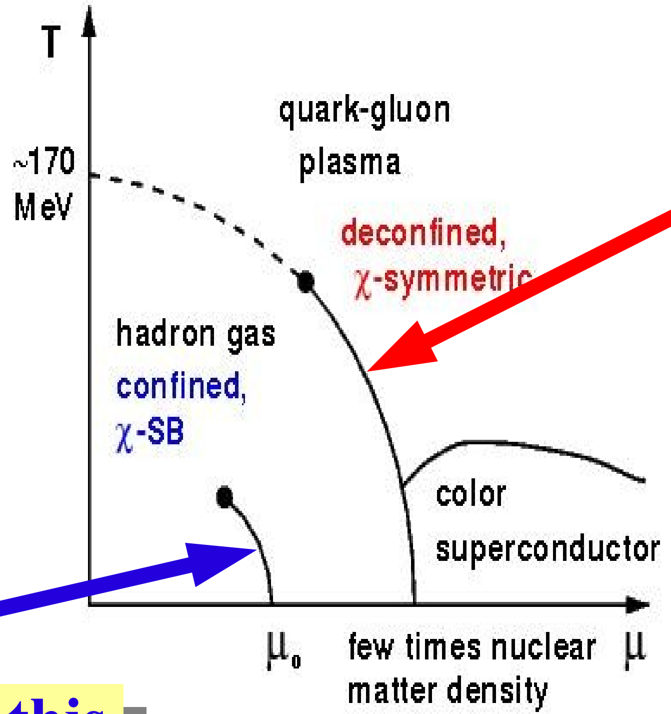
Event by Event Fluctuations

- General remarks about fluctuations
- First order, second order
- Practical aspects

Event by Event = Multi-particle correlations

Thanks to J. Randrup for sharing some of his slides

Phase diagram



Can we establish this line experimentally?

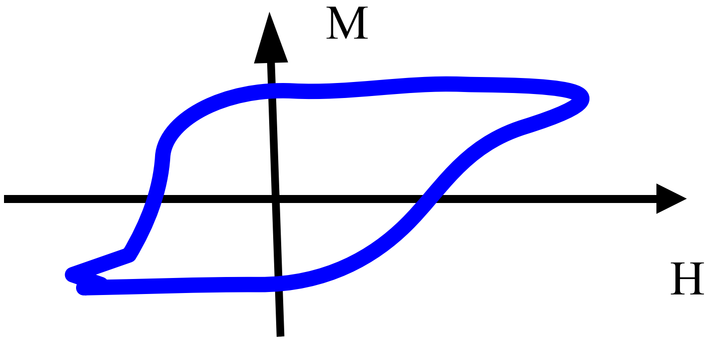
Have we established this line experimentally?

Susceptibilities

$$E = E_0 + m H + \mu Q$$

$$\langle m \rangle = \frac{d F}{d H}$$

$$\langle Q \rangle = \frac{d F}{d \mu}$$



Susceptibilities

$$\chi_m = \frac{d^2 F}{d H^2}$$

$$\chi_Q = \frac{d^2 F}{d \mu^2}$$

$$\langle \delta m \rangle = \chi_m \delta H$$

$$\langle \delta Q \rangle = \chi_Q \delta \mu$$

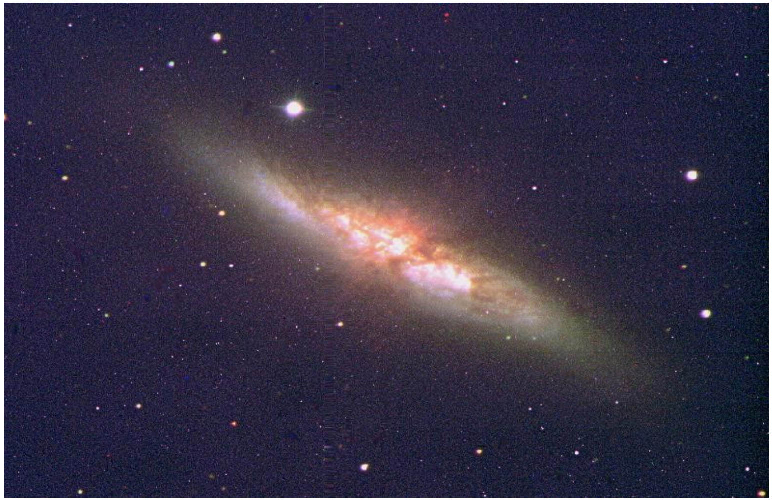
Linear response

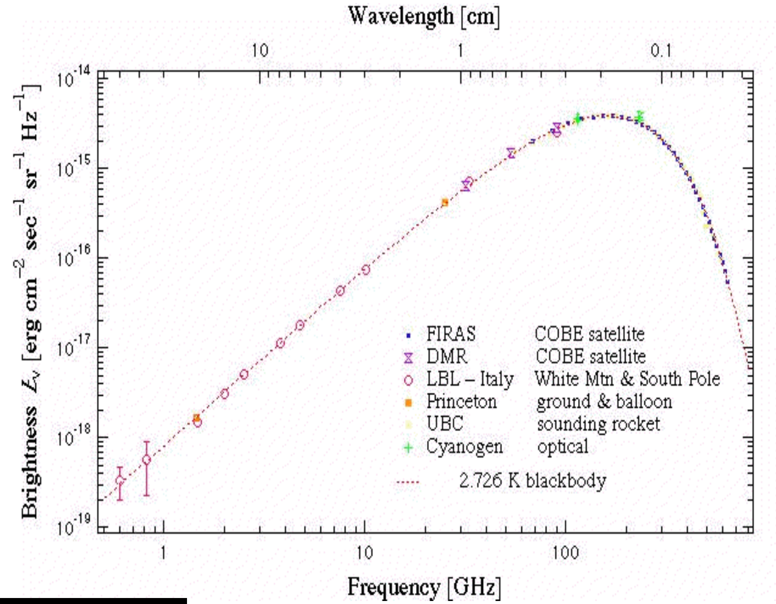
$$\langle (\delta m)^2 \rangle = \chi_m$$

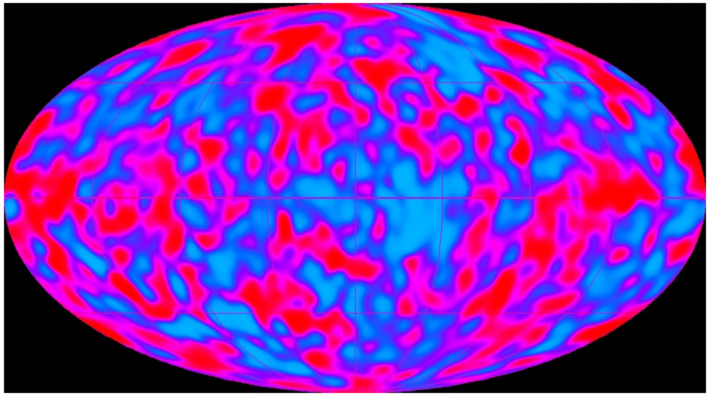
$$\langle (\delta Q)^2 \rangle = \chi_Q$$

Fluctuations

The mother of all thermal spectra and fluctuations

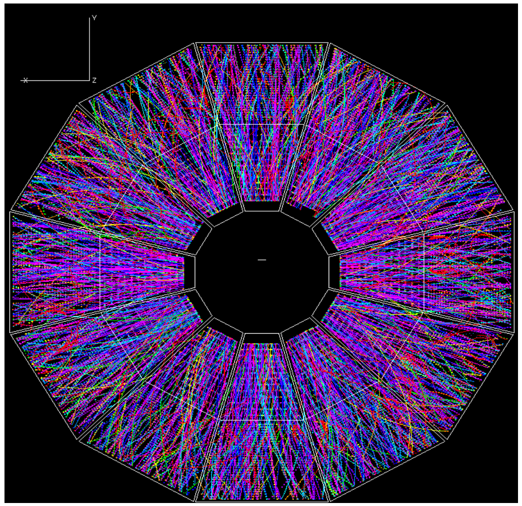


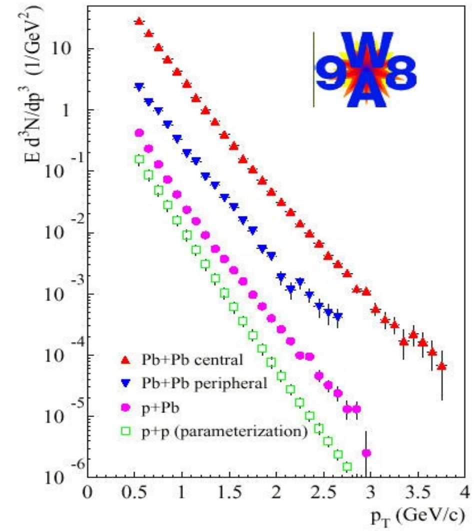


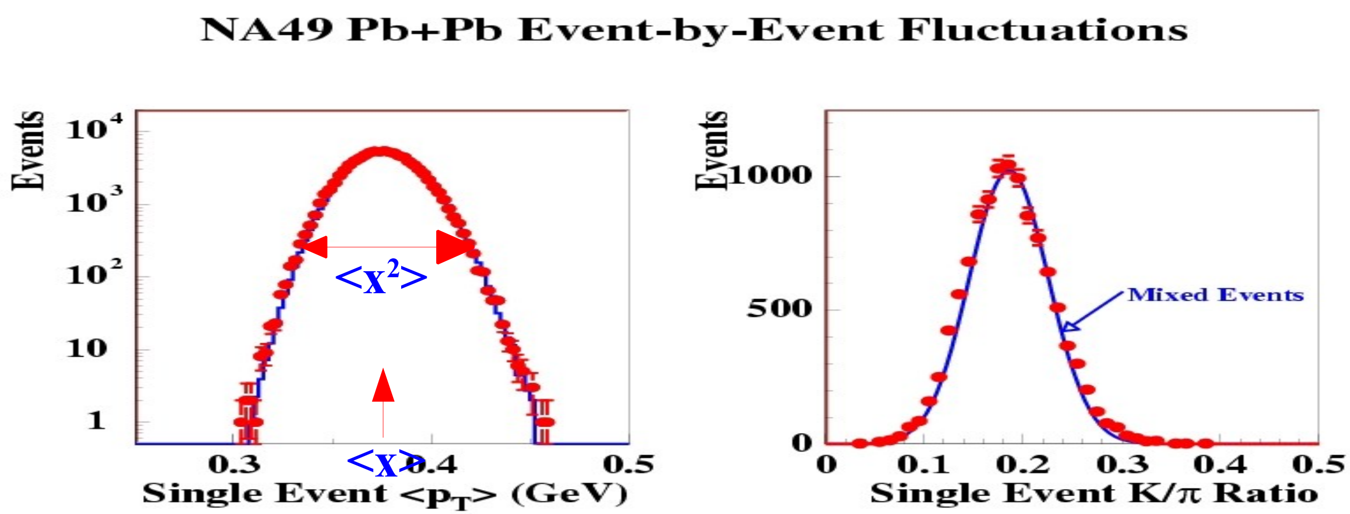


Fluctuations at the level of 10^{-5} !!!

