GSI 16 December 2005

PHASES of QCD

- Lattice Thermodynamics vs. PNJL Model -

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Principal question: can results of

LATTICE QCD THERMODYNAMICS

be understood in terms of **QUASIPARTICLE** degrees of freedom **?**

Synthesis of **POLYAKOV LOOP** dynamics and

NAMBU & JONA-LASINIO approach

(**PNJL** model)

M. Thaler, R.A. Schneider, W. Weise: Phys. Rev. **C 69** (2004) 035210

C. Ratti, W. Weise: Phys. Rev. **D 70** (2004) 054013

C. Ratti, M. Thaler, W. Weise: Phys. Rev. **D** (2005), submitted

C. Ratti, S. Rößner, W. Weise: in progress

I. Sketch of the **NJL MODEL**

... updates with applications to
HADRON PHYSICS:
U. Vogl, W. W. : Prog. Part. Nucl. Phys. 27 (1991) 195
T. Hatsuda, T. Kunihiro: Phys. Reports 247 (1994) 221

Y. Nambu, G. Jona-Lasinio: Phys. Rev. 122 (1961) 345

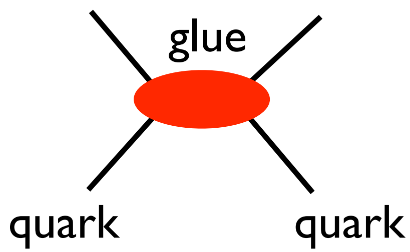
- **QUARK COLOR CURRENT**:

$$\mathbf{J}^a_\mu(x) = \bar{\psi}(x)\gamma^\mu \frac{\lambda^a}{2}\psi(x)$$

- Assume: **short correlation range** for **"color transport"** between quarks

l_c < 0.2 fm

$$G_c \sim g^2\, l_c^2$$



$$\mathcal{L}_{int} = -G_c\,\mathbf{J}^a_\mu(x)\,\mathbf{J}^\mu_a(x)$$

(chiral invariant)

$$\mathcal{L} = \bar{\psi}(x)\,(i\gamma^\mu\partial_\mu - \mathbf{m}_0)\,\psi(x) + \mathcal{L}_{int}$$

LOCAL SU(N_c) gauge symmetry of **QCD** ➞ **GLOBAL** SU(N_c) symmetry of **NJL** model

- Fierz transform (N_f = 2 flavors)

➡ **QUARK-ANTIQUARK** channels

$$\mathcal{L}_{q\bar{q}} = \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] + \dots$$

vector + axial vector
+ colour octet terms

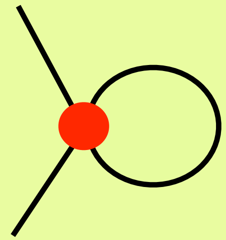
➡ **DIQUARK** channels

$$\mathcal{L}_{qq} = H\,(\bar{\psi}i\gamma_5\tau_2\lambda^A C\bar{\psi}^T)(\psi^T C i\gamma_5\tau_2\lambda^A\psi) + \dots$$

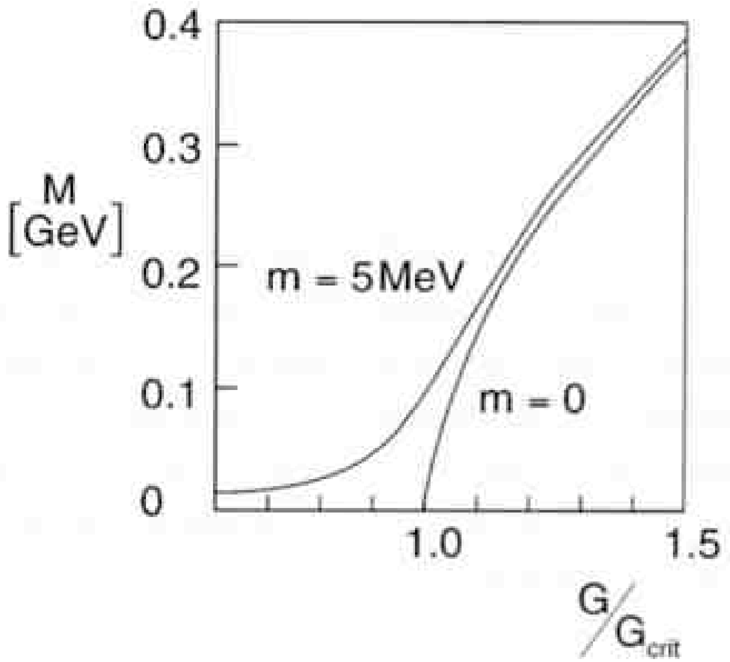
- Self-consistent **MEAN FIELD** approximation

➡ **GAP equation**:

$$M = m_o - G\langle\bar{\psi}\psi\rangle$$

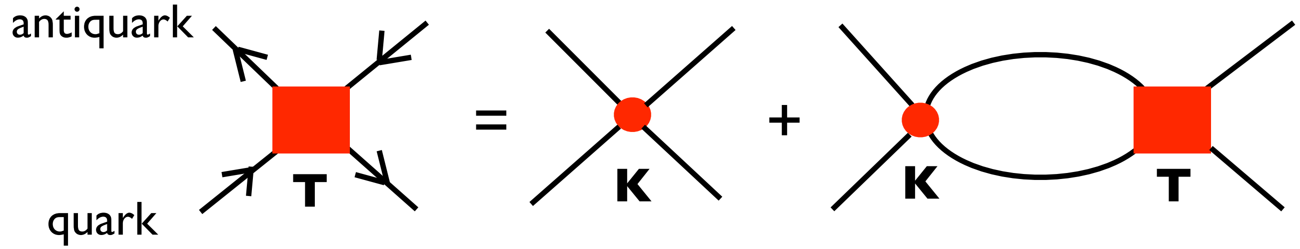


$$\langle\bar{\psi}\psi\rangle = -2iN_f N_c \int \frac{d^4p}{(2\pi)^4}\frac{M\,\theta(\Lambda^2 - \vec{p}^{\,2})}{p^2 - M^2 + i\epsilon}$$

