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Global Evolution

1st order tr.

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Appendices

Multi-Fluid Dynamics

Multi-Fluid Hydrodynamics

Hadronic Scenario as yet

Yu.B. Ivanov[1], V.N. Russkikh[1] and V.D. Toneev[2]

[1]GSI&Kurchatov Inst. (Moscow)

[2]GSI&JINR (Dubna)

CBM Workshop, GSI, Dec. 15-16, 2005

Outline

1. Motivations
2. Phys. Input
3. Simulations
4. Global Evolution
5. 1st order tr.
6. Summary
7. Appendices

Hydro: Conventional 1-Fluid versus Multi-Fluids

1-Fluid Hydro:

Energy–Momentum Conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Baryon Number Conservation

$$\partial_\mu J^\mu = 0$$

2-Fluid Hydro:

Energy–Momentum Exchange

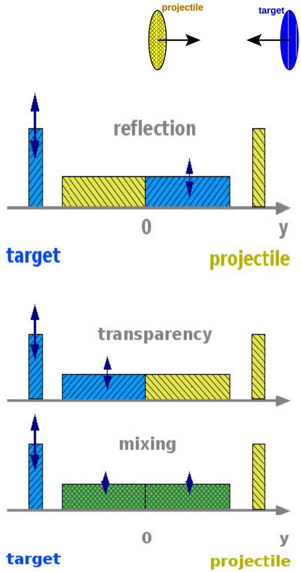
$$\partial_\mu T^{\mu\nu}_{proj.} = \text{Friction}$$

$$\partial_\mu T^{\mu\nu}_{target} = \text{Friction}$$

Total conservation: $\partial_\mu \left(T^{\mu\nu}_{proj.} + T^{\mu\nu}_{target} \right) = 0$

Baryon Number Conservation

$$\partial_\mu J^\mu_{proj.} = 0 \quad \partial_\mu J^\mu_{target} = 0$$



Gazdzicki&Gorenstein, hep-ph/0511058

From 2 to 3 Fluids

- Los Alamos 1978: 2-fluids
- GSI 1991: 2-fluids with mean-fields and hadrochemistry

2-fluids are good at $E_{lab} \approx 1$ A·GeV

At higher E_{lab} produced particles populate mid–rapidity

$\Downarrow$

3d Baryon-free (Fireball) Fluid

- Kurchatov Inst. 1988: (2+1/2)-fluids (with free-streaming pions)
- Frankfurt University 1993: 3-fluids
- **GSI 2003: 3-fluids with delayed formation of fireball**

$$\partial_\mu T^{\mu\nu}_{fireball} = \text{Delayed Source}$$

The source term is delayed due to a formation time τ

Important Physical Input

- Equation of State (EoS)
- Formation Time (τ)
- Freeze-out energy-density (ε_{frz})

  It is the same for different species
- Coalescence coefficients for fragments
- Friction: tuning factor for Friction

(!) Protons and neutrons are not distinguished!

3-Fluid Hydro
CBM/GSI, 16.12.2005
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In fact:

projectile–target friction = in terms of only *NN* cross sections (Satarov, (1990))

fireball-projectile(target) friction = in terms of only πN resonance cross sections (Russkikh, et al., (2004))

Uncertainties in Friction:

- poor usage of various cross sections
- medium effects
- multiparticle collisions

Therefore:

Projectile-Target Friction
$= \xi^2(s_{pt})$ (Friction estimated by *NN* cross sections)

$\xi^2(s_{pt})$ = tuning factor for Friction

$s_{pt} = m_N^2 \left(u_p^\nu + u_t^\nu\right)^2$ = mean invariant energy squared

Friction Fit

In fact:

projectile–target friction = in terms of only *NN* cross sections (Satarov, (1990))

fireball-projectile(target) friction = in terms of only πN resonance cross sections (Russkikh, et al., (2004))

Uncertainties in Friction:

- poor usage of various cross sections
- medium effects
- multiparticle collisions

Therefore:

PROJECTILE-TARGET FRICTION
$= \xi^2(s_{pt})$ (FRICTION ESTIMATED BY *NN* CROSS SECTIONS)