Heavy Ions: Event-by-Event







The physics is in the width

E-by-E measures
2-particle correlations

Fluctuations in thermal system

e.g. Lattice QCD

$$Z = Tr\left[\exp\left(-\beta\left(H - \mu_Q Q - \mu_B B - \mu_S S\right)\right)\right]$$

Mean : $\langle X \rangle = T \frac{\partial}{\partial \mu_X} \log(Z) = -\frac{\partial}{\partial \mu_X} F$ $X = Q, B, S$

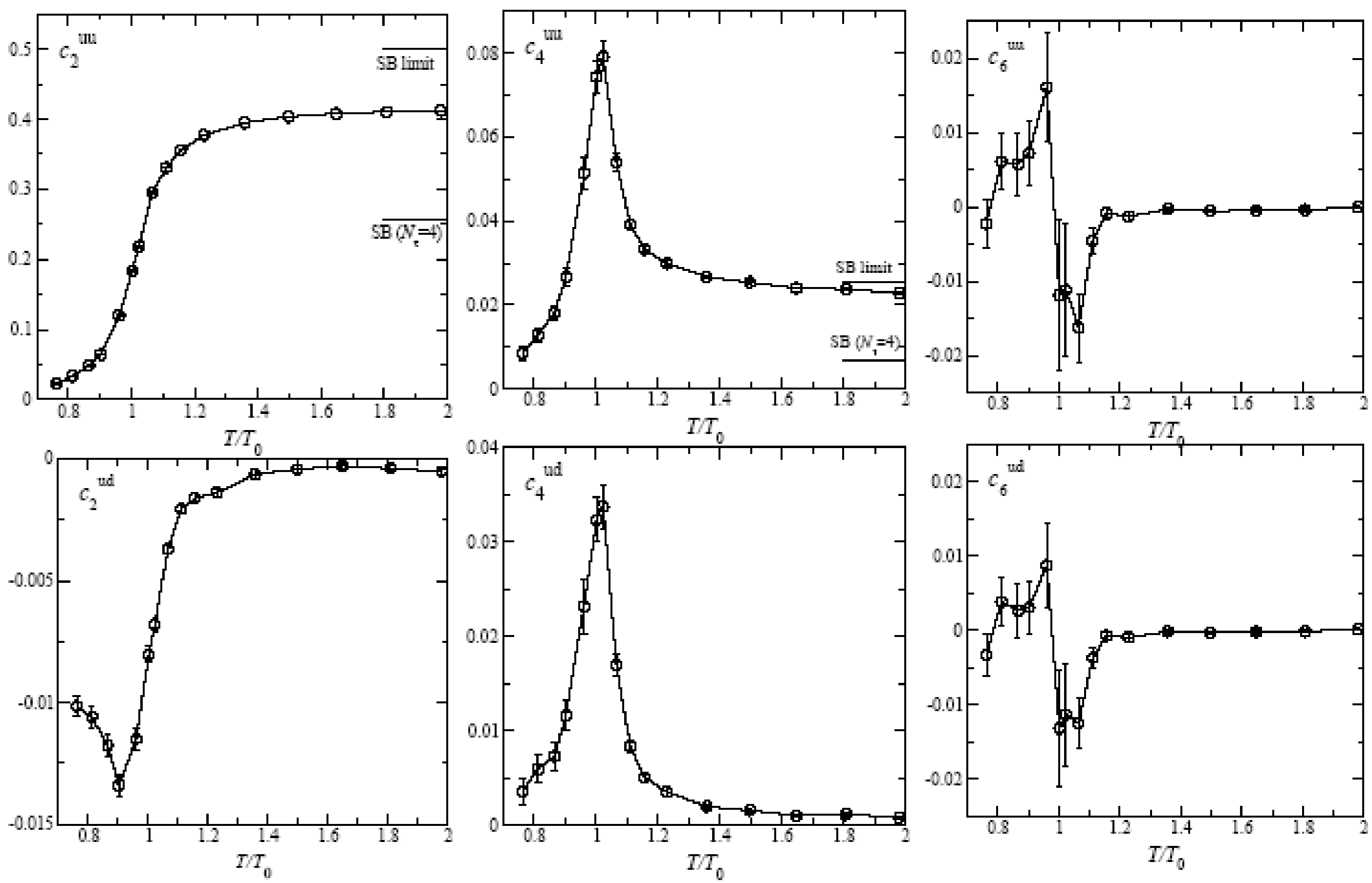
Variance: $\langle (\delta X)^2 \rangle = T^2 \frac{\partial^2}{\partial \mu_X^2} \log(Z) = -T \frac{\partial^2}{\partial \mu_X^2} F$

Co-Variance: $\langle (\delta X)(\delta Y) \rangle = T^2 \frac{\partial^2}{\partial \mu_X \partial \mu_Y} \log(Z) = -T \frac{\partial^2}{\partial \mu_X \partial \mu_Y} F$

Susceptibility: $\chi_{XY} = -\frac{1}{V} \frac{\partial^2}{\partial \mu_X \partial \mu_Y} F = -\frac{1}{V} \frac{\partial}{\partial \mu_X} \langle Y \rangle$

Lattice-QCD susceptibilities

$$\frac{\chi(T,\mu_q)}{T^2} = 2c_2 + 12\,c_4\left(\frac{\mu_q}{T}\right)^2 + 30\,c_6\left(\frac{\mu_q}{T}\right)^4 + \ldots$$



Rule of thumb:

$$c_n \sim \langle X^n \rangle$$
$$X = B, Q, S, \ldots$$

Alton et al, PRD 66 074507 (2002)

E-by-E observables

- Multiplicity fluctuations
  - interesting centrality dependence at top SPS energies
- Charge fluctuations
  - Resonance gas at RHIC
  - no sensitivity at SPS
- Transverse momentum fluctuations
  - some signal at SPS & RHIC (mostly “jets”)
- Ratio (K/π) fluctuations
  - statistical at top SPS, possible signal at low SPS

Something new: Simple Observation

Or how can we test the bs-QGP

Simple QGP: strangeness is carried by strange quarks

→ Baryon Number and Strangeness are correlated

Hadron Gas: strangeness is carried mostly by mesons

→ Baryon Number and Strangeness are uncorrelated

Bound state QGP: strangeness is carried by partonic bound states

→ Baryon Number and Strangeness should be uncorrelated

<BS> and the Bound State QGP

Define: $$C_{BS} \equiv -3\,\frac{\langle(\delta B)(\delta S)\rangle}{\langle(\delta S)^2\rangle} = -3\,\frac{\langle\left(B-\langle B\rangle\right)\left(S-\langle S\rangle\right)\rangle}{\langle\left(S-\langle S\rangle\right)^2\rangle} = -3\,\frac{\langle BS\rangle}{\langle S^2\rangle} = -3\,\frac{X_{BS}}{X_{SS}}$$

(-3) compensates baryon-number and strangenes of quarks

In a QGP phase

$$-3\langle BS\rangle = \langle n_s\rangle + \langle n_{\bar{s}}\rangle$$

$$\langle S^2\rangle = \langle n_s\rangle + \langle n_{\bar{s}}\rangle$$

At *all* T and μ

C_{BS} = 1

In hadron gas phase

$$-3\langle BS\rangle = 3\left[\Lambda + \bar{\Lambda} + \Sigma + \bar{\Sigma} + \ldots\right] + 6\left[\Xi + \bar{\Xi} + \ldots\right] + 9\left[\Omega + \ldots\right]$$

$$\langle S^2\rangle = K^+ + K^- + K^0 + \Lambda + \bar{\Lambda} + \ldots$$

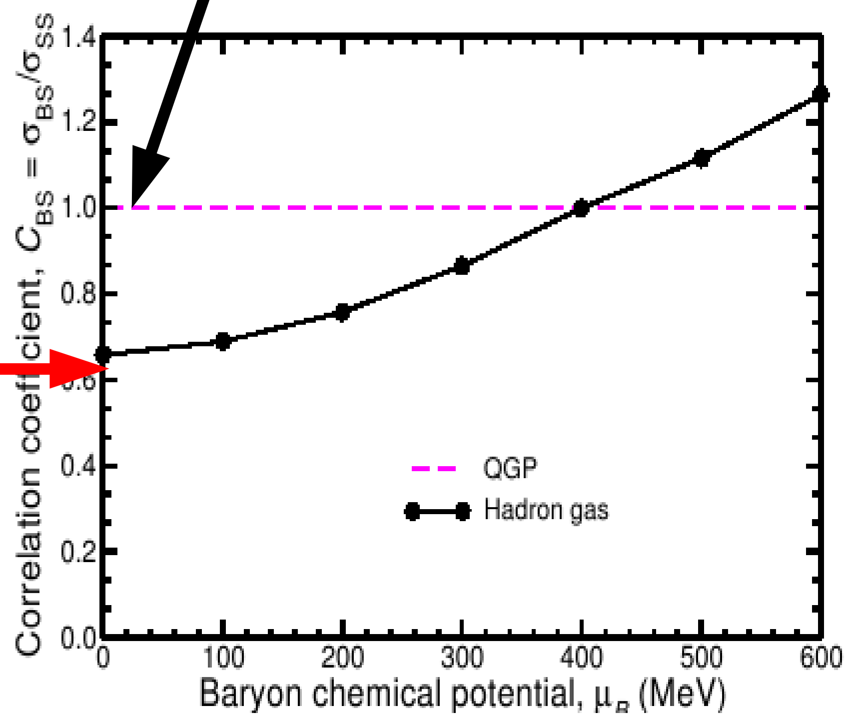
At T=170MeV, μ=0

C_{BS} = 0.66

<BS> continued

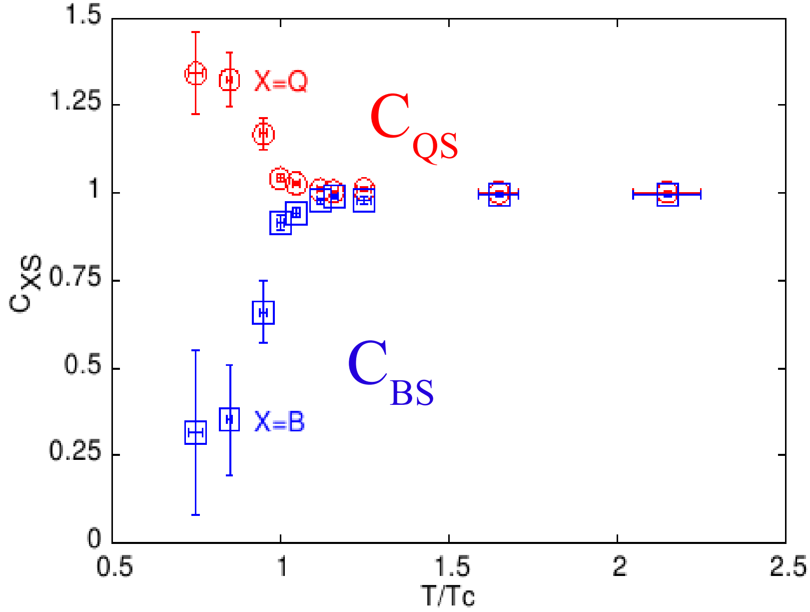
Independet quarks and
LATTICE QCD for T>1.1T_c