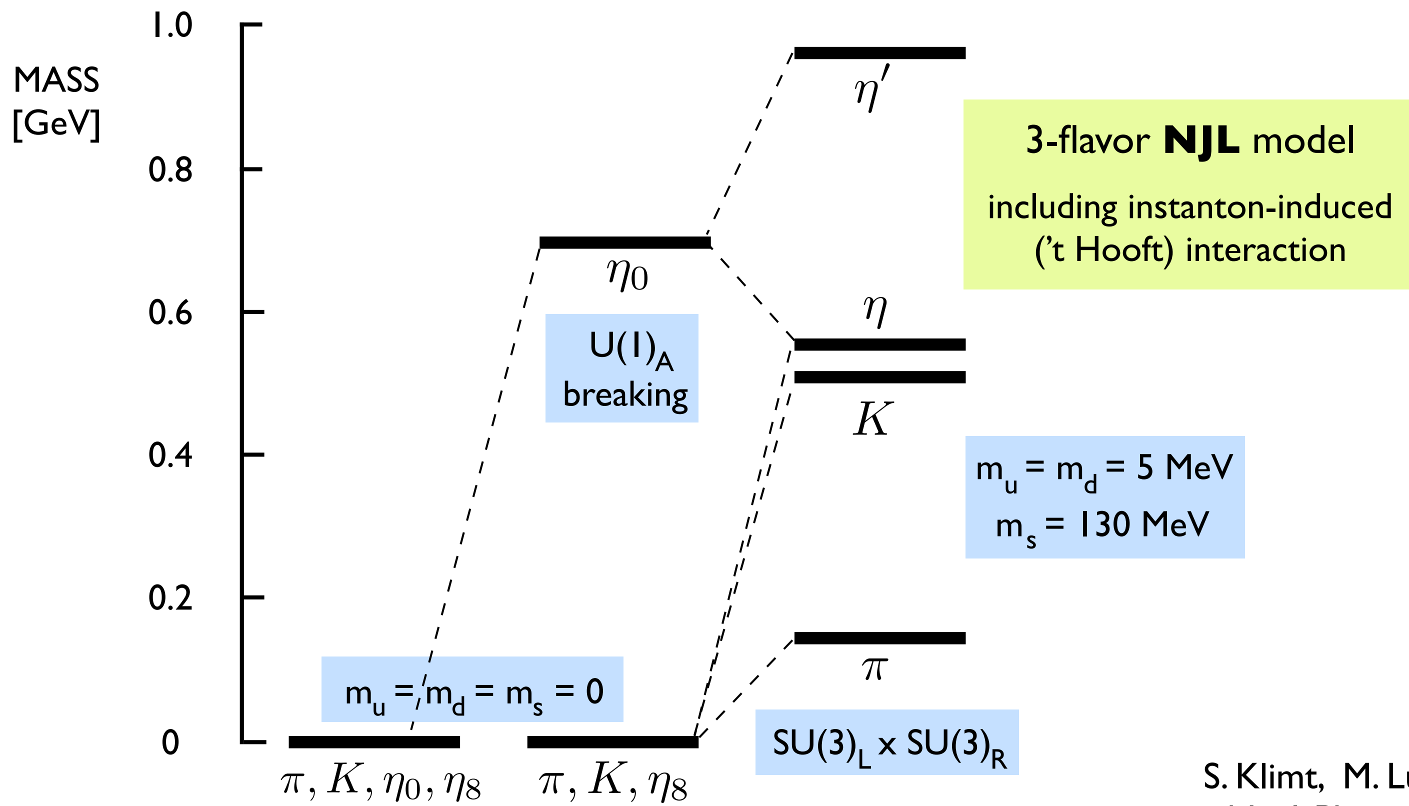


MESON sector

- Bethe-Salpeter Equation
  in (colour singlet) QUARK-ANTIQUARK channels:



PSEUDOSCALAR MESON SPECTRUM



S. Klimt, M. Lutz, U.Vogl, W.W.:
Nucl. Phys. A 516 (1990) 429

2. **Polyakov Loop** (Thermal Wilson Line)

- Order parameter of spontaneously broken $Z(N_c)$ center symmetry of $SU(N_c)$ pure gauge theory ($\longrightarrow$ **DECONFINEMENT**)

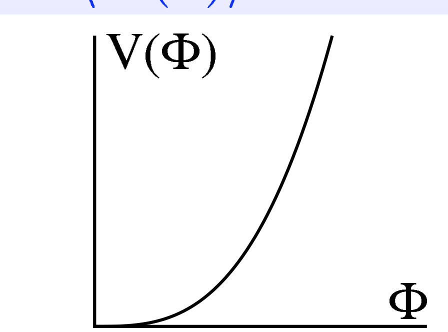
$$\mathbf{L}(\vec{\mathbf{x}}) = \mathcal{P}\exp\left(\mathbf{i}\int_0^{\beta=\frac{1}{\mathbf{T}}} \mathbf{d}\tau\,\mathbf{A_4}(\vec{\mathbf{x}},\tau)\right) \qquad \boldsymbol{\Phi}(\vec{\mathbf{x}}) = \frac{\mathbf{1}}{\mathbf{N_c}}\mathbf{Tr}\,\mathbf{L}(\vec{\mathbf{x}})$$

- Effective Potential: (R. Pisarsky (2000))

$$\frac{\mathbf{V}(\boldsymbol{\Phi},\mathbf{T})}{\mathbf{T^4}} = -\frac{\mathbf{b_2}(\mathbf{T})}{\mathbf{2}}\boldsymbol{\Phi}^\dagger\boldsymbol{\Phi} - \frac{\mathbf{b_3}}{\mathbf{6}}\left(\boldsymbol{\Phi}^\mathbf{3} + {\boldsymbol{\Phi}^\dagger}^\mathbf{3}\right) + \frac{\mathbf{b_4}}{\mathbf{4}}\left(\boldsymbol{\Phi}^\dagger\boldsymbol{\Phi}\right)^\mathbf{2}$$

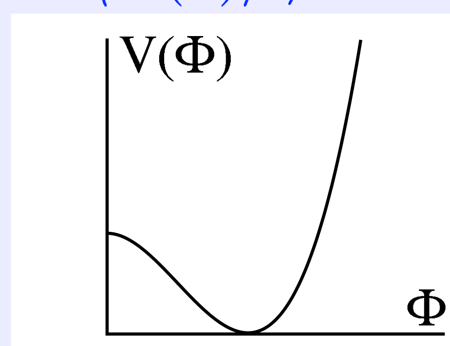
$T < T_c$

- color confinement
- $\langle\Phi\left(\vec{x}\right)\rangle = 0 \longrightarrow Z(3)$ unbroken



