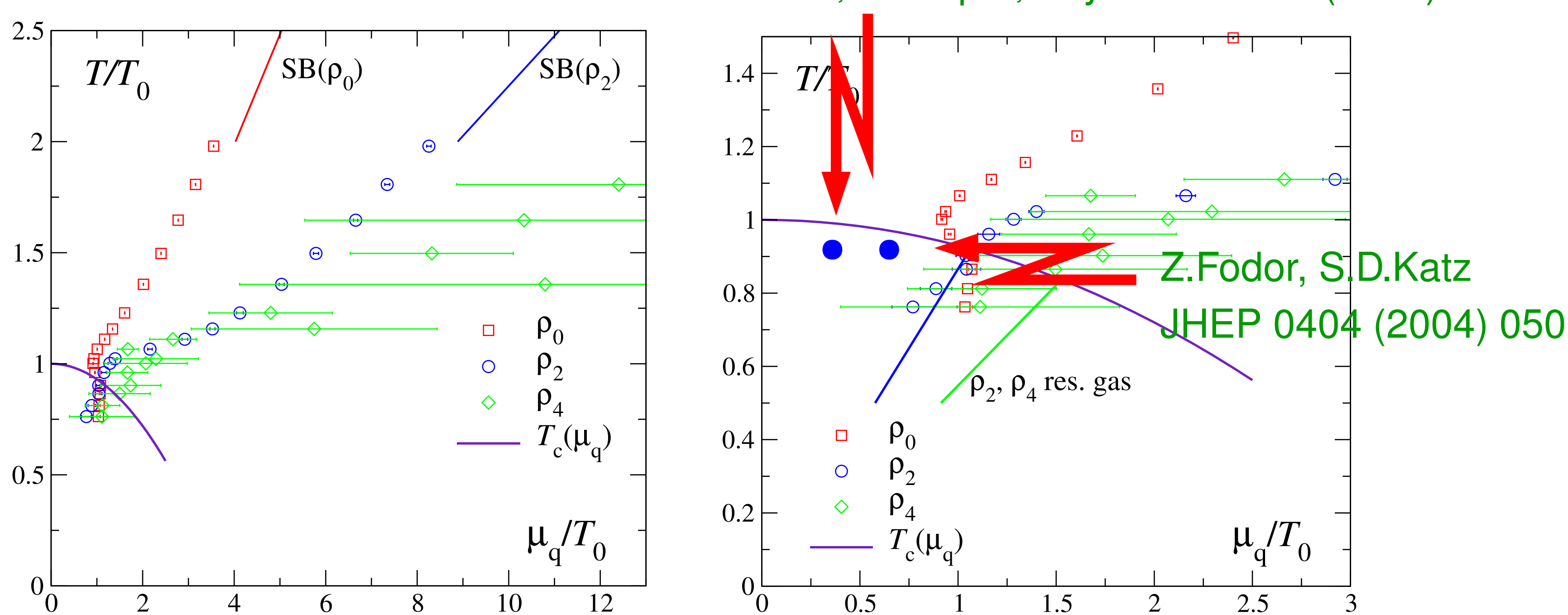
HOWEVER still consistent with resonance gas!!!

HRG analytic, LGT consistent with HRG $\Rightarrow$ infinite radius of convergence not yet ruled out

Radius of convergence: lattice estimates vs. resonance gas

- Taylor expansion $\Rightarrow$ estimates for radius of convergence $\rho_{2n} = \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}$

R.V. Gavai, S. Gupta, Phys. Rev. D71 (2005) 114014



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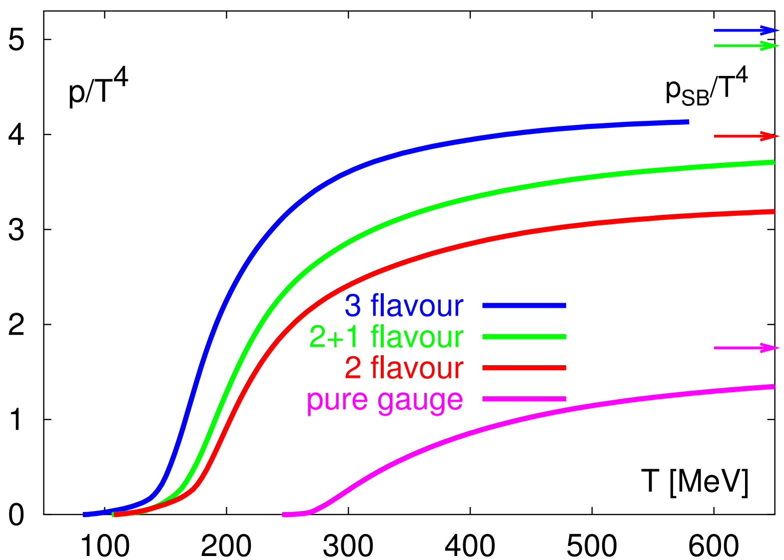
The pressure for $\mu_q/T > 0$

C.R. Allton et al. (Bielefeld-Swansea), PRD68 (2003) 014507

$\mu_q = 0, \quad 16^3 \times 4$ lattice

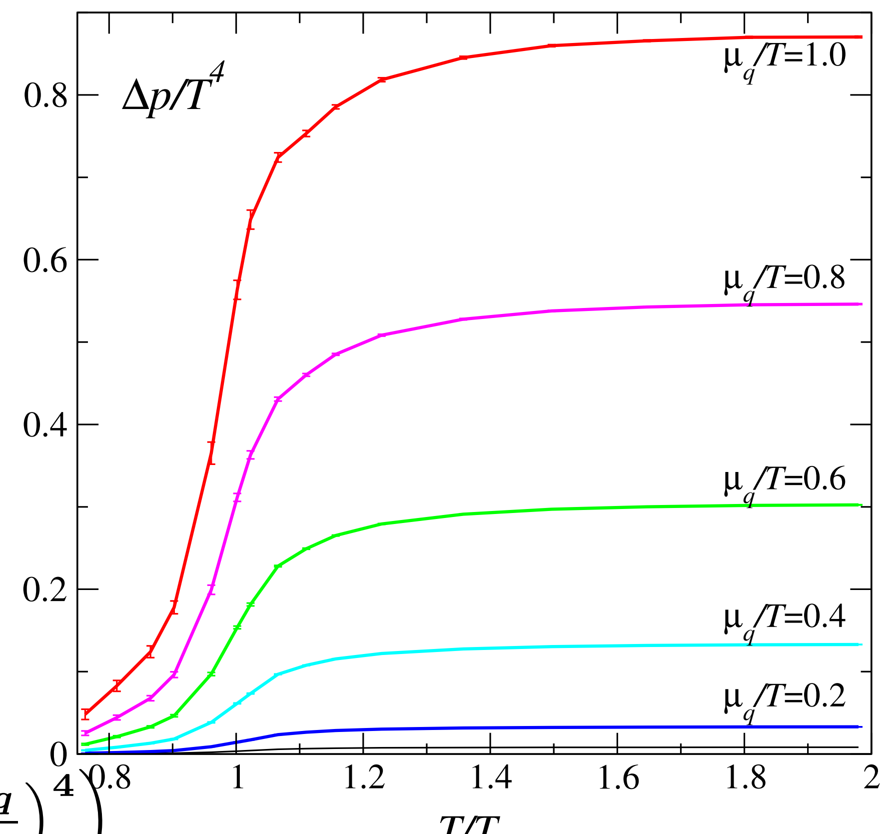
improved staggered fermions;

$n_f = 2,\ m_\pi \simeq 770\ MeV$



contribution from $\mu_q/T > 0$

Taylor expansion, $\mathcal{O}((\mu/T)^4)$



high-T, ideal gas limit

$$\frac{p}{T^4}\Big|_\infty = n_f\left(\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_q}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_q}{T}\right)^4\right)$$

similar F. Csikor et al., JHEP 0311 (2003) 070

SPS: $\mu_q/T \lesssim 0.6$ RHIC: $\mu_q/T \lesssim 0.1$

The pressure for $\mu_q/T > 0$

C.R. Allton et al. (Bielefeld-Swansea), PRD68 (2003) 014507

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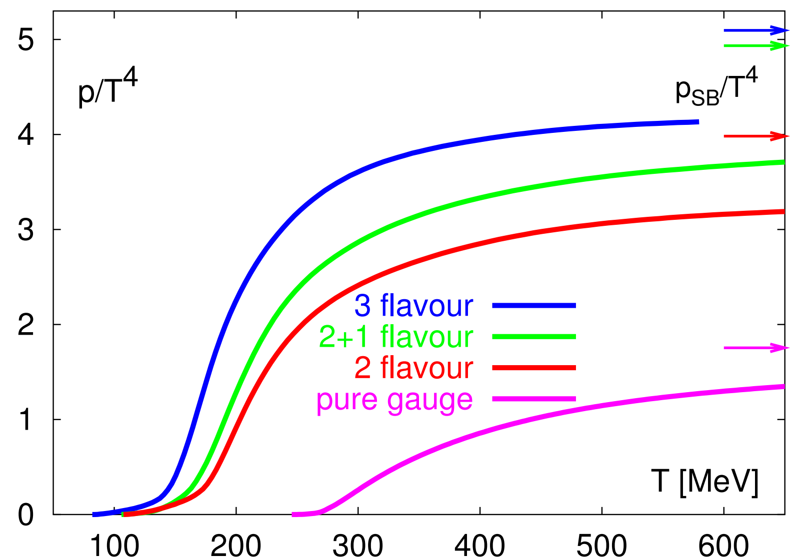
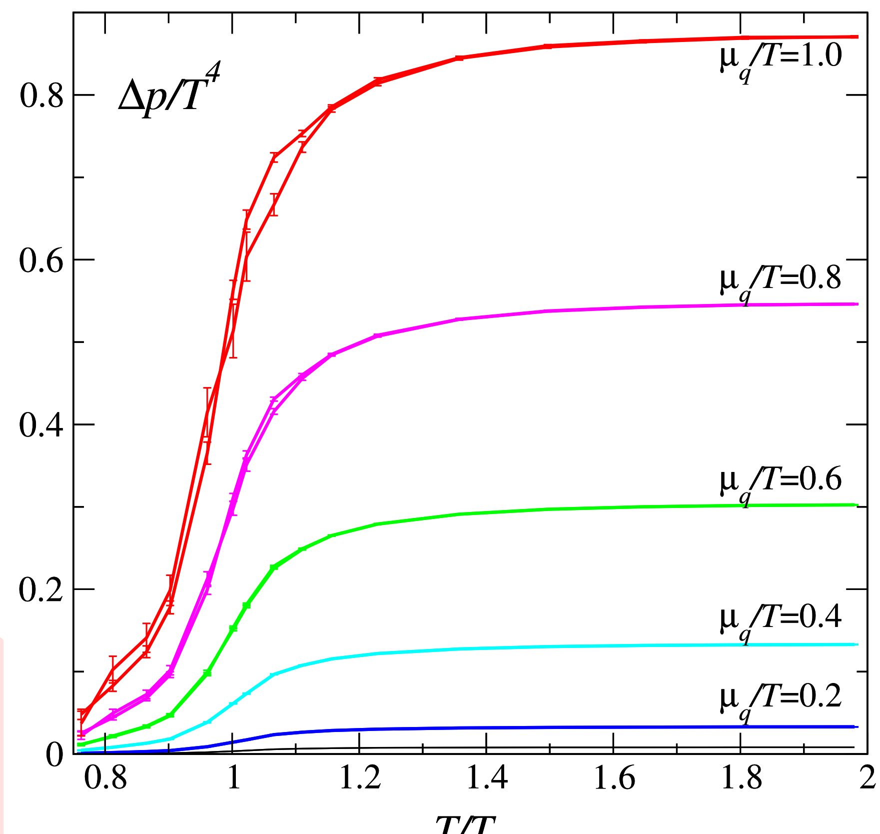