Bound state QGP

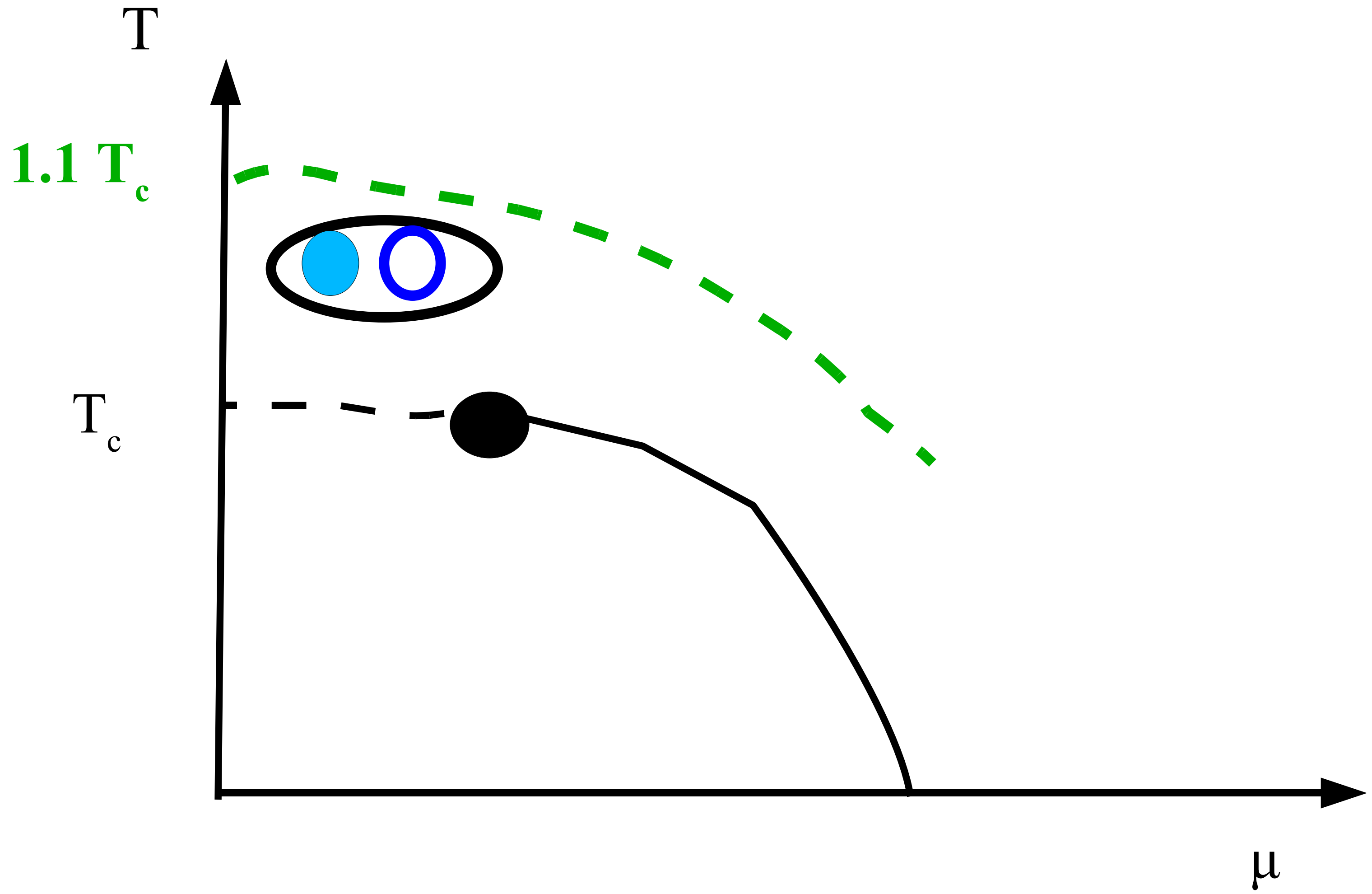


V.K, Majumder, Randrup PRL95:182301,2005

$$C_{QS} = -3\frac{\langle QS\rangle}{\langle S^2\rangle}$$



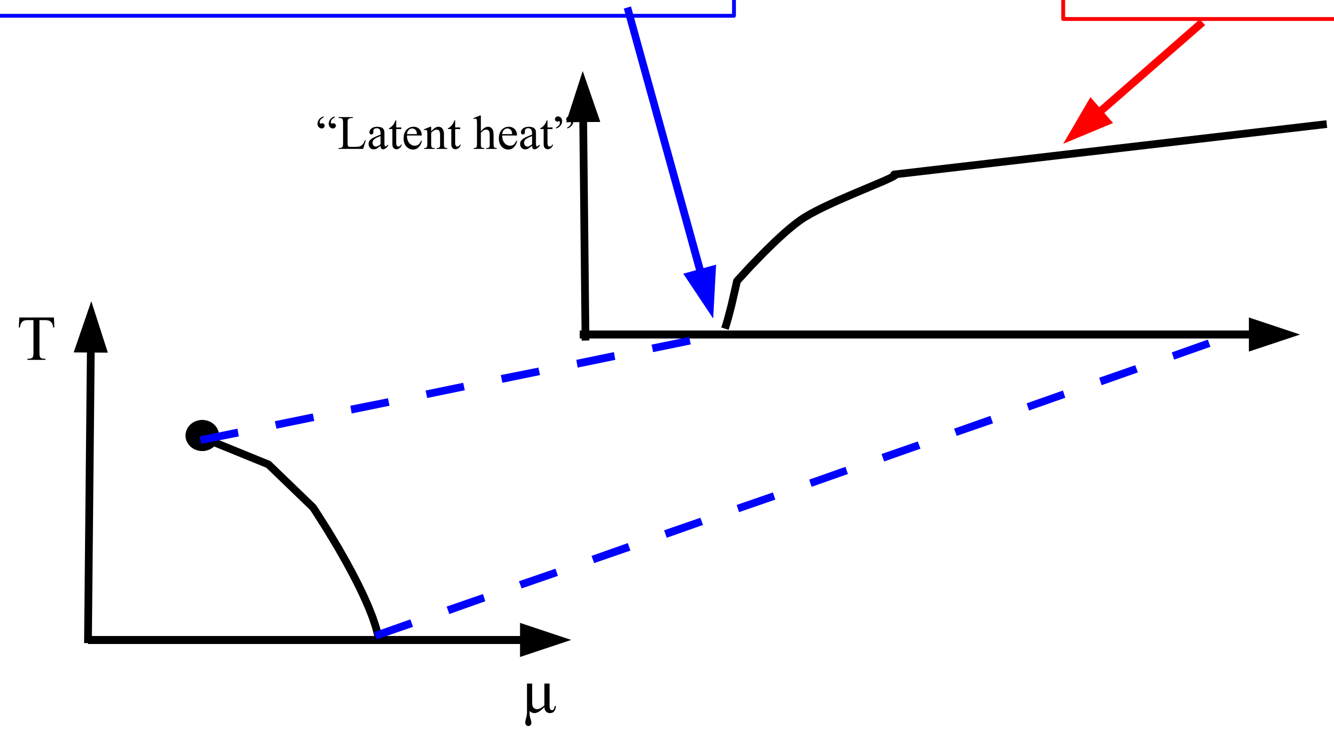
Gavai,Gupta, hep-lat/0510044


T
1.1 T_c
T_c
μ

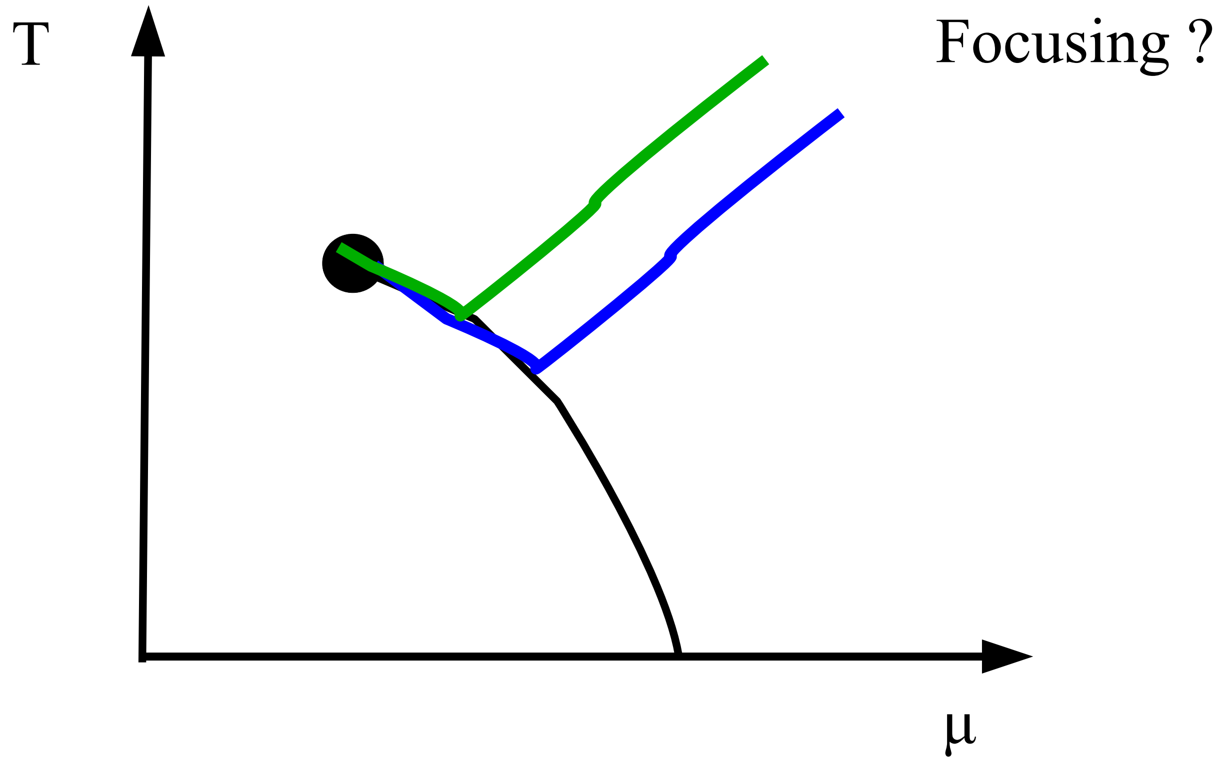
First order or second order?