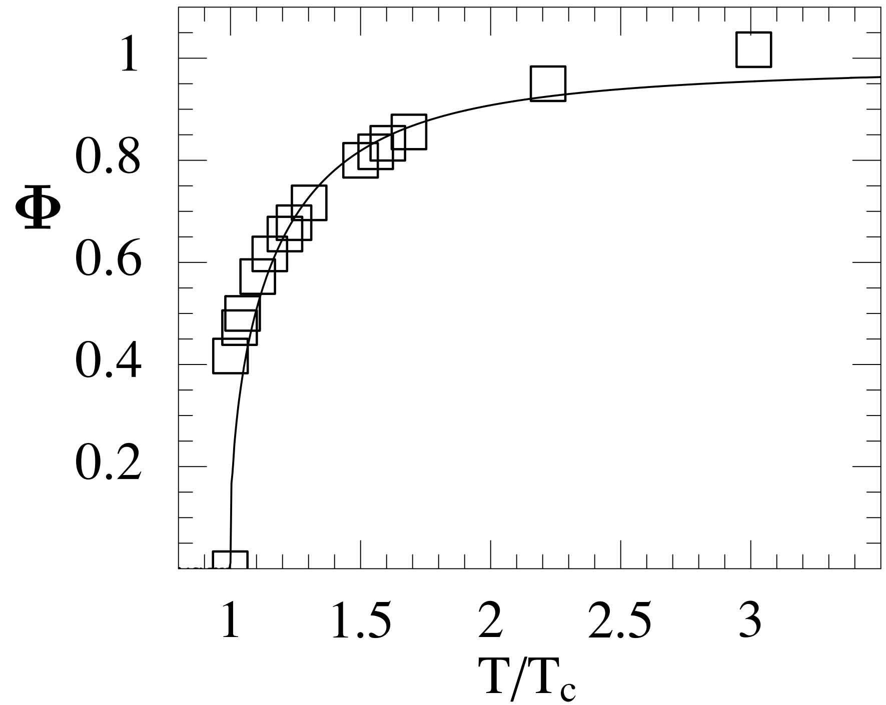
$T > T_c$

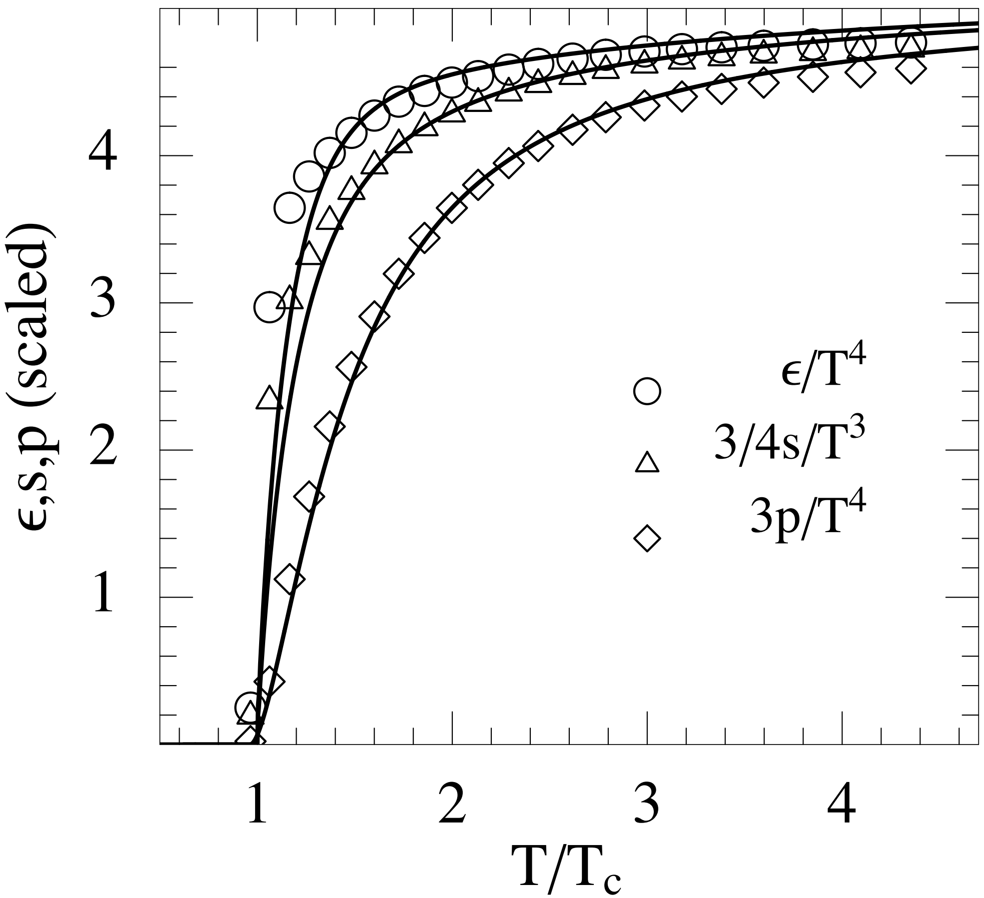
- color deconfinement
- $\langle\Phi\left(\vec{x}\right)\rangle \neq 0 \longrightarrow Z(3)$ broken



Comparison with “PURE GLUE” Lattice Thermodynamics

- Minimization of $V(\Phi, T)$: Polyakov loop expectation value as a function of T
- Comparison with lattice data from Kaczmarek *et al.* PLB 543 (2002)





$$p(T) = -V(\Phi(T), T)$$

$$s(T) = \frac{dp}{dT} = -\frac{dV(\Phi(T),T)}{dT}$$

$$\epsilon(T) = T\frac{dp}{dT} - p = Ts(T) - p(T)$$

Comparison with lattice data from Boyd *et al.* NPB 469 (1996)

3. PNJL

POLYAKOV-loop-extended Nambu & Jona-Lasinio Model

(see also: Meisinger and Ogilvie 1996, Fukushima 2004)

- Unify CONFINEMENT and spontaneous CHIRAL SYMMETRY breaking

$$\mathcal{L}_{\mathbf{PNJL}} = \bar{\psi}(i\gamma_\mu \mathbf{D}^\mu - m_0)\psi - \mathbf{V}(\mathbf{\Phi}[\mathbf{A}], \bar{\mathbf{\Phi}}[\mathbf{A}]; \mathbf{T})$$
$$+ \frac{\mathbf{G}}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \vec{\tau}\psi)^2\right]$$

gauge-covariant derivative

$$\mathbf{D}^\mu = \partial^\mu - i\mathbf{A}^\mu$$

(temporal) background gauge field

$$\mathbf{A}^\mu = \delta_{\mu 0}\,\mathbf{A}^0$$

- treat $\mathbf{\Phi}$ as classical field

$$\mathbf{\Phi} = \frac{1}{3}\mathbf{Tr}\,\mathbf{L} = \frac{1}{3}\mathbf{Tr}\exp\left(\mathbf{i}\mathbf{A}_4/\mathbf{T}\right)$$

- note: thermal expectation values

  $\langle \mathbf{\Phi} \rangle$ and $\langle \bar{\mathbf{\Phi}} \rangle$ both real but different at non-zero quark chemical potential

(Dumitru, Pisarski, Zschiesche: Phys. Rev. D 72 (2005) 065008)

PNJL Thermodynamics

Parameters

Λ [GeV]	0.65
$\mathbf{G}$ [GeV^{-2}]	10.1
m_0 [MeV]	5.5

Physical quantities

f_π [MeV]	92.4
$\lvert\langle\bar{\psi}\psi\rangle\rvert^{1/3}$ [MeV]	247
m_π [MeV]	139.3

- Thermodynamical Potential:

$$\Omega(T,\mu) = \mathbf{V}(\mathbf{\Phi},\bar{\mathbf{\Phi}};T) + \frac{\sigma^2}{2\mathbf{G}}$$

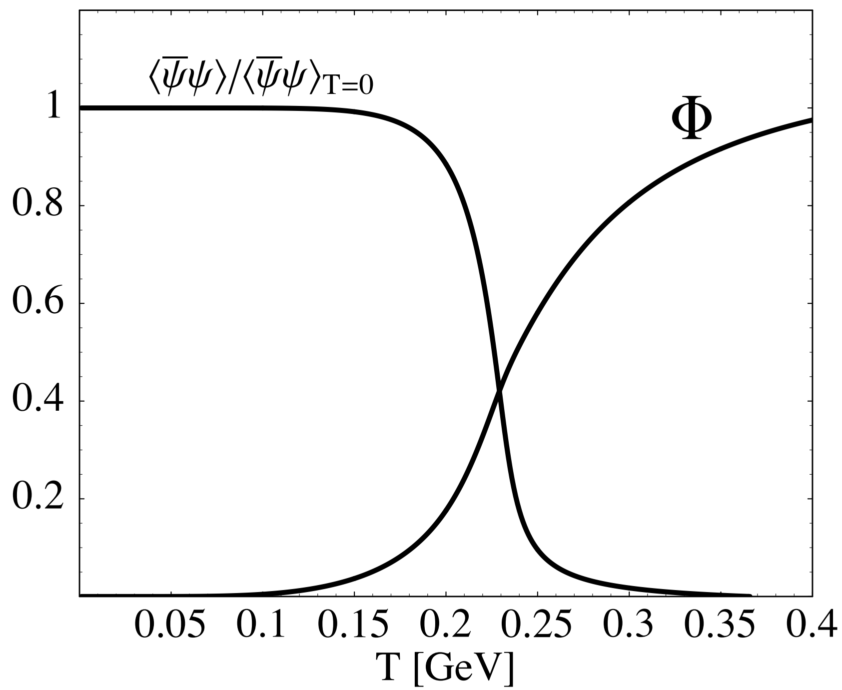
$$-2N_f\,T\int\frac{d^3p}{(2\pi)^3}\left[Tr\,ln\left(1+\mathbf{L}\,e^{-(E_p-\mu)/T}\right)+Tr\,ln\left(1+\mathbf{L}^\dagger\,e^{-(E_p+\mu)/T}\right)+\frac{E_p}{T}\right]$$