PRD71 (2005) 054508

contribution from $\mu_q/T > 0$

NEW: Taylor expansion, $\mathcal{O}((\mu/T)^6)$



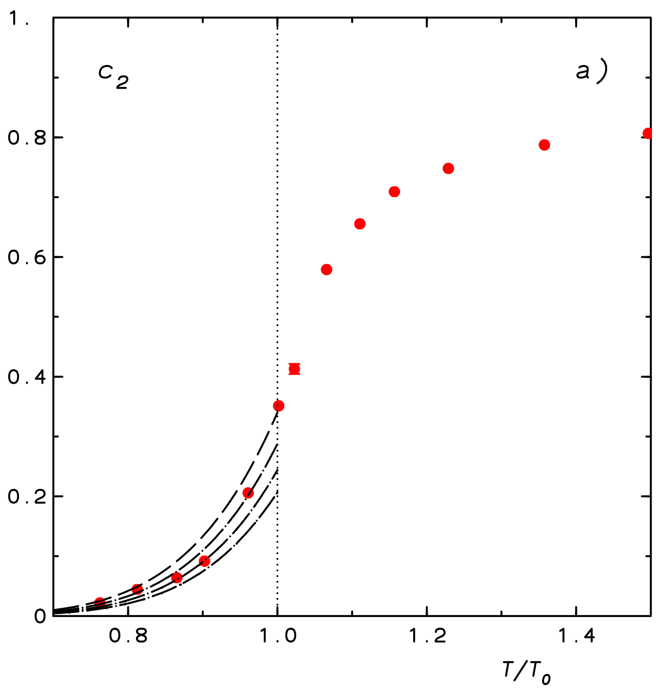
pattern for $\mu_q = 0$ and $\mu_q > 0$ similar;
quite large contribution in hadronic phase;
$\mathcal{O}((\mu/T)^6)$ correction small for $\mu_q/T \lesssim 1$

SPS: $\mu_q/T \lesssim 0.6$ RHIC: $\mu_q/T \lesssim 0.1$

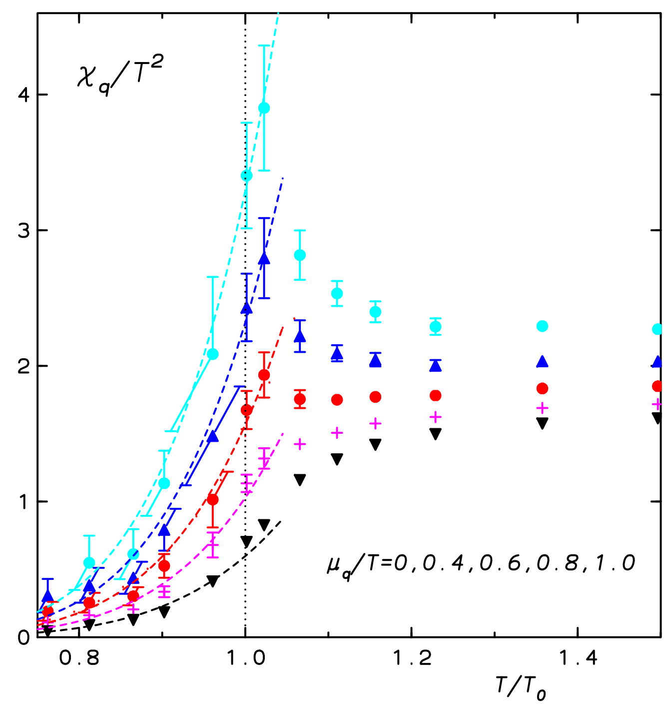
Resonance gas:
spectrum dependent consequences

- "fit" with modified spectrum $m_H(m_\pi) = m_H(0) + A \left(\frac{m_\pi}{m_H(0)}\right)^2$
  $\Rightarrow$ tests factorization

$$\frac{\chi_q}{T^2} = 9F(T)\cosh(3\mu_q/T) \sim c_2(T)\left(2 + 12\,\frac{c_4}{c_2}\left(\frac{\mu_q}{T}\right)^2 + \mathcal{O}\left(\left(\frac{\mu_q}{T}\right)^4\right)\right)$$



$A = 0.9,\ 1.0,\ 1.1,\ 1.2$



$A = 1.0$

Energy and Entropy density for $\mu_q > 0$

S. Ejiri, F. Karsch, E. Laermann and C. Schmidt, in preparation

Thermodynamics: (NB: continuum $\hat{m} \equiv m_q$
lattice $\hat{m} \equiv m_q a$, implicit T-dependence)

- pressure $$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(T, \mu_q) = \sum_{n=0}^{\infty} c_n(T, \hat{m}) \left(\frac{\mu_q}{T}\right)^n$$

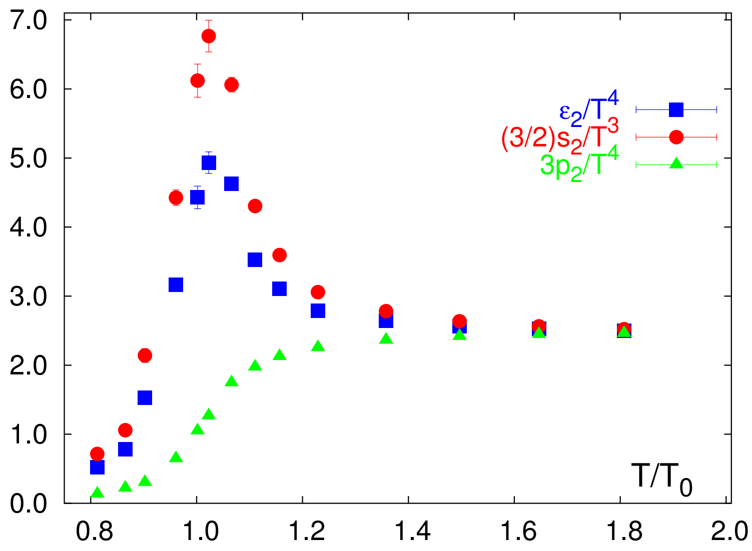
- energy density from ”interaction measure”

$$\frac{\epsilon - 3p}{T^4} = \sum_{n=0}^{\infty} c_n'(T, \hat{m}) \left(\frac{\mu_q}{T}\right)^n \quad , \quad c_n'(T, \hat{m}) \equiv T \frac{\mathrm{d}c_n(T, \hat{m})}{\mathrm{d}T}$$

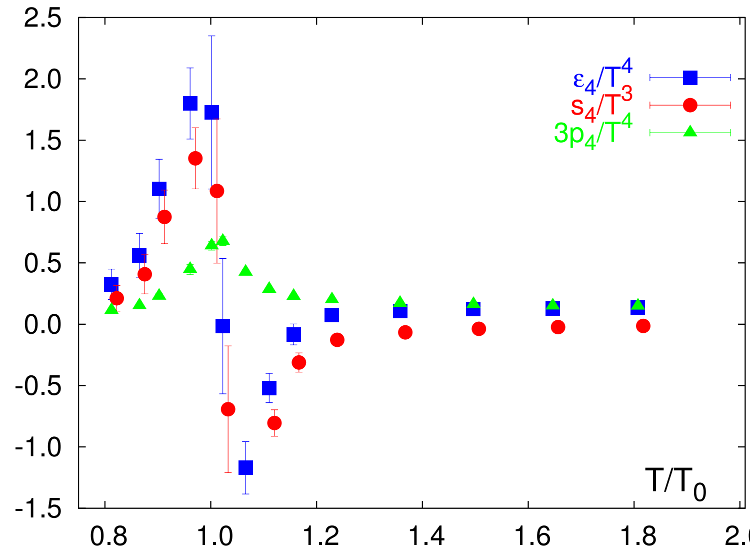
- entropy density

$$\frac{s}{T^3} \equiv \frac{\epsilon + p - \mu_q n_q}{T^4} = \sum_{n=0}^{\infty} \left((4-n)c_n(T, \hat{m}) + c_n'(T, \hat{m})\right) \left(\frac{\mu_q}{T}\right)^n$$

Taylor expansions of ϵ/T^4



$\mathcal{O}((\mu_q/T)^2)$

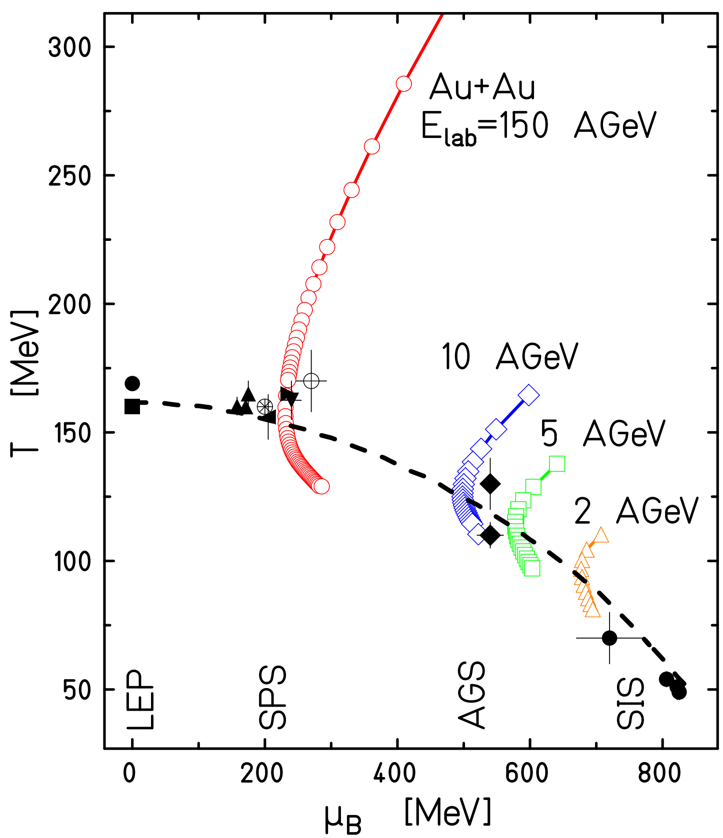


$\mathcal{O}((\mu_q/T)^4)$

- magnitudes similar for all three quantities;
- expansion of ϵ/T^4 and s/T^3 also dominated by $\mathcal{O}((\mu_q/T)^2)$ for $\mu_q/T \lesssim 1$ (except close to T_c)

EoS on HIC trajectories

- dense matter created in a HI-collision expands and cools at fixed entropy and baryon number
  $\Rightarrow$ lines of constant S/N_B in the QCD phase diagram



for example:

isentropic expansion,
”mixed phase model”:
V.D. Toneev, J. Cleymans, E.G. Nikonov,
K. Redlich, A.A. Shanenko,
J. Phys. G27 (2001) 827

EoS on HIC trajectories

- dense matter created in a HI-collision expands and cools at fixed entropy and baryon number
  $\Rightarrow$ lines of constant S/N_B in the QCD phase diagram
  - high T: ideal gas

$$\frac{S}{N_B} = 3\,\frac{\frac{32\pi^2}{45 n_f} + \frac{7\pi^2}{15} + \left(\frac{\mu_q}{T}\right)^2}{\frac{\mu_q}{T} + \frac{1}{\pi^2}\left(\frac{\mu_q}{T}\right)^3}$$