$\xi^2(s_{pt})$ = tuning factor for Friction

$s_{pt} = m_N^2 \left(u_p^\nu + u_t^\nu\right)^2$ = mean invariant energy squared

Hadronic EoS (Galitsky&Mishustin 1979)

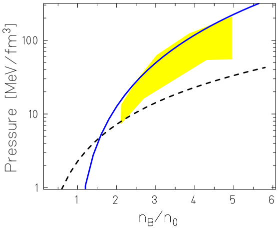
Energy density:
$$\varepsilon(n_B,T) = \underbrace{\varepsilon_{gas}(n_B,T)}_{\text{hadron gas in mean field}} + \underbrace{W(n_B)}_{\text{mean field}}$$

Pressure:
$$P(n_B,T) = \underbrace{P_{gas}(n_B,T)}_{\text{hadron gas in mean field}} \underbrace{+ n_B \frac{dW(n_B)}{dn_B} - W}_{\text{mean field}}$$

$$W(n_B) = n_B m_N \left[-b\left(\frac{n_B}{n_0}\right) + c\left(\frac{n_B}{n_0}\right)^{\gamma+1} \right]$$

$W(n_B)$ saturates the cold matter at $n_0 = 0.15\ \text{fm}^{-3}$ and $\varepsilon(n_0, T=0)/n_0 - m_N =$ -16 MeV, and provides incompressibility $K = 235$ MeV.

$T = 0$



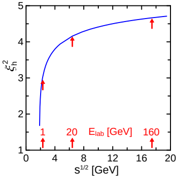
Danielewicz, Lacey, Lynch (2002)

A natural reference point for any other more elaborate EoS!

Physical Input for Hadronic Scenario

- GM Hadronic EoS
- Enhanced Friction

  fitted to observed stopping power (i.e. proton rapidity distributions)



$F_{pt} = \xi_h^2(s)$(Friction estimated by NN cross sections)

$s = m_N^2 \left(u_p^\nu + u_t^\nu\right)^2$ = mean inv. energy squared

$$\xi_h^2(s) = 1 + 3\left[\ln\left(s/(2m_N)^2\right)^{1/2}\right]^{1/4}$$

Function of 3 variables, $dN/dy(y, b, E_{lab})$, is fitted by function of 1 variable, $\xi_h^2(s)$!

- Freeze-out: $\varepsilon_{frz} \approx 0.2$ GeV/fm^3 (≈ 0.1 GeV/fm^3 at $E_{lab} < 3$ A·GeV)

  fitted to observed proton p_T spectra

  Funct. of 4 variables, $(1/2\pi m_T)d^2N/dm_T dy(m_T, y, b, E_{lab})$, is fitted by 1 parameter ε_{frz}!

- Formation Time $\tau = 2$ fm/c $\Rightarrow$ $\tau_{particle} \approx 1$ fm/c

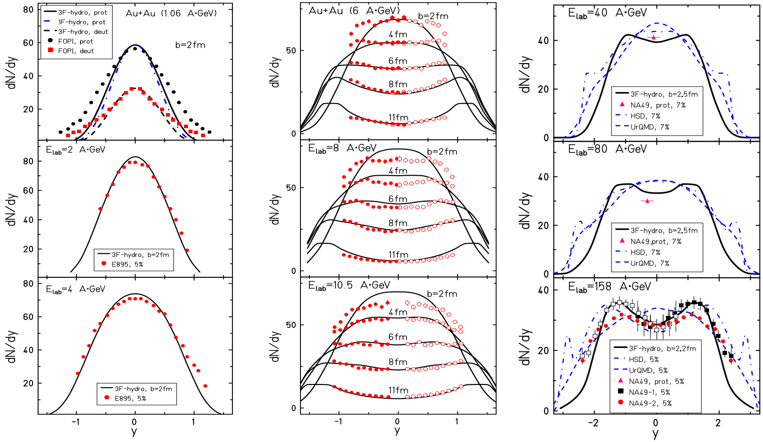
  fitted to observed pion production at $E_{lab} > 30$ A·GeV

◦ $\tau_{particle} < \tau = \int d^3p\ \tau_{particle}\gamma_{particle} f(p) \Big/ \int d^3p\ f(p)$ If $T \approx 100$ MeV, $\tau_{particle} \approx 1$ fm/c.