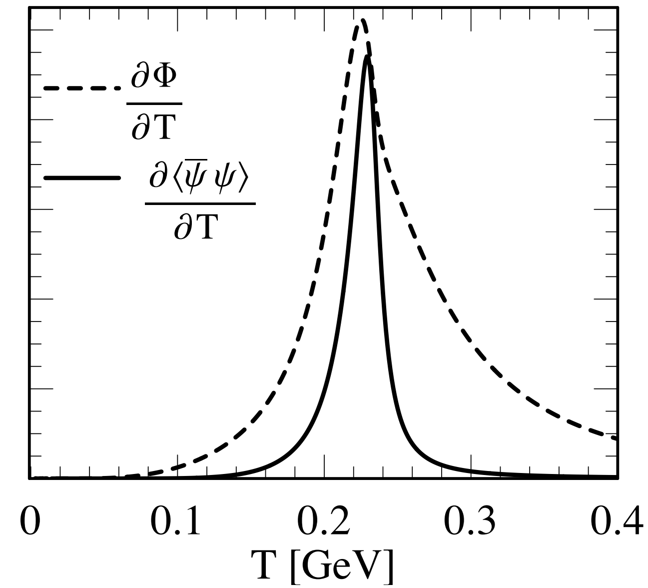
- Quasiparticle energy and mass of quarks:

$$E_p = \sqrt{\vec{p}^{\,2}+m^2} \qquad\qquad m = m_0 - \sigma = m_0 - \mathbf{G}\langle\bar{\psi}\psi\rangle$$

CONFINEMENT and CHIRAL SYMMETRY BREAKING

C. Ratti, M. Thaler, W.W. (2005)

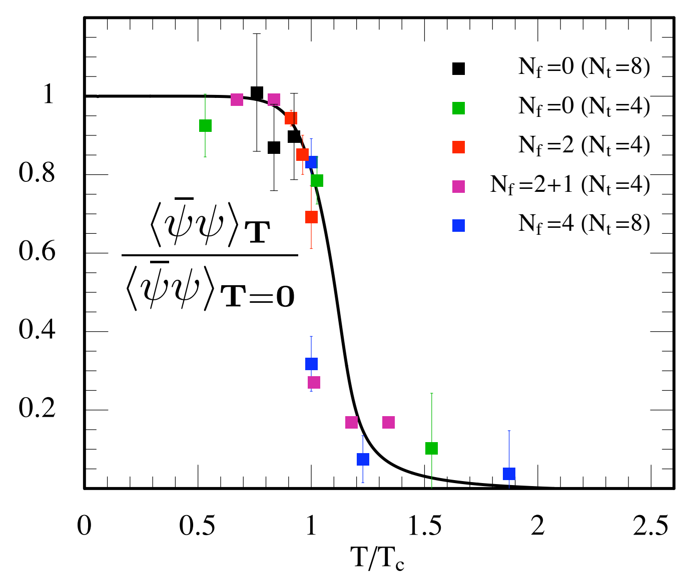




CHIRAL and **DECONFINEMENT**

transitions

almost coincide !

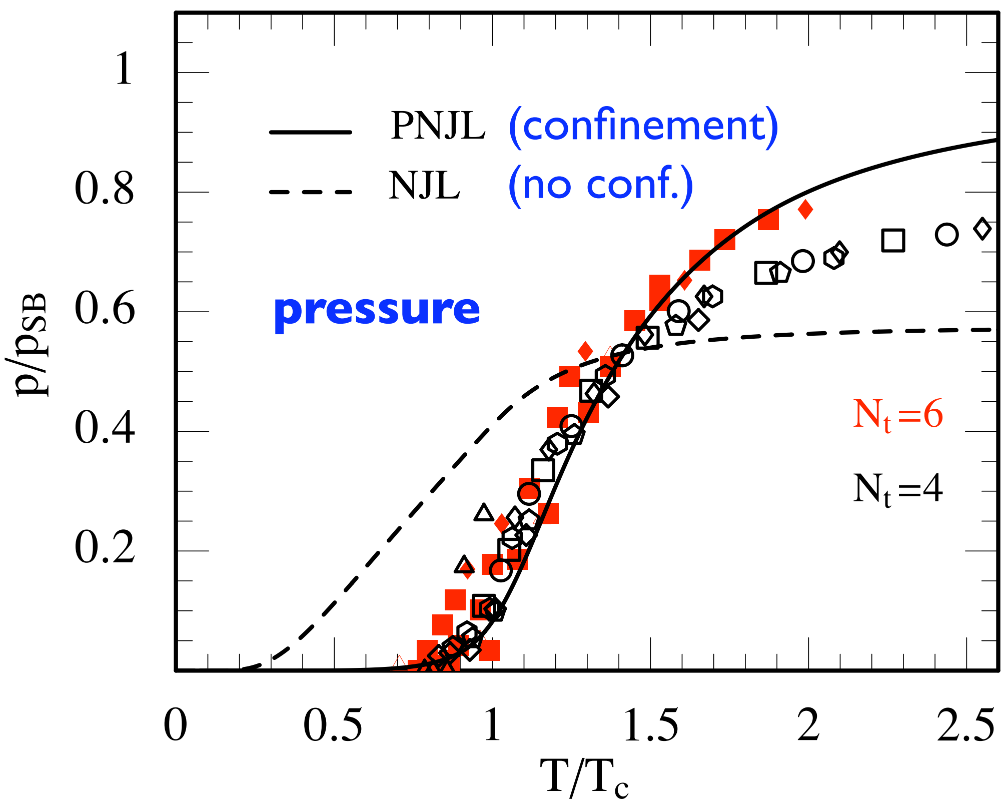


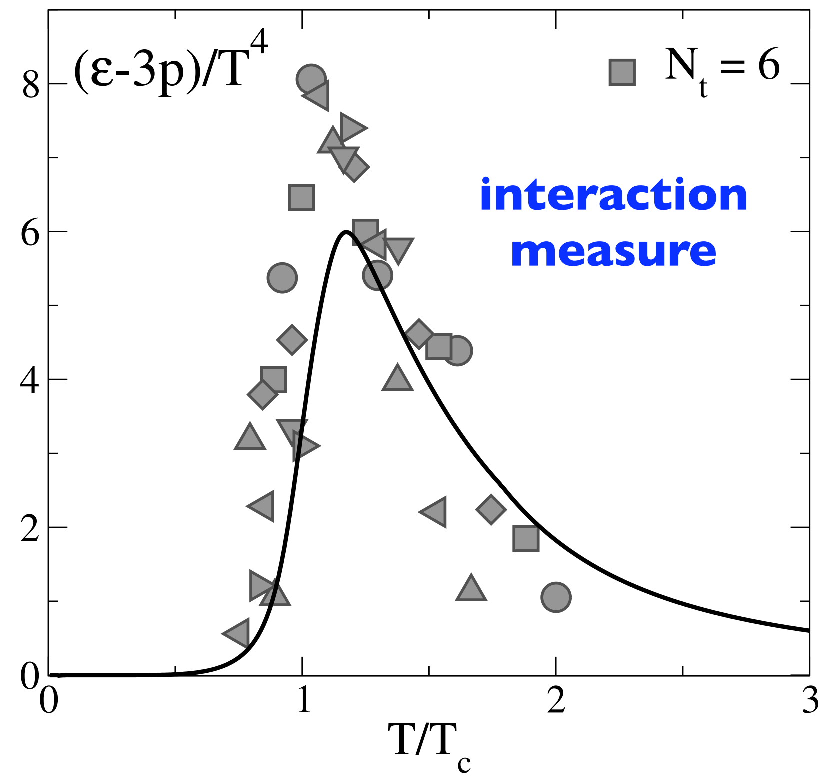
Lattice: G. Boyd et al., Phys. Lett. B349 (1995) 170