Second order:
- Critical fluctuations
- Diverging Susceptibilities

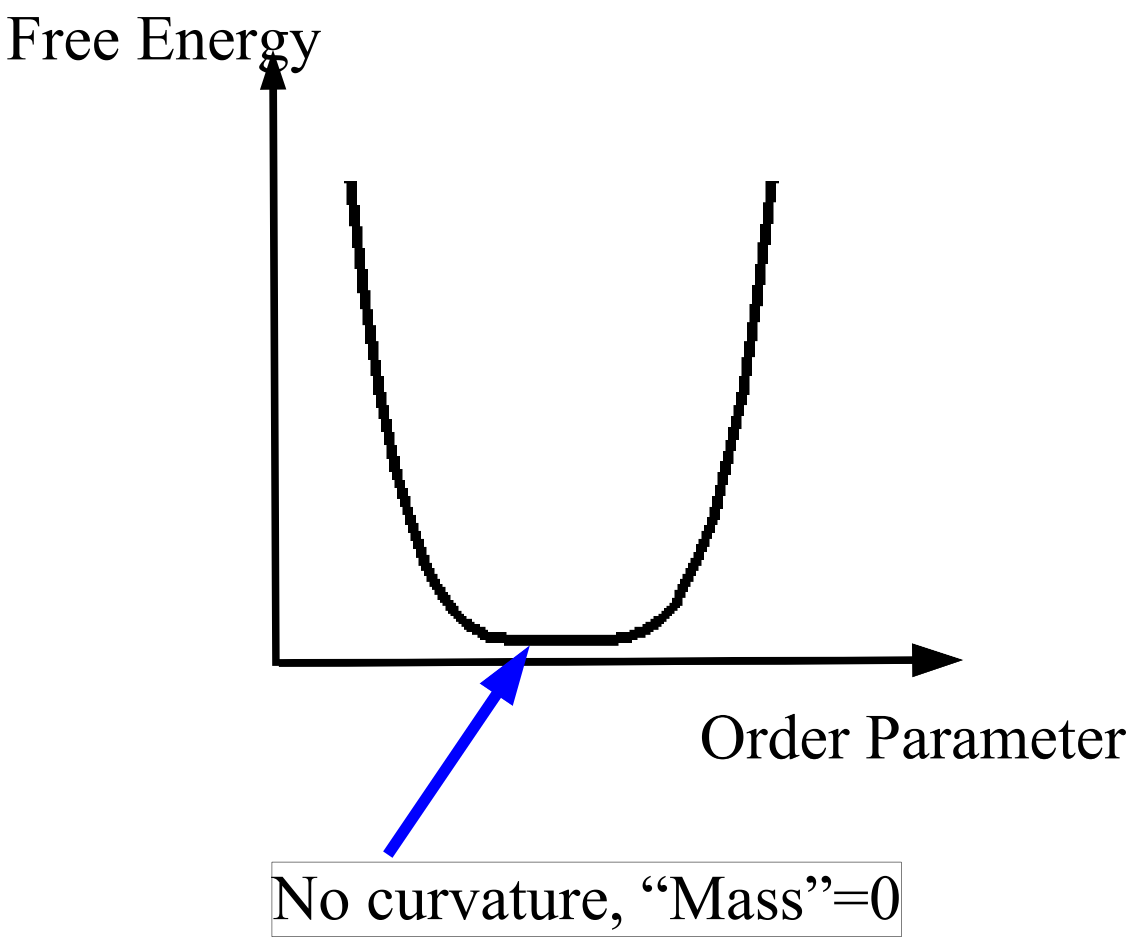
First order:
- Phase coexistence, bubbles
- Spinodal instabilities



First or second order?



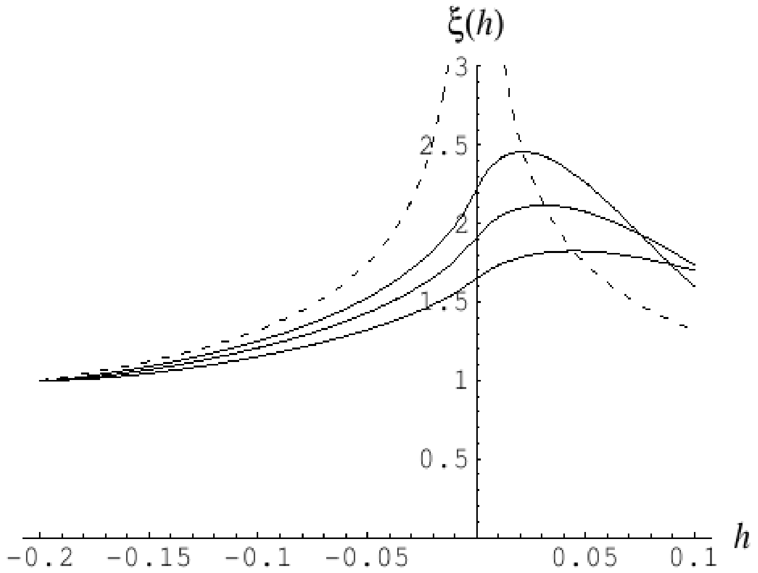
Second order



- Fluctuation of order parameter at all scales
- Diverging susceptibilities
  ~1/("Mass")^2
- Diverging correlation length
  ~1/("Mass")
- Universality
- Critical slowing down !

Second order

correlation length ~1/m_σ

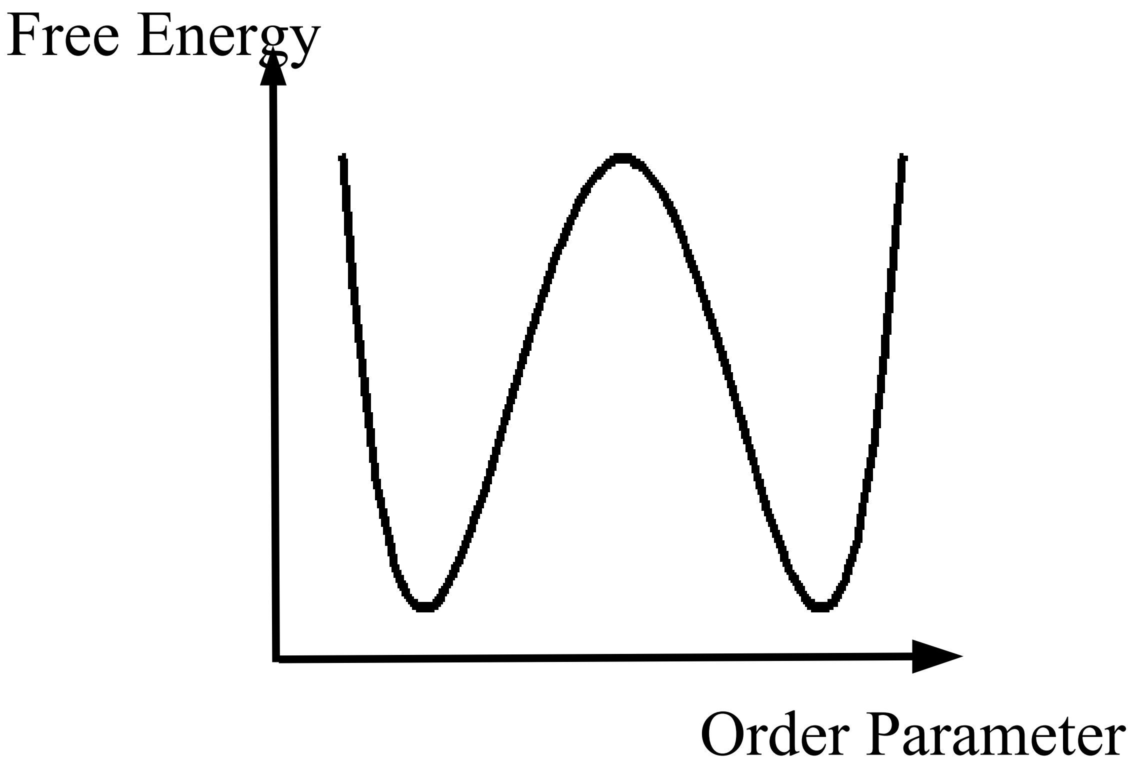


T/MeV 180 140 120

Bernikov, Rajagopal, hep-ph/9912274

- Critical slowing down
- limited sensitivity on model parameters
- Max. correlation length 2-3 fm
- Translates in **3-5%** effect in p_t-fluctuations

First order

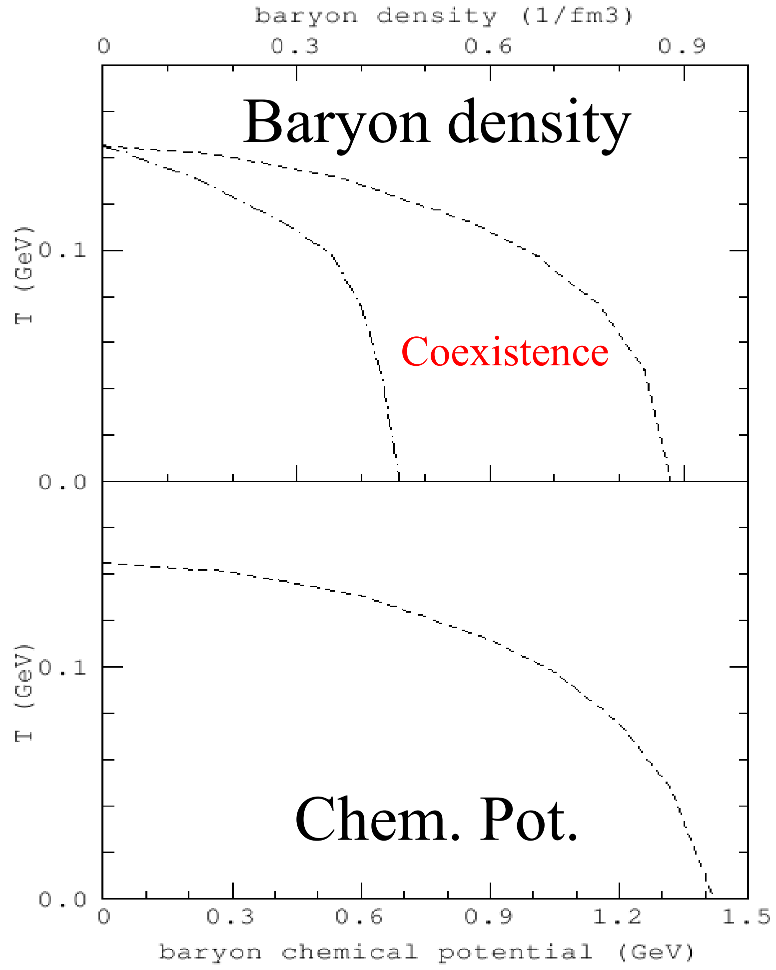


- Phase coexistence
- "Bubble" formation
  - Spatial fluctuations of order parameter
  - definite length scale
- Specific heat
- Dynamics: Spinodal instability