Proton and ($p - \bar{p}$) Rapidity Distributions

Fitted by Friction: $\xi_h^2(s)$

SIS AGS SPS



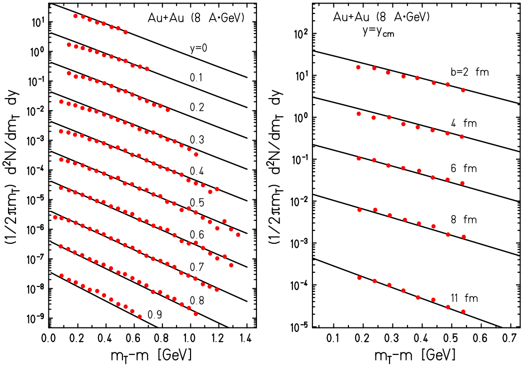
FOPI: NP **A610** (1996) 49c [Au(1.06 GeV/nucl.)+Au]
E895: PR **C68** (2003) 054905 [Au(2 and 4 GeV/nucl.)+Au]
E917: PRL **86** (2001) 1970 [Au(6, 8 and 10.5 GeV/nucl.)+Au]

NA49 (prot.): PR **C69** (2004) 024902
NA49–1: PRL **82** (1999) 2471
NA49–2(prelim.): NP **A661** (1999) 362c
Models: H. Weber, et al., PR **C67** (2003) 014904

Transverse-Mass Distributions of Protons

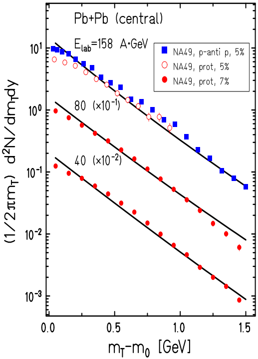
Fitted by Freeze-out: $\varepsilon_{frz} \approx 0.2$ GeV/fm^3

AGS



E917: PRL **86** (2001) 1970

SPS



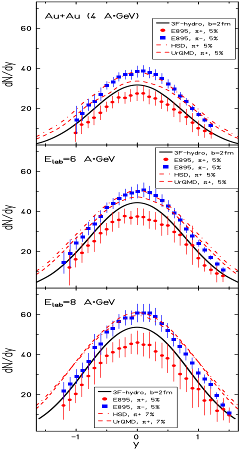
b = 2.2 fm for 158 AGeV, and b = 2.5 fm for 40 and 80 AGeV are experimental estimates

NA49: PRL **82** (1999) 2471

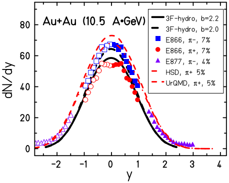
NA49: NP **A715** (2003) 166c

Pion Rapidity Spectra: almost “prediction”

Fitted by Formation Time at $E_{lab} > 30$ A·GeV: $\tau = 2$ fm/c



E895: PR **C68** (2003) 054905

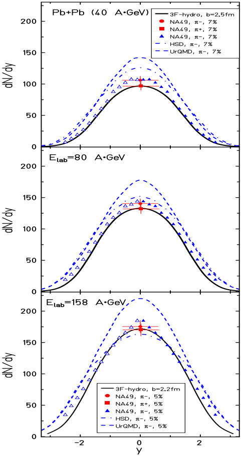


b = 2.0 fm for 7%σ and b = 1.5 fm for 4%σ are experimental estimates.

E895: PR **C68** (2003) 054905

E877: PR **C62** (2000) 024901

Transport models: H. Weber, et al., PR **C67** (2003) 014904

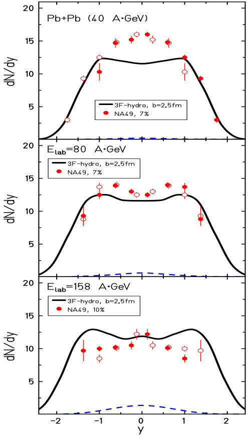


b = 2.2 fm for 158 AGeV, and b = 2.5 fm for 40 and 80 AGeV are experimental estimates