$S/N_B =$ constant $\Leftrightarrow$ μ_q/T constant

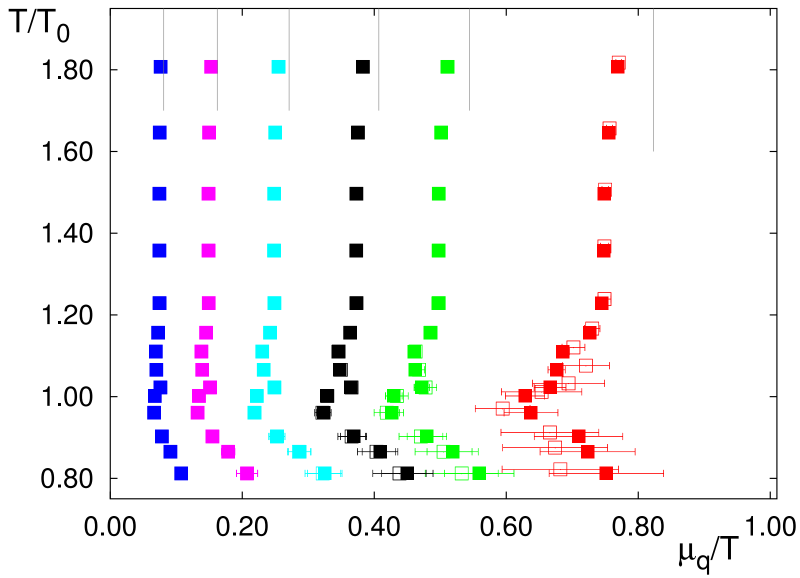
  - low T: nucleon + pion gas

$T \rightarrow 0$: $\quad \mu_q/T \sim c/T$

Lines of constant S/N_B

S/N_B = 300 150 90 60 45 30

S. Ejiri, FK, E. Laermann, C. Schmidt, in prep.



- isentropic trajectories close to ideal gas behavior for $T > T_c$
- trajectories bend towards larger μ_q for $T < T_c$
- $\mathcal{O}(\mu_q^6)$ correction (open sym.) is small for $\mu_q/T \lesssim 0.8$ (despite large errors)

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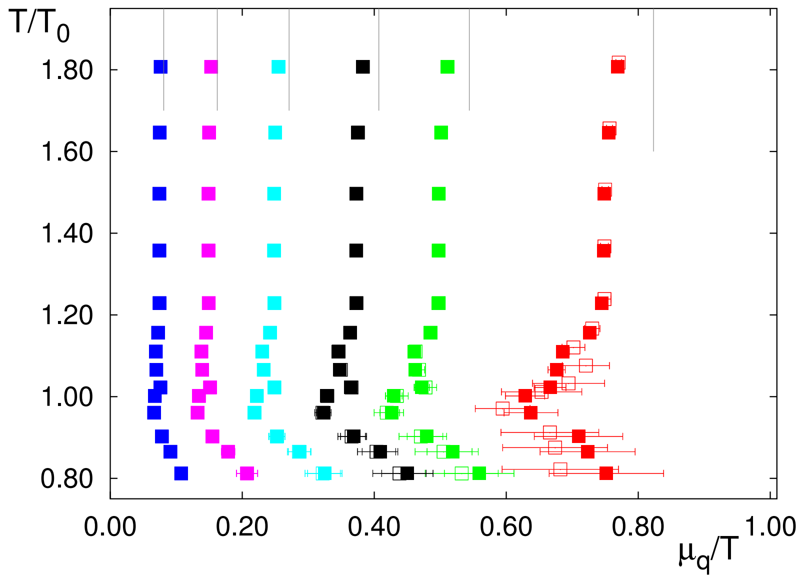

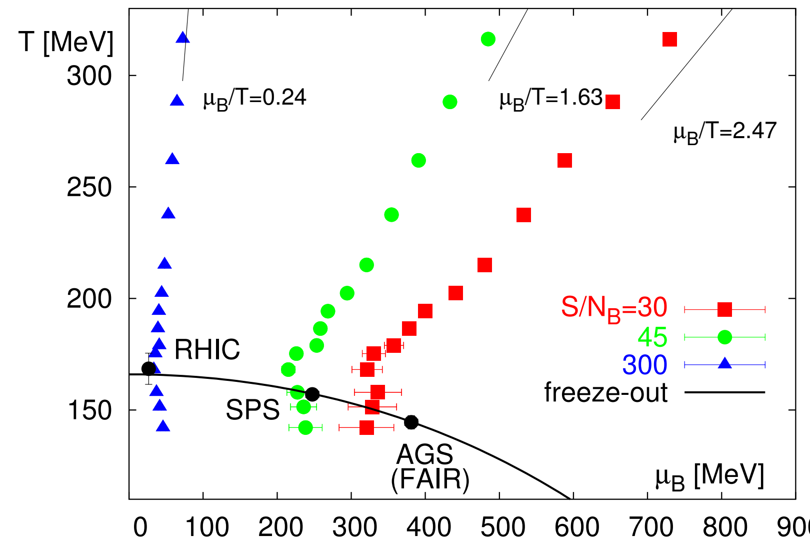

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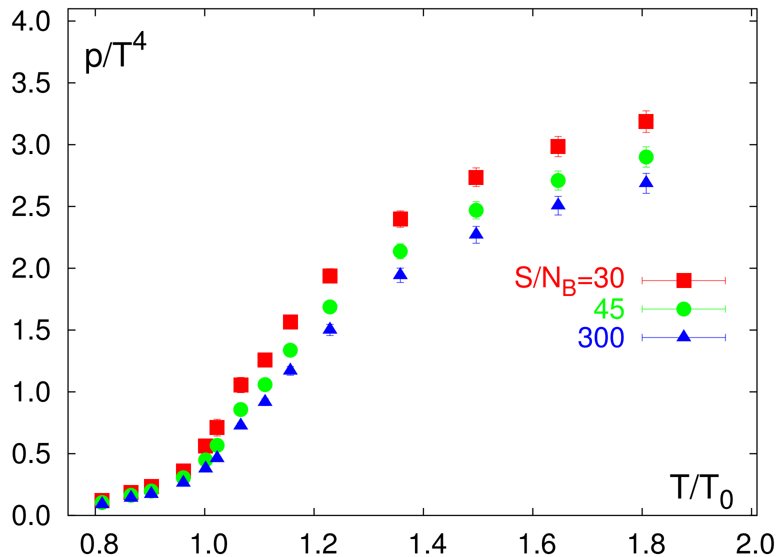
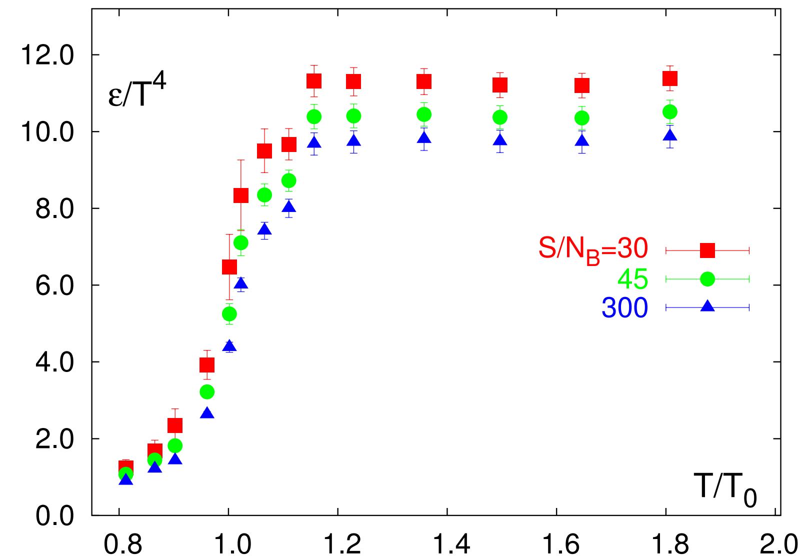
- trajectories bend towards larger μ_q for $T < T_c$
- $\mathcal{O}(\mu_q^6)$ correction (open sym.) is small for $\mu_q/T \lesssim 0.8$ (despite large errors)
- RHIC corresponds to $S/N_B \simeq 300 \simeq \infty$
- SPS corresponds to $S/N_B \simeq 45$
- FAIR will operate at $S/N_B \simeq 30$ or $\mu_q/T \lesssim 0.9$

Isentropic Equation of State

2-flavor QCD





pressure on lines of constant S/N_B

energy density on lines of constant S/N_B

- μ_q-dependent contribution added on top of the $\mu_q = 0$ result

Isentropic Equation of State

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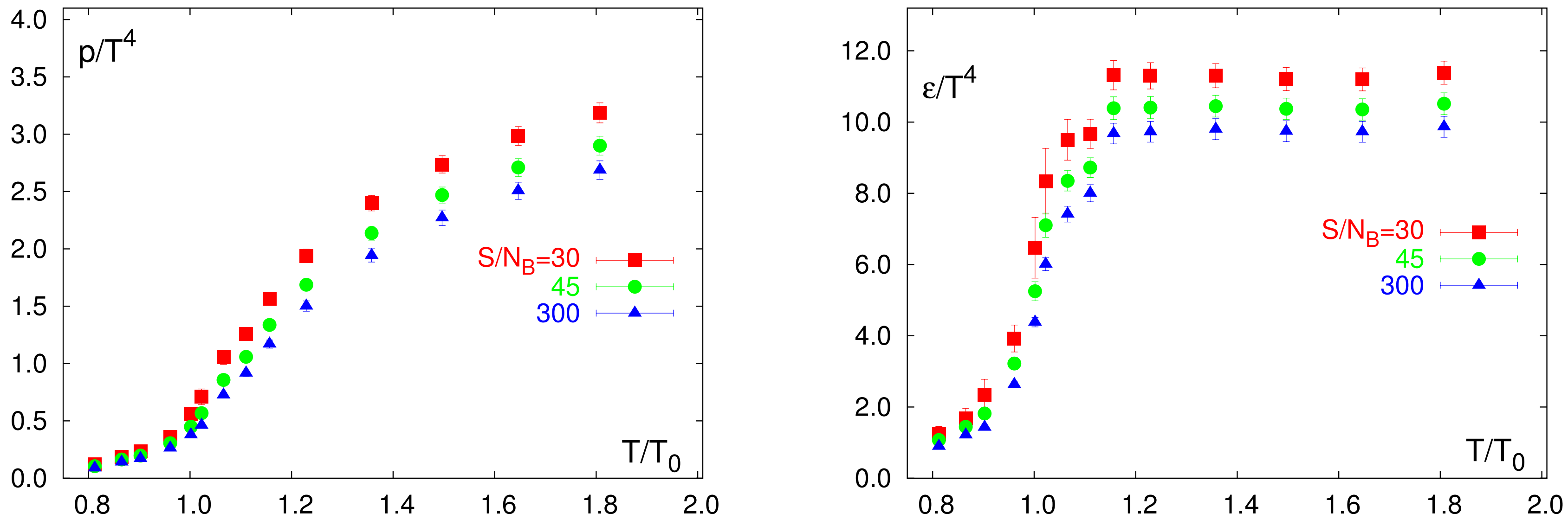
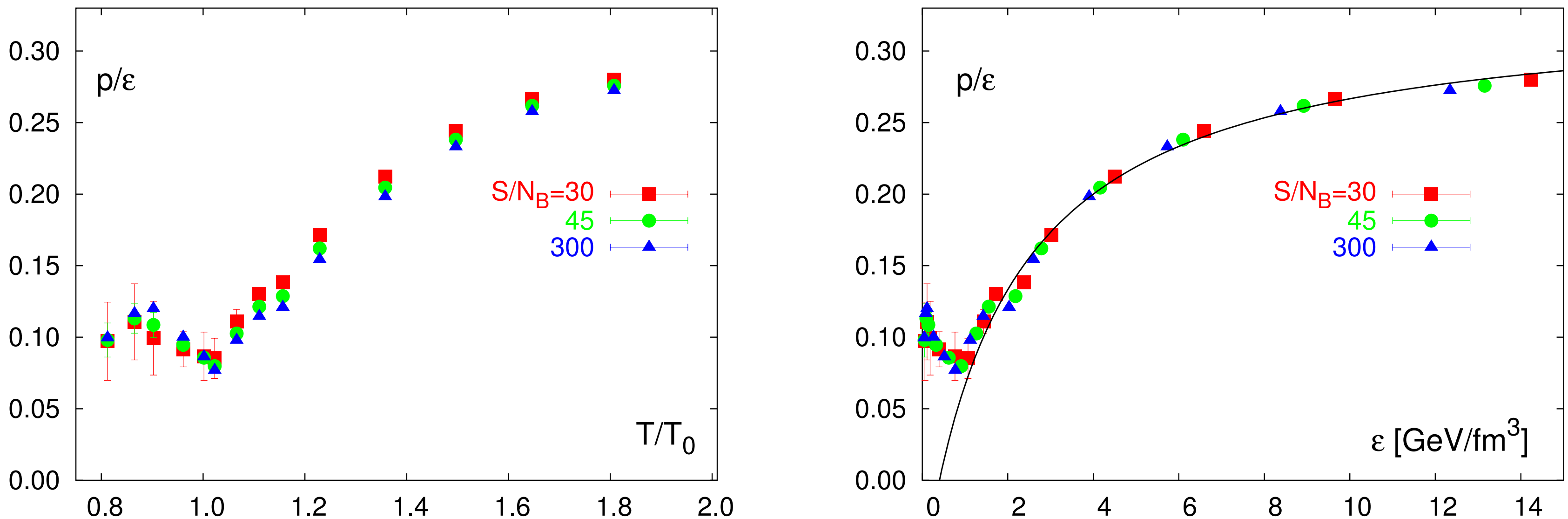