First order

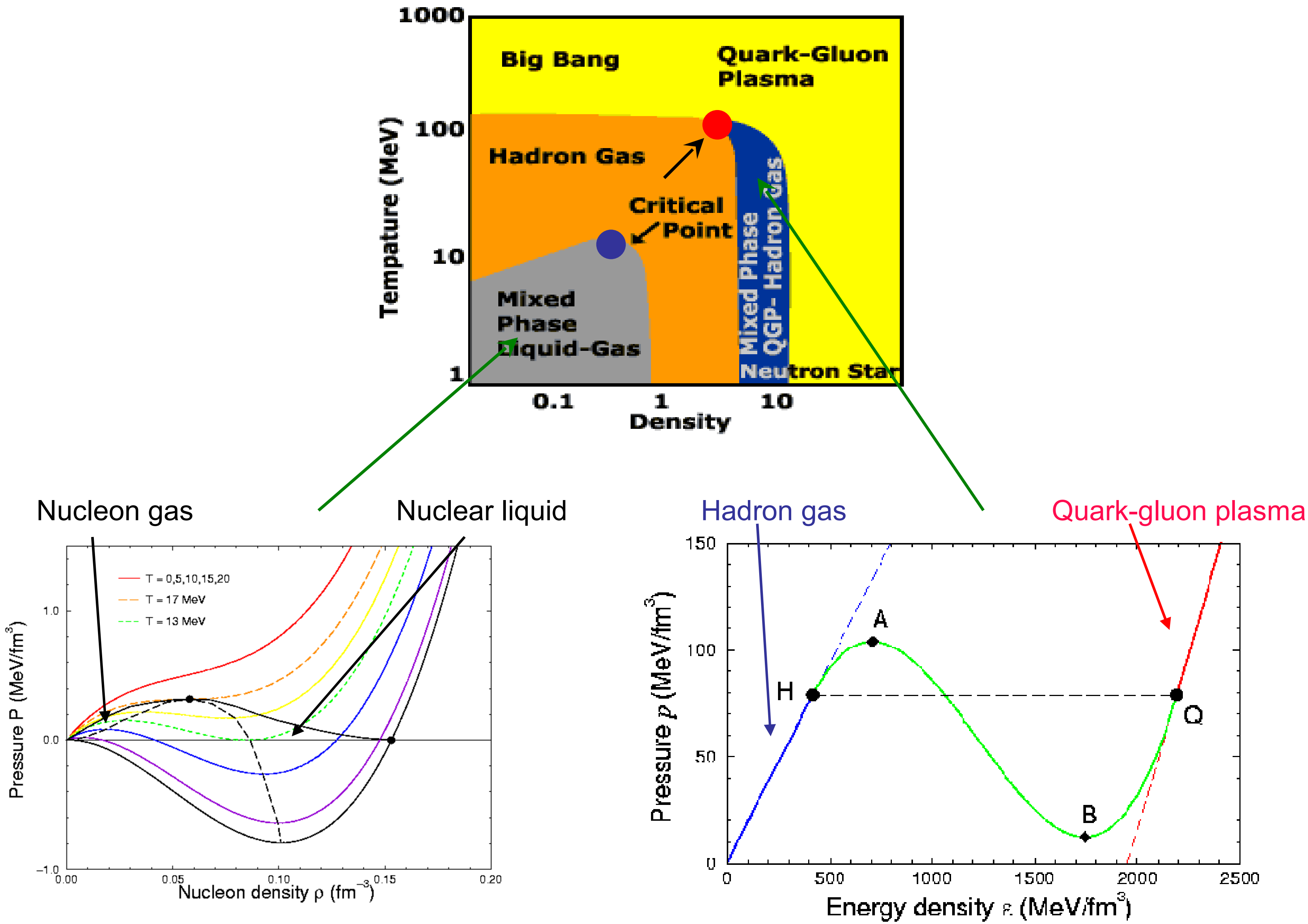
What are the phases?

“One” order parameter

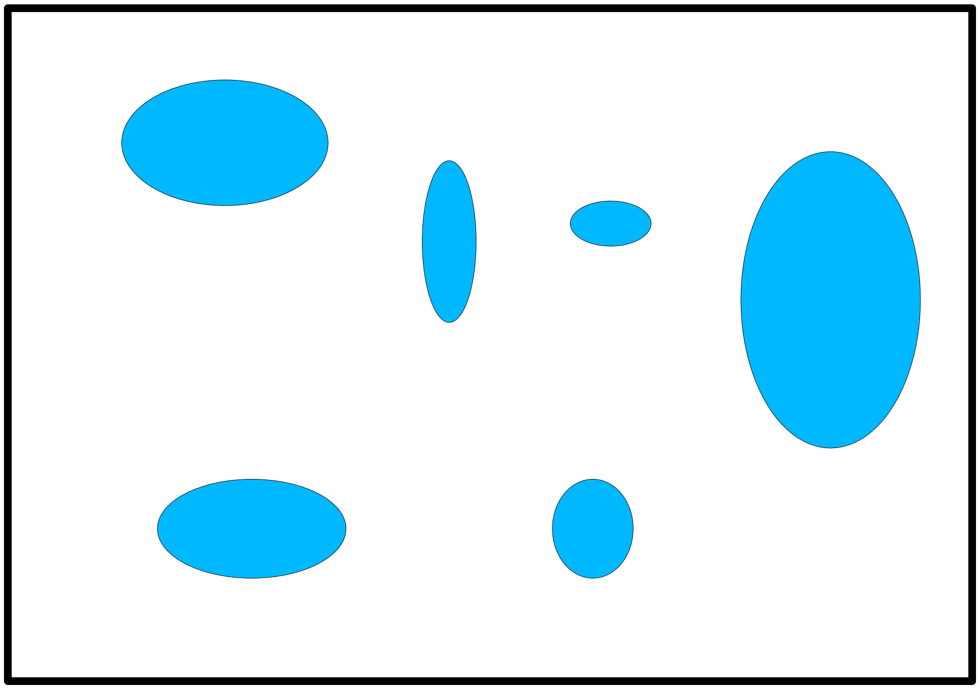


P. Braun-Munzinger and J. Stachel, Nucl.Phys.A606:320-328,1996

Phase diagram of strongly interacting matter



Baryon number fluctuations

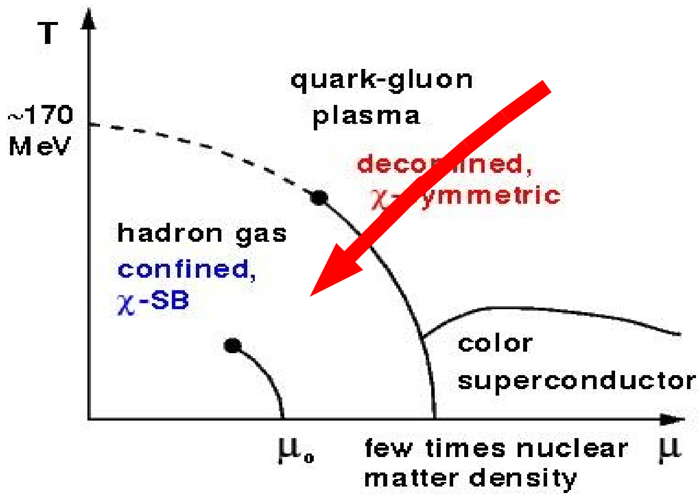


Strong spatial fluctuations

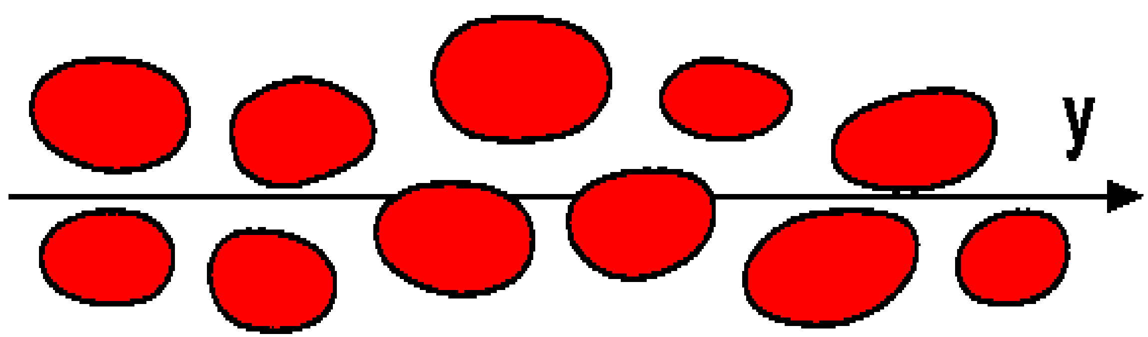
If $V_{domain} << V$, small effect on integrated Baryon Number fluctuations

$$\frac{\langle (\delta N)^2 \rangle}{\langle N \rangle} \approx \left(1 + \frac{(\Delta \rho)^2}{4 \bar{\rho}^2} \right)$$

Spinodal breakup



Spinodal decomposition:
- general phenomenon
- dynamical process
- typical "blob" size
  - depends on details of interaction



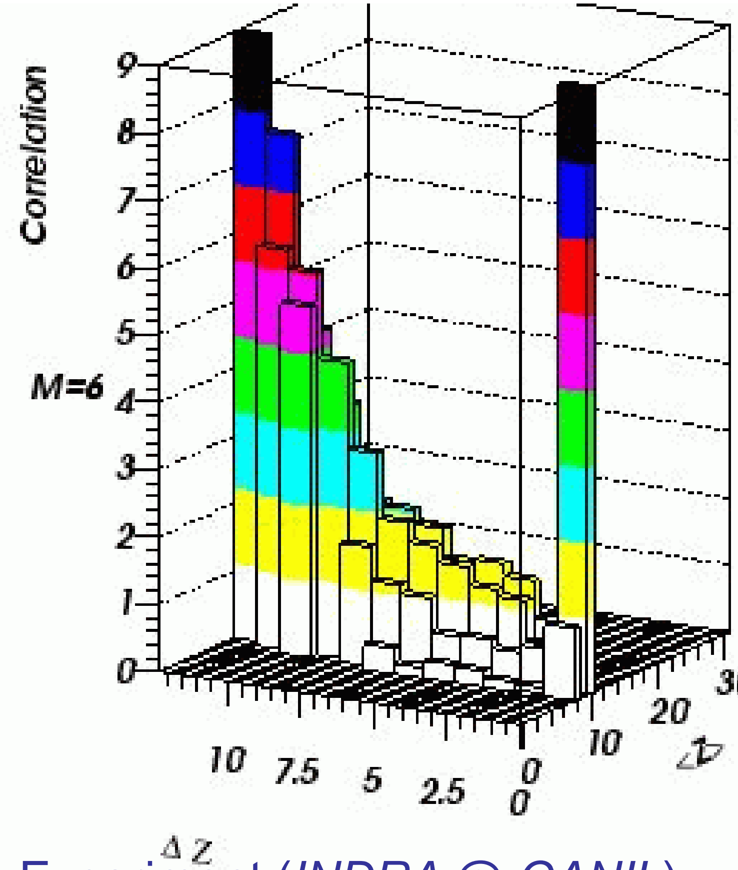
Spinodal decomposition in nuclear multifragmentation

occurs!