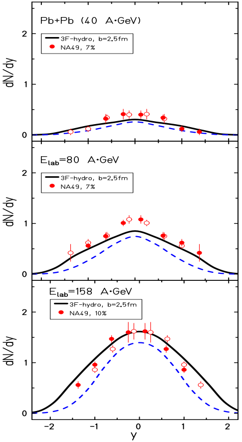
NA49: PR **C66** (2002) 054902

Rare-Particle Rapidity Distributions: “prediction”

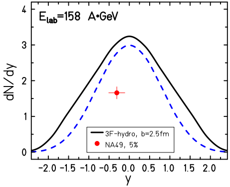
$\Lambda + \Sigma^0$



$\bar{\Lambda} + \bar{\Sigma}^0$



$\bar{p}$



NA49: NP **A661** (1999) 383c

Decays of anti-hyperons are excluded.

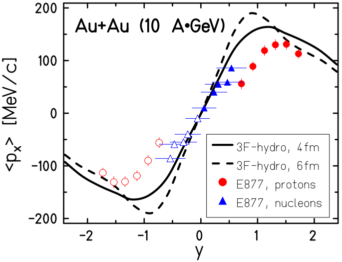
NA49: nucl-ex/0311024

dashed line = contribution from the fireball fluid

In-plane transverse momentum per hadron

$$\langle p_x \rangle (y) = \frac{\int d^2p_T \; p_x \; (dN/dy \; d^2p_T)}{\int d^2p_T \; (dN/dy \; d^2p_T)}$$

E877: PR **C56** (1997) 3254

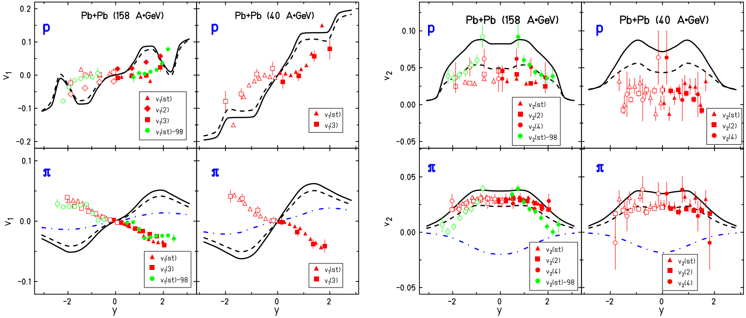


$$\text{Directed flow} = v_1(y) = \int d^2p_T \; \frac{p_x}{p_T} \; \frac{dN}{dy \; d^2p_T} \Big/ \int d^2p_T \; \frac{dN}{dy \; d^2p_T}$$

$$\text{Elliptic flow} = v_2(y) = \int d^2p_T \; \frac{p_x^2 - p_y^2}{p_T^2} \; \frac{dN}{dy \; d^2p_T} \Big/ \int d^2p_T \; \frac{dN}{dy \; d^2p_T}$$

Flow: "prediction"

directed, v_1 elliptic, v_2



3-fluid hydro: b = 5.6 fm (solid line) and b= 4 fm (dashed line) for mid-central collisions

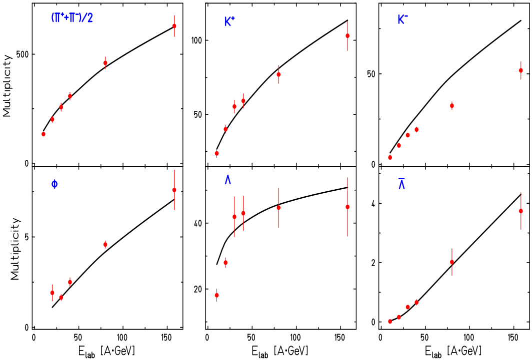
$v_{1,2}(st)$: standard method; $v_{1,2}(n)$: n-particle correlation method (N.Borghini, et al., PR **C93** (2001) 054906)

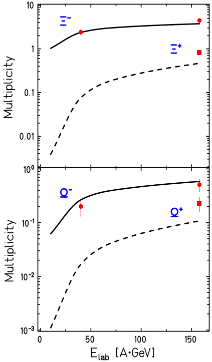
NA49: Phys. Rev. Lett. **80** (1998) 4136 NA49: Phys. Rev. **C68** (2003) 034903