pressure
on lines of constant S/N_B

energy density
on lines of constant S/N_B

- μ_q-dependent contribution added on top of the $\mu_q = 0$ result
- RHIC EoS essentially coincides with the $\mu_q = 0$ EoS
- EoS at SPS and RHIC differ at high T by less than 10%
- FAIR: changes at high $T \sim 30\%$; but need better resolution at low T

Isentropic Equation of State: p/ϵ

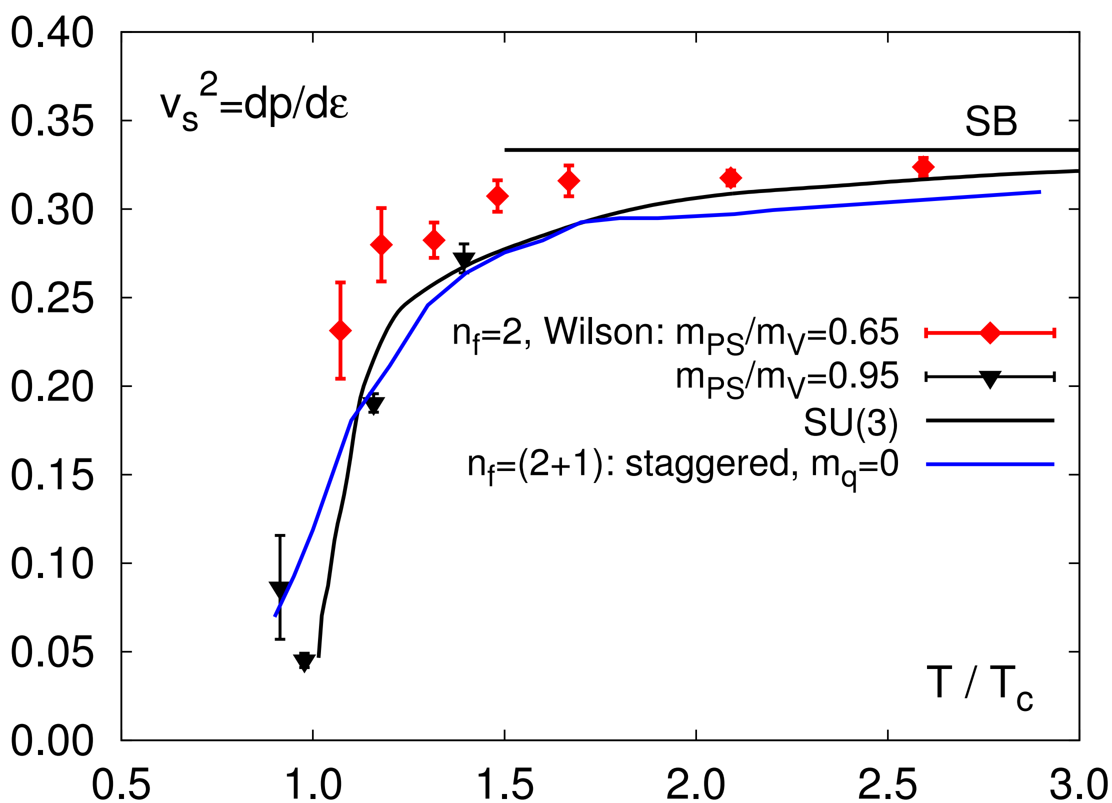


- p/ϵ vs. ϵ shows almost no dependence on S/N_B
- softest point: $p/\epsilon \simeq 0.075$
- phenomenological EoS for $T_0 \lesssim T \lesssim 2T_0$

$$\frac{p}{\epsilon} = \frac{1}{3}\left(1 - \frac{1.2}{1 + 0.5\epsilon}\right)$$

Velocity of sound

- steep EoS:
  rapid change of energy density; slow change of pressure
  $\Rightarrow$ reduced velocity of sound $\Rightarrow$ more time for equilibration



pure gauge theory:
G. Boyd et al.,
NP B469 1996

$n_f = 2$:
A. Ali Khan et al.,
PR D64 2001

$n_f = (2+1)$:
Y. Aoki et al.,
hep-lat/0510084
$(TV^{1/3} = 2)$

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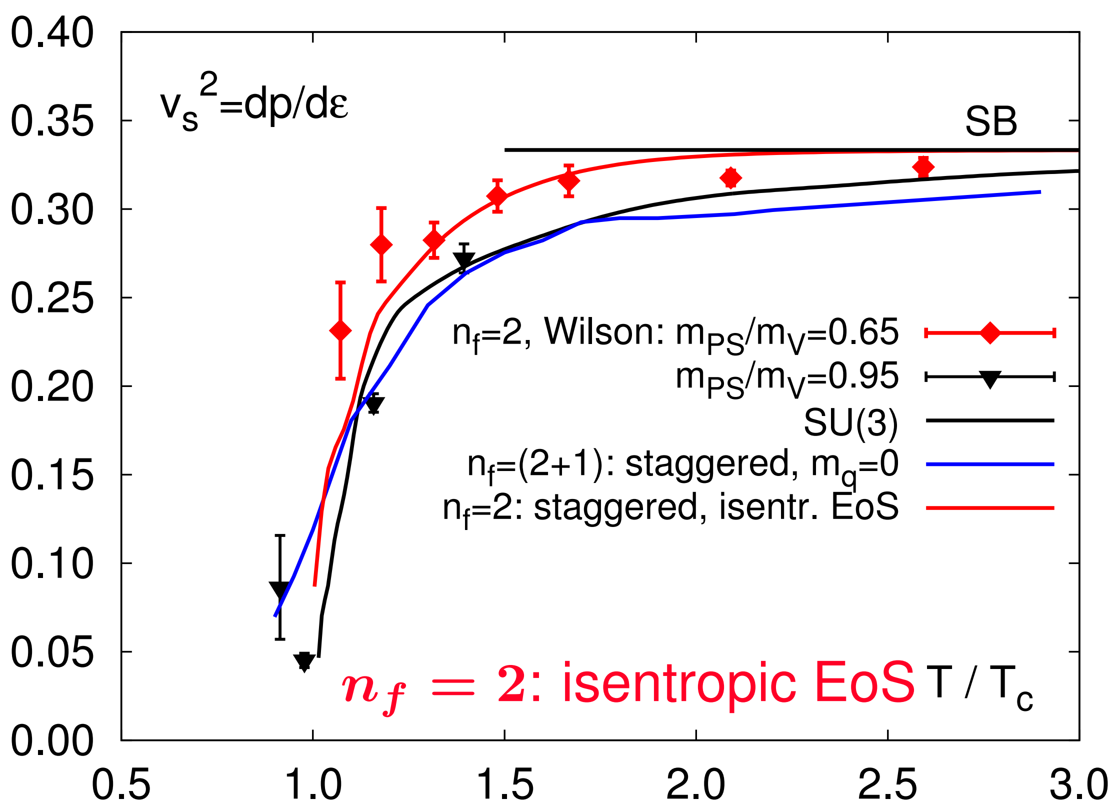

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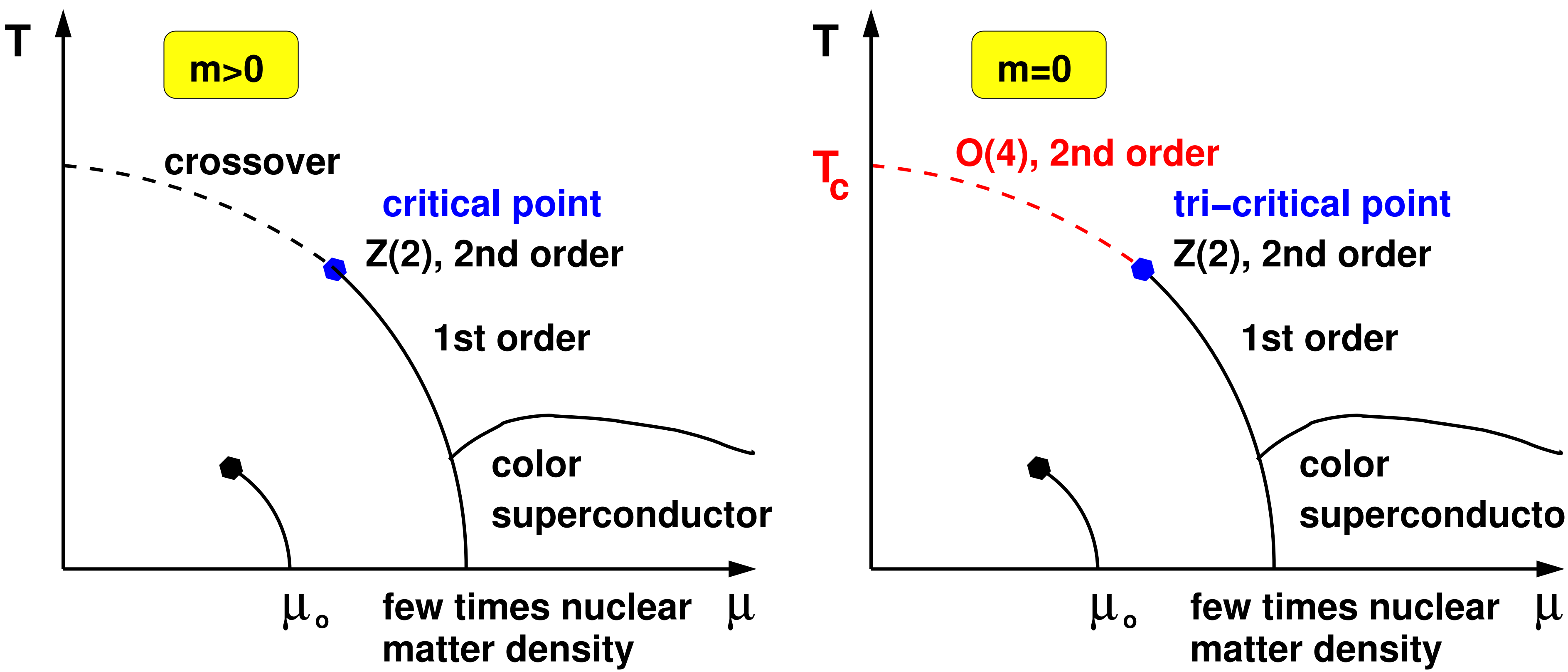
Hadronic fluctuations and chiral symmetry restoration

generic QCD phase diagram ($n_f = 2$)