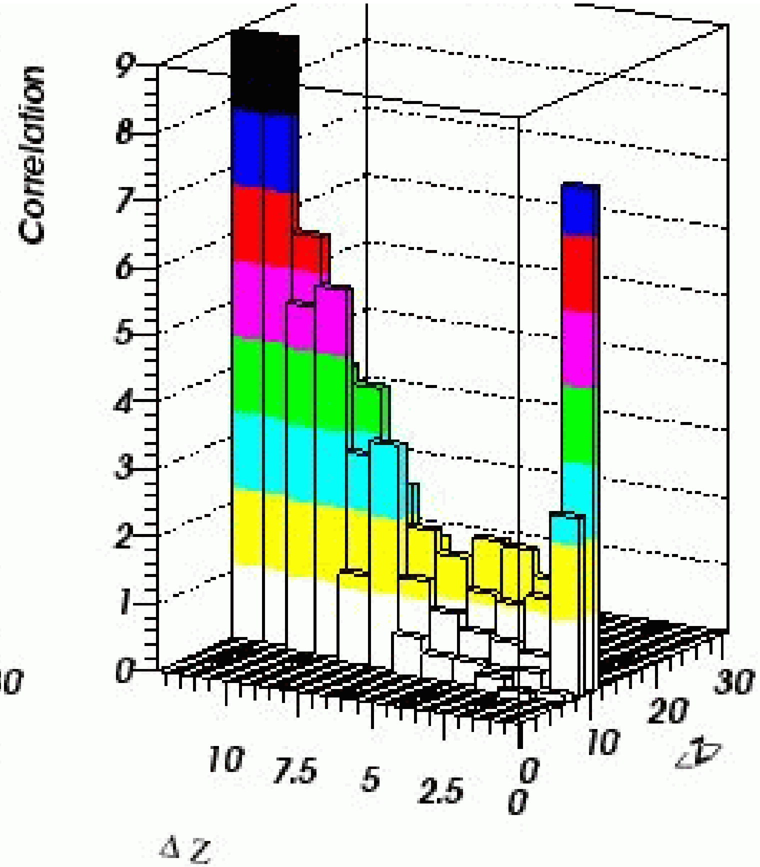
32 MeV/A Xe + Sn (b=0)

(select events with 6 IMFs)

Bin wrt { $\langle Z \rangle$: average IMF charge; ΔZ : dispersion in IMF charge }



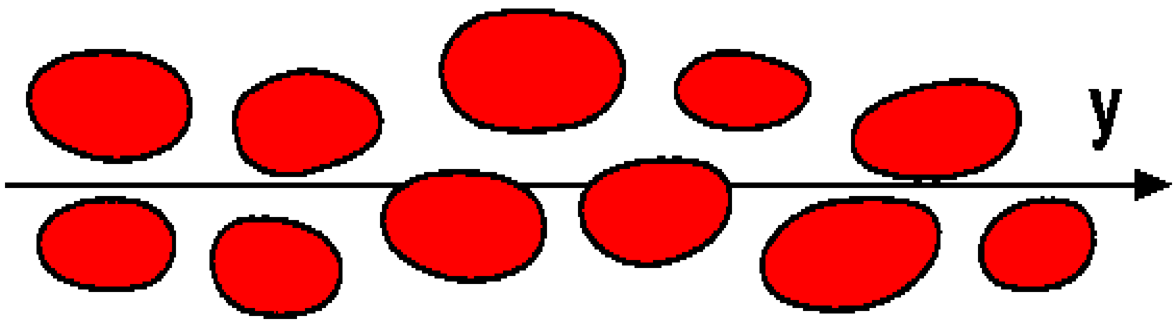
Experiment (*INDRA @ GANIL*)
Borderie *et al*, PRL 86 (2001) 3252



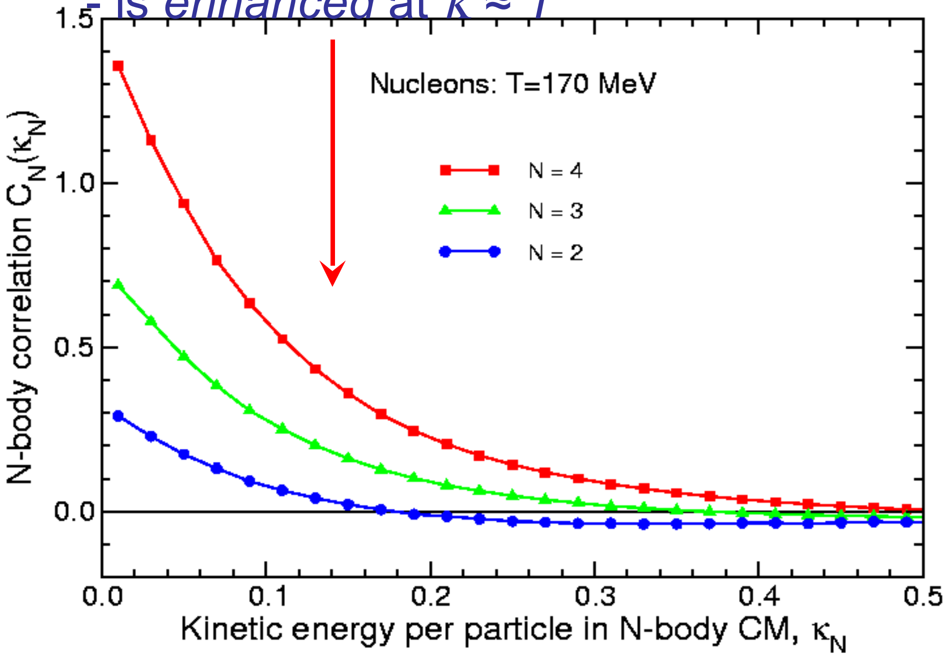
Pre-diction

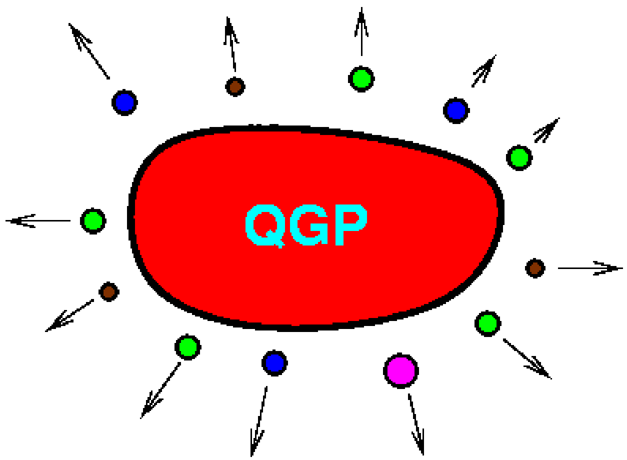
Theory (*Boltzmann-Langevin*)
Chomaz, Colonna, Randrup, …

N-particle correlations

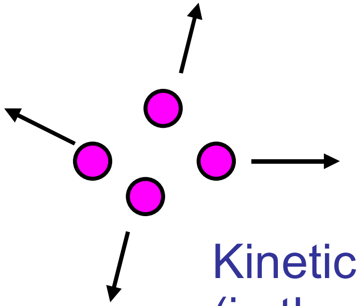


- is *enhanced* at $\kappa \approx T$





[J. Randrup, J. Heavy Ion Physics 22 (2005) 69]



Kinematic clumping =>

Invariant-mass correlations

Total four-momentum:

$$P\{\boldsymbol{p}_n\} = \sum_n (E_n, \boldsymbol{p}_n)$$

Kinetic energy per particle (in the *N*-body CM frame):

$$\kappa_N\{\boldsymbol{p}_n\} = \frac{1}{N}\left[[P\{\boldsymbol{p}_n\} \cdot P\{\boldsymbol{p}_n\}]^{\frac{1}{2}} - \sum_n m_n \right]$$

Distribution of κ:

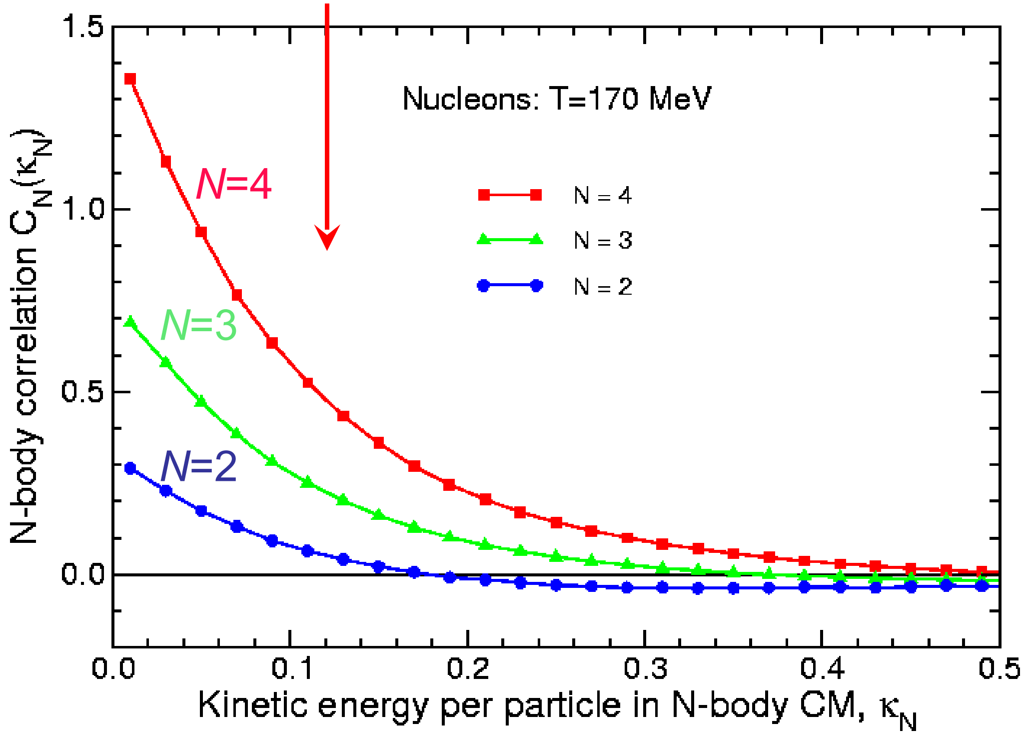
$$P_N(\kappa) \equiv \prec \delta(\kappa - \kappa_N\{\boldsymbol{p}_n\}) \succ$$

Correlation function:

$$C_N(\kappa) \equiv P_N(\kappa) \;/\; P_N^0(\kappa) - 1$$

Same event / *Mixed events*

- is *enhanced* at $\kappa \approx T$



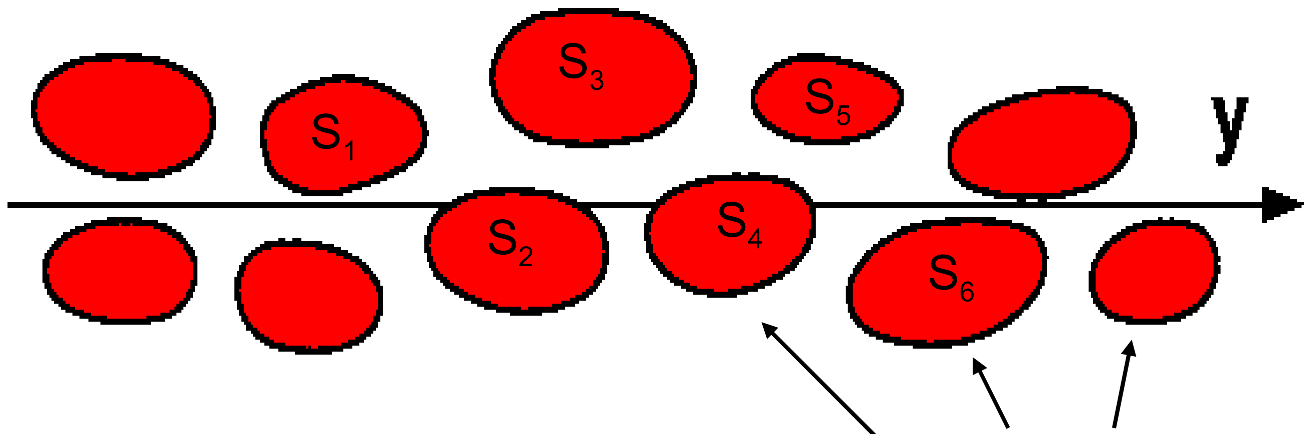
Higher-order correlations stand out more clearly!