(1) Poor reproduction of proton v_1: Hadronic GM EoS is too hard
(2) Poor reproduction of pion v_1: Pion shadowing is absent, and (1)

Multiplicities: “prediction”

non-strange | strange | multi-strange



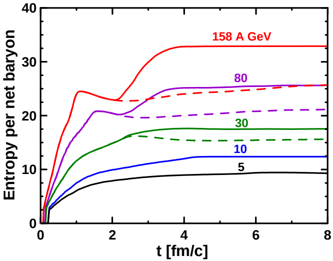


as summarized by F.Becattini, M.Gazdzicki et al., hep-ph/0310049

NA49: nucl-ex/0409004

3F-hydro: b=2.2 fm, grand canonical ensemble, unique freeze-out

Thermalization?



—— : total entropy
- - - - : baryon-rich subsyst. entropy
⟸ Dip ⇐ unformed fireball matter

(Entropy of baryon-rich subsyst.)≈const

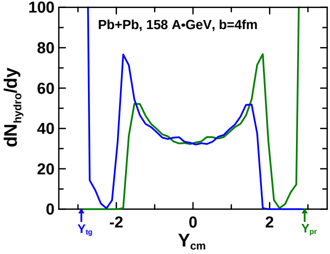
at the expansion stage

⇓

Thermalization!

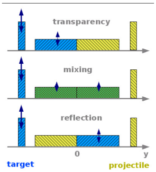
of baryon-rich subsystem

Frozen-out baryon number distribut.



in **target** and **projectile** matter

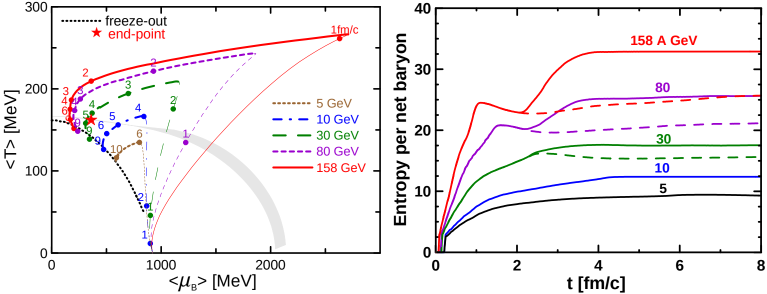
⟹



Gazdzicki&Gorenstein, hep-ph/0511058

"Mixing-type"Thermalization

central $Pb + Pb$ collisions

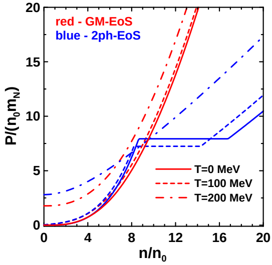


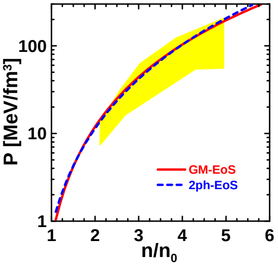
Phase-separation region only to guide an eye: No phase transition in GM EoS!

Phase-separation region: MIT-bag model + hadronic gas with repulsion (Toneev, et al., EPJ **C32**, 399 (2004))
⋆ **Critical end-point:** Z.Fodor, S.D.Katz, hep-lat/0402006.
Freeze-out curve: J.Cleymans, K.Redlich, Phys. Rev. Lett. **81** (1998) 5284.

EoS with 1st order phase transition

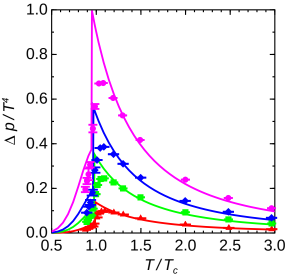
A.S.Khvorostukhin, V.V.Skokov, K.Redlich, V.D.Toneev, to be published

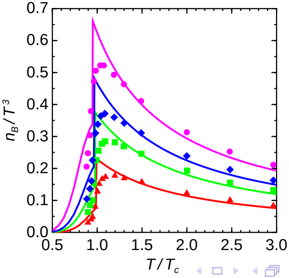




Danielewicz, Lacey, Lynch (2002)

Bag with heavy quarks ($m_{u/d} = 70MeV$, $m_s = 145MeV$) and gluons (700 MeV), to fit lattice data (Fodor et al.)