PNJL :
Comparisons with $N_c = 3$, $N_f = 2$ Lattice Thermodynamics

- PRESSURE and ENERGY DENSITY at zero chemical potential

$$\mathbf{p} = -\mathbf{\Omega}(\mathbf{T}, \mu = \mathbf{0}) \qquad \varepsilon = \mathbf{T}\frac{\partial \mathbf{p}(\mathbf{T}, \mu = \mathbf{0})}{\partial \mathbf{T}} - \mathbf{p}(\mathbf{T}, \mu = \mathbf{0})$$



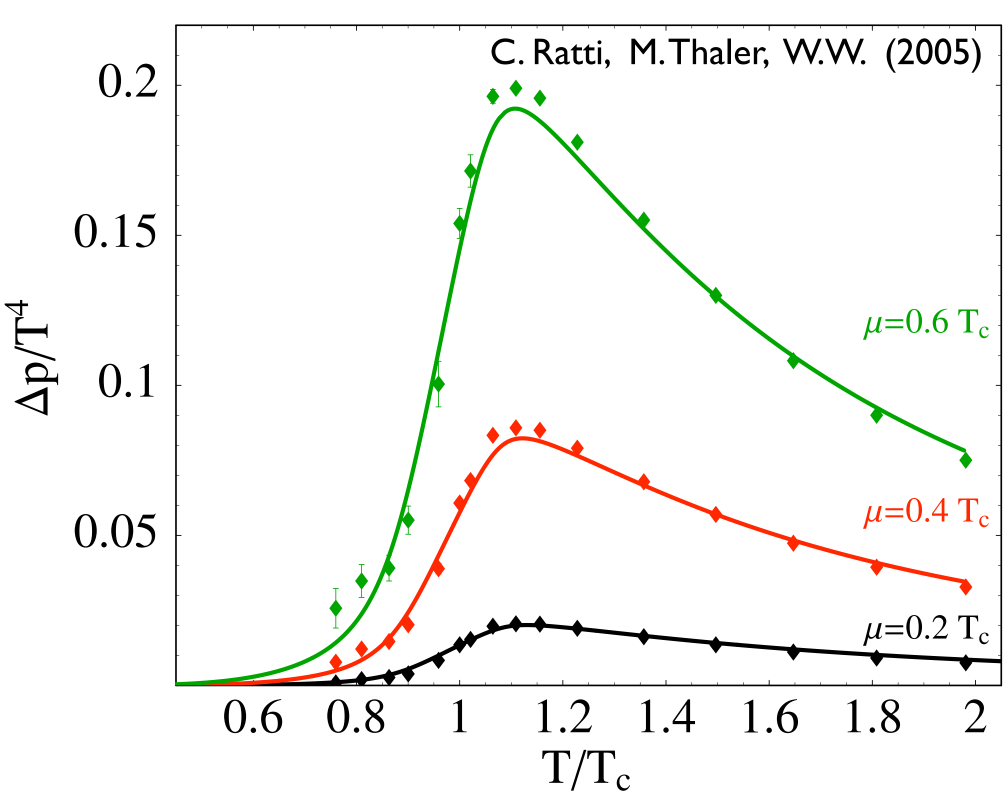


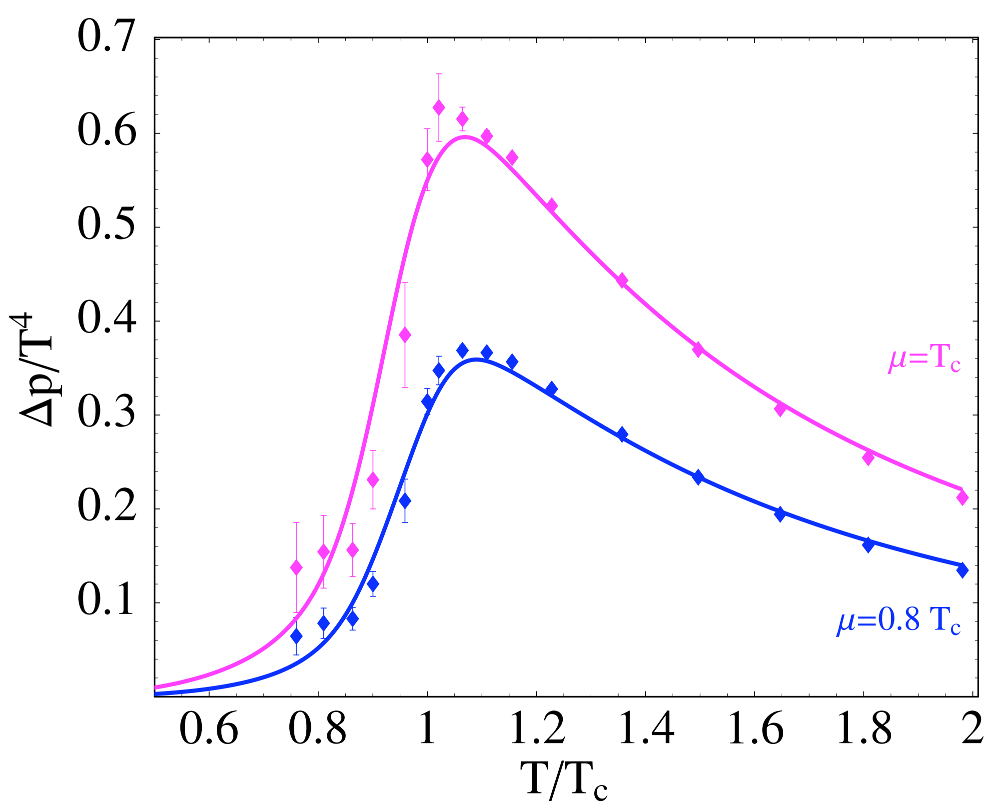
Lattice data: F. Karsch, F. Laermann, A. Peikert; Nucl. Phys. B 605 (2002) 579

Non-zero QUARK CHEMICAL POTENTIAL (part 1)

- Pressure difference:

$$\Delta\mathbf{p}(\mathbf{T},\mu) = \mathbf{p}(\mathbf{T},\mu) - \mathbf{p}(\mathbf{T},\mu = \mathbf{0})$$