Hadronic fluctuations and chiral symmetry restoration

- expect 2^{nd} order transition in 3-d, O(4) symmetry class;

  scaling field: $t = \left|\frac{T-T_c}{T_c}\right| + A\left(\left(\frac{\mu_q}{T_c}\right)^2 - \left(\frac{\mu_{crit}}{T_c}\right)^2\right)$

  singular part: $f_s(T,\mu_u,\mu_d) = b^{-1} f_s(tb^{1/(2-\alpha)}) \sim t^{2-\alpha}$

$$\frac{\partial^2 \ln Z}{\partial \mu_{q,I}^2} \sim t^{-\alpha} \quad , \quad \frac{\partial^4 \ln Z}{\partial \mu_{q,I}^4} \sim t^{-(2+\alpha)} \qquad (\mu > 0)$$

- O(4)/O(2): $\alpha < 0$, small $\Rightarrow$

  $\langle(\delta N_q)^2\rangle$ develops a cusp

  $\langle(\delta N_q)^4\rangle$ diverges on the O(4) critical line;

  strength $\sim \left(\frac{\mu_{crit}}{T_c}\right)^4$ ($\sim 10^{-4}$ at RHIC)

Hadronic fluctuations and chiral symmetry restoration

- expect 2^{nd} order transition in 3-d, O(4) symmetry class;

  scaling field: $t = \left| \frac{T - T_c}{T_c} \right| + A \left(\frac{\mu_q}{T_c} \right)^2$, $\mu_{crit} = 0$

  singular part: $f_s(T, \mu_u, \mu_d) = b^{-1} f_s(t b^{1/(2-\alpha)}) \sim t^{2-\alpha}$

$$\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_q^2} \sim t^{1-\alpha} \quad , \quad \frac{\partial^4 \ln \mathcal{Z}}{\partial \mu_q^4} \sim t^{-\alpha} \qquad (\mu = 0)$$

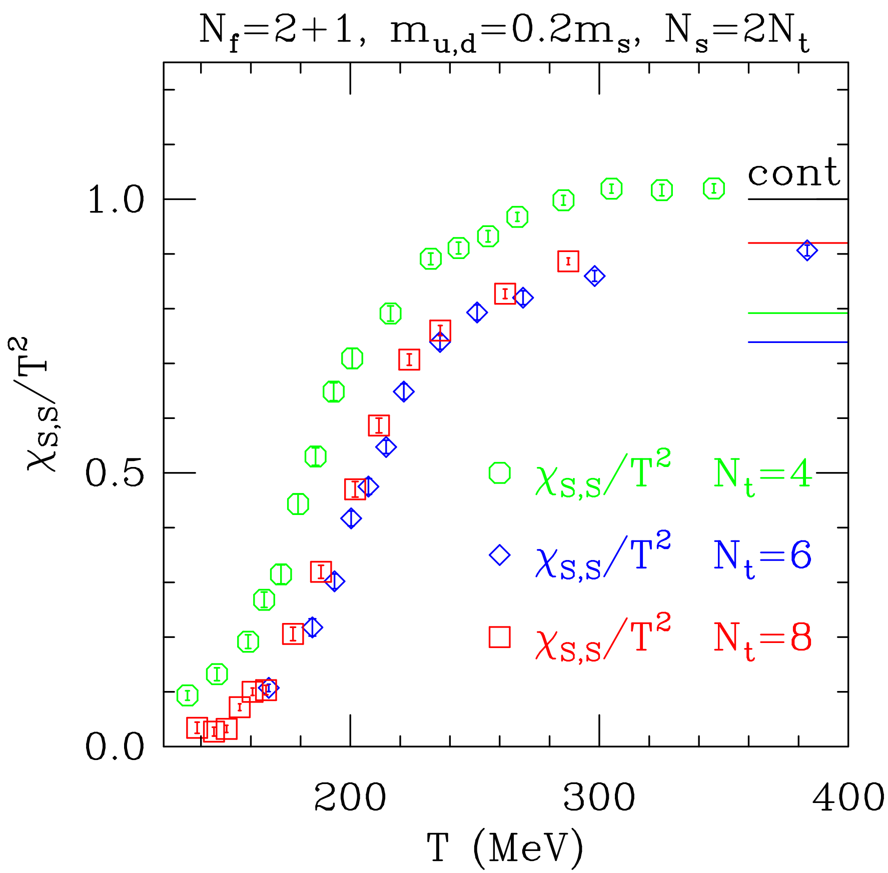
- O(4)/O(2): $\alpha < 0$, small $\Rightarrow$

  $\langle (\delta N_q)^2 \rangle$ dominated by T-dependence of regular part

  $\langle (\delta N_q)^4 \rangle$ develops a cusp

Fluctuations of the baryon number density ($\mu = 0$)

baryon number density fluctuations:
(MILC coll., hep-lat/0405029)
$\mu = 0$



$$\frac{\chi_q}{T^3} = \left(\frac{d^2}{d(\mu/T)^2} \frac{p}{T^4} \right)_{T \text{ fixed}}$$
$$= \frac{9\,T}{V} \left(\langle N_B^2 \rangle - \langle N_B \rangle^2 \right)$$

susceptibilities = integrated correlation functions
= integrated spectral functions

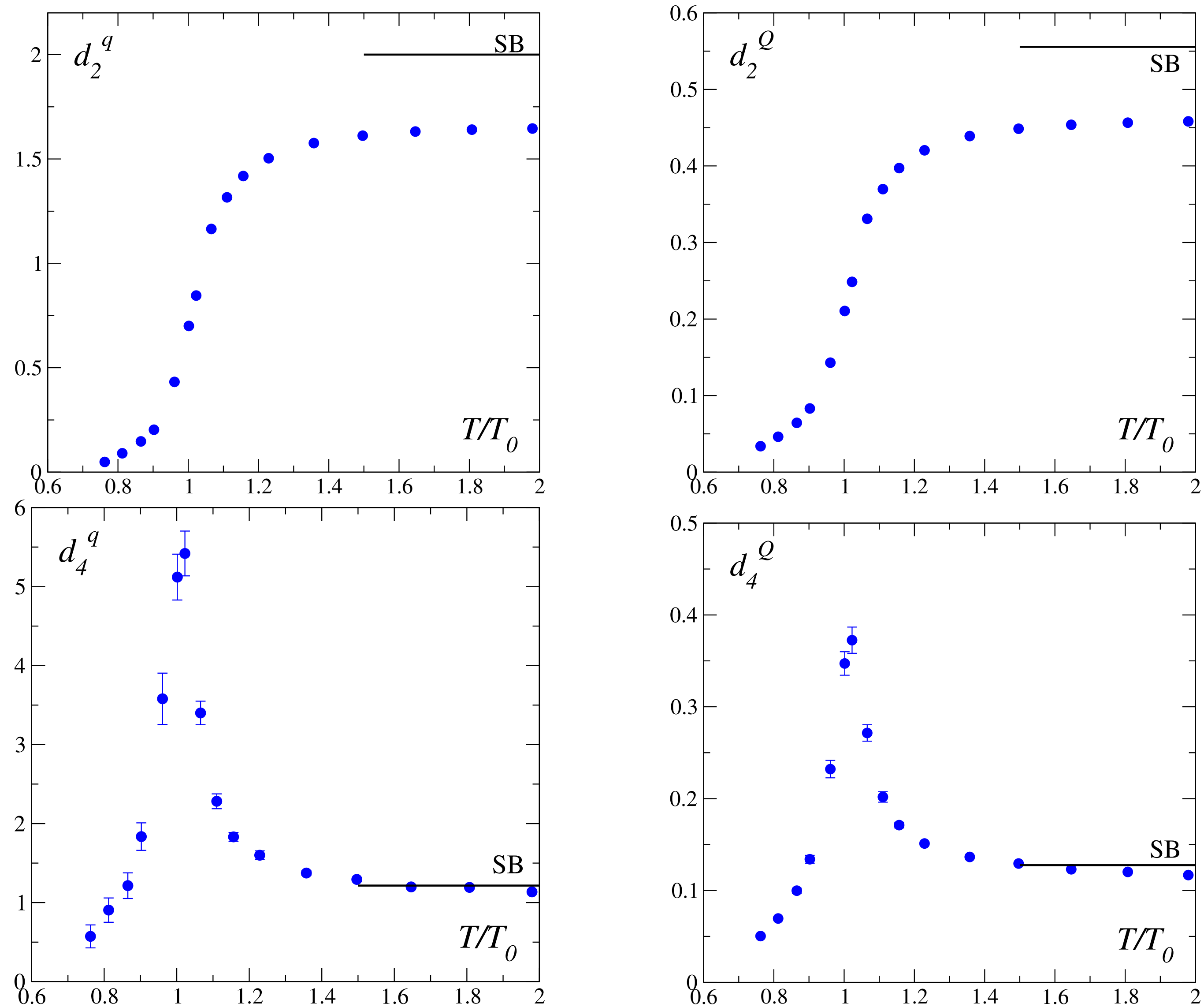
to be studied in event-by-event fluctuations

recent papers:

V. Koch, E.M. Majumder, J. Randrup, nucl-th/0505052
S. Ejiri, FK, K. Redlich, hep-ph/05090521
R.V. Gavai, S. Gupta, hep-lat/0510044

Quark number and charge fluctuations at $\mu_B = 0$; 2-flavor QCD ($m_\pi \simeq 770\ MeV$)

C. Allton et al. (Bielefeld-Swansea), PRD71 (2005) 054508



monotonic increase;
close to ideal gas value for $T \gtrsim 1.5 T_c$

develops cusp at T_c

reaches ideal gas value for $T \gtrsim 1.5 T_c$

Fluctuations of the baryon number density ($\mu \geq 0$)