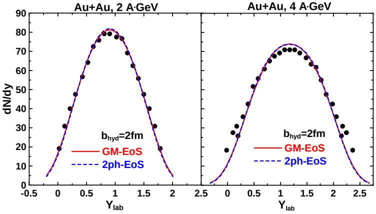


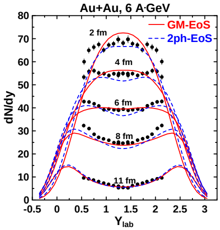


μ_b = 210, 330, 410, 530 MeV

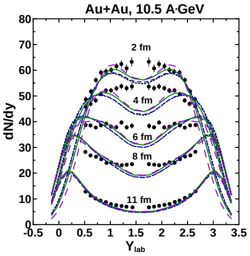
2ph-EoS Preliminarily!!!

Proton Rapidity Distributions (Preliminary!)

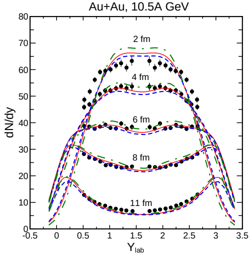




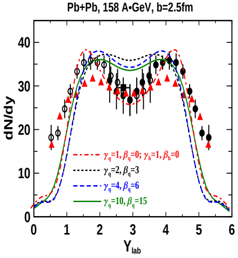
2ph-EoS: variation of friction



Hadr. EoS: variation of friction



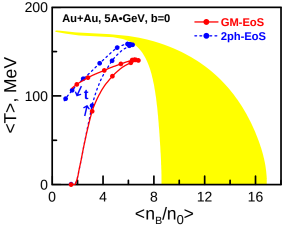
2ph-EoS: variation of friction

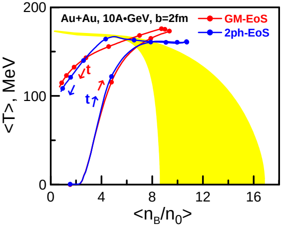


2ph-EoS Preliminarily!!!

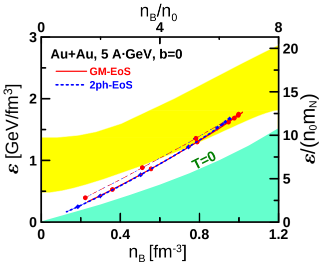
Dynamic Trajectories for central box 4 fm × 4 fm × 4 fm/γ_{cm} (Preliminary!)

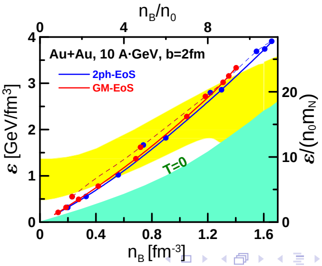
Collective motion excluded: Average over local rest frames





Collective motion included: Average over global cm frame





Summary

Hydrodynamics addresses **Equation of State**!
Simple hadronic EoS $\Longrightarrow$ Natural reference point

- Simple *hadronic EoS* reasonably reproduces observables (except flows).
  - Flows require a softer EoS and hence leave **room for phase transition to quark-gluon phase**.
- **Thermalization** of baryon-rich subsyst. at expansion stage.
- **EoS with 1st order ph. tr.**: no conclusions, only bad feelings

- Outlook
  - EoS with 1st order phase transition is comming.
  - Analysis of larger body of exp. data is comming.
  - Anybody can try her/his favorable EoS! The source code of the 3-fluid hydro is available at http://theory.gsi.de/∼mfd/

Global Dynamics: baryon density evolution in reaction plane

3-Fluid Hydro

CBM/GSI, 16.12.2005

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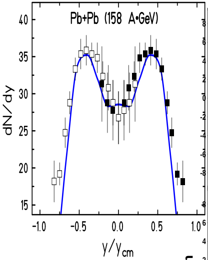