(but require larger samples)

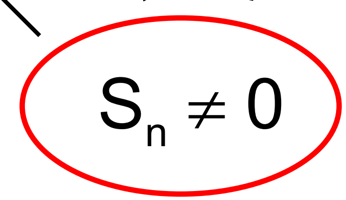
[J. Randrup, J. Heavy Ion Physics 22 (2005) 69]

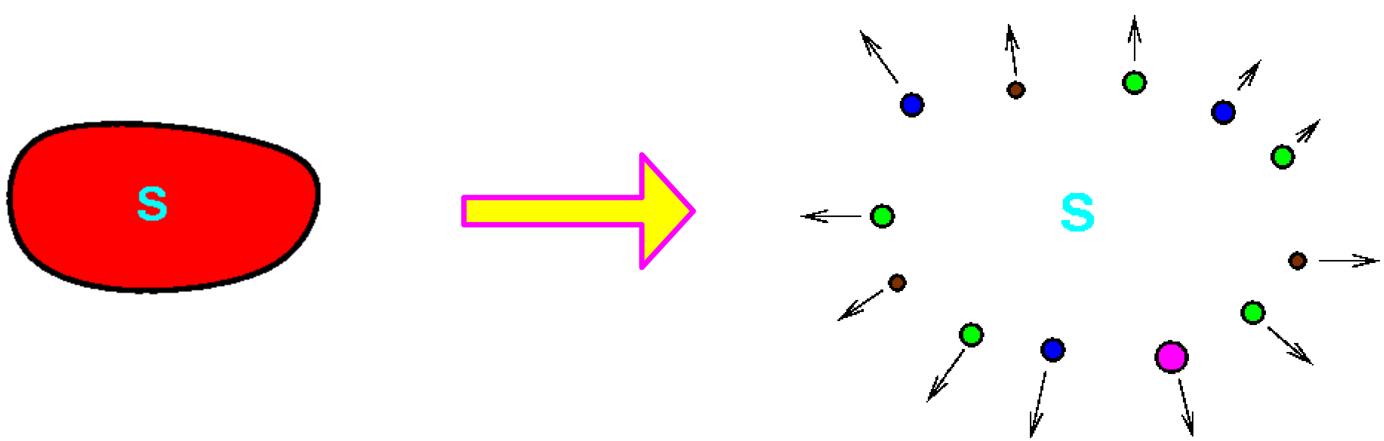
Strangeness correlations

The expanding system decomposes into plasma blobs which each contain a certain amount of strangeness:



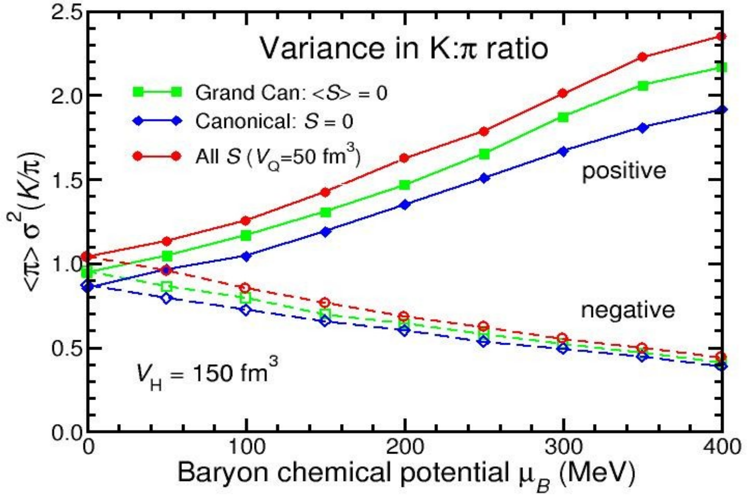
The hadronization of each isolated blob *conserves* strangeness:

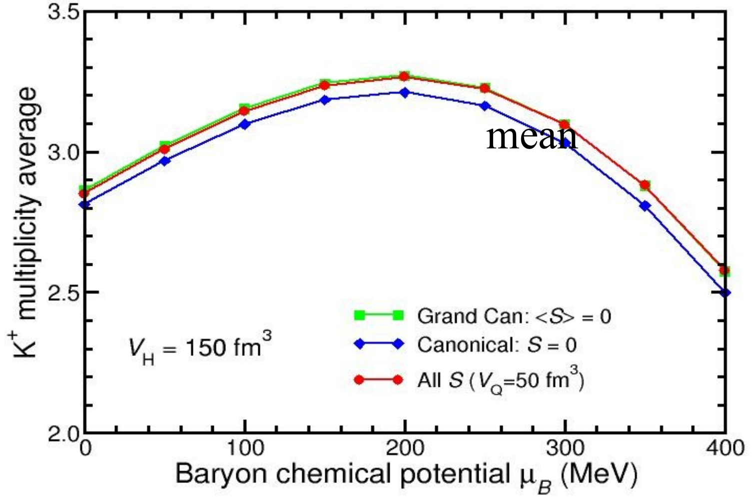




[V. Koch, A. Majumder, J. Randrup, Phys. Rev. C (in press)]

Some numbers





Variance: enhanced by ~10 %

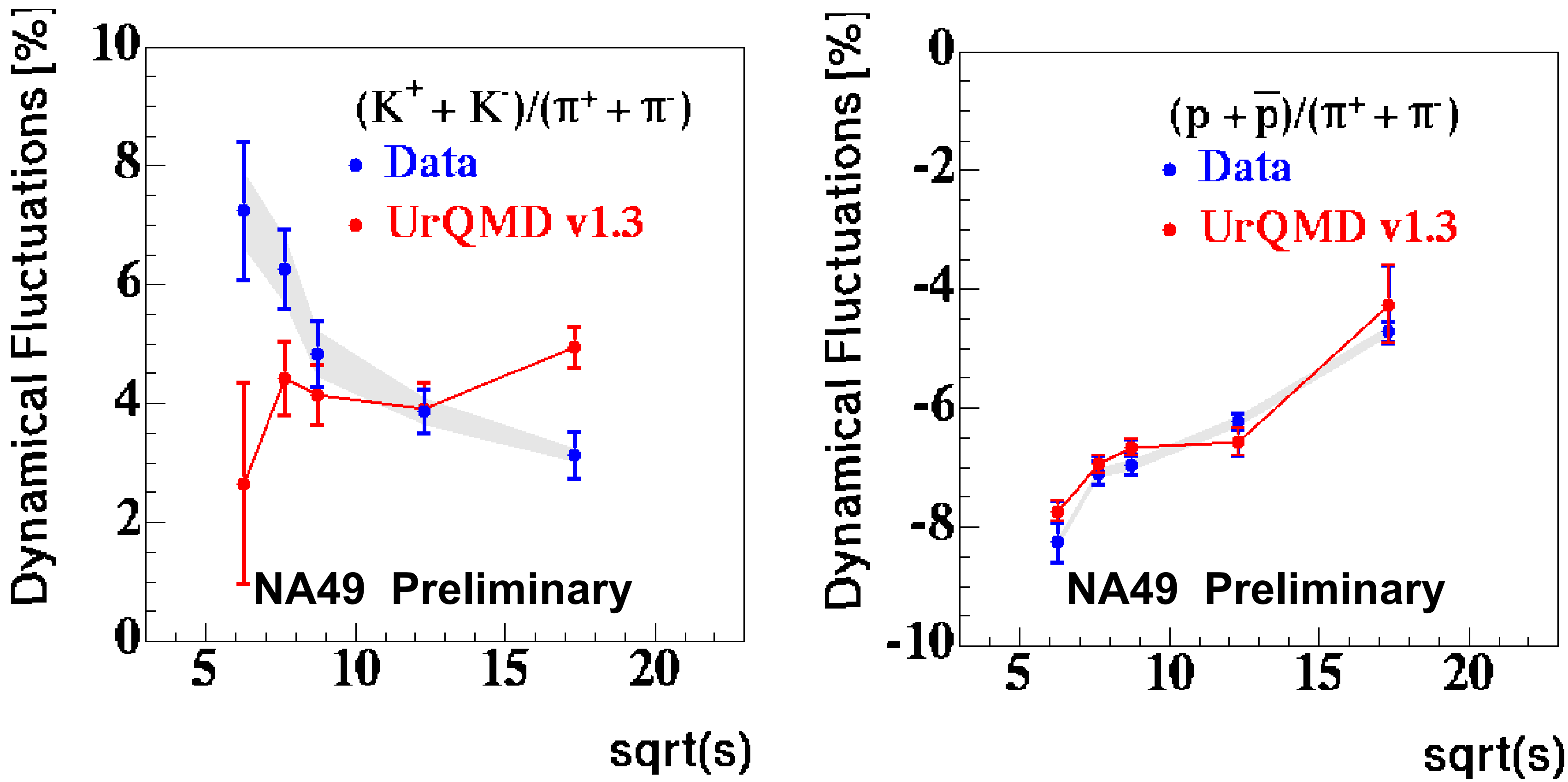
$V_{QGP} = 50\ fm^3$

$V_{hadron} = 150\ fm^3$

T = 170 MeV

Generally: variance is more enhanced than mean

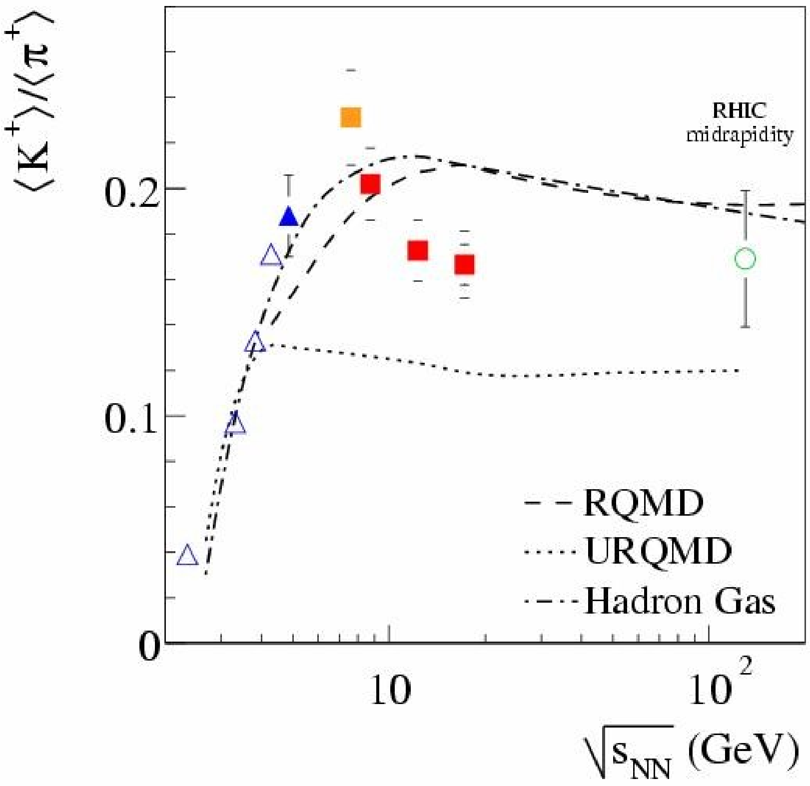
Fluctuations (NA49, QM2004)