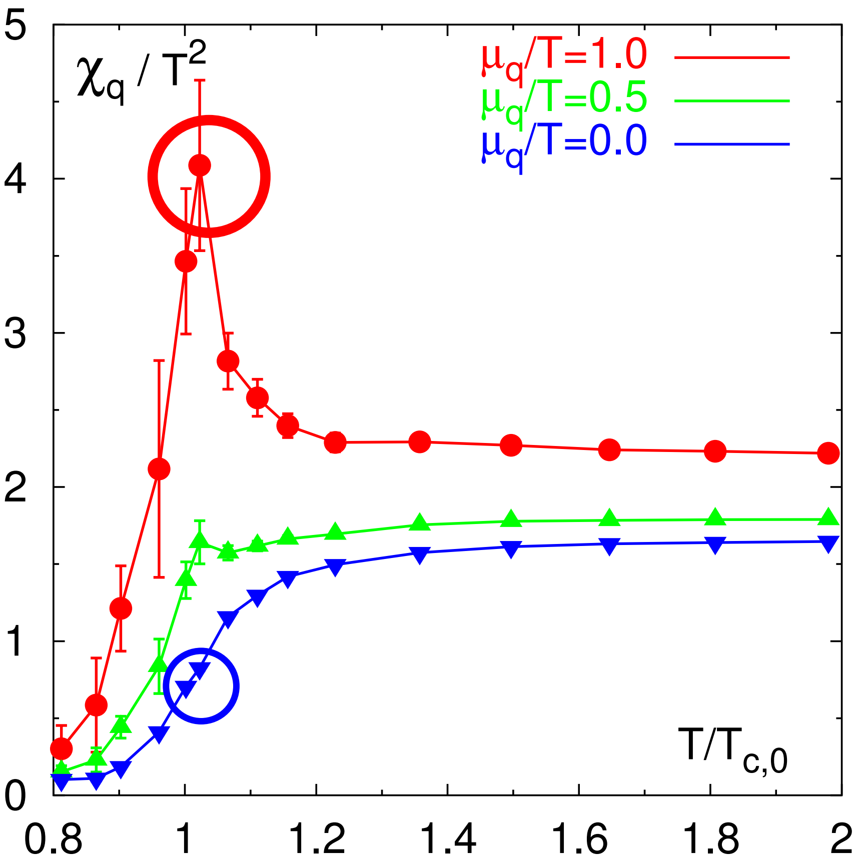
baryon number density fluctuations:

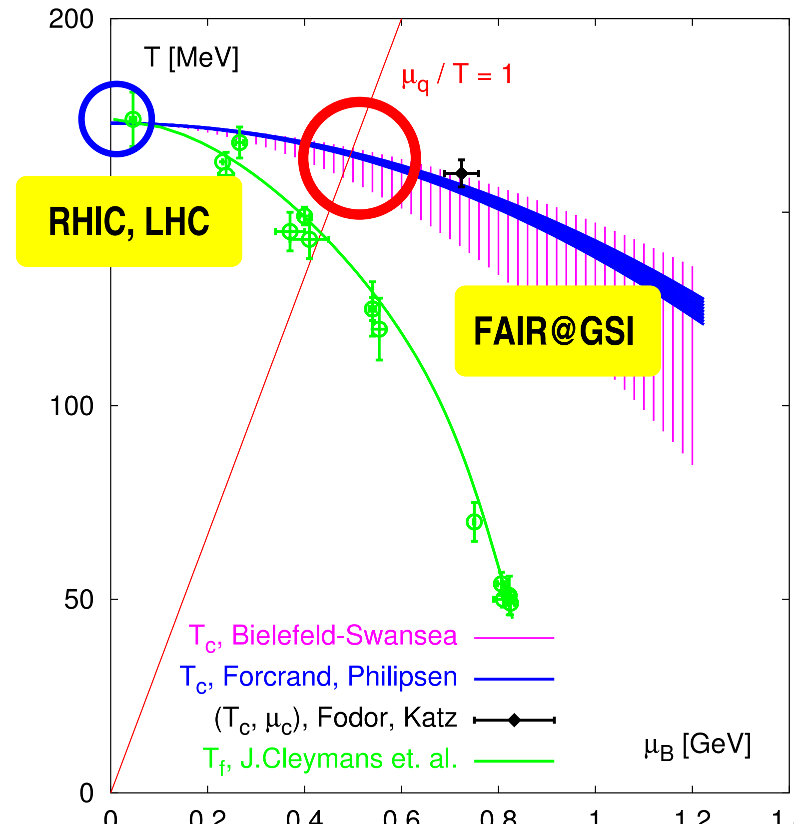
(Bielefeld-Swansea, PRD68 (2003) 014507)

$\mu \geq 0, n_f = 2$

$$\frac{\chi_q}{T^3} = \left(\frac{d^2}{d(\mu/T)^2} \frac{p}{T^4} \right)_{T \text{ fixed}}$$

$$= \frac{9\,T}{V} \left(\langle N_B^2 \rangle - \langle N_B \rangle^2 \right)$$





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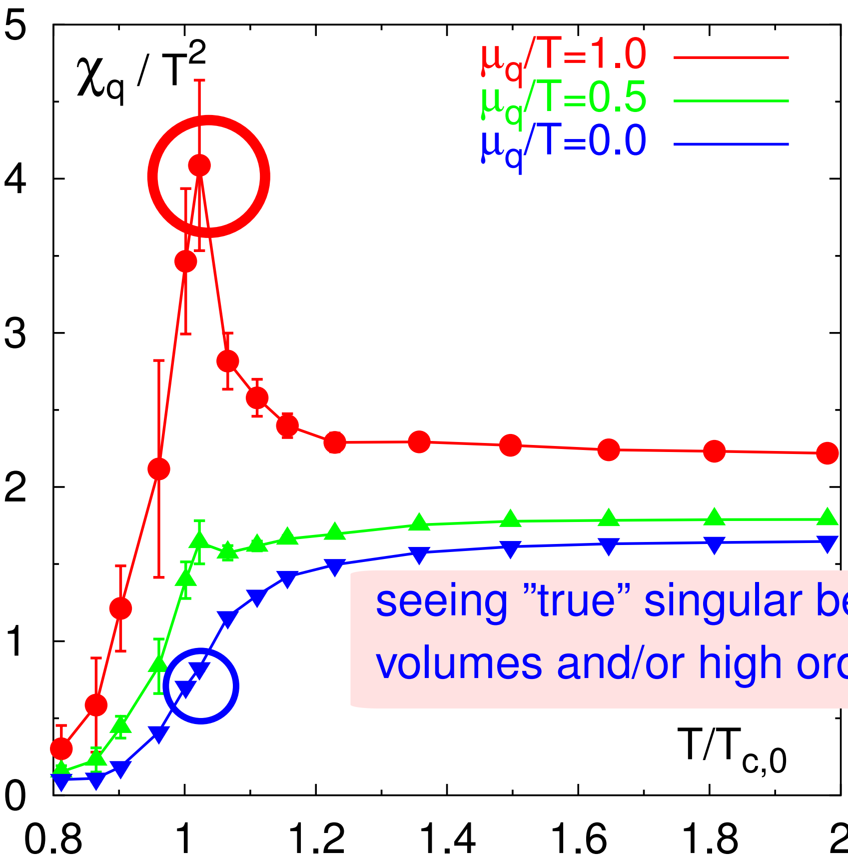
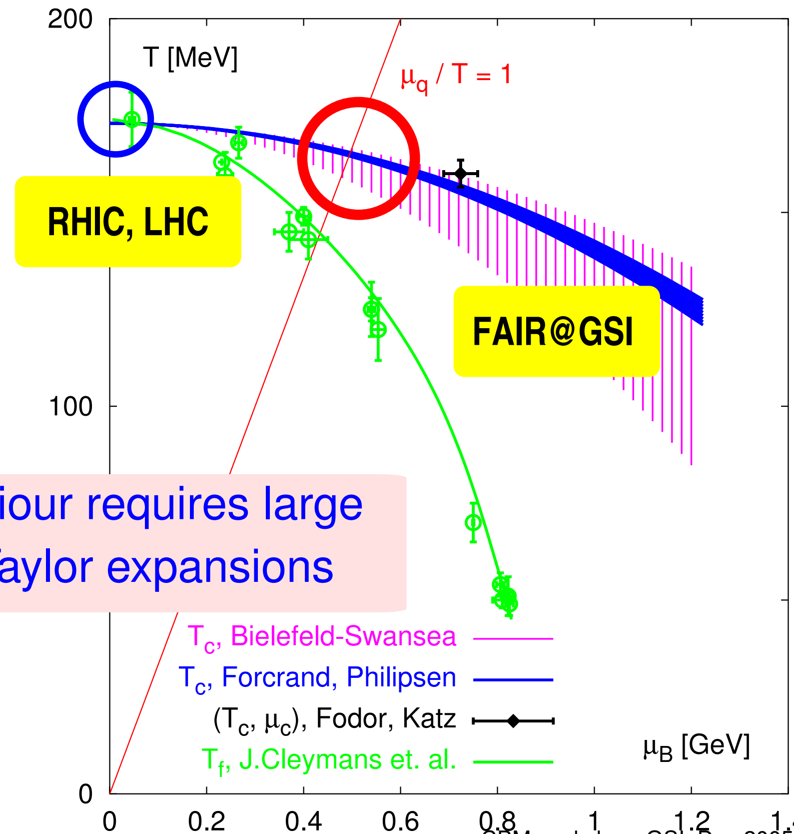




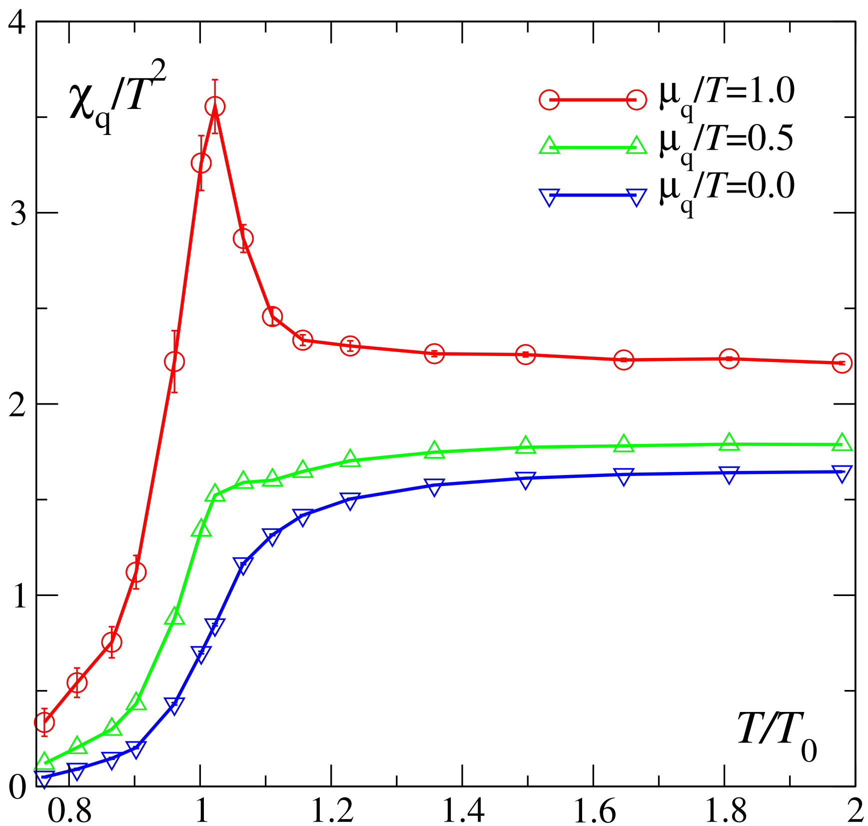
seeing "true" singular behaviour requires large volumes and/or high order Taylor expansions