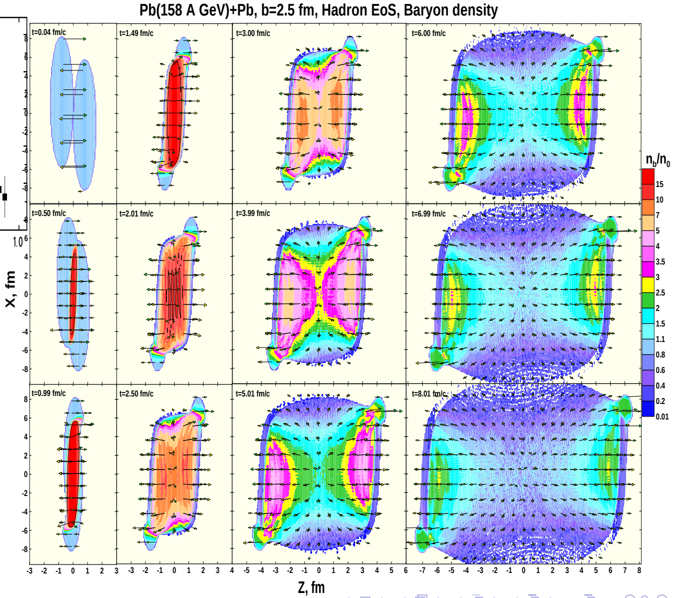

Global Dynamics: baryon density evolution in reaction plane

3-Fluid Hydro

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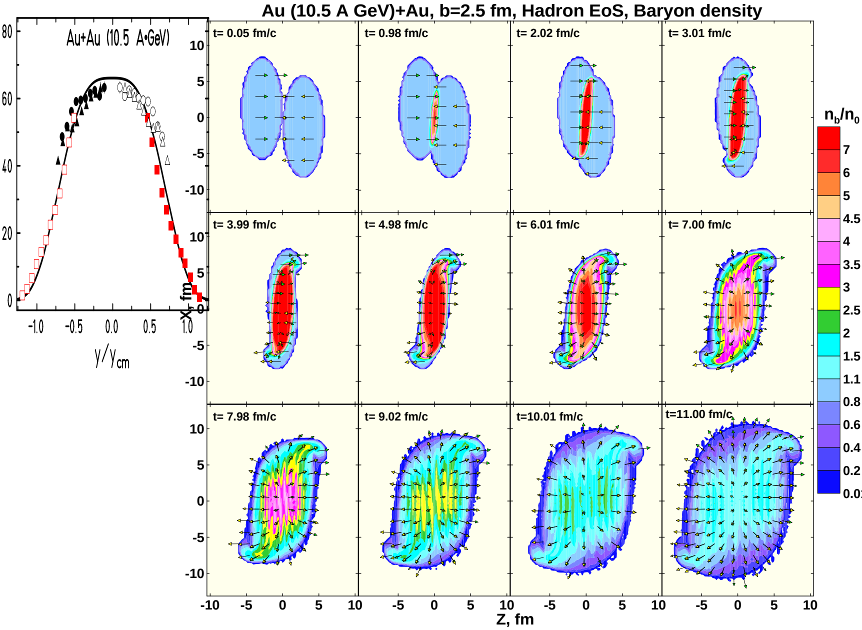
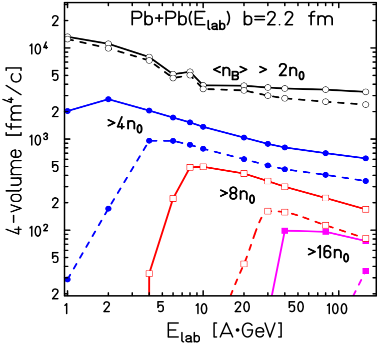
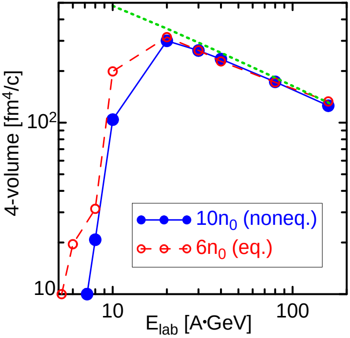


Invariant 4-Volume

How long and in which volume a quantity Q exceeds a Q_0 value?