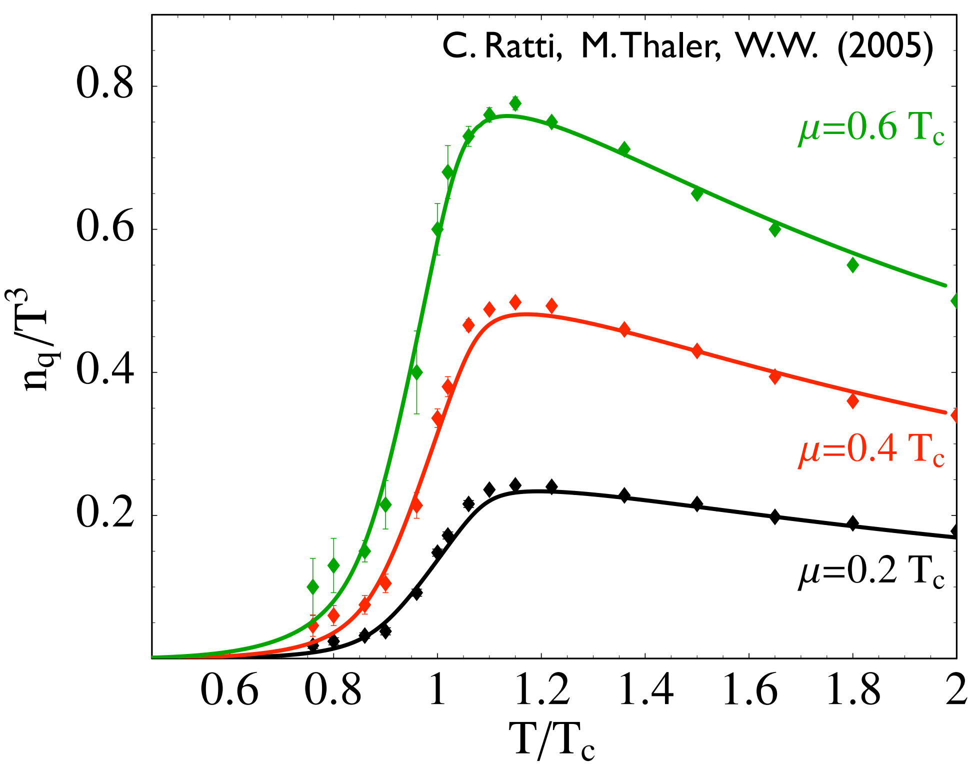


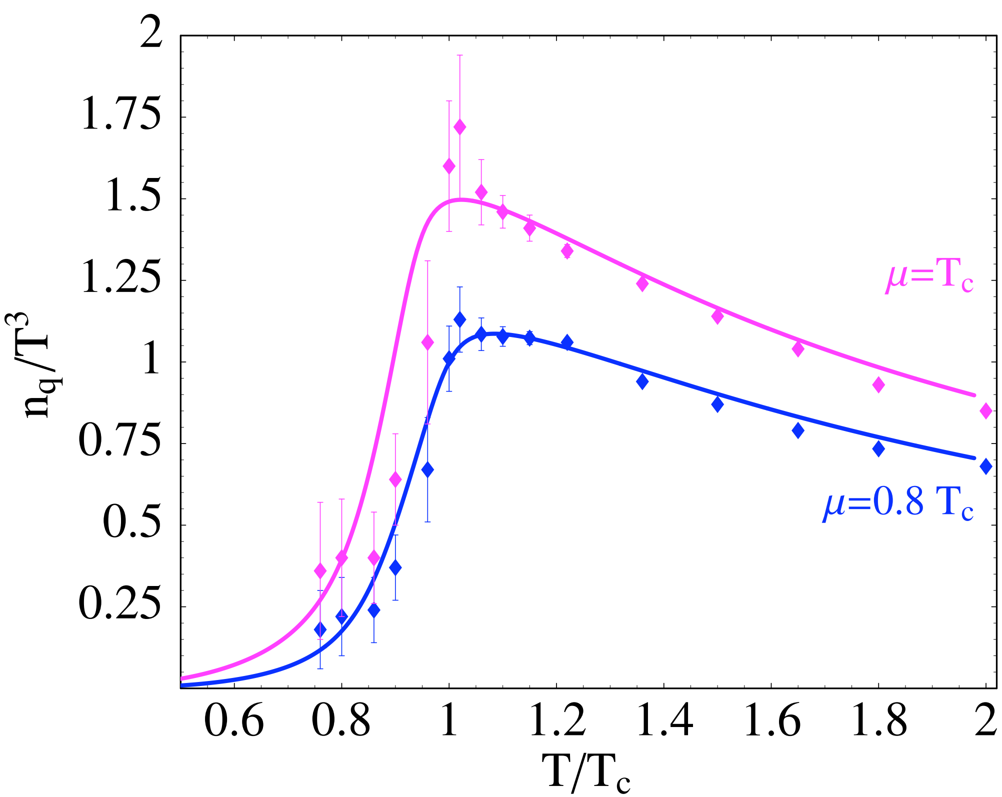
Lattice data: Allton et al. Phys. Rev. D 68 (2003)

Non-zero QUARK CHEMICAL POTENTIAL (part II)

- Quark number density:

$$\mathbf{n_q}(\mathbf{T},\mu) = -\frac{\partial \mathbf{\Omega}(\mathbf{T},\mu)}{\partial \mu}$$



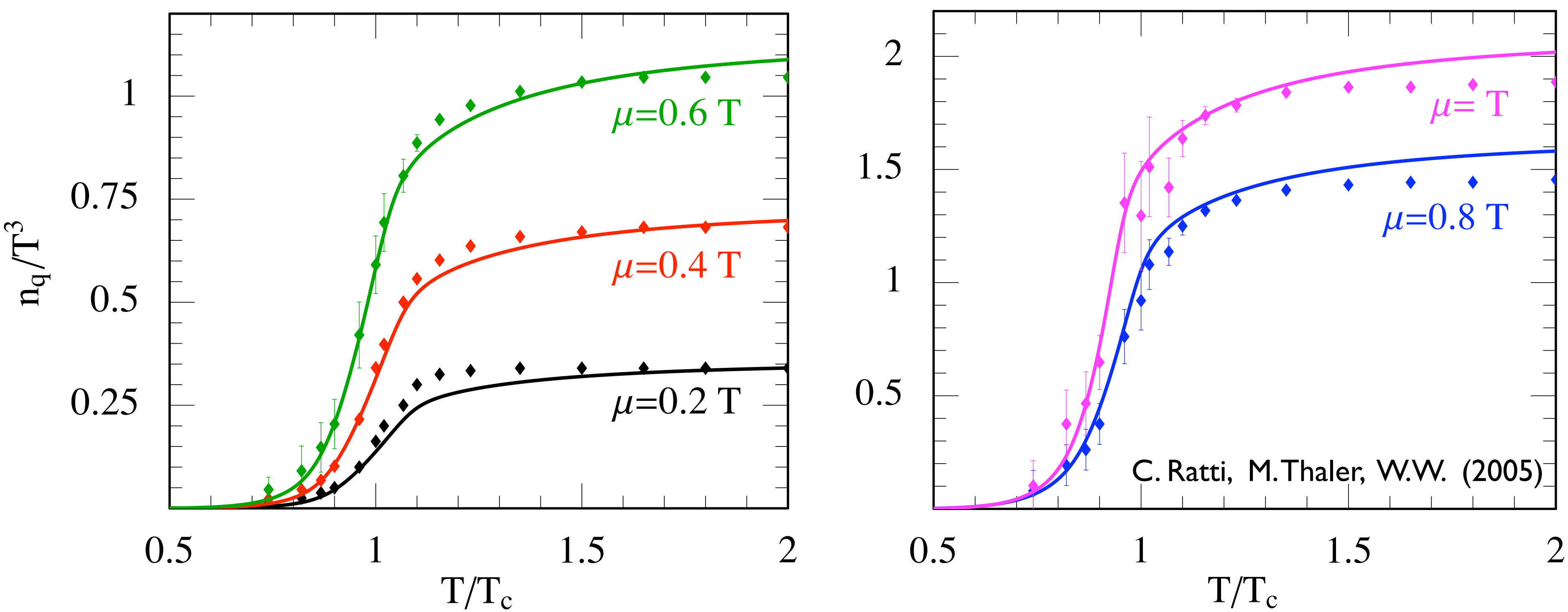


Lattice data: Allton et al. Phys. Rev. D 68 (2003)