- K/π fluctuations increase towards lower beam energy
  - Significant enhancement over hadronic cascade model
- p/π fluctuations are negative
  - indicates a strong contribution from resonance decays
  - **Where are the baryon number fluctuations????**

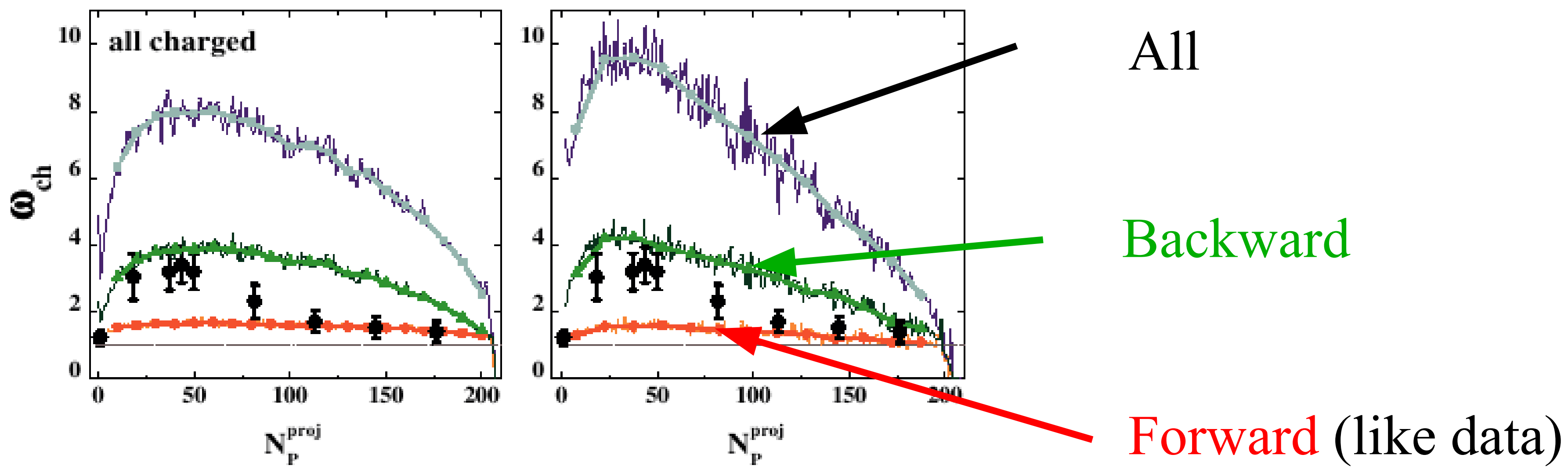
K/π Ratio

Fluctuations strong where inclusive K/π peaks!



Dynamics, event selection ...

Konchakovski et al, nucl-th/0511083



- Fluctuations are sensitive to dynamics (mixing of projectile and target material?)

- Event selection/trigger affects fluctuations → large Acceptance!

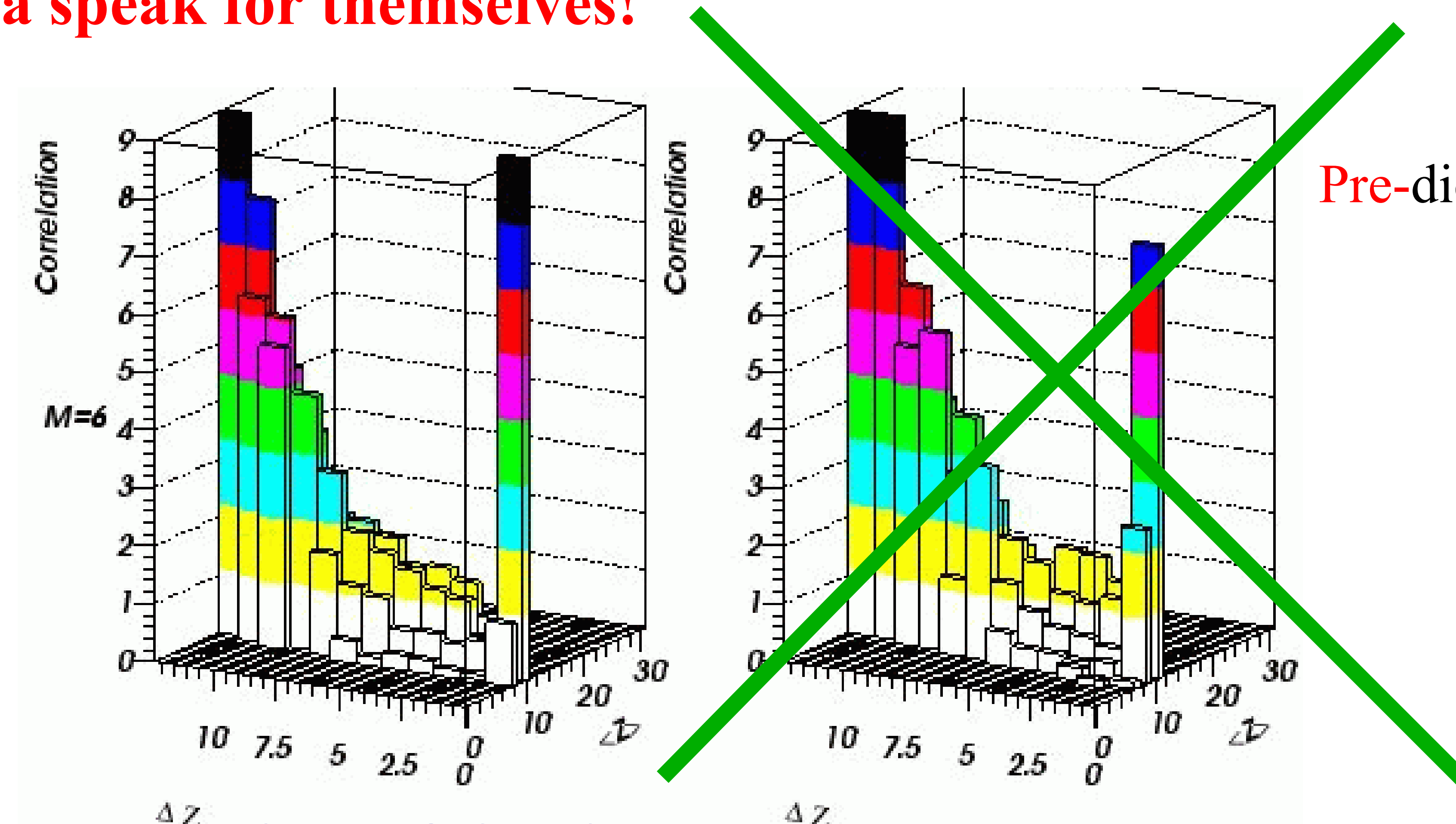
Things to do!

- Characterize the Phases
  - what are useful order parameters
- Test observables using static and dynamical models
  - Effects are small, comparable with 'trivial ones" such as quantum statistics, dynamics etc.
  - Only a well chosen observable / set of observables will prevent us from seeing Poisson
    - e.g. can we live without neutrons?

Spinodal decomposition in nuclear multifragmentation

occurs!

Data speak for themselves!



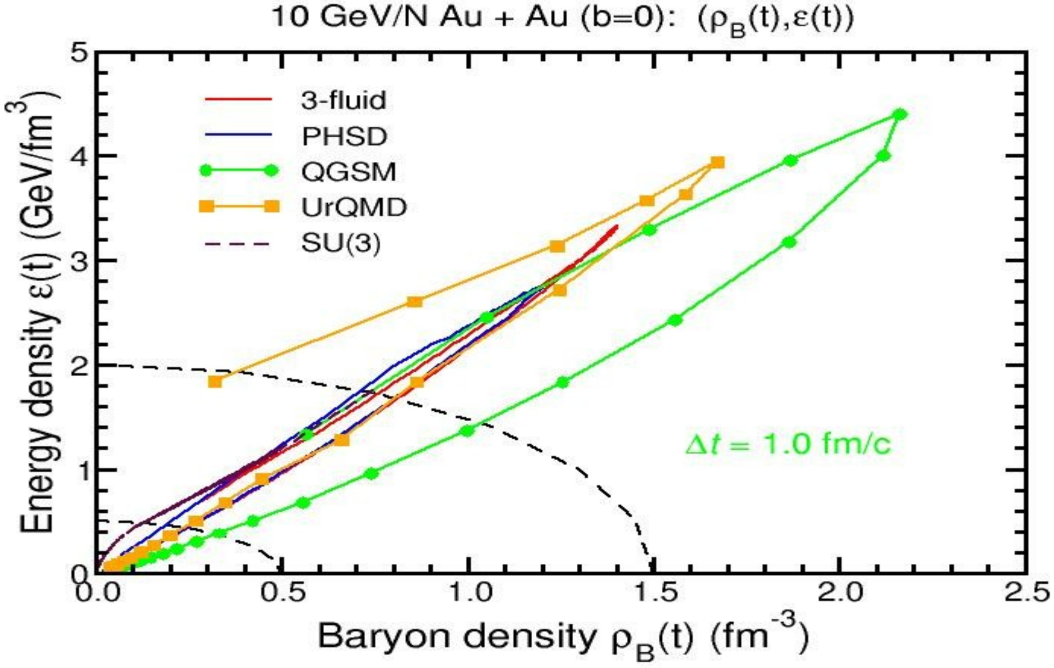
Pre-diction

Experiment (*INDRA @ GANIL*)
Borderie *et al*, PRL 86 (2001) 3252

Theory (*Boltzmann-Langevin*)
Chomaz, Colonna, Randrup, …

Phase trajectories

(thanks to J. Randrup and the dynamics working group)



10 AGeV!!!!!

Is there a chance to start experiments already with SIS 100?

Conclusions

- Fluctuations are in principle THE* probe for the phase diagram (susceptibilities).
- Need good order parameter (Baryon density?)
- Effects are expected to be few percent at best.
  - Trivial effects are of same size!
- Don't get hung up on critical point. Identification of coexistence is “good enough” as first result.
- Acceptance, Acceptance, Acceptance

* personal view