Fluctuations of the quark number density ($\mu_q > 0$)

quark number density fluctuations: $$\frac{\chi_q}{T^2} = \left(\frac{\partial^2}{\partial(\mu_q/T)^2} \frac{p}{T^4} \right)_{T \text{ fixed}}$$

$$= \frac{1}{VT^3} \left(\langle N_q^2 \rangle - \langle N_q \rangle^2 \right)$$

up to $\mathcal{O}\left((\mu_q/T)^2\right)$



high-T, massless limit: polynomial in (μ_q/T)

$$\frac{\chi_{q,SB}}{T^2} = n_f + \frac{3n_f}{\pi^2} \left(\frac{\mu_q}{T} \right)^2$$

Fluctuations of the quark number density ($\mu_q > 0$)

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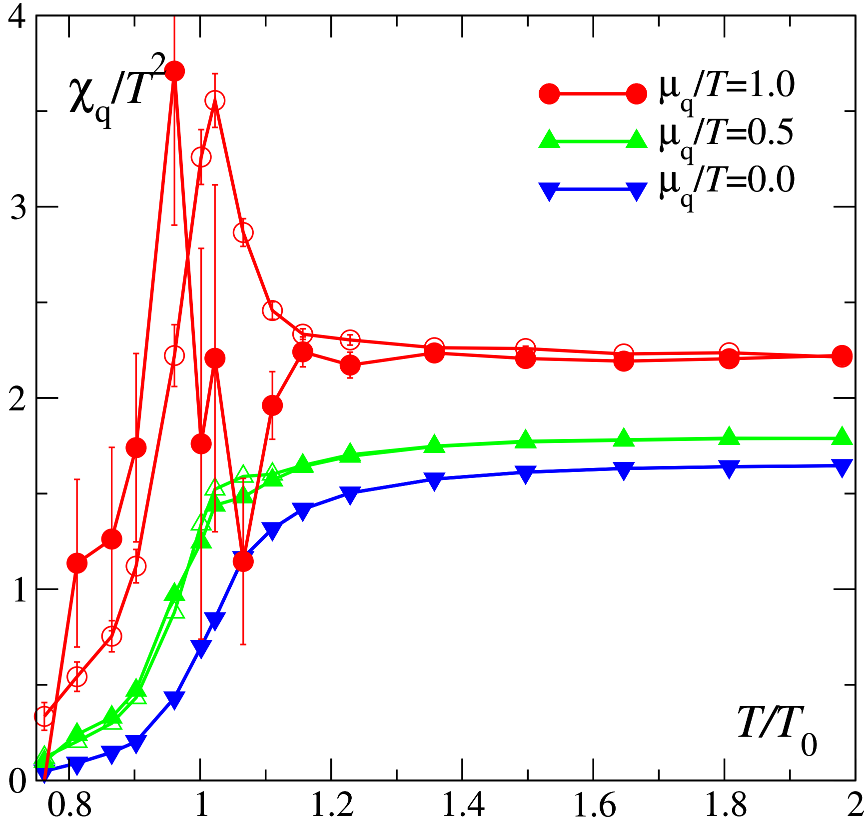

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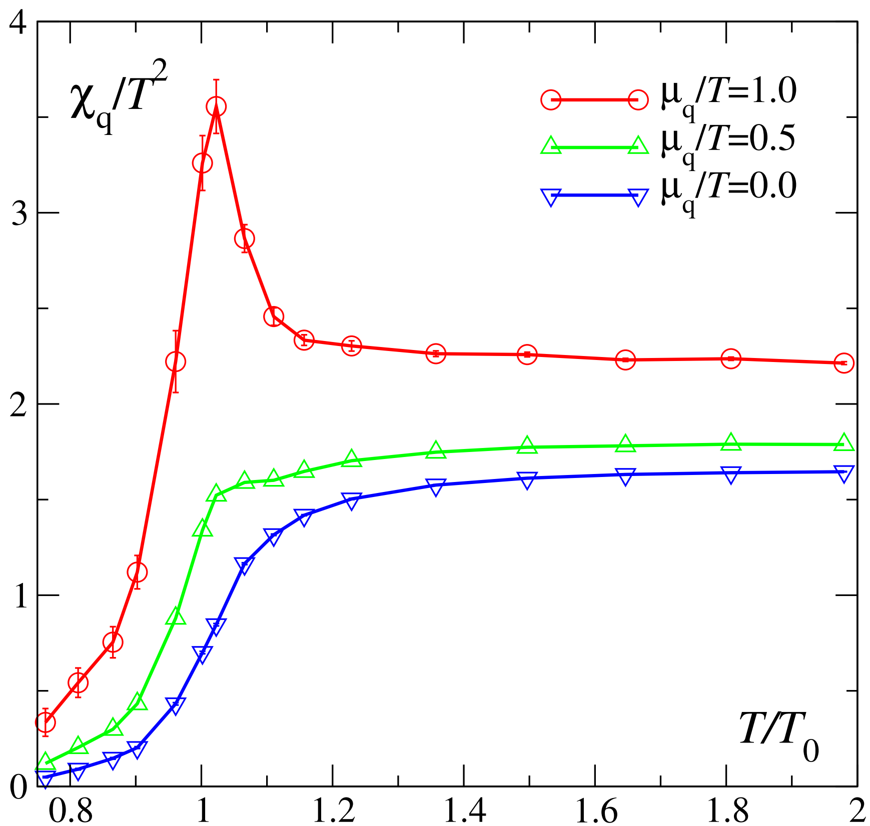


high-T, massless limit: polynomial in (μ_q/T)

$$\frac{\chi_{q,SB}}{T^2} = n_f + \frac{3n_f}{\pi^2} \left(\frac{\mu_q}{T} \right)^2$$

larger density fluctuations for $\mu_q > 0$;

coming closer to the chiral critical point?

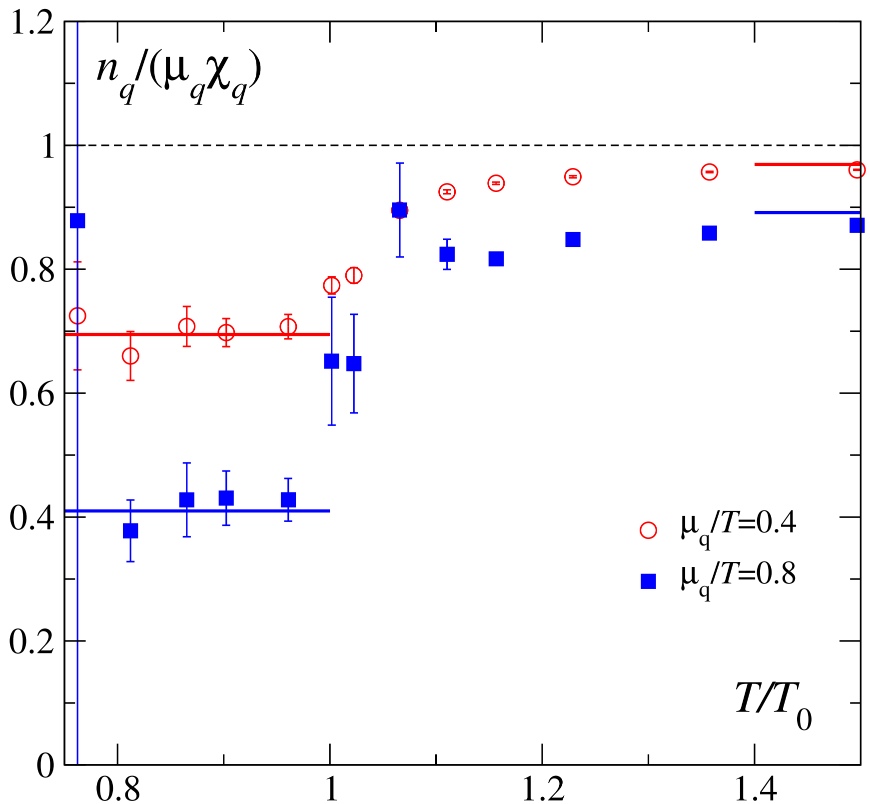
$$\left(\frac{\partial p}{\partial n_q} \right)_T = \frac{n_q}{\chi_q}$$

$\Rightarrow$ χ_q will diverge on chiral critical point

Isothermal compressibility of the quark gluon plasma

inverse compressibility:

$$\kappa_T^{-1} = \frac{n_q}{\chi_q} = \left(\frac{\partial p}{\partial n_q}\right)_{T\ \mathrm{fixed}}$$



high-T, massless limit: polynomial in (μ_q/T)

$$\frac{n_q}{\chi_q} = \mu_q + \mathcal{O}\left(\left(\frac{\mu_q}{T}\right)^3\right)$$

Isothermal compressibility of the quark gluon plasma

inverse compressibility:

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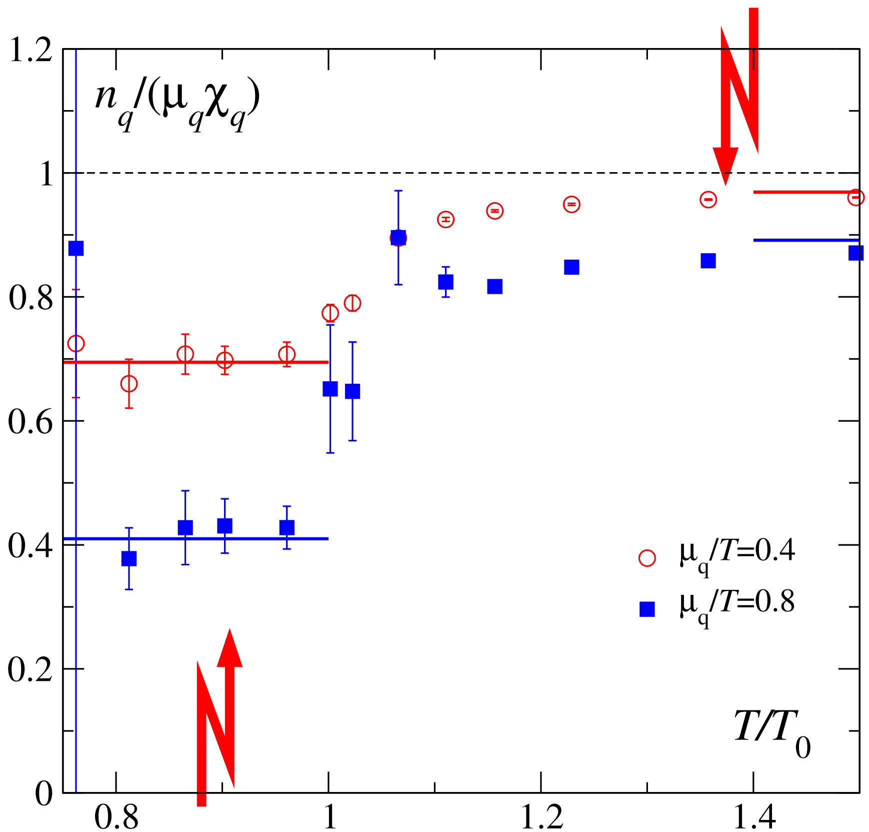
ideal $q\bar{q}$ gas



resonance gas

high-T, massless limit: polynomial in (μ_q/T)

$$\frac{n_q}{\chi_q} = \mu_q + \mathcal{O}\left(\left(\frac{\mu_q}{T}\right)^3\right)$$

large density fluctuations for $\mu_q > 0,\ T < T_c$

"saturated" by fluctuations in a

hadron resonance gas

expect: $$\left(\frac{\partial p}{\partial n_q}\right)_T = \frac{n_q}{\chi_q} = 0$$