Non-zero QUARK CHEMICAL POTENTIAL (part III)

- towards larger chemical potential

Lattice results: Allton et al. Phys. Rev. D 71 (2005)

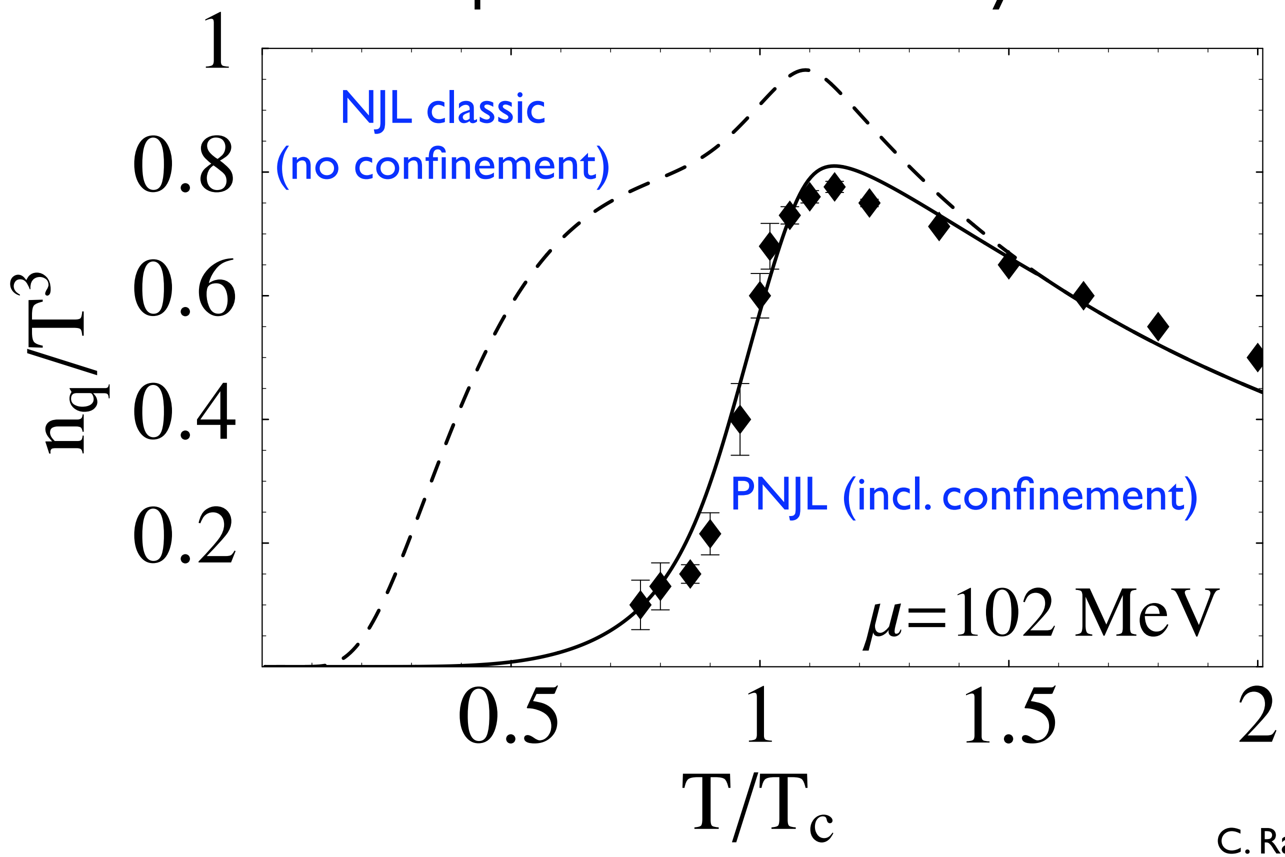


- rapid convergence in powers of $\mu/\mathbf{T}$ observed

Non-zero QUARK CHEMICAL POTENTIAL (part IV)

- Role of CONFINEMENT (POLYAKOV loop dynamics)