$$V_4(Q) = \int d^4x \; \Theta(q - Q)$$





Solid lines: $Q = n_B =$ baryonic density

Dashed lines: $Q = n_B^{(eq)} =$ baryonic density of thermalized matter

Lorentz-contracted cylinder: $V_4(E_{lab}) = \pi R^2 \, (2R/\gamma_{cm}) \cdot \delta t$ with R=4 fm, δt=3 fm/c

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Freeze-out

- Criterion: $\underbrace{\textit{Local}}_{(\text{at } x \text{ position})}$ $\underbrace{\textit{proper}}_{(\text{in local rest frame})}$ $\underbrace{\textit{energy density of matter}}_{(\text{summed over all fluids})}$

  is less than ε_{frz}

- Freeze-out shock: T_{hydro} and μ_{hydro} are mapped on T_{gas} and μ_{gas} proceeding from baryon, energy and momentum conservations.

Energy accumulated in "mean fields" is released!

- Freeze-out *a là* Milekhin

$$E\frac{dN}{d^3p} = \int f_{gas}(x,p)p^\mu d\sigma_\mu, \qquad d\sigma_\mu = u_\mu (d^3x)_{proper}$$

$u_\mu =$ hydro 4-velocity proper = in the frame, where u_μ = (1,0,0,0)

- In "space-like regions": Very similar to Cooper-Frye
- No "time-like freeze-out": Only tiny fireballs are frozen out.
- No problem with Cooper-Frye's negative contributions into particle numbers
- Baryon number, energy and momentum are exactly conserved! ($P = 0$ on the boundary)
- Problem of shadowing still persists

- **Further study of Freeze-out is needed!**

Freeze-out

- Criterion: $\underbrace{\textit{Local}}_{(at\ x\ position)}$ $\underbrace{\textit{proper}}_{(in\ local\ rest\ frame)}$ $\underbrace{\textit{energy density of matter}}_{(summed\ over\ all\ fluids)}$

  is less than ε_{frz}

- Freeze-out shock: T_{hydro} and μ_{hydro} are mapped on T_{gas} and μ_{gas} proceeding from baryon, energy and momentum conservations.

Energy accumulated in "mean fields" is released!

- Freeze-out *a là* Milekhin

$$E\frac{dN}{d^3p} = \int f_{gas}(x,p)p^\mu d\sigma_\mu, \qquad d\sigma_\mu = u_\mu(d^3x)_{proper}$$

u_μ =hydro 4-velocity   proper = in the frame, where u_μ = (1,0,0,0)

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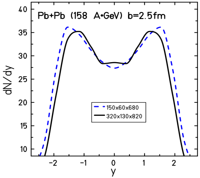
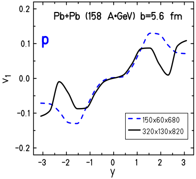
Numerics

- **Particle-in-Cell Method for Hydro**
  - Roshal'&Russkikh (1982)
  - Euler stage: transfer due to pressure gradients (on a grid)
  - Lagrange stage: transfer due to drift of the matter ($\partial_\mu J_t^\mu$, $\partial_\mu T_t^{\mu\nu}$, etc.) is simulated by test-particle motion
  - Computation in the c.m. frame

- **Careful choice of the grid** to avoid numerical diffusion:
  - $\Delta x : \Delta y : \Delta z = 1 : 1 : 1$ is best of all (Waldhauser, et al., 1992)
  - $\Delta x : \Delta t = 3.5$ is best of all (1D simulations)
  - Number of cells per Lorentz contracted diameter > 30
  - Number of test-particles per cell > 3: $\approx 10^7$ test-particles for Pb(158 GeV/nucl.) + Pb

- **Required Resources:**
  - Hydro: 30 h of CPU time at AMD Opteron 64 bit 2.0 GH and 7.5 GB for central collision Pb(158 GeV/nucl.) + Pb
  - Hydro: 10 h of CPU time at P4 2.6 GH and 0.7 GB memory for central collision Au(10 GeV/nucl.) + Au
  - Converting the hydro data into observables: few hours of CPU time





Solid Black $\Rightarrow$ 7.5 GB memory

Dashed Blue $\Rightarrow$ 1.5 GB memory