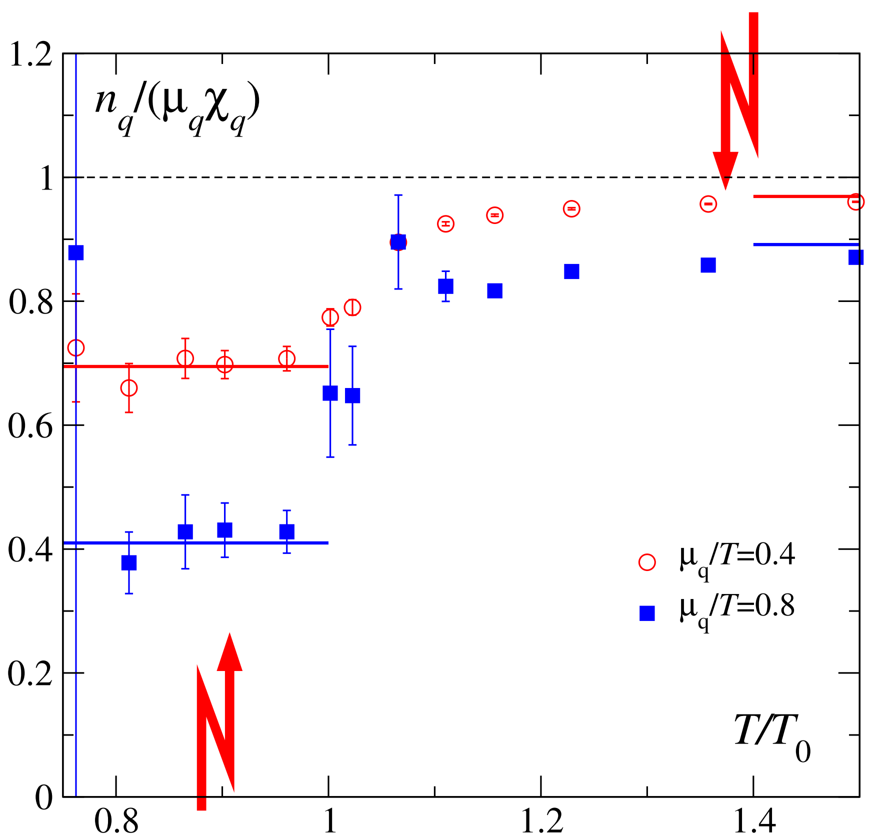
at chiral critical point

Isothermal compressibility of the quark gluon plasma

inverse compressibility:

$$\kappa_T^{-1} = \frac{n_q}{\chi_q} = \left(\frac{\partial p}{\partial n_q}\right)_{T \text{ fixed}}$$

ideal $q\bar{q}$ gas



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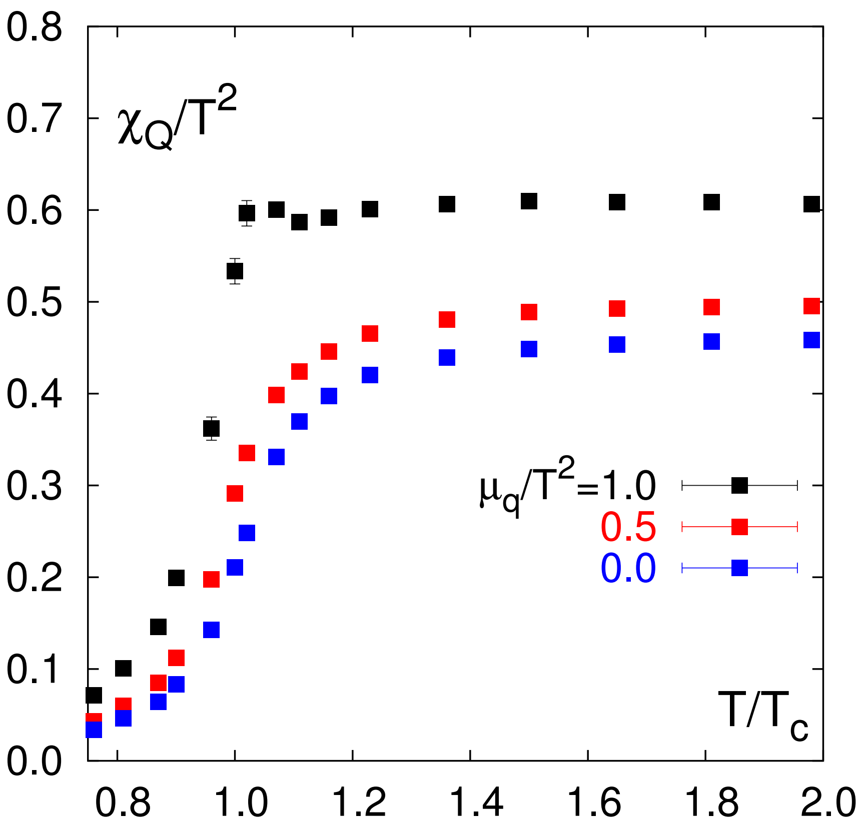
at chiral critical point

$m_\pi \simeq 770$ MeV, smaller m_q needed!!

Charge fluctuations for $\mu_q > 0$

quark number density fluctuations: $$\frac{\chi_Q}{T^2} = \frac{1}{4}\left(\frac{\chi_I}{T^2} + \frac{1}{9}\frac{\chi_q}{T^2}\right)$$

up to $\mathcal{O}\left((\mu_q/T)^2\right)$



$$= \frac{1}{VT^3}\left(\langle N_Q^2\rangle - \langle N_Q\rangle^2\right)$$

high-T, massless limit: polynomial in (μ_q/T)

$$\frac{\chi_{Q,SB}}{T^2} = \frac{5}{9} + \frac{15}{9\pi^2}\left(\frac{\mu_q}{T}\right)^2$$

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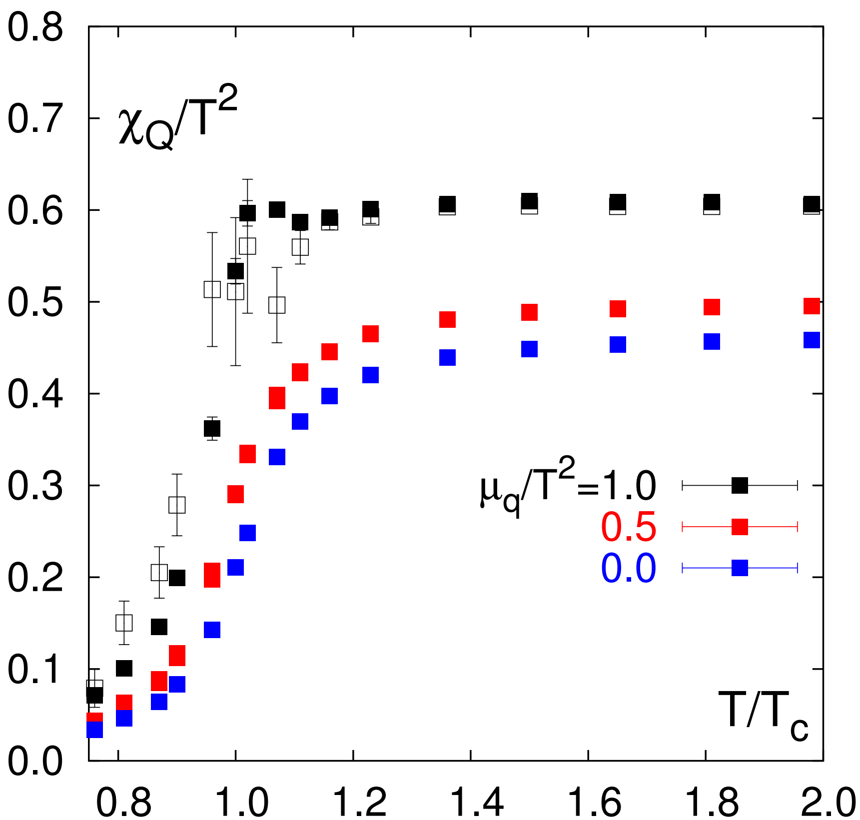

high-T, massless limit: polynomial in (μ_q/T)

$$\frac{\chi_{Q,SB}}{T^2} = \frac{5}{9} + \frac{15}{9\pi^2}\left(\frac{\mu_q}{T}\right)^2$$

charge fluctuations for $(\mu_q/T)\lesssim 1$

dominated by isospin fluctuations;

Nonetheless:

expect singularity at chiral critical point;

arises from contribution of χ_q;

χ_I is expected to be non-singular at CCP

...Status of finite density calculations

- calculations for non-vanishing chemical potential ($\mu_q > 0$) show a rapid transition from a HRG to a QGP; signaled by sudden changes in EoS and susceptibilities
- where and whether the transition becomes first order ...
  $\Rightarrow$ Z. Fodor, tomorrow

...Status of finite density calculations

- calculations for non-vanishing chemical potential ($\mu_q > 0$) show a rapid transition from a HRG to a QGP; signaled by sudden changes in EoS and susceptibilities
- where and whether the transition becomes first order ...
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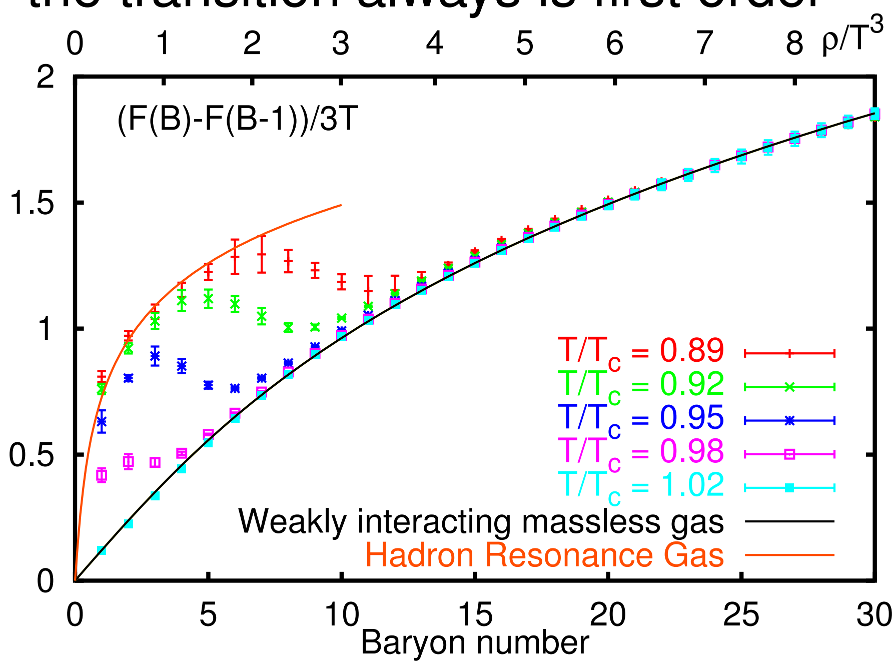
- alternative approach in the canonical ensemble ($B > 0$) looks promising; so far applied only to 4-flavor QCD where the transition always is first order



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