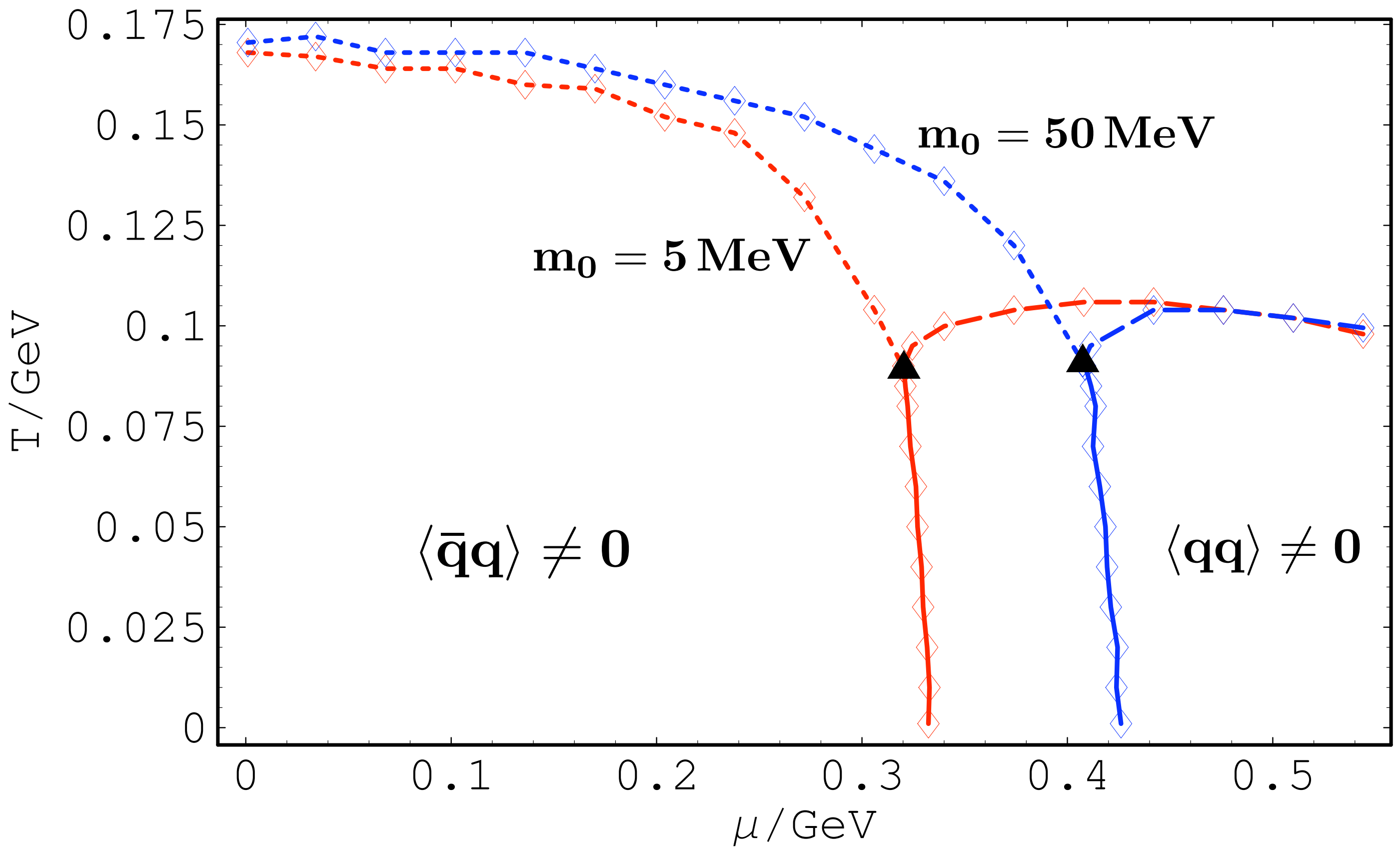
quark number density



C. Ratti, M. Thaler, W.W. (2005)

4. **Outlooks** (part 1)
... towards the PHASE DIAGRAM

- Two-flavour **PNJL** model incl. **DIQUARK** degrees of freedom



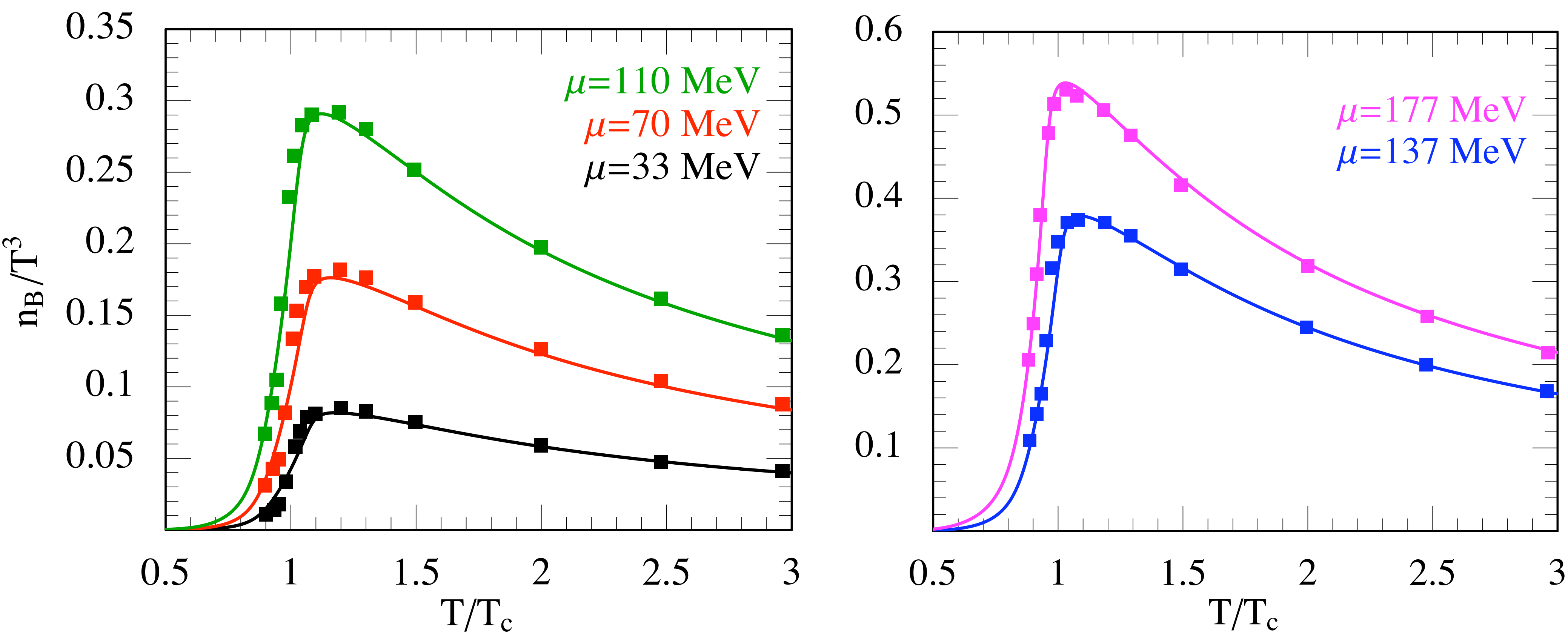
S. Rößner,
C. Ratti,
W.W.
(2005)

PRELIMINARY

- Note: strong dependence of critical point on input quark mass

Outlooks (part II)
PNJL model with 2+1 flavours

- NJL input: $\mathbf{m_{u,d} = 5.5\,MeV}$ $\mathbf{m_s = 141\,MeV}$
- AXIAL U(1) breaking by ´t Hooft interaction



Lattice results: Z. Fodor et al., Phys. Lett. B 568 (2003)

5. Summary

- **QUASIPARTICLE** approach encoding **CHIRAL SYMMETRY** and **CONFINEMENT** **(PNJL)** successful in comparison with **QCD THERMODYNAMICS** on the Lattice $(\mathbf{T \leq 2\,T_c})$

next steps:

- 2 + 1 flavors (including **DIQUARKS**)
  high quark densities but
  $\mu < \mathbf{\Lambda_{NJL}}$

- establish contact with high temperature limit
  ("Hard Thermal Loops")

Polyakov loop extended NJL model with strange quarks

$$\mathcal{L}_{PNJL} = \bar{\psi}\left(i\gamma_\mu D^\mu - \hat{m}_0\right)\psi + \frac{G}{2}\sum_{f=u,d,s}\left[\left(\bar{\psi}_f\psi_f\right)^2 + \left(\bar{\psi}_f i\gamma_5\vec{\tau}\psi_f\right)^2\right]$$

$$- \frac{K}{2}\left[\det_{i,j}\left(\bar{\psi}_i(1+\gamma_5)\psi_j\right) + \det_{i,j}\left(\bar{\psi}_i(1-\gamma_5)\psi_j\right)\right] - V\left(\Phi, T\right),$$

where:

$$D_\mu = \partial_\mu + igA_\mu \quad \text{and} \quad A_\mu = \delta_{\mu 0}A_0 \ .$$

and

$$\hat{m}_0 = \text{diag}\left[m_{0u}, m_{0d}, m_{0s}\right].$$

Parameters

Λ [GeV]	0.6023
$G\Lambda^2$	3.67
$K\Lambda^5$	24.72
$m_{0u,d}$ [MeV]	5.5
m_{0s} [MeV]	140.7

Physical quantities

f_π [MeV]	92.4
$\lvert\langle\bar{\psi}\psi\rangle_{u,d}\rvert^{1/3}$ [MeV]	241.9
$\lvert\langle\bar{\psi}\psi\rangle_{s}\rvert^{1/3}$ [MeV]	257.7
m_π [MeV]	139.3
m_K [MeV]	497.7

Final form of Ω:

$$\Omega(T,\mu) = V(\Phi,T) + \frac{\sigma_{u,d}^2}{2G} + \frac{\sigma_s^2}{4G} - \frac{K}{4G^3}\sigma_{u,d}^2\sigma_s - 2\sum_f T\int\frac{d^3p}{(2\pi)^3}\left\{\mathrm{Tr}_c \ln\left[1 + L\, e^{-(E_{p,f}-\mu_f)/T}\right]\right.$$
$$\left. + \mathrm{Tr}_c \ln\left[1 + L^\dagger\, e^{-(E_{p,f}+\mu_f)/T}\right] + 3\frac{E_{p,f}}{T}\theta\left(\Lambda^2 - \vec{p}^{\,2}\right)\right\}.$$